Calculations of statistical properties of atomic nuclei

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M2-Subatomic Physics and Astroparticles – Students' Project

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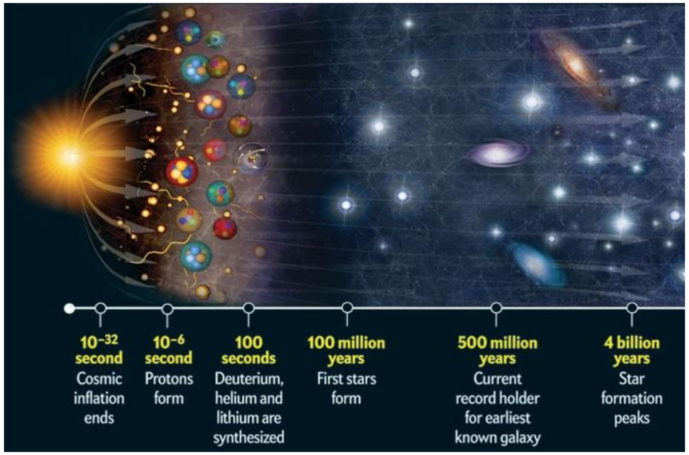
Summary

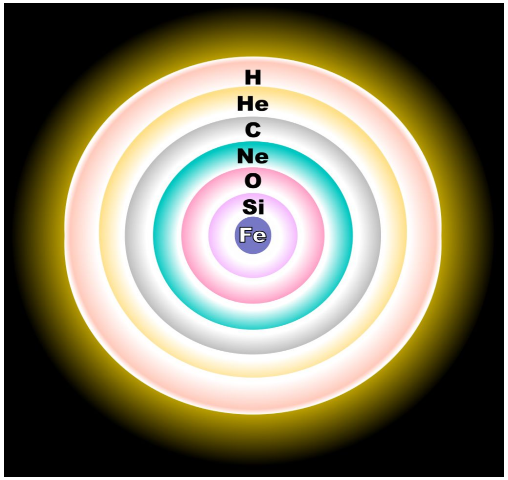
1) **Introduction**
2) **Theory**
3) **Work done**
4) **Results**
5) **Conclusion**
6) **Resources**

Nuclear Astrophysics and nucleosynthesis:

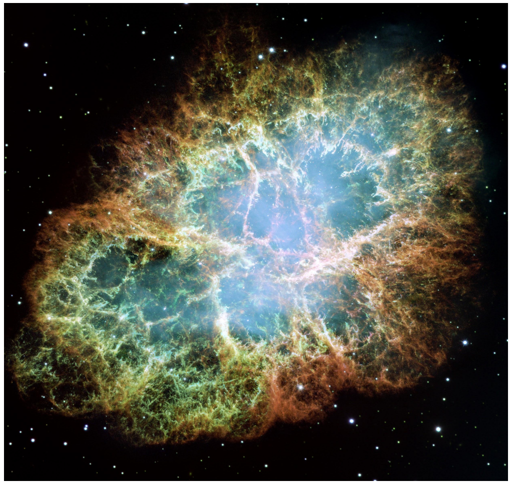
- Three main types of nucleosynthesis:



Big bang nucleosynthesis - during early stages after Big bang



Stellar nucleosynthesis - pp chain, CNO cycle and fusion of heavier nuclei upto Fe



Explosive nucleosynthesis - supernovae and mergers of compact objects

Figure 01

- Understanding of neutron capture reactions are crucial for the understanding of nucleosynthesis of heavy elements as well as in the design of critical assemblies.

Image resources: Google image feed

1. Introduction

Neutron Capture:

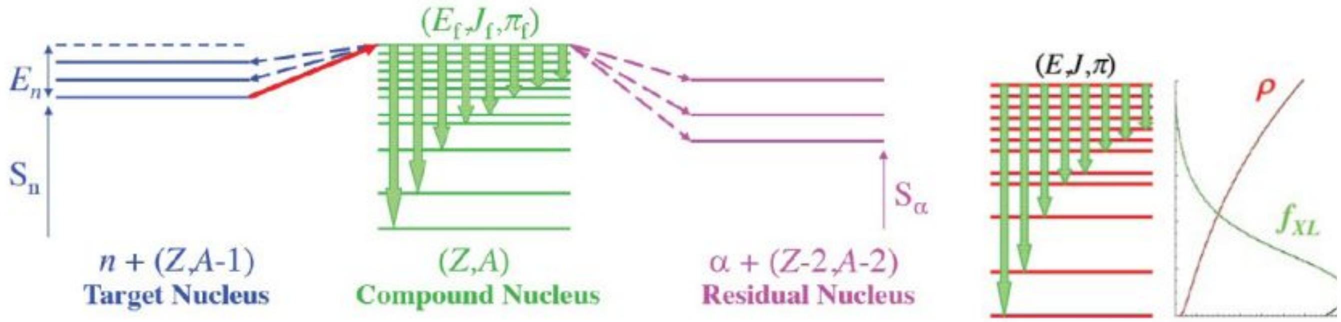


Figure 02

In the Hauser-Feshbach model for radiative neutron capture:

$$\sigma(n,\gamma) \approx \sum_{J,\pi} T_\gamma(J^\pi) = \sum_{J^\pi XL} \int 2\pi\epsilon_\gamma^{2L+1} f_{XL}(\epsilon_\gamma)\rho(E,J,\pi)d\epsilon_\gamma \qquad (1)$$

We need to know or predict the level density and gamma-decay strength function.

The calculations are done under a basic assumption called Brink-Axel hypothesis which states that the gamma-decay is independent of the initial state.

Image resources: from slides of K. Sieja

2. Theory

Neutron capture cross-sections:

Main statistical properties required to provide inputs for the calculation:

1. **Nuclear level density - $\rho(E_i, J_i, \pi)$**

   The nuclear level density of a nucleus with A nucleons and an excitation energy E is defined as,

$$\rho_{\mathrm{nucl}}(E) = \frac{N(E+\Delta E) - N(E)}{\Delta E} \xrightarrow{\Delta E \to 0} \frac{dN}{dE} \longrightarrow (2)$$

   with N (E) being t

2. **Reduced Transition probability - $B(E1(M1):J_i,\pi,E_\gamma)$**

   A general definition can be given as,

$$B(E\mathrm{L}, J_i \to J_f) = \frac{1}{2J_i+1} \left| \langle f \,||\, \hat{Q}_{\mathrm{L}} \,||\, i \rangle \right|^2$$
$$B(M\mathrm{L}, J_i \to J_f) = \frac{1}{2J_i+1} \left| \langle f \,||\, \hat{M}_{\mathrm{L}} \,||\, i \rangle \right|^2 \longrightarrow (3)$$

   where $\mathbf{Q}_L$ and $\mathbf{M}_L$ represent the electric and magnetic multipole operators respectively.

   In the case of our work the value is found in the input spectrum files per each transition, but our program applies an average over the considered excitation energy range (or "bin"), and is summed over J,π and E_γ ranges.

2. Theory

Radiative strength functions - $f_{xl}(E_i, J_i, \pi, E_\gamma)$

The photon strength function, radiative strength function, or gamma strength function $\mathbf{f_{XL}}$ is defined by the average value of partial radiation width $\mathbf{\Gamma_{ab}}$ for the γ-decay from an initial level "a" with the quantum numbers $\mathbf{(E_a, J_a, \pi_a)}$ to a final level "b" with the quantum numbers $\mathbf{(E_b, J_b, \pi_b)}$.

$$\overline{\Gamma}^{\gamma}_{ab}(\mathrm{XL}) = \frac{f_{\mathrm{XL}}(E_\gamma) E_\gamma^{2L+1}}{\rho(E_a, J_a, \pi_a)} \longrightarrow \quad (4)$$

An alternative definition that allows us to combine the aforementioned level density and the reduced transition probabilities is given as,

$$f_{\mathrm{XL}}(E_\gamma, E_i, J_i, \pi) = 16\pi/(9\hbar c)^3 \langle B(\mathrm{XL})\rangle \rho(E_i, J_i, \pi) \quad (5)$$

Problem:

"Handle the tables of large-scale shell model (LSSM) results (spectra, transition probabilities) and compute the level densities and the radiative strength functions of magnetic and electric dipole type in ^{128}Te and ^{44}Sc"

3. Work done

Structure of the program/solution:

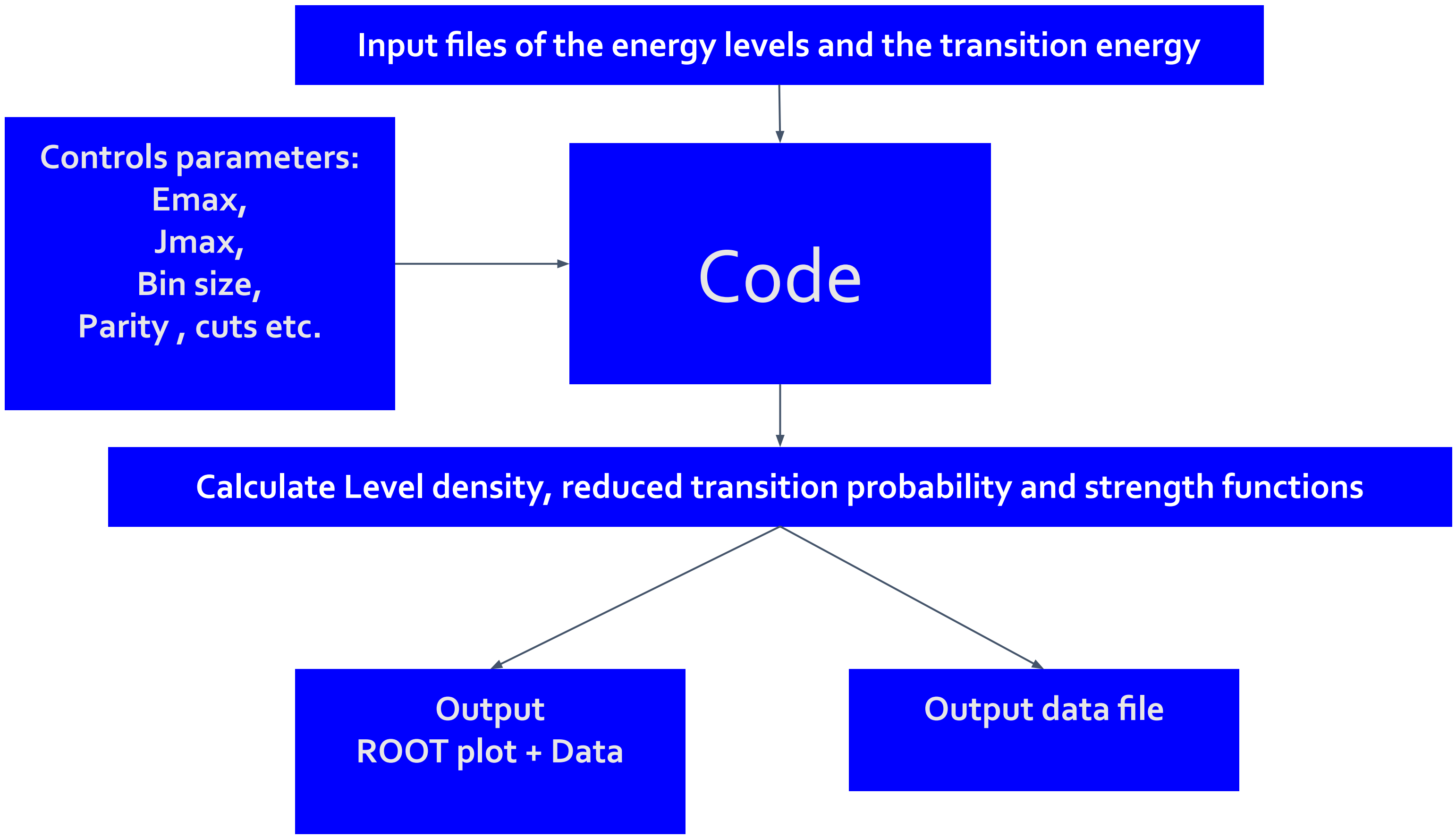


Figure 03

3. Work done

Input file formats:

Need of a specific program structure to be able to handle the input data:

```
N,Z=  2 26 2*J= 0 P=0 N=   1 C= 0  EXC   =    0.00000  E=  -282.14173
N,Z=  2 26 2*J= 4 P=0 N=   1 C= 0  EXC   =    0.83070  E=  -281.31102
N,Z=  2 26 2*J= 8 P=0 N=   1 C= 0  EXC   =    1.59369  E=  -280.54804
N,Z=  2 26 2*J= 4 P=0 N=   2 C= 0  EXC   =    1.64601  E=  -280.49571
N,Z=  2 26 2*J= 4 P=0 N=   3 C= 0  EXC   =    1.79686  E=  -280.34487
N,Z=  2 26 2*J=12 P=0 N=   1 C= 0  EXC   =    1.93282  E=  -280.20891
N,Z=  2 26 2*J= 0 P=0 N=   2 C= 0  EXC   =    2.04064  E=  -280.10109
N,Z=  2 26 2*J= 2 P=0 N=   1 C= 0  EXC   =    2.09963  E=  -280.04209
N,Z=  2 26 2*J= 4 P=0 N=   4 C= 0  EXC   =    2.11942  E=  -280.02230
N,Z=  2 26 2*J= 8 P=0 N=   2 C= 0  EXC   =    2.13861  E=  -280.00312
N,Z=  2 26 2*J= 6 P=0 N=   1 C= 0  EXC   =    2.24828  E=  -279.89345
N,Z=  2 26 2*J= 0 P=0 N=   3 C= 0  EXC   =    2.25790  E=  -279.88382
N,Z=  2 26 2*J= 4 P=0 N=   5 C= 0  EXC   =    2.28277  E=  -279.85896
N,Z=  2 26 2*J= 8 P=0 N=   3 C= 0  EXC   =    2.33461  E=  -279.80712
N,Z=  2 26 2*J= 6 P=0 N=   2 C= 0  EXC   =    2.36661  E=  -279.77511
N,Z=  2 26 2*J= 8 P=0 N=   4 C= 0  EXC   =    2.46621  E=  -279.67552
N,Z=  2 26 2*J= 0 P=0 N=   4 C= 0  EXC   =    2.47071  E=  -279.67101
```

Figure 04: Snapshot of a Level Scheme data file

J_i	J_f	E_i	E_γ	$B(XL)_{i\to f}$	$B(XL)_{f\to i}$
2	0	-278.48468510	-2.54334492	0.00201063	0.00603189
2	0	-278.48468510	-2.52844609	0.00020074	0.00060222
2	0	-278.48468510	-2.51579759	0.00120412	0.00361237
2	0	-278.48468510	-2.49883606	0.00026376	0.00079129
2	0	-278.48468510	-2.49022415	0.00382762	0.01148285
2	0	-278.48468510	-2.48315809	0.00038931	0.00116793
2	0	-278.48468510	-2.46700899	0.00034799	0.00104398
2	0	-278.48468510	-2.44459468	0.00137583	0.00412748
2	0	-278.48468510	-2.43636155	0.00224311	0.00672934
2	0	-278.48468510	-2.42793375	0.00008338	0.00025015
2	0	-278.48468510	-2.42340565	0.00005012	0.00015035
2	0	-278.48468510	-2.41106481	0.00039432	0.00118295
2	0	-278.48468510	-2.38688695	0.00012069	0.00036206
2	0	-278.48468510	-2.38495763	0.00008988	0.00026964
2	0	-278.48468510	-2.37563121	0.00065228	0.00195684
2	0	-278.48468510	-2.37087781	0.00018370	0.00055110
2	0	-278.48468510	-2.35126434	0.00010203	0.00030610

Figure 05: Snapshot of a Spectrum data file

3. Work done

The outputs:

One of the codes gives us a data + ROOT graph.
The other code compile the data in a file by the C++ program to be drawn by another program.

```
0; 4.54839e-05
0.2; 4.63661e-05
0.4; 4.6225e-05
0.6; 3.99873e-05
0.8; 4.70204e-05
1; 4.42895e-05
1.2; 4.40632e-05
1.4; 4.53709e-05
1.6; 5.00917e-05
1.8; 4.5673e-05
2; 5.10059e-05
2.2; 5.70797e-05
2.4; 5.45602e-05
2.6; 6.16396e-05
2.8; 5.61528e-05
3; 5.69562e-05
3.2; 6.36173e-05
3.4; 5.05408e-05
3.6; 4.5389e-05
3.8; 5.15446e-05
4; 4.85767e-05
4.2; 4.54952e-05
4.4; 4.9581e-05
4.6; 4.40324e-05
4.8; 4.8617e-05
5; 3.56528e-05
5.2; 4.6486e-05
5.4; 4.02043e-05
5.6; 4.2093e-05
5.8; 3.72277e-05
6; 4.91108e-05
6.2; 4.77169e-05
```

Figure 06

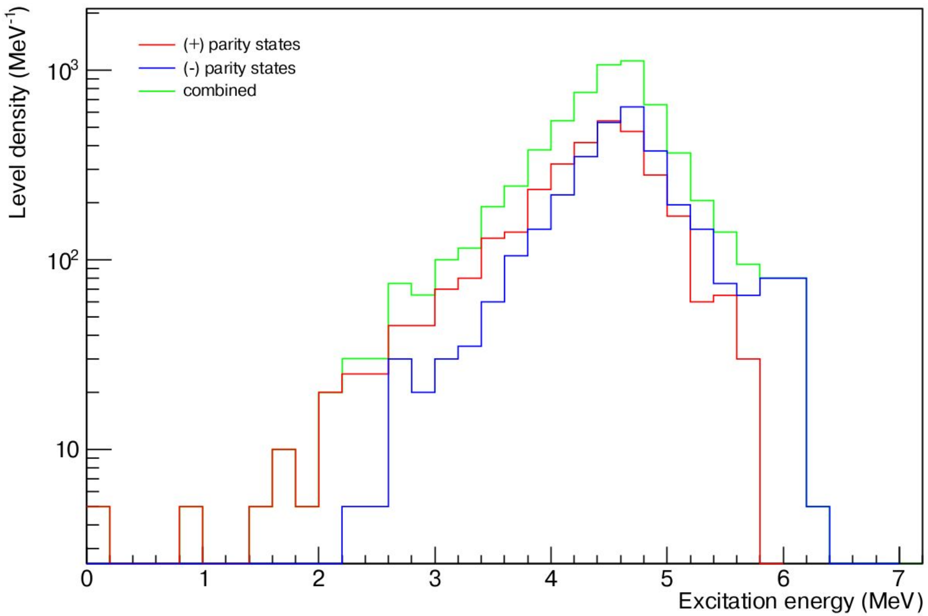


Figure 07

4. Results

Level density:

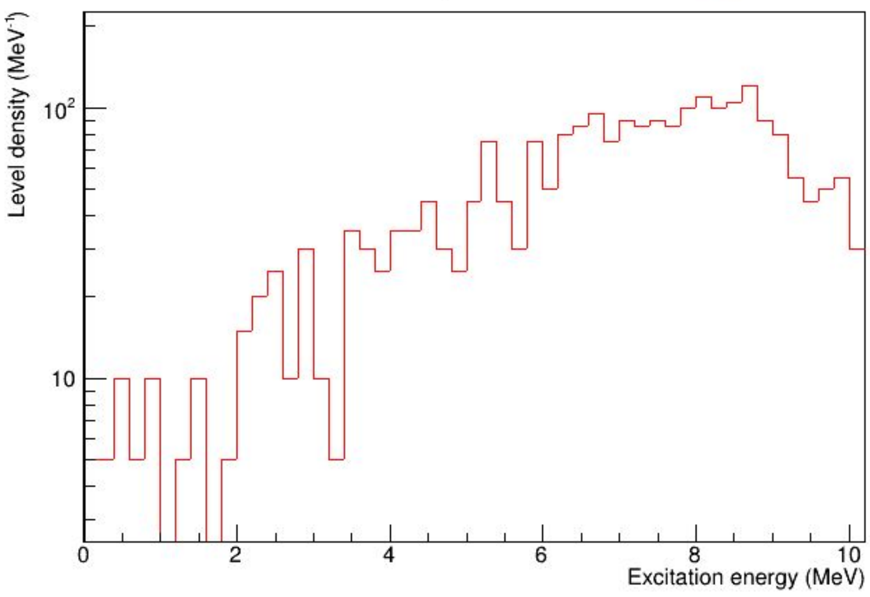


Figure 08: Level density for Sc-44

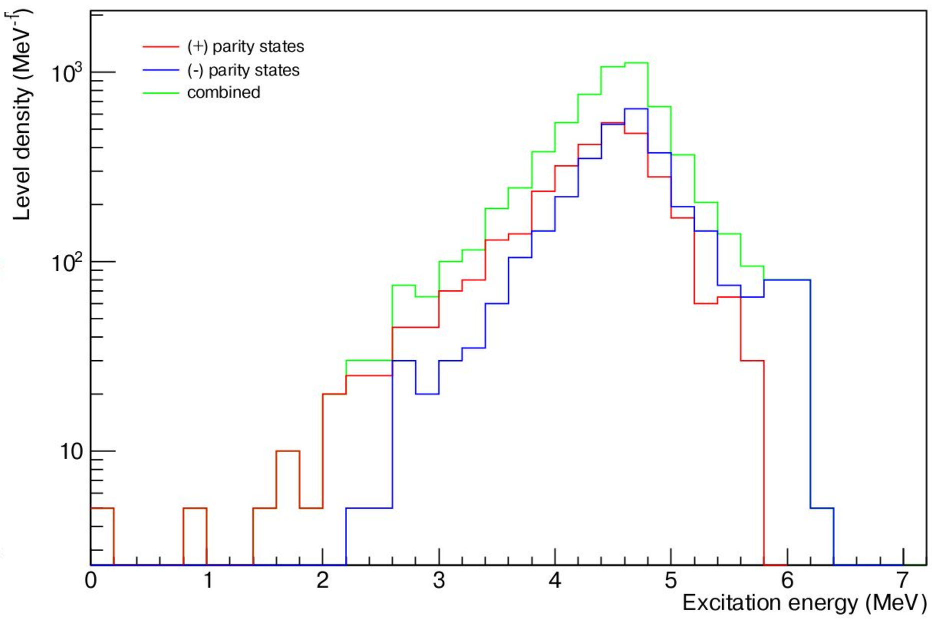


Figure 09: Level density of Te-128

Reduced transition probability:

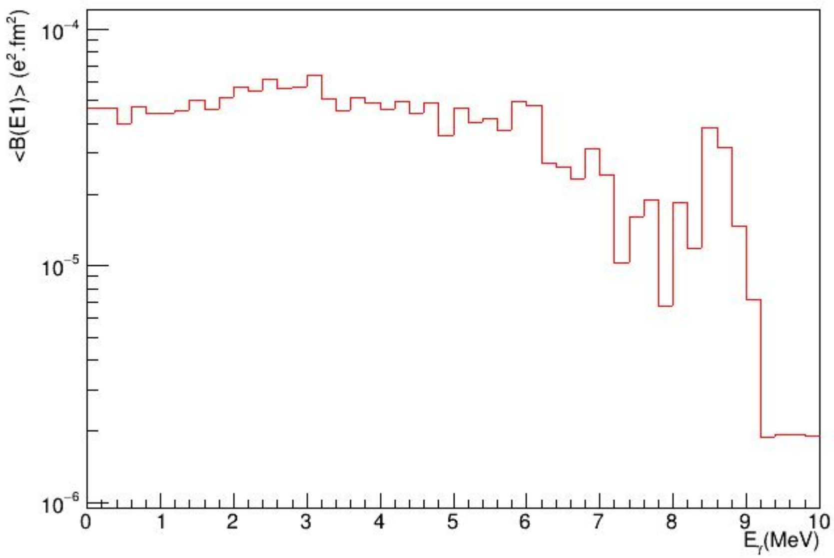


Figure 10: Averaged Reduced transition probability (<B(E1)>) for Sc-44

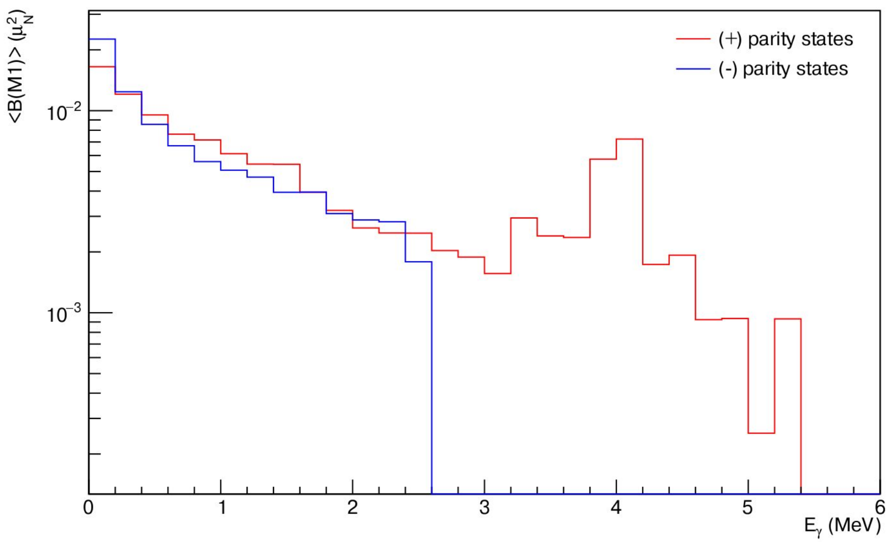


Figure 11: Averaged Reduced transition probability (<B(M1)>) for Te-128

4. Results

Strength Functions:

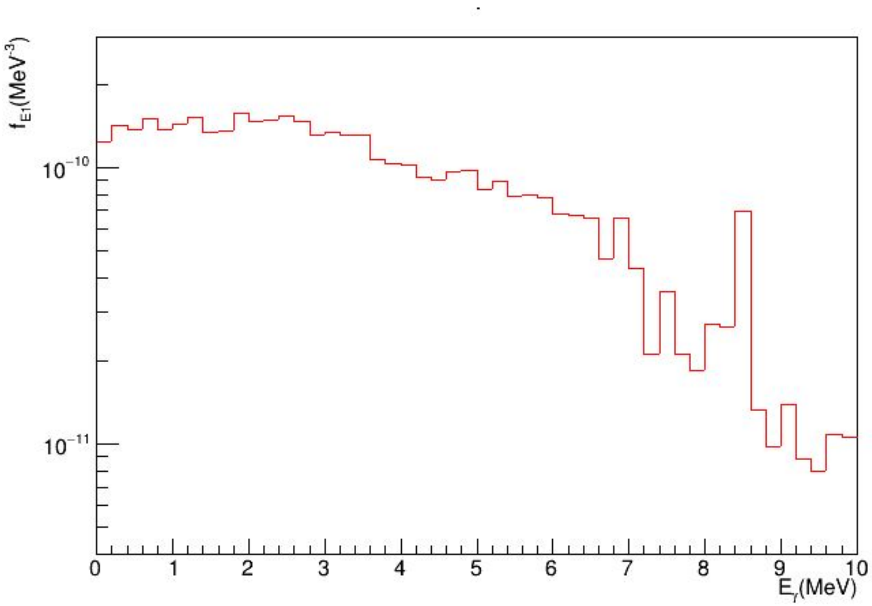


Figure 12: De-excitation Strength function for Sc-44

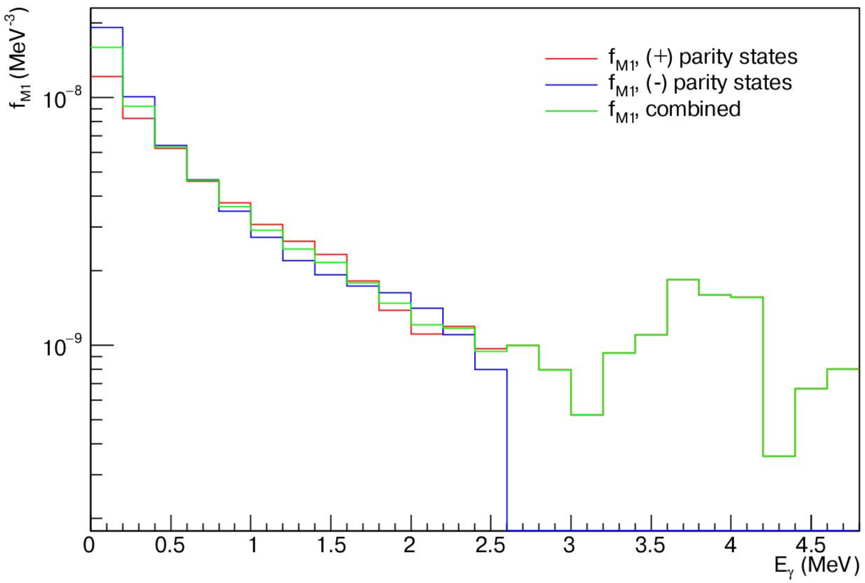


Figure 13: De-excitation Strength function for Te-128

5. Conclusion

General idea:

- Knowledge of de-excitation strength functions of nuclei is important in the study of neutron capture processes involved in explosive nucleosynthesis
- **Goal**: To produce a computer program to handle the tables of large-scale shell model (LSSM) results (spectra, transition probabilities) and compute the level densities and the radiative strength functions of magnetic and electric dipole type in ^{128}Te and ^{44}Sc
- Inputs were studied and the programs were designed to be able to handle the input data structure, and thus the goal was achieved

Difficulties:

- Social distancing - less effective collaboration compared to an in-person activity
- Two different nuclei, two physically different (but quite logically equivalent) processes

How we overcame:

- Zoom/messenger for frequent exchange of thoughts and ideas
- Github for sharing the project space
- Overleaf.com and Google for the report and the presentation

Possible improvements:

- Developing the program to be able to handle more other nuclei
- Graphical improvements
- Improvements of versatility
- Minor bug fixes

6. Resources

- **K. Sieja,** Phys. Rev. Lett. (PRL 119, 052502) Electric and Magnetic Dipole Strength at Low Energy, July 31, 2017.
- **K. Sieja,** Phys. Rev. Lett. (C98, 064312), Shell-model study of the M1 dipole strength at low energy in the A>100 nuclei, December 10, 2018.
- **H.P.Loens,** Microscopic radiative strength functions and fission barriers for r-process nucleosynthesis (2011)
- **The project repository,** https://github.com/Deshan332/Student_Project_2021.git
- Google image feed

Thank you!