Axion hot dark matter bound, reliably

RJP 2021

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Based on *L. Di Luzio, G. Martinelli, GP* [arXiv: **2101.10330**]

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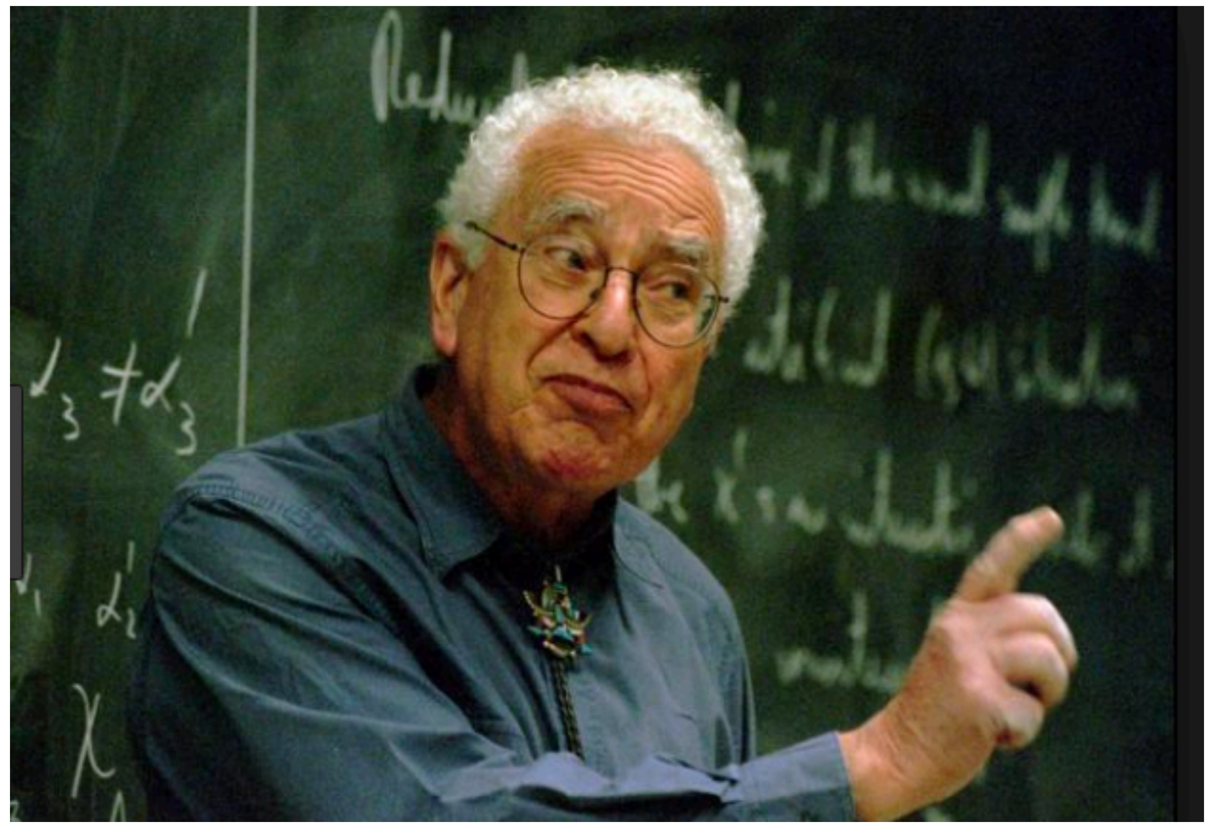
Outline

1. Strong CP problem & The Axion
2. The HDM bound
3. Axion-Pion Effective Lagrangian: Leading Order
4. Why HDM bound is not reliable: $a\pi \leftrightarrow \pi\pi$ thermalization rate at NLO
5. Conclusions

Strong CP problem



"Any process which is not forbidden by a conservation law actually does take place with appreciable probability"

[M. Gell-Mann,1956]

QCD vacuum structure

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}\left(i\not{D} - m_q e^{i\theta_q}\right) q - \frac{1}{4} G^{a\,\mu\nu} G^a_{\mu\nu} + \theta \frac{g_s^2}{32\pi^2} G^{a\,\mu\nu} \tilde{G}^a_{\mu\nu} + h.c.$$

CP-V: Expectation $\bar{\theta} \equiv \theta + \theta_q \sim \mathcal{O}(1)$, but $|\bar{\theta}| \lesssim 10^{-10}$ from **nEDM**

$$d_n = 2.4\,(1.0) \cdot 10^{-16}\,\bar{\theta}\; e\ \mathrm{cm} \qquad |d_n^{\rm exp}| < 3.0 \cdot 10^{-26}\; e\ \mathrm{cm}$$

WHY?

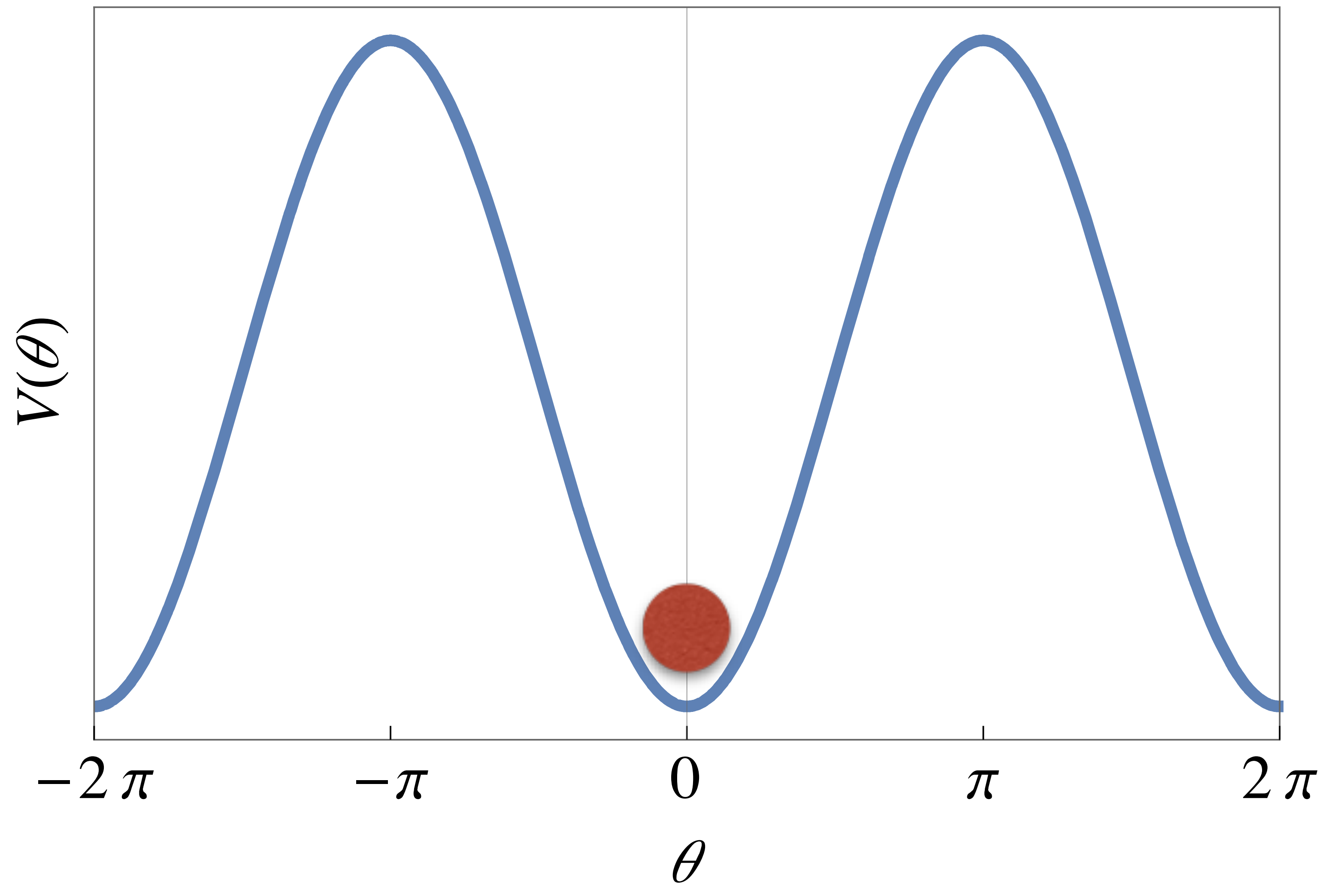
[Pendlebury, et al. arX:1509.04411]

The Axion

- Pseudo-Goldstone Boson of a spontaneously broken global symmetry, with QCD anomaly.
- Solves the Strong CP problem: θ-term dynamically set to zero

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \mathcal{L}(\partial_\mu a, \psi) + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G} + \theta\frac{g_s^2}{32\pi^2}G^{a\,\mu\nu}\tilde{G}^a_{\mu\nu}$$

$a \rightarrow a - \theta f_a$ with $\langle a \rangle = 0$ [C. Vafa, E. Witten 1984]



$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

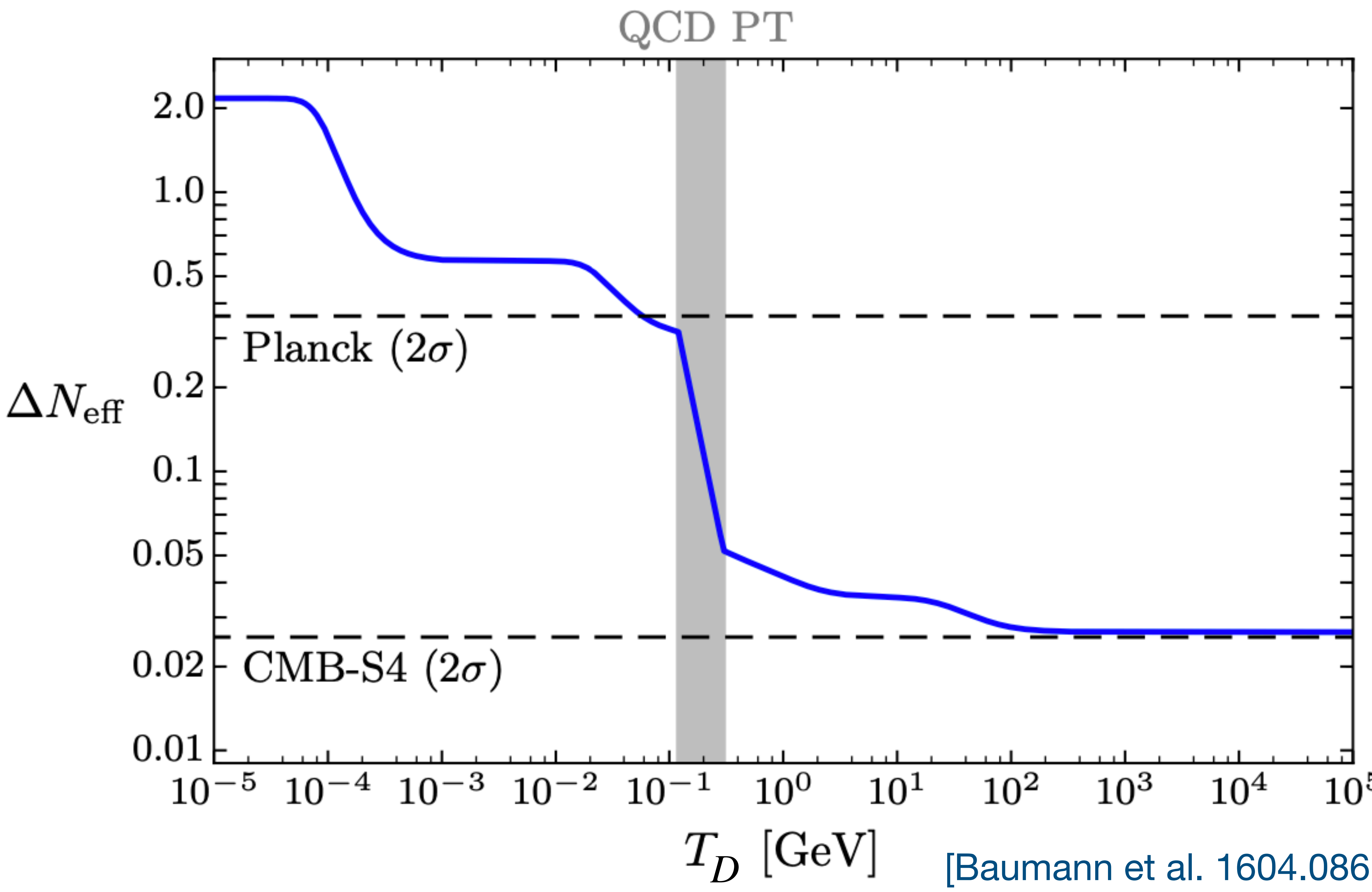
[Peccei, Quinn ’77]
[Wilczek ’78]
[Weinberg ’78]

- Rich phenomenology (Dark Matter, Astrophysics, Cosmology)

A possible discovery channel for the axion!

Axion contributes to the Number of extra relativistic species

- Axions once in equilibrium with SM bath contribute to the radiation density of the Universe
- T_D depends on the strength of the axion interactions set by f_a
- Full range of allowed $\Delta N_{\rm eff}$ will be covered by CMB-S4



(2015)

(2029 ?)

[Baumann et al. 1604.08614]

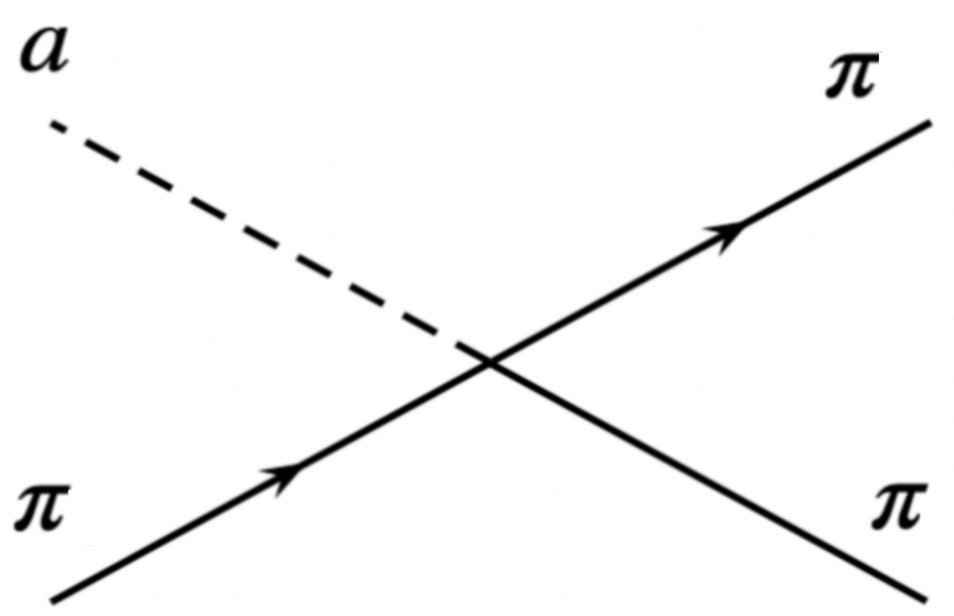
A possible discovery channel for the axion!

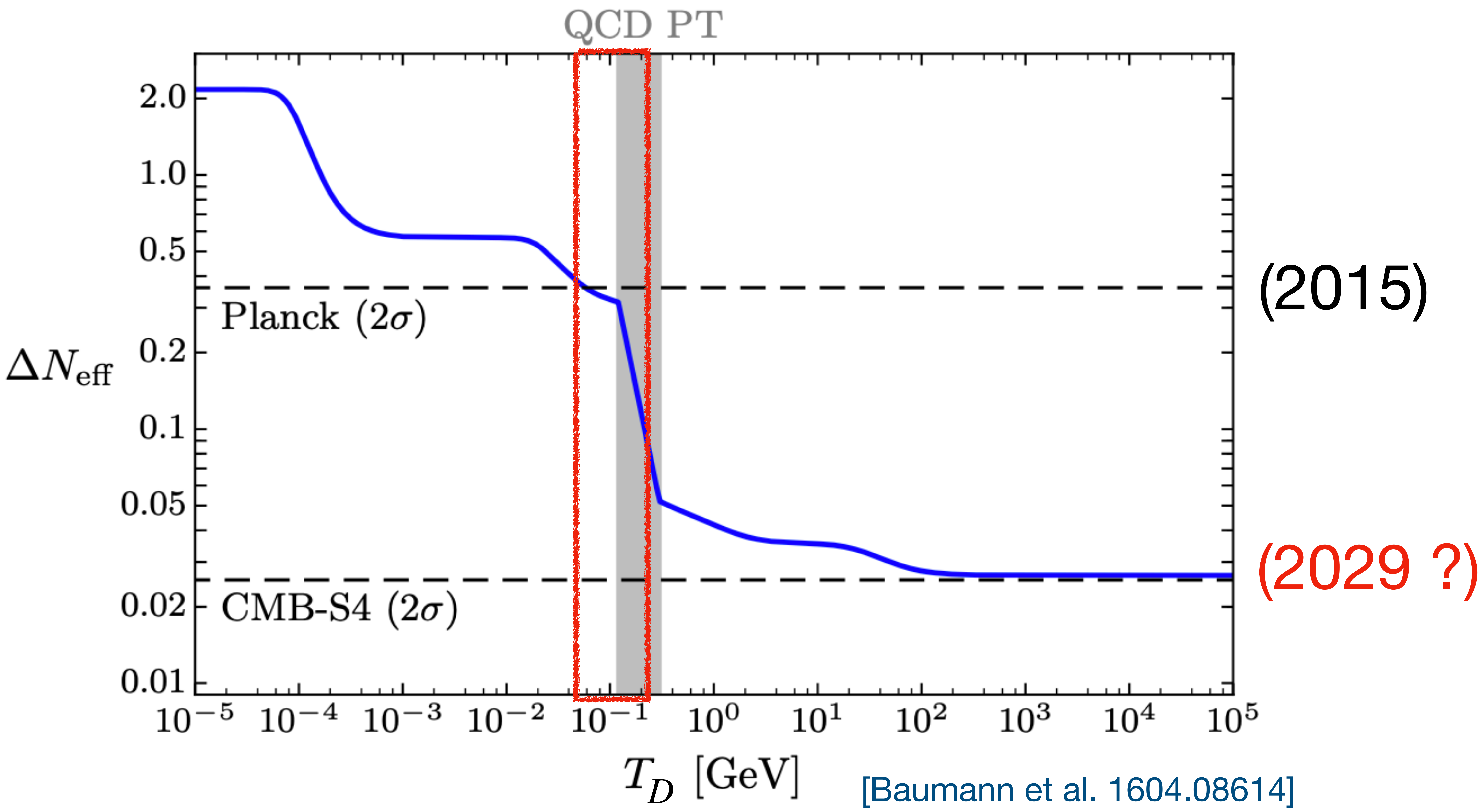
Axion contributes to the Number of extra relativistic species

- Axions once in equilibrium with SM bath contribute to the radiation density of the Universe

$T_D \sim [50\ \text{MeV}\ ,\ 200\ \text{MeV}]$

- Below 200 MeV the main thermalization channel is

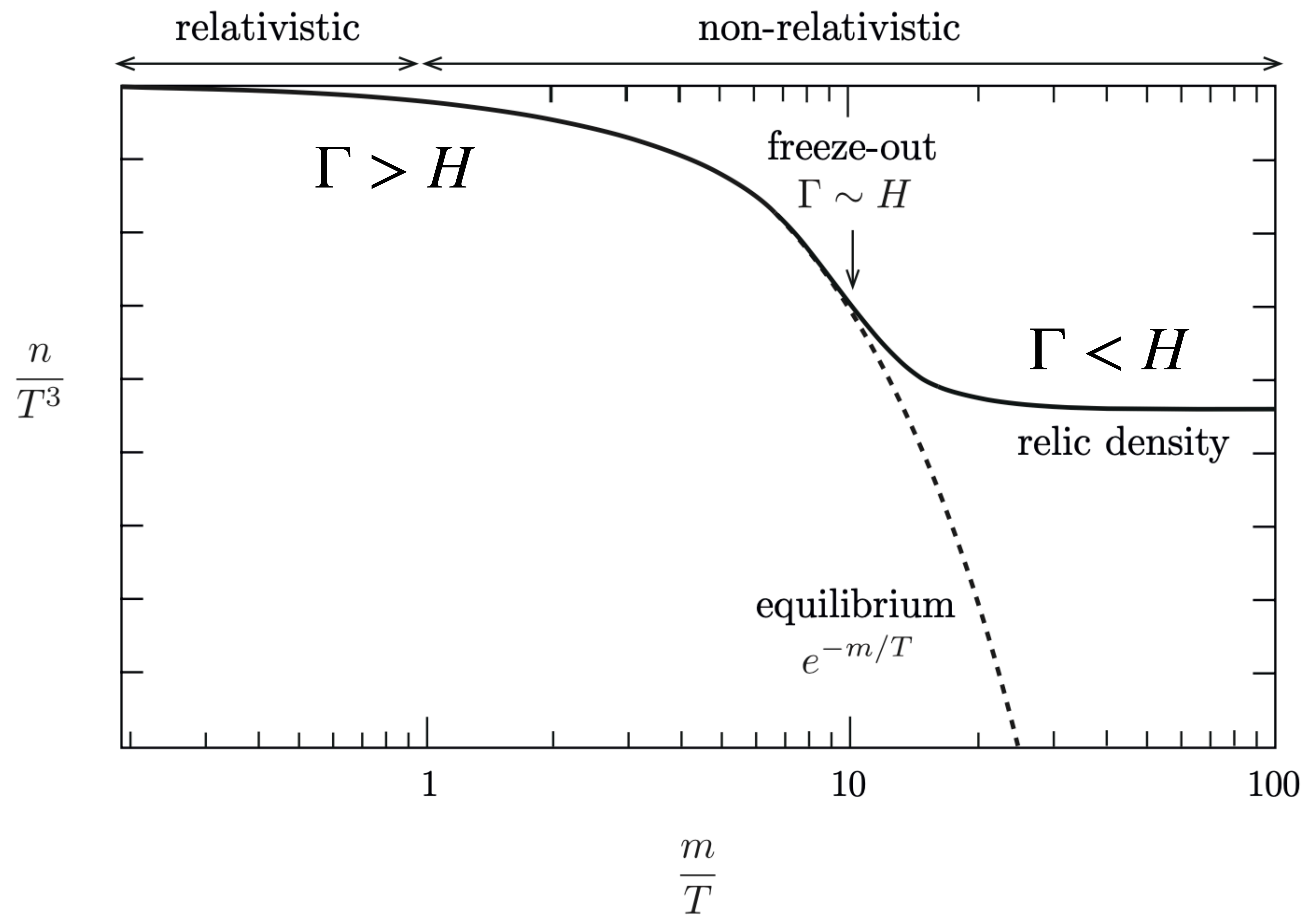




[Baumann et al. 1604.08614]

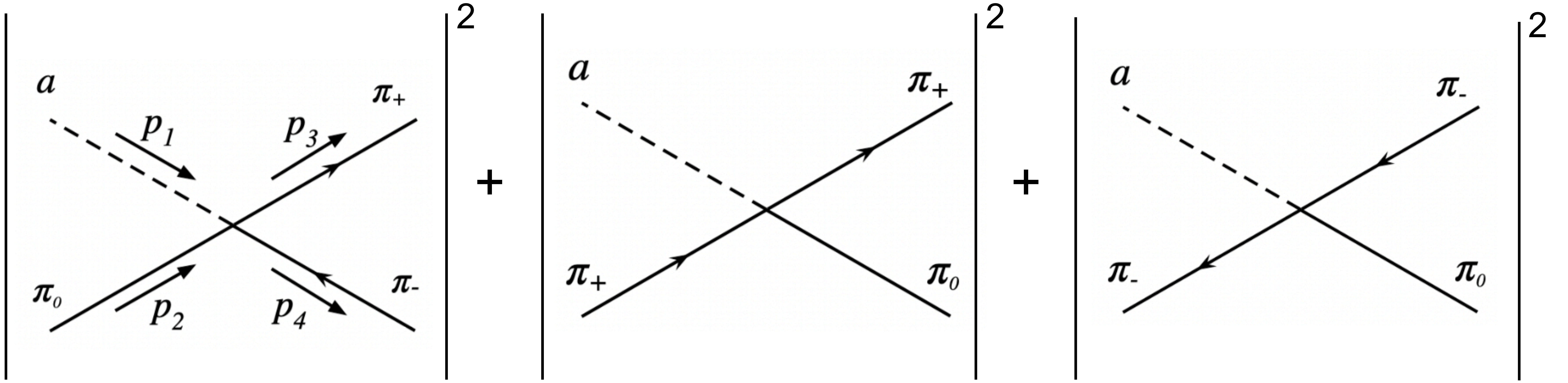
Axion thermal production in the Early Universe

- The evolution of the axions number density is controlled by the ratio Γ/H



Leading order scattering amplitude

$$\mathcal{L}_{a\pi}^{(\mathrm{LO})} = \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a\Big(2\partial_\mu\pi_0\pi_+\pi_- - \pi_0\partial_\mu\pi_+\pi_- - \pi_0\pi_+\partial_\mu\pi_-\Big)$$



$$\sum |\mathcal{M}|^2_{\mathrm{LO}} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4}\left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

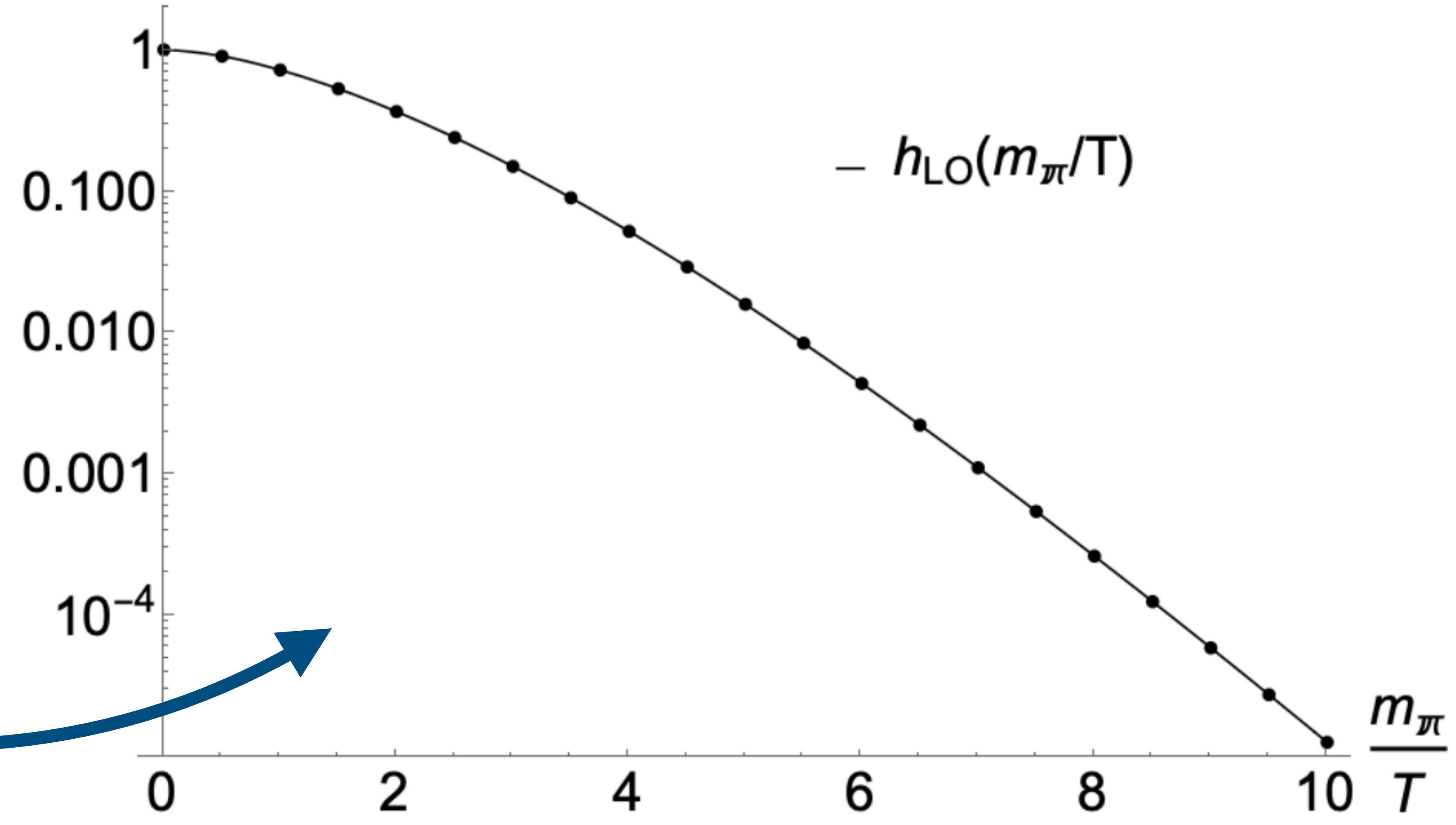
Thermal scattering rate

$$\Gamma = \frac{1}{n_a^{\mathrm{eq}}} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \boxed{\sum |\mathcal{M}|^2}$$
$$(2\pi)^4 \delta^4 \left(p_1 + p_2 - p_3 - p_4\right) f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

$$\sum |\mathcal{M}|^2_{\mathrm{LO}} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4} \left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

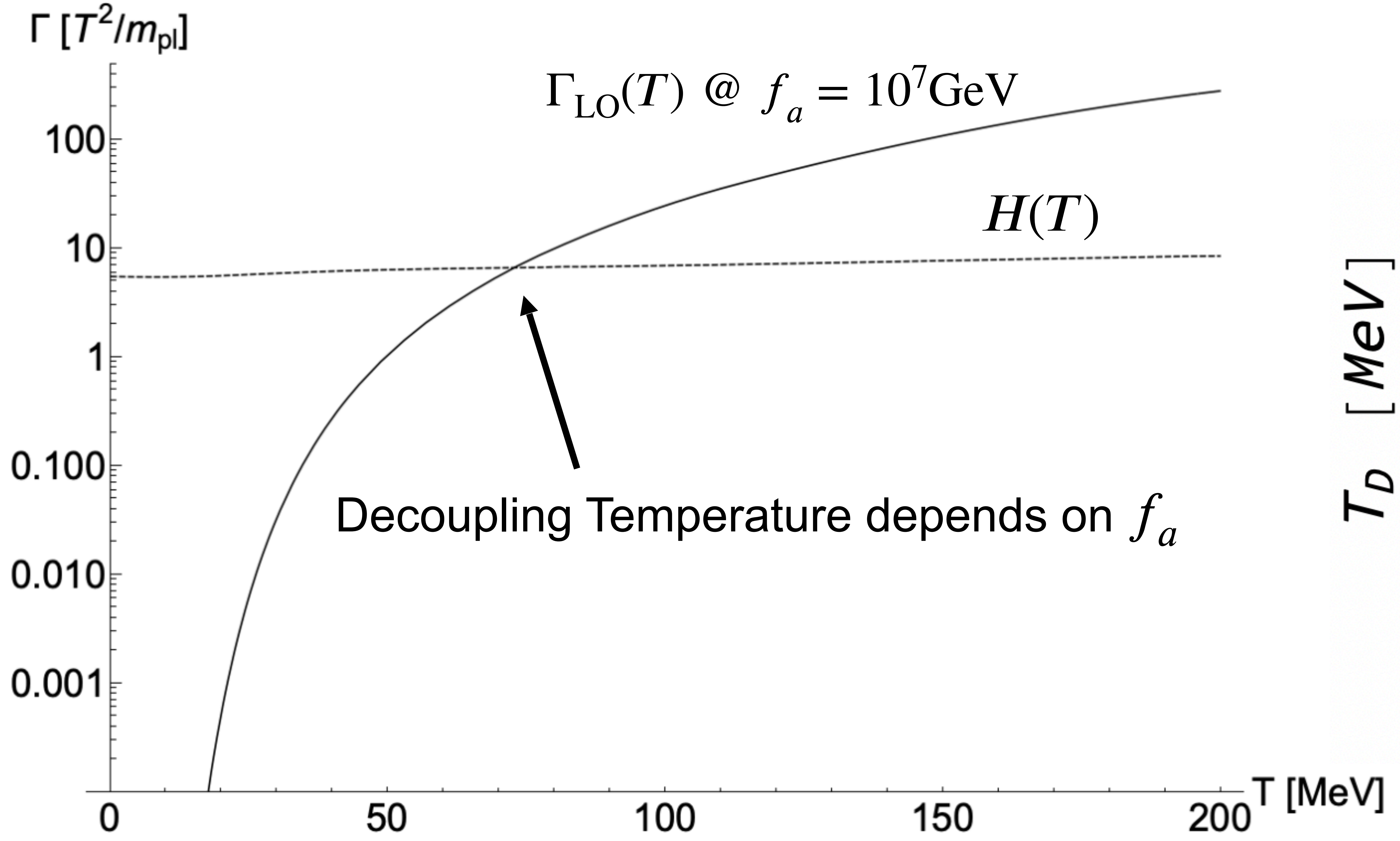
Numerically integrating:

$$\boxed{\Gamma(T) = 0.212 \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 T^5 h_{\mathrm{LO}}(m_\pi/T)}$$

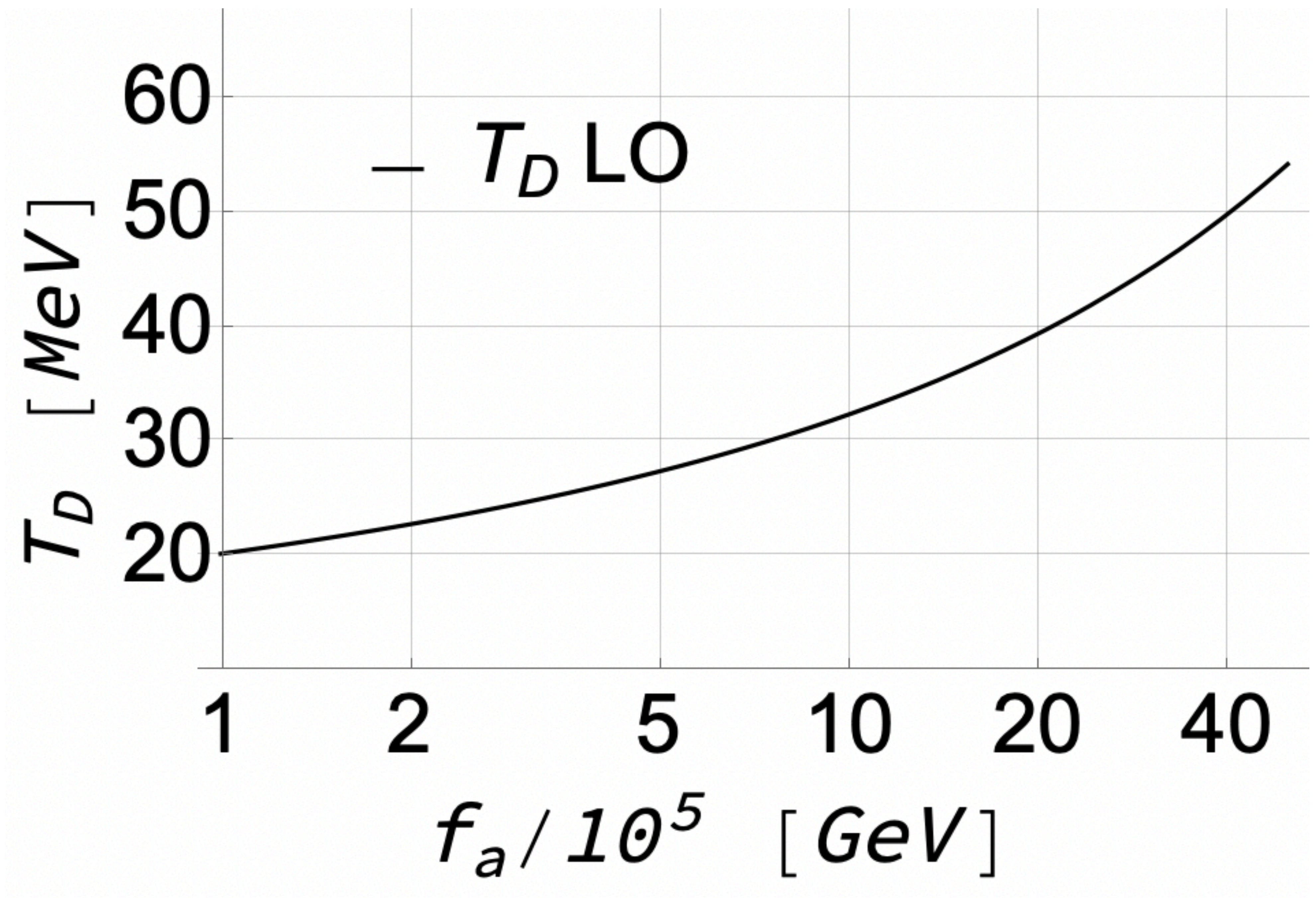


see also:
[Chang, Choi, hep-ph/9306216]
[Hannestad, Mirizzi, Raffelt, hep-ph/0504059]

Γ vs H



$\Gamma\ [T^2/m_{\rm pl}]$
$\Gamma_{\rm LO}(T)$ @ $f_a = 10^7$GeV
$H(T)$
Decoupling Temperature depends on f_a
100
10
1
0.100
0.010
0.001
0
50
100
150
200
T [MeV]

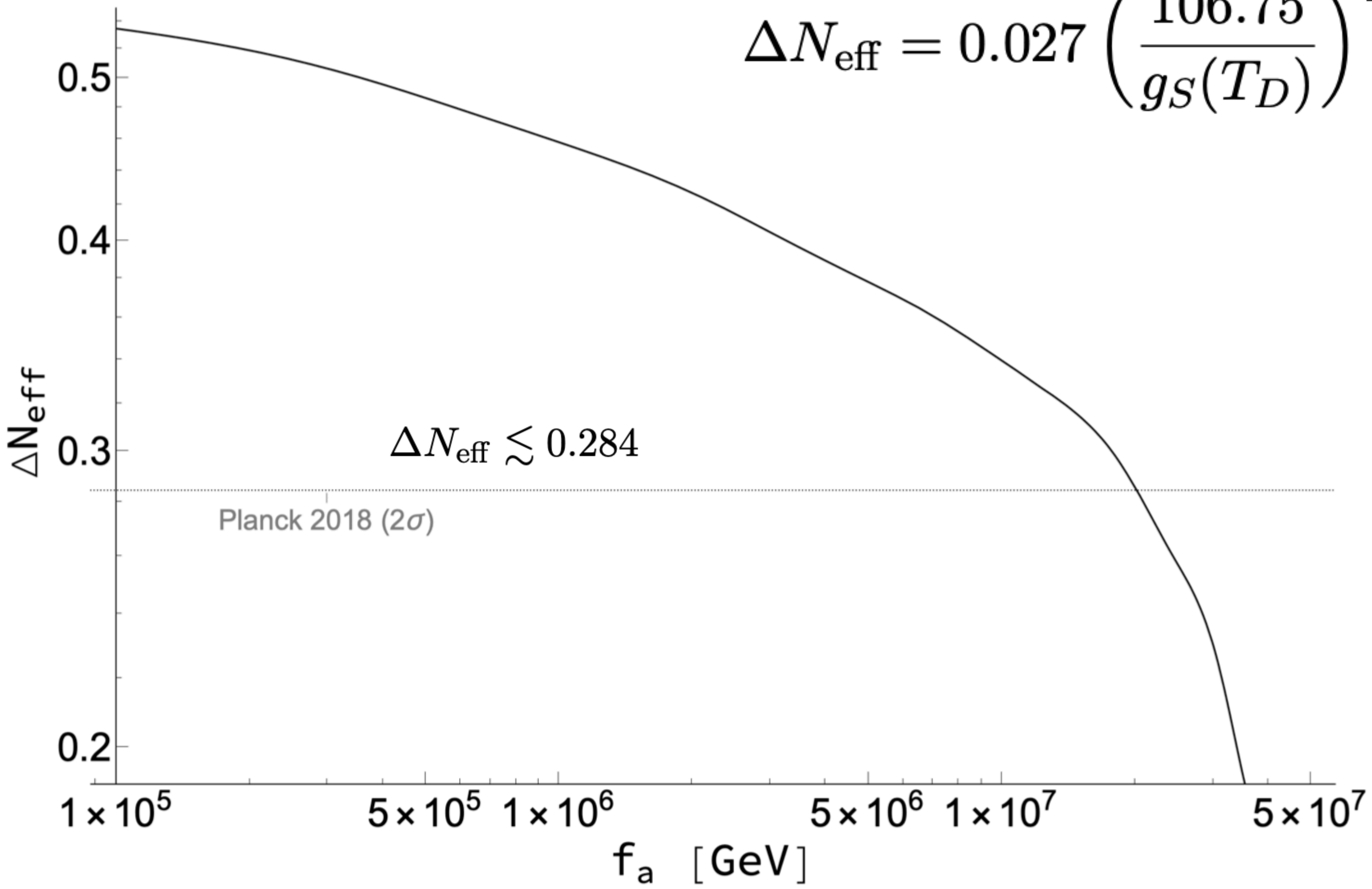


— T_D LO
T_D [MeV]
60
50
40
30
20
1
2
5
10
20
40
$f_a/10^5$ [GeV]

Hot Dark Matter bound on Axion mass, LO

Axion contribution to the Number of relativistic species

$$\Delta N_{\rm eff} = 0.027 \left(\frac{106.75}{g_S(T_D)} \right)^{4/3}$$



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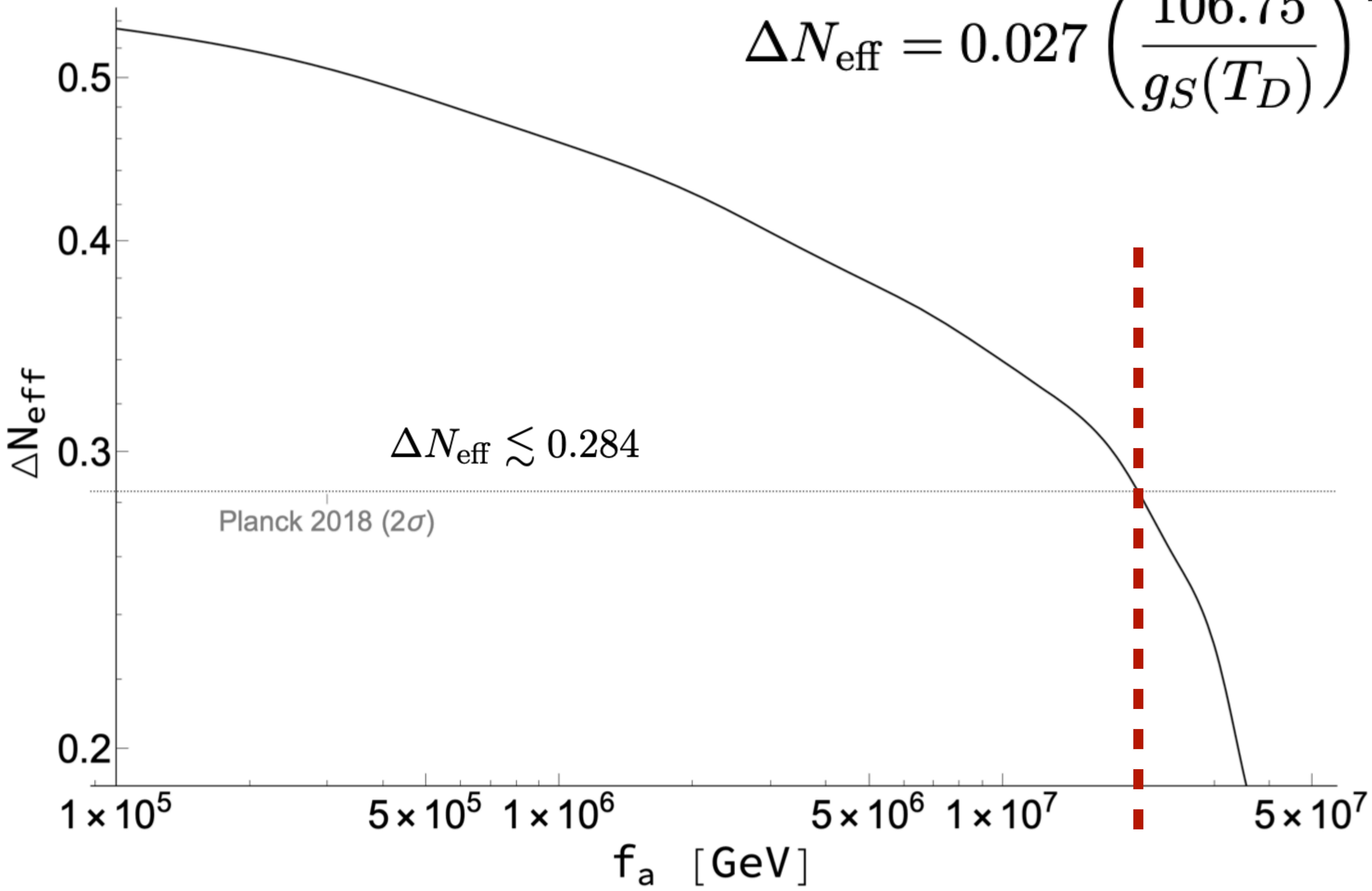


$$f_a \gtrsim 2.03 \times 10^7 \text{ GeV}$$

$$m_a \simeq 5.7 \left(\frac{10^{12}}{f_a \text{ [GeV]}}\right) \mu\text{eV} \lesssim 0.28 \text{ eV}$$

Naïve way to set the bound

See:
[Melchiorri, Mena, Slosar, arXiv:0705.2695]
[Hannestad, Mirizzi, Raffelt, Wong, arXiv:0803.1585]
[Hannestad, Mirizzi, Raffelt, Wong, arXiv:1004.0695]
[Di Valentino, Giusarma, Lattanzi, Mena, Melchiorri, Silk, arXiv:1507.08665]

But… is ChPT valid?

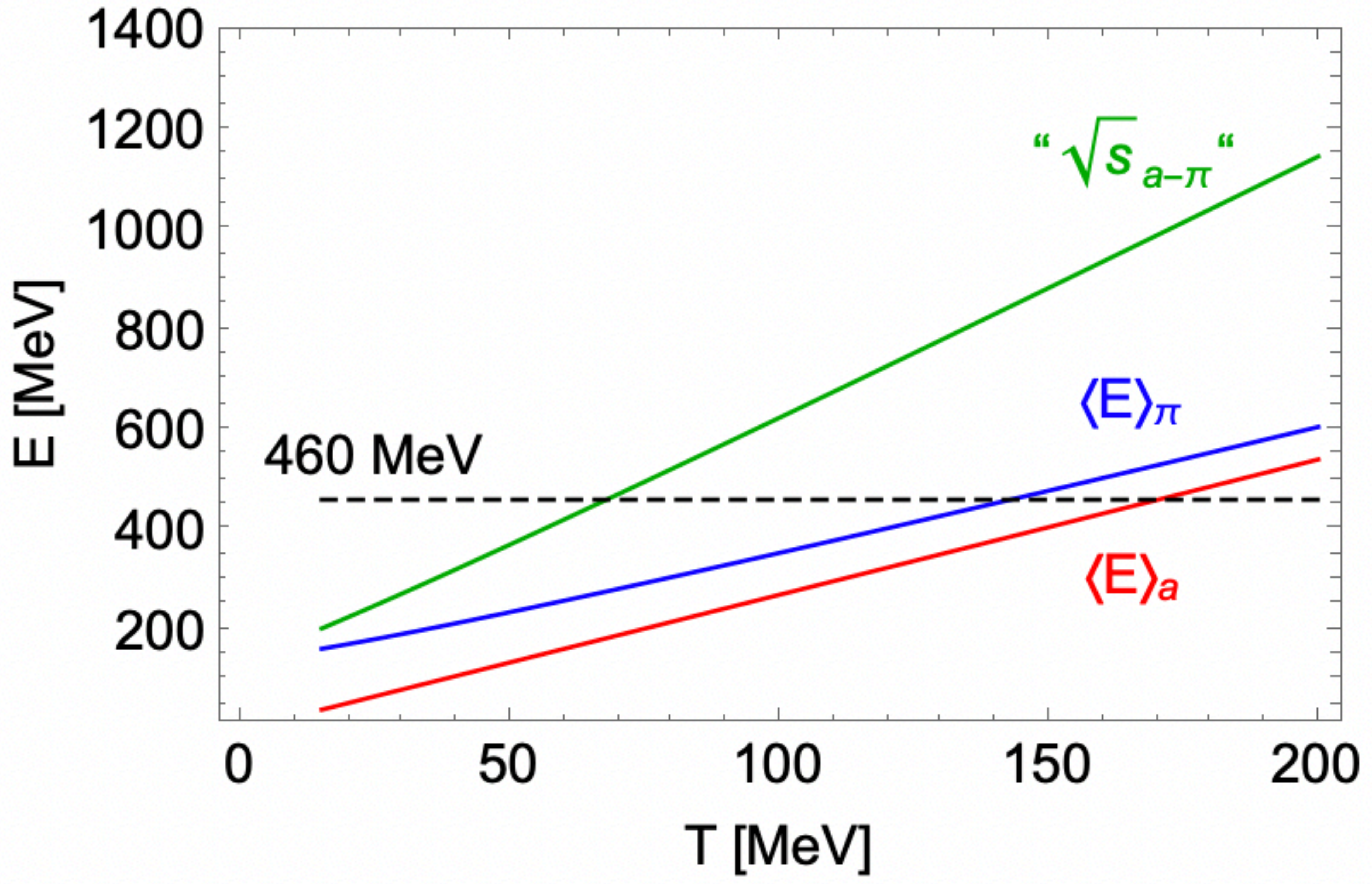
The mean energy of π, a at
$T \simeq 80\ \mathrm{MeV}$ is
$\langle E \rangle \equiv \rho / n \simeq 305\ \mathrm{MeV},\ 220\ \mathrm{MeV}$

BUT

ChPT is valid for $E \lesssim 460\ \mathrm{MeV}$

[Donoghue et al., PhysRevD.86.014025]

$$\langle E \rangle (T) \sim \frac{\rho(T)}{n(T)}$$



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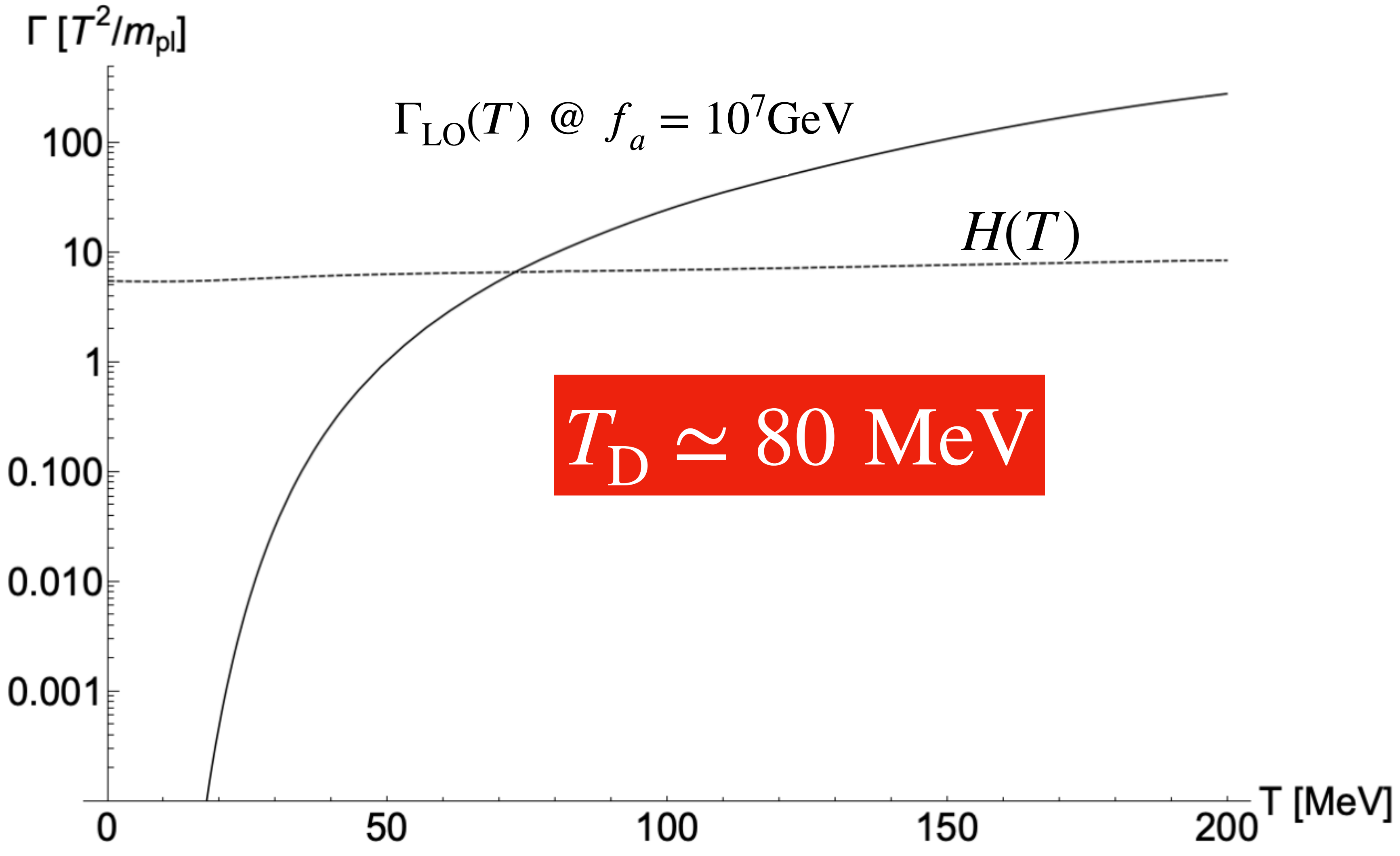
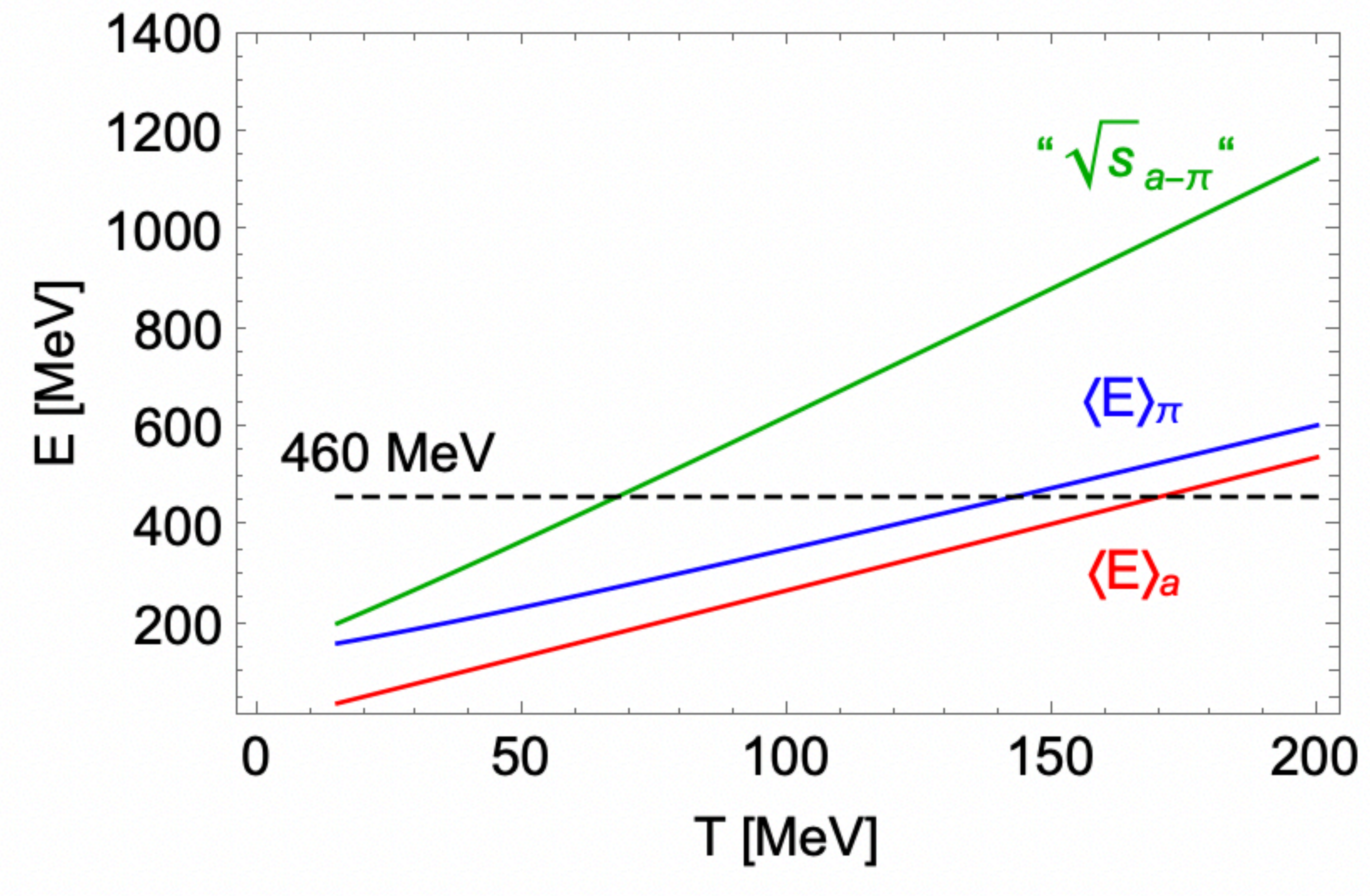




The bound extracted from ChPT is not reliable

NLO axion production rate

Axion-Pion scattering: Next-to-Leading Order

Ingredients

Tree-level graph from NLO Lagrangian and loop amplitudes from LO Lagrangian contributes to the same Order

$$
\begin{aligned}
\mathcal{L}_{\mathrm{NLO}} = {} & \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\,\mathrm{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\mathrm{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right] \\
& + \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\,\mathrm{Tr}\left[D_\mu U\left(D^\mu \chi\right)^\dagger + D_\mu \chi\left(D^\mu U\right)^\dagger\right] \\
& + l_5\left[\mathrm{Tr}\left(f^R_{\mu\nu} U f_L^{\mu\nu} U^\dagger\right) - \frac{1}{2}\,\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)\right] \\
& + i\frac{l_6}{2}\,\mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right] \\
& - \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1 + h_3}{4}\,\mathrm{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1 - h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right. \\
& \left. + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)
\end{aligned}
$$

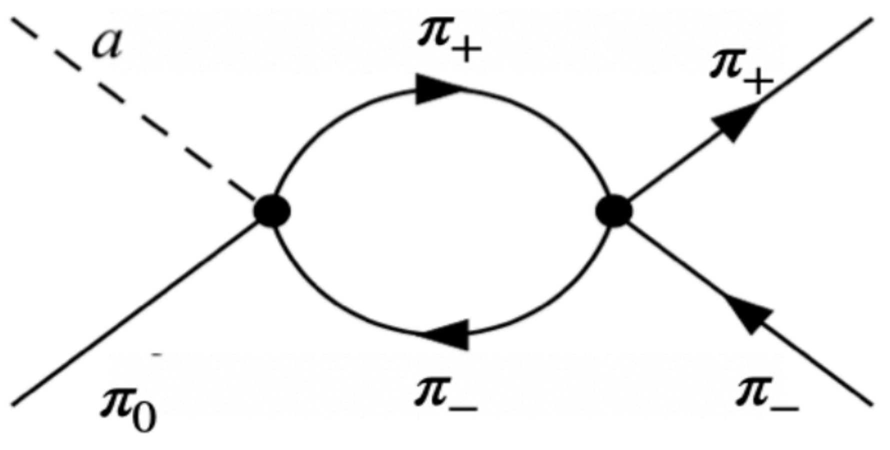
NLO Lagrangian

+

NLO chiral axial current J^a_μ

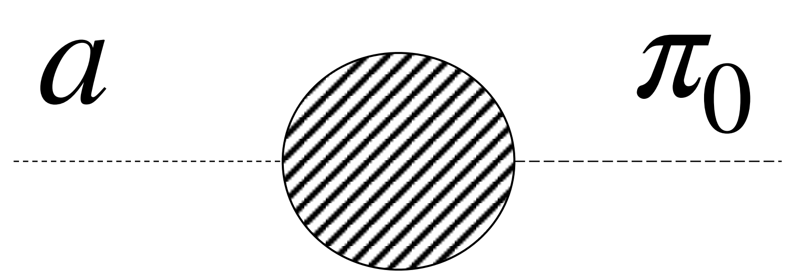
$$\mathcal{L}_a^\chi \supset \frac{\partial^\mu a}{f_a}\mathrm{Tr}\frac{1}{2}\left[c_q \sigma^a\right] J^a_\mu$$

+



1-loop amplitudes from LO

+



$a - \pi_0$ diagonalization

Amplitudes

- After renormalization & including NLO corrections to f_π

$$\mathcal{M}^{\rm NLO}_{a\pi_0\to\pi_+\pi_-} = \frac{C_{a\pi}}{192\pi^2 f_\pi^3 f_a}\Bigg\{15m_\pi^2(u+t) - 11u^2 - 8ut - 11t^2 - 6\overline{\ell_1}\left(m_\pi^2 - s\right)\left(2m_\pi^2 - s\right)$$
$$-6\overline{\ell_2}\left(-3m_\pi^2(u+t) + 4m_\pi^4 + u^2 + t^2\right) + 18\overline{\ell_4}m_\pi^2(m_\pi^2 - s)$$
$$+3\left[3\sqrt{1-\frac{4m_\pi^2}{s}}s\left(m_\pi^2 - s\right)\ln\left(\frac{\sqrt{s\left(s-4m_\pi^2\right)}+2m_\pi^2 - s}{2m_\pi^2}\right)\right.$$
$$+\sqrt{1-\frac{4m_\pi^2}{t}}\left(m_\pi^2(t-4u) + 3m_\pi^4 + t(u-t)\right)\ln\left(\frac{\sqrt{t\left(t-4m_\pi^2\right)}+2m_\pi^2 - t}{2m_\pi^2}\right)$$
$$\left.+\sqrt{1-\frac{4m_\pi^2}{u}}\left(m_\pi^2(u-4t) + 3m_\pi^4 + u(t-u)\right)\ln\left(\frac{\sqrt{u\left(u-4m_\pi^2\right)}+2m_\pi^2 - u}{2m_\pi^2}\right)\right]\Bigg\}$$
$$+\frac{4\ell_7 m_\pi^2 m_d\left(s-2m_\pi^2\right)m_u\left(m_d - m_u\right)}{f_\pi^3 f_a\left(m_d+m_u\right)^3},$$

- Other pionic channels obtained via $s \to t, u$

NLO Thermalization rate

$$\sum |\mathcal{M}|^2 = |\mathcal{M}_{\rm LO}|^2$$

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.212\, T^5 \Big[h_{\rm LO}(m_\pi/T)$$

NLO Thermalization rate

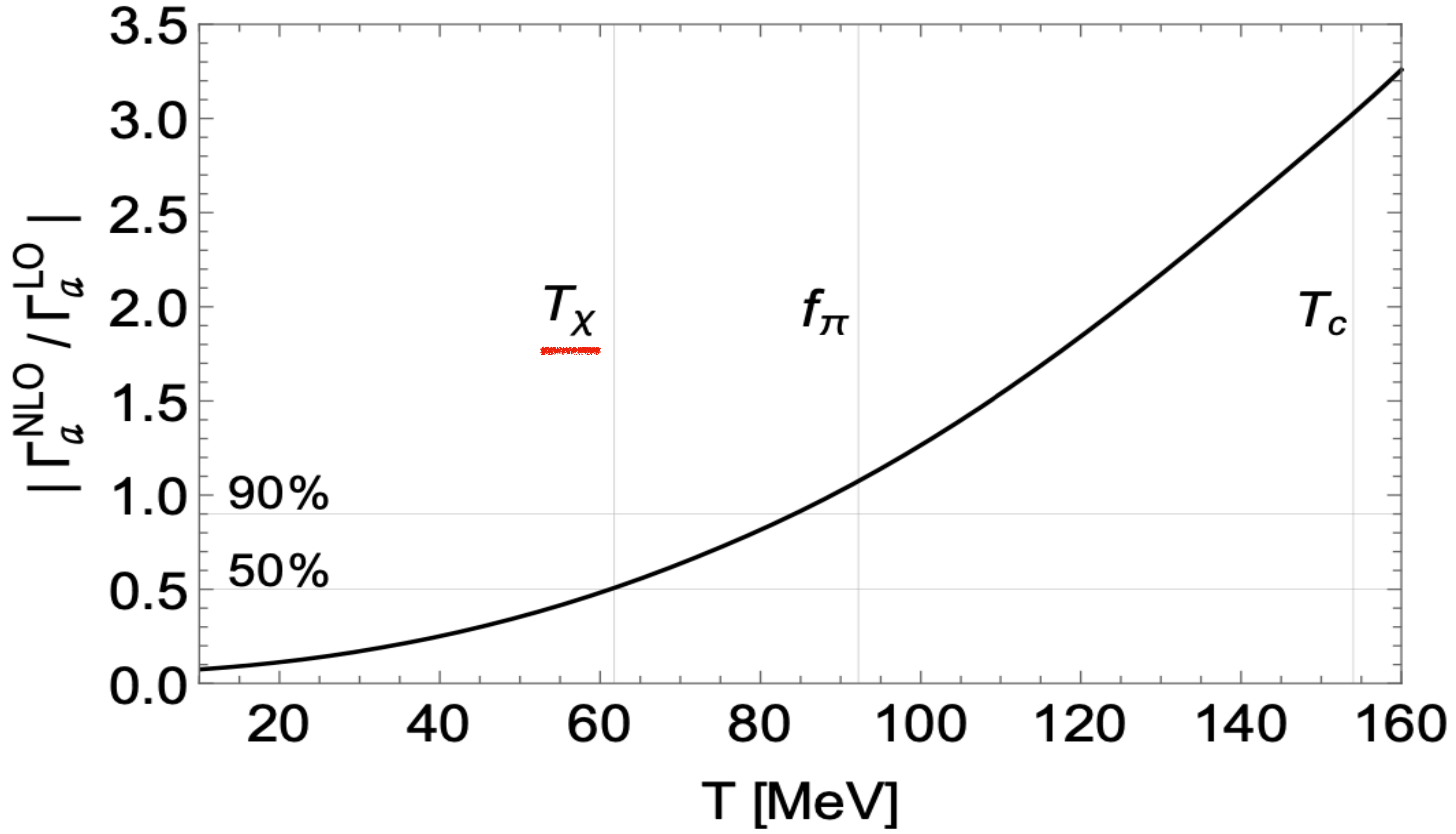
$$\sum |\mathcal{M}|^2 = |\mathcal{M}_{\rm LO}|^2 + 2{\rm Re}[\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO}]$$

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.212\, T^5 \left[h_{\rm LO}(m_\pi/T) - 2.92\frac{T^2}{f_\pi^2}\, h_{\rm NLO}(m_\pi/T)\right]$$

Different T-behavior between LO and NLO contributions

Correction of $\sim 50\,\%$ already at $T_\chi \equiv 62$ MeV

Convergence problem!



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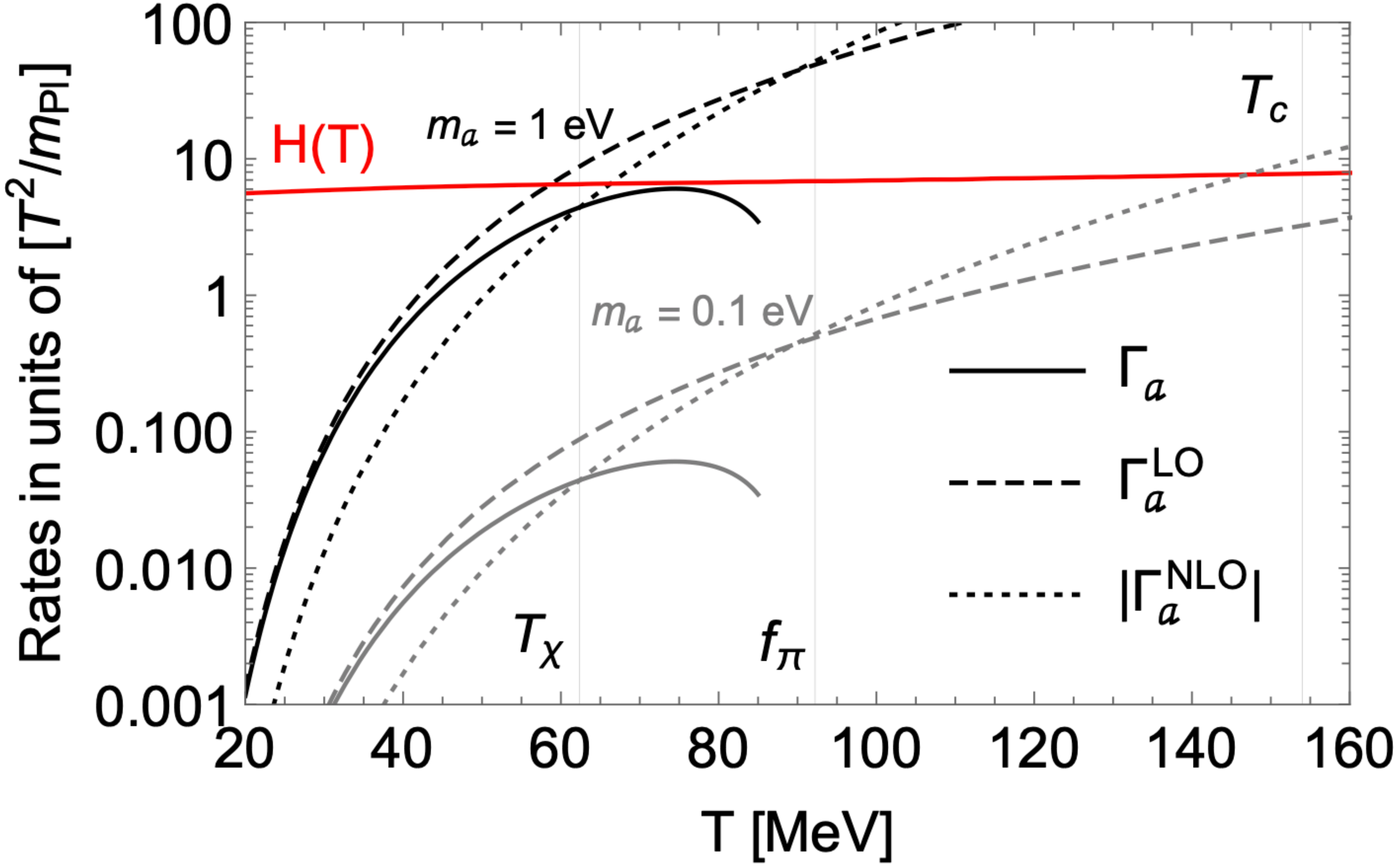
Γ vs H , **NLO**

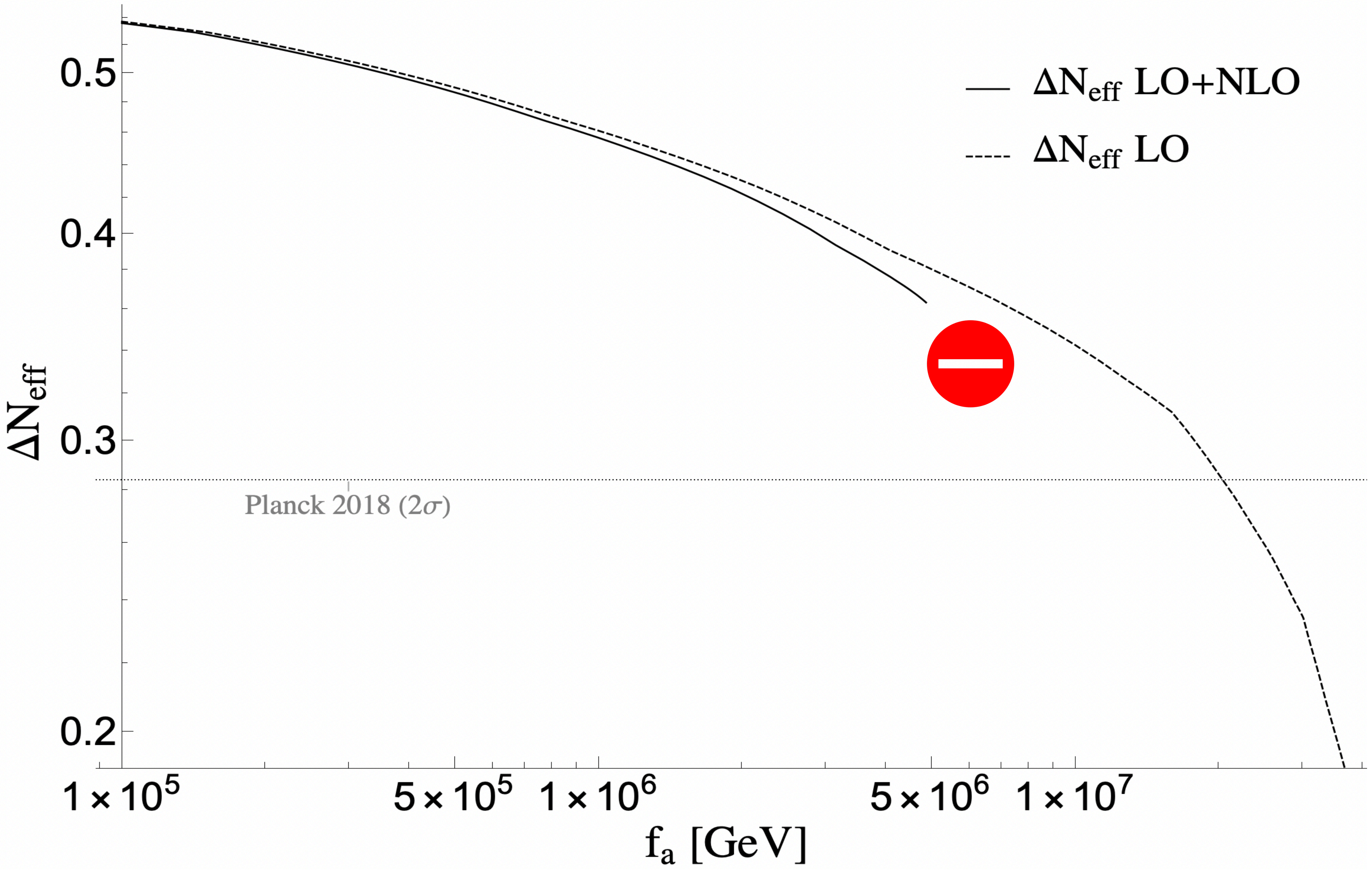
- $m_a = 1$ eV: the most conservative HDM bound
- $m_a = 0.1$ eV: typical reach of future CMB-S4 experiments
- $T_\chi \sim 62$ MeV: boundary of validity of the chiral expansion



$\Delta N_{\rm eff}$ including NLO correction

$T_{\rm D}$ cannot be extracted in the region of interest since the *axion-pion EFT theory does not converge*

$$f_a \gtrsim 4.9 \times 10^6 \text{ GeV} \rightarrow \text{T}_{\rm D} \gtrsim 60 \text{ MeV}$$



Conclusions

- The present HDM bound $m_a \lesssim 0.2\ \mathrm{eV}$ is not reliable;
- In the mass range of interest, $m_a \in [0.1,1]\ \mathrm{eV}$, the decoupling temperature and consequently the axion HDM bound cannot be extracted within the chiral Lagrangian, since thermal effects push energies above EFT scale;
- A reliable computation is needed to set targets for future CMB surveys (lattice QCD, …?)

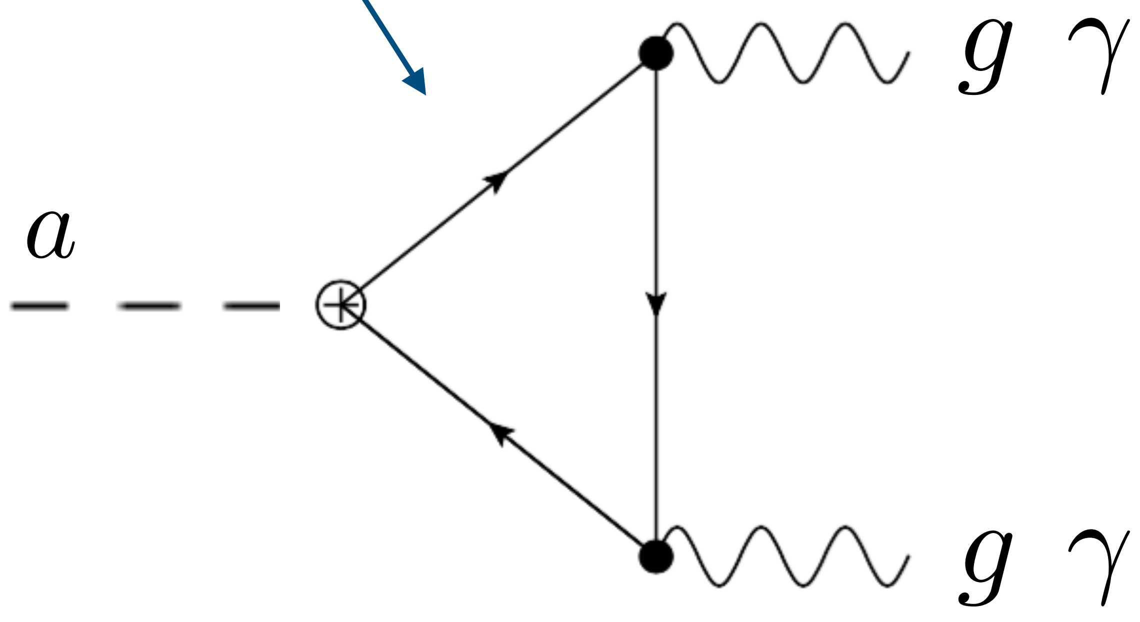
Thanks for the attention!

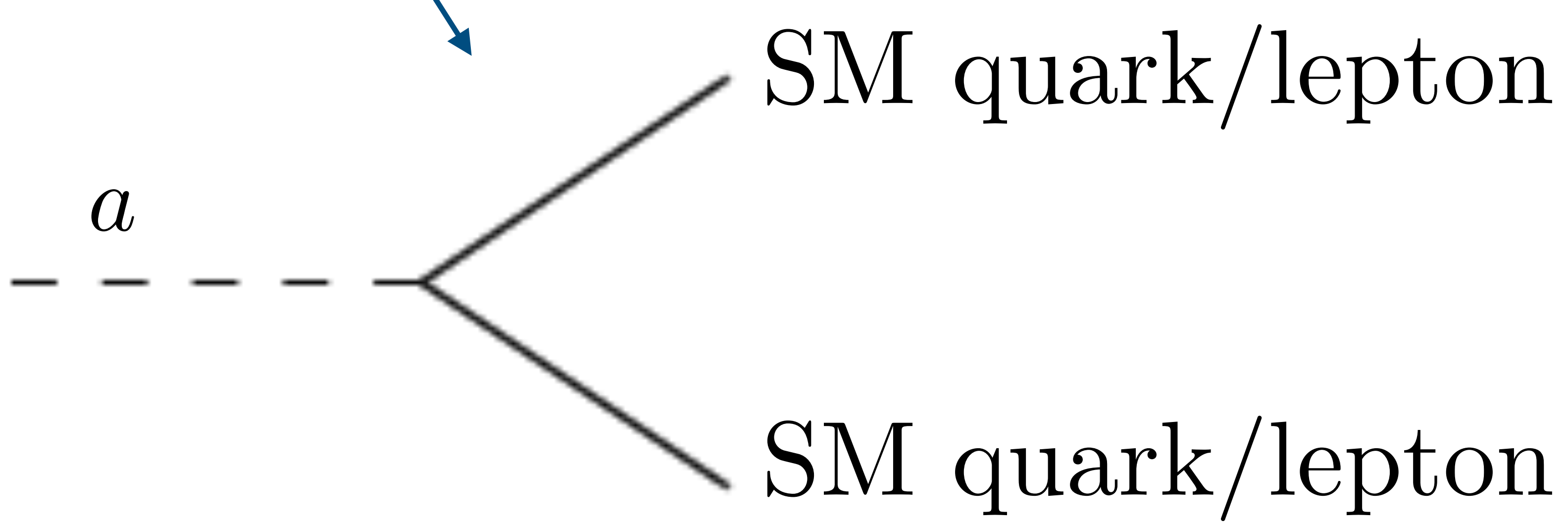
Backup

Axion EFT

$E > \Lambda_{QCD}$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)^2 + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\widetilde{G} + \frac{1}{4}g^0_{a\gamma}aF\widetilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c^0_q\gamma^\mu\gamma_5 q - \bar{q}_L M_q q_R + h.c.$$





Axion-Pion Effective Lagrangian: Leading Order

$E \gtrsim 1$ GeV

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)^2 + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\widetilde{G} + \frac{1}{4}g^0_{a\gamma}aF\widetilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c^0_q\gamma^\mu\gamma_5 q - \bar{q}_L M_q q_R + h.c.$$

1. Rotate away $aG\widetilde{G}$ through $q \to e^{i\gamma_5 \frac{a}{2f_a}Q_a}q$,
2. Write an EFT with Pions and Axions as Goldstone Bosons

$E \lesssim 1$ GeV

$$\mathcal{L}_a^\chi = \frac{f_\pi^2}{4}Tr\left[(D^\mu U)^\dagger D_\mu U + U\chi^\dagger + \chi U^\dagger\right] + \frac{\partial^\mu a}{f_a}\frac{1}{2}Tr\left[c_q\sigma^a\right]J^a_\mu$$

$$U = e^{i\pi^a\sigma^a/f_\pi}$$

$$\chi = 2B_0\, e^{i\frac{a}{2f_a}Q_a}M_q e^{i\frac{a}{2f_a}Q_a}$$

$$J^a_\mu = \frac{i}{4}f_\pi^2 Tr\left[\sigma^a\{U,(D^\mu U)^\dagger\}\right]$$

$$c_q = \begin{pmatrix} c_u^0 - \frac{m_d}{m_u+m_d} & 0 \\ 0 & c_d^0 - \frac{m_u}{m_u+m_d}\end{pmatrix}$$

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Axion Chiral Lagrangian, NLO

$$\mathcal{L}_{\rm NLO} = \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\mathrm{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right]$$
$$+ \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U\left(D^\mu \chi\right)^\dagger + D_\mu \chi\left(D^\mu U\right)^\dagger\right]$$
$$+ l_5\left[\mathrm{Tr}\left(f^R_{\mu\nu} U f^{\mu\nu}_L U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(f^L_{\mu\nu} f^{\mu\nu}_L + f^R_{\mu\nu} f^{\mu\nu}_R\right)\right]$$
$$+ i\frac{l_6}{2}\mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right]$$
$$- \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1 + h_3}{4}\mathrm{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1 - h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right.$$
$$\left.+ \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\mathrm{Tr}\left(f^L_{\mu\nu} f^{\mu\nu}_L + f^R_{\mu\nu} f^{\mu\nu}_R\right)$$

[J. Gasser and H. Leutwyler, Annals Phys. **158** (1984)]
[S. Scherer, arXiv:hep-ph/0210398]

And, differentiating the Lagrangian with respect to the external fields…

Axial current to NLO

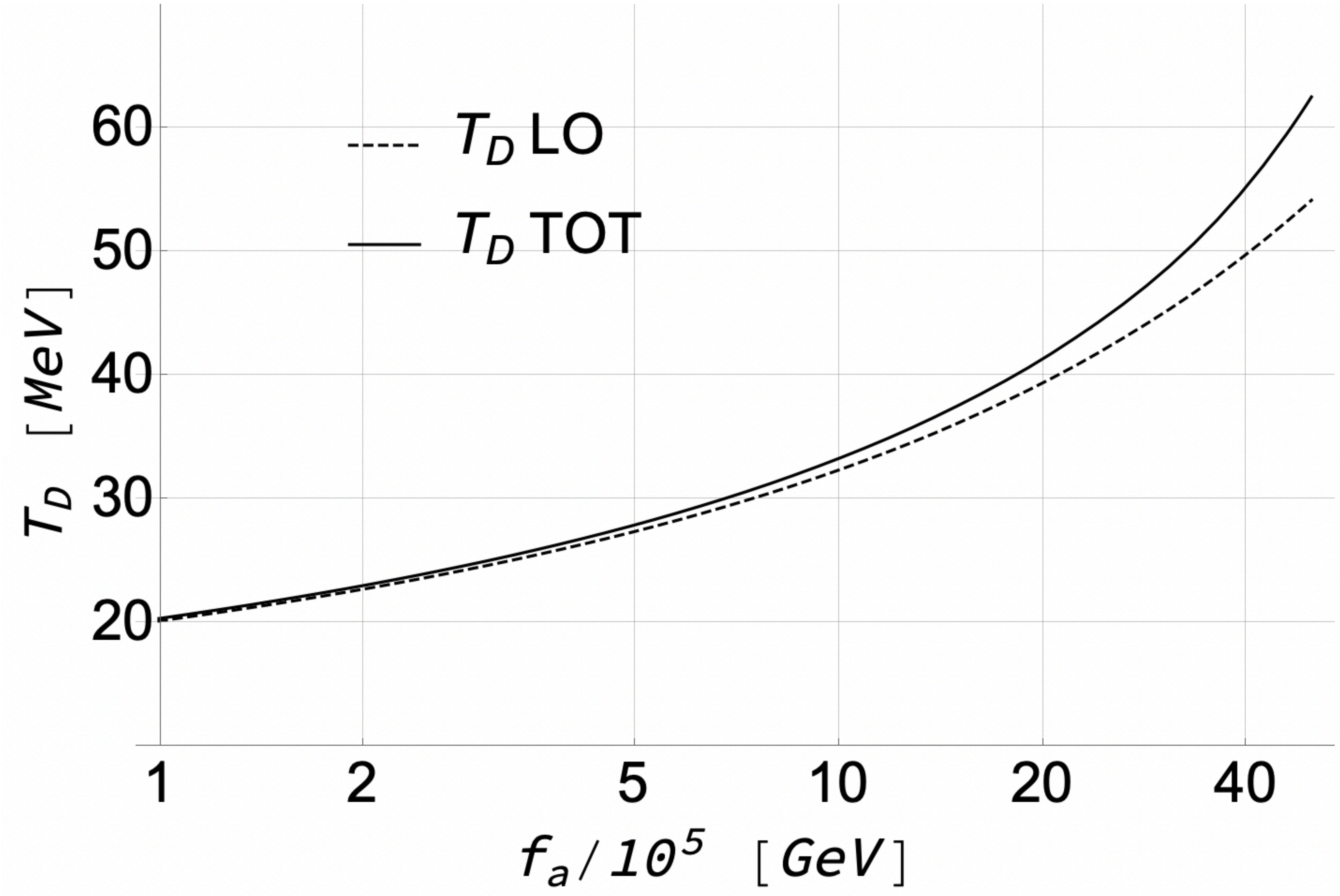
$$\frac{\partial^\mu a}{4f_a}\mathrm{Tr}\left[c_q\sigma^a\right]\left(J_\mu^{a\ (\mathrm{LO})}+J_\mu^{a\ (\mathrm{NLO})}\right)$$

↓

$$J_{A\ (\mathrm{NLO})}^{\mu,a}=+i\frac{l_1}{2}\left\langle\sigma^a\left\{D^\mu U^\dagger,U\right\}\right\rangle\left\langle D_\nu U D^\nu U^\dagger\right\rangle$$
$$+i\frac{l_2}{4}\left\langle\sigma^a\left\{D^\nu U^\dagger,U\right\}\right\rangle\left\langle D^\mu U D_\nu U^\dagger+D_\nu U D^\mu U^\dagger\right\rangle$$
$$-i\frac{l_4}{8}\left\langle\sigma^a\left\{D^\mu U,\chi^\dagger\right\}-\sigma^a\left\{U,D^\mu\chi^\dagger\right\}+\sigma^a\left\{D^\mu\chi,U^\dagger\right\}-\sigma^a\left\{\chi,D^\mu U^\dagger\right\}\right\rangle$$
$$+\frac{l_6}{4}\left\langle f_{\mu\nu}^R\left[\sigma^a,D^\nu U\right]U^\dagger+f_{\mu\nu}^R U\left[D^\nu U^\dagger,\sigma^a\right]+f_{\mu\nu}^L U^\dagger\left[\sigma^a,D^\nu U\right]+f_{\mu\nu}^L\left[D^\nu U^\dagger,\sigma^a\right]U\right\rangle$$

T_D vs f_a

Decoupling temperature for the LO and LO+NLO case, as a function of f_a



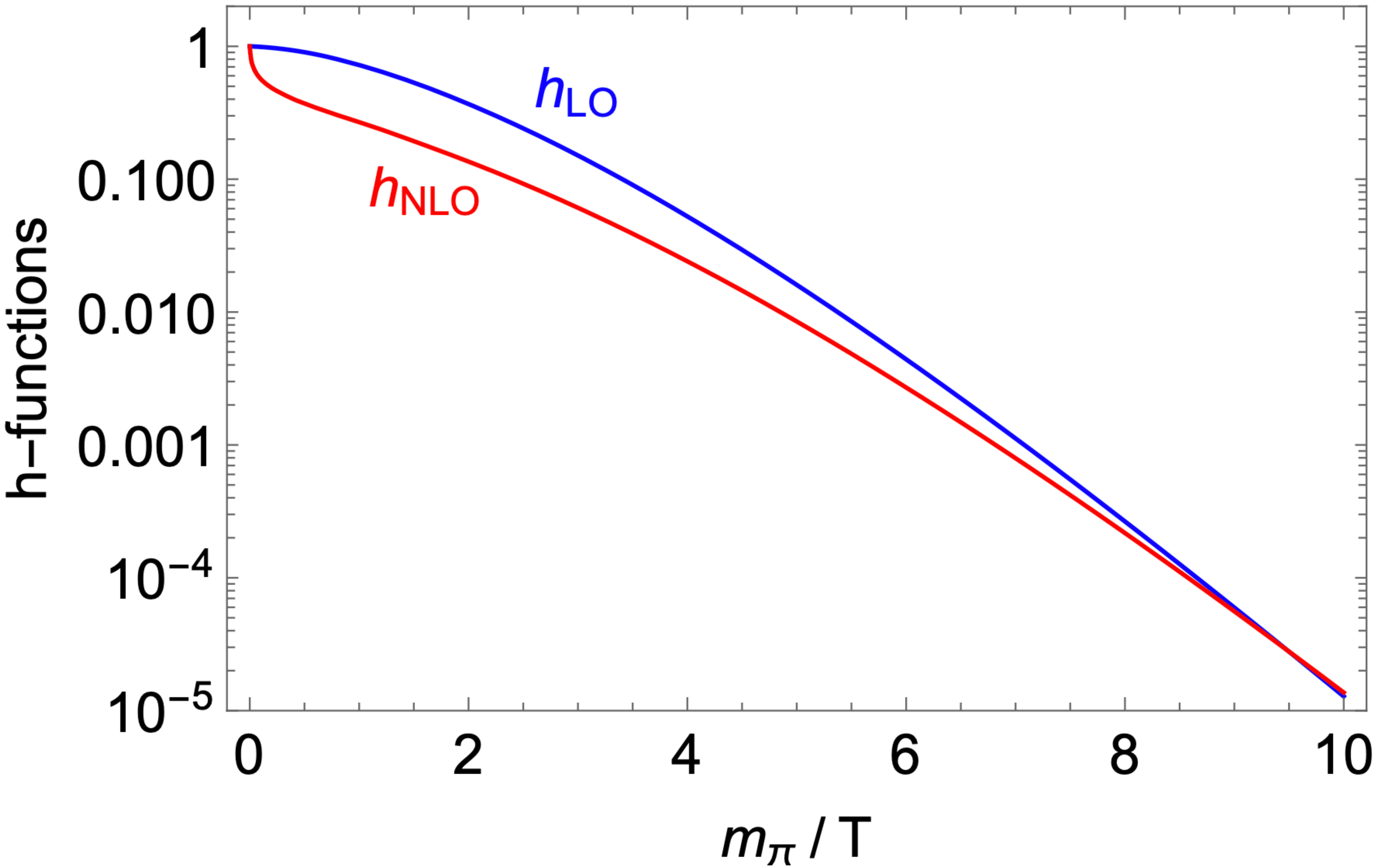
$$T_D < T_\chi$$

if

$$f_a \lesssim 4.9 \times 10^6 \text{ GeV}$$

$$m_a \gtrsim 1.16 \text{ eV}$$

h functions



$\Delta N_{\rm eff}$, the origins

$$\rho = \rho_\gamma + \rho_\nu + \rho_a$$
$$= \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}^{\rm SM} + \frac{1}{2}\left(\frac{T_a}{T_\gamma}\right)^4\right]$$
$$= \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]$$

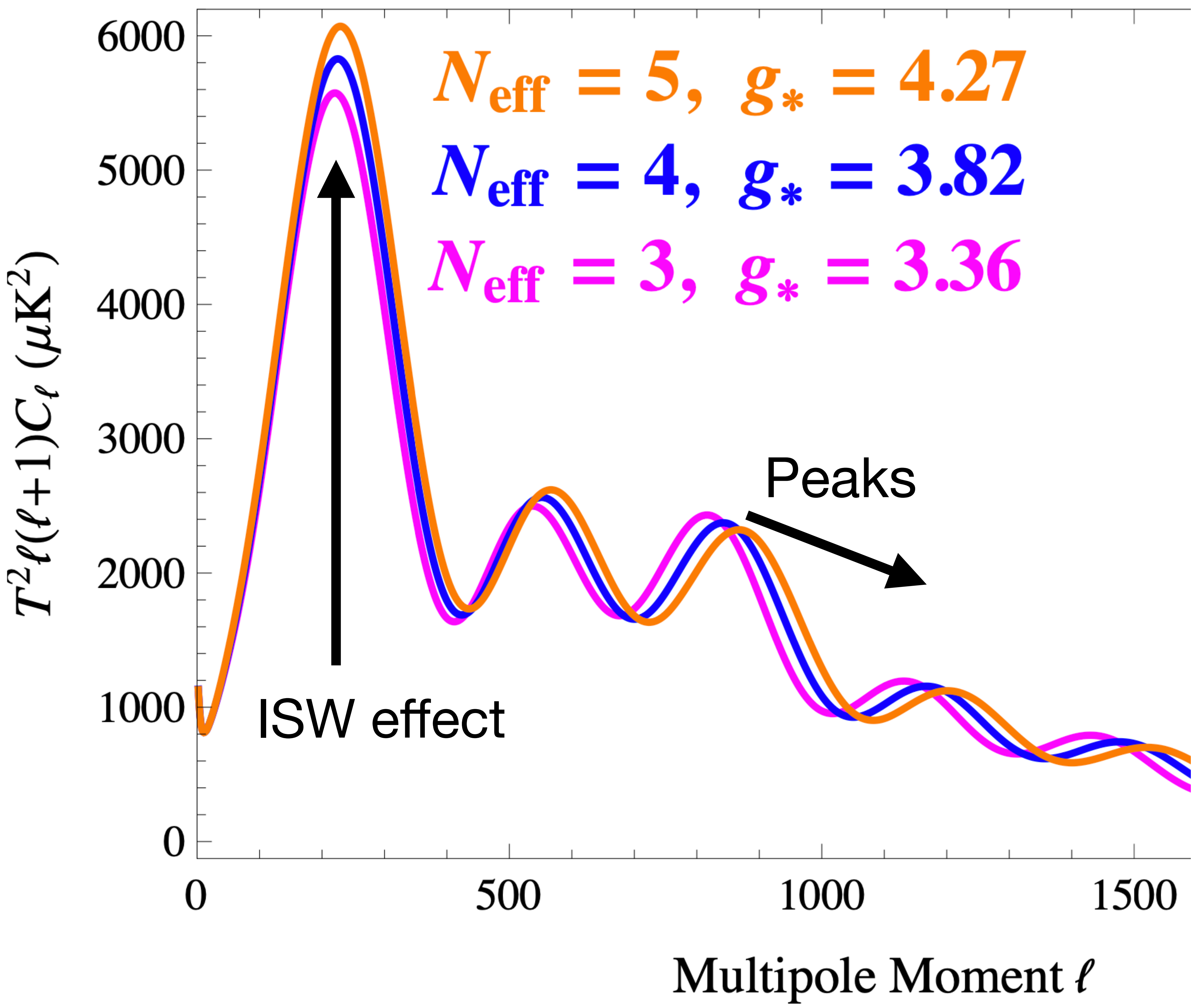
$$\Delta N_{\rm eff} = N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4$$

$$\frac{T_a}{T_\nu} = \left(\frac{43}{4 g_S(T_D)}\right)^{1/3}$$

$$\Delta N_{\rm eff} = 0.027\left(\frac{106.75}{g_S(T_D)}\right)^{4/3}$$

Effects of $N_{\rm eff}$ on the CMB

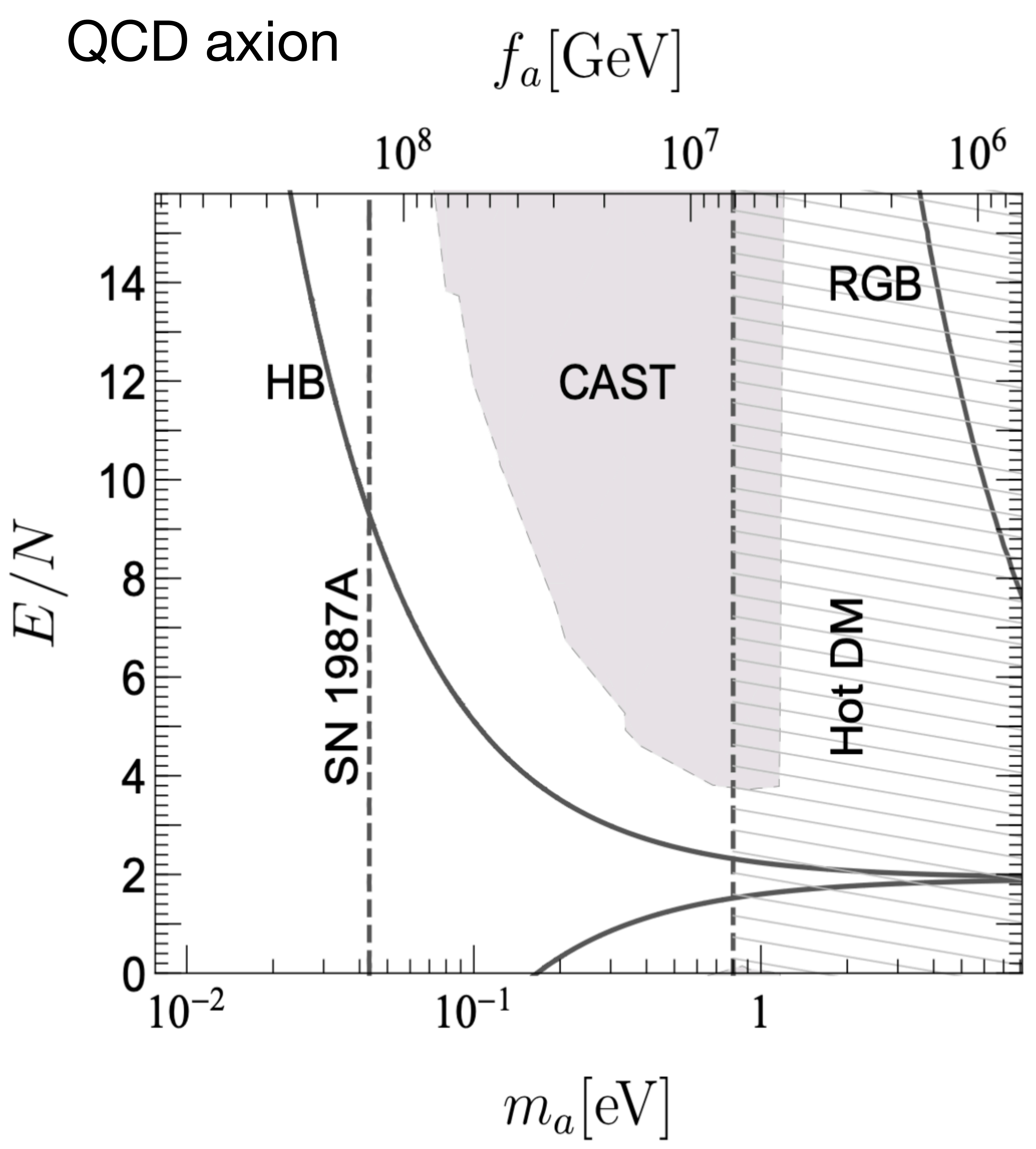
- $N_{\rm eff} \uparrow \Rightarrow H \uparrow$, time for photons diffusion in the plasma decreases, reducing Silk damping and restricting it to higher ℓ. $\ell_{\rm dump} \uparrow$
- $H \uparrow$ Acoustic oscillation length scale decreases, increasing the sound horizon. $\ell_{\rm sound} \uparrow$
- Overall less dumping but more peaks dumped. $H \uparrow \Rightarrow \ell_s / \ell_d \uparrow$
- Also, gravitational red/blue shift increased on 1st peak scales (ISW)



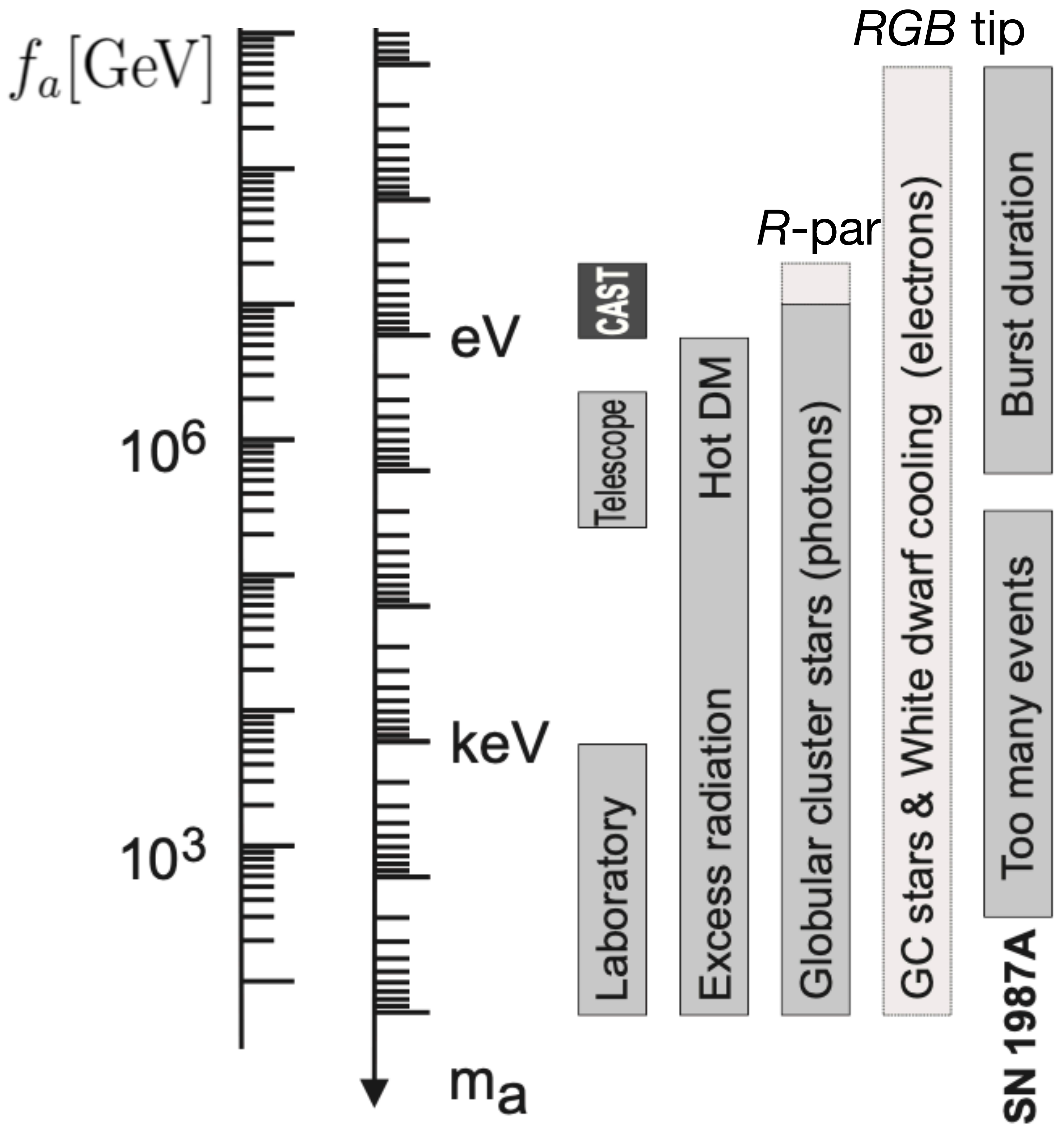
[Brust, Kaplan, Walters, arXiv:1303.5379]

[Silk, *Astrophys.J.* 151 (1968)]
[Sachs, Wolfe, *Astrophys. J.* 147 (1967)]
[Bowen, Hansen, Melchiorri, Silk, Trotta, arXiv: astro-ph/0110636]
[Brust, Kaplan, Walters, arXiv:1303.5379]

ASTRO Bounds



[Di Luzio et al., Phys. Rept. **870** (2020)]



- $g_{ae}^0 = 0$ in KSVZ models
- SN bound not solid from astrophysics
  [Bar, Blum, D'Amico 1907.05020]
- $g_{a\gamma}$ can be accidentally suppressed
  [Di Luzio, Mescia, Nardi, arXiv:1705.05370]