SURVIVAL OF A RUN-AND-TUMBLE PARTICLE IN THE PRESENCE OF A DRIFT

RENCONTRE DES JEUNES PHYSISIEN.NE.S 2021

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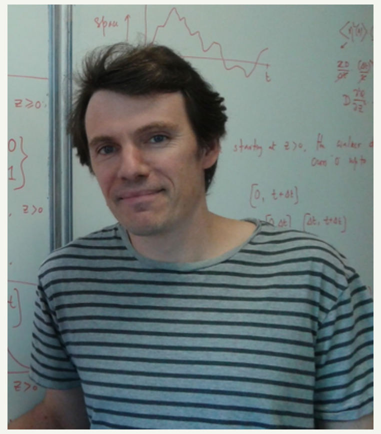
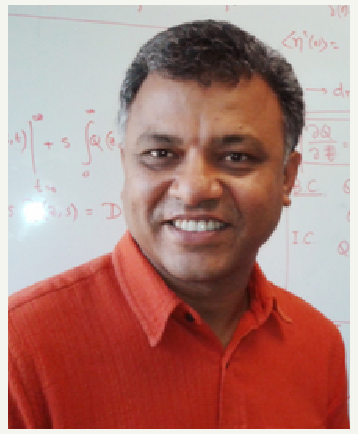
Fonds National de la Recherche Luxembourg

WORK DONE IN COLLABORATION WITH



Grégory Schehr
Laboratoire de Physique Théorique et Hautes Energies.



Satya N. Majumdar
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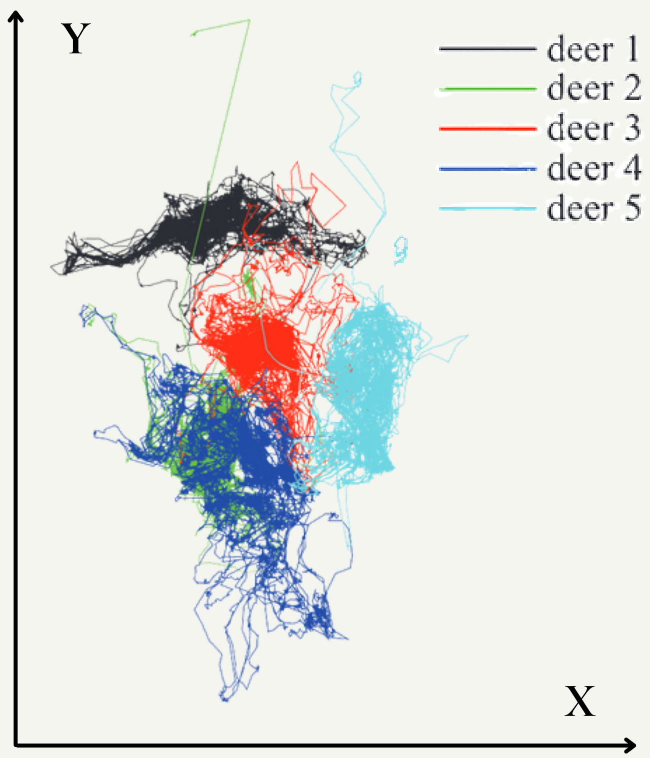
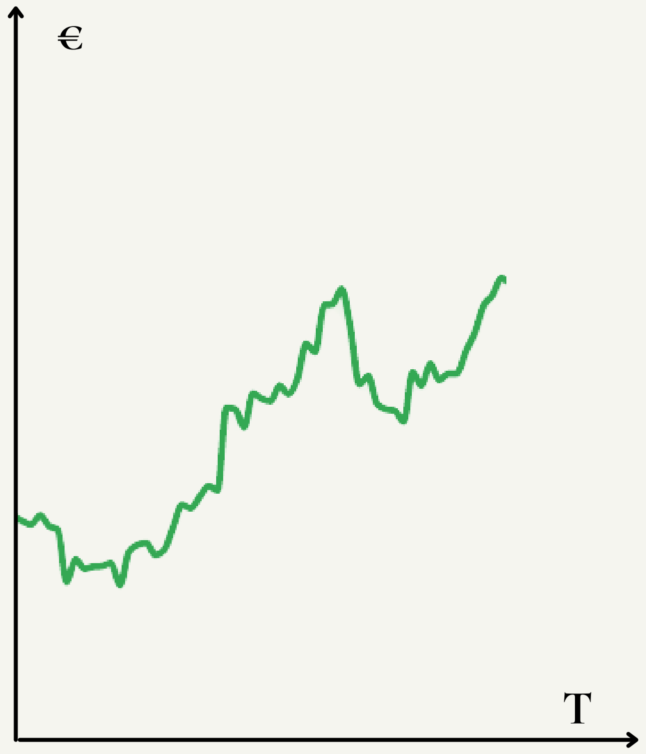
OUTLINE

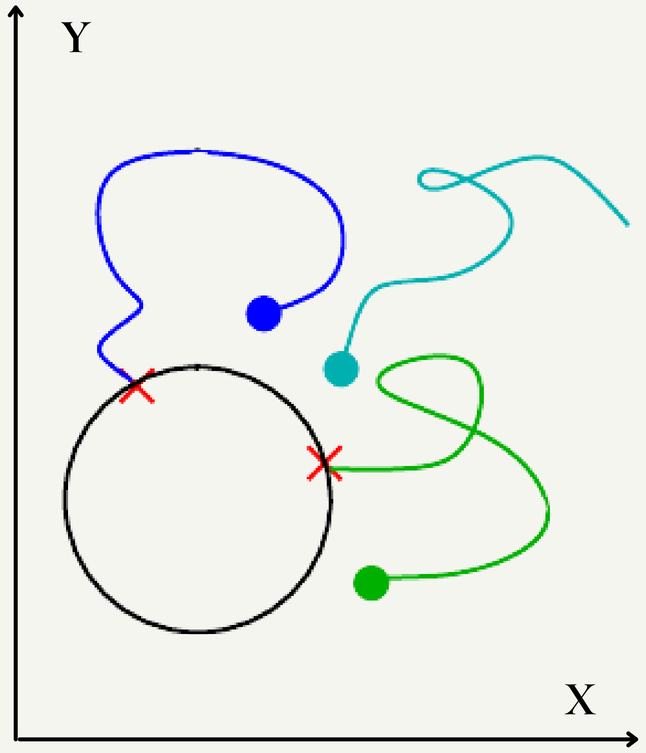
SURVIVAL PROBABILITY MOTIVATIONS



ANIMAL FORAGING[1]



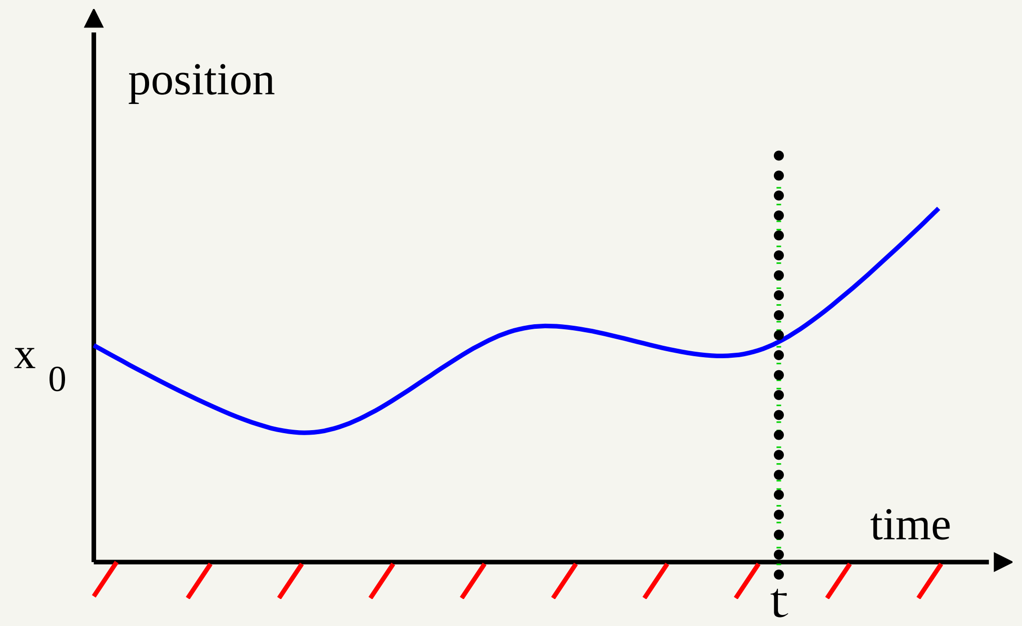
STOCK MARKETS [2]



CHEMICAL REATIONS

[1] Reproduced from Berthelot G, Saïd S & Bansaye V 2020 *bioRxiv.*
[2] Bitcoin price in EUR over the last months from Google Finance.

CLASSICAL RESULTS ON BROWNIAN MOTION



Survival probability $S(x_o, t)$
Probability that the walker did not cross the origin up to time t given that it started at x_o.

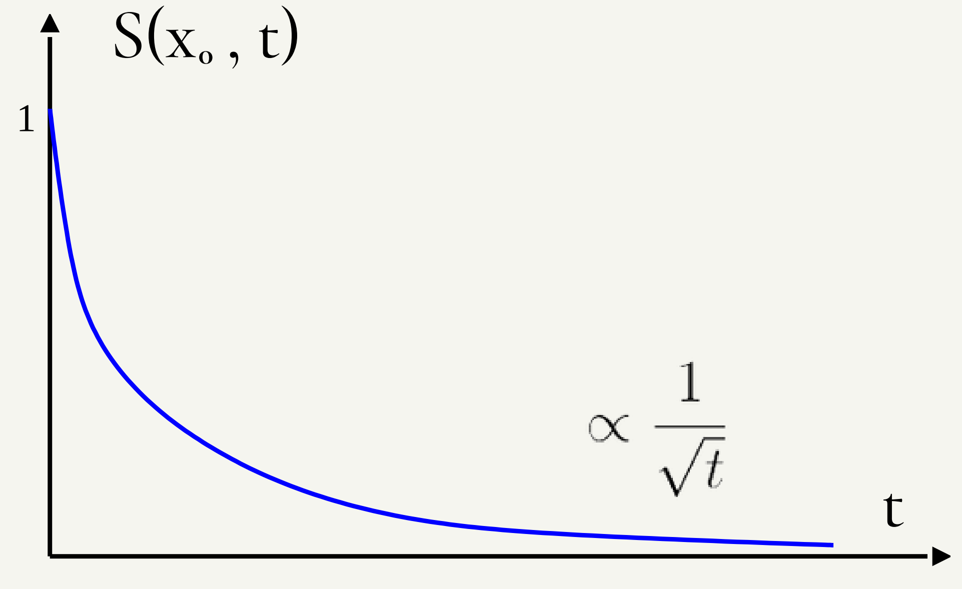
White noise

$$\dot{x}(t) = \sqrt{2\,D}\,\eta(t)$$

Diffusion coefficient

Brownian motion
The particle is driven by uncorrelated with noise. The process is **Markovian**.

$$\langle \eta(t)\,\eta(t') \rangle = \delta(t - t')$$

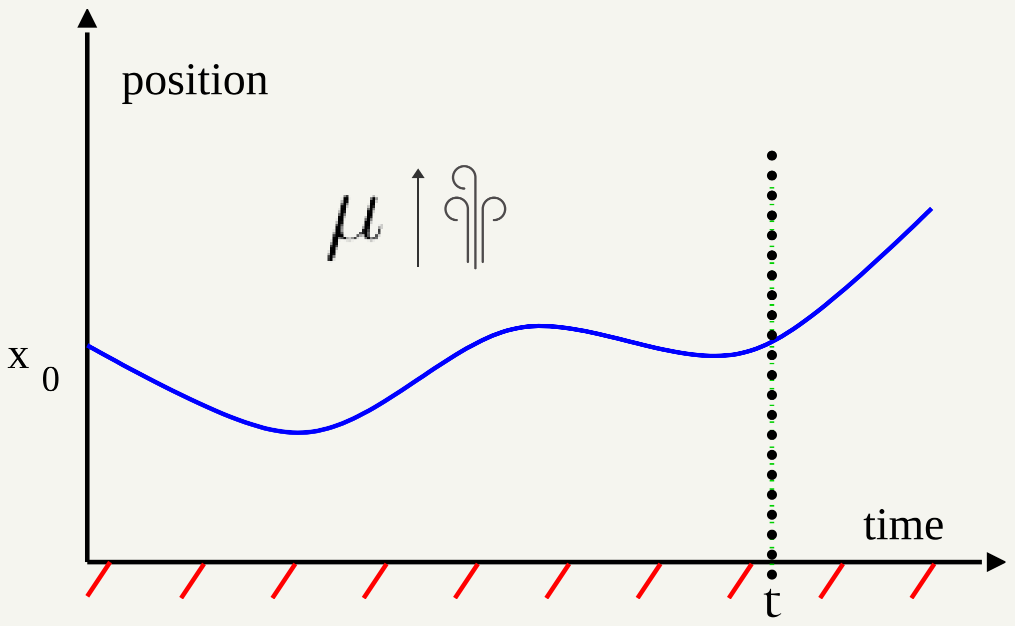


Brownian motion eventually crosses the origin
The survival probability decays as a power law (see [1,2] for reviews).

$$S(x_0, t) = \mathrm{erf}\left(\frac{x_0}{\sqrt{4\,D\,t}} \right)$$

[1] Bray A J, Majumdar S N & Schehr G 2013 Persistence and first-passage properties in nonequilibrium systems Advances in Physics **62** 225-361.
[2] Redner S 2001 *A guide to first-passage processes* Cambridge University Press.

CLASSICAL RESULTS ON BROWNIAN MOTION

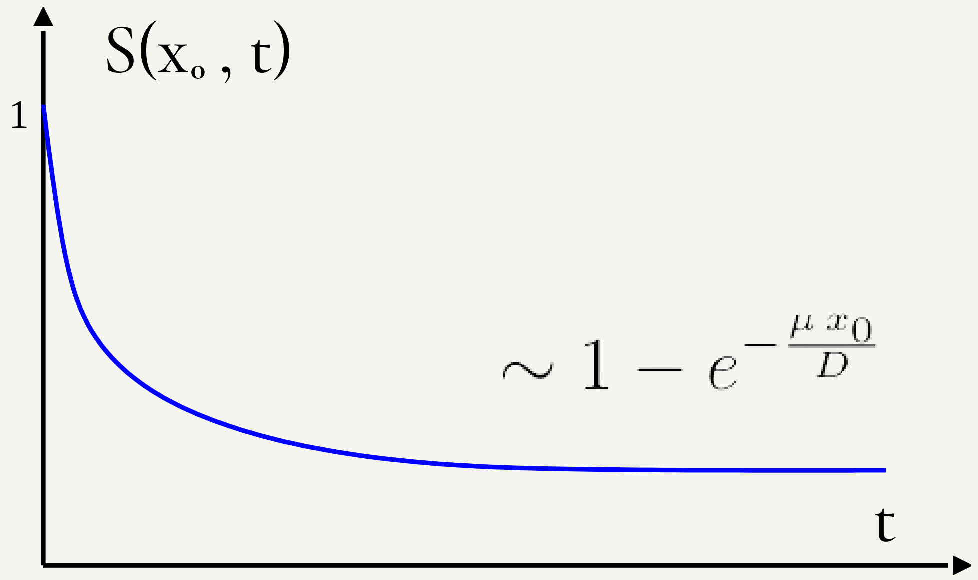


Survival probability $S(x_o, t)$
Probability that the walker did not cross the origin up to time t given that it started at x_o.

Constant positive drift

$$\dot{x}(t) = \sqrt{2\,D}\,\eta(t) + \mu$$

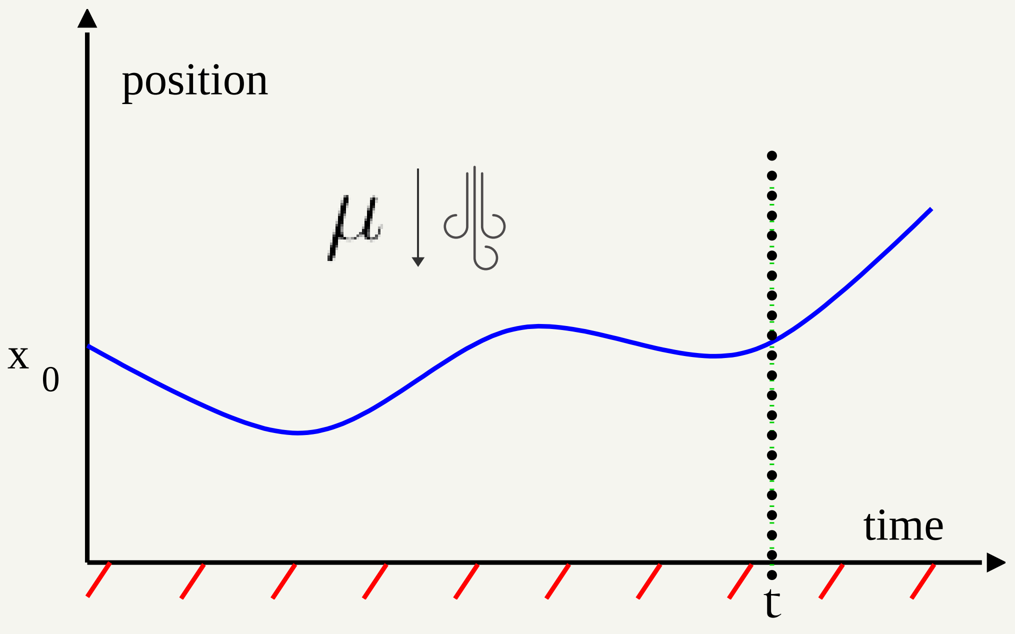
Brownian motion with a positive drift
A drift is added to the white noise.



The particle has a finite escape probability! (see [1,2] for reviews).

[1] Bray A J, Majumdar S N & Schehr G 2013 Persistence and first-passage properties in nonequilibrium systems Advances in Physics **62** 225-361.
[2] Redner S 2001 *A guide to first-passage processes* Cambridge University Press.

CLASSICAL RESULTS ON BROWNIAN MOTION

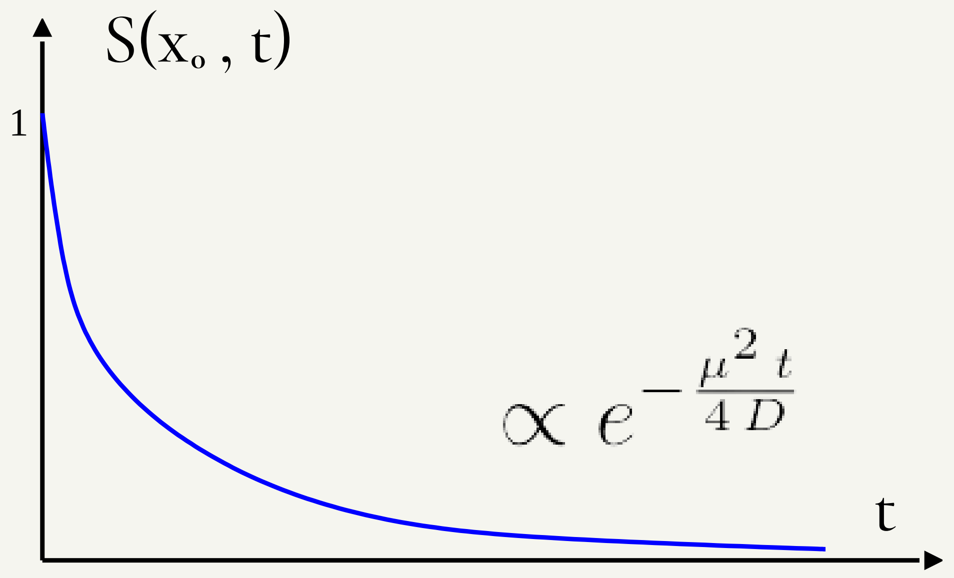


Survival probability $S(x_o, t)$
Probability that the walker did not cross the origin up to time t given that it started at x_o.

Constant **negative** drift

$$\dot{x}(t) = \sqrt{2\,D}\,\eta(t) - \mu$$

Brownian motion with a negative drift
A drift is added to the white noise.

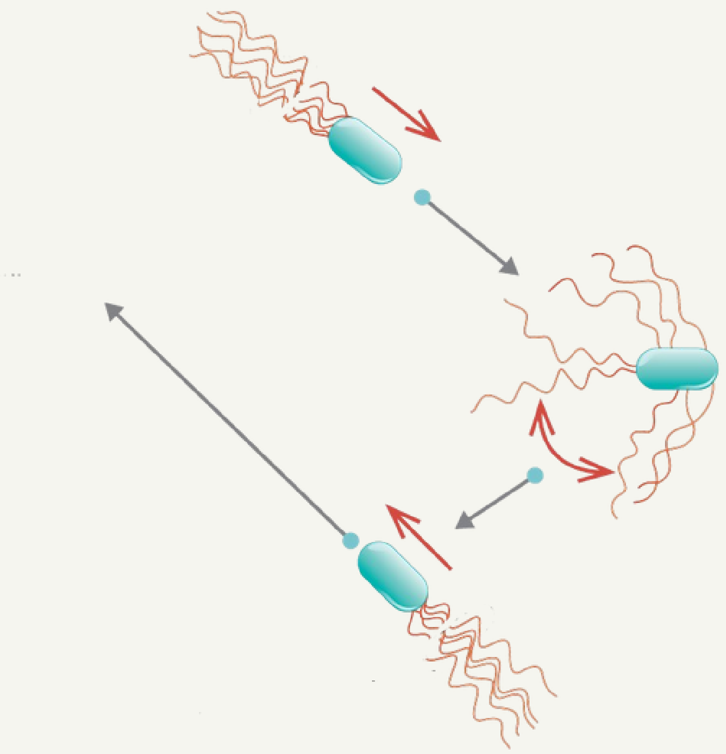


The particle eventually crosses the origin.
The survival probability decays exponentially (see [1,2] for reviews).

[1] Bray A J, Majumdar S N & Schehr G 2013 Persistence and first-passage properties in nonequilibrium systems Advances in Physics **62** 225-361.
[2] Redner S 2001 *A guide to first-passage processes* Cambridge University Press.

IS EVERYTHING CLEAR?

We will now turn to recent results on the survival probability of the run-and-tumble particle.

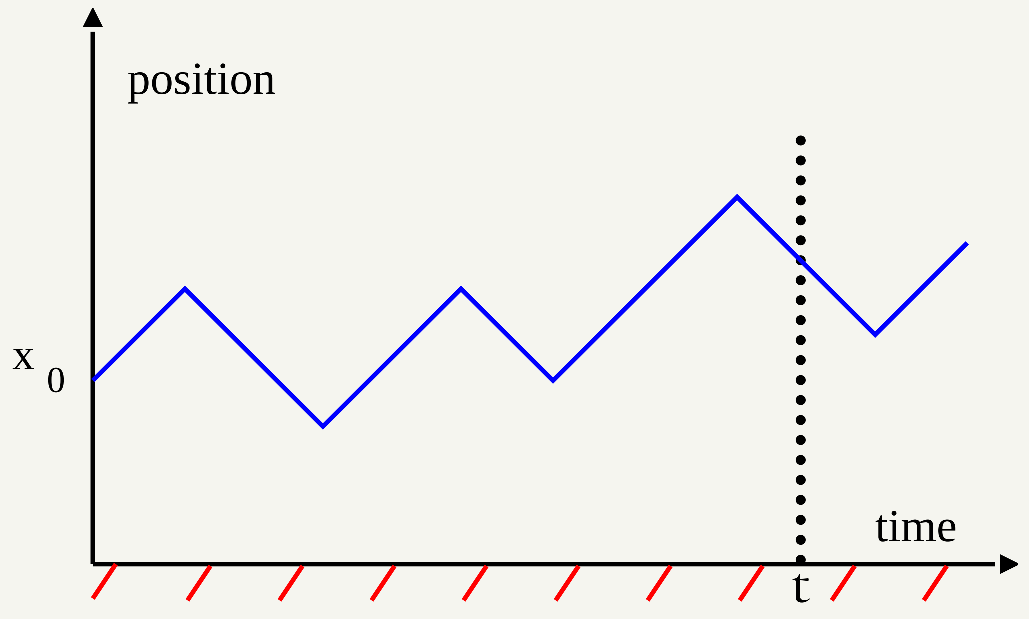


RUN-AND-TUMBLE PARTICLE [1-3]

[1] Furth R 1917 *Annals of Physics* **53,177**.
[2] Kac M 1974 T*he Rocky Mountain Journal of Mathematics* **497-509**.
[3] Marchetti M et al 2013 *Reviews of Modern Physics* **1143**.

Figure from https://opentextbc.ca/microbiologyopenstax/chapter/unique-characteristics-of-prokaryotic-cells./

RECENT RESULTS ON THE RUN-AND-TUMBLE PARTICLE



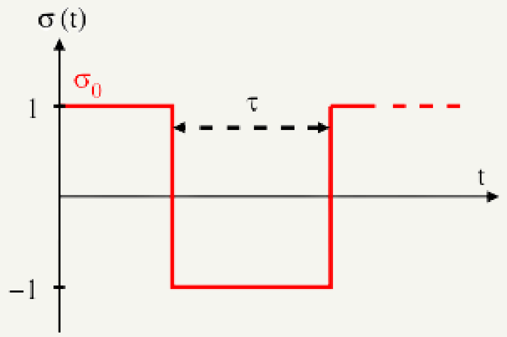
Survival probability $S(x_o, t)$

Probability that the walker did not cross the origin up to time t given that it started at x_o.

Velocity of the particle

$$\dot{x}(t) = v_0\,\sigma(t)$$

Telegraphic noise



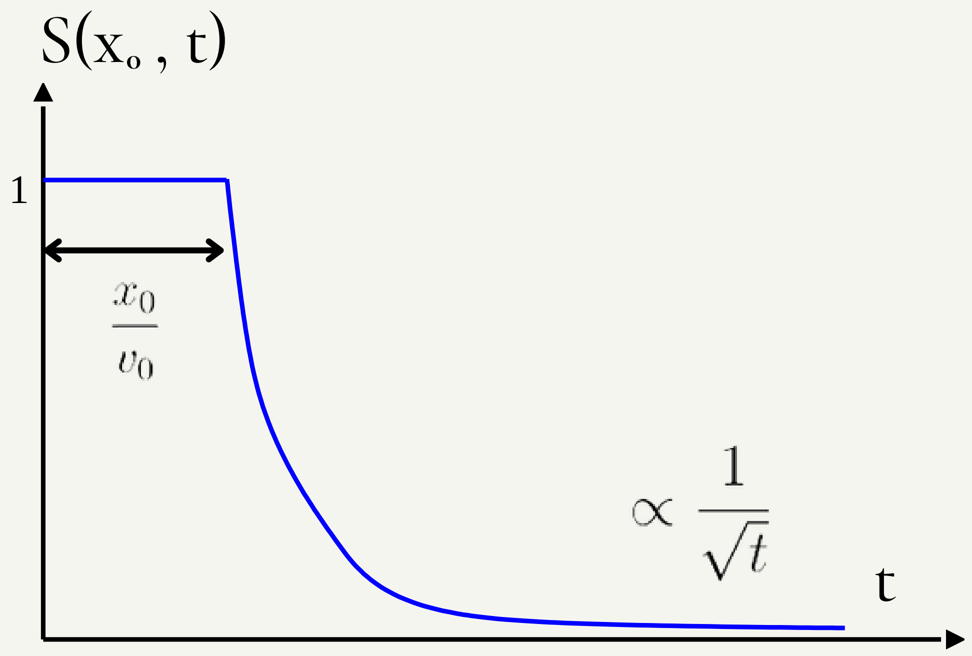
Run-and-tumble particle

The particle is driven by a correlated noise.

The process is **non-Markovian.**

$$\langle \sigma(t)\sigma(t') \rangle = e^{-\gamma(t'-t)}$$

γ^{-1} is the persistence time



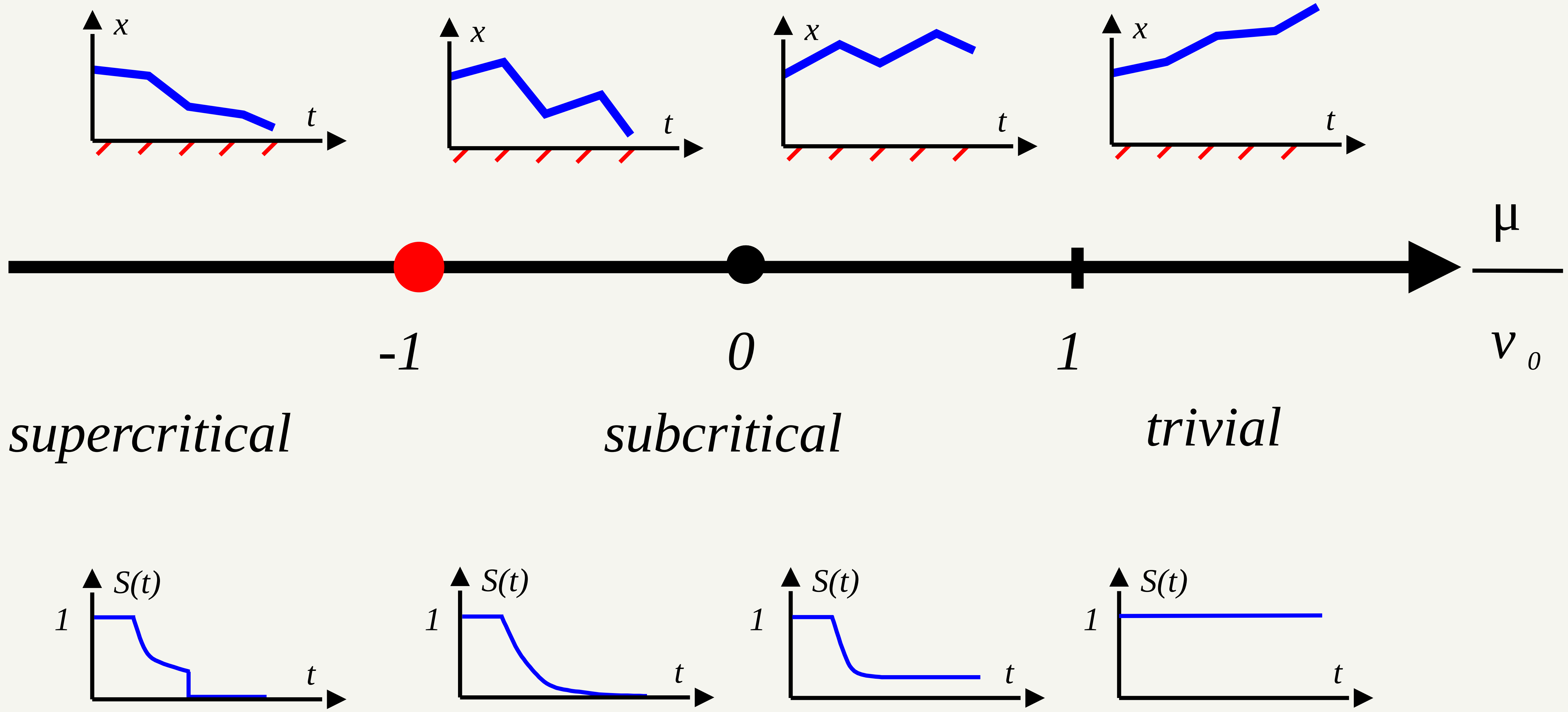
The particle eventually crosses the origin [1].

The particle needs a minimum time to reach the origin. Contrary to Brownian motion, it can survive even if it starts at the origin [2].

[1] Malakar K et al 2018 *Journal of Statistical Mechanics: Theory and Experiment* **4** 043215.
[2] Mori F, Le Doussal P, Majumdar S N & Schehr G 2020 *Physical review letters* **124** 090603.

NEW RESULTS ON THE RUN-AND-TUMBLE PARTICLE WITH DRIFT

$$\dot{x}(t) = v_0\, \sigma(t) + \mu$$



$$S_-(x,t) = \begin{cases} 1\,, & t < t_m\,, \\ 1 - \mathrm{e}^{-t_m} - \int_{t_m}^{t} dt'\ \mathrm{e}^{-t'} \frac{x}{h(t',x)}\, I_1\left[h(t',x)\right]\,, & t \geq t_m\,, \end{cases}$$

$$t_m = \frac{x}{1-\mu}\,,$$
$$f(t,x) = t\,(1-\mu) - x\,,$$
$$g(t,x) = t\,(1+\mu) + x\,,$$
$$h(t,x) = \sqrt{f(t,x)\, g(t,x)}\,.$$

[1] De Bruyne B, Majumdar S N & Schehr G 2021 *arXiv:2101.11895.*

CONCLUSION

Survival probability of a run-and-tumble particle in the presence of a drift

Simple model that encapsulate 3 levels of difficulty:

- Non-Markovian,
- Drift,
- Survival probability.

Exactly solvable model

The model is exactly solvable and has an unexpected rich phase diagram. The model will serve as a benchmark for more realistic models of the run-and-tumble particle.

Next steps

The model discussed here is at single-particle level. It would be very interesting to study a multi-particle system with interactions.

THANK YOU FOR YOUR ATTENTION.

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