

non-linear QCD meets data

*:: from proton to nuclei ::*

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[AAM(Q)S]

PRD 80:034031, 2009  
DIS2009 Proceedings

Workshop on Nuclear Parton Distribution Functions  
– 22 Feb 2010 –

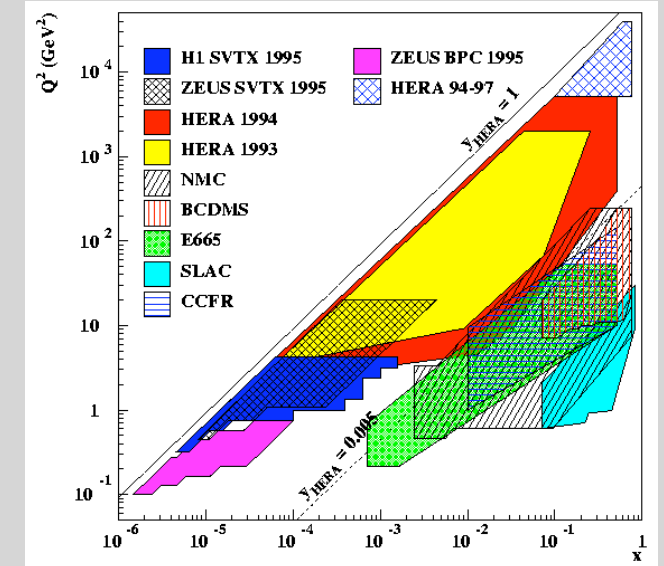
# ⌚ parton distribution functions [DGLAP]

- ‘standard’ approach
  - *collinear* factorization allows for identification of universal [process independent] pdfs
  - pdfs extracted via global fits to data
    - ↪ parameterized non-perturbative initial condition
  - systematically improvable in perturbation theory [NLO, NNLO,...]

# proton vs. nuclear pdfs

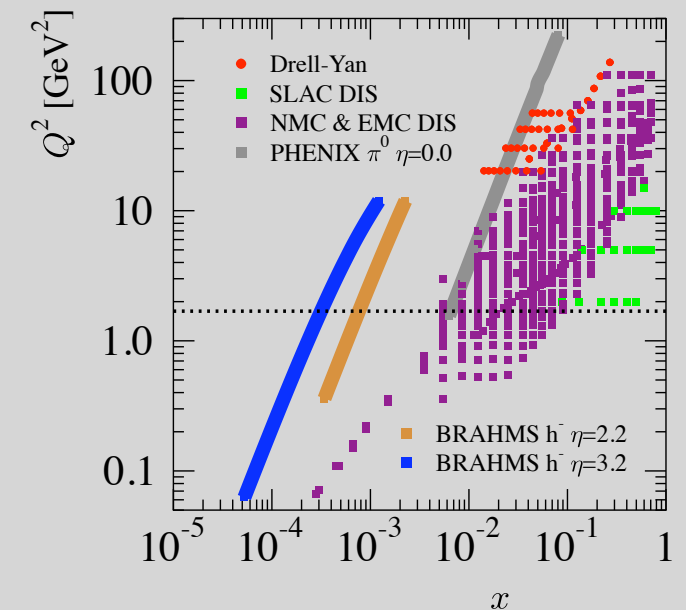
- *proton case*

- collinear factorizability proven
- wealth of data (DIS, DY, jets)
  - ↪ very reliable pdfs in 'data covered' kinematical range



- *nuclear case*

- collinear factorizability is a working assumption
  - ↪ for testability, see Paloma's talk
- relatively scarce data
- standardly encoded as nuclear modification of proton pdfs [inherits proton pdf uncertainties]



$$f_{i/A}(x, Q^2) = R_i^A(x, Q^2) f_{i/p}(x, Q^2)$$

pdf of parton in  
nucleon inside nucleus

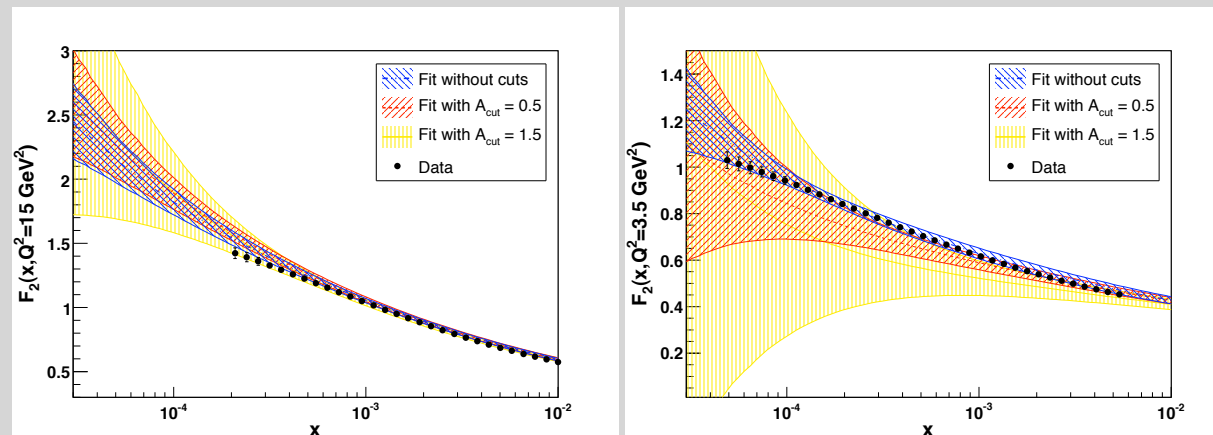
nuclear modification

pdf of parton in  
free nucleon

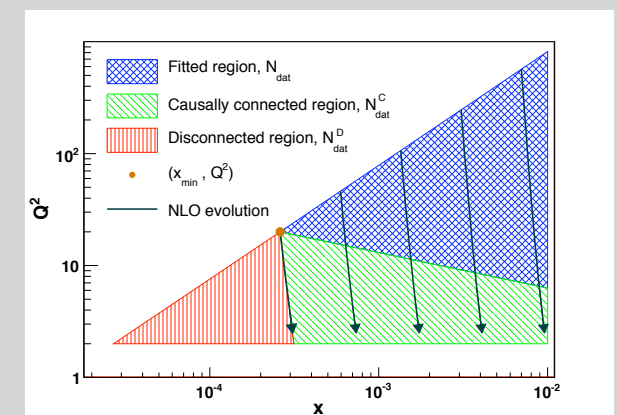
# ↻ small-x and DGLAP

- at small-x *novel* [non-linear parton density induced] effects come into play

- deviations from NLO-DGLAP identified in [of all places...] HERA data for  $F_2$  Caola, Forte, Rojo 0910.3143 [hep-ph]



:: not *NLLO*, not only heavy quark, ... ::



:: initial condition independent statement ::

—○ BFKL resummed DGLAP

↪ accounts for small-x evolution linearly

□ an interim fix...

—○ ***DGLAP is not a predictive tool at small-x***

—○ nuclei probe smaller x at lower energy [as compared to proton]

# ↻ non-linear QCD approach

- $k_t$  factorization + dipole formulation of high energy QCD
  - limited by  $k_t$  factorizability and its compatibility with dipole formulation
    - ↪ no access to ‘standard’ pdfs
      - rather, unintegrated gluon distribution is the relevant object
  - recall François’ talk for nuclear case
- first principle QCD calculation of  $x$ -evolution of dipole scattering amplitude
  - running coupling BK
    - ↪ best, numerically implementable, incarnation of non-linear QCD
    - ↪ unlike most [phenomenological] ‘dipole models’
  - ↪ first proton then nuclei

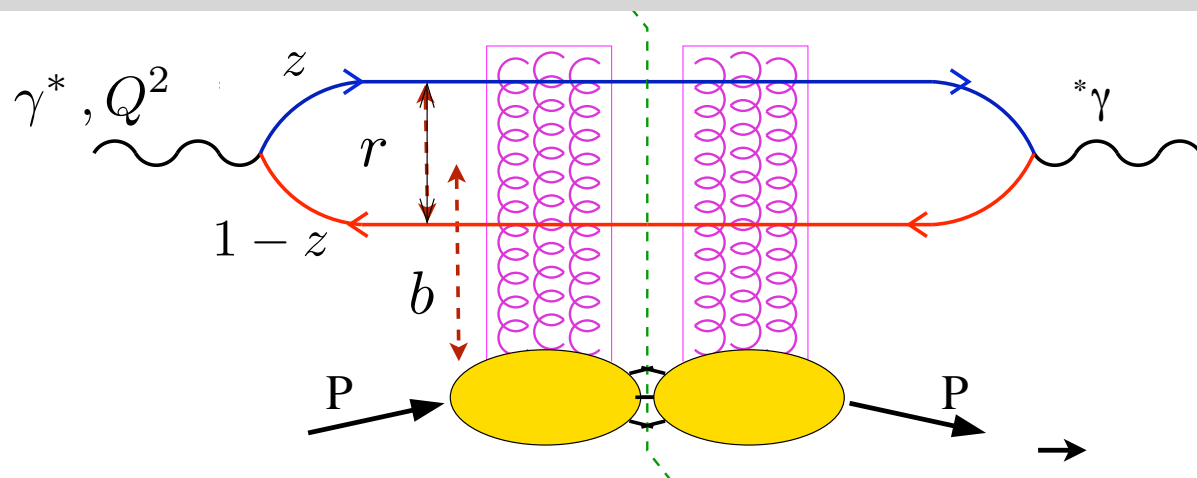
# dipole QCD [in DIS]

- at high energy [ $x \ll 1$ ] the coherence length of the virtual photon fluctuation

$$l_c \sim (2m_N x)^{-1} \simeq 0.1/x \text{ fm} \gg R_N$$

- total virtual photon-proton cross section can be factorized as

$$\sigma_{T,L}(x, Q^2) = \int_0^1 dz \int d\mathbf{b} d\mathbf{r} \underbrace{|\Psi_{T,L}(z, Q^2, \mathbf{r})|^2}_{\text{QED calculation}} \underbrace{\mathcal{N}(\mathbf{b}, \mathbf{r}, x)}_{\text{[imaginary part of] dipole-target scattering amplitude}}$$




QED calculation

[imaginary part of]  
dipole-target scattering amplitude  
:: all QCD information  
:: all  $x$  dependence  
:: non-perturbative, but  $x$ -evolution  
computable from first principles

# 🔄 impact parameter dependence

- b-dependence governed by long-distance non-perturbative physics
- AAMS 1.0 resorts to translational invariance approximation
  - proton homogeneous in transverse plane

$$\sigma_{T,L}(x, Q^2) = \int_0^1 dz \int d\mathbf{b} d\mathbf{r} |\Psi_{T,L}(z, Q^2, \mathbf{r})|^2 \mathcal{N}(\mathbf{b}, \mathbf{r}, x)$$


$$\sigma_{T,L}(x, Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r} |\Psi_{T,L}(z, Q^2, \mathbf{r})|^2 \mathcal{N}(r, Y)$$



‘b-integration’  
fit parameter

:: if factorized structure [of x, r and b dependence]  
unchanged by evolution, then related to  
t-dependence in diffractive events

- exclusive observables require more sophisticated treatment of b-dependence

# ↻ from A to B

- want the best possible, numerically tractable, incarnation of non-linear evolution
  - with Dense-Dense effects and NLO
    - ↪ RFT-QCD contains all Dense-Dense effects
      - no known strategy for numerical implementation
      - NLO [running coupling] should trump Dense-Dense [toy model]  
*[Dumitru, Iancu, Portugal, Soyez and Triantafyllopoulos, JHEP 0708:062, 2007]*
- ‘safely’ neglect Dense-Dense if NLO formulation available
  - B-JIMWLK
    - ↪ no Dense-Dense, no NLO, numerically challenging ...
    - ↪ but BK [large N, mean field] solutions deviate only 0.1% from full B-JIMWLK  
*[Kovchegov, Kuokannen, Rummukainen, Weigert, NPA 823, 47 (2009)]*
- ‘safely’ replace full B-JIMWLK by BK
  - LO-BK not consistent with data [unless coupling very small]
  - NLO-BK computed *[Balitsky, Chirilli, Kovchegov, Weigert]*
  - running coupling part numerically tractable

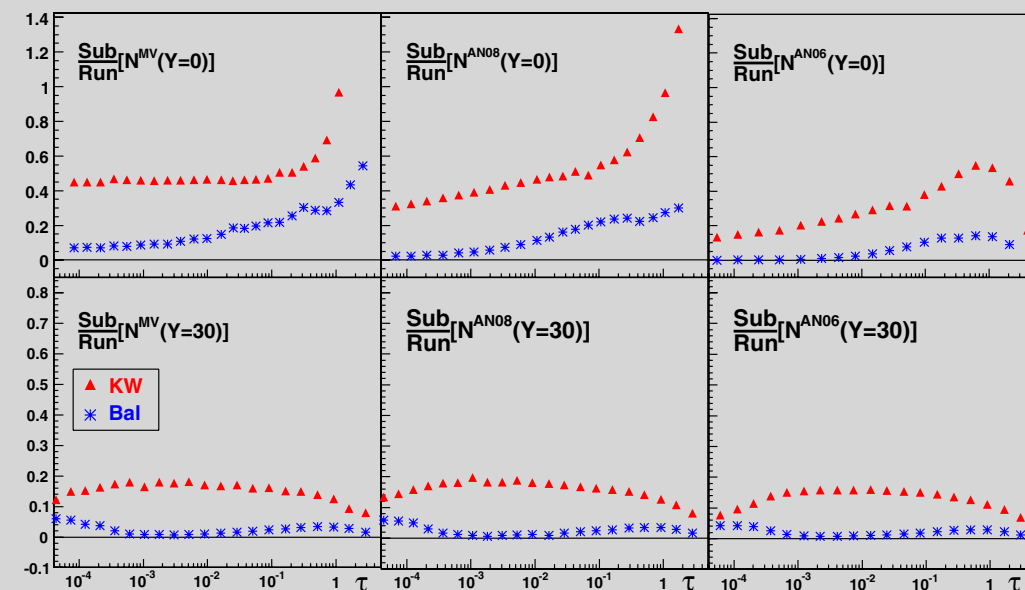


# on why B is B'

- NLO-BK = all orders in  $\alpha_s N_f$  + other
  - all orders in  $\alpha_s N_f$  = rc + subtraction

$$\frac{\partial \mathcal{N}(r, Y)}{\partial Y} = \mathcal{R}[\mathcal{N}] - \mathcal{S}[\mathcal{N}]$$

- subtraction piece numerically demanding
  - ↪ running coupling contribution dominant over conformal piece in  $\alpha_s N_f$  piece



[Albacete, Kovchegov]

- scheme dependent definition of 'subtraction' piece
  - Balitsky's (Bal) scheme minimizes 'subtraction' contribution
- [other] yet to be numerically computed :: challenging ::

# ⌚ BK evolution with running coupling [Bal scheme]

$$\frac{\partial \mathcal{N}(r, Y)}{\partial Y} = \int d\mathbf{r}_1 K^{\text{Bal}}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) [\mathcal{N}(r_1, Y) + \mathcal{N}(r_2, Y) - \mathcal{N}(r, Y) - \mathcal{N}(r_1, Y) \mathcal{N}(r_2, Y)]$$

modified kernel

same structure as LO-BK

$$K^{\text{Bal}}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

$$\Lambda_{QCD} = 0.241 \text{ GeV}$$

$$\text{---} \bigcirc \quad r < r_{fr}, \quad \alpha_s(r^2) = \frac{12\pi}{(11N_c - 2N_f) \ln \left( \frac{4C^2}{r^2 \Lambda_{QCD}^2} \right)}$$

$$\alpha_s(M_Z) = 0.1176$$

$$\text{---} \bigcirc \quad r > r_{fr}, \quad \alpha_s(r_{fr}^2) \equiv \alpha_{fr} = 0.7$$

regulated IR behaviour

$C^2$  —● uncertainty in FT from momentum to coordinate space  
:: fit parameter

# ↻ initial condition[s]

- 2 families of initial conditions
  - generalized with anomalous dimension GBW and MV forms

$$\mathcal{N}^{GBW}(r, Y=0) = 1 - \exp \left[ - \left( \frac{r^2 Q_{s0}^2}{4} \right)^\gamma \right]$$

$$\mathcal{N}^{MV}(r, Y=0) = 1 - \exp \left[ - \left( \frac{r^2 Q_{s0}^2}{4} \right)^\gamma \ln \left( \frac{1}{r \Lambda_{QCD}} + e \right) \right]$$

- differ in UV behaviour

- fit parameters

↪ initial saturation scale  $Q_{s0}^2$

↪ anomalous dimension  $\gamma$

- anomalous dimension in GBW form set to one after initial tests

# data

- all available  $F_2(x, Q^2)$  data [before 'new' combined H1/ZEUS data]
  - with  $x \leq 10^{-2}$
  - no cut on  $Q^2$   $:: 0.045 \text{ GeV}^2 \leq Q^2 \leq 800 \text{ GeV}^2$
- no  $F_L$  data included, but shown to be consistent with fit results
- 847 data points
  - statistical and systematic uncertainties added in quadrature
  - normalization uncertainties not considered
- redefinition of Bjorken  $x$  as to go smoothly to photoproduction

$$\tilde{x} = x \left( 1 + \frac{4m_f^2}{Q^2} \right) \quad \text{with } m_f = 0.14 \text{ GeV}, \text{ only light quarks}$$

# summary [for fit]

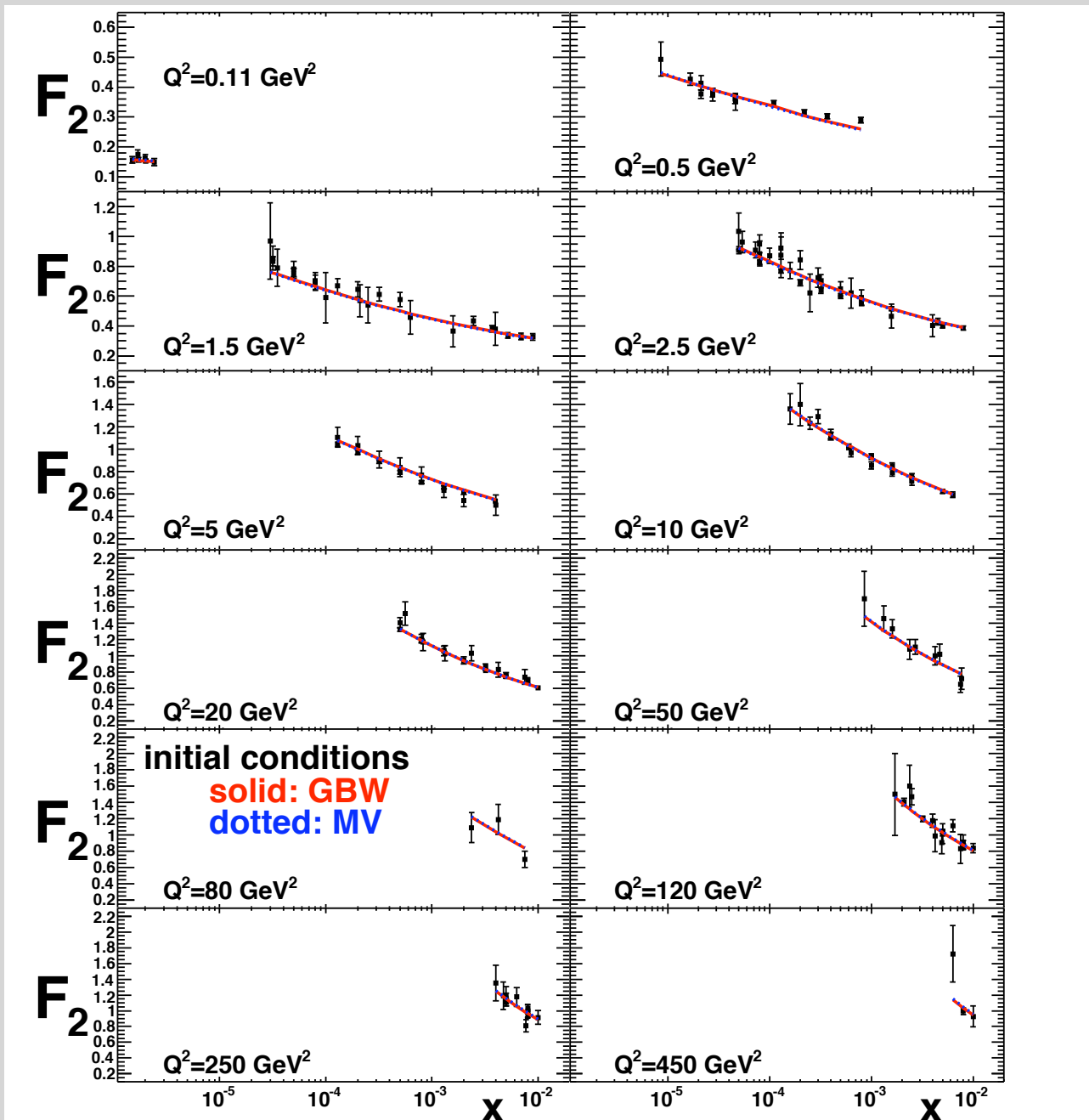
- $F_2$  calculated from

$$F_2(x, Q^2) = \frac{Q^2}{4 \pi^2 \alpha_{em}} (\sigma_T + \sigma_L)$$

$$\sigma_{T,L}(x, Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r} |\Psi_{T,L}(z, Q^2, \mathbf{r})|^2 \mathcal{N}(r, Y)$$

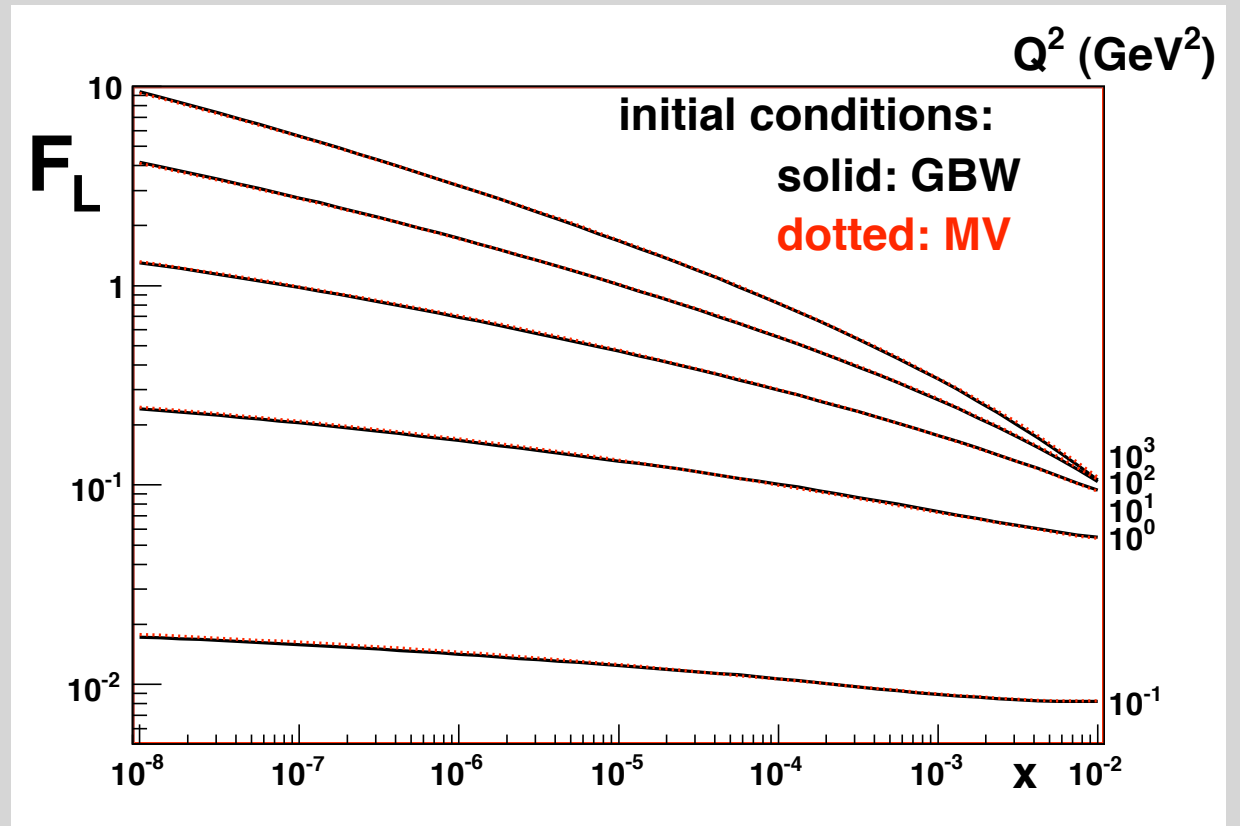
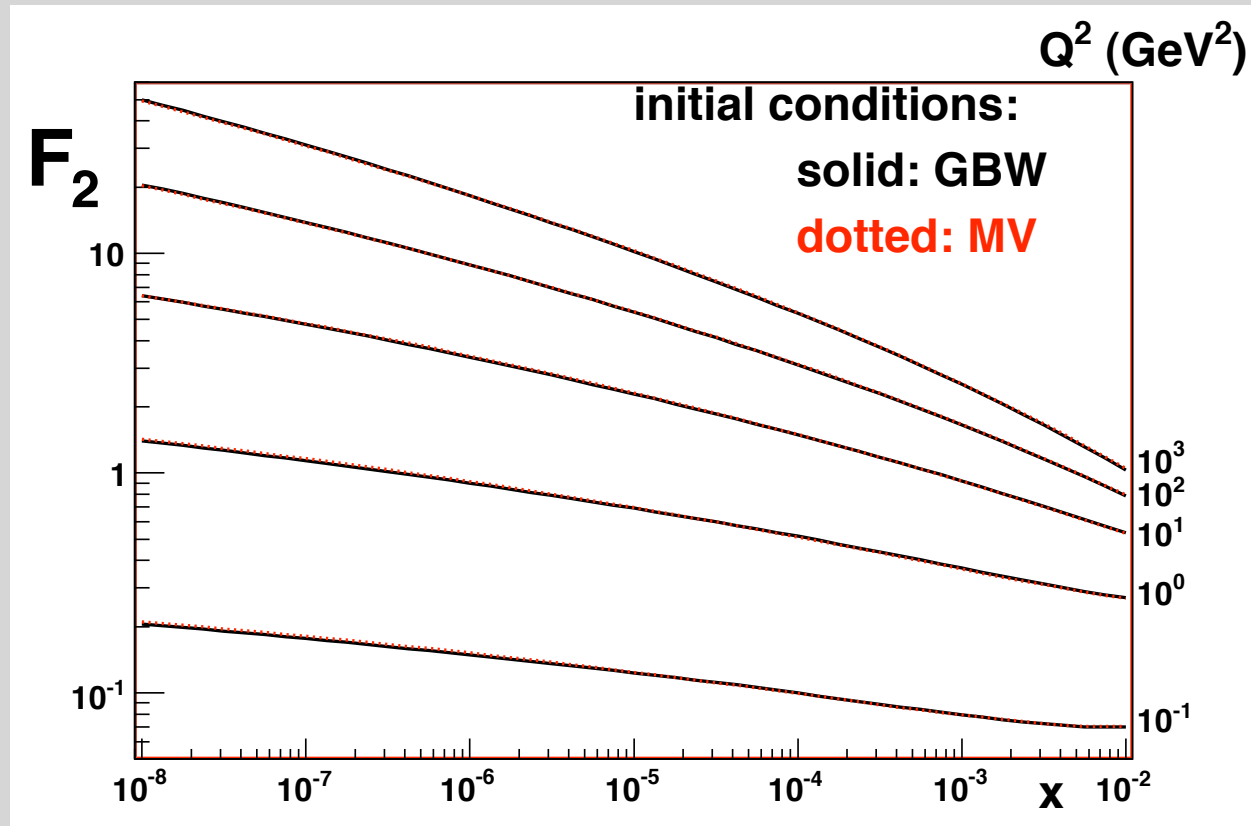
- 4 fit parameters [3 for GBW ic]
  - total normalization of cross section [b-integration]
  - IR uncertainty in running coupling [from FT]
  - initial saturation scale [in ic]
  - anomalous dimension [in ic] :: MV only

# fit results



Initial condition	$\sigma_0$ (mb)	$Q_{s0}^2$ ( $\text{GeV}^2$ )	$C^2$	$\gamma$	$\chi^2/\text{d.o.f.}$
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075

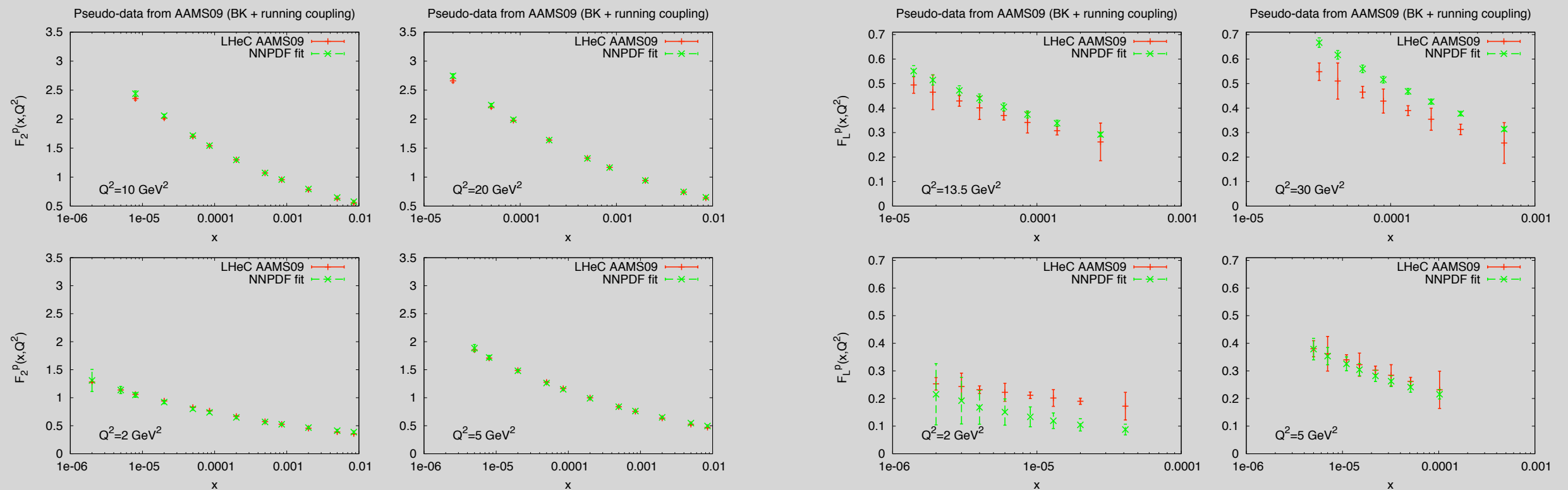
# predictions



- $F_2$  and  $F_L$  extrapolated to LHeC and UHECR kinematical conditions
  - near independence on [tested] initial conditions
  - first principle approach allows for credible extrapolation
    - ↪ ‘all’ relevant physics included

# ↻ vs. DGLAP

- AAMS  $F_2$  and  $F_L$  cannot be fitted by NLO-DGLAP



- i.e., pseudo-data (for LHeC) generated from AAMS is inconsistent with NLO-DGLAP
- differences cannot be absorbed into initial condition



# 🔄 current release ::AAMS 1.0 ::

home	IGFAE and USC Phenomenology Group .....
People	Software
Jobs	Conferences

## ...Phenomenology Group

### Dipole-proton cross section

The imaginary part of the dipole-proton scattering amplitude is available as a FORTRAN routine for public use. This quantity has been fitted to lepton-proton data using the Balitsky-Kovchegov evolution equations with running coupling. More details can be found at

J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, [arXiv:0902.1112](https://arxiv.org/abs/0902.1112)

Please refer to this publication when using the routine.

In order to compute the dipole cross section, simply multiply the output from the routine by the corresponding constant values

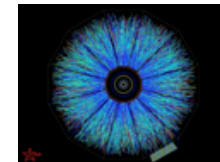
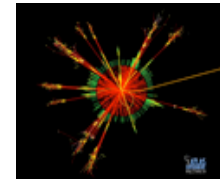
$\sigma_0=31.59$  mb for GBW initial conditions

$\sigma_0=32.77$  mb for MV initial conditions

To download the code, please follow [this link](#)

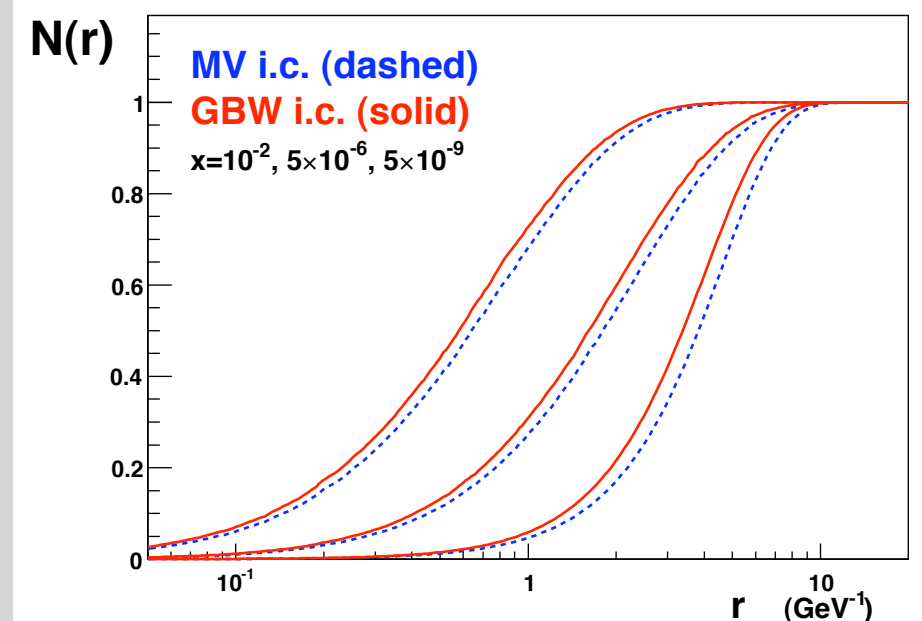
#### NEWS

The code has been updated to work properly with some old compilers. If you find any problem, please, [let us know](#)



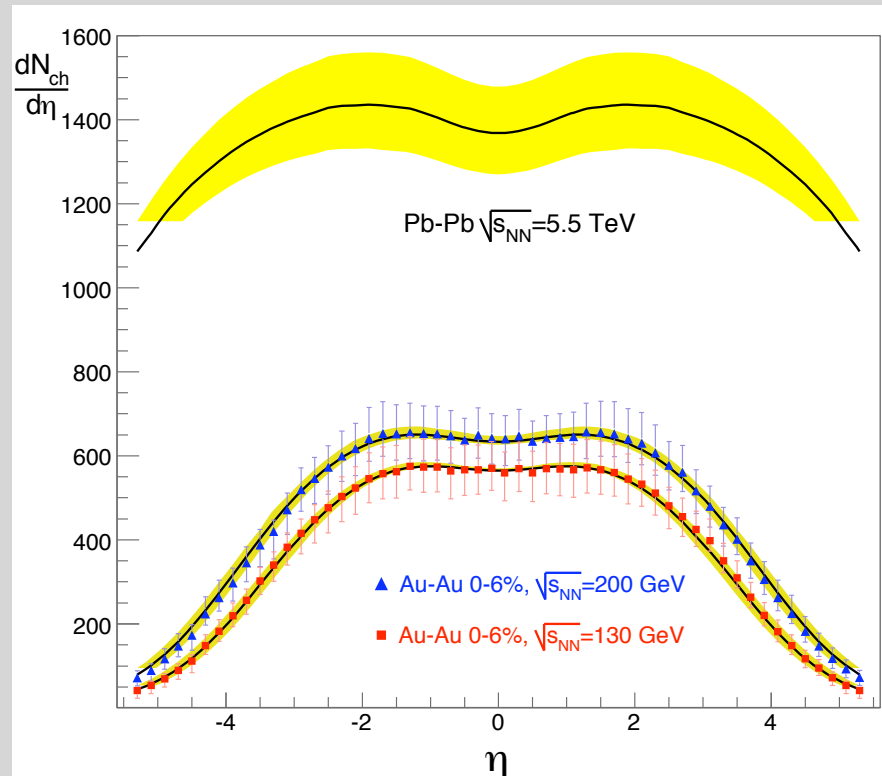
<http://www-fp.usc.es/phenom/rcbk/>

$$10^{-12} \leq x \leq 10^{-2}$$



## ↻ also used in ...

- mid-rapidity multiplicities in AA [*Albacete PRL 99, 262301*]
  - preceded present work



$$\frac{dN_{ch}}{dy d^2b} = C \frac{4\pi N_c}{N_c^2 - 1} \int \frac{d^2 p_t}{p_t^2} \int^{p_t} d^2 k_t \alpha_s(Q) \varphi\left(x_1, \frac{|k_t + p_t|}{2}\right) \varphi\left(x_2, \frac{|k_t - p_t|}{2}\right)$$

$$\varphi(Y, k) = \int \frac{d^2 r}{2\pi r^2} \exp\{i \underline{r} \cdot \underline{k}\} \mathcal{N}(Y, r)$$

- diffractive and forward hadron production in pp [*Betemps, Gonçalves and Santana Amaral*]
- hadron and direct photon production in ep and pA [*Rezaeian and Schafer*]
- long range 2-particle rapidity correlation in AA [*Dusling, Gelis, Lappi and Venugopalan*]
- single inclusive hadron production in pp, pA and AA [*Albacete and Marquet*]

# 🔄 new release [AAMQS] :: very soon

- proton
  - include H1/ZEUS combined data
    - ↪ fit reduced cross-section directly [much better, extraction indep.]
  - include  $F_2^{\text{charm}}$  and  $F_2^{\text{beauty}}$  data
  - include heavy quarks in calculation
  - improved treatment of running coupling
    - ↪ matched over mass thresholds
  - NO EXTRA PARAMETERS
- nuclei
  - direct fit of nuclear DIS data with proton parameters [2 nuclear params]

$$Q_{s,A}^2 = c A^\delta Q_{s,p}^2$$

# 🔄 harder...

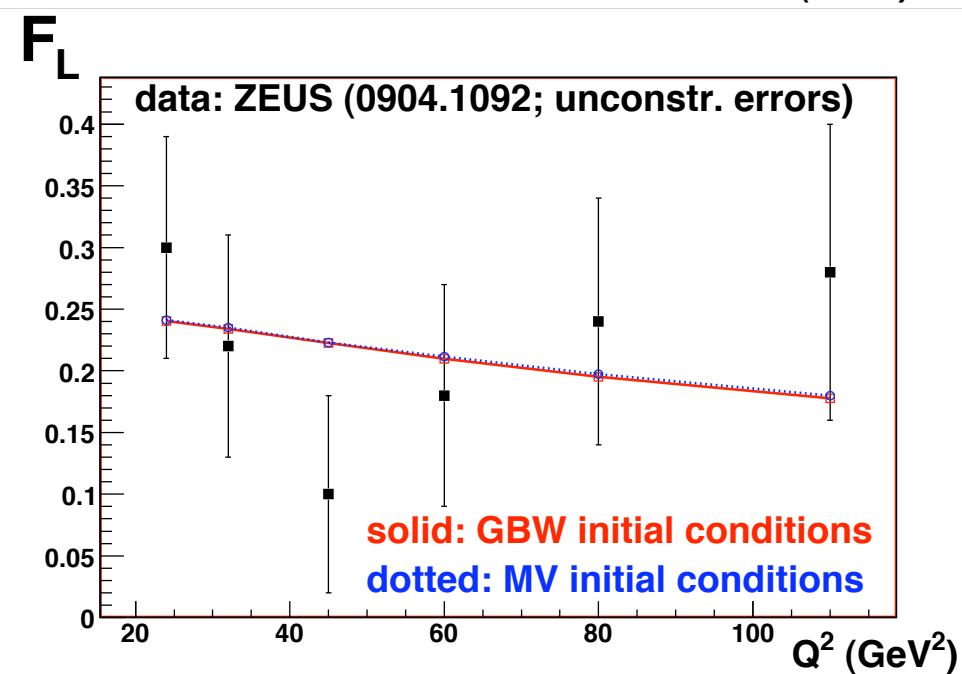
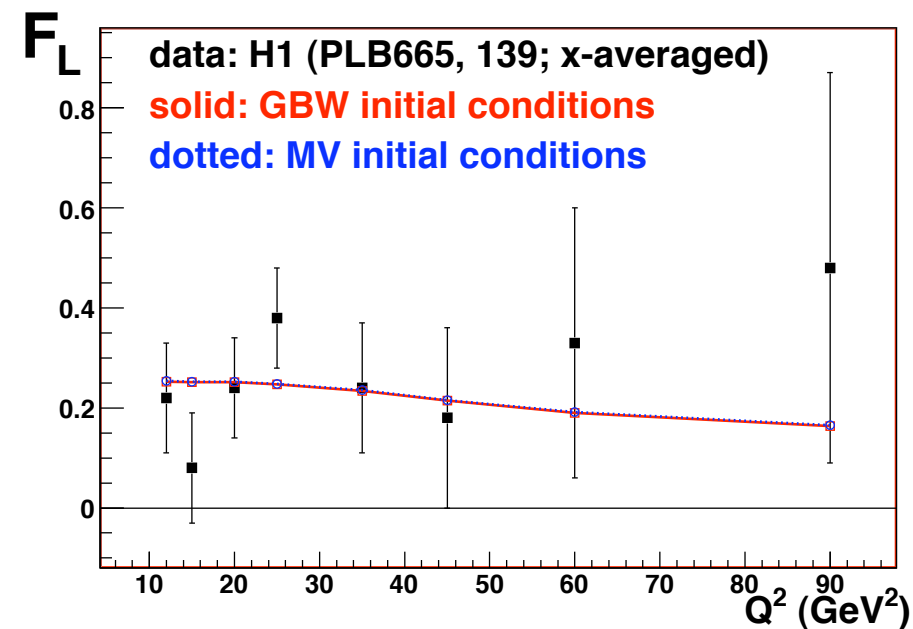
- impact parameter dependence [for both nuclei and proton]
  - access to diffractive and non-inclusive observables
  - centrality dependence for AA
- speculative...
  - extract integrated [standard] gluon distribution
    - ↪ no unique meaningful procedure

# abstract

- small- $x$  effects cannot be neglected at the LHC [and more so for nuclei]
- DGLAP is not an appropriate tool to address small- $x$  physics
- useful, phenomenologically usable, parametrizations can be obtained within  $k_t$  factorized approach
- they are easy to use...

# backups

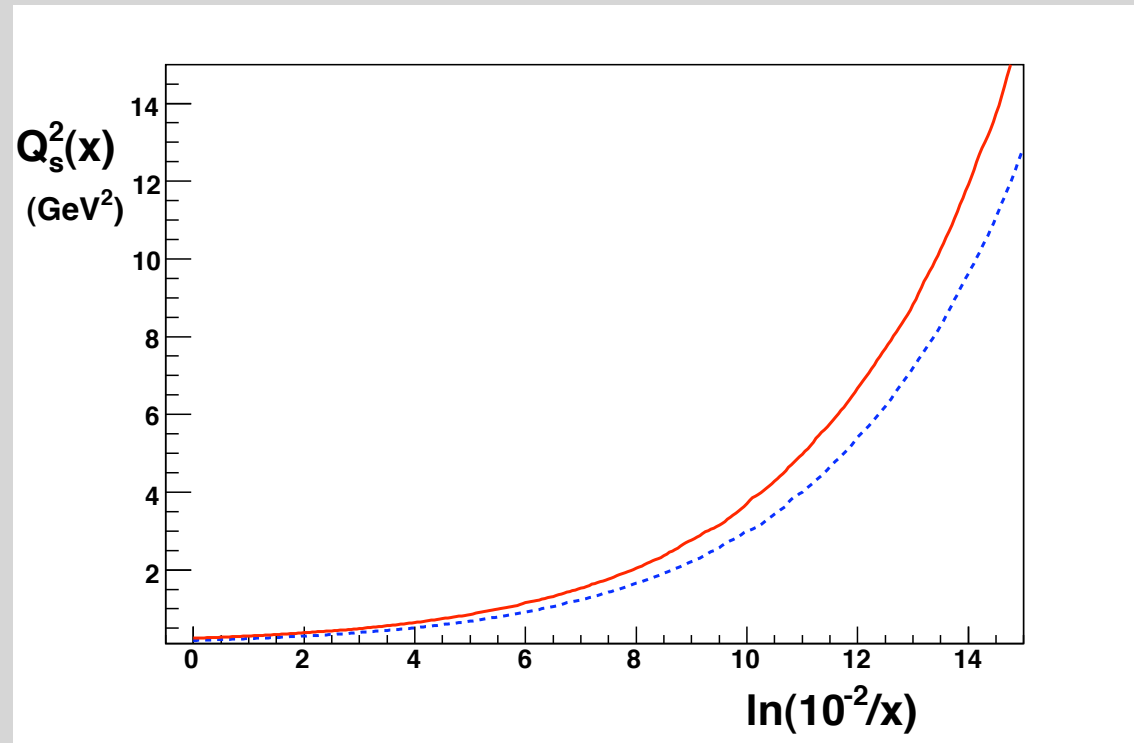
$\circlearrowleft F_L$



$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \sigma_L$$

- $F_L$  data not included in the fit
- consistently described  
[error bars too large for meaningful statement]

# ↻ saturation momentum, geometric scaling



$$\mathcal{N}(r = 1/Q_s(x), x) = 1 - \exp[-1/4]$$

- large [perturbative] saturation scale for forward region in pp at the LHC

$$x = (2 M/\sqrt{s})e^{-y} \qquad Q_s^2 \simeq 3 \div 4 \text{ GeV}^2 \qquad y = 6$$

- geometric scaling in DGLAP ?? [no scale]