Gluon saturation and nuclear parton distributions

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Outline

1. **Generalities on factorization**
2. **Gluon saturation at small x**
3. **Saturation and factorization**

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What is factorization?

- QCD is the fundamental theory of strong interactions among quarks and gluons
- Experiments involve hadrons in their initial and final states, not quarks and gluons
- Hadrons cannot be described perturbatively in QCD
- Scattering amplitudes with time-like on-shell momenta cannot be computed on the lattice

  ▷ How can we compare theory and experiments?

▷ **Factorization :** separation of short distances (perturbative) and long distance (non perturbative)

What is factorization?

- At a superficial level, factorization means that :

$$\mathcal{O}_{\text{hadrons}} = F \otimes \mathcal{O}_{\text{partons}}$$

  - F = parton distribution
  - $\mathcal{O}_{\text{partons}}$ = observable at the partonic level (calculable in perturbation theory)

- For this to be useful, F must be universal (i.e. independent of the observable $\mathcal{O}$)

- In order to test QCD experimentally, measure as many observables as possible, and try to find common F's that fit all the data

  Note : at this stage, by looking at only one observable, it is impossible to perform any meaningful test, since it is generally possible to adjust F so that it works

What is factorization?

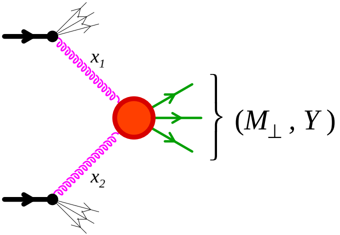
- Some loop corrections in $\mathcal{O}_{\rm partons}$ are enhanced by large logarithms, e.g.

$$\alpha_s \ln\left(\frac{M^2}{m_H^2}\right) \quad , \qquad \alpha_s \ln\left(\frac{s}{M^2}\right) \sim \alpha_s \ln\left(\frac{1}{x}\right)$$

  Note : the log that occurs depends on the details of the kinematics
  - Bjorken limit: $s, M^2 \to +\infty$ with s/M^2 fixed
  - Regge limit: $s \to +\infty$, M^2 fixed
- These logs upset a naive application of perturbation theory when $\alpha_s \ln(\cdot) \sim 1$ ▷ they must be resummed
- This resummation can be performed analytically
  - the result of the resummation is universal
  - all the leading logs can be absorbed in F
    ▷ the factorization formula remains true
    ▷ this summation dictates how F evolves with M^2 or x

What is factorization?

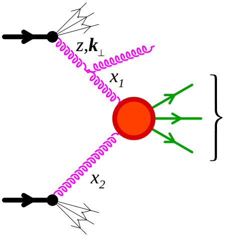
- These logarithms tell us that the relevant parton distributions depend on the resolution scales (in time and in transverse momentum) associated to a given process
- Calculation of some process at LO :



$$\left\{ \begin{array}{l} x_1 = M_\perp\, e^{+Y}/\sqrt{s} \\ x_2 = M_\perp\, e^{-Y}/\sqrt{s} \end{array} \right.$$

What is factorization?

- These logarithms tell us that the relevant parton distributions depend on the resolution scales (in time and in transverse momentum) associated to a given process
- Radiation of an extra gluon :

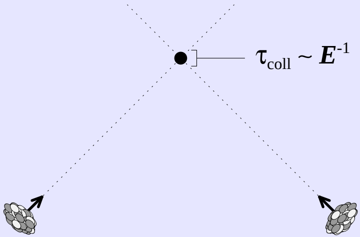


$$\Big\} \, (M_\perp, Y) \quad \Longrightarrow \quad \alpha_s \int_{x_1} \frac{dz}{z} \int^{M_\perp} \frac{d^2\vec{k}_\perp}{k_\perp^2}$$

- Practical consequence : pQCD predicts not only $\mathcal{O}_{\rm partons}$ but also the evolution $\partial_M F$ (or $\partial_x F$)

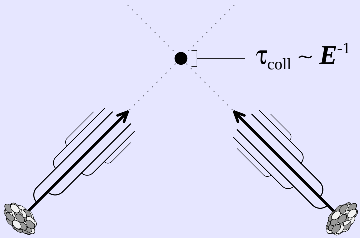
  ▷ the only required non-perturbative input is $F(x, M_0)$ or $F(x_0, M)$

Why factorization works



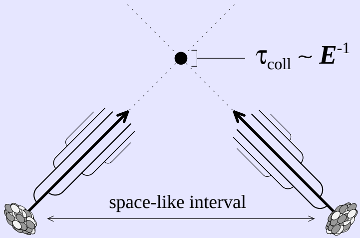
- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

Why factorization works



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we need to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

Why factorization works



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we need to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Common forms of factorization: 1. DGLAP

- Logs of $M_\perp \Longrightarrow$ DGLAP. Important when :
  - $M_\perp \gg \Lambda_{QCD}$, while x_1, x_2 are rather large

- Cross-sections read :

$$\frac{d\sigma}{dYd^2\vec{\boldsymbol{P}}_\perp} \propto F(x_1, M_\perp^2)\, F(x_2, M_\perp^2)\; |\mathcal{M}|^2$$

with $x_{1,2} = M_\perp \exp(\pm Y)/\sqrt{s}$

- Note : there are convolutions in x_1 and x_2 if some particles are integrated out in the final state

- The factorization of logarithms has been proven to all orders for sufficiently inclusive quantities (see Collins, Soper, Sterman, 1984–1985)

Common forms of factorization: 2. BFKL

Collins, Ellis (1991), Catani, Ciafaloni, Hautmann (1991)

- Logs of $1/x \Longrightarrow$ BFKL. Important when :
  - $M_\perp$ remains moderate, while x_1 or x_2 (or both) are small
- The BFKL equation is non-local in transverse momentum
  ▷ it applies to non-integrated gluon distributions $\varphi(x, \vec{\boldsymbol{k}}_\perp)$

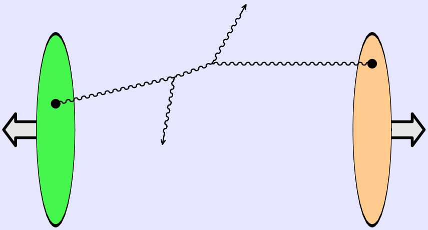
$$xG(x, Q^2) = \int^{Q^2} \frac{d^2\vec{\boldsymbol{k}}_\perp}{(2\pi)^2}\ \varphi(x, \vec{\boldsymbol{k}}_\perp)$$

▷ the matrix element must calculated for off-shell gluons with $\vec{\boldsymbol{k}}_\perp \neq \vec{0}$

- In this framework, cross-sections read :

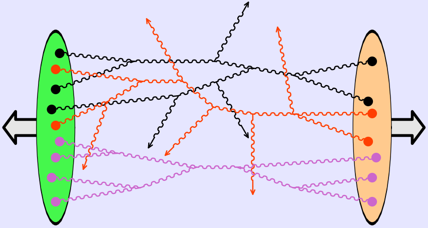
$$\frac{d\sigma}{dYd^2\vec{\boldsymbol{P}}_\perp} \propto \int_{\vec{\boldsymbol{k}}_{1\perp}, \vec{\boldsymbol{k}}_{2\perp}} \delta(\vec{\boldsymbol{k}}_{1\perp} + \vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{P}}_\perp)\ \varphi_1(x_1, k_{1\perp})\ \varphi_2(x_2, k_{2\perp})\ \frac{|\mathcal{M}|^2}{k_{1\perp}^2 k_{2\perp}^2}$$

Conditions of validity



- Dilute regime : one parton in each projectile interact (what the standard PDFs are made for)

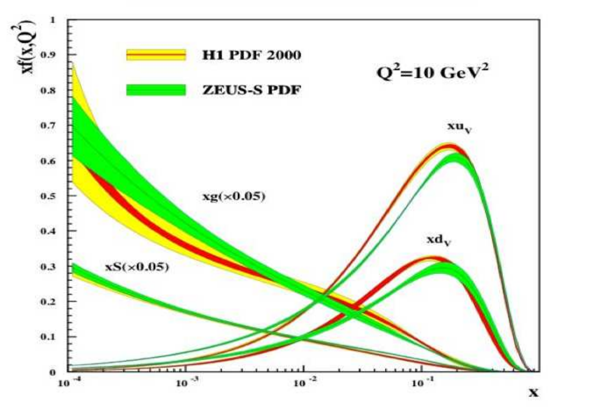
Conditions of validity



- Dilute regime : one parton in each projectile interact (what the standard PDFs are made for)
- Dense regime : multiparton processes become crucial
  ▷ standard forms of factorization break down
  ▷ new distributions are required

Growth of the gluon distribution at small x

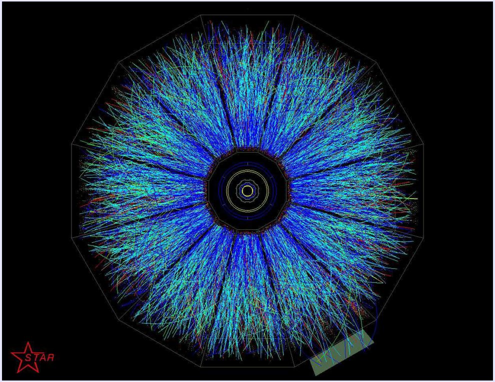
Gluon distribution at small x



- Note: gluons have been divided by 20
- Gluons dominate at any $x \lesssim 10^{-1}$

Longitudinal momentum fraction in AA collisions

Nucleus-Nucleus collision



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

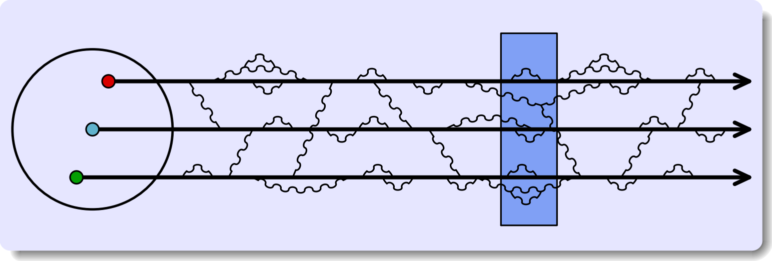
▷ partons at small x are the most important

1. Generalities on factorization
   - What is factorization?
   - Causality
   - DGLAP
   - BFKL
   - Limitations

2. **Gluon saturation at small x**
   - What is a hadron?
   - Gluon evolution
   - Saturation domain
   - Multiple scatterings
   - Color Glass Condensate

3. Saturation and factorization
   - Inclusive DIS
   - AA collisions
   - DGLAP and saturation
   - Dense-dilute limit

Nucleon partonic structure



At low energy:

- Fluctuations at all space-time scales smaller than its size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

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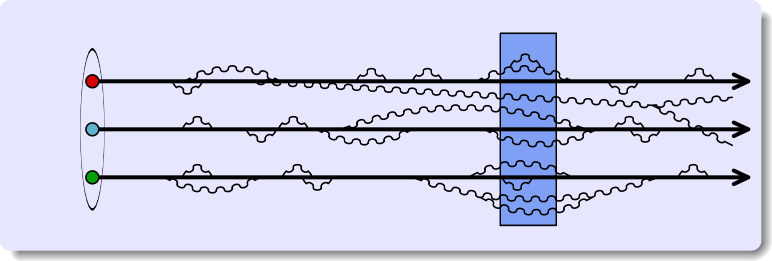
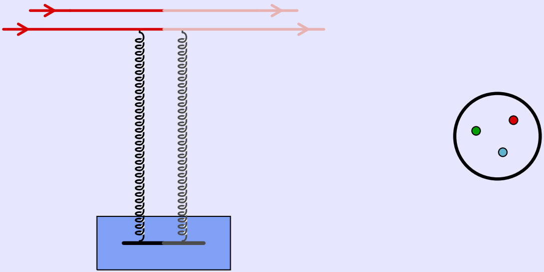
Nucleon partonic structure



At high energy:

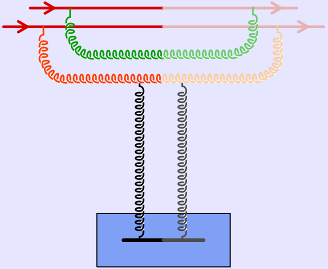
- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe
  ▷ the nucleon appears denser at small x
- Pre-existing fluctuations are frozen over the time-scale of the probe, and act as static sources of new partons

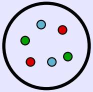
Gluon saturation



- at low energy, the probe sees mostly the valence quarks

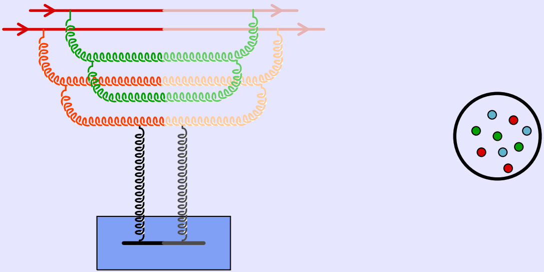
Gluon saturation





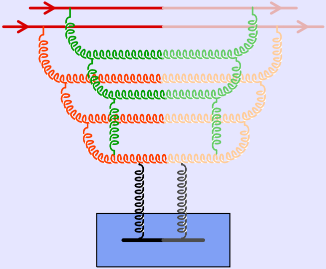
- when energy increases, new partons are emitted
- the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon
- at small-x (i.e. high energy), these logs need to be resummed

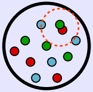
Gluon saturation



- as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Gluon saturation





- eventually, the partons start overlapping in phase-space
- parton recombination becomes favorable
- after this point, the evolution is non-linear:
  the number of new partons depends non-linearly on the number of partons at the previous step

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

Recombination cross-section :

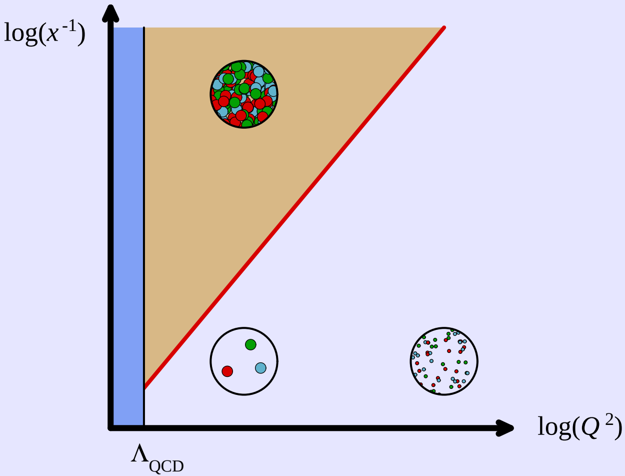
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Note: At a given energy, the saturation scale is larger for a nucleus (for $A = 200$, $A^{1/3} \approx 6$)

Saturation domain



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Multiple scatterings

- Power counting :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2}$$

- When this ratio becomes ~ 1, all the rescattering corrections become important
  - ▷ one must resum all $\left[Q_s/M_\perp\right]^n$
- These effects are not accounted for in DGLAP or BFKL

Color Glass Condensate: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad\qquad (0 < x^- < 1/\Lambda^+)$$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ they must be treated as standard gauge fields

  Eikonal coupling to the current J^μ : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution functional $W_{\Lambda^+}[\rho]$, with Λ^+ the longitudinal momentum that separates "soft" and "hard"

Color Glass Condensate: RG evolution

Evolution equation (JIMWLK) :

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H}\ W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\alpha(\vec{\boldsymbol{y}}_\perp)} \eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta}{\delta\alpha(\vec{\boldsymbol{x}}_\perp)}$$

where $\alpha(\vec{\boldsymbol{x}}_\perp) = \frac{1}{\nabla_\perp^2} \rho(1/\Lambda^+, \vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

1 Generalities on factorization

What is factorization?
Causality
DGLAP
BFKL
Limitations

2 Gluon saturation at small x

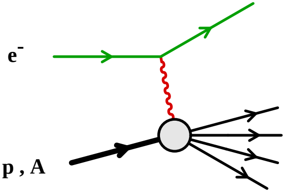
What is a hadron?
Gluon evolution
Saturation domain
Multiple scatterings
Color Glass Condensate

3 Saturation and factorization

Inclusive DIS
AA collisions
DGLAP and saturation
Dense-dilute limit

Reminder on DIS

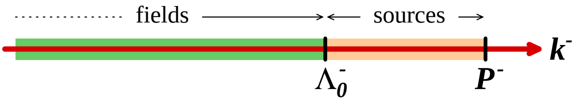
- Collision between an electron and a nucleon or nucleus, by exchange of a virtual photon



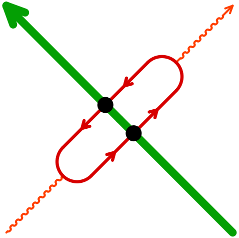
- Variant : collision with a neutrino, by exchange of $Z^0, W^\pm$

Inclusive DIS at Leading Order

- CGC effective theory with cutoff at the scale Λ_0^- :



- At Leading Order, DIS is an interaction between the target and a $q\bar{q}$ fluctuation of the virtual photon :



Inclusive DIS at Leading Order

- Forward dipole amplitude at leading order:

$$T_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp) = 1-\frac{1}{N_c}\,{\rm tr}\,(\underbrace{U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)}_{\text{Wilson lines}})$$

$$U(\vec{\boldsymbol{x}}_\perp) = {\rm P}\,\exp\, i g\int^{1/xP^-} dz^+\,\mathcal{A}^-(z^+,\vec{\boldsymbol{x}}_\perp)$$

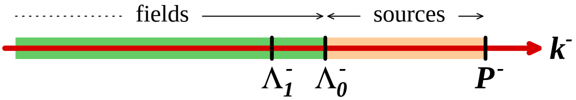
$$[\mathcal{D}_\mu,\mathcal{F}^{\mu\nu}] = \delta^{\nu-}\,\rho(x^+,\vec{\boldsymbol{x}}_\perp)$$

▷ at LO, the scattering amplitude on a saturated target is entirely given by classical fields

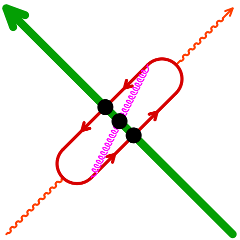
- Note: the $q\bar{q}$ pair couples only to the sources up to the longitudinal coordinate $z^+ \lesssim (xP^-)^{-1}$. The other sources are too slow to be seen by the probe

Inclusive DIS at NLO

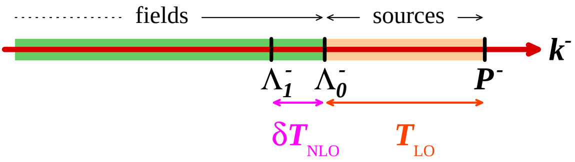
- Consider now quantum corrections to the previous result, restricted to field modes with $\Lambda_1^- < k^- < \Lambda_0^-$ (the upper bound prevents double-counting with the sources):



- At NLO, the $q\bar{q}$ dipole must be corrected by a gluon, e.g. :



Inclusive DIS at NLO



- At leading log accuracy, the contribution of the quantum modes in that strip is :

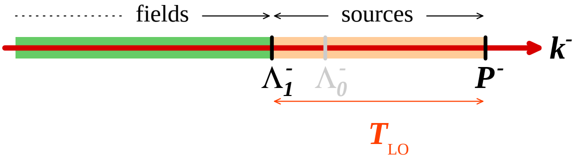
$$\delta T_{\rm NLO}(\vec{x}_\perp, \vec{y}_\perp) = \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}\; T_{\rm LO}(\vec{x}_\perp, \vec{y}_\perp)$$

Inclusive DIS at NLO

- These NLO corrections can be absorbed in the LO result,

$$\left\langle T_{\rm LO} + \delta T_{\rm NLO} \right\rangle_{\Lambda_0^-} = \left\langle T_{\rm LO} \right\rangle_{\Lambda_1^-}$$

provided one defines a new effective theory with a lower cutoff Λ_1^- and an extended distribution of sources $W_{\Lambda_1^-}[\rho]$:



$$W_{\Lambda_1^-} \equiv \left[1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}\right] W_{\Lambda_0^-}$$

(JIMWLK equation for a small change in the cutoff)

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Inclusive DIS at Leading Log

- Iterate the previous process to integrate out all the slow field modes at leading log accuracy:

Inclusive DIS at Leading Log accuracy

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp)$$

$$\sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp) \equiv 2\int d^2\vec{\boldsymbol{X}}_\perp \int [D\rho]\, W_{xP^-}[\rho]\, \boldsymbol{T}_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$$

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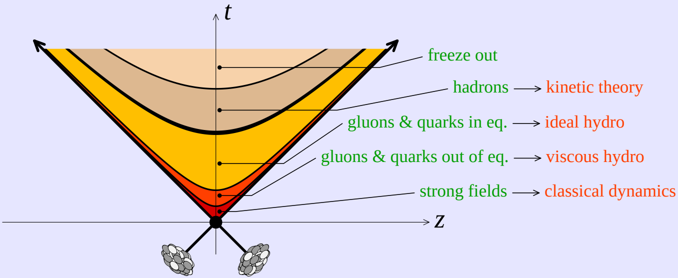
Stages of an AA collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$
- Subsequent stages are usually described as fluid dynamics

Reminder on hydrodynamics

Equations of hydrodynamics = energy-momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0$$

Inputs from the underlying microscopic theory :

$$\text{EoS}: \quad p = f(\epsilon) \quad , \quad \text{Transport coefficients}: \quad \eta, \zeta, \cdots$$

- Required initial conditions : $T^{\mu\nu}(\tau = \tau_0, \eta, \vec{\boldsymbol{x}}_\perp)$

Initial conditions from CGC: Leading Order

- $T^{\mu\nu}$ has two remarkable properties:
  - only connected graphs contribute
  - simpler expression in terms of retarded propagators

- Small coupling expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

- The Leading Order contribution is given by classical fields :

$$T^{\mu\nu}_{\rm LO} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

with $\underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\rm Yang-Mills\ equation}$, $\lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$

Initial conditions from CGC: Leading Log resummation

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, large logarithms of the CGC cutoffs appear at NLO
- Like in DIS, the coefficients of the logs are given by the action of the JIMWLK Hamiltonian on the LO observable:

$$\delta T^{\mu\nu}_{\rm NLO} = \left[\ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right)\mathcal{H}_2\right] T^{\mu\nu}_{\rm LO}$$

- By iterating this process, one arrives at:

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right] W_1\left[\rho_1\right] W_2\left[\rho_2\right] \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed }\rho_{1,2}}$$

(FG, Lappi, Venugopalan (2008))

Correlations in η and $\vec{\boldsymbol{x}}_\perp$

- The factorization valid for $\big\langle T^{\mu\nu}\big\rangle$ can be extended to multi-point correlations :

$$\Big\langle T^{\mu_1\nu_1}(\tau,\eta_1,\vec{\boldsymbol{x}}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta_n,\vec{\boldsymbol{x}}_{n\perp})\Big\rangle_{\rm LLog} =$$
$$= \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]$$
$$\times\; T^{\mu_1\nu_1}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_{n\perp})$$

$\triangleright$ For each $\rho_{1,2}$, solve the Yang-Mills equations to get the classical field $\mathcal{A}^\mu$, then compute $\boldsymbol{T}^{\mu\nu}_{\rm LO}$ from $\mathcal{A}^\mu$

By sampling the distributions $W_{1,2}[\rho_{1,2}]$, one gets all the correlations at leading log accuracy

DGLAP and saturation

- Question 1: can one define standard PDFs in the saturated regime?

  ▷ yes, but they are insufficient to do any calculation, because they do not provide any information about multi-parton correlations

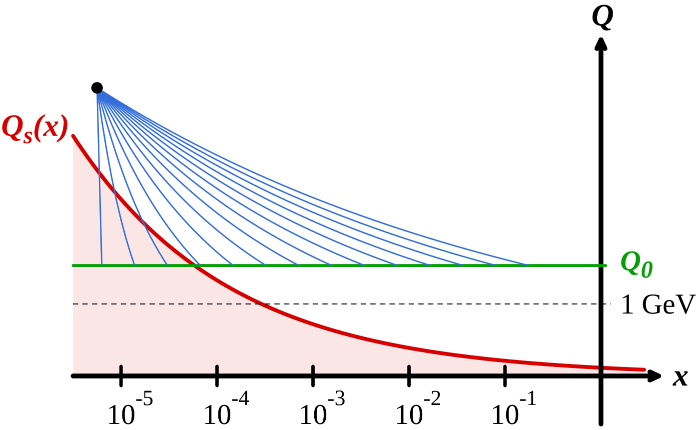
- Question 2: can one define modified PDFs that would encode these correlations?

  ▷ for a given process, maybe. But these functions would not be universal

DGLAP and saturation

- Question 3: my favorite observable is not in the saturated regime: I should be fine with the usual PDFs?

  ▷ maybe not... PDFs may have been contaminated by an improper evolution from a smaller Q:



François Gelis

Dense-dilute limit

- Factorization in the saturated regime:

$$\langle \mathcal{O} \rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\; W_1\left[\rho_1\right]\; W_2\left[\rho_2\right]\; \mathcal{O}[\rho_{1,2}]$$

($\mathcal{O}[\rho_{1,2}]$ can only be calculated numerically)

- But if ρ_1 is a weak source (projectile 1 is dilute):

$$\mathcal{O}[\rho_{1,2}] = \int_{\vec{\boldsymbol{k}}_{1\perp}} \rho_1^2(\vec{\boldsymbol{k}}_{1\perp})\; \mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp}, \rho_2] + \rho_1^4(\vec{\boldsymbol{k}}_{1\perp})\; \mathcal{O}_4[\vec{\boldsymbol{k}}_{1\perp}, \rho_2] + \cdots$$

and $\mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp}, \rho_2]$ has a compact analytical expression

Dense-dilute limit

- One gets the non-integrated gluon distribution:

$$\int [D\rho_1]\, W_1[\rho_1]\, \rho_1^2(\vec{\boldsymbol{k}}_{1\perp}) \equiv \varphi_1(\vec{\boldsymbol{k}}_{1\perp})$$

- The expectation value of $\mathcal{O}$ can be rewritten as

$$\left\langle\mathcal{O}\right\rangle_{\rm LLog} = \int_{\vec{\boldsymbol{k}}_{1\perp}} \varphi_1(\vec{\boldsymbol{k}}_{1\perp}) \int \big[D\rho_2\big]\, W_2\big[\rho_2\big]\, \mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp},\rho_2]$$

- This can be further simplified by noting that $\mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp},\rho_2]$ contains only simple correlators of Wilson lines

  ▷ one can replace the JIMWLK equation by the much simpler BK equation (mean field approximation)

Dense-dilute limit: heavy quarks production in pA collisions

Pair production cross-section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp} - \vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times\Big\{ \int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\phi_A^{(4)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp)$$

$$+\int_{\vec{\boldsymbol{k}}_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{(2)}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\varphi_1(\vec{\boldsymbol{k}}_{1\perp})$$

▷ standard factorization broken for the nucleus: one needs three different “distributions” in order to describe the target

François Gelis

Target correlators

- Target "gluon distributions":

$$\phi_A^{(2)}(\vec{k}_{2\perp}) \propto \int_{\vec{x}_\perp,\vec{y}_\perp} e^{i\vec{k}_{2\perp}\cdot(\vec{x}_\perp-\vec{y}_\perp)}\ \mathrm{tr}\left\langle U(\vec{x}_\perp)U^\dagger(\vec{y}_\perp)\right\rangle$$

$$\phi_A^{(3)}(\vec{k}_{2\perp}|\vec{k}_\perp) \propto \int_{\vec{x}_\perp,\vec{y}_\perp,\vec{z}_\perp} e^{i\left[\vec{k}_\perp\cdot\vec{x}_\perp+(\vec{k}_{2\perp}-\vec{k}_\perp)\cdot\vec{y}_\perp-\vec{k}_{2\perp}\cdot\vec{z}_\perp\right]} \times\ \mathrm{tr}\left\langle \widetilde{U}(\vec{x}_\perp)t^a\widetilde{U}^\dagger(\vec{y}_\perp)t^bU_{ba}(\vec{z}_\perp)\right\rangle$$

$$\phi_A^{(4)}(\vec{k}_{2\perp}|\vec{k}_\perp,\vec{k}'_\perp) \propto \int_{\vec{x}_\perp,\vec{y}_\perp,\vec{x}'_\perp,\vec{y}'_\perp} e^{i\left[\vec{k}_\perp\cdot\vec{x}_\perp-\vec{k}'_\perp\cdot\vec{x}'_\perp+(\vec{k}_{2\perp}-\vec{k}_\perp)\cdot\vec{y}_\perp-(\vec{k}_{2\perp}-\vec{k}'_\perp)\cdot\vec{y}'_\perp\right]} \times\ \mathrm{tr}\left\langle \widetilde{U}(\vec{x}_\perp)t^a\widetilde{U}^\dagger(\vec{y}_\perp)\widetilde{U}(\vec{y}'_\perp)t^a\widetilde{U}(\vec{x}'_\perp)\right\rangle$$

Limit of Kt factorization

Fujii, FG, Venugopalan (2005)

- The quark cross-section factorizes if the the 3-point and 2-point functions are related by:

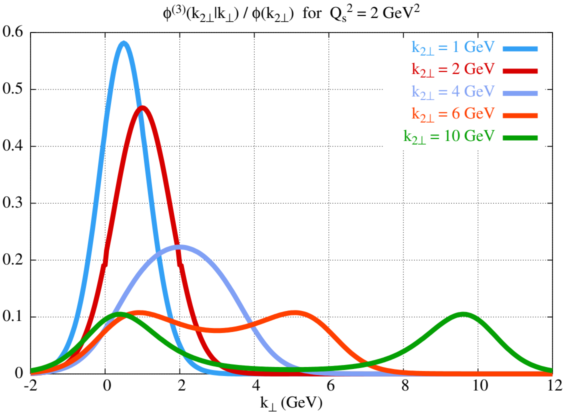
$$\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = (2\pi)^2 \frac{1}{2}\left[\delta(\vec{\boldsymbol{k}}_\perp) + \delta(\vec{\boldsymbol{k}}_\perp - \vec{\boldsymbol{k}}_{2\perp})\right]\phi_A^{(2)}(\vec{\boldsymbol{k}}_{2\perp})$$

- This relation would be satisfied if the $Q\overline{Q}$ pair interacts with the target in such a way that all the momentum exchanged goes to the quark or to the antiquark

- The ratio $\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)/\phi_A(\vec{\boldsymbol{k}}_{2\perp})$ should be close to the sum of two delta functions for factorization to be approximately valid

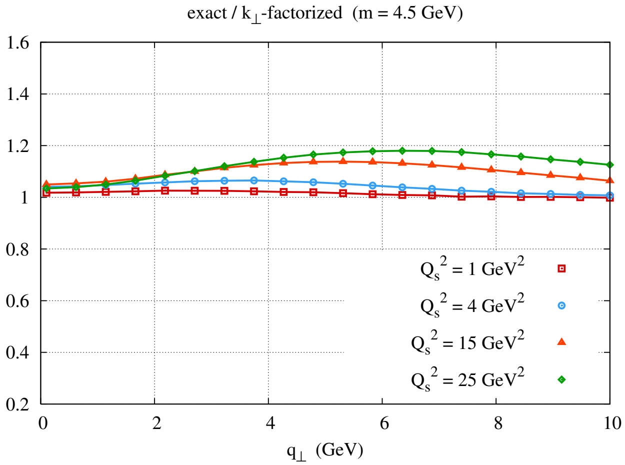
3-point correlator

- 3-point function/2-point function (in the MV model):



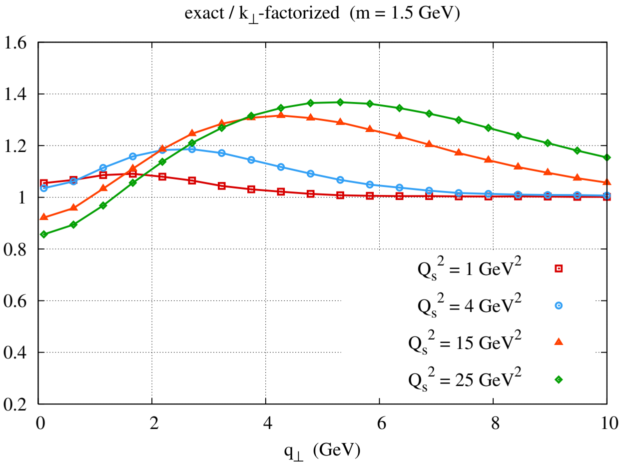
Factorization violation for b quarks

- Single *b*-quark cross-section :



Factorization violation for c quarks

- Single *c*-quark cross-section :



Summary

- Gluon saturation is enhanced in nuclei, and is reached earlier than in nucleons

- An effect of saturation is to break the standard forms of factorization (in particular the one based on DGLAP evolution)

  ▷ This may lead to an apparent non universality of the parton distributions

- In the saturated non-linear regime, there exist some universal distributions $W[\rho]$ that describe the dense projectiles both in DIS and AA collisions

  - Resums the logs of $1/x$ at leading log accuracy
  - Applies to sufficiently inclusive observables

  (but this factorization framework is hard to implement in practical calculations...)