The Phase Diagram of QCD From Heavy-ion collisions to compact stars

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QCD under extreme conditions

Understanding the dynamics of the strong interaction under extreme conditions of temperature and density!

The QCD phase diagram connects to

- cosmology -> evolution of the early universe
- compact stars at high net-baryon density
- strongly coupled quantum fluids



GSI Helmholtzzentrum für Schwerionenforschung

Connect first-principle QCD calculations with experimental observables via a realistic modeling of heavy-ion collisions and astrophysical events!

QCD under extreme conditions

Understanding the dynamics of the strong interaction under extreme conditions of temperature and density!

The QCD phase diagram features

- deconfinement
- chiral symmetry restoration
- a strongly coupled quantum fluid



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QCD under extreme conditions

Understanding the dynamics of the strong interaction under extreme conditions of temperature and density!

Important questions:

- onset of deconfinement and chiral symmetry restoration?
- properties of the strongly coupled QGP?
- existence of a phase transition with critical end point?
- dof in the core of compact stars?



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QCD at finite T and $\boldsymbol{\mu}$

- Lattice QCD:
 - Crossover at μ_B = 0 and T = [145, 165] MeV WB JHEP1009 (2010), HotQCD PoS LATTICE2010 (2010)
 - Fermionic sign problem at $\mu_{_{\rm B}} \neq 0$
 - Methods to extend to finite μ_B , e.g. Taylor expansion, etc. \Rightarrow no critical point for small $\mu_B / T < 1$.

Success story at $\mu_{\rm B}$ = 0 :

the QCD EoS is known!

- Advance new lattice techniques, eg. Complex Langevin!
- Functional Renormalization Group (FRG):
 - A critical point observed at larger densities
 - Extend truncation schemes for larger densities!
- Low-energy effective models of QCD:
 - e.g. PNJL, PQM models or quasi-particle models
 - Constrain parameters by the lattice QCD for $\mu_B / T \lesssim 1$ and experimental data from HIC and astrophysical observations at larger μ_B !







EoS at finite density

At $\mu_B \neq 0$ there are many unknowns in the QCD EoS...

- Toward finite net-baryon density the phase transition is supposed to change from crossover to a first-order phase transition via a critical end point.
- Beyond the onset density the hadronic gas description is not valid anymore => nuclear liquid or a mixed phase.
- The correct inclusion of the hadronic degrees of freedom is crucial and constraints from the nuclear ground state need to be taken into account.
- Constrain parameters by experimental results from HIC and astrophysical observations.

Need to extend the validity of the approaches to the QCD EoS to $\mu_{\rm B} \neq 0!$



EoS at high density & low temperature - astrophysics



HIC and the QCD phase diagram

- Highest energies at LHC/RHIC: PbPb/AuAu at $\sqrt{s_{_{NN}}} = 0.2 5$ TeV \Rightarrow Energy deposition \Rightarrow handle on the temperature.
- Lower energies at GSI/FAIR, SPS/CERN, JPARC: AuAu at √s_{NN} = 2 20 GeV
 ⇒ Baryon stopping → handle on the baryon chemical potential.



- Initial state
- Pre-equilibrium phase -> rapid thermalization, fluidization, etc.
- Expansion of the QGP -> fluid dynamical description
- Hadronization and Particlization -> Cooper-Frye prescription
- Final hadronic interactions -> microscopic transport

At $\mu_B \neq 0$ the physics in all of these stages changes... Current models need to be improved!

Fluctuations - messengers of the QCD phase diagram

• Reflect change in degrees of freedom.

Asakawa, Heinz, Müller, Phys.Rev.Lett.85 (2000), Jeon, Koch, Phys.Rev.Lett.85 (2000)

- Highlight phase transitions (critical point, first-order phase transition).
 Stephanov, Rajagopal, Shuryak, Phys.Rev.Lett.81 (1998); Karsch, Redlich, Phys.Lett.B.695 (2011)
- Connect with chemical freeze-out conditions. Braun-Munzinger, Stachel, Wetterich, Phys.Lett.B.596 (2004); Alba et al., Phys.Lett.B.738 (2014)
- Are important in small systems at high multiplicity, e.g. p+p and p+A.
- Affect the determination of the EoS and transport coefficients in fluid dynamics: $\eta \sim \int d^3x dt < T^{ij}(x,t)T^{ij}(0,0) >$

Included in fluid dynamics

NOT included in fluid dynamics

$$G_{R, \text{shear-shear}}^{xyxy}(\omega, \mathbf{0}) = -\frac{7T}{90\pi^2}\Lambda^3 - i\omega\frac{7T}{60\pi^2}\frac{\Lambda}{\gamma_{\eta}} + (i+1)\omega^{3/2}\frac{7T}{90\pi^2}\frac{1}{\gamma_{\eta}^{3/2}}$$

cutoff-dependent fluctuation contribution to the pressure

cutoff-dependent correction to η

frequency-dependent contribution to η and τ_{π}

Comparison of theory with experiment meaningless without inclusion of fluctuations and their dynamics !

Fluctuations at the phase transition

- Fluctuation observables, in particular higher-order cumulants of net-baryon number, are sensitive to critical phenomena at the phase transition.
- So far predictions are based on grand-canonical thermodynamics, but HIC are highly dynamical!



=> Need to include the fluctuation dynamics into descriptions of HIC!



- Propagate fluctuations of the chiral order parameter and fluid dynamical fields coupled to the regular evolution.
- Highly non-trivial due to conceptual challenges, e.g. renormalization of the EoS and the transport coefficients.
- Numerical implementation requires new algorithms!

So far, largely unexplored territory!

Fluctuations - Projects and Applications

- Embed Advanced Fluid Dynamics into a HIC event generator, e.g. EPOS.
- Fluctuation observables for the critical point in HIC!
- What is the effect on probes of the QGP (jets, heavy-quarks) and the QGP transport coefficients?
- Applications to the cosmological scale.
- Application for cold atomic gases, e.g. spin diffusion ⇒ hot topic for industrial applications: spintronics, storage and transport of information!

Work in progress Attieh, Pihan, Touroux, Bluhm, Nahrgang, Sami 2020–

Need people with the expertise of writing, maintaining, running and analysing high-performance computing codes.



Time for two heavy nuclei to collide and produce particles: ~ 10⁻²³ seconds

Time for a simulation of two colliding heavy nuclei and particle production: **~ 1 hour**

- EoS from first-principle/effective models to QCD
- Modelling of astrophysical events and collaboration with HIC community
- Fluctuations in heavy-ion collisions
 - Include dynamical fluctuations in fluid dynamics and the simulation of HIC
 - Propagate fluctuations of the chiral order parameter explicitly
 - Renormalization of FD in theory and practice
 - New numerical approaches needed
- Small systems
 - Fluid dynamics and fluctuations and probes
- Bayesian statistical analysis for unbiased determination of all parameters
 - promote it to a standard procedure for tuning any HIC (pp, pA) simulation tool
 - set up the numerical environment with a sufficient background in statistical analysis
 - requires large computational power
- Interdisciplinarity with cold atomic gases
 - fluctuating anisotropic fluid dynamics at the phase transition
- Collaboration with scientific computing facilities

IPN Orsay, IP2I, Subatech

IPhT Saclay, LLR, Subatech

Subatech

IP2I,Subatech

austical analysi

Summary

What: QCD under extreme conditions from HIC and astrophysics

- Why:
- Strongly coupled QGP
- Deconfinement and chiral symmetry restoration
- Phase transition
- DoF in the interior of compact stars
- How: First-principle and effective models of QCD
 - Improved modeling of HIC
 - Improved modeling of astrophysical events
 - Collaborating!

Who: • France: Subatech, IP2I, ... WG2 of GDR QCD

- France: Astrophysics, Cold Gases, Scientific computing community
- Strong ties with experiments/observation
- International collaborations

Plans:

- Extend effective model calculations by including experimental data, HIC and Astro
 - Improve modeling of HIC -> fluid dynamical fluctuations!
 - Applications beyond HIC: astrophysics and cold gases

APPENDIX

Connections to related systems

- The QGP shares features with other strongly coupled quantum systems, like **ultracold atomic gases**.
- UAG: unique experimental playground to study fermionic many-body problems: variable density, temperature and interatomic interaction strength
- Elliptic flow measurements: release the UAG from the trapping potential and take pictures of the expansion.
- Can theoretically be well described by anisotropic fluid dynamics:



M. Bluhm et al, PRA92 (2015)

 $\begin{array}{c} 1.5 \\ \hline & & \text{ideal hydrodynamics} \\ \hline & & \text{free streaming} \\ \hline & & \text{...} & \text{Naive: Soles (} (\alpha = 0.1) \\ \circ & \text{...} \text{Hydro} (\alpha = 1) \\ \text{Hydro} (\alpha = 1) \\ \circ & \text{...} \text{Hydro} (\alpha = 1) \\ \text{Hydro} (\alpha = 1)$

0.6

0.4

T/E

0.2

← BCS

BEC -

- Include fluctuations in anisotropic fluid dynamics and study the BCS-BEC crossover and the second-order superfluid phase transition - in equilibrium and dynamically!
- Connect to further research groups in France: e.g. IPN Orsay (superfluidity in UAG and neutron stars) or IP2I Lyon!



K.M. O'Hara et al. Science 298 (2002)



Equation of state (EoS)

Success story! At μ_{R} = 0 the QCD EoS is known!

- Lattice QCD results on the EoS are far from the Stefan-Boltzmann-limit of a non-interacting gas => strongly coupled system
- Thermodynamic quantities change characteristically at the phase transition.
- Energy density, pressure and entropy increase at the crossover transition due to the liberation of color degrees of freedom \rightarrow quark-gluon plasma (QGP)
- Speed of sound has a minimum at the phase transition/ crossover
- Naive pQCD poorly convergent => find more convergent gauge-invariant schemes (e.g. HTLpt, resummed DR)



Equation of state (EoS) Success story! At $\mu_B = 0$ the QCD EoS is known!

- Not only is the EoS precisely known from lattice QCD calculations.
- It has also been validated in a model-to-data statistical analysis.
- This combined (Bayesian analysis) tunes model and physics parameters simultaneously!



EoS at high density & low temperature - astrophysics

Phenomenological

Approaches to the EoS at high density

Aqnostic (no Lagrangian, no matter composition)

> Semi-aqnostic (no Lagrangian, matter composition)

All nuclear models are based on the phenomenology at nuclear ground state density and extrapolated to higher densities!

Make use of meta-modelization (using Bayes' statistics) to connect lower density (pheno. Lagrangian, matter composition) measurements to very-high density

Microscopic

observables! (reproduce NN XS, predict dense matter

properties)

QCD based (LQCD, chiral EFT, holography)

Gravitational wave signal, calculated with a nuclear EoS showing the presence of a transient supermassive neutron star:







NS merger, simulated with a hybrid EoS containing a holographic QCD part

EoS from holography and the link to astrophysics

- The holographic correspondence is a duality between gauge theories and string theories, prototypical example of N=4 supersymmetric gauge theory in 4 space-time dimensions is equivalent to string theory in 10 dimensions on $AdS_{F}xS^{5}$ J. M. Maldacena, IJTP 38 (1999)
- Apply holography to construct semi-phenomenological models for 4d Yang-Mills theory and QCD using the gravitational description
- Detailed map between observables in the QFT (correlation functions) and in gravity (S-matrix in asymptotical AdS spaces)
- Improved Holographic QCD: confinement and asymptotic freedom fix the asymptotics of the potential
 U. Gursov et al, JHEP0802 (2008)
- Full Holographic QCD: includes quark degrees of freedom

Matching parameters in hybrid EoS not well constraint and the treatment of baryons remains approximative!



HIC and the QCD phase diagram

Do HIC have something to do with the QCD phase diagram?

- Particle yields at chemical freeze-out
- success of statistical hadronization
- chemical equilibration



- local thermal equilibration
- Success of fluid dynamics



G. Denicol et al, PRC98 (2018)

 \Rightarrow Heavy-ion collisions are connected to the QCD phase diagram!





- Color Glass Condensate, EPOS, MC Glauber, ERKT, MC-KLN, etc...
- Nuclear shadowing
- Pre-equilibrium phase -> rapid thermalization, fluidization, etc.
- Expansion of the QGP -> fluid dynamical description
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Transport coefficients - shear viscosity

- Close to equilibrium the QGP can be characterized by transport coefficients, such as the shear and bulk viscosities, charge conductivities and diffusion or energy loss coefficients for probe particles.
- Real-time dynamics needed => difficult for lattice QCD (Euclidean space)
- E.g. the shear viscosity probes how easily fluctuations in momentum density propagate in the medium, can be calculated from:



Bayesian analysis of shear & bulk viscosity at $\mu_{_{\rm R}}$ = 0

- Parametric temperature dependence of shear and bulk viscosity.
- Model parameters tuned simultaneously.
- Likelihood distributions

An UNBIASED model-to-data comparison should be standard for all simulations of HIC!



Bayesian analysis of shear & bulk viscosity at $\mu_{_{\rm R}} \neq 0$

- Collision energy dependence of the shear viscosity.
- Model parameters tuned simultaneously.
- Likelihood distributions



J. Auvinen et al, PRC97 (2018)

An UNBIASED model-to-data comparison should be standard for all simulations of HIC!

Bayesian determination of HQ diffusion coefficient



Bayesian determination of HQ diffusion coefficient



• Set of parameter values chosen according to a Latin hypercube algorithm.



- Design points X are evenly spread across the n-dimensional parameter space.
- "Prior" parameter distribution is uniform.
- **Y** = Physics Model(**X**) gives the "prior".

• Gaussian process emulator used as a fast surrogate for physics model.



- Calculation of exp. observables needs approx. 5000 runs of the Physics Model.
- $\mathcal{O}(1000)$ steps in the random walk are to be performed for calibration.
- Use a GP emulator for the top 8 principal components.
- Mapping from a n-dimensional input space to a normal distributed output.
- Interpolates and predicts the model output, provides the uncertainties of its prediction.

Y. Xu, MN et al., PRC97 (2018)

• Bayes' theorem: posterior \propto likelihood \times prior

 $P(\mathbf{x}|\mathbf{X},\mathbf{Y},\mathbf{y}_{\exp}) = P(\mathbf{X},\mathbf{Y},\mathbf{y}_{\exp}|\mathbf{x}) P(\mathbf{x})$

• Likelihood
$$\propto \exp\left((\mathbf{y} - \mathbf{y}_{exp}) \Sigma^{-1} (\mathbf{y} - \mathbf{y}_{exp})^{\mathsf{T}}\right)$$

- Covariance matrix: $\Sigma^{-1} = \text{diag}(\sigma_{\text{stat}}^2) + \text{diag}(\sigma_{\text{sys}}^2) + \text{diag}(\sigma_{\text{GP}}^2)$
- Uncorrelated experimental systematic errors assumed so far.
- Markov Chain Monte Carlo random walk is performed.
- Each step of the MCMC is accepted or rejected according to the likelihood.
- After equilibration → posterior distribution.

Prior:



red: calibrated on all energies

Y. Xu, MN et al., PRC97 (2018)



Heavy-ion collisions at finite density



- Especially at low densities HIC span over large regions in the phase diagram.
- Already in $\sqrt{s_{NN}}$ =72GeV rather large $\mu_{B}/T > 1$ are probed.



The initial state of HIC at finite density

- Boost invariance is not a good approximation anymore -> thick pancakes!
- Increasing penetration time for colliding NUClei. J. Auvinen and H. Petersen, PRC88 (2013)
- CGC picture does not work anymore.
- Dynamical fluidization: Y. Akamatsu et al, PRC98 (2018)
 Need to evolve pre-fluid and fluid in parallel!
- Multi-fluid approach: fluid dynamical description from the initial state on.
 P. Batyuk, MN et al, PRC94 (2016)
 Need to couple the fluid description dynamically to the corona formation!
- EPOS: currently all collisions happen in parallel, for application at finite density a cascade with sequential collisions needs to be implemented!



Fluid dynamical simulations of HIC



- Fluid dynamical simulations of heavy-ion collisions describe successfully, particle spectra and anisotropic flow coefficients v_n
- Many improvements in recent years have started:
 - viscosities and coupling terms between viscosities done
 - initial state fluctuations done
 - anisotropic fluid dynamics preliminary
 - external fields (magnetic field, order parameters, etc) preliminary
- NOT included so far: thermal fluctuations!

No code has it all (yet)!

Role of fluctuations



Susceptibilities relate to fluctuations in multiplicities

$$\begin{split} \chi_1 &= \frac{1}{VT^3} \langle N \rangle \,, \quad \chi_2 = \frac{1}{VT^3} \langle (\Delta N)^2 \rangle \,, \quad \chi_3 = \frac{1}{VT^3} \langle (\Delta N)^3 \rangle \,, \\ \chi_4 &= \frac{1}{VT^3} \langle (\Delta N)^4 \rangle_c \equiv \frac{1}{VT^3} \left(\langle (\Delta N)^4 \rangle - 3 \langle (\Delta N)^2 \rangle^2 \right) \,. \end{split}$$

- Derivatives reveal more details about the phase transition.
- Derivatives of thermodynamic quantities are related to fluctuations.
- => Obtain more information about the phase transition from fluctuation observables!

To zeroth order in volume fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$
variance Skewness Kurtosis

Can be applied to freeze-out conditions, the critical point search and fluid dynamics! 39

Main axes - dynamics of fluctuations

Predictions based on equilibrium thermodynamics !

- Very rough estimates for finite time effects.
- No coupling of fluctuations to the evolution.
- Simplification of highly dynamical HIC.

Stephanov et al., PRL81 (1998), PRD60 (1999) Asakawa, Heinz, Müller, PRL85 (2000)





NA49 Collab. JPG 35 (2008)

Main axes - dynamics of fluctuations



- No renormalization of the lattice spacing dependence performed!
- No systematic numerical implementation applied!
- Typically no coupling to all sources of fluctuations considered!

Main axes - dynamics of fluctuations

Fluctuating dissipative fluid dynamics with

- renormalized equation of state and transport coefficients,
- thoroughly tested numerical algorithms,
- coupling to initial state fluctuations and final state hadrons.



Event-by-event applications to

- HIC (pp, pA) to constrain parameters by Bayesian analysis,
- HIC at highest energy in order to extract transport coefficients,
- HIC at lower energies in order to study critical point observables,
- smaller systems in order to capture deviations from (conventional) fluid dynamics,
- Other strongly coupled quantum fluids, like cold atomic gases!

Full results from EPOS

- Mapping to a viscous fluid dynamical evolution Tmunu
 - Parameter eta/s = ...
 - EoS from lattice QCD
- Cooper Frye Freeze-out (works well for averages, not obvious for fluctuations!)
- Hadronic final state interactions from UrQMD.



K. Werner et al, Phys. Rept. 350 (2001), PRC85 (2012), PRL112 (2014), PRC89 (2014), PRC92 (2015)

Probes of the QGP

D meson RAA

Good qualitative/ semi-quantitative description of various models

Finite v2 of Jpsi Some description at low pT Jet shape function Including medium response

2.76 TeV. R= 0.3

PbPb (Shower)

PbPb (Hydro)

DD (PYTHIA)

0.6

V

PbPb(CMS, 0-30 %)

PbPb (Shower+Hydro)

0.8



S. Cao et al, PRC99 (2019)

Large community effort to compare the different ingredients of the models! ALICE Coll., Quark Matter 2017 New theoretical ideas needed! Proper implementation into dynamical models challenging! Y. Tachibana, PRC95 (2017)

How can we connect jet observables to properties of QCD?

What about proton-proton and proton-nucleus?

• From low-multiplicity pp to pA to central AA - all is QCD!

In EPOS:

- => increasing multiplicity
- => increasing number of pomerons
- => increasing Qs (individual saturation scale for each pomeron)
- => harder pomerons
- => harder strings
- => more high pT particles
- => strong increase of <pT>
- => strong nonlinear increase of D meson or J/psi multiplicity
 - Can also be tested with resonance production



The role of numerics and computational resources

