TITLE: **Theoretical modeling of the neutron-star crust**

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The present contribution concerns the following research fields:

☒ Nuclear astrophysics

☐ Nuclear structure and reactions

☐ Hadronic physics

☐ Particle physics

☐ Astro-particle physics

☐ Cosmology and gravitation

Abstract: In the next decade, neutron-star observations will provide data with unprecedented precision, which will allow us to learn more about the properties of nuclear matter at extreme conditions which cannot be reached in experiments. To that end, observations have to be confronted with models. Although the neutron-star crust has only sub-saturation density, its modeling is very challenging since many different length scales are involved. While the crust equation of state, which has observable consequences for the mass-radius relation, can be described with nuclear-theory methods, other observable phenomena such as cooling or glitches require in addition techniques from condensed-matter physics. For instance, we plan work on the properties of dilute matter and their consequences on the crust composition, the superfluid fraction of the inner crust, the dynamical properties of superfluid vortices, and the limits of elasticity of the crust.

1 Context

Neutron stars are formed in core-collapse supernovae at the end of the life of intermediate-mass stars. They are very compact objects with densities up to several times nuclear saturation density. Furthermore, neutron-star matter is very neutron rich. Therefore, in addition to their astrophysical interest, neutron stars can be seen as a laboratory for the study of nuclear matter under extreme conditions that cannot be reached in terrestrial experiments [Weber1999].
In the last decade, the data on neutron stars obtained from observations in radio, visible, X-ray, and gravitational waves has reached unprecedented precision, and this trend will continue in the next decade. New observational facilities coming online (such as the Five hundred meter Aperture Spherical Telescope and the Square Kilometre Array precursors) will revolutionize the search for pulsars (rotating neutron stars emitting beamed radiation) by accessing thousands more. The timing data available is expected to increase at an unprecedented rate, while NICER is already delivering important information on the radius of neutron stars [Bogdanov2019], one of the key parameters to put constraints on the equation of state of dense nuclear matter. Moreover, multi-messenger observations of neutron star mergers have the potential to

revolutionize nuclear astrophysics, improving our understanding of nucleosynthesis in the kilonova environment (especially r-processes), and providing insights about the equation of state of strongly interacting matter at high densities. The first observation of GWs from GW170817 can be combined with modern calculations of the EOS to extract useful insights about the EOS for neutron stars matter [Tews2019].

Roughly speaking, neutron stars have a very dense liquid core (a mixture of Cooper paired superfluid neutrons, superconducting protons, degenerate electrons and muons and possibly hyperons or even quarks) which is surrounded by the solid inner and outer crust. The outer crust has a crystalline structure made of nuclei immersed in a degenerate relativistic electron gas. The inner crust contains, in addition, a gas of unbound neutrons which is supposed to form a superfluid as the temperature drops below the critical one of $\sim 10^{10}$ K.

As a consequence, the theory of the neutron-star crust is particularly rich, because it combines elements of nuclear physics, condensed matter physics and general relativity.

2 Brief overview of our recent activities in the field

2.1 Composition and equation of state of the crust

Due to the presence of the degenerate electron gas, whose Fermi energy can reach tens of MeV, the stability condition with respect to beta decay or electron capture implies that nuclei in the crust are much more neutron rich than ordinary nuclei on earth. We studied the composition of the crust in various approaches based on density functional theory, using microscopic Hartree-Fock-Bogoliubov (HFB) [Pearson2018] or semiclassical Extended Thomas-Fermi approaches [Aymard2014, Martin2015]. We showed that it is important to develop unified equations of state (EOS) for crust and core [Pearson2018]. Concerning the inner crust, we found that the densities of the neutron gas and of the nuclei (clusters) satisfy to a good approximation the liquid-gas phase coexistence conditions [Martin2015]. While these calculations assume that the composition of the crust will be the energetically most favorable one, the crystallization of hot matter will inevitably give rise to impurities, which have important consequences for transport properties [Fantina2020, Carreau2020].

2.2 Neutron superfluidity in the inner crust

The superfluidity of the neutron gas manifests itself in a curious phenomenon called “glitches”, i.e., sudden jumps of the rotation frequency of a rotating neutron star (pulsar). Moreover, superfluidity is also known to affect the cooling of neutron stars. Both these aspects are linked as the rotational and thermal evolution of a neutron star are coupled: as the pulsar spins down (because of electromagnetic emission), the friction between the superfluid and the rest of the star (mediated by the presence of quantized vortices) heats up the star [Antonelli2020].

The modeling of pairing in the inner crust proceeds in two steps. First, one needs an effective in-medium pairing interaction that describes pairing of uniform matter, and second, additional complications arise from the non-uniform structure due to the presence of nuclear clusters. Even the question of pairing in a uniform neutron gas is quite complicated, as the bare nucleon-nucleon interaction undergoes screening in the medium [Urban2020] and also BCS-BEC crossover effects beyond BCS theory may play a role [Durel2020]. The easiest way to describe pairing in the presence of clusters in the gas is to use the Local-Density Approximation (LDA) which was for instance used to compute the heat capacity [Burello2015]. Besides the spatial dependence of the pairing gap, the clusters also affect the superfluid properties of the neutron gas through the entrainment effect (a non-dissipative coupling between the superfluid and the normal component), which has important consequences for the interpretation of glitch observations [Martin2016, Delsate2016, Montoli2021], see [Sourie2017] for a global glitch model in general relativity.

2.3 Collective modes

Because of the pairing gap, the heat capacity due to neutron quasiparticles is strongly suppressed. However, the superfluid still has low-lying collective modes [Khan2008, Urban2015]. Furthermore, the periodic structure of the crystal in the inner crust gives also rise to longitudinal and transverse lattice phonons [Durel2018]. All these collective modes contribute to the heat capacity and to transport properties such as heat conductivity and viscosity, which are essential to model both the rotational and cooling evolution of neutron stars.

3 Selected topics to be addressed in the next decade

3.1 Properties of dilute matter

In a large part of the inner crust, clusters are very sparse and the neutron gas can be almost regarded as uniform. Furthermore, the equation of state of dilute matter is decisive for the composition of the crust which can be seen as a coexistence of a liquid (clusters) and a gas phase (free neutrons). While the form of the most common (e.g., Skyrme) energy-density functionals, even of those whose parameters have been fitted to ab-initio calculations of the neutron-matter EOS, is not well suited to describe very dilute matter, the recently developed YGLO and ELYO functionals [Yang2016, Grasso2017, Bonnard2020]) interpolate correctly between the low-density regime and the usual behavior around nuclear saturation density. The consequences of these improved functionals for the neutron-drip density (i.e., the transition between the outer and the inner crust) and the composition of the inner crust will be studied. These functionals will be also used to include finite-temperature effects in the equation of state, which are expected to have a strong impact in core-collapse supernova and neutron star merger simulations. In parallel, we plan to improve our understanding of low-density matter, in particular of pairing. For that purpose, we plan to apply ab-initio methods using renormalisation-group based low-momentum interactions, which have already been very successful in recent nuclear-structure calculations.

3.2 Entrainment and superfluid fraction

As mentioned above, the superfluid free neutrons in the inner crust are responsible for the glitches. However, they are entrained with the nuclear clusters, which can be expressed as an effective reduction of the superfluid density. To date, our knowledge of the entrainment effect is quite uncertain, even though glitch observations could be used to constrain the average entrainment parameter in the crust. While the superfluid hydrodynamic approach [Martin2016] predicts a rather weak entrainment, band-structure calculations predict much stronger entrainment [Delsate2016]. To reconcile both approaches, we plan to solve numerically the HFB equations in the periodic lattice for the case of a small superfluid velocity. In this way, one should be able to correctly interpolate between superfluid hydrodynamics in the limit of very strong pairing and the usual band-structure result in the limit of weak pairing. This will finally allow us to decide whether there are enough superfluid neutrons in the inner crust to explain the observed glitch activity of the Vela pulsar [Montoli2021].

3.3 Dynamical properties of quantized vortices

A superfluid rotates by forming an array (or a tangle) of vortices of quantized circulation (up to 10^{19} in a pulsar), which mediates the aforementioned dissipative coupling known as mutual friction. To date, the macroscopic modeling of rotating neutron stars is based on the famous Hall-Vinen-Bekarevich-Khalatnikov (HVBK) equations, which are an extension of the two-fluid hydrodynamics of Tisza and Landau for superfluid helium. However, this kind of two-fluid description is unsatisfactory for neutron stars. In fact, the mutual friction term in the HVBK model cannot be a realistic approximation of the hydrodynamic force acting on the neutron fluid in both the inner crust and outer core. The reason is that in a neutron star the vortices are immersed in a non-homogeneous medium, which provides a way to pin the vortices, so that the mutual friction is suppressed [Antonelli2020]. Understanding the pinning-unpinning transition in the vortex configuration is the key to interpret timing observations of pulsar glitches. This could be achieved by means of extended dynamical simulations of many vortices immersed in the crustal lattice.

3.4 Limits of elasticity

The elastic response of the solid crust plays an important role in neutron star physics. For example, the breaking stress of crustal matter (the maximum of the stress-strain curve) is related to a number of astrophysical phenomena, including the emission of gravitational waves from static mountains and magnetic flares triggered by crustquakes and pulsar glitches [Giliberti2020].

In particular, the nuclear pasta phase (whose properties depend crucially on the nuclear symmetry energy J) theorized near the limit between crust and core should impact some observed properties of neutron stars, like the torsional oscillations and the maximum sustainable quadrupole moment (and its associated emission of gravitational waves). For example, in the range of experimentally acceptable values of the nuclear parameter L, the pasta phase can alter the crust oscillation frequencies by up to a factor of three and decrease the maximum quadrupole ellipticity sustainable by the crust by up to an order of magnitude. Furthermore, studying the elastic response of the crust is also relevant for understanding the maximum rotation frequency of millisecond pulsars.

To date, microscopic estimates of the breaking stress are obtained thanks to classical molecular dynamics simulations that, however, are not accurate in the high density regime of the inner crust and give unreasonably high values. Hence, it will be important to develop more realistic methods to account for a better microscopic description of matter in the inner crust to validate the current estimates of the breaking strain of the nuclear cluster lattice.

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