



IN2P3 Prospectives Théorie 2021

Unitarity Limit & Universality

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Few-Body

Many-Body

Two-Body Unitarity & Universality

$$T_2(k) \equiv \frac{i\pi}{\mu k} (1 - S_2(k)) \equiv \frac{2\pi}{\mu k} (\cot \delta_2(k) - i)^{-1} \simeq \frac{2i\pi}{\mu k} \left(1 + \mathcal{O}\left(\frac{1}{ka_2}, kR\right) \right) \rightarrow \begin{cases} S_2(k) \simeq -1 \\ \delta_2(k) \simeq \pi/2 \end{cases}$$

$k \equiv \sqrt{2\mu E} \ll 2\mu$
(nonrelativistic)

S matrix

phase shift

scattering length finite range

regardless of short-range details (also atomic physics, etc.)

Two-component fermions

Three- or more-component fermions, bosons

(continuum) scale invariance

RG invariance \rightarrow limit-cycle three-body force with single dimensionful parameter
e.g. three-body ground-state energy $E_{3,0} = -B_{3,0}$

discrete scale invariance

↑
degree of universality

Does the additional symmetry improve our understanding of nuclear structure/reactions?

Neutrons

- how close are intermediate-density neutron systems to unitarity?
- what are the implications to neutron-star matter?

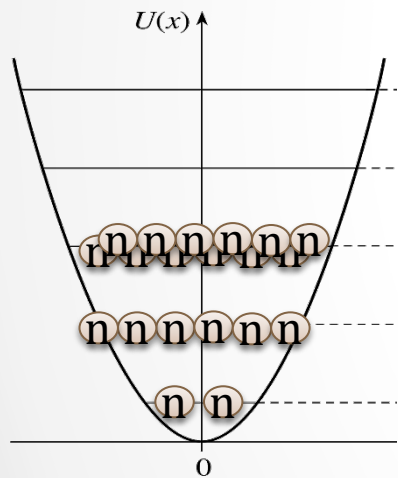
scale invariance

no bound states except for external interaction/trap

e.g.
$$\left. \frac{E_N^{(0)}}{N} \right|_{N \rightarrow \infty} = \frac{3k_F^2}{10m} \left(\xi + \mathcal{O}\left(\frac{1}{k_F a_2}, k_F R\right) \right) \quad \text{universal number Bertsch '99}$$

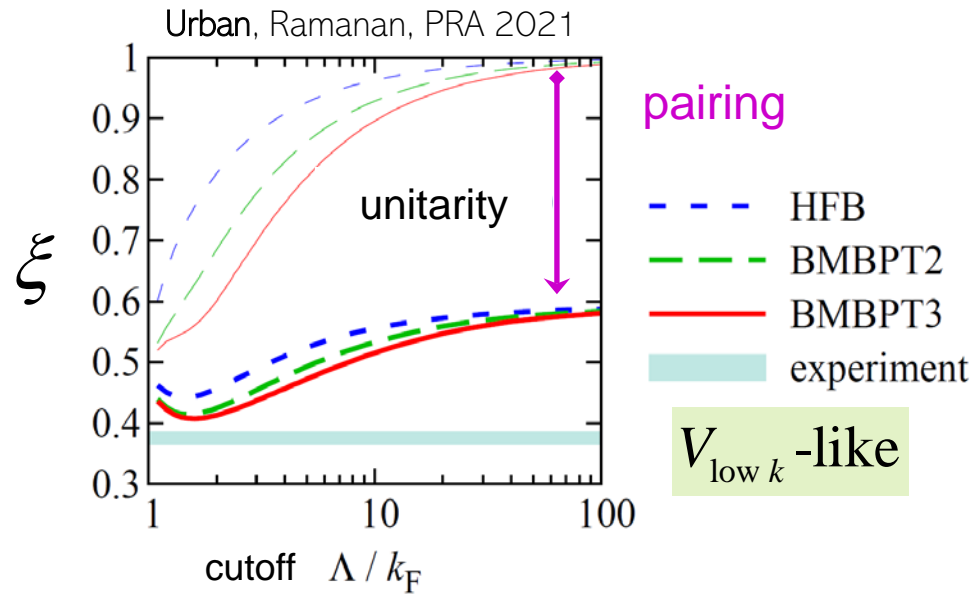
$$k_F = (3\pi^2 \rho)^{1/3}$$

free gas

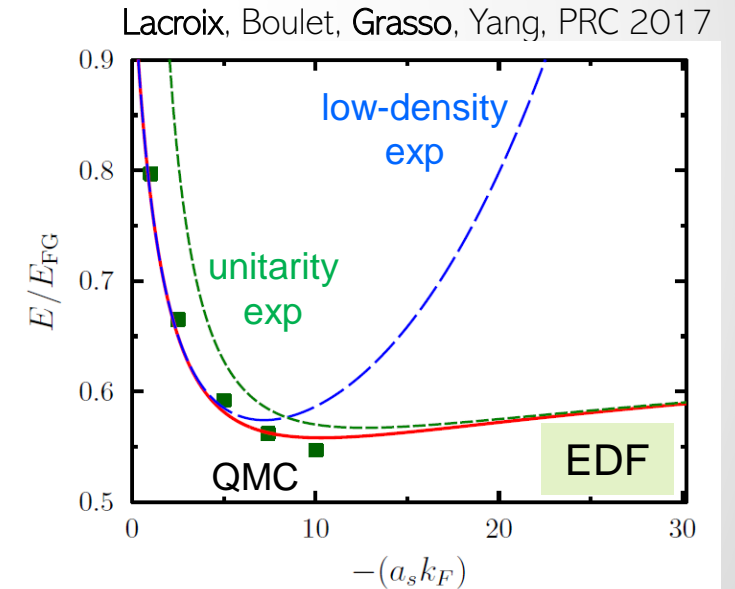


Neutron drops:
gateway to larger systems

EFT + QMC
(Contessi, Somà)



- To be added (Urban)
- induced three-body interactions
 - higher orders in BMBPT
 - nonperturbative resummations



- new energy density functionals with unitarity constraints (Grasso, Lacroix)

Bosons

simplest systems with discrete scale invariance

discrete scale invariance

1) Geometric towers of excited states

2) Ground-state correlations

$$\frac{B_{N,0}^{(0)}}{N} = \kappa_N \frac{B_{3,0}^{(0)}}{3}$$

universal numbers

$N = 3$

Efimov '70

$N = 4$

Hammer, Platter '07

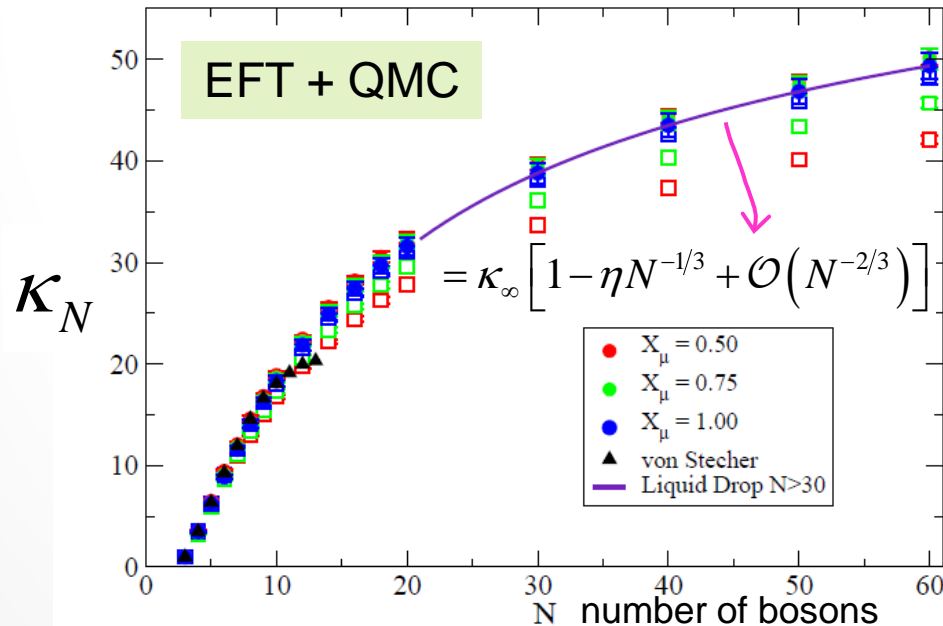
$N \geq 5$

von Stecher '10'11

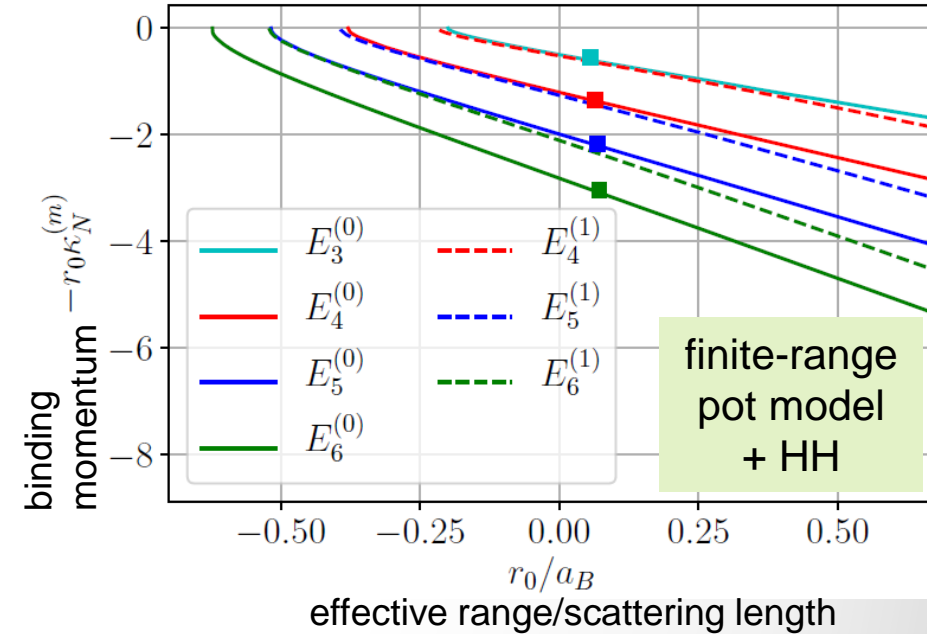
Gattobigio *et al.* '11'12'14'20...

towers multiply

Carlson, Gandolfi, Vitiello, vK, PRL 2017



Kievsky, Girlanda, Gattobigio, Viviani, ARNPS 2021

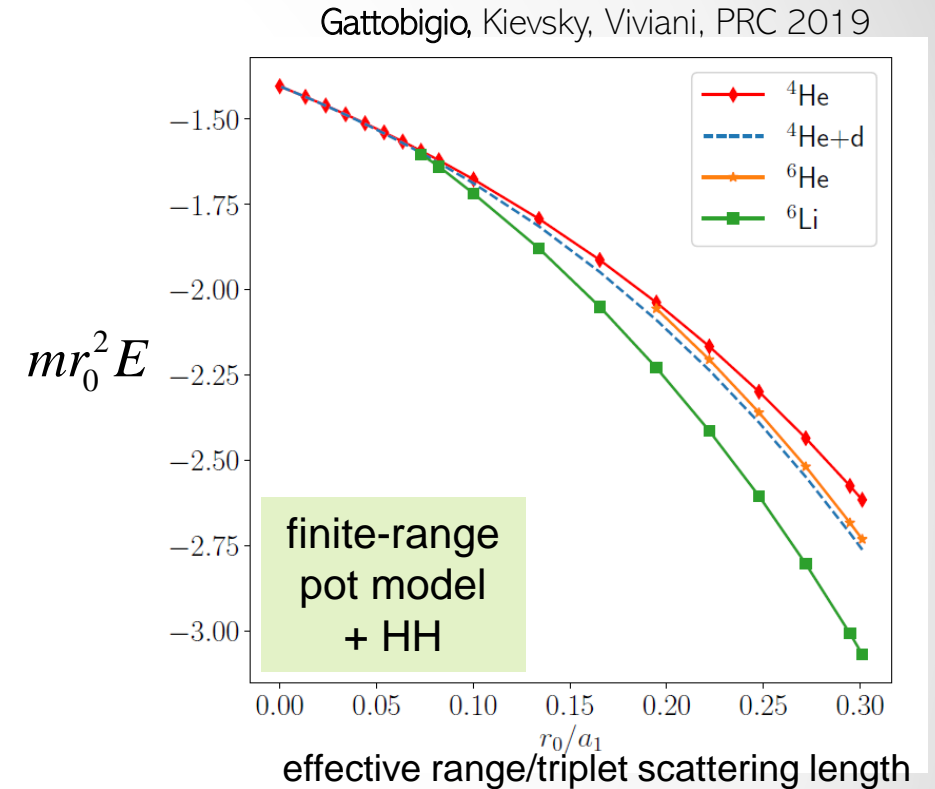
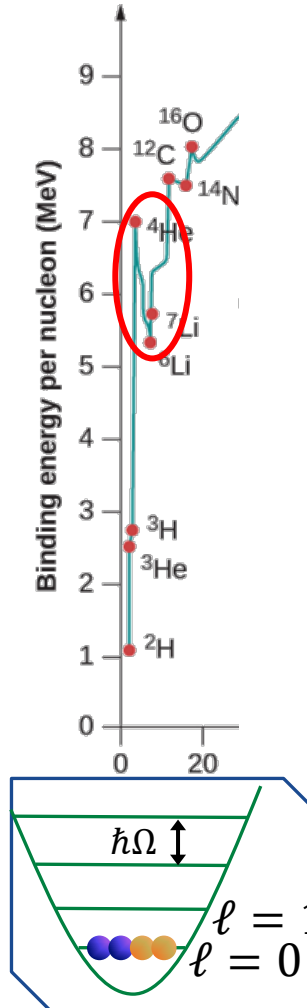
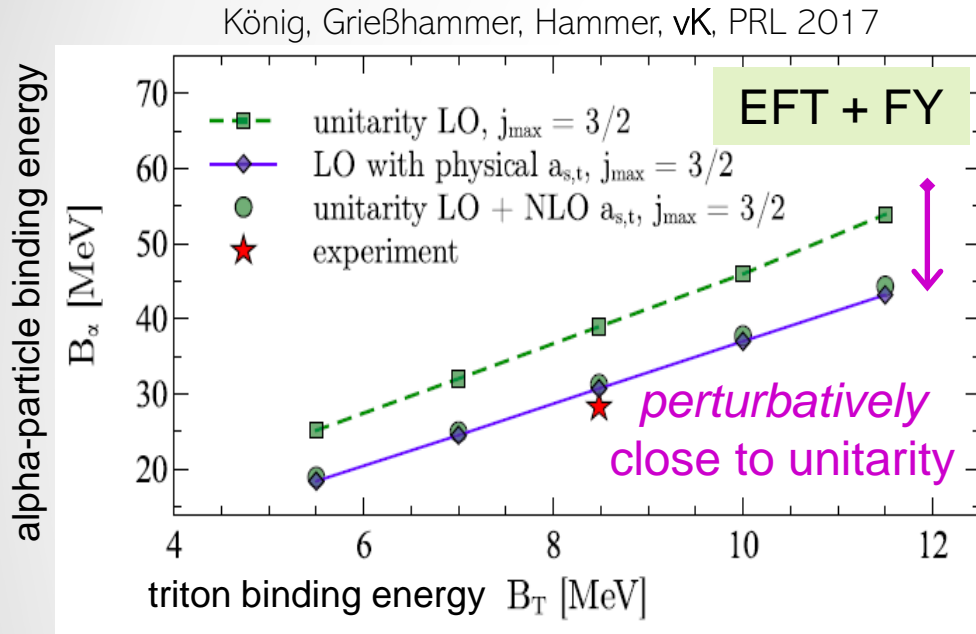


- why do towers multiply?
- what are the corrections to unitarity?
- how close are ^4He atomic clusters to unitarity?

(Contessi, Gattobigio, vK)

Nucleons

~ bosons + shell effects



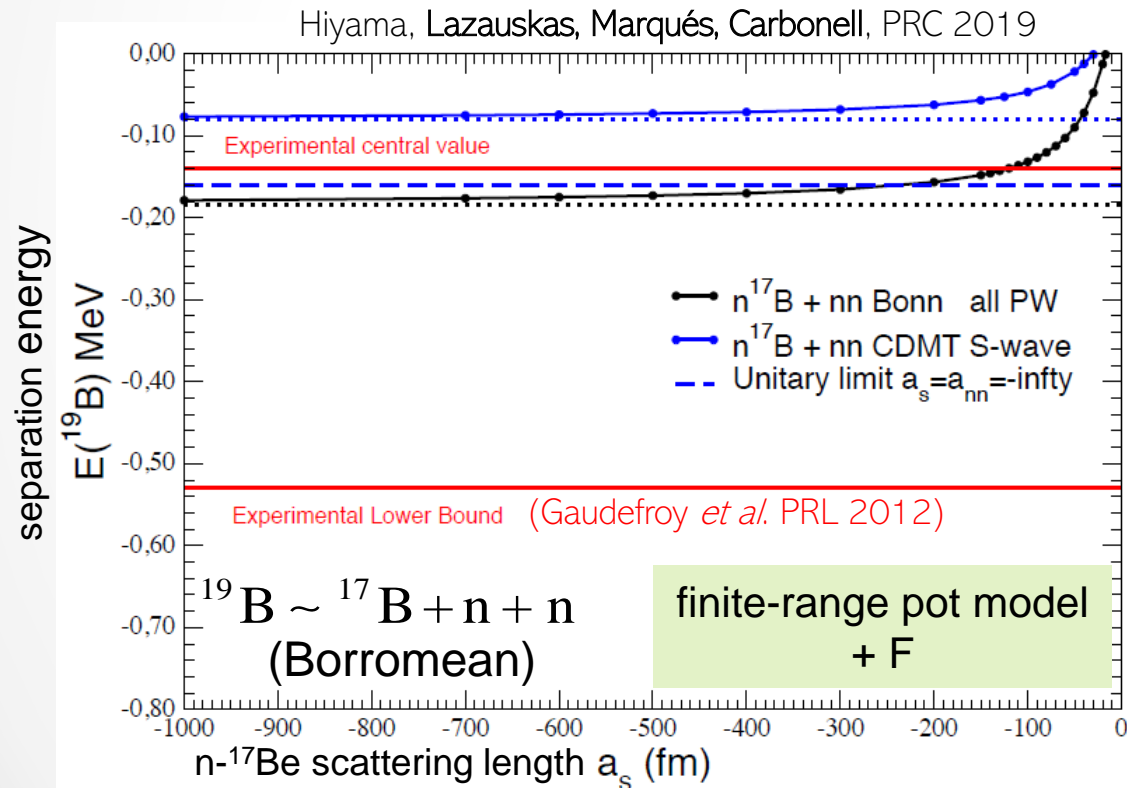
□ how does the first-excited unitarity state become a $p+{}^3\text{H}$ resonance?

□ can we get p, \dots -shell physics in expansion around unitarity?
 ${}^4\text{He}$ p-wave states, ${}^5\text{He}$, ${}^6\text{He}$, ${}^6\text{Li}$,
 \dots , ${}^{16}\text{O}$, \dots

Halo/Cluster Nuclei

~ boson-fermion or fermion-fermion mixtures

□ Is an expansion around unitarity useful also for halo/cluster nuclei?



- more neutrons: ^{20}B , ^{21}B
(cf. Leblond *et al.* PRL 2018)
- other cores
- multiple cores, e.g. ^8Be

(Carbonell, Lazauskas, vK)

Conclusion

Unitarity and universality **simplify** description of physical systems

Expansion around unitarity **works** for light nuclei
and intermediate-density neutron matter

Is it useful also for larger nuclei?

Could the saturation mechanism be similar to that for unitary bosons?

Can it inspire better energy density functionals?