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Emergent Gravity and Cosmology

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Introduction

• Cosmology has brought forth the most important problems for the fundamental theory:

♠ The cosmological constant problem: here the quantum theory seems to disagree with gravity. It has several aspects:

– All inflationary models are fine-tuned.

– All dark energy models are fine-tuned.

♠ The baryon asymmetry problem.

♠ The dark matter problem.

• The first class of problems points to a clash between the quantum theory and gravity.

• This is a central problem in a theory of quantum gravity.

• There are other issues that seem problematic: it is difficult to control, until now, massless quantum theories in de Sitter space.

• It is not known whether this is due to the de Sitter space' instabilities or to physics becoming non-perturbative.

• There are many attempts to modify classical gravity in the IR and amend the problems.

• I do not think that these can be successful without understanding at the same time the quantum completion of the modified theory.

(Quantum) Gravity

• String theory provides for a perturbative (and semiclassical) theory of quantum gravity.

• It did not, however, help us to solve the cosmological constant problem.

• However, this may be because we were not using it properly.

• The AdS/CFT correspondence already maps string theory to QFT and gives a non-perturbative definition of string theory.

• But most importantly, it suggests that the gravitons of the dual string are bound states of gluons.

• And although the gravitons are massless in the higher dimension, due to the non-trivial background, they can be massive in the lower dimensions.

Composite gravitons?

• The analogy with QCD is suggestive.

• The low-energy theory of the strong interactions is the IR-free (but non-renormalizable) theory of pions.

• In that theory, it was eventually understood, that one can quantize the low-energy degrees of freedom (pions) in the chiral Lagrangian, but this description has a cutoff, $\Lambda \sim GeV$ and a large number of counterterms are needed.

• Instead, the high-energy degrees of freedom (quarks+gluons) are different and the QFT associated to them is UV complete

• Taking this as clue, it would suggest that the non-renormalizability of the graviton appears because of its compositeness: the graviton is a low-energy bound-state.

• This idea is old and many attempts were made in the past to construct gravity theories where the graviton is a composite field.

• All such attempts failed to go beyond the classical and provide a dynamical explanation of why the bound state appears “feature-less” at low energies, because the theories had to be strongly coupled.

The energy momentum tensor

• The composite graviton is generated out of the vacuum by the (conserved) energy-momentum tensor of the QFT

$$T_{\mu\nu}(p)|0\rangle \equiv |\epsilon_{\mu\nu}, p\rangle$$

• In weakly-coupled theories, this is a spin-2 multi-particle state and therefore its interactions are expected to be non-local.

• If however, the interactions are strong and make this state a true tightly-bound state with a "size" L, then maybe we can reproduce gravity at scales $\gg L$.

• In particular, in the limit of infinitely-strong interactions we would expect to obtain a good point-like interaction theory for this bound-state graviton.

• If the theory is conformal, such states will form a continuum.

• This is the case in AdS/CFT which provided the first concrete and workable example of a composite/emergent graviton.

The energy-momentum tensor vev as a dynamical metric

• The action that describes the dynamics of the emergent graviton in a generic QFT, is (not surprisingly) the effective action for the energy-momentum tensor.

• Consider the Schwinger functional $W(g_{\mu\nu}, J)$:

$$e^{-W(g_{\mu\nu},J)} = \int \mathcal{D}\phi \; e^{-S(\phi,g_{\mu\nu},J)}$$

• $g_{\mu\nu}$ is an arbitrary background metric, ϕ are the "quantum fields".

• The emergent (dynamical) graviton is proportional to the vev of the stress tensor:

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{\det g}} \frac{\delta W(g, J)}{\delta g^{\mu\nu}}$$

and we use it to define the associated effective action via a Legendre transform.:

$$\Gamma(h, J, \mathbf{g}) \equiv -W(g, J) + \int d^4x \sqrt{g}\; h_{\mu\nu}\; (g^{\mu\nu} - \mathbf{g}^{\mu\nu})$$

- Γ is extremal,

$$\left.\frac{\delta\Gamma(h_{\mu\nu}, J)}{\delta h_{\mu\nu}}\right|_{g=\mathbf{g}} = 0 \quad , \quad \left.\Gamma(h^*_{\mu\nu}, J)\right|_{g=\mathbf{g}} = W(\mathbf{g}, J)$$

- The description above in terms of the energy-momentum tensor "effective action" is a theory of (classical) dynamical gravity.

- This description is diff-invariant.

- The related theory is a bi-gravity as it involves a dynamical metric $h_{\mu\nu}$ and a fixed fiducial metric, $\mathbf{g}_{\mu\nu}$ on which the QFT is defined.

- $\Gamma(h, \mathbf{g})$ describes spin-two and spin-0 particles, the last is associated to to the energy-momentum tensor trace: it is the dilaton.

• Independent of quantum corrections, if the original QFT is on a flat metric, then a flat metric is always a solutions to the emergent gravitational equations!

• The emergent interaction of energy sources can be shown to have the tensor structure of massive gravity.

• The mass of the emergent graviton is controlled by the poles of the energy-momentum tensor of the QFT.

• For generic QFTs, the dynamics of the emergent metric, is far away from what we understand as “gravity”.

Gravitons from (holographic) hidden sectors

• In the real world, the graviton that couples to the SM stress tensor must be an additional dynamical field beyond the SM.

♠ It can emerge in a similar way from a "hidden sector".

♠ The hidden sector will be coupled to the SM at some high scale.

♠ Only a few interactions must survive in the IR between the two theories.

• If we want this graviton to be tightly bound and weakly coupled, then this hidden sector theory must be a large-N, strongly coupled (ie holographic) QFT.

The brane-world picture

• Once the hidden theory is holographic, there is a dual gravitational description of hidden-theory$\times$ SM:

$$S_{total} = S_{bulk} + S_{brane}$$

$$S_{bulk} = M_P^3 \int d^5x \sqrt{G} \left[-V(\phi) + R_5(G) - \frac{1}{2}(\partial\phi)^2 + \cdots \right]$$

$$S_{brane} = M_P^2 \int dz \delta(z-z_0) \left(\int d^4x \sqrt{\gamma} \left[-W_B(\phi) + U_B(\phi)\ R_4(\gamma) - \frac{1}{2} Z_B (\partial\phi)^2 + \cdots \right] + S_{SM}(\gamma, \phi) \right)$$

• Bulk equations plus Israel conditions give all dynamical equations.

• These have been studied recently and shown to generically have "self-tuning" solutions if the boundary metric is a flat metric.

Charmousis+Kiritsis+Nitti

• This implies that even if $W_B(\phi) \neq 0$, the brane is stabilized at a fixed bulk position $z = z_0$ with an induced flat metric.

• This is the holographic dual of the property of emergent gravity we mentioned before.

• Moreover, if one allows the brane to move in the bulk, then we obtain a cosmological evolution on the brane (known in this context as "mirage cosmology")

P. Kraus, Kehagias+Kiritsis

• The minimum of the potential for this brane motion is the "self-tuning" solution.

Amariti+Charmousis+Forcella+Kiritsis+Nitti

• Moreover, when the brane is in a near-AdS region, the brane metric is de Sitter.

• However, the cosmological possibilities remain still unexplored.

The brane graviton

• The general analysis of this setup indicates that the brane graviton is always massive.

• The mass is controlled by properties of the hidden theory and the standard model, but it scales as $N^{-\frac{2}{3}}$ and can be therefore made naturally small as $N \to \infty$.

• The gravitational interaction on the brane is four-dimensional at long and short distances. It may become five dimensional at intermediate distances depending on scales.

Dvali+Gabadadze+Porrati, Kiritsis+Tetradis+Tomaras

• There is a VdVZ discontinuity, and the fate of the Vainshtein mechanism needs to be investigated further.

• There is also generically a dilaton. Its masses and couplings are generically similar to that of the graviton.

Open Problems

• It is clear that there are many open problems before this approach can be brought to agree with data.

♠ It is not yet clear how to bridge the ultimate IR non-linear theory with one of the known massive graviton theories.

♠ The non-linear structure and the Vainshtein mechanism must be investigated.

♠ The dilaton, like in string theory, is a major problem. When can it be made massive enough or weakly-coupled enough to avoid problems?

♠ This must be constrained by the S-matrix bootstrap. Can one make the lowest pole in the spin-0 part much heavier than in the spin 2 part?

♠ How the main eras in cosmology (primordial inflation, reheating, and late time acceleration) fit in this framework?

♠ How the structure of the Standard Model and its extensions affect this gravitational history?

New opportunities

- We can integrate-in many other fields. Most of them however will have large masses of $\mathcal{O}(M) \sim M_P$. The only generically protected ones, are the graviton, the universal axion and global conserved currents (graviphotons).
Anastasopoulos+Betzios+Bianchi+Consoli+Kiritsis

- In all emergent graviton theories, after including all quantum corrections the background fiducial flat metric is always a solution. Therefore there is no standard CC problem.

- This maps into the self-tuning solutions of the brane-world description.

- Adding also the natural (bulk) axion one can, in principle, correlate the self-tuning of the brane CC to a solution of the hierarchy problem.
Hamada+Kiritsis+Nitti+Witkowski

- There seems to be a "dark energy" that originates in the hidden theory.

- Additional sources in the hidden theory may provide new sources of "dark" components: energy, matter etc.

• Black holes depend on non-linear dynamics. The black-hole dynamics here is similar to brane-worlds and therefore has novel features.

• In emergent gravity, even the signature of the metric can change. The metric of the energy-momentum tensor vev depends on the state of the hidden theory, and there are states in which it has a Euclidean signature.

THANK YOU!

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The Weinberg-Witten Theorem

- The WW theorem assumes Lorentz invariance and a conserved Lorentz-covariant Energy-Momentum tensor.

- It proceeds to prove that no massless particle with spin $S > 1$ can couple to the stress tensor and no massless particles with $S > 1/2$ to a global conserved current.

- This does not rule out a theory that contains a "fundamental" massless graviton, as there exists a loop-hole: The stress tensor is not conserved in the presence of a metric, and projecting on helicity-2 is also non-covariant in a general metric.

- There are also other ways of avoiding the theorem:

• In the case of massless vectors the statement says that no massless (non-abelian) vectors can couple to a conserved Lorentz-covariant global current. It seems that Yang-Mills theory is excluded.

• This is avoided in standard non-abelian gauge theories as the conserved current is not Lorentz-covariant (only up to a gauge transformation).

• There are more interesting counter-examples:

$*$ At the lower end of the conformal window in N=1 sQCD: the ρ-mesons become massless but also develop a gauge invariance at the same time.
Komargodski

• These are the "magnetic" gauge bosons of Seiberg.

• Their effective theory is renormalizable (being a standard non-abelian gauge theory).

• A final caveat: Lorentz invariance is crucial: otherwise the notion of masslessness is not well-defined. (even in dS or AdS the notion changes)

• In conclusion: WW can be evaded but it is a serious litmus test for all emergent graviton theories.

• We shall find that although the essence of the WW theorem remains true, the effective theories for composite gravitons are very rich.

The Weinberg-Witten loop-hole

• In GR the stress tensor is not conserved but covariantly conserved.

• One can add corrections to the tress tensor (involving also the flat metric) to make is strictly conserved and Lorentz covariant. This is however NOT a tensor under general coordinate transformations (but this is OK with WW).

• To make a pure helicity-two state, we must project out the (unphysical) helicity 1 and 0 states. This projection is NOT Lorentz covariant (but only up to a gauge transformation).

• We may appeal to diff-invariance to decouple the helicity 0 and 1 states but then we are stuck: $T_{\mu\nu}$ is now NOT fully covariant.

• Therefore GR and many other theories with an explicit dynamical graviton avoid the WW theorem.

The AdS/CFT paradigm

• AdS/CFT relates QFT to string theory and therefore to a theory of "quantum gravity"

• That a gauge theory at large-N can be described by a weakly-coupled string theory was anticipated since the work of 't Hooft.

• Emergent dimensions are the avatar of the large N limit. Eigenvalue distributions become continuous extra dimensions as it was already seen in simpler matrix models.

• It is still a puzzle however, why the higher-dimensional theory has diffeomorphism invariance.

• The masslessness of the higher-dimensional graviton, as we understand it now, is related to energy conservation of the dual QFT.

Kiritsis, Adams+Aharony+Karch

• The holographic duality essentially implements what we discussed already: the graviton (and all other bulk fields) are composites of (generalized) gluons.

• Strong coupling in the QFT, and the higher dimensionality, as expected, is important in making the gravitational theory local (by suppressing string corrections)

• The other important ingredient is the large-N limit. It makes bulk fields (composites) interact weakly (despite the fact that the constituents interact strongly)

We have learned that:

♠ Strong coupling in QFT makes gravitons tightly bound states.

♠ Large N makes gravitons weakly interacting.

and both of the above give an effective semiclassical theory of (composite) quantum gravity.

• We believe that the duality can be used to define string theory and gravity non-perturbatively, by using the QFT to define the physics beyond the obvious cutoff of the string theory.

• This however, needs to be understood much better and it is a very difficult question, as in many cases it requires controlling non-perturbative physics

The stress tensor vev as a (classical) dynamical metric

• We would like to implement directly the idea of an emergent graviton as the state generated by the energy-momentum tensor.

• We will construct the theory that describes the dynamics of such a graviton in any QFT.

• As a warm-up, we consider a translationally invariant QFT at a fixed background metric $\mathbf{g}_{\mu\nu}$ and a scalar source J coupled to a scalar operator O (for purposes of illustration).

• The presence of an arbitrary background metric $\mathbf{g}_{\mu\nu}(x)$ breaks translation invariance.

• A redefinition of the derivative→ covariant derivative "restores" energy-momentum conservation (in the absence of other non-constant sources):

$$T_{\mu\nu} \equiv \frac{1}{\sqrt{\mathbf{g}}} \frac{\delta S(\mathbf{g}, J)}{\delta \mathbf{g}^{\mu\nu}} \quad , \quad \nabla^{\mu}_{\mathbf{g}} \langle T_{\mu\nu} \rangle \sim \partial_\nu J$$

where $S(\mathbf{g}, J)$ is the action of the theory coupled to the fixed metric $\mathbf{g}$ and to the scalar source J.

• Consider the Schwinger functional $W(g_{\mu\nu}, J)$:

$$e^{-W(g_{\mu\nu}, J)} = \int \mathcal{D}\phi \; e^{-S(\phi, g_{\mu\nu}, J)}$$

• $g_{\mu\nu}$ is an arbitrary background metric, ϕ are the "quantum fields".

• $W(g_{\mu\nu}, J)$ has (naive) diffeomorphism invariance.

• We assume the presence of a cutoff that preserves diff invariance so that the quantities above are finite.

• This is tricky business but for the moment we can have dim-reg in mind.

• Sometimes $W(g, J)$ is unique (modulo renormalization) at the linearized level, sometimes it is not (improvement).

• Moreover there are ambiguities at the non-linear level.

• One can add diff-invariant functionals of the curvature for example.

• These correspond to "improvements" (ie alternative definitions of the stress tensor), both at the linear as also the non-linear level.

• We will call all of this "the scheme dependence" of the Schwinger functional.

• $W(g,J)$ is now diff-invariant if the original theory is translation invariant*:

$$W(g'_{\mu\nu}(x'),J(x')) = W(g_{\mu\nu}(x),J(x)) \quad , \quad g'_{\mu\nu} = g_{\rho\sigma}\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}$$

• The interaction energy between energy-momentum sources $t_{\mu\nu}$ with $g_{\mu\nu} = \mathbf{g}_{\mu\nu} + t_{\mu\nu}$ is encoded in $W(t)$.

• The (quantum) vev of the stress tensor is:

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{\det g}}\frac{\delta W(g,J)}{\delta g^{\mu\nu}}$$

and we will use it to define the associated effective action:

$$\Gamma(h,J,\mathbf{g}) \equiv -W(g,J) + \int d^4x\sqrt{g}\; h_{\mu\nu}\;(g^{\mu\nu} - \mathbf{g}^{\mu\nu})$$

via a modified Legendre transform.

• Γ is the generating functional of 1-PI energy-momentum tensor correlators and is extremal,

$$\left.\frac{\delta\Gamma(h_{\mu\nu},J)}{\delta h_{\mu\nu}}\right|_{g=\mathbf{g}} = 0 \quad , \quad \left.\Gamma(h^*_{\mu\nu},J)\right|_{g=\mathbf{g}} = W(\mathbf{g},J)$$

• The description above in terms of the energy-momentum tensor "effective action" is a theory of (classical) dynamical gravity.

• The dynamical metric is (almost) the energy-momentum tensor vev, $h_{\mu\nu}$.

• Other sources like J represent energy-momentum carrying sources.

• This description is diff-invariant by construction. The related theory is a bi-gravity as it involves a dynamical metric $h_{\mu\nu}$ and a fixed fiducial metric, $\mathbf{g}_{\mu\nu}$.

• The interactions mediated by this (emergent) graviton are essentially summarizing exchanges of the energy-momentum tensor as we had postulated.

• The emergent graviton propagator (by construction) is generated by the poles of the energy-momentum tensor two-point function in the original theory.

We obtain at quadratic order, around flat space, by definition

$$S_{int} = \int \frac{d^4k}{(2\pi)^4}\ t_{\mu\nu}(k)\ \langle T^{\mu\nu} T^{\rho\sigma}\rangle\ t_{\rho\sigma}(-k) \qquad (1)$$

where the general form of the TT two-point function in momentum space with $\mathbf{g}_{\mu\nu} = \eta_{\mu\nu}$ is

$$\langle T_{\mu\nu} T_{\rho\sigma}\rangle(k) = -\frac{V}{2}\left(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right)$$

$$+ B_2(k)\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right] + \frac{B_0(k)}{3}\ \pi_{\mu\nu}\pi_{\rho\sigma}$$

$$B_0 = \frac{\pi^2}{40} k^4\ \int_0^\infty d\mu^2 \frac{\rho_0(\mu^2)}{k^2+\mu^2} \quad , \quad B_2 = \frac{3\pi^2}{80} k^4\ \int_0^\infty d\mu^2 \frac{\rho_2(\mu^2)}{k^2+\mu^2}$$

where

$$\langle T_{\mu\nu}\rangle \equiv V\eta_{\mu\nu} \quad , \quad \pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \quad , \quad k^\mu \pi_{\mu\nu} = 0\ . \qquad (2)$$

• There are three types of contact terms in $\langle TT \rangle$. The $O(k^0)$ are fixed by the translational Ward identity.

• There are $O(k^2)$ terms

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3\pi^2}{80}\ k^2\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]\delta_2 + \frac{\pi^2}{120}k^2\ \pi_{\mu\nu}\pi_{\rho\sigma}\ \delta_0$$

For IR regularity:

$$6\delta_2 + \delta_0 = 0$$

• There are $O(k^4)$ terms (scheme dependent)

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]k^4 A_2 + \frac{B_0(k)}{3}\pi_{\mu\nu}\pi_{\rho\sigma}k^4 A_0$$

• Ignoring the contact terms, the interaction mediated by $T_{\mu\nu}$ is given at the quadratic level by

$$W_2^{nc} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[2B_2^{nc}(k)\left(t^{\mu\nu}(k)t_{\mu\nu}(-k) - \frac{1}{3}t(k)t(-k)\right) + \frac{B_0^{nc}(k)}{3}t(k)t(-k)\right]$$

The tensor structure is that of a massive spin-2 exchange. For the non-contact contributions at small k

$$B_2(k) = c_{IR}^{(2)}k^4 \log\frac{k^2}{M^2} + \mathcal{O}(k^6) \quad , \quad B_0 \simeq c_{IR}^{(0)}k^4 \log\frac{k^2}{M^2} + \mathcal{O}(k^6)$$

• The interaction depends crucially on the structure of $B^{nc}_{2,0}$. If there is a mass gap and discrete states, then near a pole we can approximate

$$B_{2,0} \simeq \frac{R_{2,0}}{k^2 + m^2_{2,0}}$$

where the residue $R_{2,0}$ has mass dimension six as B has mass dimension four.

• The resulting interaction involves a massive spin-2 particle of mass m_2 and a massive spin-0 particle with mass m_0.

• In a unitary theory all residues are positive and the exchanges are never ghostlike.

• By an appropriate rescaling of the interacting densities, we find the associated "Planck scales" to be given by

$$M^2_{0,2} \sim \frac{V^2}{R_{2,0}}$$

• The associated field theory is a bi-gravity theory.

A low-energy effective action

• To try to discern the non-linear theory, we consider a theory with a gap and we write the most general Schwinger functional valid below the gap energy. We keep the metric $g_{\mu\nu}$, and a scalar source, ϕ

$$S_{\text{Schwinger}}(g,\phi) = \int d^4x\sqrt{g}\left[-V(\phi) + M^2(\phi)\,R - Z(\phi)\,(\partial\phi)^2 + \mathcal{O}(\partial^4)\right]$$

We now define $h_{\mu\nu}$ as the expectation value of the stress tensor

$$h_{\mu\nu} \equiv \langle T_{\mu\nu}\rangle = \frac{V}{2}g_{\mu\nu} + M^2 G_{\mu\nu} - \mathcal{T}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4)$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} \quad , \quad \mathcal{T}^{\phi}_{\mu\nu} \equiv T^{\phi}_{\mu\nu} + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)M^2$$

and the (emergent) dimensionless metric $\tilde{h}_{\mu\nu}$ as

$$\tilde{h}_{\mu\nu} \equiv \frac{2}{V} h_{\mu\nu} = g_{\mu\nu} + \frac{2M^2}{V} G_{\mu\nu}(g) - \frac{2}{V} \mathcal{T}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4) \ .$$

• We can now solve g as a function of $\tilde{h}$:

$$g_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{2M^2}{V} \tilde{G}_{\mu\nu} + \frac{2}{V} \tilde{\mathcal{T}}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4) \ .$$

• This is the dynamical equation stemming from the effective action Γ for the emergent metric that we rewrite as,

$$M^2 \ \tilde{G}_{\mu\nu} = \frac{V}{2} (\tilde{h}_{\mu\nu} - \mathbf{g}_{\mu\nu}) + \tilde{\mathcal{T}}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4) \ .$$

• This is the equation of a bi-gravity theory with $\mathbf{g}_{\mu\nu}$ as the fiducial metric.

• Here we see a general property of emergent gravity: $\tilde{h}_{\mu\nu} \sim \mathbf{g}_{\mu\nu}$ is always a solution if $\mathbf{g}_{\mu\nu}$ is a constant curvature metric and corresponds to the vacuum solution of the theory (independently of any quantum corrections) .

• However, and not surprisingly, when we linearize this equation and calculate the interaction it mediates, we obtain

$$S_{int}(T,T') = \frac{T^{\mu\nu}T'_{\mu\nu} - \frac{1}{3}TT'}{M^2(p^2-\Lambda)} - \frac{1}{6}\frac{TT'}{M^2\left(p^2+\frac{\Lambda}{2}\right)} \quad , \quad \Lambda = \frac{V}{M^2} \, ,$$

• From this interaction we conclude that the spin-zero mode is always a ghost. Moreover, depending on the sign of the vev Λ, either the spin-2 or the spin-0 exchange behaves as a tachyon.

• This discrepancy exists because expanding in derivatives the Schwinger functional, computing Γ and then computing the induced interaction back, mixes contact terms with pole terms, and therefore misidentifies masses and residues.

The linearized coupling

• We consider a hidden theory and a visible theory defined on the Minkowski metric $\mathbf{g}_{\mu\nu} = \eta_{\mu\nu}$.

• We consider a coupling between the "hidden theory" and the "visible theory" of the form

$$S_{int} = \int d^4x \left(\lambda \mathsf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x) + \lambda' \mathsf{T}(x) \widehat{\mathsf{T}}(x) \right)$$

at a high scale M where $\mathsf{T} \equiv \eta^{\mu\nu}\mathsf{T}_{\mu\nu}$.

This is an irrelevant coupling with $\lambda, \lambda' \sim M^{-4}$.

• $\mathsf{T}_{\mu\nu}$ is the SM energy-momentum tensor, $\widehat{\mathsf{T}}_{\mu\nu}$ is the hidden one.

• We also define

$$\mathfrak{c} \equiv \frac{\lambda'}{\lambda} \quad , \quad \mathbf{T}_{\mu\nu} \equiv \mathsf{T}_{\mu\nu} + \mathfrak{c}\, \mathsf{T}\, \eta_{\mu\nu}$$

so that

$$S_{int} = \lambda \int d^4x\, \mathbf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x)$$

• Note that the expectation value of the hidden energy momentum tensor, acts as an external metric for the SM.

$$\int d^4x \; \mathbf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x) \quad \to \quad \int d^4x \; \mathbf{T}_{\mu\nu}(x) h^{\mu\nu}$$

• We assume that $\langle \widehat{T}_{\mu\nu} \rangle = \widehat{\Lambda} \eta_{\mu\nu}$.

• The coupling has introduced the following effective interactions in the visible theory:

$$\delta S_{vis} = \lambda \widehat{\Lambda} \int d^4x \, \mathbf{T}(x) - \frac{1}{2}\lambda^2 \int d^4x_1 d^4x_2 \, \mathbf{T}_{\mu\nu}(x_1)\, \mathbf{T}_{\rho\sigma}(x_2)\, \widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(x_1 - x_2)$$

• The second term is an induced quadratic energy-momentum interaction in the visible theory.

• This interaction can be reformulated in terms of a classical spin-2 field $h_{\mu\nu}$

$$\delta S^{TT}_{eff} = \int d^4k \left[-h_{\mu\nu}(-k)\mathbf{T}^{\mu\nu}(k) + \frac{(2\pi)^4}{2\lambda^2} h_{\mu\nu}(-k) \quad \mathcal{P}^{\mu\nu,\rho\sigma}(k) \quad h_{\rho\sigma}(k) \right]$$

• The inverse propagator $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ of the emerging spin-2 field is the inverse of the hidden sector 2-point function $\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$.

• It remains to examine under what circumstances $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ is well-defined and what tensor structures it involves.

• We assume that the hidden theory is a Lorentz-invariant QFT.

$$\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k) = \widehat{\Lambda}\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) + \widehat{\mathsf{b}}(k^2)\Pi^{\mu\nu\rho\sigma}(k) + \widehat{\mathsf{c}}(k^2)\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\pi^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \quad , \quad \Pi^{\mu\nu,\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k)$$

• The only combination of tensor structures which is analytic at quadratic order in momentum, in the long-wavelength limit $k^2 \to 0$, is the one that has

$$\widehat{\mathsf{b}}(k^2) = \widehat{\mathsf{b}}_0 \; k^2 + \mathcal{O}(k^4) \; , \quad \widehat{\mathsf{c}}(k^2) = -2\widehat{\mathsf{b}}_0 \; k^2 + \mathcal{O}(k^4)$$

• If $\widehat{\Lambda} = 0$, the two-point function has zero modes which are proportional to k^μ and is therefore not invertible.

• In this case, one must invert in the space orthogonal to the zero modes. This gives rise to a non-local effective theory for the graviton.

• Up to quadratic order in the momentum expansion

$$\mathcal{P}^{\mu\nu\rho\sigma}(k) = -\frac{1}{4\widehat{\Lambda}}\left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$

$$+2\widehat{b}_0\widehat{\Lambda}^{-2}\left[\frac{k^2}{8}\left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)\right.$$

$$\left.+\frac{1}{8}(\eta^{\nu\sigma}k^\mu k^\rho + \eta^{\nu\rho}k^\mu k^\sigma + \eta^{\mu\sigma}k^\nu k^\rho + \eta^{\mu\rho}k^\nu k^\sigma)\right] + \mathcal{O}(k^4)$$

Emergent quadratic gravity

- We now re-define:

$$h_{\mu\nu} = -\mathfrak{h}_{\mu\nu} + \frac{1}{2}\mathfrak{h}\,\eta_{\mu\nu} + \lambda\widehat{\Lambda}\,\eta_{\mu\nu}\ , \quad \mathfrak{h} = \mathfrak{h}^{\rho\sigma}\eta_{\rho\sigma}$$

$$\mathfrak{T}^{\mu\nu} \equiv \mathbf{T}^{\mu\nu} - \frac{1}{\lambda}\left(1 + \frac{1}{2\lambda\,\widehat{\Lambda}}\right)\eta^{\mu\nu}\ , \quad \mathfrak{T} = \mathfrak{T}^{\mu\nu}\eta_{\mu\nu}$$

- The full effective action of the visible QFT at this order in the λ-expansion and at the two-derivative level is

$$S_{eff} = S_{vis} + \int d^4x\,\left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) + \frac{1}{16\pi G}\int d^4x\left[\sqrt{g}\,(R + \Lambda)\right]^{(2)}_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

with the identification of parameters

$$\Lambda = \frac{\widehat{\Lambda}}{\widehat{b}_0}\ , \quad \frac{1}{16\pi G} \equiv M_P^2 = -\frac{(2\pi)^8\ \widehat{b}_0}{\lambda^2\widehat{\Lambda}^2}$$

- The sign of Newton's constant is positive when $\widehat{b}_0$ is negative

• This seems to be the case with simple QFTs but we have no general proof.

• The second term, which describes the coupling of the visible QFT to the emergent graviton, can be expressed in terms of the original energy-momentum tensor of the visible QFT

$$\int d^4x \left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) = \int d^4x\sqrt{g}\ g^{\mu\nu}\ \mathfrak{T}_{\mu\nu}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

• There is a non-trivial shift of the energy due to the coupling of the two theories.

• Because of the presence of "dark energy" the flat (fiducial) metric is always a solution to the equations of emergent gravity.

• However as before, we will do the computation without expanding in momenta.

Emergent quadratic gravity II

We can compute the (non-contact part of the) induced interaction between SM sources without expanding in momenta

$$L_{int} = -\frac{\lambda^2}{2}\left[2B_2(k)\left(T_{\mu\nu}(-k)T^{\mu\nu}(k) - \frac{1}{3}T(-k)T(k)\right) + \frac{(1+3\mathfrak{c})^2}{3}B_0(k)\right] + \cdots$$

where $\mathfrak{c}$ is defined by

$$S_{int} = \lambda \int d^4x \left[\hat{T}_{\mu\nu}T^{\mu\nu} + \mathfrak{c}\ \hat{T}T\right]$$

- The tensor structure of the interaction is that of massive gravity.

- At the special (integrable) value $\mathfrak{c} = -\frac{1}{3}$ the scalar dilaton decouples.

Taylor

• Both the spin-2 and spin-0 interactions are always attractive and stable.

• Around a massive pole, of mass m_2 (assuming $R \sim m_2^6$) we obtain

$$M_P^2 \sim \frac{M^8}{R_2} \sim M^2 \left(\frac{M}{m_2}\right)^6$$

• A generalization of the formalism of the effective action allows us to (formally) construct the full non-linear theory.

The non-linear analysis

• We start again from the Schwinger functional of the coupled QFTs $W\left(\mathcal{J},\hat{\mathcal{J}},\mathbf{g}\right)$

• The interaction is defined as general as possible:

$$S_{int} = \int d^4x\sqrt{\mathbf{g}}\sum_i \lambda_i\,\mathcal{O}_i(x)\,\widehat{\mathcal{O}}_i(x)$$

• Via similar techniques a functional $S_{eff}(h)$ can be constructed and satisfies

$$\left.\frac{\delta S_{eff}}{\delta h_{\mu\nu}}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = 0$$

♠ These are the emerging non-linear gravitational equations.

♠ When evaluated in the solution of the above equation gives the original action.

• Therefore, $S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g})$ is the emergent gravity action that generalizes the linearized computation.

The holographic hidden QFT

• The general action is

$$S = S_{hidden} + S_{T\hat{T}} + S_{visible}$$

• Using the holographic correspondence

$$\langle e^{iS_{T\hat{T}}}\rangle_{hidden} = \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\mathbf{g}_{\mu\nu}} \mathcal{D}G\ e^{iS_{\text{bulk}}[G]+i\lambda\int d^4x\,\sqrt{\mathbf{g}}\,\widehat{T}_{\mu\nu}\mathbf{T}^{\mu\nu}}$$

with $z_0 \sim \frac{1}{M}$.

• It is also true that

$$\langle e^{iS_{T\hat{T}}}\rangle_1 = \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}} \mathcal{D}G\ e^{iS_{\text{bulk}}[G]}$$

• By a series of formal manipulations we can show that this is equivalent to a brane (visible theory) coupled to the holographic bulk, but with Neumann bcs.

The brane graviton

- The induced interaction due to the transverse-traceless fluctuation is

$$S_{int} = -\frac{1}{2M^3}\int d^4x\, d^4x'\, G(r_0, x; r_0, x') \left(T^{\mu\nu}(x)T_{\mu\nu}(x') - \frac{1}{3}T(x)T(x')\right)$$

$$G(r, x; r_0, x') = \frac{1}{\frac{1}{G_{bulk}(r,x;r_0,x')} + G_{brane}(x, x')}$$

Dvali+Gabadadze+Porrati

- This should be contrasted with the field-theoretical formula

$$Interaction\ \ of\ \ energy\ \ sources = \frac{1}{\frac{1}{\lambda^2\langle\hat{T}\hat{T}\rangle_{hidden}} + \langle TT\rangle_{SM}} = \frac{\langle\hat{T}\hat{T}\rangle_{hidden}}{1 + \langle\hat{T}\hat{T}\rangle_{hidden}\langle TT\rangle_{S}}$$

- As $\langle\hat{T}\hat{T}\rangle_{hidden} \sim \mathcal{O}(1)$, $\lambda \sim \mathcal{O}(N^{-1})$ the SM corrections shift slightly the poles of $\langle\hat{T}\hat{T}\rangle_{hidden}$ that are at $m \sim \mathcal{O}(1)$.

• There are the following characteristic distance scales.

• The *transition scale* r_t around which $G_{bulk}(r_0, p)$ changes from small to large momentum asymptotics:

• The DGP scale, r_c:

$$r_c \equiv \frac{U_0}{2};$$

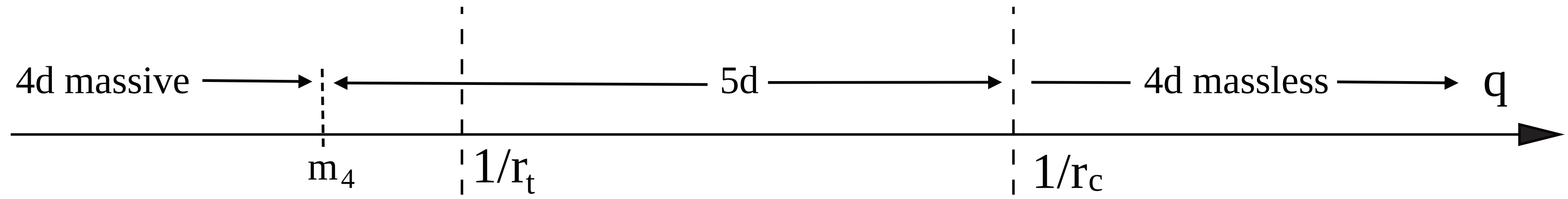
This scale determines the crossover between 5-dimensional and 4-dimensional behavior, and enters the 4D Planck scale and the graviton mass.

• The *gap scale* d_0

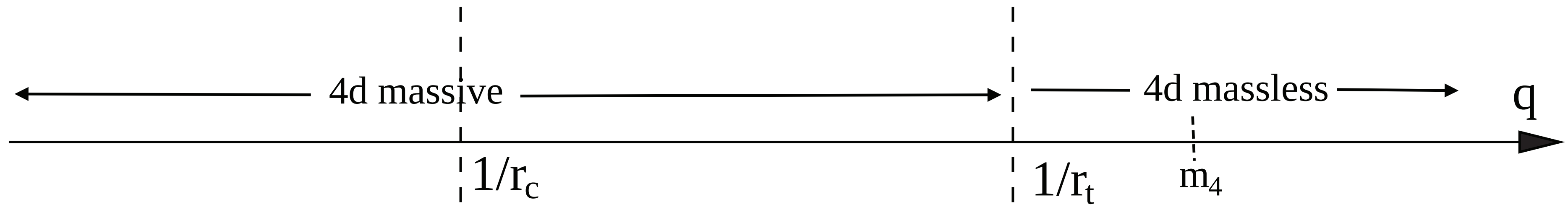
$$d_0 \equiv G_{bulk}(r_0, 0) = \int_0^{r_0} dr' e^{-3A_{UV}(r')},$$

which governs the propagator at the largest distances (in particular it sets the graviton mass).

• When $r_t > r_c$ we have three regimes for the gravitational interaction on the brane:



• Massive 4d gravity ($r_t < r_c$)



• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive. It should be cancelled by the Vainshtein mechanism.

Scalar Perturbations

• The equations for the scalar perturbations can be derived and they are complicated.

• There are two scalar modes on the brane:

• In general the two scalar modes couple to two charges:

(a) the "scalar charge" and

(b) the trace of the brane stress tensor.

• The mode that couples to the trace of the stress-tensor has a mass that is of order the graviton mass and is the lightest of the two scalars.

• All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

Translation Ward identity

• We consider a theory with Lagrangian $\mathcal{L}$. For concreteness, we focus on four-dimensional QFTs.

• Under an infinitesimal diffeomorphism generated by a vector ξ_μ

$$\delta_\xi \mathcal{L} = \frac{1}{2} (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu) T^{\mu\nu}$$

$$\delta_\xi T^{\mu\nu} = \xi^\sigma \partial_\sigma T^{\mu\nu} + T^{\sigma\nu} \partial^\mu \xi_\sigma + T^{\mu\sigma} \partial^\nu \xi_\sigma$$

• The invariance of the partition function $Z = e^{i \int d^4x\, \mathcal{L}}$ under the infinitesimal translation implies the conservation equation

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

• Similarly, the invariance of the one-point function of the energy-momentum tensor

$$\langle T^{\rho\sigma}(y)\rangle = \frac{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}\, T^{\rho\sigma}(y)}{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}}$$

under the infinitesimal translations implies the Ward identity

$$-i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle + \delta(x-y)\langle\partial^\nu T^{\rho\sigma}(x)\rangle + \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$-\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) - \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

• In addition, Lorentz invariance implies that the one-point function of the energy-momentum tensor is

$$\langle T^{\mu\nu}(x)\rangle = a\eta^{\mu\nu}$$

where a is a dimensionfull constant.

Consequently, we set

$$\langle\partial^\nu T^{\rho\sigma}(x)\rangle = 0$$

and use it to simplify the Ward identity

$$i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle - \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$+\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) + \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

• In momentum space we obtain instead:

$$k_\mu\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle = ia\left(-k^\nu\eta^{\rho\sigma} + k^\rho\eta^{\nu\sigma} + k^\sigma\eta^{\rho\nu}\right)$$

• This allows us to deduce the 2-point function as **??**

$$\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle$$

$$= ia\left(-\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) + b(k^2)\,\Pi^{\mu\nu\rho\sigma}(k) + c(k^2)\,\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\Pi^{\mu\nu\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k) \quad , \quad \pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

Aside: String theory vs the swampland

• Conjectures talk about "quantum gravity" but everyone means "string theory"

• The (plausible) assumption that string theory is the space of large-N strongly coupled QFTs, has an automatic avatar:

• The "swampland" corresponds to QFTs that are either weakly-coupled, or are not at large N.

• This explains for example, the generic towers of states that appear at the boundaries of moduli spaces.

• It also suggests why there might be no de Sitter solution in "string theory".

• The notion of string theory used above is certainly more general that the conventional one based on 2d CFTs

• It involves also 3, 4, 5 and 6-dimensional CFTs.

• It might be illuminating to try to see the swampland conjectures via this point of view.

Higher spin

• It is one of the obvious next questions to ask: what about doing this for other operators of your QFT:

• For fields up to $S = 1/2$ this is a standard procedure, and has been done in many contexts.

• The case of $S = 1$ is interesting as it would describe emergent gauge theory. It is qualitatively different than the gravity case.

• When $S > 2$ one can again do the same procedure as here.

• In that case however for interacting theories, higher spin fields are not conserved. The effective theory one obtains will be massive, with characteristic mass the overall cutoff (in string theory this is the string scale).

• They are therefore less interesting for low-energy physics.

• In a free QFT however they are conserved and then one can construct massless actions (of an infinite number of them)

Douglas+Razamat, Leigh

WW versus AdS/CFT

• Is AdS/CFT compatible with the WW theorem?

• The WW theorem involves a subtle limit to define the helicity amplitudes that determine the couplings of massless states to the stress tensor or a local current.

• This limiting procedure is not valid in theories where the states form a continuum.

• This is the case in AdS/CFT.

• From the point of view of the QFT, the effective gravitational coupling is non-local.

• Therefore the WW-theorem does not apply to this case.

• What about non-CFTs?

WW versus nAdS/nCFT

- Consider a familiar example: four-dimensional, large-N YM theory.

- Its string-theory dual is stringy (and nearly tensionless) near the AdS-boundary (weak QFT coupling).

- We expect a gravitational description at low energies (strong QFT coupling).

- The theory has a gap and a discrete spectrum and therefore the emergent gravitational interactions must be local.

- Also gravity must be weakly coupled (and it is, due to large N limit).

• The low energy spectrum contains two stable (lightest) massive scalars (0^{++}, 0^{-+}), and a stable massive graviton (2^{++}). All other glueballs are resonances, and are not asymptotic states.

• The higher cousins of the graviton are unstable.

• A massive graviton is compatible with WW.

• It is also compatible with a fully diff invariant theory of a massless graviton in 5 dimensions.

• The 4d graviton mass is due to the non-trivial 5d background, hence a gravitational "Higgs effect".

• The above gives some credence to the idea that heavy-ion collisions form (unstable) black holes of a massive gravity theory that quickly Hawking evaporate.

Nastase, Kiritsis+Taliotis

An explicit IR parametrization

- We assume that the theory has a uniform mass gap for simplicity.

- We will now parametrize the Schwinger functional W in an IR expansion below the mass gap as

$$W(g,J)=\int\sqrt{g}\left[-V(J)+M^2(J)R(g)-\frac{Z(J)}{2}(\partial J)^2+\mathcal{O}(\partial^4)\right]$$

- We calculate

$$h_{\mu\nu}=\frac{V}{2}g_{\mu\nu}+M^2G_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)M^2-\frac{1}{2}T_{\mu\nu}+\cdots$$

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}R\;g_{\mu\nu}\quad,\quad T_{\mu\nu}=Z(J)\left(\partial_\mu J\partial_\nu J-\frac{1}{2}g_{\mu\nu}(\partial J)^2\right)$$

- The $h_{\mu\nu}$ appears uniquely determined, but there is a initial+boundary condition dependence in this formula.

- Note that for arbitrary external source J, this energy-momentum tensor vev is not conserved.

$$\nabla_g{}^\mu h_{\mu\nu}=\frac{1}{2}\left[V(J)'-Z(J)\Box_g J-\frac{1}{2}Z'(J)(\partial J)^2-(M(J)^2)'\;R\right]\partial_\nu J$$

• We may now solve $g_{\mu\nu}$ as a function of $h_{\mu\nu}$:

$$g_{\mu\nu} = \tilde{h}_{\mu\nu} - \delta\tilde{h}_{\mu\nu} \quad , \quad \tilde{h}_{\mu\nu} = \frac{2}{V} h_{\mu\nu}$$

$$\delta\tilde{h}_{\mu\nu} = \frac{2}{V}\left[M^2 \tilde{G}_{\mu\nu} - (\tilde{\nabla}_\mu \tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M^2\right] - \frac{1}{V}\tilde{T}_{\mu\nu} + \cdots$$

• All the tensors above are written in terms of $\tilde{h}_{\mu\nu}$.

• $\tilde{h}_{\mu\nu}$ is dimensionless and plays the role of the emergent dynamical metric.

• We may rewrite it as an Einstein equation coupled to "matter"

$$M^2\ \tilde{G}_{\mu\nu} = \frac{V(J)}{2}\left(\tilde{h}_{\mu\nu} - \mathbf{g}_{\mu\nu}\right) + \frac{1}{2}\tilde{T}_{\mu\nu}(J) + (\tilde{\nabla}_\mu \tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M(J)^2 + \cdots$$

• The effective gravitational equation above is equivalent to $\frac{\delta\Gamma}{\delta h_{\mu\nu}} = 0$.

• The background metric $\mathbf{g}_{\mu\nu}$ appears as an external source and contributes like a cosmological constant.

• This is an "unusual" bigravity theory.

• Other sources act as sources of energy and momentum.

• This description is non-singular only if $V \neq 0$.

• If $V = 0$, then the gravitational theory is non-local but can be constructed.

• Note that when $J(x) \neq 0$ the original QFT is not translationally invariant and its energy-momentum tensor is not conserved.

• The emergent gravity theory is however still diff. invariant, and the diff. invariance is broken "spontaneously" because of the presence of the scalar source $J(x)$ and the fixed (fiducial) metric of the original QFT.

BACK

Emerging quadratic gravity:Comments

• A coupling of stress tensors between two theories induces gravity at the quadratic level.

• This is true in the generic case: $\widehat{\Lambda} \neq 0$.

• Otherwise the graviton theory is non-local.

• There is always an effective cosmological constant for the emerging gravity in the local case.

• There is also a shift of the stress tensor giving a "dark" energy. It is a reflection of the coupling to the hidden theory.

• We parametrize $\lambda = \frac{1}{NM^4}$ where M a large scale controlling the coupling of the two theories and N the number of colors of the hidden theory.

• Also from calculations

$$\hat{b}_0 = -\kappa N^2 m^2 \quad , \quad \kappa \sim O(1) \quad , \quad \hat{\Lambda} = \epsilon\ N^2\ m^4 \quad , \quad \epsilon = \pm 1 \tag{3}$$

We may now calculate the relevant ratios of scales

$$\frac{\Lambda}{M_P^2} = -\frac{\epsilon}{\kappa^2 x^2} \quad , \quad \frac{\Lambda_{dark}}{M_P^2} = -\frac{\frac{N}{x} + \frac{\epsilon}{2(2\pi)^4}}{(1+4\mathfrak{c})\kappa^2\ x^2} \quad , \quad \frac{m}{M_P} = \frac{1}{\sqrt{\kappa}\ x} \tag{4}$$

$$\frac{\Lambda_{dark}}{\Lambda} = \frac{\epsilon\frac{N}{x} + \frac{1}{2(2\pi)^4}}{(1+4\mathfrak{c})} \quad , \quad \frac{M^4}{M_P^4} = \frac{1}{\kappa^2 x^3} \quad , \quad x \equiv \frac{M^4}{m^4} \gg 1 \tag{5}$$

• We always have semiclassical gravity, $\Lambda \ll M_P^2$.

- If $N \lesssim x$ then

$$\Lambda \sim \Lambda_{\rm dark} \sim \mathcal{O}(m^2) \ll M^2 \ll M_P^2$$

- If $x \ll N \ll x^{\frac{3}{2}}$ then

$$\Lambda \ll \Lambda_{\rm dark} \ll M^2 \ll M_P^2$$

- If $x^{\frac{3}{2}} \ll N \ll x^3$ then

$$\Lambda \ll M^2 \ll \Lambda_{\rm dark} \ll M_P^2$$

- If $N \gg x^3$ then

$$\Lambda \ll M^2 \ll M_P^2 \ll \Lambda_{\rm dark}$$

- For phenomenological purposes $x \lesssim 10^{20}$ so that the messenger scale is above experimental thresholds.

- Note that so far the SM quantum effects are not included.

Renormalization and contact terms in $\langle TT \rangle$

• There can exist various issues when trying to formulate a spectral representation of correlators in momentum space and the integral over the spectral factor will generically exhibit divergences.

In momentum space we have (without the constant contact term)

$$\langle T_{\mu\nu} T_{\rho\sigma} \rangle (k) = \frac{(d-1)^2 \mathcal{A}_d}{2\Gamma(d)} \; k^4 \; \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{d-1}\pi_{\mu\nu}\pi_{\rho\sigma} \right] \bar{G}_2 +$$

$$+ \frac{\mathcal{A}_d}{\Gamma(d)} k^4 \; \pi_{\mu\nu}\pi_{\rho\sigma} \; \bar{G}_0 \equiv$$

$$\equiv B_2(k) \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{d-1}\pi_{\mu\nu}\pi_{\rho\sigma} \right] + \frac{B_0(k)}{3}\pi_{\mu\nu}\pi_{\rho\sigma}$$

with

$$\bar{G}_i(k) = \int_0^\infty d\mu^2 \frac{\rho_i(\mu^2)}{k^2 + \mu^2} \quad , \quad i = 0, 2$$

$$\mathcal{A}_d = \frac{2\pi^{d/2}}{(d+1)2^{d-1}\Gamma(d/2)} \quad , \quad \pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \quad , \quad k^\mu \pi_{\mu\nu} = 0$$

• In $d=4$ the spectral functions $B_{2,0}$ are related to the rest as

$$B_0 = \frac{\pi^2}{40} k^4 \ \bar{G}_0(k) \quad , \quad B_2 = \frac{3\pi^2}{80} k^4 \bar{G}_2$$

• Typically, the integral over μ^2 does not converge either at zero or infinity.

• We can rearrange the integral so that we can separate the UV and IR divergences by using the identity

$$\frac{\rho_i(\mu^2)}{k^2+\mu^2} = \frac{\rho_i(\mu^2)}{\mu^2 + {m_{IR}}^2} - (k^2 - {m_{IR}}^2) \frac{\rho_i(\mu^2)}{(\mu^2 + {m_{IR}}^2)(k^2+\mu^2)}$$

and rewrite

$$\bar{G}_i(k) = A_i - (k^2 - m_{IR}^2) \int_0^\infty \frac{d\mu^2}{(\mu^2 + m_{IR}^2)} \frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

with

$$A_i \equiv \int_0^\infty d\mu^2 \frac{\rho_i(\mu^2)}{\mu^2 + m_{IR}^2}$$

• m_{IR} acts as an IR cutoff and is needed if the theory in question is massless

• Convergence in the IR assumes that $\lim_{\mu\to 0}\mu^2\rho_i(\mu^2)=0.$. This happens if the IR CFT is non-empty.

• On the other hand, all UV divergences are now hidden in A_i. We may introduce a UV cutoff Λ and define

$$A_i^c(\Lambda, m_{IR}) \equiv \int_0^{\Lambda^2} d\mu^2 \frac{\rho_i(\mu^2)}{\mu^2+m_{IR}^2}$$

so that the cutoff spectral functions are

$$\bar{G}_i^c(k) = A_i^c - (k^2-m_{IR}^2)\int_0^{\infty}\frac{d\mu^2}{(\mu^2+m_{IR}^2)}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

• As $\Lambda\to\infty$, we have a finite number of divergent terms, starting with a single logarithm in $d=4$,

$$A_i^c \simeq c_i^{UV}\Lambda^{d-4}+d_i^{UV}\Lambda^{d-6}+\cdots+e_i^{UV}\log\Lambda^2+\cdots \quad , \quad d\geq 4 \quad , \quad d=even$$

$$A_i^c \simeq c_i^{UV}\Lambda^{d-4}+d_i^{UV}\Lambda^{d-6}+\cdots+e_i^{UV}\Lambda+\cdots \quad , \quad d>4 \quad , \quad d=odd$$

• We then define the renormalized A_i by subtracting the divergences and eventually a finite piece, and then taking the UV cutoff to infinity.

$$A_i^{ren}(m_{IR}) = \lim_{\Lambda\to\infty} (A_i^c - \text{UV} \quad \text{divergences})$$

• $A_i^{ren}(m_{IR})$ is now a finite contact term that still depends in general on m_{IR}, if the IR theory is a non-trivial CFT.

• It is important to mention that the UV divergences do not depend on m_{IR}, and therefore the subtracted piece does not depend on m_{IR}. This will guarantee that the final renormalized density is m_{IR} -independent.

• Finally the renormalized $\bar{G}_i$ is given by

$$\bar{G}_i^{ren} \equiv A_i^{ren}(m_{IR}) - (k^2 - m_{IR}^2) \int_0^\infty \frac{d\mu^2}{(\mu^2 + m_{IR}^2)} \frac{\rho_i(\mu^2)}{(k^2 + \mu^2)}$$

and is independent of m_{IR}.

• For a CFT_4 we have $\rho_i(\mu^2) = c_i$ and we obtain

$$A_i^c = c_i \log \frac{\Lambda^2 + m_{IR}^2}{m_{IR}^2}$$

- This can be renormalized by subtracting the leading UV divergence

$$A_i^{ren} \equiv \lim_{\Lambda\to\infty}\left(A_i^c - c_i \log\frac{\Lambda^2}{M^2}\right) = c_i \log\frac{M^2}{m_{IR}^2}$$

- The scheme dependence is associated with the value of M.

- The renormalized $\bar{G}$ for a CFT$_4$ is then

$$\bar{G}_i^{ren} = A_i^{ren} - (k^2 - m_{IR}^2)\int_0^\infty \frac{d\mu^2}{(\mu^2 + m_{IR}^2)}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)} =$$

$$= c_i\left[\log\frac{M^2}{m_{IR}^2} - (k^2 - m_{IR}^2)\int_0^\infty \frac{d\mu^2}{(\mu^2+m_{IR}^2)(k^2+\mu^2)}\right] = -c_i \log\frac{k^2}{M^2}$$

where M is a renormalization group scale.

- The appearance of the arbitrary scale M in the momentum space correlator is another avatar of the conformal anomaly.

- For a theory with a mass gap, we can set the scale $m_{IR} = 0$ and we can rewrite

$$\bar{G}_i^{ren} \equiv A_i^{ren} - k^2\int_0^\infty \frac{d\mu^2}{\mu^2}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

• In $d = 4$ the A_i^{ren} are dimensionless contact terms whose value depends on the renormalization scheme.

• The low momentum expansion becomes

$$\bar{G}_i^{ren} \equiv A_i^{ren} - B_i\ k^2 + \mathcal{O}(k^4) \quad , \quad B_i \equiv \int_{m_0^2}^{\infty} \frac{d\mu^2}{\mu^4} \rho_i(\mu^2)$$

where m_0 is the mass gap of the correlator.

• For a general four-dimensional theory without a mass gap we have that

$$\rho_i(\mu^2) \simeq c_i^{UV} \qquad for \qquad \mu \to \infty$$

while

$$\rho_i(\mu^2) \simeq c_i^{IR} \qquad for \qquad \mu \to 0$$

• We pick two scales, $m_1 \to 0$ so that it is much smaller that all the scale of the theory, while $m_2 \to \infty$ is much larger than all scales of the theory (except the UV cutoff) and write

$$\bar{G}_i^{ren} \equiv A_i^{ren}(m_{IR}) - I_{IR}^i - I_{UV}^i - I_{inter}^i$$

with

$$I^i_{IR} \equiv (k^2 - m^2_{IR}) \int_0^{m_1^2} \frac{d\mu^2}{(\mu^2 + m^2_{IR})} \frac{\rho_i(\mu^2)}{(k^2 + \mu^2)} \simeq c_i^{IR}(k^2 - m^2_{IR}) \int_0^{m_1^2} \frac{d\mu^2}{(\mu^2 + m^2_{IR})(k^2}$$

$$= c_i^{IR} \left[\log \frac{(m_1^2 + k^2)}{k^2} + \log \frac{m^2_{IR}}{(m_1^2 + m^2_{IR})} \right]$$

$$I^i_{UV} \equiv (k^2 - m^2_{IR}) \int_{m_2^2}^{\infty} \frac{d\mu^2}{(\mu^2 + m^2_{IR})} \frac{\rho_i(\mu^2)}{(k^2 + \mu^2)} \simeq c_i^{UV}(k^2 - m^2_{IR}) \int_{m_2^2}^{\infty} \frac{d\mu^2}{(\mu^2 + m^2_{IR})(k^2}$$

$$= c_i^{UV} \log \frac{m_2^2 + k^2}{m_2^2 + m^2_{IR}}$$

and

$$I_{inter} \equiv (k^2 - m^2_{IR}) \int_{m_1^2}^{m_2^2} \frac{d\mu^2}{(\mu^2 + m^2_{IR})} \frac{\rho_i(\mu^2)}{(k^2 + \mu^2)}$$

- From these expressions, we deduce that I_{inter} is a regular power series in k^2 for k^2 small.

- Therefore in $\bar{G}_i$ there is only a log k^2 divergence that is appearing due to the IR CFT.

• For a gapless theory, we can write a small k^2 expansion that is of the form

$$\bar{G}_i = c_i^{IR} \log \frac{M^2}{k^2} + \text{regular} \quad \text{expansion} \quad \text{in} \quad k^2$$

and where M^2 is some scale of the theory.

• On the other hand, as $k^2 \to \infty$ we obtain

$$I_{IR} \simeq \text{regular} \quad \text{series} \quad \text{in} \quad \frac{1}{k^2} \quad , \quad I_{UV} \simeq c_i^{UV} \log k^2 + \text{regular} \quad \text{series} \quad \text{in} \quad \frac{1}{k^2}$$

$$I_{inter} = \text{regular} \quad \text{series} \quad \text{in} \quad \frac{1}{k^2}$$

so that

$$\bar{G}_i = c_i^{UV} \log \frac{k^2}{M'^2} + \text{regular} \quad \text{expansion} \quad \text{in} \quad \frac{1}{k^2}$$

as $k^2 \to \infty$.

• As $k^2 \to 0$, $\bar{G}_i^{ren}$ are regular functions of k^2 with an exception of a $\log k^2$ appearance, if the theory is gapless.

• There is, however, a set of contact terms, compatible with stress tensor conservation and IR regularity that are not included.

$$\bar{G}_i^{ren}(k) \to G_i^{ren} + \frac{\delta_i}{k^2}$$

Then

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3\mathcal{A}_4}{4}\; k^2 \;\left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]\delta_2 + \frac{\mathcal{A}_4}{6}k^2\; \pi_{\mu\nu}\pi_{\rho\sigma}\; \delta_0$$

and the absence of the $\frac{k_\mu k_\nu k_\rho k_\sigma}{k^2}$ term implies that

$$6\delta_2 + \delta_0 = 0$$

and inserting in $\langle TT\rangle$ we obtain

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3\pi^2\delta_2}{80}\left[k^2(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\mu\rho} - 2\delta_{\mu\nu}\delta_{\rho\sigma}) - \right.$$

$$\left. -(\delta_{\mu\rho}k_\nu k_\sigma + \delta_{\nu\sigma}k_\mu k_\rho + \delta_{\mu\sigma}k_\nu k_\rho + \delta_{\nu\rho}k_\mu k_\sigma) + 2\delta_{\mu\nu}k_\rho k_\sigma + 2\delta_{\rho\sigma}k_\mu k_\nu\right]$$

• It is clear that if $\delta_2 > 0$, then $\delta_0 < 0$ and the spin-zero piece of this particular term is ghost-like.

• Summarizing, the explicit contact contributions in the renormalized stress tensor functions $\bar{G}_i^{ren}$ in four-dimensions are

$$\bar{G}_2^{ren,contact}(k) = A_2^{ren} + \frac{\delta_2}{k^2} \quad , \quad \bar{G}_0^{ren,contact}(k) = A_0^{ren} - \frac{6\delta_2}{k^2}$$

Mixing with contact terms

We consider a quadratic source functional

$$W(J) = \int d^4p \; J(-p)G(p)J(p) \quad , \quad G(p) = G_0 + \frac{R}{p^2 - m^2}$$

where we took the two-point correlator to have a pole and a constant contact term. It is clear that the interaction of the source J contains an innocuous contact term contribution and the effect of the exchange of a particle of mass m and residue R.

• Consider now the following sequence of steps. Expand $W(J)$ up to $\mathcal{O}(p^2)$, construct the effective action Γ to order $\mathcal{O}(p^2)$ and then recompute the interaction of sources.

$$W(J) = \int d^4p \; J(-p)J(p) \left[\tilde{G}_0 - \frac{Rp^2}{m^4} + \mathcal{O}(p^4)\right] \quad , \quad \tilde{G}_0 = G_0 - \frac{R}{m^2}$$

$$h(p) = \frac{\delta W}{\delta J(-p)} = 2J(p) \left[\tilde{G}_0 - \frac{Rp^2}{m^4} + \mathcal{O}(p^4)\right]$$

$$\Gamma(h) = \int Jh - W = \frac{1}{4}\int d^4p \; h(-p)\left[\tilde{G}_0 - \frac{Rp^2}{m^4} + \mathcal{O}(p^4)\right]^{-1} h(p) =$$

$$= \frac{1}{4\tilde{G}_0}\int d^4p \; h(-p)\left[1 + \frac{Rp^2}{m^4\tilde{G}_0} + \mathcal{O}(p^4)\right] h(p)$$

Recomputing the original interaction we obtain instead

$$W(J) = \frac{m^4\tilde{G}_0^2}{R}\int d^4p \; \frac{J(-p)J(p)}{p^2 + \frac{m^4}{R}\tilde{G}_0} + \mathcal{O}(p^4)$$

• Comparing we observe that now both the residue and the position of the pole has changed.

• The reason is that the position of the pole is now not reliable in the momentum expansion. Moreover, depending on the sign and size of the initial contact term, G_0, the pole now may become a tachyon.

The non-linear analysis

- We start again from the Schwinger functional of the coupled QFTs

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi]\,[D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\mathbf{g},\hat{\mathcal{J}}\right)-S_{int}\left(\mathcal{O}^i,\widehat{\mathcal{O}}^i,\mathbf{g}\right)}$$

- Φ^i and $\widehat{\Phi}^i$ are respectively the (quantum) fields of the visible QFT and the hidden $\widehat{\text{QFT}}$.

- $\mathcal{J}$ and $\hat{\mathcal{J}}$ are (scalar) sources in the visible and hidden theories respectively.

- The interaction part is defined as:

$$S_{int} = \int d^4x\sqrt{\mathbf{g}}\sum_i \lambda_i\,\mathcal{O}_i(x)\,\widehat{\mathcal{O}}_i(x)$$

- For energies $E \ll M$, we can integrate out the hidden theory and obtain

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi][D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\hat{\mathcal{J}},\mathbf{g}\right)-S_{int}}$$

$$= \int [D\Phi]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-\mathcal{W}(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g},)}$$

• We now put the full theory on a curved manifold with metric $g_{\mu\nu}$ and define again the generating functional in the presence of the background metric as

$$e^{-W(\mathcal{J},g,\hat{\mathcal{J}})} = \int [D\Phi]\, e^{-S_{visible}(\Phi,\mathcal{J},g) - \mathcal{W}(\mathcal{O}^i + \hat{\mathcal{J}}^i, g)}$$

• We define

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{g}} \frac{\delta \mathcal{W}\left(\mathcal{O}^i, g, \hat{\mathcal{J}}\right)}{\delta g^{\mu\nu}}\Big|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = \langle \widehat{\mathsf{T}}_{\mu\nu} \rangle$$

• This will eventually play the role of an emergent metric for the visible theory.

• The diffeomorphism invariance of the functional $W(\mathcal{J}, g, \hat{\mathcal{J}})$ is reflecting (as usual) the translational invariance of the underlying QFT.

• We define the Legendre-transformed functional

$$S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g}) = S_{vis}(\mathbf{g},\Phi,\mathcal{J}) - \int d^4x \sqrt{g(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)}\, h_{\mu\nu}\times$$

$$\times\left[g^{\mu\nu}(\mathcal{O}^i+\hat{\mathcal{J}}^i,h) - \mathbf{g}^{\mu\nu}\right] + \mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i, g(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)\right)$$

We can show that:

♠ This functional satisfies

$$\left.\frac{\delta S_{eff}}{\delta h_{\mu\nu}}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = 0$$

♠ These are the emerging non-linear gravitational equations.

♠ When evaluated in the solution of the above equation gives the original action.

$$, \quad \left. S_{eff}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = S_{visible} + \mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g}\right)$$

• Therefore, $S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g})$ is the emergent gravity action that generalizes the linearized computation.

The brane-bulk setup

- The general action is

$$S = S_{hidden} + S_{T\hat{T}} + S_{visible}$$

- Using the holographic correspondence

$$\langle e^{iS_{T\hat{T}}} \rangle_{hidden} = \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\mathbf{g}_{\mu\nu}} \mathcal{D}G \ e^{iS_{\text{bulk}}[G]+i\lambda\int d^4x \sqrt{\mathbf{g}}\,\widehat{T}_{\mu\nu}\mathbf{T}^{\mu\nu}}$$

with $z_0 \sim \frac{1}{M}$.

- It is also true that

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}} \mathcal{D}G \ e^{iS_{\text{bulk}}[G]}$$

- By inserting a functional δ-function we may rewrite it as

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int \mathcal{D}\chi\mathcal{D}h \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\chi_{\mu\nu}} \mathcal{D}G \ e^{iS_{\text{bulk}}[G]-i\int d^4x h^{\mu\nu}(x)(\chi_{\mu\nu}(x)-\mathbf{g}_{\mu\nu}-\lambda\mathbf{T}_{\mu\nu}(x))}$$

• The total Schwinger functional is represented semi-holographically by substituting the previous equation into

$$e^{iW(\mathbf{g})} = \int \mathcal{D}\Phi_{vis}\, e^{iS_{visible}(\Phi_{vis},\mathbf{g})}\, \langle e^{iS_{T\hat{T}}} \rangle$$

• We now change perspective and integrate $\chi_{\mu\nu}(x)$ first in the path integral

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int \mathcal{D}\chi \int \mathcal{D}h e^{i\int d^4x\ h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda \mathbf{T}_{\mu\nu}(x))} \times$$

$$\times \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\chi_{\mu\nu}} \mathcal{D}G\ e^{iS_{\text{bulk}}[G]-i\int d^4x\ h^{\mu\nu}(x)\chi_{\mu\nu}(x)}\,.$$

This is equivalent to

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int \mathcal{D}h e^{i\int d^4x\ h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda \mathbf{T}_{\mu\nu}(x))} \int \mathcal{D}\chi\, e^{iW_{hid}(\chi)-i\int d^4x h^{\mu\nu}(x)\chi_{\mu\nu}(x)}$$

$$= \int \mathcal{D}h e^{i\int d^4x\ h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda \mathbf{T}_{\mu\nu}(x))}\, e^{i\Gamma^{eff}_{hid}(h)}\,,$$

that involves the effective action $\Gamma^{eff}_{hid}(h)$ of the (hidden) bulk theory.

• At the saddle point, this reduces to the Legendre transform of the Schwinger functional of the bulk graviton.

• This corresponds in holography to switching boundary conditions at the AdS boundary from Dirichlet to Neumann for the graviton.

Compere+Marolf

• We can then rewrite the effective action part, holographically, using Neumann boundary conditions

$$\langle e^{iS_{12}}\rangle_1 = \int_{G_{\mu\nu}(x,z_0)\,:\,N.B.C.} \mathcal{D}G_{MN}(x,z)\mathcal{D}h_{\mu\nu}(x)\ e^{iS_N[G]+i\int h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}(x))}$$

and hence

$$e^{iW(\mathbf{g})} = \int \mathcal{D}h_{\mu\nu} \int_{G_{\mu\nu}(x,z_0)\,:\,N.B.C.} \mathcal{D}G_{MN}\mathcal{D}\Phi_{SM}\ e^{iS_N[G]+i\int h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}(x))+iS}$$

• This setup corresponds to our linearized computation and describes a four-dimensional visible QFT, whose stress tensor ($\mathbf{T}_{\mu\nu}$) is linearly coupled to a dynamical boundary graviton denoted by $h_{\mu\nu}(x)$.

• In addition, the original background metric $\mathbf{g}$, plays the role of a “Dark” stress energy tensor that shifts the SM stress energy tensor $\mathbf{T}_{\mu\nu}$.

• The non-linear completion is quite simple and just involves setting $\mathbf{g}_{\mu\nu} = \mathbf{g}_{\mu\nu}(h)$ in the SM action so that the total system is self-consistently coupled to the dynamical boundary metric $h_{\mu\nu}(x)$.

• The end-result is that we obtain a holographic bulk with a SM brane embedded, coupled to the bulk fields, but with Neumann bcs

The characteristic scales

• There are the following characteristic distance scales that play a role, besides r_0 set by the brane position.

• The *transition scale* r_t around which $D(r_0, p)$ changes from small to large momentum asymptotics:

$$D(r_0, p) \simeq \begin{cases} \dfrac{1}{2p} & p \gg \frac{1}{r_t}, \\ \\ d_0 + O(p^2) & p \ll \frac{1}{r_t} \end{cases}$$

• The transition scale r_t depends on r_0 and the bulk QFT dynamics.

• The *crossover scale,* or DGP scale, r_c:

$$r_c \equiv \frac{U_0}{2};$$

This scale determines the crossover between 5-dimensional and 4-dimensional behavior, and enters the 4D Planck scale and the graviton mass.

• The *gap scale* d_0

$$d_0 \equiv D(r_0, 0) = e^{3A_0} \int_0^{r_0} dr' e^{-3A_{UV}(r')},$$

which governs the propagator at the largest distances (in particular it sets the graviton mass as we will see).

• In generic cases, $d_0 \lesssim r_0$

• In confining bulk backgrounds we have instead

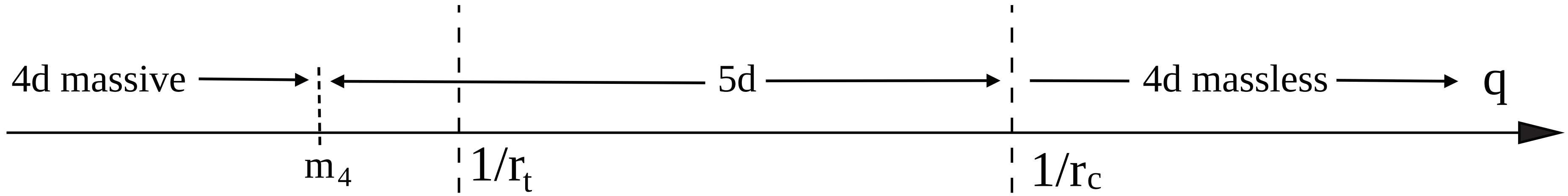
$$d_0 \simeq \frac{1}{6\Lambda_{QCD}^2\ r_0}$$

• In the far IR, $\Lambda r_0 \gg 1$ and d_0 can be made arbitrarily small.

DGP and massive gravity

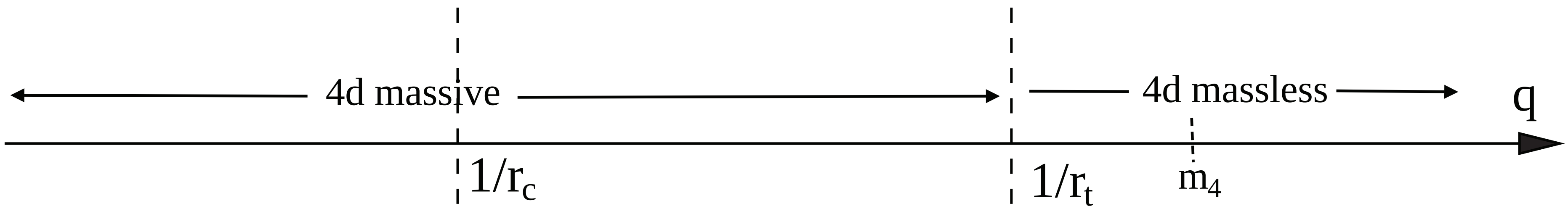
- When $r_t > r_c$ we have three regimes for the gravitational interaction on the brane:

$$\tilde{G}_4(p) \simeq \begin{cases} -\frac{1}{2M_P^2} \quad \frac{1}{p^2} & p \gg \frac{1}{r_c}, \qquad , M_P^2 = r_c M^3 \\ -\frac{1}{2M^3} \quad \frac{1}{p} & \frac{1}{r_c} \gg p \gg m_0 \\ -\frac{1}{2M_P^2} \quad \frac{1}{p^2 + m_0^2} & p \ll m_0, \qquad m_0^2 \equiv \frac{1}{2r_c d_0} \end{cases}$$



• Massive 4d gravity ($r_t < r_c$)

• In this case, at all momenta above the transition scale, $p \gg 1/r_t > 1/r_c$, we are in the 4-dimensional regime of the DGP-like propagator.



• Below the transition, $p \ll 1/r_t$, we have again a massive-graviton propagator.

• The behavior is four-dimensional at all scales, and it interpolates between massless and massive four-dimensional gravity.

Kiritsis+Tetradis+Tomaras

More on scales

- Scales depend on the bulk dynamics=the nature of the RG flow.

- They depend on "SM" data (the brane potential and the cutoff scale Λ).

- They can depend on boundary conditions = the UV coupling constant of the bulk QFT.

- Φ_0 at the position of the brane is fixed by the Israel conditions and is independent of boundary conditions.

- The two important parameters for 4d gravity do not depend on b.c.

$$\frac{m_0}{M_P} \sim \left(\frac{M}{\Lambda}\right)^2 \frac{1}{N^{\frac{2}{3}}} \quad , \quad m_0 \, M_P = \left(\frac{M^3}{\bar{d}}\right)^{\frac{1}{2}}$$

- $\bar{d}$ is the "rescaled" value of the bulk propagator at $p = 0$ at the position of the brane (so that it is independent of boundary conditions). It depends only on the bulk action.

• The choice of a small ratio $\frac{m_0}{M_P} \sim 10^{-60}$ is (technically) natural from the QFT point of view.

• There is important numerology to be analyzed for typical classes of holographic theories.

The brane graviton

The graviton fluctuation satisfies

$$\partial_r \left(e^{3A(r)} \partial_r \widehat{h} \right) + \left[e^{3A(r)} + U_0 \delta(r - r_0) \right] \partial_\mu \partial^\mu \widehat{h} = \delta(r - r_0) \frac{\widehat{T}}{M^3}$$

- Then, the solution is given by:

$$\widehat{h}_{\mu\nu}(x, r) = \frac{1}{M^3} \int d^d x' \, G(r, x; r_0, x') \widehat{T}_{\mu\nu}(x'),$$

- The induced interaction is

$$S_{int} = -\frac{1}{2M^3} \int d^4x \, d^4x' \, G(r_0, x; r_0, x') \left(T^{\mu\nu}(x) T_{\mu\nu}(x') - \frac{1}{3} T(x) T(x') \right)$$

$$G(r, x; r_0, x') = \frac{1}{\frac{1}{G_{bulk}(r,x;r_0,x')} + G_{brane}(x, x')}$$

Dvali+Gabadadze+Porrati

- This should be contrasted with the field-theoretical formula

$$Interaction \;\; of \;\; energy \;\; sources = \frac{1}{\frac{1}{\langle \widehat{T}\widehat{T} \rangle_{hidden}} + \langle TT \rangle_{SM}} = \frac{\langle \widehat{T}\widehat{T} \rangle_{hidden}}{1 + \langle \widehat{T}\widehat{T} \rangle_{hidden} \langle TT \rangle_{SM}}$$

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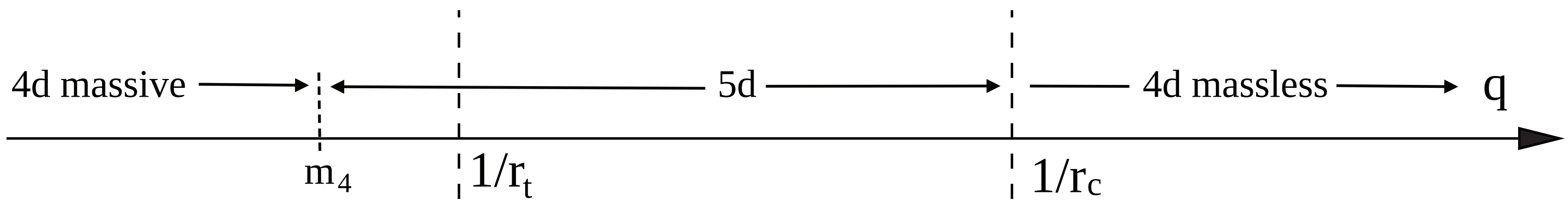

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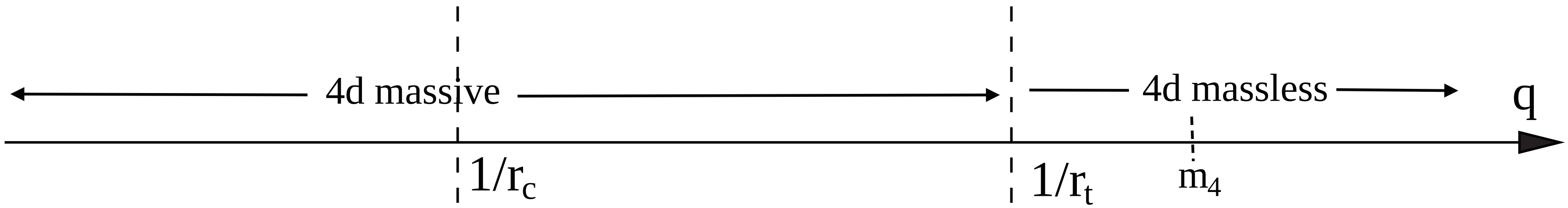

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Kiritsis+Tetradis+Tomaras

• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive. It should be cancelled by the Vainshtein mechanism.

Scalar Perturbations

• The scalar perturbations are of interest, as they might destroy the equivalence principle.

• The equations for the scalar perturbations can be derived and they are complicated.

• Unlike previous analysis of similar systems they cannot be factorized to a relatively simple system as the graviton.

• There are two scalar modes on the brane:

• In one gauge, the brane bedding mode can be "eliminated" but the scalar perturbation is discontinuous on the brane.

• In another gauge the perturbation is continuous but the brane bending mode is present.

The effective quadratic interactions for the scalar modes are of the form

$$S_4 = -\frac{\mathcal{N}}{2}\int d^4x\sqrt{\gamma}((\partial\phi)^2 + m^2\phi^2)$$

- We need both $\mathcal{N} > 0$ and $m^2 > 0$.

- In general the two scalar modes couple to two charges:

(a) the "scalar charge" and

(b) the trace of the brane stress tensor.

- The mode that couples to the scalar charge has a "heavy" mass of the order of the cutoff/Planck Scale.

- The mode that couples to the trace of the stress-tensor has a mass that is of order the graviton mass.

- All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

Scalar Perturbations

• The next step is to study the scalar perturbations. They are of interest, as they might destroy the equivalence principle.

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• The mode that couples to the scalar charge has a "heavy" mass of the order of the cutoff/Planck Scale.

• The mode that couples to the trace of the stress-tensor has a mass that is O(1) in cutoff units (like the graviton mass).

• All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

• They can be investigated further from the known parameter dependence of the vacuum energy in the SM.

Kounnas+Pavel+Zwirner, Dimopoulos+Giudince+Tetradis

• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive.

• It should be cancelled by the Vainshtein mechanism. To derive the relevant constraints on parameters, we must study the non-linear interactions of the scalar-graviton modes.

Scalar Perturbations (details)

We introduce perturbations of the metric and scalar field, on each side of the brane, in the form:

$$ds^2 = a^2(r)\left[(1+2\phi)dr^2 + 2A_\mu dx^\mu dr + (\eta_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu\right], \quad \varphi = \bar{\varphi}(r) + \chi$$

where the fields $\phi, A_\mu, h_{\mu\nu}, \chi$ depend on r, x_μ.

• We further decompose the 5 dimensional bulk modes into tensor, vector and scalar perturbations with respect to the 4 dimensional diffeomorphism group,

$$A_\mu = \partial_\mu W + A^T_\mu, \quad h_{\mu\nu} = 2\eta_{\mu\nu}\psi + 2\partial_\mu\partial_\nu E + 2\partial_{(\mu}V^T_{\nu)} + \widehat{h}_{\mu\nu}$$

with $\partial^\mu A^T_\mu = \partial^\mu V^T_\mu = \partial^\mu \widehat{h}_{\mu\nu} = \widehat{h}^\mu_\mu = 0$. All indices μ, ν are raised and lowered with the flat Minkowski metric $\eta_{\mu\nu}$.

• Therefore, we have one bulk tensor $\widehat{h}_{\mu\nu}$, two bulk transverse vectors (A^T_μ, V^T_μ), five bulk scalars (ϕ, ψ, χ, W, E) (plus one brane scalar, describing brane bending as we will see later).

• At the linearized level, general coordinate transformations $(\delta r, \delta x^\mu) = (\xi^5, g^{\mu\nu}\xi_\nu)$ act as gauge transformations, under which:

$$\delta\psi = -\frac{a'}{a}\xi^5 \qquad \delta\phi = -(\xi^5)' - \frac{a'}{a}\xi^5$$

$$\delta B = -\xi' - \xi^5, \qquad \delta E = -\xi, \qquad \delta\chi = -\bar{\varphi}'\xi^5, \tag{6}$$

$$\delta A^T_\mu = -(\xi^T_\mu)', \qquad \delta V^T_\mu = -\xi^T_\mu$$

$$\delta\widehat{h}_{\mu\nu} = 0 \tag{7}$$

where we have introduced a decomposition of the diffeomorphism parameter ξ_μ in its transverse and longitudinal components, i.e. $\xi_\mu = \xi^T_\mu + \partial_\mu\xi$ with $\partial^\mu\xi^T_\mu = 0$.

• The tensor mode $\widehat{h}_{\mu\nu}$ is gauge-invariant, and gauge symmetry plus constraints allow to eliminate the two vectors and four of the bulk scalars.

• The remaining physical bulk scalar can be identified with the gauge-invariant combination:

$$\zeta = \psi - \frac{1}{z}\chi,$$

where $z(r)$ is the background quantity:

$$z \equiv \frac{a\bar{\Phi}'}{a'}.$$

- However ζ is not continuous along the brane so we choose to work with ψ by setting $\chi = 0$.

- We also use a residual transformation to set the brane bending mode to zero at the expense of making ψ discontinuous.

- The bulk gauge-invariant fluctuations satisfy the second order equations:

$$\widehat{h}''_{\mu\nu} + (d-1)\frac{a'}{a}\widehat{h}'_{\mu\nu} + \partial^\rho\partial_\rho\widehat{h}_{\mu\nu} = 0 \qquad (8)$$

$$\zeta'' + \left[(d-1)\frac{a'}{a} + 2\frac{z'}{z}\right]\zeta' + \partial^\rho\partial_\rho\zeta = 0. \qquad (9)$$

• After solving the constraints for E and ϕ, and after eliminating the brane-bending field ρ, one is left with only the scalar mode ψ, which satisfies the bulk field equation (on each side of the brane) as well as the Israel conditions

$$\begin{pmatrix} \psi'_{UV}(r_0) \\ \psi'_{IR}(r_0) \end{pmatrix} = (\Gamma_1 + \Gamma_2 \partial^\mu \partial_\mu) \begin{pmatrix} \psi_{UV}(r_0) \\ \psi_{IR}(r_0) \end{pmatrix}$$

where the matrices Γ_1 and Γ_2 are given by:

$$\Gamma_1 = \frac{a_0 \tilde{\mathcal{M}}^2}{[z]^2} \begin{pmatrix} -z_{IR}^2 & z_{IR}^2 \\ -z_{UV}^2 & z_{UV}^2 \end{pmatrix},$$

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$$\Gamma_2 = \frac{1}{[z]^2 a_0} \begin{pmatrix} -12 z_{IR} \frac{dU_B}{d\Phi}\Big|_{\Phi_0} + \tau_0 + Z_0 z_{IR}^2 & 6 z_{IR} \left(\frac{z_{IR}}{z_{UV}} + 1\right) \frac{dU_B}{d\Phi}\Big|_{\Phi_0} - \tau_0 \frac{z_{IR}}{z_{UV}} - Z_0 z_{IR}^2 \\ -6 z_{UV} \left(\frac{z_{UV}}{z_{IR}} + 1\right) \frac{dU_B}{d\Phi}\Big|_{\Phi_0} + \tau_0 \frac{z_{UV}}{z_{IR}} + Z_0 z_{UV}^2 & 12 z_{UV} \frac{dU_B}{d\Phi}\Big|_{\Phi_0} - \tau_0 - Z_0 z_{UV}^2 \end{pmatrix}$$

where

$$\tilde{\mathcal{M}}^2 = \frac{d^2 W_B}{d\Phi^2}\Big|_{\Phi_0} - \left[\frac{d^2 W}{d\Phi^2}\right], \qquad \tau_0 = 6\left(6 \frac{W_B}{W_{IR} W_{UV}}\Big|_{\Phi_0} - U_0\right).$$

Detailed plan of the presentation

- Title page 0 minutes
- Introduction 2 minutes
- (Quantum) Gravity 3 minutes
- Composite Gravitons? 4 minutes
- The energy-momentum tensor 5 minutes
- The energy-momentum tensor vev as an emergent metric 7 minutes
- Gravitons from (holographic) hidden sectors 9 minutes
- The brane-world picture 12 minutes
- The brane graviton 13 minutes
- Open problems 14 minutes
- New opportunities 15 minutes
- Bibliography

Back-up Slides

- The Weinberg-Witten theorem 17 minutes
- Subtleties of WW 18 minutes
- The AdS/CFT paradigm 20 minutes
- The stress-tensor state as a (classical) dynamical graviton 31 minutes
- The low-energy effective action 32 minutes
- The linearized coupling 39 minutes
- Emergent quadratic gravity 43 minutes
- Emergent quadratic gravity, II 45 minutes
- The non-linear analysis 51 minutes
- The holographic hidden QFT 52 minutes
- The brane graviton 54 minutes
- The scalar perturbations 55 minutes
- Translation Ward Identities 56 minutes

- Aside: String theory and the Swampland 57 minutes
- Higher Spin 58 minutes
- WW versus AdS/CFT 59 minutes
- WW versus nAdS/nCFT 60 minutes
- An explicit IR parametrization 65 minutes
- Emerging quadratic gravity:Comments 69 minutes
- Renormalization and contact terms in $\langle TT \rangle$ 73 minutes
- Mixing with contact terms 77 minutes
- The non-linear analysis 80 minutes
- The brane bulk setup 84 minutes
- The characteristic scales 88 minutes
- DGP and massive gravity 92 minutes
- More on scales 96 minutes
- The brane graviton 102 minutes
- The scalar perturbations 108 minutes
- Scalar Perturbations 112 minutes
- Scalar perturbations (details) 116 minutes