Effective approach to lepton observables

Michele Frigerio

Laboratoire Charles Coulomb, CNRS & Université de Montpellier

Physique théorique des deux infinis

7-8 Juin 2021

New physics in the lepton sector ?

- non-zero neutrino masses → tiny lepton number violation (likely)

$$m_\nu \lesssim \Lambda_{NP} \lesssim m_{GUT}$$

- no direct indication of new particles presently, and progress at the energy frontier may be very slow

For more, see A.Abada et al.

- yet, lepton observables are extremely sensitive to higher scales:
  - electroweak precision tests
  - charged lepton flavour violation
  - dipole moments
  - (semi-)leptonic decays of hadrons

- powerful indirect tests of $\Lambda_{NP} \gg v_{EW} \simeq 250\mathrm{GeV}$

Effective Field Theory recipe

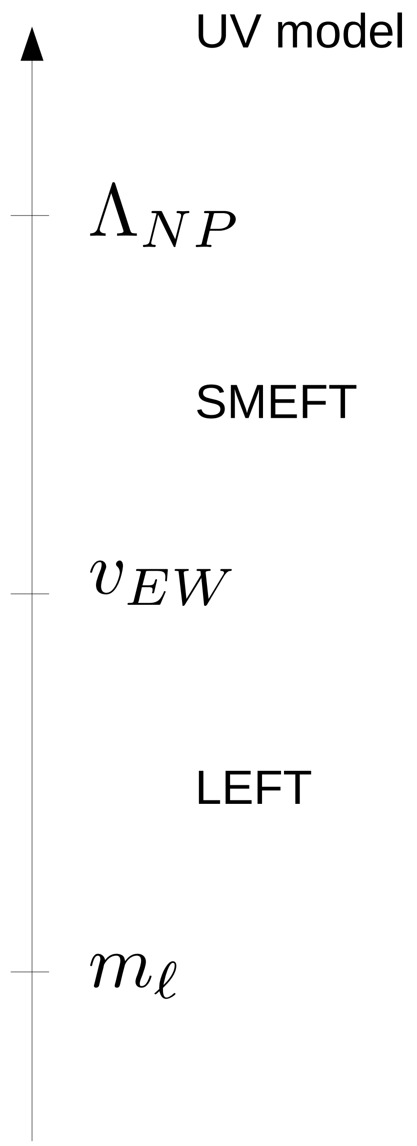
- start from any new physics model in the UV

- "integrate out" all heavy degrees of freedom, whose dynamics is irrelevant for low-energy observables

- each model is reduced to a set of low-energy Wilson Coefficients (WCs), which have to be estimated / computed

- each low-energy observable constrains a model-independent combination of WCs

- straightforward to (i) compare constraints & (ii) compare models



Do we learn something new ?

Estimating WCs :

Spurion analysis & Naive dimensional analysis are very instructive

- decide whether the size of some effect is within reach or not, even at very high order in the EFT expansion
  - e.g. in the type-I seesaw model, lepton Electric Dipole Moment arises only from a dim-10 operator, at 2 loops

Coy, Frigerio '18

$$|d_e| \sim \frac{2e}{(16\pi^2)^2} \left(\frac{v}{\sqrt{2}}\right)^4 \frac{\mathrm{Im}\left(\left[\hat{S}Y_e^\dagger Y_e \hat{S}, \hat{S}\right]_{ee}\right)}{m_W^6} m_e$$
$$\simeq 5.7 \times 10^{-28}\, \mathrm{Im}\left(\hat{S}_{e\tau}\hat{S}_{\tau\mu}\hat{S}_{\mu e}\right) e\ \mathrm{cm}$$

$$[\ \hat{S} \equiv m_W^2 Y^\dagger M^{-1*} M^{-1} Y\ ]$$

For more, see C.Smith et al.

Do we learn something new ?

Estimating Wcs :

Spurion analysis & Naive dimensional analysis are very instructive

- decide whether the size of some effect is within reach or not, even at very high order in the EFT expansion

- deal with strongly-coupled new physics models, where perturbative computations are not feasible
  - e.g. in the partial compositeness scenario for leptons

Frigerio, Nardecchia, Serra, Vecchi ‘18

$$\mathcal{L}_{m_*} \supset y^e_{ij}\overline{\ell_{Li}}e_{Rj}H + \frac{C^{e\gamma}_{ij}}{\Lambda^2}\overline{\ell_{Li}}\sigma^{\mu\nu}e_{Rj}F_{\mu\nu}H$$

Yukawa coupling vs e.g. dipole operator

$$y^e_{ij} = c^e_{ij}\,\epsilon^\ell_i\,\epsilon^e_j\,g_*$$

$$\frac{C^{e\gamma}_{ij}}{\Lambda^2} = \frac{c^{e\gamma}_{ij}}{m_*^2}\left(\frac{g_*}{4\pi}\right)^2\epsilon^\ell_i\,\epsilon^e_j\,e\,g_*$$

order one coefficients

lepton composite fraction

Higgs coupling

scale dimension

strong loop-factor

photon coupling

careful Yukawa diagonalisation to derive WCs in flavour basis

Frigerio, Nardecchia, Serra, Vecchi '18

Effective operator	Wilson coefficient
$Q_{eW}^{ij} = \left(\bar{\ell}_L^i \sigma^{\mu\nu} e_R^j\right) \sigma^I H W_{\mu\nu}^I$	$\frac{C_{ij}^{eW}}{\Lambda^2} = \frac{1}{16\pi^2} \frac{g_*^3}{m_*^2} \epsilon_i^\ell \epsilon_j^e g\, c_{ij}^{eW} = \frac{1}{16\pi^2} \frac{g_*^2}{m_*^2} \frac{\epsilon_i^\ell}{\epsilon_j^\ell} \frac{\sqrt{2} m_j^e}{v} g\, c_{ij}^{eW}$
$Q_{eB}^{ij} = \left(\bar{\ell}_L^i \sigma^{\mu\nu} e_R^j\right) H B_{\mu\nu}$	$\frac{C_{ij}^{eB}}{\Lambda^2} = \frac{1}{16\pi^2} \frac{g_*^3}{m_*^2} \epsilon_i^\ell \epsilon_j^e g'\, c_{ij}^{eB} = \frac{1}{16\pi^2} \frac{g_*^2}{m_*^2} \frac{\epsilon_i^\ell}{\epsilon_j^\ell} \frac{\sqrt{2} m_j^e}{v} g'\, c_{ij}^{eB}$
$Q_{eH}^{ij} = (H^\dagger H)\left(\bar{\ell}_L^i e_R^j H\right)$	$\frac{C_{ij}^{eH}}{\Lambda^2} = \frac{g_*^3}{m_*^2} \epsilon_i^\ell \epsilon_j^e\, c_{ij}^{eH} = \frac{g_*^2}{m_*^2} \frac{\epsilon_i^\ell}{\epsilon_j^\ell} \frac{\sqrt{2} m_j^e}{v}\, c_{ij}^{eH}$
$Q_{H\ell}^{(1)ij} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{\ell}_L^i \gamma^\mu \ell_L^j\right)$	$\frac{C_{ij}^{H\ell(1)}}{\Lambda^2} = \frac{g_*^2}{m_*^2} \epsilon_i^\ell \epsilon_j^\ell\, c_{ij}^{H\ell(1)}$
$Q_{H\ell}^{(3)ij} = \left(H^\dagger \sigma^I i \overleftrightarrow{D}_\mu H\right)\left(\bar{\ell}_L^i \sigma^I \gamma^\mu \ell_L^j\right)$	$\frac{C_{ij}^{H\ell(3)}}{\Lambda^2} = \frac{g_*^2}{m_*^2} \epsilon_i^\ell \epsilon_j^\ell\, c_{ij}^{H\ell(3)}$
$Q_{He}^{ij} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{e}_R^i \gamma^\mu e_R^j\right)$	$\frac{C_{ij}^{He}}{\Lambda^2} = \frac{g_*^2}{m_*^2} \epsilon_i^e \epsilon_j^e\, c_{ij}^{He} = \frac{1}{m_*^2} \frac{2 m_i^e m_j^e}{v^2} \frac{1}{\epsilon_i^\ell \epsilon_j^\ell}\, c_{ij}^{He}$
$Q_{\ell\ell}^{ijmn} = \left(\bar{\ell}_L^i \gamma_\mu \ell_L^j\right)\left(\bar{\ell}_L^m \gamma^\mu \ell_L^n\right)$	$\frac{C_{ijmn}^{\ell\ell}}{\Lambda^2} = \frac{g_*^2}{m_*^2} \epsilon_i^\ell \epsilon_j^\ell \epsilon_m^\ell \epsilon_n^\ell\, c_{ijmn}^{\ell\ell}$
$Q_{\ell e}^{ijmn} = \left(\bar{\ell}_L^i \gamma_\mu \ell_L^j\right)(\bar{e}_R^m \gamma^\mu e_R^n)$	$\frac{C_{ijmn}^{\ell e}}{\Lambda^2} = \frac{g_*^2}{m_*^2} \epsilon_i^\ell \epsilon_j^\ell \epsilon_m^e \epsilon_n^e\, c_{ijmn}^{\ell e} = \frac{1}{m_*^2} \frac{2 m_m^e m_n^e}{v^2} \frac{\epsilon_i^\ell \epsilon_j^\ell}{\epsilon_m^\ell \epsilon_n^\ell}\, c_{ijmn}^{\ell e}$
$Q_{ee}^{ijmn} = \left(\bar{e}_R^i \gamma_\mu e_R^j\right)(\bar{e}_R^m \gamma^\mu e_R^n)$	$\frac{C_{ijmn}^{ee}}{\Lambda^2} = \frac{g_*^2}{m_*^2} \epsilon_i^e \epsilon_j^e \epsilon_m^e \epsilon_n^e\, c_{ijmn}^{ee} = \frac{1}{g_*^2 m_*^2} \frac{4 m_i^e m_j^e m_m^e m_n^e}{v^4 \epsilon_i^\ell \epsilon_j^\ell \epsilon_m^\ell \epsilon_n^\ell}\, c_{ijmn}^{ee}$

Do we learn something new ?

Computing WCs :

In weakly-coupled models, an explicit, perturbative computation of WCs is possible. This allows to identify

– accidental (gauge/Lorentz) cancellations

– log enhancements

– order-one factors

Models can be analysed and compared, in principle, up to any desired level of accuracy

Relevant ratios of new physics parameters (with various flavour indexes)

$$\epsilon \equiv \frac{Y v}{\sqrt{2} M} \ll 1$$

Coy, Frigerio, to appear

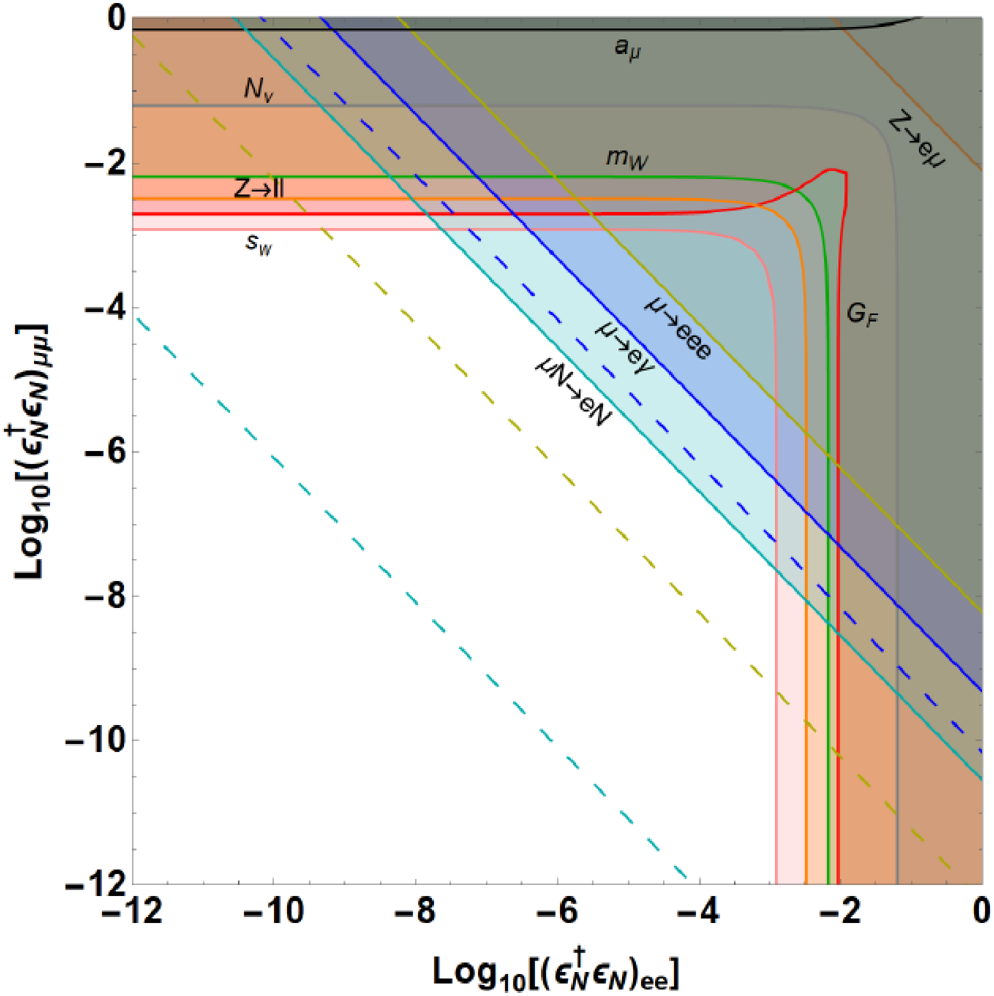
WCs	Seesaw I	Seesaw III	Zee	Leptoquarks
c^{ll}_{abcd}	$\frac{g_1^2-g_2^2}{24}(\epsilon_N^\dagger\epsilon_N)_{ab}\delta_{cd}$ $+\frac{g_1^2-g_2^2}{24}\delta_{ab}(\epsilon_N^\dagger\epsilon_N)_{cd}$ $+\frac{g_2^2}{12}(\epsilon_N^\dagger\epsilon_N)_{ad}\delta_{cb}$ $+\frac{g_2^2}{12}\delta_{ad}(\epsilon_N^\dagger\epsilon_N)_{cb}$ $+2(c^{W\dagger})_{ac}(c^W)_{bd}$	$\frac{3g_1^2+g_2^2}{24}(\epsilon_\Sigma^\dagger\epsilon_\Sigma)_{ab}\delta_{cd}$ $+\frac{3g_1^2+g_2^2}{24}\delta_{ab}(\epsilon_\Sigma^\dagger\epsilon_\Sigma)_{cd}$ $-\frac{g_2^2}{12}(\epsilon_\Sigma^\dagger\epsilon_\Sigma)_{ad}\delta_{cb}$ $-\frac{g_2^2}{12}\delta_{ad}(\epsilon_\Sigma^\dagger\epsilon_\Sigma)_{cb}$ $+2(c^{W\dagger})_{ac}(c^W)_{bd}$	$(\epsilon_\delta^\dagger)_{ac}(\epsilon_\delta)_{db}$	$\frac{g_1^2}{6}(\epsilon_D^\dagger\epsilon_D)_{ab}\delta_{cd}$ $+\frac{g_1^2}{6}\delta_{ab}(\epsilon_D^\dagger\epsilon_D)_{cd}$ $+\frac{g_2^2}{2}(\epsilon_L^\dagger\epsilon_L)_{ad}\delta_{bc}$ $+\frac{g_2^2}{2}\delta_{ad}(\epsilon_L^\dagger\epsilon_L)_{cb}$ $+\frac{g_1^2-3g_2^2}{12}(\epsilon_L^\dagger\epsilon_L)_{ab}\delta_{cd}$ $+\frac{g_1^2-3g_2^2}{12}\delta_{ab}(\epsilon_L^\dagger\epsilon_L)_{cd}$
c^{le}_{abcd}	$\frac{g_1^2}{6}(\epsilon_N^\dagger\epsilon_N)_{ab}\delta_{cd}$	$\frac{g_1^2}{2}(\epsilon_\Sigma^\dagger\epsilon_\Sigma)_{ab}\delta_{cd}$	$-\frac{1}{2}(\epsilon_2^\dagger)_{ad}(\epsilon_2)_{cb}$	$\frac{3}{2}(\epsilon_L^\dagger y_u^T\epsilon_R)_{ad}(y_e)_{cb}$ $+\frac{3}{2}(y_e^\dagger)_{ad}(\epsilon_R^\dagger y_u^*\epsilon_L)_{cb}$ $+\frac{g_1^2}{3}(\epsilon_L^\dagger\epsilon_L)_{ab}\delta_{cd}$ $+\frac{2g_1^2}{3}(\epsilon_D^\dagger\epsilon_D)_{ab}\delta_{cd}$ $+\frac{2g_1^2}{3}\delta_{ab}(\epsilon_R^\dagger\epsilon_R)_{cd}$
c^{ee}_{abcd}			$\frac{g_1^2}{3}(\epsilon_2\epsilon_2^\dagger)_{ab}\delta_{cd}$ $+\frac{g_1^2}{3}\delta_{ab}(\epsilon_2\epsilon_2^\dagger)_{cd}$	$\frac{2g_1^2}{3}(\epsilon_R^\dagger\epsilon_R)_{ab}\delta_{cd}$ $+\frac{2g_1^2}{3}\delta_{ab}(\epsilon_R^\dagger\epsilon_R)_{cd}$

Coy, Frigerio, to appear

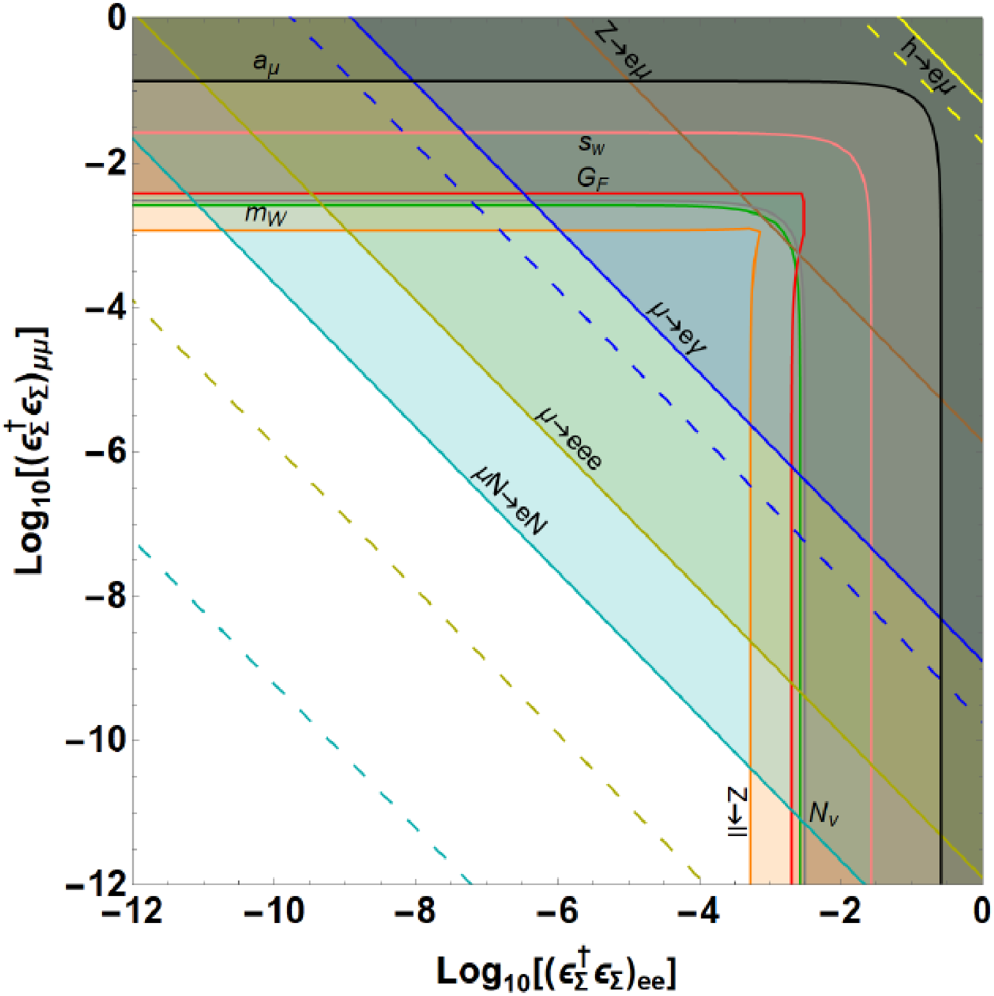
N_ν	$0.58(c^{HD} - c^{G}) + 11.1c^{He} - 24.8c^{Hl(1)} - 0.82c^{Hl(3)} \in [-0.019, 0.011]$ [32, 46, 50]	2σ
$\tau^- \to e^-\mu^+e^-$	$\lvert c^{le}_{e\tau e\mu}\rvert^2 + \lvert c^{le}_{e\mu e\tau}\rvert^2 + 2\lvert c^{ll}_{e\tau e\mu} + c^{ll}_{e\mu e\tau}\rvert^2 + 2\lvert c^{ee}_{e\tau e\mu} + c^{ee}_{e\mu e\tau}\rvert^2 \lesssim 8.4(0.2) \times 10^{-8}$ [54] ([55])	90%
$\tau^- \to \mu^-e^+\mu^-$	$\lvert c^{le}_{\mu\tau\mu e}\rvert^2 + \lvert c^{le}_{\mu e\mu\tau}\rvert^2 + 2\lvert c^{ll}_{\mu\tau\mu e} + c^{ll}_{\mu e\mu\tau}\rvert^2 + 2\lvert c^{ee}_{\mu\tau\mu e} + c^{ee}_{\mu e\mu\tau}\rvert^2 \lesssim 9.5(0.2) \times 10^{-8}$ [54] ([55])	90%
$\mu \to e\gamma$	$\sqrt{\lvert c^{e\gamma,obs}_{e\mu}\rvert^2 + \lvert c^{e\gamma,obs}_{\mu e}\rvert^2} \lesssim 6.4(2.4) \times 10^{-12}$ [33] ([60])	90%
$\tau \to e\gamma$	$\sqrt{\lvert c^{e\gamma,obs}_{e\tau}\rvert^2 + \lvert c^{e\gamma,obs}_{\tau e}\rvert^2} \lesssim 7.1(2.1) \times 10^{-8}$ [61] ([55])	90%
$\tau \to \mu\gamma$	$\sqrt{\lvert c^{e\gamma,obs}_{\mu\tau}\rvert^2 + \lvert c^{e\gamma,obs}_{\tau\mu}\rvert^2} \lesssim 8.2(1.2) \times 10^{-8}$ [61] ([55])	90%
a_e	$\lvert \mathrm{Re}\ c^{e\gamma,obs}_{ee}\rvert \lesssim 3 \times 10^{-8}$ [41–43]	4.6
a_μ	$\mathrm{Re}[c^{e\gamma,obs}_{\mu\mu} + 4.3 \times 10^{-7}(c^{G} - c^{HD})] \in [-0.5, 4.6] \times 10^{-7}$ [38, 39]	4.6
d_e	$\lvert \mathrm{Im}\ c^{e\gamma,obs}_{ee}\rvert \lesssim 1.5 \times 10^{-14}$ [62]	90%
d_μ	$\lvert \mathrm{Im}\ c^{e\gamma,obs}_{\mu\mu}\rvert \lesssim 2.5 \times 10^{-4}$ [63]	95%

Coy, Frigerio, to appear

Seesaw type-I $\overline{N_R} Y_N \tilde{H}^\dagger l_L$



Seesaw type-III $\overline{\Sigma_R^A} Y_\Sigma \tilde{H}^\dagger \sigma^A l_L$

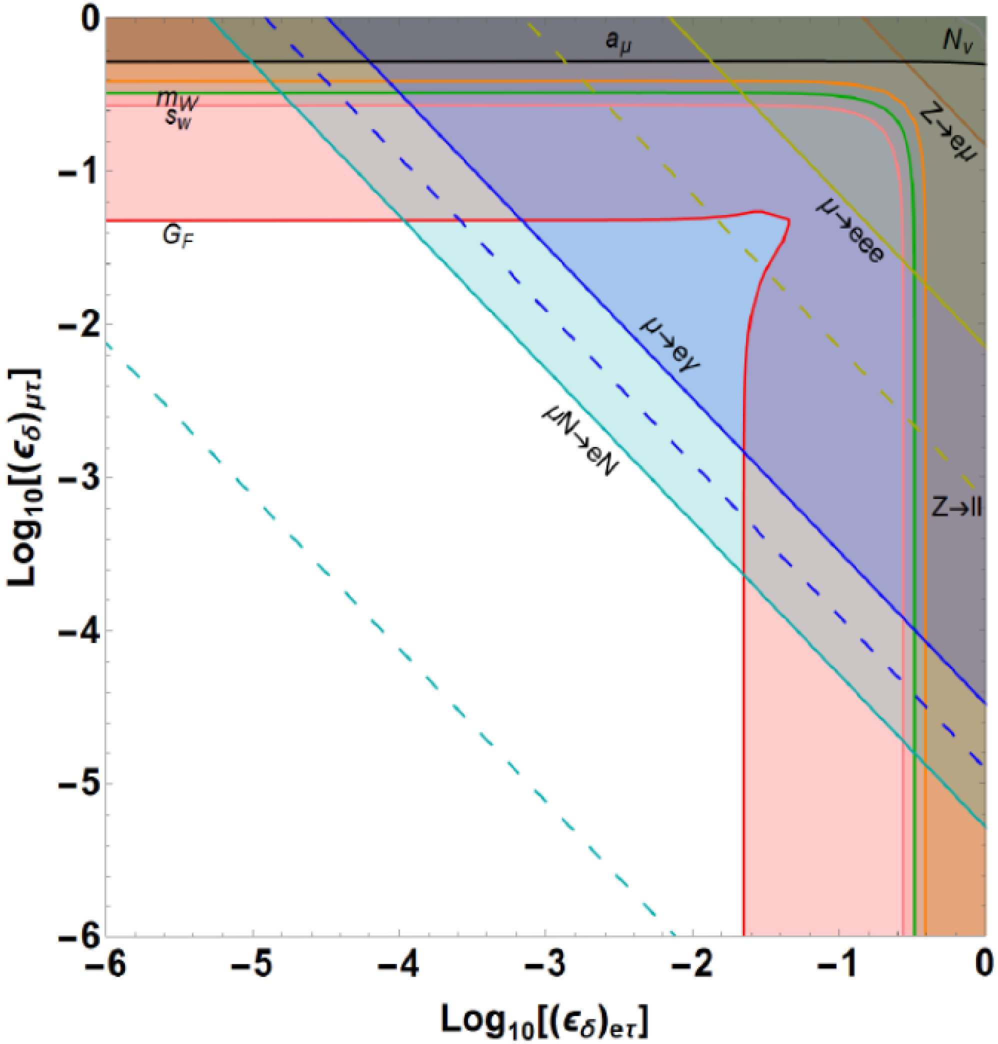


Here the **e-mu plane** in parameter space, similarly e-tau & mu-tau planes...

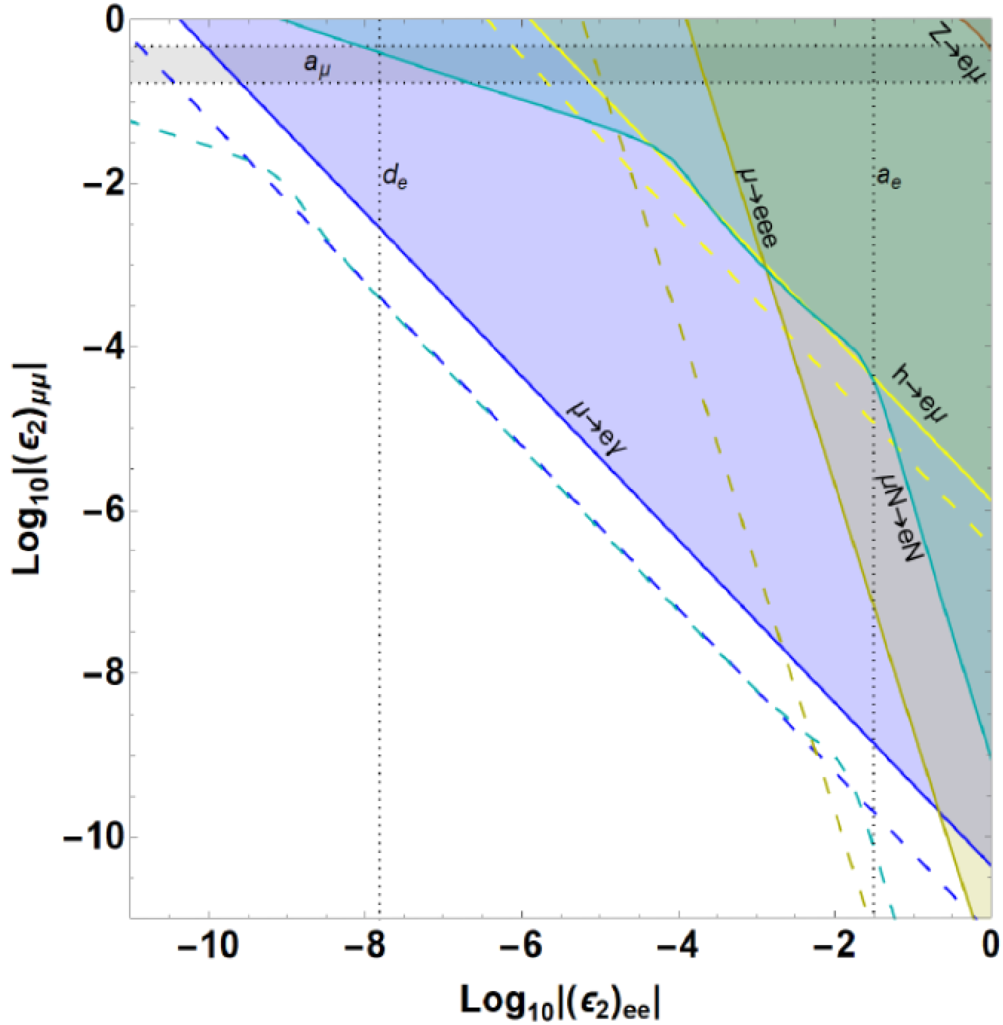
Coy, Frigerio, to appear

Zee model

$$\overline{l_L^c} Y_\delta \, i\sigma_2 l_L \delta$$



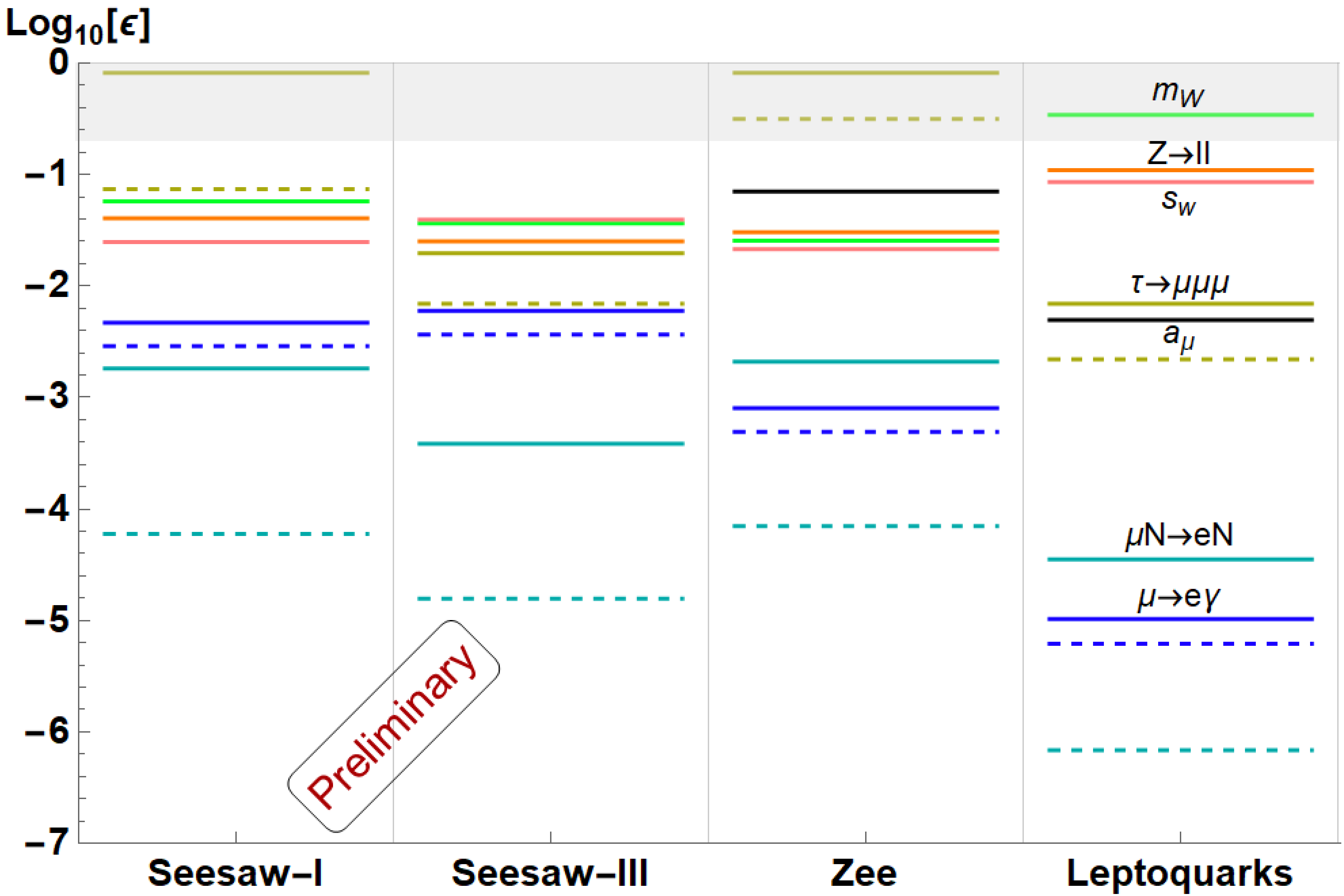
$$\overline{e_R} Y_2 H_2^\dagger l_L$$



Here the **e-mu plane** in parameter space, similarly e-tau & mu-tau planes...

Coy, Frigerio, to appear

Comparison of neutrino mass model constraints



Lepton EFT perspectives

- Expected exp progress: within the EFT regime of validity ?

  – orders of magnitude in mu-to-e transitions

  For more, see A.Teixeira et al.

  – discovery of leptonic CP violation in oscillations and elsewhere?

  – two sticking anomalies: muon g-2 & lepton-flavour non-universality in b → sll transitions

- Computation tools for WCs: tree-level matching trivial, one-loop leading-log known, one-loop matching laborious → automatisation ongoing, two-loops …
  Not obvious whether more precision can be exploited theoretically

- EFT bottom-up: exploit all possible exp observables to disentangle as many WCs as possible.
  Is some piece of analysis missing?

  For more, see S.Davidson et al.

- While waiting for a clear new-physics signal, model-builders should privilege (1) theory motivations (2) minimality → correlated predictions