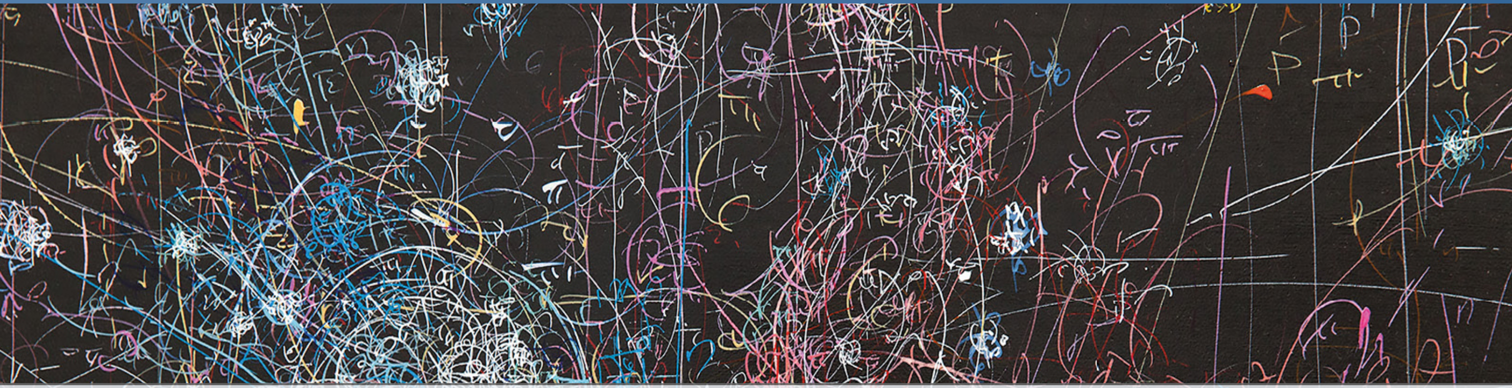

Flavor physics and Lattice QCD

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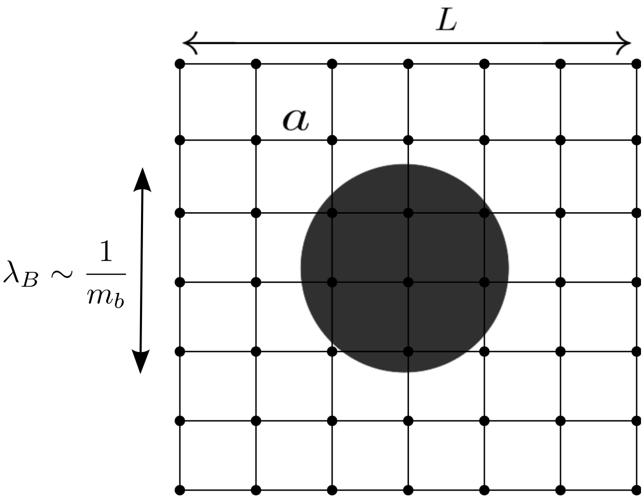
- Lattice QCD :
  - UV regulator $\Lambda_{\mathrm{UV}} = a^{-1}$
  - IR regulator $\Lambda_{\mathrm{IR}} = L^{-1}$

- Direct simulations using an heavy meson B :

$$L^{-1} \ll m_\pi, m_B \ll a^{-1}$$

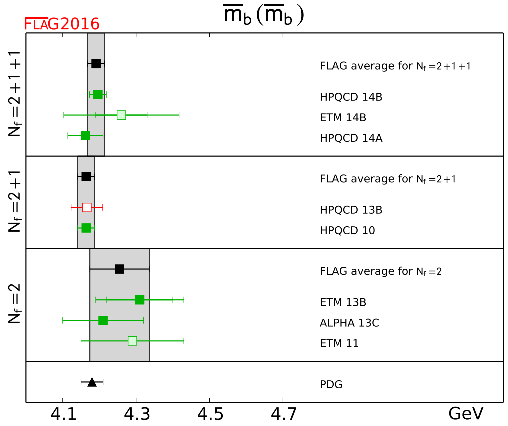
$\mathcal{O}\left(e^{-m_\pi L}\right)$	$am_B < 1$
$L \gg 6$ fm	$a \ll 0.05$ fm
Volume effect	Discretization effect



- Lattice resolution $L/a \gg 120$: not yet possible

- Methods used by the lattice community :
  - Effective field theories (Heavy Quark Effective Theory, Non-Relativistic QCD)
  - Step scaling method
  - Highly-improved actions (to reduce discretization effects)

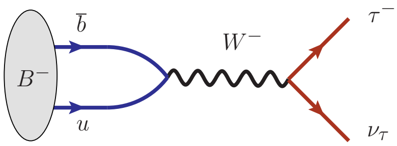
- Non perturbative matching of Heavy Quark Effective Theory (HQET) with QCD
- Strategy OK to pass quality criteria established by the Flavour Lattice Averaging Group.

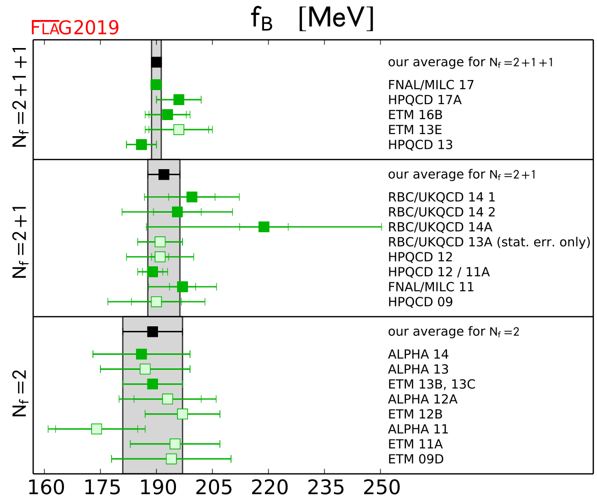


- Determination of the b quark mass
  - → systematic control of cut-off effects
  - → Effective theory : static + $O(1/m_b)$ corrections

- B and B_s decay constants

$$\mathcal{B}(B^- \to \tau^- \nu_\tau) \propto |V_{ub}|^2 \, f_B^2$$





[B. Blossier, A. Gérardin et al : 2014]

- Series of (dynamical) heavy quark masses m_{h_i} to reach the b quark mass

$$\frac{m_{h_{i+1}}}{m_{h_i}} = \left(\frac{m_{B_q}}{m_{D_q}}\right)^{1/N} \,, \qquad i \in [1:N]$$

- Observable (decay constant, ...)

$$A(m_{B_q}) = A(m_{D_q}) \ \Pi_{i=1}^{N} R'_i \,, \qquad R'_i = \frac{A(m_{h_{i+1}})}{A(m_{h_i})} \propto R_i^{\mathrm{HQET}}$$

$$R_i^{\mathrm{HQET}} = 1 + \frac{X_1}{m_{h_i}} + \frac{X_2}{(m_{h_i})^2}$$

- Each step : continuum extrapolation + chiral extrapolation to the physical pion mass
  - ✓ ratio are expected to have smaller discretization effects
  - ✓ smoother chiral extrapolation
  - X The statistical error accumulates at each step !

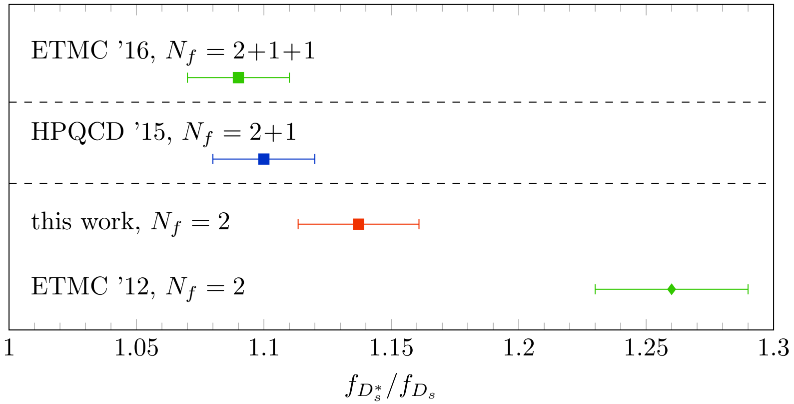
Preliminary study : charm physics

[B. Blossier, V. Morenas et al : 2018, 2020]

- A dynamical c quark can be simulated on the lattices

  $\rightarrow$ study of excited state contaminations

  $\rightarrow$ study of the size of discretization effects



- $m_{D_s^*} = 2.111(10)(13)$ GeV

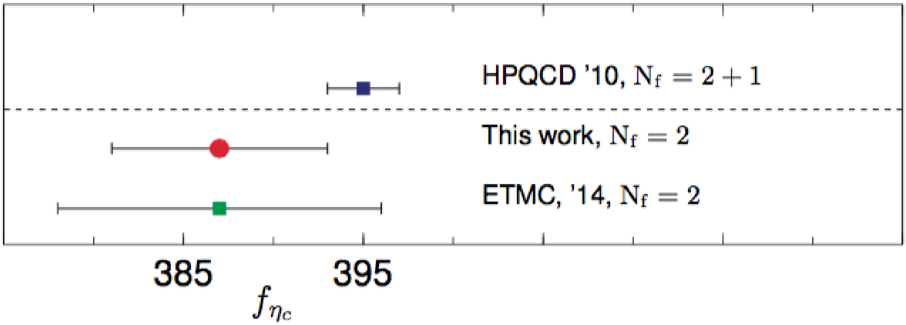
  $\rightarrow$ Very good agreement with experiment

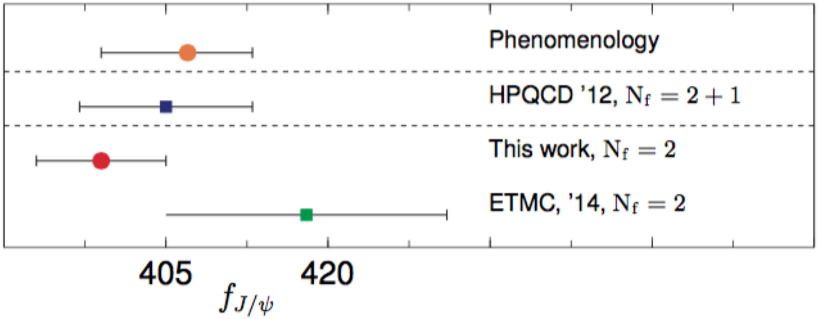
- $f_{D_s^*}/f_{D_s} = 1.14(2)$

  $\rightarrow$ Discretization effects $< 10\%$

- Next step : study transitions such as $B_s \rightarrow D_s^{(*)}$

- Also some new results with charmonia : η_c (pseudoscalar) and J/Ψ (vector)

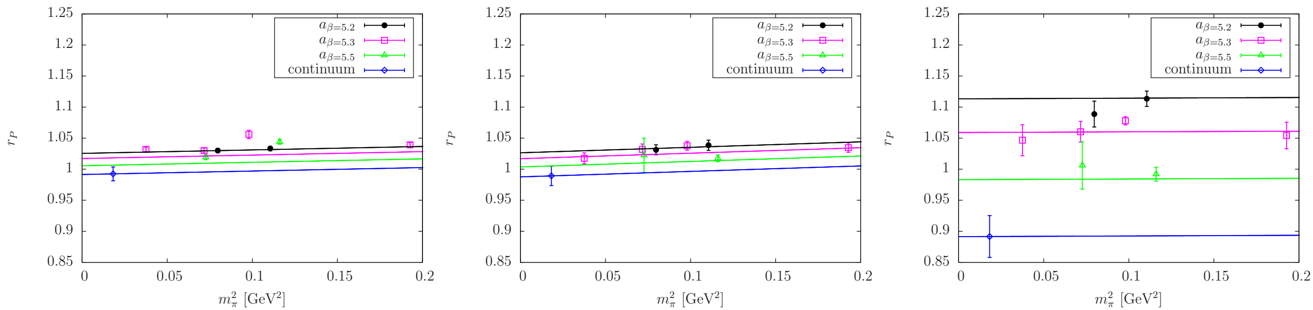




- Results included in the on-going FLAG report
- Next step : study transitions such as $B_c \to J/\psi\ \ell\bar{\nu}$, $B_c \to \eta_c \ell\bar{\nu}$
- Method : step scaling functions to reach the physical b quark mass (ETMC strategy)

- $N = 5$ steps are used to reach the b quark mass

First step — Third step — Last step



$\rightarrow$ discretization effects become important at large masses

$\rightarrow$ smooth chiral behavior (simulations at physical pion mass challenging)

- Difficulty : the statistical error accumulates with the number of iterations

$$\frac{f_{B_s}}{f_{D_s}} = 0.87(4)(^{+1}_{-4}) \ \Rightarrow \ f_{B_s} = 212(10)(2)(^{+2}_{-10}) \text{ MeV}$$

$\rightarrow$ Compatible with [ALPHA '14]) : $f_{B_s} = 224(14)$ MeV (same ensemble, using HQET)

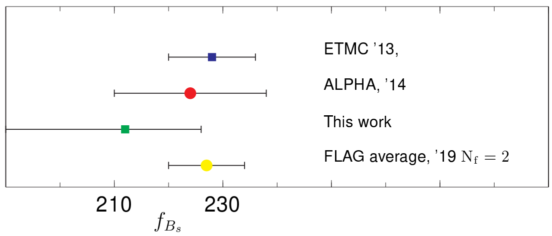
- Advantage : One can directly study the $1/m_h$ corrections

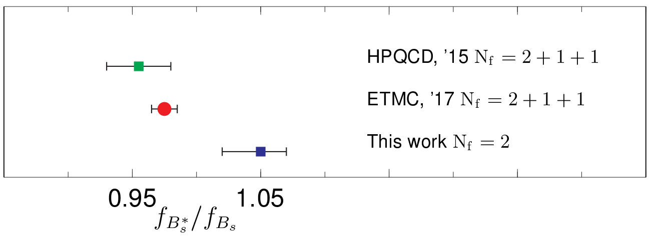
- Similar strategy in the vector channel : $B^*_{(s)}$

$$\frac{m_{B_s^*}}{m_{B_s}} = 1.0061(4)(5)\ , \quad \frac{f_{B_s^*}}{f_{B_s}} = 1.05(2)$$

$\rightarrow$ $\frac{f_{B_s^*}}{f_{B_s}} > 1$ quenching of the strange quark ?

$\rightarrow$ similar observation on ETMC lattice [Becirevic et al '14]

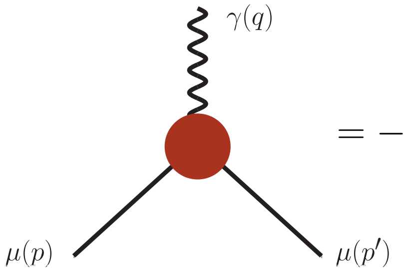




FLAG average of 2016

- Conclusion : the method works (but challenging)
- Study of the form factors for $B_s \rightarrow D_s^{(*)}\ell\nu$ has started [B. Blossier, S. Zafeiropoulos et al, 2021]
- Future : plan to use another strategy (relativistic heavy quarks)

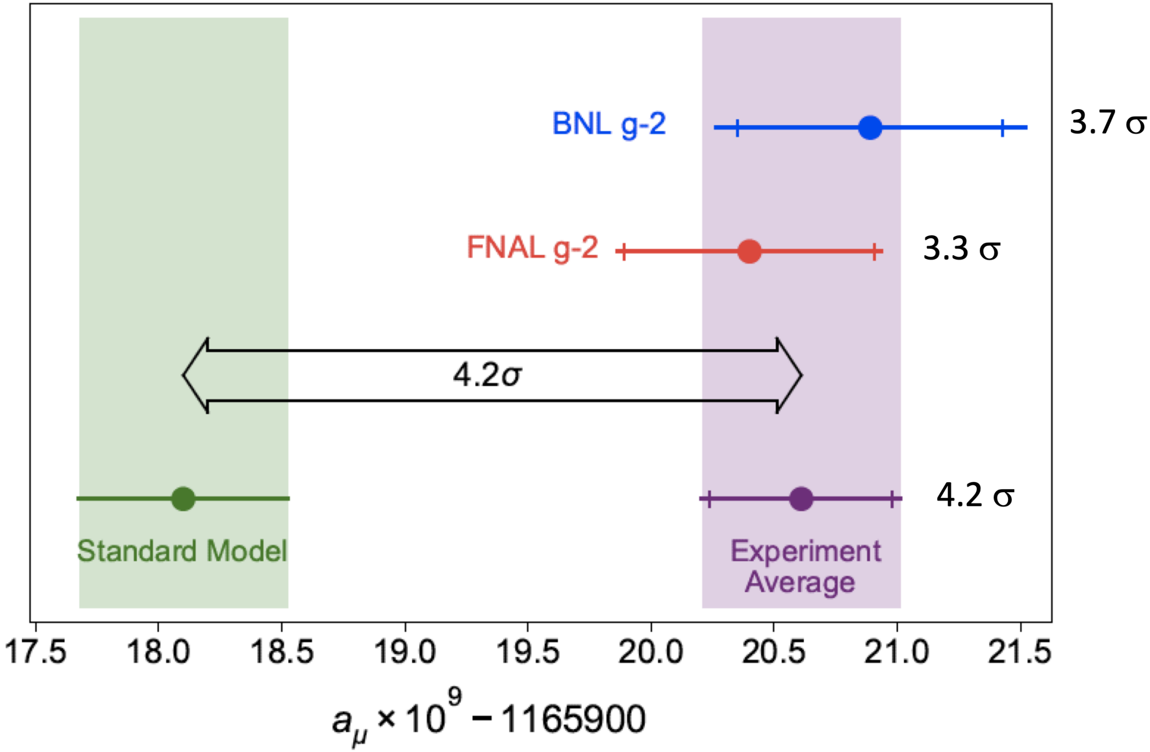
The anomalous magnetic moment of the muon



$$= -ie\,\overline{u}(p',\sigma')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p,\sigma)$$

$a_\mu = \frac{g-2}{2} = F_2(0) =$ quantum corrections

Pheno.	$a_\mu = (116\ 591\ 810 \pm 43) \times 10^{-11}$	
Exp.	$a_\mu = (116\ 592\ 061 \pm 41) \times 10^{-11}$	[BNL + Fermilab 2021]

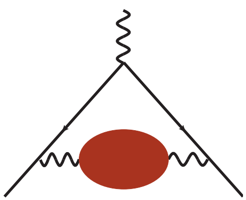


- Theory estimate from the White Paper (posted 10 June 2020)
  The anomalous magnetic moment of the muon in the Standard Model [Phys.Rept. 887 (2020) 1-166]

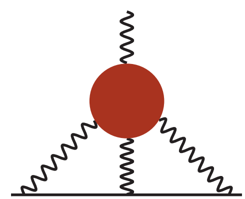
Why lattice QCD ?

Contribution	$a_\mu \times 10^{11}$	
- QED (leptons, 5^{th} order)	$116\ 584\ 718.95 \pm 0.08$	[Aoyama et al. '12]
- Electroweak	153.6 ± 1.0	[Gnendiger et al. "13]
- Strong contributions		
HVP (LO)	$6\ 931 \pm 40$	[DHMZ 19, KNT 20]
HVP (NLO)	-98.3 ± 0.7	[Hagiwara et al. 11]
HVP (NNLO)	12.4 ± 0.1	[Kurtz et al. '14]
HLbL	94 ± 19	[See WP]
Total (theory)	$116\ 591\ 810 \pm 43$	

- **Theory error dominated by hadronic uncertainties**
  - → Dominated by low energy QCD
  - → Require non-perturbative methods



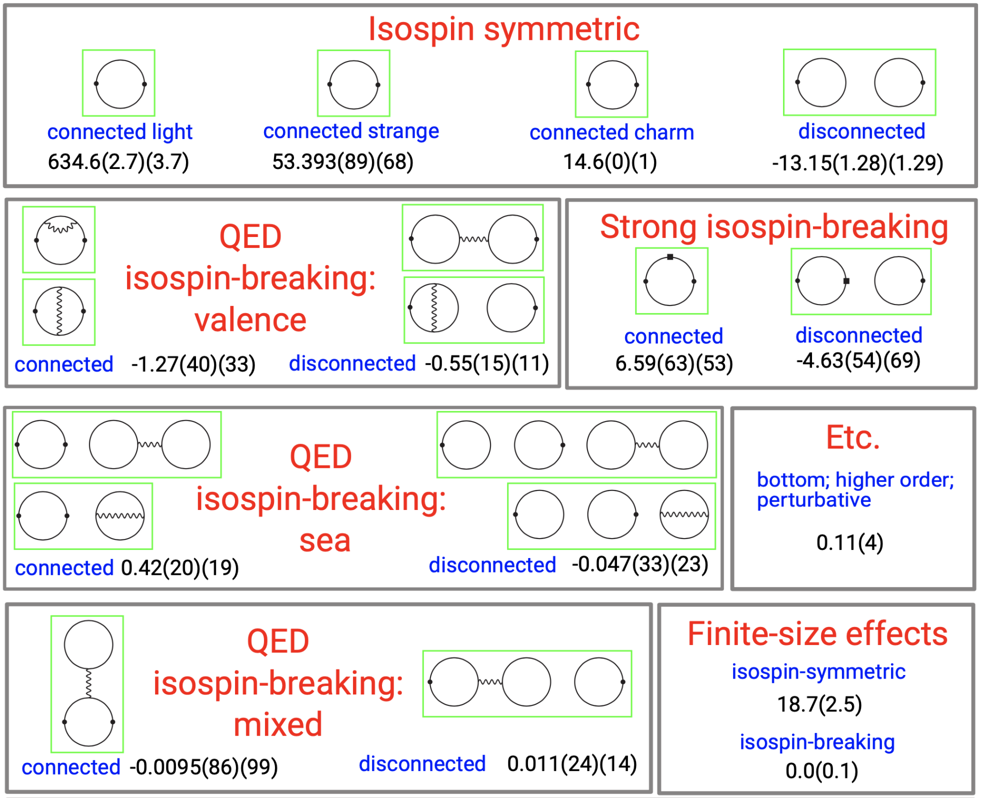
Hadronic Vacuum Polarisation (LO-HVP, α^2)



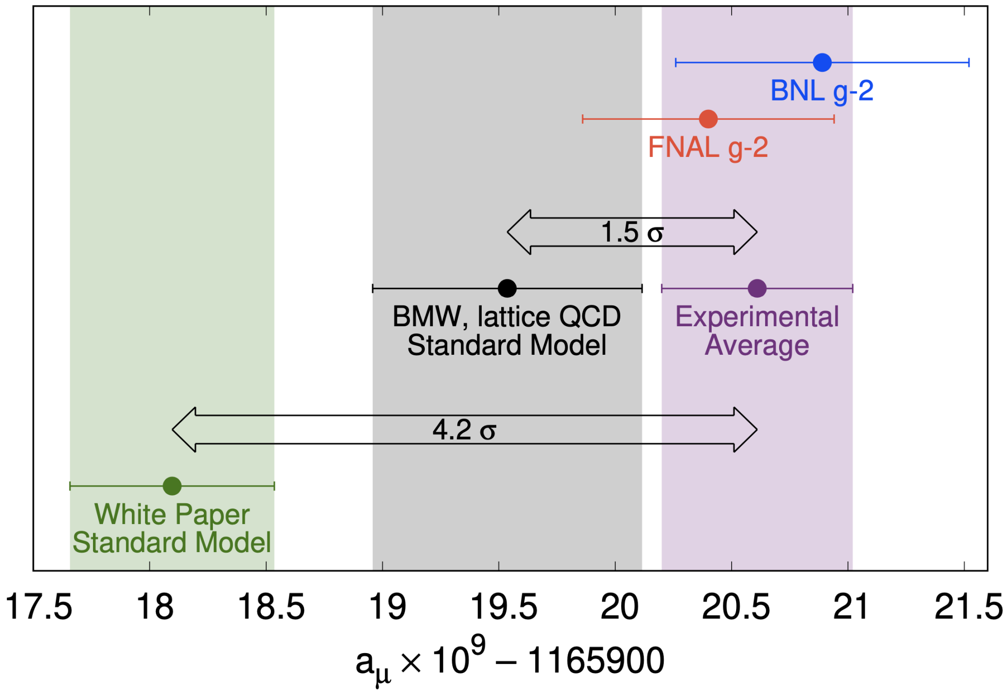
Hadronic Light-by-Light scattering (HLbL, α^3)

- **Contribution from unknown particles / interactions (?)** $a_\ell^{\mathrm{NP}} = \mathcal{C}\,\frac{m_\ell^2}{\Lambda^2}$

- **First complete calculation** [BMW Collaboration : Nature 593 (2021)]
  - → first lattice calculation below 1% precision
  - → include all isospin-braking and QED corrections

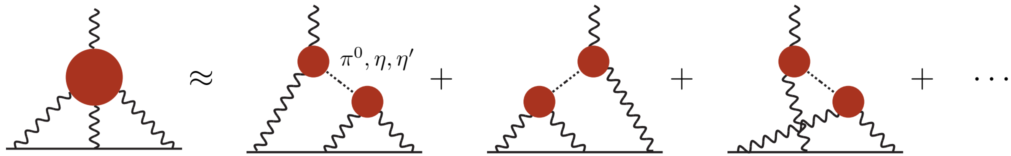


$$10^{10} \times a_\mu^{\text{LO-HVP}} = 708.7(2.8)_{\text{stat}}(4.5)_{\text{sys}}[5.3]_{\text{tot}}$$

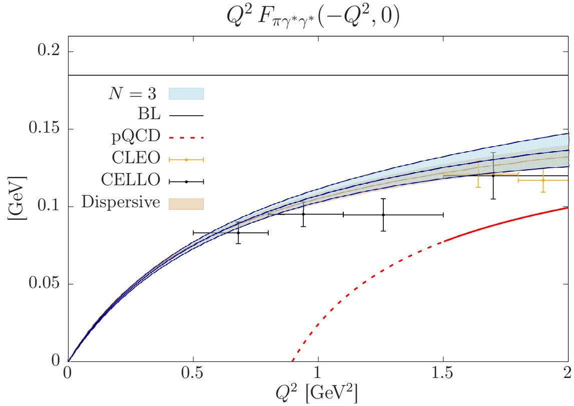


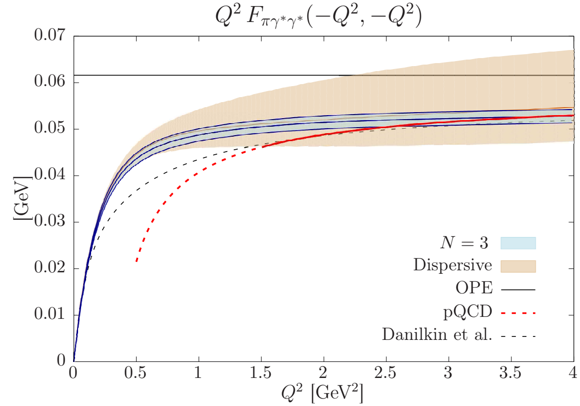
- The BMW result reduces the tension with SM
- The BMW result increases the tension with R-ratio data
- Future : s further reduction by a factor at least 3 is needed to reach the future exp. precision at Fermilab

- Main contribution from light pseudoscalar poles



- Pion transition form factor [A. G et al, Phys.Rev. D94 (2016)] [A. G et al, Phys.Rev. D100 (2019)]





- First ab-initio calculation of the pion-pole contribution

$$a_\mu^{\mathrm{HLbL};\pi^0} = (59.9 \pm 3.6) \times 10^{-11}$$

→ Compatible with the dispersive result

$$a_\mu^{\mathrm{HLbL};\pi^0} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$ [Hoferichter et al. '18]

→ Contribution under control

→ Next step : η and η' !

- Flavor physics in the quark sector
  - → Many results in the charm sector
  - → Test of the step-scaling method to simulate the b-quark
  - → Form factors $B_s \to D_s^{(*)} \ell\nu$ are under investigation
- Lepton sector : anomalous magnetic moment of the muon
  - → strong expertise in the calculation of the hadronic vacuum polarization
  - → first sub-percent result for the HVP (Marseille)
  - → on-going efforts on form factors for the hadronic light-by-light contribution
- Computer ressources
  - → simulations with light pion masses, small lattice spacings are computationally expensive