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Irreducible representations of $\mathbb{Z}_2^2$-graded supersymmetry algebra and their applications

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Recently, Lie superalgebras with a grading higher than $\mathbb{Z}_2$ attract wide range of interest. One of the simple systems having such symmetry is the $\mathbb{Z}_2^2$-graded version of supersymmetric quantum mechanics by Bruce and Duplij. It is a quantum mechanical realization of the $\mathbb{Z}_2^2$-graded extension of the $\mathcal{N} = 1$ supersymmetry algebra in $(0+1)$-dimensional spacetime. It is also known that the result of Bruce and Duplij is extendable to higher values of $\mathcal{N}$.

In the present work we investigate irreducible representations of $\mathbb{Z}_2^2$-graded SUSY algebra of $\mathcal{N} = 1, 2$. The irreps are specified by two parameters and $\mathcal{N} = 1$ algebra has only four dimensional irreps. While, $\mathcal{N} = 2$ algebra has four and eight dimensional irreps depending on the values of the parameters. As an application of the results, we consider $\mathbb{Z}_2^2$ supersymmetric classical mechanics in which the four dimensional irreps are used to define $\mathbb{Z}_2^2$-SUSY transformation. It is observed that one of the conserved Noether charge vanishes for a particular choice of auxiliary variables.

Quantum geodesics in quantum mechanics.

Edwin Beggs

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We show that the standard Heisenberg algebra of quantum mechanics admits a noncommutative differential calculus Ω^1 depending on the Hamiltonian $p^2/2m + V(x)$, and a flat quantum connection with torsion such that a previous quantum-geometric formulation of flow along autoparallel curves (or geodesics) is exactly Schrodingers equation. The connection ∇ preserves a non-symmetric quantum metric given by the canonical symplectic structure lifted to a rank $(0, 2)$ tensor on the extended phase space where we adjoin a time variable. We also apply the same approach to obtain a novel flow generated by the Klein Gordon operator on Minkowski spacetime with a background electromagnetic field, by formulating quantum geodesics on the relativistic Heisenberg algebra with proper time for the external geodesic parameter. Examples include quantum geodesics that look like a relativistic free particle wave packet and a hydrogen-like atom.

Dimensional flow from nonlocality: some results in Tensor Field Theory

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Tensor Field Theory (TFT), one of the simplest field theoretic counterparts of tensor models, enjoys two remarkable features: a large N expansion and nonlocality. Mixing these ingredients in a balanced way is of course what makes nontrivial the renormalization group analysis of TFT. On the other hand, it is a known fact that, e.g., the Functional Renormalization Group analysis of TFT with configuration space a compact group has one peculiarity: its sets of beta-functions becomes non autonomous. In the simplest instances, the large N limit (equivalently, the large radius size of the group) helps in setting up a notion dimension of the coupling constants. This makes the beta system autonomous and therefore computable. This presentation will explain how not taking the large N limit and letting the RG flow fully non autonomous leads to a genuine definition of dimensional flow. Some results will be given for a particular TFT endowed with cyclic melonic interactions at arbitrary but finite valence.

A combinatorial interpretation of the Kronecker coefficient via quantum mechanics of bipartite ribbon graphs

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he action of subgroups on a cartesian product of symmetric groups allows one to enumerate different families of graphs. In particular, bipartite ribbon graphs (with at most n edges) enumerate as the orbits of the adjoint action on two copies of the symmetric group (of order n!). These graphs form a basis of an algebra, which is also a Hilbert space for a certain sesquilinear form. Acting on this Hilbert space, we define operators which are Hermitians. We are therefore in the presence of a quantum mechanical model. We show that the multiplicities of the eigenvalues of these operators are precisely the Kronecker coefficients which counts the multiplicities of irreducible representation in the tensor product of irreducible representations of the symmetric group. We then prove that there exists an algorithm that delivers the Kronecker coefficients and allow us to interpret those as the dimension of a sub-lattice of the lattice of the ribbon graphs. Thus, this provides an answer to Murnaghans question (Amer. J. Math, 1938) on the combinatorial interpretation of the Kronecker coefficient.

Dirac operators, vector variables and bases for parastatistical Fock space representations

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The raising and lowering operators of m parabosons and n parafermions generate the Lie superalgebra $\mathfrak{osp}(2m+1|2n)$ or the $\mathbb{Z}_2\times\mathbb{Z}_2$-graded Lie superalgebra $\mathfrak{pso}(2m+1|2n)$ depending on whether relative relations of parafermi or parabose type are used.

We show that the relative parafermi (relative parabose) parastatistical Fock space representation of $\mathfrak{osp}(2m+1|2n)$ ($\mathfrak{pso}(2m+1|2n)$) can be realized as a subspace of the space of Clifford algebra valued super-antisymmetric (super-symmetric) forms by letting the raising and lowering operators act as vector variables and Dirac operators.

Using this realization we construct bases for the Fock spaces consisting of polynomials in the raising operators acting on the vacuum.

Kac-Moody Exceptional field theory

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General relativity and more generally supergravity solutions with commuting isometries admit hidden symmetries. Eleven-dimensional supergravity with more than six commuting isometries admits in particular exceptional group hidden symmetries E6, E7, E8, up to the Geroch type affine Kac-Moody symmetry E9. Following ideas of West, Hohm and Samtleben, we defined a formulation of eleven-dimensional supergravity with E11 Kac-Moody invariance in collaboration with Kleinschmidt and Sezgin. I will introduce this theory and explain how it provides an alternative formulation of maximal supergravity allowing for generalised geometries.

How Kontsevich's (affine) star product is associative up to order 6 (respectively 7)

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Joint work with: Arthemy Kiselev

Kontsevich proved that his star product built by using graphs is associative—but how? We contrast the associativity mechanisms at orders $\leqslant 6$ against order 7. We show that the expansion of the Kontsevich star product $\star$ mod $\bar{o}(\hbar^6)$ found by Banks–Panzer–Pym (2018) is associative up to $\bar{o}(\hbar^6)$. We find and reduce the formula $\star_{\text{aff}}$ mod $\bar{o}(\hbar^7)$ for the expansion of the Kontsevich star-product restricted to affine Poisson brackets; it is associative up to $\bar{o}(\hbar^7)$. But, whereas all the needed consequences of the Jacobi identity are contained in the associator itself for all orders $\leqslant 6$, this is no longer the case at $\hbar^7$. The results are obtained using the newly developed free software package gcaops (*Graph Complex Action on Poisson Structures*) for SageMath; see `https://github.com/rburing/gcaops`.

Hilbert Space Structure of the Low Energy Sector of U(N) Quantum Hall Ferromagnets and Their Classical Limit

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Abstract

Using the Lieb–Mattis ordering theorem of electronic energy levels, we identify the Hilbert space of the low energy sector of $\mathrm{U}(N)$ quantum Hall/Heisenberg ferromagnets at filling factor M for L Landau/lattice sites with the carrier space of irreducible representations of $\mathrm{U}(N)$ described by rectangular Young tableaux of M rows and L columns, and associated with Grassmannian phase spaces $\mathrm{U}(N)/\mathrm{U}(M)\times\mathrm{U}(N-M)$. We embed this N-component fermion mixture in Fock space through a Schwinger–Jordan (boson and fermion) representation of $\mathrm{U}(N)$-spin operators. We provide different realizations of basis vectors using Young diagrams, Gelfand–Tsetlin patterns and Fock states (for an electron/flux occupation number in the fermionic/bosonic representation). $\mathrm{U}(N)$-spin operator matrix elements in the Gelfand–Tsetlin basis are explicitly given. Coherent state excitations above the ground state are computed and labeled by complex $(N-M)\times M$ matrix points Z on the Grassmannian phase space. They adopt the form of a $\mathrm{U}(N)$ displaced/rotated highest-weight vector, or a multinomial Bose–Einstein condensate in the flux occupation number representation. Replacing $\mathrm{U}(N)$-spin operators by their expectation values in a Grassmannian coherent state allows for a semi-classical treatment of the low energy (long wavelength) $\mathrm{U}(N)$-spin-wave coherent excitations (skyrmions) of $\mathrm{U}(N)$ quantum Hall ferromagnets in terms of Grasmannian nonlinear sigma models.

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Symmetry algebras of ODEs and the embedding problem for Lie algebras

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The representation theory of semisimple Lie algebras $\mathfrak{s}$ is used to characterize n-dimensional systems of ordinary differential equations of degree k admitting $\mathfrak{s}$ as exact Lie point symmetry algebra. The branching rules of representations are combined with the embedding problem in order to determine conditions that ensure that a given subalgebra appears as symmetry algebra of a system in a given realization by differential operators. The analysis is extended to some classes of inhomogeneous Lie algebras.

Generalisation of affine Lie algebras on compact real manifolds

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Joint work with: M. de Montigny (U. Alberta) and M. Rausch de Traubenberg (Université de Strasbourg, CNRS)

A generalised notion of Kac-Moody algebra is defined using smooth maps from a compact real manifold $\mathcal{M}$ to a finite-dimensional Lie group, by means of complete orthonormal bases for Hermitian inner products on the manifold and Fourier expansions. The Peter–Weyl theorem for the case of manifolds related to compact Lie groups and coset spaces is discussed, and appropriate Hilbert bases for the space $L^2(\mathcal{M})$ of square-integrable functions are constructed. It is shown that such bases are characterised by the representation theory of the compact Lie group, from which a complete set of labelling operator is obtained. The existence of central extensions of generalised Kac-Moody algebras is analysed using a duality property of Hermitian operators on the manifold, and the corresponding root systems are constructed. Several examples are given.

The Klein paradox in phase space quantum mechanics

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The Klein paradox is considered in the context of quantum mechanics in phase space. The external degrees of freedom are represented together with the internal degrees of freedom in the Hilbert space $L^2 2(R) \otimes C^2$. The tunneling coefficients are extrapolated with the help of a continuity equation newly formulated in terms of a density operator.

Geometry of regular and not regular separation: the example of the bi-Helmoltz equation

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Joint work with: B. Jayyusi and R.G. McLenaghan

Separation of variable is a standard usefull ansatz in order to determine some solutions of a PDE. We give the geometrical framework of two different types of separation, called by Kalnins and Miller "regular" and "non-regular" separation. The geometric interpretation of non regular separation provides an effictive method to understand and characterize some known examples, as well as the so called fixed energy separation or the constrained separation for Schroedinger equation. Furthermore, we will see how to apply this tool for an exploration of multiplicatively separable solutions of the bi-Helmoltz equation $\Delta^2 f = Ef$. This equation appears classically in the theory of sound because it is used as a model for the vibrations of a (thin) solid plate. Even if it will be shown that regular separation never occurs, however the equation naturally admits families of separated solutions (coinciding with separated solutions of Helmholtz equation). In some examples we geometrically characterize the coordinate systems where additional separated solutions are determined.

Optimal programming of quantum gates

Giulio Chiribella

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A universal quantum processor is a device that can approximately implement any desired quantum gate on a given system. The specification of the desired gate is provided by a program, which in most implementations of quantum computing consists of classical data. From the foundational point of view, however, it is interesting to explore the more general scenario where the program is itself a quantum system. In the past two decades, a major open question has been to determine how the size of the smallest quantum program scales with the required accuracy in the implementation of the desired gate. Here we answer the question, by proving a bound on the size of the program and designing a concrete protocol that attains the bound in the asymptotic limit. Our result is based the representation theory of the special unitary group. It provides improved bounds on the estimation of unitary gates, and on the implementation of quantum protocols subject to conservation laws.

A minimal representation of the exceptional Lie superalgebra $D(2,1,\alpha)$

Claerebout, Sam

Ghent University

We construct two infinite-dimensional irreducible representations for $D(2,1;\alpha)$: a Schrödinger model and a Fock model. Further, we also introduce an intertwining isomorphism. These representations are similar to the minimal representations constructed for the orthosymplectic Lie supergroup and for Hermitian Lie groups of tube type. The intertwining isomorphism is the analogue of the Segal-Bargmann transform for the orthosymplectic Lie supergroup and for Hermitian Lie groups of tube type.

On Old Relations of Lie Theory, Classical Geometry and Invariant Theory

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We discuss briefly an embedding of gauge theory, low dimensional Lie symmetries, and affine geometry advocated by Weyl into (linear) projective geometry and classical, well-known representations. There, we show in the case of special and general relativity some geometrical relations of the quaternary quadratic form $x_\mu x^\mu = 0$, its transfer to dual geometrical objects $p_\mu p^\mu = 0$, and to differential geometry. In this context, the Dirac equation and Clifford algebras emerge naturally, as well as skew transformation reps of SU(2)×SU(2) or subgroups thereof which nowadays are usually interpreted in terms of Lie algebras, groups, and their representation theory, and which are usually treated by Riemannian geometry and Geometric Analysis. So the linear approach yields Lie algebras, exponentials and groups as it should.

The general background, however, leads back to Lie's sphere geometry as well as Lie's and Klein's work on line Complexe, and thus to null systems and P^5. Whereas the Plücker-Klein quadric describes the photons/the massless gauge sector in terms of special linear line Complexe, the 'full' P^5 has to include general linear Complexe with non-vanishing invariants, and advocates the identification with massive states. Last not least, the (quadratic) tetrahedral Complex shows up in a central rôle and provides the central link between geometry and typical 'quantum' theories in terms of SU(4), or SU*(4), both related to projective quaternion reps, and paves the way to 'classical' invariant theory. As one striking result, the symmetric **<u>20</u>** of SU(4) we've discussed so far in terms of SU(4) and chiral SU(2)×SU(2) ('Chirons') to describe the nucleon-Delta system can be identified immediately with the quaternary cubic null forms as discussed by Hilbert in his work on full invariant systems, i.e. as a different rep to handle classical spatial geometry and its associated symmetries.

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Reduction of contact Hamiltonian systems

Manuel de León

ICMAT-CSIC and Real Academia de Ciencias

In recent years there has been increasing interest in the study of contact Hamiltonian systems, which lead to dissipative properties instead of the usual conservative properties of contact Hamiltonian systems. In this paper we will present several ways to reduce a contact Hamiltonian system and we will also discuss the Hamilton-Jacobi equation for them, as well as the reduction procedure using the notion of Legendrian submanifold.

Symmetry Groups, Quantum Mechanics and Generalized Hermite Functions

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Abstract

In this communication a generalisation of Euclidean and pseudo-Euclidean groups is presented. Also the well known in quantum mechanics Weyl-Heisenberg group is involved. A new family of groups is obtained including the above mentioned groups as subgroups. Properties like self-similarity and invariance with respect to the orientation of the axes are properly included in the structure of this new family of groups. Generalized Hermite functions on multidimensional spaces, which serve as orthogonal bases of Hilbert spaces supporting unitary irreducible representations of these new groups, are introduced. By extending these Hilbert spaces, we obtain representations on rigged Hilbert spaces.

New perspectives in Gravity Mediated SUSY Breaking

Robin Ducrocq, Groupe Théorie, IPHC, Strasbourg

Work with Gilbert Moultaka (L2C) & Michel Rausch de Traubenberg (IPHC)

I will present a pedagogical review of supersymmetry & supergravity and emphasize the main differences between these two theories. As known, supersymmetry and supergravity cannot be a real symmetry of the universe and must be broken. In this talk, we investigate Gravity Mediated Supersymmetry Breaking. New solutions are exhibited.

The Harmonic Oscillator, Enhancements and Applications

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University of Virginia

The simple harmonic oscillator is one of the first models taught in undergraduate courses. As simple as it is there are many real-world applications (Foucault pendulum, childrens swings etc.). The quantum version is an instructive model with Hermite functions as wavefunctions, and by using a classical asymptotic formula one can see how the quantum model tends to the classical one as the energy increases. Next, add a potential of inverse-square type and invariant under a reflection group to the N-fold product of the quantum oscillator. The simplest case is the group $\mathbb{Z}_2$ of sign changes of the real line; both the classical and quantum models can be solved, the latter with Laguerre functions as wavefunctions and again there are interesting asymptotic relations. Several studies of the N-dimensional oscillator with the abelian group $\mathbb{Z}_2^N$ of sign-changes (thus the coordinate hyperplanes repel the particle) have been carried out. By constructing different bases of wavefunctions interesting algebras arise in the problem of expressing one basis in terms of another. As well there are results about super- integrability and relativistic modifications. In contrast to the abelian symmetry groups consider a model where the reflection group is nonabelian - in particular the dihedral situation. Here the plane is divided into "pie slices" with equal angles. We present more details about this model, solving for the wavefunctions and describing associated algebras of operators.

What comes after Cappelli-Itzykson-Zuber's A-D-E?

Terry Gannon

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One of the most celebrated results in Conformal Field Theory is the classification by Cappelli-Itzykson-Zuber in 1987 of the conformal field theories with sl(2) symmetry. They found that it falls mysteriously into an A-D-E pattern. A few years later the analogous result for sl(3) was obtained; these have a mysterious connection with Jacobians of Fermat curves. Until recently, very little else was known. The hard part of the problem concerns the possible extensions of the rational vertex operator algebras coming from Lie algebras. This can be thought of as the nonabelian version of extending lattices. My talk is aimed at nonexperts. It describes the history of the problem, and a recent breakthrough. And it will ask for your help in seeing what comes after A-D-E and Fermat curves.

An algebraic approach to intertwined quantum phase transitions in the Zr isotopes[a]

Strasbourg 2022, abstract submission

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Abstract

In this talk I will discuss the notion of intertwined quantum phase transitions (IQPT), for which a QPT involving a crossing of two configurations (Type II QPT) is accompanied by a shape evolution of each configuration with its own separate QPT (Type I QPT). We demonstrate the relevance of IQPTs to the zirconium isotopes, with A =92−110, which have one of the most intricate evolutions of structure in the nuclear chart. We employ a calculation using the interacting boson model with configuration mixing (IBM-CM). Such an algebraic framework enables us to examine a large range of experimental data such as energy levels, two neutron separation energies, $E2$ and $E0$ transition rates, isotope shifts and magnetic moments. We consequently find the occurrence of Type II QPT between the normal and intruder configurations. Alongside the Type II QPT, we find that the Type I QPT takes place within the intruder configuration, which changes from weakly-deformed to prolate-deformed and finally to γ-unstable shapes, associated with the U(5), SU(3) and SO(6) dynamical symmetry limits of the IBM, respectively. In such a situation, both Types I and II have a critical-point near $A \approx 100$. The good agreement of our calculation with the vast empirical data along the chain of isotopes demonstrates the relevance of IQPTs to the zirconium isotopes, and can serve as a case study to set path for new investigations of IQPTs in other nuclei and other physical systems.

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Dark matter as a QCD effect in an Anti de Sitter background (Cosmogonic implications of de Sitter, Anti de Sitter and Poincaré symmetries)

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Joint work with: Gilles Cohen-Tannoudji, CEA Saclay

The ΛCDM standard model of cosmology involves two dark components of the universe, dark energy and dark matter. Whereas dark energy is usually associated with the (positive) cosmological constant associated with a de Sitter geometry, we propose [1] to explain dark matter as a pure QCD effect, namely a gluonic Bose Einstein condensate. This effect is due to the trace anomaly viewed as a Anti de Sitter positive curvature (negative cosmological constant) accompanying baryonic matter at the hadronization transition from the quark gluon plasma phase to the colorless hadronic phase. Our approach not only allows us to assume a ratio Dark/Visible equal to 11/2 but also provides gluons (and di-gluons, viewed as quasi-particles) with an extra mass of vibrational nature. Such an interpretation would comfort the idea that, apart from the violation of the matter/antimatter symmetry satisfying the Sakharovs conditions, the reconciliation of particle physics and cosmology needs not the recourse to any ad hoc fields, particles or hidden variables.

1. Gilles Cohen-Tannoudji and Jean-Pierre Gazeau https://www.mdpi.com/2218-1997/7/11/402 (ArXiv https://arxiv.org/abs/2111.01130v2) and references therein

QUANTUM MODELS À LA GABOR FOR SPACE-TIME METRIC
(*QUANTUM FIELD THEORY WITH NO PLANCK CONSTANT*)

JEAN-PIERRE GAZEAU

Abstract. As an extension of Gabor signal processing, the covariant Weyl-Heisenberg integral quantization is implemented to transform functions on the eight-dimensional phase space (x, k) into Hilbertian operators. The $x = (x^\mu)$'s are space-time variables and the $k = (k^\mu)$'s are their conjugate wave vector-frequency variables. The procedure is first applied to the variables (x, k) and produces canonically conjugate essentially self-adjoint operators. It is next applied to the metric field $g_{\mu\nu}(x)$ of general relativity and yields regularised semi-classical phase space portraits $\check{g}_{\mu\nu}(x)$. The latter give rise to modified tensor energy density. Examples are given with the uniformly accelerated reference system and the Schwarzschild metric. Interesting probabilistic aspects are discussed.

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ICGTMP-Group 34-Strasbourg (July, 2022) Hyperquaternions and Physics

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Abstract. Clifford algebras constitute a major tool in theoretical physics [1-2]. Defining hyperquaternions as a tensor product of quaternion algebras (or a subalgebra thereof) the paper presents a hyperquaternionic representation of Clifford algebras. Examples of hyperquaternions are quaternions $\mathbb{H}$ $(e_1 = i, e_2 = j)$, biquaternions $\mathbb{H} \otimes \mathbb{C}$ $(e_1 = iI, e_2 = jI, e_3 = kI$ with $I = 1 \otimes i)$, tetraquaternions $\mathbb{H} \otimes \mathbb{H}$ $(e_0 = j, e_1 = kI, e_2 = kJ, e_3 = kK)$ and so on. Since $\mathbb{H} \otimes \mathbb{H} \simeq m(4, \mathbb{R})$, $[\mathbb{H} \otimes \mathbb{H}] \otimes \mathbb{C} \simeq m(4, \mathbb{C})$, $[\mathbb{H} \otimes \mathbb{H}] \otimes \mathbb{H} \simeq m(4, \mathbb{H})$, it follows that hyperquaternions yield all real, complex and quaternionic square matrices. A hyperconjugation gives the matrix transpose, adjoint and transpose quaternion conjugate. Hyperquaternions allow a good representation of major symmetry groups of physics: pseudo-orthogonal, unitary, unitary symplectic as well as the Poincaré and conformal groups [3-6]. First, the paper retraces historically, the uses (and frequent misuses) of hyperquaternions and their links to symmetry groups up to the present day. The term hyperquaternions was coined by Moore a hundred years ago to designate Lipschitz' algebras (isomorphic to even Clifford algebras) [7]. Secondly, the paper develops a new hyperquaternionic representation of the conformal group in $3D$ $(T + 2D)$ hyperbolic space $(+ - -)$ as a subalgebra of $\mathbb{H} \otimes \mathbb{H} \otimes \mathbb{H} \simeq Cl(2, 4)$. A numerical implementation thereof is provided together with a canonical decomposition into simple planes. Potential applications include in particular modeling and imaging. Finally, as an outlook, hyperquaternions are proposed as a unifying tool of physics.

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The Prediction of Anyons: Its History and Wider Implications

Gerald A. Goldin

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"Anyon" are quantum particles or excitations in two space dimensions whose exchange statistics can be intermediate between bosons and fermions. They are associated with surface phenomena in the presence of magnetic flux. Theoretical applications include explaining the quantum Hall effect, describing quantum vortices in superfluids, and relevance to quantum computing. In 2020 experimentalists succeeded in creating anyonic excitations, focusing new attention on these fascinating possibilities. Prediction of the anyon, which required fundamental changes in our understanding of quantum statistics, is often attributed exclusively and incorrectly to Frank Wilczek. This talk first outlines the early history, from predecessor ideas to the first clear, independent predictions in papers by Leinaas & Myrheim (1977) and Goldin, Menikoff, & Sharp (1980-81), followed by Wilczeks 1982 articles and subsequent important group-theoretical insights by Goldin, Menikoff, & Sharp (1983, 1985). Then I shall discuss some wider implications for physics, its teaching, and its presentation to the public. Why did the possibility of intermediate statistics elude physicists for over half a century after bosons and fermions were understood, when the concept now seems so easy? What then led to independent predictions within a short time of each other? What can we learn from this about the teaching of mathematics and physics? I shall conclude by addressing the painful implications of scientists and journalists systemic failure to accurately attribute scientific achievements breaches of integrity occurring even when there is no dispute. The anyon case is not unique. The social consequences of such failure include non-recognition and career obstacles disproportionately hurting women, minorities, and scientists in developing countries, as well as intimidation and disillusionment of young scientists who have much to lose by speaking out.

Twisted Affine Integrable Hierarchies and Soliton Solutions

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Joint work with: Y. F. Adans, G. V. Lobo and A. H. Zimerman

A systematic construction of a class of integrable hierarchy is discussed in terms of the twisted affine $A_2^{(2)}$ Lie algebra. The zero curvature representation of the time evolution equations are shown to be classified according to its algebraic structure and according to its vacuum solutions. It is shown that a class of models admit both zero and constant (non zero) vacuum solutions. Another, consists essentially of integral non-local equations and can be classified into two sub-classes, one admitting zero vacuum and another of constant, non zero vacuum solution. The two dimensional gauge potentials in the vacuum plays a crucial ingredient and are shown to be expanded in powers of the vacuum parameter v_0. Soliton solutions are constructed from vertex operators, which for the non zero vacuum solutions, correspond to deformations characterized by v_0.

Traceless projection of tensors of any rank and symmetry via Brauer algebra

Yegor Goncharov

Université de Tours, Université de Mons

Traceless tensors are widely encountered in field theory in the context of irreducible representations of orthogonal groups (for example, Lorentz group and conformal group in $d \geq 2$). We present the construction of a traceless projector for tensors of any rank in any dimension. The projector is constructed in terms of the Brauer algebra, where its representation theory gives the possibility of obtaining the result in a closed from. Application of the projector to a tensor commutes with permutations of indices, so the irreducible tensors can be obtained by additional application of Young projectors equally either before or after subtracting traces.

Do the solutions of the generalized Cattaneo-Vernotte equation vanish outside the compact region?

Katarzyna Gorska

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We consider the generalized Cattaneo-Vernotte equation (GCVE) with the time derivatives smeared through convoluting them with some memory kernels which choice to be power-like $t^{-a}, 0 < \ \leq 1$, reduces it to the time fractional equation governed by the Caputo derivatives which highest order is 2. To invert the solutions from the Fourier-Laplace domain to the space-time domain we use the Efross theorem and find out that solutions we are looking for are given by integral decompositions obeying a kind of duality they tangle the fundamental solutions either of the standard Cattaneo-Vernotte equation or the diffusion equation with some functions which non-negativity depends on the shape of memory kernels. Methodology arising from the theory of complete Bernstein functions allows us to find sufficient conditions for their non-negativity and if it happens to interpret the integral decompositions as subordination. This clarifies puzzling situation occurring if one considers the kernels $t^{-a}, 1/2 < a \leq 1$, for which the subordination based on the Brownian motion fails. But the crucial benefit of our research comes from the observation that the existence of solutions which vanish outside the compact region is correlated with the presence of the second rank time derivative in the equation we begin with.

Reflection Equation Algebra as quantum analog of $U(gl(N))$

Dimitri Gourevithc

UPHF

One usually considers the Quantum Group $U_q(sl(N))$ as a quantum analog of the enveloping algebra $U(sl(N))$. Nevertheless, this QG is isomorph to a completion of the algebra $U(sl(N)$ and only the coalgebraic structure is deformed indeed. The Reflection Equation Algebra, corresponding to the Hecke symmetry coming from the QG $U_q(sl(N))$, is a deformation of the algebraic structure of $U(gl(N))$. The properties of this and other REA, corresponding to different Hecke symmetries, are similar to these of the algebra $U(gl(N))$. I plan to exhibit certain of these properties and some applications of REA in algebra and geometry.

Standard Model symmetries from Cayley-Dickson algebras

Liam Gourlay, Abhinav Varma, Niels Gresnigt*

Abstract

The four normed division algebras, as very generative mathematical objects, have frequently been proposed as the algebraic source responsible for the gauge symmetries and particle content of the Standard Model. The many promising results to date are predominantly restricted to a single generation. Despite numerous attempts to describe three generations, none has brought clear success, suggesting that the most suitable algebraic structure has not yet been found.

We propose that the algebra of sedenions, generated from the octonions via the Cayley-Dickson process, has suitable properties to describe the symmetries and particle content of three generations. Three octonion subalgebras can be isolated within the sedenions, each of which describes a single generation. Two possible models based on such a construction are outlined: one in which generations are distinguished by their unbroken electrocolour symmetries, the other by their chiral weak symmetry.

*corresponding author

Entanglement measures in parity adapted coherent states for symmetric multiquDits

J. Guerrero

University of Jaén

We pursue the use of information measures (in particular, information diagrams) for the study of entanglement in symmetric multi-quDit systems. We use generalizations to U(D) of spin U(2) coherent states and their adaptation to parity (multicomponent Schrdinger cats) and we analyse one- and two-quDit reduced density matrices. We use these correlation measures to characterize quantum phase transitions occurring in Lipkin-Meshkov-Glick models of D = 3-level identical atoms and we propose the rank of the corresponding reduced density matrix as a discrete order parameter.

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Noncommutative spaces of worldlines from quantum groups: Construction and phenomenological implications

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(Joint work with A. Ballesteros and F. J. Herranz, Universidad de Burgos, Spain)

In this talk I will describe a canonical procedure to introduce noncommutativity in the space of geodesics of a maximally symmetric spacetime, starting from a quantum group. I will show that by construction this space of noncommutative geodesics, as well as the associated noncommutative spacetime, is covariant under the corresponding quantum group. As an example of this general framework I will present the noncommutative space of time-like geodesics associated to the κ-Poincaré quantum group [1] and the noncommutative space of light-like geodesics associated to the light-like deformation [2]. Moreover, I will comment on some phenomenological implications derived form the noncommutative structure of the space of worldlines [3].

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Investigation of the Dunkl-Schrödinger equation for Position Dependent Mass in the presence of a Lie algebraic approach

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Abstract

Recent studies have shown that the use of Dunkl derivatives instead of ordinary derivatives leads to deriving parity-dependent dynamic solutions. According to this motivation in this manuscript, we formulate the Dunkl-Schrödinger equation within the position-dependent mass formalism and derive an algebraic solution out of it. Our systematic approach lets us observe some new findings in addition to the earlier ones. For example, we find that the solution of the Dunkl-Schrödinger equation with position-dependent mass cannot be considered independent from the choice of parameters. Similarly, through the sl(2) algebra, the energy spectrum and the corresponding wave functions are derived in terms of possible Dunkl, (μ), and mass, (α), parameters.

Keywords: Dunkl derivative; Position-dependent mass; Quasi-Exactly Solvable (QES); sl(2) Lie algebra.

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From quantum deformations to noncommutative spaces: Application to quantum (A)dS and Poincaré groups

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(Joint work with A. Ballesteros and I. Gutierrez-Sagredo, Universidad de Burgos, Spain)

The aim of this contribution is twofold. Firstly, we present a systematic and general procedure that allows the construction of a noncommutative analogue (endowed with quantum group invariance) of any classical homogeneous space G/H with Lie group G and isotropy subgroup H [1]. And secondly, we apply this approach to obtain new noncommutative (A)dS and Minkowskian spacetimes in (3+1) dimensions by imposing to keep the Lorentz sector as a quantum subgroup [2]. In contrast to the well-known kappa-Minkowski space, all these new noncommutative Lorentzian spacetimes are non-Lie algebraic. Finally, some of their properties are also discussed.

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A family of three-dimensional classical Hamiltonian systems in magnetic fields

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Joint work with: Doc Ing. Libor Šnobl

The talk presents a family of three-dimensional classical Hamiltonian systems with vector potential, i.e. in a magnetic field. We consider more general structure of its quadratic commuting integrals of motion and construct explicitly quadratic commuting integrals in several magnetic and electric fields. We show how under certain conditions the integrals split into independent integrals of motion, which may ensure the superintegrability of the system. We also demonstrate their symmetry algebras.

Subordination and memory dependent kinetics in diffusion and relaxation phenomena

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Recent years have brought the idea of subordination to the standard set of mathematical tools used in the theory of anomalous diffusion and non-exponential relaxations. Grace to intuitively understood concepts of parent and leading processes subordination not only enables us to attribute physical interpretation to integral decompositions representing solutions to anomalous diffusion and relaxation problems but also to study mathematical requirements underlying these solutions. To solve the time-nonlocal , i.e., memory dependent, evolution equations we use the Efross theorem of operational calculus which appears an universal method directly leading to integral decompositions. Our scheme confirms the memory-stemmed origin of subordination and builds a bridge between the analysis-based methods and well established stochastic and probabilistic approaches. Results are illustrated on several models of anomalous diffusion and relaxation phenomena.

Newton mechanics, Galilean relativity and special relativity in α-deformed binary operation setting

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Abstract

We define new velocity and acceleration having dimension of $(Length)^\alpha/(Time)$ and $(Length)^\alpha/(Time)^2$, respectively, based on the fractional addition rule. We discuss the formulation of fractional Newton mechanics, Galilean relativity and special relativity in the same setting. We show the conservation of the fractional energy, characterize the Lorentz transformation and group, and derive the expressions of the energy and momentum. The two body decay is discussed as a concrete illustration.

Differential realization of ladder operators for the Rosen-Morse systems

Véronique Hussin

Université de Montral, Montr'eal, Canada

The Rosen-Morse systems in one dimension are well-known as completely solvable leading to an energy spectrum which is rational in the quantum number n. As a special case, we have the Pschl-Teller system for which the ladder operators are well-known. They are realized as first order differential operators in the coordinate x and depend also on the number operator N. We show that these operators are not easy to find if we just try to generalize the preceding ones in the case of Rosen-Morse. We explain how to make the connection in this presentation.

Schwingers picture of Quatum Mechanics: Groupoids and their algebras

Alberto, Ibort

Universidad Carlos III de Madrid

The recently proposed groupoidal description of Schwingers picture of Quantum Mechanics will be succinctly reviewed. The construction of elementary quantum systems will be discussed from this perspective and some non-trivial applications concerning their topological properties will be discussed.

Kontsevich's universal graph flows on the spaces of Nambu–Poisson brackets: their hidden symmetry

Arthemy Kiselev

University of Groningen

Joint work with: R.Burin

Can we deform a given Poisson bracket –universally for all finite-dimensional Poisson manifolds– in such a way that it stays Poisson at least infinitesimally and there is no a priori mechanism for the deformation to be trivial in the respective Poisson cohomology? Although the question might sound too general, Kontsevich answered it in the affirmative (1996) by finding a source of solutions: built from suitable cocycles in the Kontsevich graph complex, these deformations are encoded by directed graphs. Willwacher (2010–15) established that there are infinitely many generators of nonlinear proper infinitesimal symmetries of the Jacobi identity; namely, countably many graph cocycles are obtained from the generators of the Grothendieck–Teichmueller Lie algebra $\mathfrak{grt}$ (in turn, introduced by Drinfeld around 1990). The tetrahedral graph cocycle gives an example of degree-four order-three differential-polynomial flow on the spaces of Poisson bi-vectors.
In this talk, we discover new properties of such flows' restrictions –for the Kontsevich tetrahedral graph cocycle and for the Kontsevich–Willwacher pentagon-wheel cocycle– to the spaces of Nambu–Poisson "determinant" brackets in dimensions 3 and 4. We examine the analytic and combinatorial structures now arising from the graph formula, and we detect a hidden symmetry of the Poisson cocycles for this class of highly-nonlinear Poisson brackets. We establish that for the class of Nambu-determinant Poisson bi-vectors, the tetrahedral graph flow on the affine space $\mathbb{R}^3$ appears to be trivial in the second Poisson cohomology; we examine the combinatorics and symmetry of the highly-nonlinear trivialising vector field.

Kontsevich's universal graph flows on the spaces of Nambu–Poisson brackets: their hidden symmetry

Arthemy Kiselev

University of Groningen

Joint work with: R.Burin

Matrices, vector fields, and graphs give us three natural examples of Lie algebras: the vector space of finite graphs with wedge ordering of edges is a differential graded Lie algebra (dgLa). Its Lie superbracket is the difference of consecutive insertions of an entire graph into a vertex of the other graph, and the differential $d = [\text{ o–o, - }]$ blows up each consecutive vertes to an edge.
The construction of the dgLa of graphs goes in parallel with construction of the Lie superalgebra of endomorphisms on the space of multidifferential operators (e.g., multivectors) on a manifold; the Nijenhuis–Richardson bracket is the Lie superalgebra structure –with the same grading signs as in the Jacobi identity for graphs– and the Schouten bracket is the Maurer–Cartan element (whereas the stick graph o–o satisfies the master-equation [o–o, o–o] = 0 in the world of graphs).
This parallel between the dgLa of graphs and dgLa of multidifferential operators' endomorphisms results in the graph orientation mapping which associates, in particular, infinitesimal symmetries of the spaces of Poisson brackets –that is, second Poisson cohomology classes– with suitable cocycles in the graph complex. We discover that the combinatorial topology of graphs encodes the supermathematics of Poisson geometry and of the endomorphisms: this is where all the rules of signs come from to superanalysis.
The aim of this talk is to explain and motivate new open problems about construction of graph cocycles and of the respective universal symmetries of the Jacobi identity for Poisson brackets. More problems are about the observed properties of these structures. Based on recent joint work with R.Buring (Mainz), the talk will serve an introduction into the subject and outline future perspectives.

Associating quantum vertex algebras with quantum affine algebras

Slaven Kožić
University of Zagreb

One important problem in the vertex algebra theory is to associate certain vertex algebra-like objects, the so-called quantum vertex algebras, to various classes of quantum groups, such as quantum affine algebras or double Yangians. Roughly speaking, the goal is to establish a correspondence between these structures that goes in parallel with the already established connection between affine Kac–Moody Lie algebras and vertex algebras. In this talk, I will discuss this problem in the context of Etingof–Kazhdan's quantum vertex algebra associated with the trigonometric R-matrix in type A. More specifically, in this setting, the usual notion of (quantum) vertex algebra module does no longer seem to be suitable, so we use Li's notion of ϕ-coordinated module instead. This allows us to prove that a certain broad class of modules for the (suitably completed) quantum affine algebra in type A coincides with the class of ϕ-coordinated modules for the aforementioned quantum vertex algebra.

Yang-Mills solutions on Minkowski space via non-compact coset spaces

Kumar, Kaushlendra

Leibniz University Hannover

We compute solutions of the Yang–Mills equations on Minkowski space by foliating different parts of it with non-compact coset spaces arising from the Lorentz group $SO(1,3)$. The interior of the lightcone is foliated with hyperbolic space H^3 which is isomorphic to $SO(1,3)/SO(3)$, while the exterior of the lightcone is foliated with de Sitter space dS_3 which is isomorphic to $SO(1,3)/SO(1,2)$. The lightcone is parametrized with $SO(1,3)/ISO(2)$. The equivariant reduction of the $SO(1,3)$ Yang–Mills system on the three coset spaces is discussed. On the foliated spaces, the reduction yields a mechanical system with inverted potential admitting analytic solutions, including the kink. On the lightcone, only the pure gauge solution remains. We also present the energy-momentum tensor for the obtained solutions.

Spin degrees of freedom incorporated in conformal group

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In the conformal transformation of a multi-component field $\psi(x)$, where spin degrees of freedom are incorporated, it may be customary to remove the translation generator P_μ from the conformal algebra. Here we introduce the translation generator in such a way that the conformal algebra acting on the field $\psi(x)$ is the same as the original conformal algebra acting on the corresponding spin-zero field. Assuming that the free Dirac equation is invariant under the new translation generator $P_\mu + \pi_\mu$ as $\psi'(x) = \mathrm{e}^{-\mathrm{i}a^\mu(P_\mu+\pi_\mu)}\psi(x)$, we find that it is necessary to decompose the range of the map $\psi : \mathbb{R}^4 \to \mathbb{C}^4$ into positive and negative energy states, and that it is natural to decompose the domain of ψ into space-like and time-like regions.

Monogenic representations of the algebra of symmetries of the generalised Dirac operator

Group 34, Strasbourg

Alexis Langlois-Rémillard

May 6, 2022

We study the representations of the algebra of symmetries of the generalised Dirac operator. It is the centraliser of a realisation of the Lie superalgebra $\mathfrak{osp}(1|2)$ by a Dirac-like operator and its dual symbol. A motivating example is given by the study of the Dunkl–Dirac equation, where the partial derivatives are changed to Dunkl derivatives, a deformation by a reflection group. The polynomial null solutions of the Dunkl–Dirac equations, the monogenic polynomials, form representation of this algebra of symmetries. We present a class of representations of which the monogenic polynomials form an example. For a class of reflection groups, the algebra admits a subalgebra exhibiting a triangular structure. We present a construction of representations exploiting this additional structure. Moreover, for arbitrary reflection groups, we construct the monogenic polynomials using generalised symmetries.

Polynomial algebras of superintegrable systems separating in Cartesian coordinates from higher order ladder operators

Danilo Latini

Abstract

We will discuss recent results concerning the general polynomial algebras characterizing a class of higher-order superintegrable systems that separate in Cartesian coordinates. The construction relies on underlying polynomial Heisenberg algebras and their defining higher-order ladder operators and applies, among others, to models involving exceptional orthogonal polynomials. As an explicit example, we will describe a new three-dimensional superintegrable system related to Hermite exceptional orthogonal polynomials of type III. In particular, we will show how the degeneracies of the model can be characterized in terms of finite-dimensional irreducible representations of the polynomial algebra.

This is joint work with Ian Marquette and Yao-Zhong Zhang.

Emergent symmetries in nuclei: Probing physics beyond the standard model

K.D. Launey[1], G.H. Sargsyan[1], A. Mercenne[2], M. Burrows[1], O.M. Molchanov[1], T. Dytrych[1,3], J.P. Draayer[1]

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[2]Center for Theoretical Physics, Yale University, USA
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Abstract

Dominant shapes naturally emerge in atomic nuclei from first principles (Fig. 1), thereby establishing the shape-preserving symplectic Sp(3,R) symmetry [2,3] as remarkably ubiquitous and approximate symmetry in nuclei [1]. In this talk, I will discuss the critical role of this symmetry in enabling machine-learning descriptions of heavy nuclei [4], *ab initio* modeling of α clustering and collectivity, as well as tests of beyond-the-standard-model physics [5]. I will report recent results, in the *ab initio* symmetry-adapted no-core shell model, that place unprecedented constraints on recoil corrections in the ^{8}Li→^{8}Be β decay and help high-precision experiment establish the most stringent limit on tensor current contribution to the weak interaction to date, while explaining the Gamow-Teller β-decay discrepancy in the mass-8 systems [5]. [Supported by the U.S. NSF (PHY-1913728) and the Czech Science Foundation (22-14497S) & benefitted from HPC resources provided by LSU, NERSC, and Frontera.]

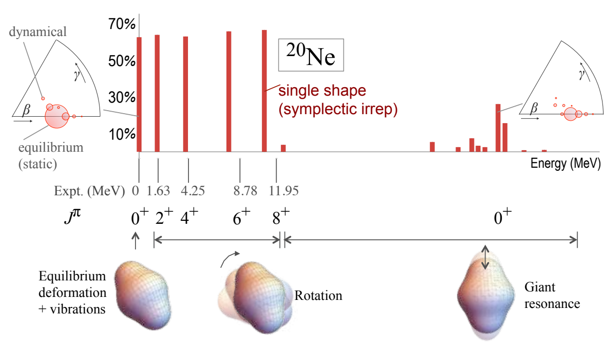

Figure 1. Emergence of almost perfect symplectic Sp(3,R) symmetry in nuclei from first principles, enabling *ab initio* descriptions of collectivity and clustering [1].

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L'évolution de la théorie des groupes et ses applications en sciences

J.-M. Lévy-Leblond

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Deux siècles de théorie des groupes : fondations, évolutions et applications. La notion de groupe est certainement l'un des plus importants apports des mathématiques du XIXe siècle. Généralisant et formalisant la notion intuitive de symétrie, elle est à la fois très simple quant à son essence et étonnamment riche quant à ses développement. La théorie des groupes a trouvé des prolongements remarquables en mathématiques pures, des appllications essentielles en physique et en chimie, voire dans les arts visuels. On s'efforcera d'en donner une présentation élémentaire et d'en dresser un panorama aussi concret que possible.

On the unexpected fate of scientific ideas

J.-M. Lévy-Leblond

Université Côte d'Azur, France

I will start from a recent personal experience. In 1965, I published a paper, exhibiting an hitherto unknown limit of the Lorentz group, which I called the « Carroll group » because of its seemingly paradoxical physical contents.Since I saw it as more curious than than relevant, so that I published in French in a journal far from the mainstream. It was most gratifying to notice the quite unexpected favour this paper started to enjoy a few years ago, so that a so-called « Carrollian physics » is now developing, with applications in various domains of forefront theoretical physics, such as supersymmetry, string theory, etc. Generalizing the theme, and drawing examples from group theory, I will reflect on the very diverse time scales with which scientific ideas develop — or not.

$N = 2$ supersymmetry, quantization and quantum superbundles

Mara A. Lledó

Universitat de Valncia and IFIC (CSIC-UVEG)

We consider N=2 superpaces and its properties of projectivity. In the projective case we succeed in quantizing it using quantum group formalism and show that this quantization is a quantum superbunde.

Integrability for Feynman Integrals

Florian Loebbert

Bonn , Germany

Since the rise of the AdS/CFT duality, integrability has become an important tool to advance our understanding of quantum field theory beyond two dimensions. One of the most challenging tasks in quantum field theory is still the evaluation of higher loop Feynman integrals. In this talk we review how large families of individual Feynman integrals inherit an infinite dimensional Yangian symmetry from the planar AdS/CFT correspondence. Surprisingly, this symmetry also extends to massive integrals, which suggests to further search for integrability in massive instances of AdS/CFT. We demonstrate how the Yangian can be used to bootstrap Feynman integrals in various spacetime dimensions from scratch. For the example of fishnet integrals in two dimensions this leads to a curious interplay between algebra and geometry.

Non-conservative systems can have conserved quantities!

Symmetries, reduction and Hamilton-Jacobi theory for forced mechanical systems

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Abstract

As it is well-known, Noether's theorem associates a constant of the motion to each symmetry of a Lagrangian system, and conversely. If the symmetries of a physical system form a Lie group, one can reduce its phase space by taking the quotient with respect to the Lie group action. Additionally, the Hamilton-Jacobi theory is a powerful technique to integrate the dynamics of a Hamiltonian system with symmetries, specially if it is an integrable system. Many physical systems require an external force besides the Hamiltonian or Lagrangian function (e.g., a dissipative force) to describe their dynamics. Moreover, external forces arise after performing a process of reduction in a nonholonomic system with symmetries. It is thus interesting to extend the results above to forced mechanical systems.

In this talk, I will show how some well-known results for conservative systems are naturally extended to forced systems in the framework of symplectic geometry. Our approach is based on geometric mechanics: we regard the space of positions as a differentiable manifold Q and make use of the symplectic structure of the phase space T^*Q and the tangent structure of the velocity space TQ. Geometrically, external forces are characterized by semibasic 1-forms on TQ or T^*Q, A Noether's theorem, a generalization of the Symplectic point reduction theorem and a Hamilton-Jacobi theory for forced Lagrangian and Hamiltonian systems will be presented. Our results are particularized for the so-called Rayleigh forces and illustrated with several examples.

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[1] Joint work with Manuel de León and Manuel Lainz (ICMAT-CSIC)

p-adic qubits from irreducible representations of $SO(3)_p$

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Abstract

A qubit at its most basic is the state space of a two-level quantum system, i.e. one with two-dimensional complex Hilbert space $\mathcal{H}$. This abstracts from all physical characteristics such a system might have, be it an electron spin, or that of a neutron, the polarization of a photon, a quantum dot, etc. Its characteristic is that the symmetry group $SU(2)$ acts transitively on all pure states

$$\mathcal{P}(\mathcal{H}) = \{\rho = |\psi\rangle\langle\psi| \,:\, \rho \geq 0, \mathrm{Tr}\,\rho = 1,\, \rho \text{ rank-one}\},$$

which geometrically is the surface of the Bloch sphere, $\mathbb{R}^3$, and the conjugation action on the density matrices is equivalent to the action of $SO(3)$ on the Bloch sphere. Furthermore, for a spin-$\frac{1}{2}$ particle, this action coincides with the one induced by the spatial rotations, and as an irreducible representation (irrep) gives rise to all other irreps by tensor powers $U^{\otimes n}$ and their Clebsch-Gordan decomposition into irreps. This has been long noticed, and even be taken as a possible foundation of quantum mechanics [1].

However, this link of the most basic quantum system to the geometry and indeed symmetry of physical space opens up other possibilities. For why should we accept a priori that physical space is described by the Euclidean $\mathbb{R}^3$? Indeed, the real field is not the only possible metric completion of the rationals: by Ostrowski's Theorem [2], there is a countable family of other completions, those of the p-adic numbers $\mathbb{Q}_p$. The fractal geometry of the p-adic numbers made them interesting for attempts at "new" quantum physics with a space-time that is decidedly non-Euclidean at very short or very large distances, as seems possible in quantum gravity [3]. In these approaches, the focus is on the translation symmetry of phase space, which in ordinary quantum mechanics leads to the framework of the Heisenberg-Weyl group of position and momentum displacements, parametrized by real numbers, but which here are displacements by p-adic amounts [4].

Here, we consider the rotation symmetry of $\mathbb{Q}_p^3$, encoded in the special orthogonal group $SO(3)_p$ in dimension three. Recently, we have characterized the geometry of this group [5], basing on the crucial observation that, up to equivalence, there is a unique quadratic form on $\mathbb{Q}_p^3$ with a compact symmetry group. Motivated by the analogy with the real case, our programme is to find its unitary complex projective representations, which should constitute a p-adic theory of quantum angular momentum. In particular the irreps of dimension two are crucial as p-adic analogues of spin-$\frac{1}{2}$ particles, which we interpret as p-adic qubits. In this spirit, we call any pair $(\mathcal{H}, \psi)$, where $\mathcal{H}$ is a two-dimensional Hilbert space and ψ is a projective irrep on $\mathcal{H}$, a qubit. We have not yet carried out this entire programme, but according to [6] we present the construction of the simplest possible two-dimensional irreps of $SO(3)_p$ for $p = 2, 3, 5$. The method to derive them could lead to examples of qubit representations for every prime p.

In particular, we recall the basic definitions and facts about the three-dimensional special orthogonal groups $SO(3)_p$, among them the crucial insight that the coefficients of special orthogo-

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nal matrices are p-adic integers. We exploit the latter property to define a hierarchy of projected groups by reducing the matrix entries modulo p^k, $k \geq 1$; each of them is a finite group, offering an easy way to constructing irreps. Then, we turn our attention in particular to the smallest reduction, $SO(3)_p \mod p$, for odd primes $p > 2$; we show that each has a homomorphism to some dihedral group, which is onto for $p = 3, 5$ and which we conjecture to be onto for all odd primes; this implies in particular that they all have two-dimensional irreps. Next, we explicitly compute the groups $SO(3)_3 \mod 3$ and $SO(3)_5 \mod 5$, and their representation theory, in both cases exhibiting qubits, i.e. two-dimensional irreps. Finally, we do the same for $SO(3)_2 \mod 2$, which turns out to be isomorphic to $\mathbb{S}_3$, the permutation group of three elements.

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Exact solvability and superintegrability : Algebraic constructions

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Abstract

It was discovered how polynomial algebras appear naturally as symmetry algebra of superintegrable quantum systems. They provide insight into their degenerate spectrum, in particular for models involving Painlevé transcendents. I will discuss an alternative perspective on those algebraic structures based on Lie algebras and their related enveloping algebras. I will discuss how the symmetry algebra of the generic superintegrable systems on the 2-sphere can be generated from an underlying Lie algebra. I will also discuss how exact solvability also can also be connected with such approach and how it differs from using differential operator realizations. The talk is based on the following recent works.

D Latini, I Marquette, YZ Zhang, Construction of polynomial algebras from intermediate Casimir invariants of Lie algebras, arXiv:2204.06840

R Campoamor-Stursberg, Ian Marquette, Quadratic algebras as commutants of algebraic Hamiltonians in the enveloping algebra of Schrödinger algebras, Annals of Physics. 437 168694 (2022) https://doi.org/10.1016/j.aop.2021.168694

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Jordan meets Freudenthal : a black hole exceptional story

Alessio Marrani

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I will review some aspects of the electric-magnetic duality in extremal black hole solutions of Maxwell-Einstein-scalar theories (which can be regarded as the purely bosonic sector of ungauged extended supergravity) in four space-time dimensions. Such aspects include the attractor mechanism, the geometry of the scalar manifolds, the duality orbits and the moduli space associated to the various classes of attractors. In particular, I will highlight the role of Jordan algebras of rank three, of the corresponding reduced Freudenthal triple systems and of their exceptional symmetries, and I will elucidate the relation between the Hessian of the black hole entropy and the pseudo-Euclidean, rigid special (pseudo)Kähler metric of the pre-homogeneous spaces associated to the duality orbits. I will then introduce the Freudenthal duality map acting on the electric-magnetic fluxes, and present the non-linear invariance of the Bekenstein-Hawking black hole entropy. I will then consider the axiomatization of groups "of type E_7" as introduced by Brown, highlighting their role as electric-magnetic duality groups, as well as their relation to pre-homogeneous vector spaces. Finally, relying on some results of Dynkin and Solomon, I will present various novel (numerably) infinite classes of groups "of type E_7". I will conclude with an outlook on further developments.

Vinberg special T-algebras : from black hole entropy to exceptional periodicity

Alessio Marrani

University of Murcia, ES.

By exploiting the so-called "magic star" projection (also named "G_2 decomposition" by Mukai), I will unravel various physical applications of the novel mathematical framework of "exceptional periodicity", which generalizes exceptional Lie algebras, especially in relation to black hole entropy in supergravity theories, Lie superalgebras, higher-dimensional Yang-Mills theories, spin factors, Bott periodicity, triality, and Vinberg special T-algebras.

Locality and Conservation laws: How, in the presence of symmetry, locality restricts realizable unitaries

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Abstract

According to a fundamental result in quantum computing, any unitary transformation on a composite system can be generated using so-called 2-local unitaries that act only on two subsystems. Beyond its importance in quantum computing, this result can also be regarded as a statement about the dynamics of systems with local Hamiltonians: although locality puts various constraints on the short-term dynamics, it does not restrict the possible unitary evolutions that a composite system with a general local Hamiltonian can experience after a sufficiently long time. In this talk, I show that this universality does not hold in the presence of conservation laws and global continuous symmetries: generic symmetric unitaries on a composite system cannot be implemented, even approximately, using local symmetric unitaries on the subsystems. In the context of quantum thermodynamics this no-go theorem implies that generic energy-conserving unitaries cannot be realized using local energy-conserving unitaries.

I also argue that in some cases this no-go theorem can be circumvented using ancilla qubits. For instance, any rotationally-invariant unitary on qubits can be realized using the Heisenberg exchange interaction, which is 2-local and rotationally-invariant, provided that the qubits in the system interact with a pair of ancilla qubits. Finally, I briefly present some results on qudit systems with SU(d) symmetry, which reveal a surprising distinction between the case of $d = 2$ and $d > 2$.

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On Möbius Gyrogroups

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Joint work with: Ali Reza Ashrafi

Gyrogroups are new algebraic structures that appeared in 1988 in the study of Einstein's velocity addition in the special relativity theory [1]. These new algebraic structures were studied intensively by Abraham Ungar [2,3]. The first gyrogroup was considered into account is the unit ball of Euclidean space R3 endowed with Einstein's velocity addition. The second geometric example of a gyrogroup is the complex unit disk $D = \{z \in \mathbb{C}, |z| = 1\}$. To define a gyrogroup structure on D we choose two elements $z_1, z_2 \in D$ Define the Möbius addition $z_1 \oplus z_2 = \frac{z_1+z_2}{1+\dot{z}_1 z_2}$ by. Then $(D, \oplus)$ is a gyrocommutative gyrogroup [4]. It is well-known that the gyrators of this gyrogroup is not closed under composition of gyrators. The aim of this talk is to report our recent results on the Möbius gyrogroup. All subgyrogroups containing gyrators will be considered into account.

Keywords: Mbius gyrogroup, gyrogroup, gyrocommutative gyrogroup, gyrator.

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Role of mixed permutation symmetry sectors in the thermodynamic limit of critical three-level Lipkin-Meshkov-Glick atom models

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(Dated: April 27, 2022)

Abstract

We introduce the notion of Mixed Symmetry Quantum Phase Transition (MSQPT) as singularities in the transformation of the lowest-energy state properties of a system of identical particles inside each permutation symmetry sector μ, when some Hamiltonian control parameters λ are varied. We use a three-level Lipkin-Meshkov-Glick (LMG) model, with $U(3)$ dynamical symmetry, to exemplify our construction. After reviewing the construction of $U(3)$ unirreps using Young tableaux and Gelfand basis, we firstly study the case of a finite number N of three-level atoms, showing that some precursors (fidelity-susceptibility, level population, etc.) of MSQPTs appear in all permutation symmetry sectors. Using coherent (quasi-classical) states of $U(3)$ as variational states, we compute the lowest-energy density for each sector μ in the thermodynamic $N \to \infty$ limit. Extending the control parameter space by μ, the phase diagram exhibits four distinct quantum phases in the λ-μ plane that coexist at a quadruple point. The ground state of the whole system belongs to the fully symmetric sector $\mu = 1$ and shows a four-fold degeneracy, due to the spontaneous breakdown of the parity symmetry of the Hamiltonian. The restoration of this discrete symmetry leads to the formation of four-component Schrödinger cat states.

Keywords: Quantum phase transitions, many-body systems, tensor-products and direct-sum Clebsch-Gordan decompositions, mixed permutation symmetries, coherent states.

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Relativistic Kinematics in Flat and Curved Space-Times

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Almost immediately after the seminal papers of Poincaré (1905, 1906) and Einstein (1905) on special relativity, wherein Poincaré established the full covariance of the Maxwell-Lorentz equations under the Poincaré group and Einstein explained the Lorentz transformation based on the assumption that the one-way speed of light in vacuo is constant and the same for all inertial observers (Einstein's second postulate), attempts were made to get at the Lorentz transformations from basic properties of space and time but avoiding Einsteins second postulate. Various such approaches usually involve general consequences of the relativity principle, such as a group structure to the set of all admissible inertial transformations and also assumptions about causality and homogeneity of space-time combined with isotropy of space.

The first such attempt is usually attributed to von Ignatowsky in 1911. It was followed shortly thereafter by a paper of Frank and Rothe published in the same year. Since then, papers have continued to be written on the subject even up to the present, with relevant contributions including those of V. Lalan (Bull. Soc. Math. France, 1937), A. D. Alexandrov and V. V. Ovchinikova (1953) and E.C. Zeeman (1964), H. Bacri and J. M. Lévi-Leblond (1968), V. Berzi and V. Gorini (1969) and V. Gorini (1971), H. J. Borchers and G. C. Hegerfeld (1972) and J. M. Lévi-Leblond (1976). We elaborate on some of the results of these papers paying special attention to the Bacri and Lévy-Leblond paper where possible kinematical groups include the de Sitter and anti-de Sitter groups and lead to special relativity in de Sitter space (S. Cacciatori et al., Ann. Phys. (Berlin) 17 728-768 (2008)).

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The orthogonal branching problem for symplectic monogenics

Guner Muarem (joint work with D. Eelbode)

April 20, 2022

Abstract

An important tool in understanding the representation theory for a Lie algebra $\mathfrak{g}$ is the study of restrictions of an irreducible representation for $\mathfrak{g}$ to a Lie sub algebra $\mathfrak{h}$. This so-called restricted representation will in general no longer be irreducible as it decomposes into a direct sum of $\mathfrak{h}$-irreducible representations for the sub algebra. The algebraic tool which allows us to describe which irreducible summands show up and what their multiplicities are, is called a branching rule. In this talk we will in particular focus on the spaces of homogeneous solutions for the symplectic Dirac operator (known as symplectic monogenics) as they provide models for irreducible $\mathfrak{sp}(2m)$-representations of infinite dimension. In detail, we will focus on the $k = 1$ case and derive the branching of these modules with respect to the sub algebra so(m). To arrive at this result we will use the notion of a transvector algebra and tensor products of Verma modules.

INTEGRAL QUANTIZATION FOR DISCRETE CYLINDER

JEAN PIERRE GAZEAU AND ROMAIN MURENZI

Abstract. Covariant integral quantization is established for systems whose phase space is $\mathbb{Z} \times \mathbb{S}^1$, i.e., for system moving on the circle. The symmetry group of this phase space is the discrete & compact version of the Weyl-Heisenberg group, namely the central extension of the abelian group $\mathbb{R} \times \mathrm{SO}(2)$, and the phase space can be viewed as the left coset of the group with its center. The non-trivial unitary irreducible representation of this group, as acting on $L^2(\mathbb{S}^1)$, is square integrable on the phase phase. We show how to derive covariant integral quantizations from (weight) functions on the phase space. Among the latter we recover as particular cases quantizations with de Bièvre-del Olmo-Gonzales-Kowalski-Rembielevski-Papaloucas coherent states on the circle. Another straightforward outcome of our approach is the Mukunda Wigner transform. We look at the specific cases of coherent states built from shifted gaussians, van de Mises, Poisson, and Fejer kernels.

Conflict of interest

The authors declare that they have no conflict of interest. The ideas and opinions expressed in this article are those of the authors and do not necessary represent the view of UNESCO.

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Wigner Function Analysis of Finite Matter-Radiation Systems

Eduardo Nahmad-Achar, Ramón López-Peña, Sergio Cordero, and Octavio Castaños

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(National University of Mexico, UNAM)

Abstract

We show that the behaviour of the Wigner function associated to the electromagnetic modes carry the information of both, the entanglement properties between matter and field, and the regions in parameter space where quantum phase transitions take place.

A finer classification for the continuous phase transitions is obtained through the computation of the surface of maximum Bures distance [1, 2].

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Non Commutative Gauge Field Theory on Approximately Finite C*-algebras and GUT :

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Joint work with: T. Masson

Non Commutative Geometry is a powerful framework to reformulate the Standard Model of Particles Physics (SMPP), taking also into account General Relativity, into a single "geometric picture" based on Non Commutative Gauge Field Theories (NCGFT). This achievement opens the door to investigate many possibilities to build new models beyond the SMPP, like Grand Unified Theories (GUT). The present talk aims at showing an elegant way recently developed by Thierry Masson and myself which propose a general scheme to build GUT in the field of NCGFT (Int. J. Geom. Methods Mod. Phys. 18.13 (Oct. 2021), p. 2150213 and paper in preparation). This concerns NCGFT based on Approximately Finite (AF) C*-algebras using spectral triples. The defining inductive sequence of an AF C*-algebra is lifted to enable the construction of a sequence of NCGFT of Yang–Mills–Higgs types. The main benefits of this framework is to control, using suitable conditions, the interplay of the degrees of freedom along the inductive sequence on top of the AF algebra, and to suggest a way to recover GUT-like models similars to Georgi–Glashow model.

Group Theoretical Derivation of Consistent Free Particle Theories

Giuseppe Antonio Nisticò, Università della Calabria, Italy. INFN, Italy

Relativistic quantum theories obtained by canonical quantization, e.g. Klein-Gordon theory, turned out to be plagued by well known problems, such as negative values of probability densities. So, Theoretical Physics turned on Quantum Field Theory to model particle physics. In order to develop relativistic quantum theories of single free particle without the problems that plagued the early theories, we have pursued an approach, based on group theoretical methods, that develops the theory deductively from two physical principles: *invariance ot the theory under Poincarè group* and *covariance of the position observable*. These methodological commitments prevent from the mentioned inconsistencies. In so doing, six inequivalent complete consistent theories for spin 0 and positive mass particles have been derived, amomg which the theories for particles of Klein-Gordon kind [1], but free from inconsistencies.

We have specialized the approach to the case of an *elementary massless systems*. Our method leads to determine definite constraints implied by the invariance principle; they were ignored by some past investigations that, as a consequence, turn out to be not consistent with the invariance principle. These contraints establish which representations must be discarded and which are consistent. As results, new classes of consistent theories for massless isolated systems are explicitly determined. Also the problem of the *localizability* for massless systems is reconsidered within the new theoretical framework, obtaining a generalization and a deeper detailing of previous results.

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Cosmic Galois group and ϕ^4 theory

Erik Panzer

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Periods are integrals of rational functions over domains bounded by polynomial inequalities. Such integrals appear in many places in mathematical physics, including the Feynman integrals of perturbative quantum field theory. The theory of motives leads to a generalization of classical Galois theory for algebraic numbers, to the action of a group on the space of periods. In conclusion, the latter, 'cosmic' Galois group acts on Feynman integrals. I will present aspects of my joint work with Oliver Schnetz on the coaction conjecture, drawing on Francis Brown's Feynman motives, to illustrate

Clocking mechanism from a minimal spinning particle model

Tobiasz Pietrzak

26.05.2022

Abstract:
The clock hypothesis is the fundamental assumption in the relativity theory and says that duration measured by an ideal clock is proportional to the length of its timelike worldline. To test this hypothesis, a model of an ideal clock as a dynamical system described by position, a single null direction, and with both its Casimir invariants of the Poincare group being fixed parameters rather than constants of motion, was introduced by Staruszkiewicz. First, I will present the model and its interpretation as a clock. Then I will discuss uniqueness and singularity of the proposed Lagrangian form, which turns out equivalent to one of the universal spinning particle model introduced earlier in a different context by Lyakhovich Segal and Sharapov. In view of this singularity (which is fatal for that model as a clock) I will investigate the inverse Legendre transform leading from the Hamiltonian (obtained with the Dirac method for constrained systems) to the Lagrangian. One then obtains a new possible Lagrangian not investigated so far, describing a particle characterized by intrinsic circular motion with the speed of light and whose clocking rate is fixed.

INFINITE GROUPS AND SYMMETRIES IN BIOPHYSICS AT THE DNA/RNA SCALE

MICHEL PLANAT† AND KLEE IRWIN‡

ABSTRACT. Infinite groups arise in the description of biophysics at the DNA/RNA scale as fundamental groups for the synthax of the 4 nucleic bases [1] and of the 20 amino acids [2, 3]. We report on our recent progress connecting DNA/RNA biophysics to free groups, Hecke groups, aperiodicity and algebraic surfaces (though character varieties of such infinite groups). We mainly focus on transcription factors and telomers and the applications to the understanding of health/diseases.

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Jordan algebra and conformal dynamical symmetries

Todor Popov

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The Newton-Hooke duality between the 2D hydrogen atom and the Landau problem is explained via the Tits-Kantor-Koecher construction of the conformal symmetries of the Jordan algebra of real symmetric 2x2 matrices. The connection between the Landau problem and the 3D hydrogen atom then is elucidated by the reduction of a Dirac spinor to a Majorana one in the Kustaanheimo-Stiefel spinorial regularization.

Tensor powers of vector representation of quantum sl(2) at roots of unity

Olga Postnova

Saint Petersburg department of Steklov Mathematical Institute

We consider statistics of tilting modules in tensor powers of the fundamental representations of quantum sl2 at an even root of unity. We compute multiplicities of tilting modules in the N -th tensor powers of vector representation of quantum sl2 explicitly as sums of binomial coefficients and describe them in terms of lattice walks with filters.

This research is supported by RSF 21-11-00141

Renormalizable Extension of the Abelian Higgs-Kibble Model with a Dim. 6 Derivative Operator

D. Binosi[1] and A. Quadri (speaker)[2,*]

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(Dated: April 15, 2022)

Abstract

We discuss a renormalizable extension of the Abelian Higgs-Kibble (HK) model supplemented by a dimension 6 derivative-dependent operator. The latter operator is controlled by the parameter z and violates power-counting renormalizability. At $z = 0$ one recovers the usual power-counting renormalizable Abelian HK model. A field-theoretic representation of the physical Higgs scalar by a gauge-invariant variable is used in order to formulate the theory by exploiting a novel differential equation controlling the dependence of the quantized theory on z. The Green's functions of the theory are uniquely defined by the z-differential equation in terms of the Feynman amplitudes of the power-counting renormalizable theory at $z = 0$. The Slavnov-Taylor identities, ensuring the fulfillment of physical unitarity of the theory, are studied and shown to hold true separately in the grading induced by the number of internal physical Higgs propagators. We show that these results hold true as a consequence of subtle cancellations of UV divergences encoded in the z-differential equation and an additional set of symmetries that are fulfilled by the one-particle irreducible (1-PI) amplitudes of the theory. The construction paves the way to the consistent subtraction in terms of a finite number of physical parameters of a class of non-power-counting renormalizable models that were previously considered only as effective field theories and that might be of direct relevance to the study of the Higgs potential at the upcoming LHC experimental program.

Based on D.Binosi, A.Q., in preparation; JHEP 05 (2020) 141 e-Print: 2001.07430; Eur.Phys.J.C 80 (2020) 9, 807 e-Print: 1904.06693

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PT-symmetries, dynamical symmetries and superintegrability

Christiane Quesne

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A two-dimensional model with quadratic complex interaction, which was previously shown to be nonseparable and nondiagonalizable, but exactly solvable due to its shape invariance property in supersymmetric quantum mechanics, is re-examined in order to show that it has a hidden algebraic structure very similar to that of the two-dimensional real harmonic oscilla- tor. The analysis is based on the introduction of a pair of ladder operators B^+ and B^- , completing the operators A^+ and A^- coming from the shape invariant supersymmetric approach. Generators of $gl(2), sp(4)$, and $osp(1/4)$ algebras are constructed in terms of these four operators.

Complete separability of the Hamilton-Jacobi equation for the charged particle orbits in a Liénard-Wiechert field

Rastelli,Giovanni

Università di Torino

We classify all orthogonal coordinate systems in four-dimensional Minkowski space, allowing complete additively separated solutions of the HamiltonJacobi equation for a charged test particle in the LienardWiechert field generated by any possible given motion of a point-charge Q. We prove that only the CavendishCoulomb field, corresponding to the uniform motion of Q, admits separation of variables, precisely in cylindrical spherical and cylindrical conical-spherical coordinates. We show also that for some fields, the test particle with motion constrained into certain planes admits complete orthogonal separation, and we determine the separable coordinates.

Two dimensional QCD revisited again

Nicolai Reshetikhin

YMSC Tsinghua University

In the first part of the talk quantum N-point spin Calogero-Moser (CM) systems will be introduced. In the second part I will show that the partition function for the 2D Yang-Mills model on a surface with corners is a solution to quantum CM evolution with special initial conditions. The relation of these topics to quantum Chern-Simons theory, invariants of 3-manifolds and quantization of moduli spaces will be outlined in the third part of the talk.

A Mathematical Model for the Statistics of Tri-Stable Potentials

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Abstract

Proteins are structures formed by chains of amino acids responsible for several biological functions, and their functionality is associated with the spatial arrangement of these amino acids that compose them. This spatial arrangement defines whether the protein structure will have an active function in the medium. The protein's folding state refers to the structure that has an active function. We have developed a consistent mathematical model to solve the FP equation which enabled the analysis of the protein folding process, through the construction of the temporal evolution of the probability density and the evaluation of the characteristic times of the intermediate states of the system described by a tri-stable potential function V(x). The lateral minima have the same depth (symmetrical wells) and are interpreted, respectively, as the unfolded (left well) and folded states (right well) of the protein and the central minimum is related to a set of intermediate conformations of the protein. The protein folding process, considered a diffusion process, is described by the Fokker-Planck equation, FPE, which can be mapped to a Schrödinger type equation, SE, which turns this probabilistic process into a spectral problem. The Supersymmetric Quantum Mechanics methodology, SQM, associated with the variational method enabled an approximate analytical treatment for obtaining the spectrum of functions and energies of the SE and the evaluation of the time dependent probability function $P(x, x_0, t)$ where x is the reaction coordinate. Thus, the diffusion process was characterized by the calculation of the particle population of the right well in terms of the probability. The time necessary for the evolution of the system population from its initial state to the third well is used as the characteristic passage time of the system. Also, a mapping of the diffusion dependence and its influence on the symmetric free energy profiles was performed aiming to evaluate the way the increase of the diffusion impacts the time it takes to pass through the intermediate state.

Beyond M-theory with Nested Braneworlds

Michael Rios

Dyonica ICMQG

Using special T-algebras of Exceptional Periodicity (EP) and higher super-Poincar algebras, we show how M-theory embeds into a nested braneworld structure. The infinite limit of this structure is suggested as the full theory of quantum gravity.

Global symmetry and conformal bootstrap in the two-dimensional $O(n)$ model

Hubert Saleur

I will discuss the full solution of the two dimensional critical $O(n)$ model using bootstrap techniques, with particular emphasis on the interplay between " $O(n)$ symmetry" (for n complex) and conformal invariance in the non-unitary case.

Splitting-merging transitions in a tensor-vectors system in exact large-N limits

Naoki Sasakura

Yukawa Institute for Theoretical Physics

Matrix models have phase transitions in which distributions of variables change topologically like the Gross-Witten-Wadia transition. In a recent study, similar splitting-merging behavior of distributions of dynamical variables was observed in a tensor-vectors system by numerical simulations. In this paper, we study the system exactly in some large-N limits, in which the distributions are discrete sets of configurations rather than continuous. We find cascades of first- order phase transitions for fixed tensors, and first- and second-order phase transitions for random tensors, being characterized by patterns of replica symmetry breaking. The system has at least threefold interests: The splitting dynamics plays essential roles in emergence of classical spacetimes in a tensor model of quantum gravity; The splitting dynamics automatically detects the rank of a tensor in the tensor rank decomposition of data analysis; The system provides a variant of the spherical p-spin model for spin glasses with a new parameter. We discuss some implications of the results from these perspectives. The results are compared with some numerical simulations to check the large-N convergence and the assumptions made in the analysis.

Group invariants for Feynman diagrams

Christian Schubert

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It is well-known that the symmetry group of a Feynman diagram can give important information on possible strategies for its evaluation, and the mathematical objects that will be involved. Here I will discuss the usefulness of introducing a basis of invariants of the symmetry group of a diagram in Feynman parameter space.

On a Reformuation of Bohmian Mechanics

Dieter Schuch[1] and Moise Bonilla-Licea[2]

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In Bohmian mechanics it is assumed that quantum objects are particles that follow real physical paths. These Bohmian trajectories are obtained by integrating the velocity field that occurs in the continuity equation. However, in the course of deriving these trajectories, the independent position variable x, appearing in the velocity field, is replaced without justification by the time-dependent trajectory $q(t)$. It will be shown under what constraints such a replacement can be justified, leading to a totally different interpretation of Bohmian trajectories. They are not real paths of physical particles but borders of regions of constant probability, so-called quantiles, and must therefore be seen in the context of descriptive statistics. Thus, they provide a complementary statistical aspect to the one adopted in the conventional formulation of quantum mechanics.

Noether's theorem, conservation of momentum, and gravitation in multiparticle systems

Walter Smilga

retired (alumnus LMU Munich and Max Planck Institute for Physics)

I report on a correspondence between conservation of momentum in multiparticle systems and gravitation: The conservation of total momentum forces the particles into trajectories that form a pseudo-Riemannian manifold described by the field equations of conformal gravity. This manifold can be quantised without problems, leading to a consistent quantum gravity.

Pairs of commuting quadratic elements in the universal enveloping algebra of Euclidean algebra and integrals of motion

Libor Snobl

Czech Technical University in Prague

Motivated by the consideration of integrable systems in three spatial dimen- sions in Euclidean space with integrals quadratic in the momenta we classify three-dimensional Abelian subalgebras of quadratic elements in the universal enveloping algebra of the Euclidean algebra under the assumption that the Casimir invariant p l vanishes in the relevant representation. We show by means of an explicit example that in the presence of magnetic field, i.e. terms linear in the momenta in the Hamiltonian, this classification allows for pairs of commuting integrals whose leading order terms cannot be written in the famous classical form of [Makarov A A, Smorodinsky J A, Valiev K and Winternitz P Il Nuovo Cimento (1967) A 10 106184]. We consider limits simplifying the structure of the magnetic field in this example and corresponding reductions of integrals, demonstrating that singularities in the integrals may arise, forcing structural changes of the leading order terms.

Inspiration and need for this work came from numerous discussions with Pavel Winternitz over several years. Unfortunately, while he was alive we always found other more urgent research directions to follow together. Thus we dedicate it to his memory.

A class of representations of the infinite-rank $\mathbb{Z}_2 \times \mathbb{Z}_2-$ graded Lie superalgebra $\mathfrak{pso}(\infty|\infty)$

N. Stoilova

INRNE, BAS

We investigate a system consisting of infinite number of parafermions and parabosons, satisfying the mutual paraboson relations. The parastatistics Fock spaces of order of such a system correspond to a class of lowest weight representations of the -graded Lie superalgebra This class of representations is constructed explicitly.

Towards higher super-σ-model categories

Rafał R. Suszek (Department of Mathematical Methods in Physics, University of Warsaw)

The standard 2d σ-model with an un-graded target space is largely determined by a geometrisation of the Kalb-Ramond 3-form component of the tensorial background of the loop dynamics known as a gerbe, which – as was first understood by Gawędzki - gives the topological Wess-Zumino term of the σ-model the meaning of a distinguished Cheeger-Simons differential character and canonically induces a prequantisation of the 2d field theory. (1-)gerbes form a bicategory, and its 0-, 1- and 2-cells are exactly what is needed to set up a rigorous description of the (multi-phase) σ-model in the presence of worldsheet defects whose relevance stems from the fact that they capture prequantisable symmetries of the field theory as well as dualities between theories (such as, *e.g.*, T-duality). In the talk, we review a transcription of the above classic scheme into the Z/2Z-graded setting of the Green-Schwarz-type super-σ-models of super-*p*-branes on homogeneous spaces of Lie supergroups in which the cohomology to be geometrised by (*p*-)gerbes and derived structures is the Cartan-Eilenberg cohomology of the supersymmetry group. Upon reconstructing the relevant super-*p*-gerbes through a generalisation of the scheme of `superspace extension' due to de Azcárraga and subsequently explaining the higher-supergeometric nature of the fundamental κ-symmetry of the super-σ-models, we discuss a class of prototypical supersymmetric defects (and the associated super-gerbe bimodules) in tight analogy with and drawing inspiration from the study of a simplicial higher-geometric structure behind the maximally symmetric defects of the Wess-Zumino-Witten σ-model, the latter defects encoding nontrivial information on the fusion ring of the 2d CFT through an intricate relation to the underlying 3d Chern-Simons TFT, which shall also be revealed.

Generalized Nijenhuis tensors and integrable systems

Piergiulio Tempesta

Universidad Complutense de Madrid and ICMAT Spain

Joint work with: D. Reyes Nozaleda and G. Tondo

We propose a new, infinite family of generalized tensor fields, whose first representatives are the celebrated Nijenhuis and Haantjes tensors. This new class of tensors possesses many interesting geometric and algebraic properties.Our main result is a theorem stating that for an operator on a differentiable manifold,the vanishing of a generalized Haantjes torsion is a sufficient condition for the Frobenius integrability of its eigen-distributions. This new condition, which does not require the explicit knowledge of the spectral properties of the given operator, generalizes the celebrated Haantjes theorem, because it provides us with an effective integrability criterion applicable to operators whose standard Nijenhuis or Haantjes torsions are non-vanishing.
The notion of generalized Haantjes algebras is also introduced. In particular, given a family of commuting operators with a vanishing higher-level Haantjes torsion, we prove that all of the operators of the family, in a suitable local chart, can be written simultaneously in a block-diagonal form. This result represents a new contribution to the Courant problem of determining normal forms of systems of PDEs.

Towards a self-dual super-chiral QFT of the weak interactions gauging the simple Lie-Kac superalgebra $SU(2/1)$

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Joint work with: Peter Jarvis (University of Tasmania, Hobart, Australia)

A natural classification of leptons and quarks, graded by their chirality, is provided by the simple representations of the SU(2/1) Lie-Kac superalgebra. Its indecomposable representations explain generation mixing. To construct a QFT of the electroweak interactions with internal chiral SU(2/1) structure, we propose to replace the Lie algebra-valued connection one-form A, by a superalgebra-valued polyform A' mixing exterior-forms of all degrees, and satisfying the chiral self-duality condition A' = *A' chi where chi denotes the superalgebra grading operator. Under the super-chiral projection, the the connection polyform contains vectors in the adjoint representation, and complex scalars and self-dual Adveev-Chizov two-forms, all coupling only to CP positive Fermions. The BIM mechanism, which ensures the cancellation of the anomalies, then implies that couplings of the scalars and Avdeev-Chizhov fields induced by the quantum loops are governed by the super-Killing metric, and by the symmetric structure constants of the superalgebra. We present the current state of the model and the open problems.

Chiral Casimirs and indecomposable representations of the sl(m/n) superalgebras

Jean Thierry-Mieg[1], Peter Jarvis[2]

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Joint work with: Peter Jarvis (University of Tasmania, Hobart, Australia), Jerome Germoni (Lyon)

Given any Kac module of dimension D of the Sl(m/n) or OSp(2/2n) superalgebras and a positive number N, we construct an indecomposable module of dimension ND with N-1 free parameters, which can be interpreted as generalized Cabibbo angles. We then classify and construct the indecomposable representations of Sl(2/1) and show that their Casimir C and ghost Casimir T are proportional C = T chi and that their chriral combination T'=(C-T)/2 is factorizable : T' = Z Z', where chi is the super-identity defining the supertrace STr(M) = Tr(chi M).

Generalized Willmore Energies and Elastic Surfaces with Applications to Biophysics

Toda, Magdalena

Texas Tech University

This study focuses on the theory and applications of curvature functionals for submanifolds, which we refer to as elastic energies or generalized Willmore energies. This will be discussed in the context of Integrable Systems. Biological applications include protein folding, red blood cells and biomembranes. We present recently published and unpublished results on critical points of these energies, stability, and related generalized Willmore flows. Cases of generalized Willmore surfaces with fixed and free boundaries will also be presented if time permits.

Inequivalent multiparticle quantizations from nontrivial braidings

Francesco Toppan

CBPF

The First Quantization defined by a Hopf algebra endowed with a braided tensor product induce paraparticles in the multiparticle sector of a QM model.
I discuss three recent applications of this framework: the proofs of the detectability of the Rittenberg-Wyler Z2xZ2-graded parafermions and parabosons, and the braiding of the Majorana fermions regarded as Z2-graded qubits.
The talk is based on:
F.T., Z2xZ2-graded parastatistics in multiparticle quantum Hamiltonians, J. Phys. A: Math. Theor. 54, 115203 (2021).
F. T., Inequivalent quantizations from gradings and Z2xZ2 parabosons, J. Phys. A: Math. Theor. 54, 355202 (2021).
F.T., First quantization of braided Majorana fermions, Nucl. Phys. B 980 (2022) 115834.

On the spin contents of the massless Rarita–Schwinger system

Mauricio Valenzuela

Centro de Estudios Cientificos (CECs)

Joint work with: J. Zanelli

In this talk we analyze the Rarita–Schwinger (RS) massless theory in the Lagrangian and Hamiltonian approach. As we shall see, the system possesses primary and secondary first class constraints. The gauge orbit generated by the first class primary constraints can be fixed in agreement with the standard vanishing gamma-trace constraint. We will show that at this stage there are not gauge orbits left in the system. Hence the Dirac conjecture-which says that secondary first class constraints generate gauge orbits-does not apply. The obtained equations of motion, either Euler-Lagrange or Hamilton, describe two massless particles, of spin-1/2 and spin-3/2. In earlier references, instead, the Dirac conjecture is assumed axiomatically (without a test), which serves to justify the introduction of new external conditions that forces the spin-1/2 field to vanish.

An overview of perturbation theory and its applications

José Antonio Vallejo

Universidad Autnoma de San Luis Potosí

I will review the basics of perturbation theory as applied to Hamiltonian systems, putting an emphasis on systems admitting some group of symmetries. Perturbed Hamiltonian systems are ubiquitous in Physics, and their study involves many interesting mathematical constructions, some of which will be exemplified. The talk is aimed at non-specialists.

Seniority and particle-hole conjugation in atomic nuclei

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Nuclear forces between nucleons in an atomic nucleus favour the formation of pairs of neutrons or pairs of protons with anti-parallel spins coupled to angular momentum zero (J=0). This pairing property implies a symmetry with an associated quantum number seniority, the number of nucleons not in J=0 pairs [1]. The concept of seniority is particularly fruitful to elucidate the structure of so-called semi-magic nuclei, which have either neutrons or protons in the valence shell [2].

In this talk it is shown that conservation of the seniority quantum number is the consequence of the existence of a geometric phase associated with particle-hole conjugation, which becomes gauge invariant and therefore observable in a (particle-hole) self-conjugate nucleus where nucleons half-fill the valence shell. The observational consequences of this geometric phase are discussed with particular emphasis on the neutron-rich semi-magic nucleus 213Pb, which recently was studied in a fragmentation reaction of a relativistic uranium beam [3].

1. G. Racah, Theory of Complex Spectra. III, Phys. Rev. 63 (1943) 367.
2. I. Talmi, Simple Models of Complex Nuclei (Harwood, Chur, 1993).
3. J.J. Valiente-Dobn et al., Manifestation of the Berry phase in the atomic nucleus 213Pb, Phys. Lett. B 816 (2021) 136183.

Noncommutative spaces at finite resolution

Walter van Suijlekom

Radboud University, Netherlands

Joint work with: Alain Connes

We extend the traditional framework of noncommutative geometry in order to deal with two types of approximation of metric spaces. On the one hand, we consider spectral truncations of geometric spaces, while on the other hand, we consider metric spaces up to finite resolution. In our approach, the traditional role played by C*-algebras is taken over by so-called operator systems. Essentially, this is the minimal structure required on a space of operators to be able to speak of positive elements, states, pure states, etc. We illustrate our methods in concrete examples obtained by spectral truncations of the circle and of metric spaces up to finite resolution. The former yield operator systems of finite-dimensional Toeplitz matrices, and the latter give suitable subspaces of the compact operators. We also analyze the cones of positive elements and the pure-state spaces for these operator systems, which turn out to possess a very rich structure.

Parabosons: basis constructions

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Joint work with: Asmus K. Bisbo

Parabosons, characterized by certain triple relations, have continued to inspire scientists because of the interesting mathematical structures related to them. A long time ago it was shown that a set of n paraboson creation and annihilation operators generate the Lie superalgebra $\mathfrak{osp}(1|2n)$. The paraboson Fock space of order p corresponds to an infinite-dimensional irreducible lowest weight representation $L(p)$ of $\mathfrak{osp}(1|2n)$. In this representation, the parabosons can also be realized in terms of np ordinary bosons and elements of a Clifford algebra on p generators. The explicit construction of basis vectors for $L(p)$ has been a long-standing problem. There is nowadays a description of a Gelfand-Zetlin basis for $L(p)$, which has the advantage that actions of paraboson creation and annihilation operators on basis elements can be given explicitly. The Gelfand-Zetlin basis provides however only a labelling of a set of basis vectors in terms of a Gelfand-Zetlin array, and no explicit construction of these basis vectors. In this contribution, we present some recent progress in constructing bases for $L(p)$. Using combinatorial methods (Young tableaux, Young subgroups) we construct a new basis for $L(p)$ in such a way that the basis vectors are expressed as actions of polynomials of paraboson creation operators on the vacuum vector. We also use extremal projectors and the theory of Mickelsson-Zhelobenko algebras to relate the new basis to the known Gelfand-Zetlin basis for $L(p)$.

Entanglement of free Fermions on graphs

Luc Vinet

CRM & IVADO

Entanglement refers to the correlation of a subsystem with its complement. It is an essential resource in quantum information and many-body physics. I will discuss the entanglement of free Fermions on graphs. I will focus on those of the Hamming and Johnson schemes. I shall make use of the parallel with time and band limiting problems in signal processing. I will indicate the role of the Terwilliger algebra in the identification of an operator of Heun type that commutes with the truncated correlation matrix and allows access to the entanglement entropy.

The founders of the Montreal school in mathematical physics - Remembering Jiri Patera and Pavel Winternitz

Luc Vinet

CRM & IVADO

I will recall the life and the work of Jiri Patera and Pavel Winternitz and their lasting scientific impact on the CRM and the ICGTMP community.

A study of multipartite quantum systems based on Markov matrices and the Gini index

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An expansion of row Markov matrices in terms of matrices related to permutations with repetitions, is introduced. It generalises the Birkhoff-von Neumann expansion of doubly stochastic matrices in terms of permutation matrices (without repetitions). An interpretation of the formalism in terms of sequences of integers that open random safes described by the Markov matrices, is presented. Various quantities that describe probabilities and correlations in this context, are discussed. The Gini index is used to quantify the sparsity (certainty) of various probability vectors. The formalism is used in the context of multipartite quantum systems with finite dimensional Hilbert space, which can be viewed as quantum permutations with repetitions or as quantum safes. Many novel probabilistic quantities that describe the statistics of multipartite quantum systems, are introduced. Local and global Fourier transforms are used to define locally dual and also globally dual statistical quantities.

Optimal Universal Quantum Error Correction via Bounded Reference Frames

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Joint work with: Yuxiang Yang, Yin Mo, Joseph M. Renes, Giulio Chiribella

Error correcting codes with a universal set of transversal gates are a desideratum for quantum computing. Such codes, however, are ruled out by the Eastin-Knill theorem. Moreover, the theorem also rules out codes which are covariant with respect to the action of transversal unitary operations forming continuous symmetries. In this work, starting from an arbitrary code, we construct approximate codes which are covariant with respect to the entire group of local unitary gates in dimension $d\,(<\infty)$, using quantum reference frames. We show that our codes are capable of efficiently correcting different types of erasure errors. When only a small fraction of the n qudits upon which the code is built are erased, our covariant code has an error that scales as $1/n^2$, which is reminiscent of the Heisenberg limit of quantum metrology. When every qudit has a chance of being erased, our covariant code has an error that scales as $1/n$. We show that the error scaling is optimal in both cases. Our approach has implications for fault-tolerant quantum computing, reference frame error correction, and the AdS-CFT duality.

Quantum Fluctuations of a Particle's Arrival Time

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We demonstrate that a real scalar quantum field has properties that can be interpreted as a field with vibrations of matters in time and space. Based on this interpretation, the formulations have a more symmetrical treatment of time and space in a matter field. A self-adjoint internal time operator can be defined without contradiction with Pauli's theorem. The particles observed are oscillators in proper time. In motion, the proper time oscillation translates to the oscillations of a particle in both time and space. A particle is oscillating back and forth along its trajectory. Therefore, two particles with the same initial average velocity can reach a target at different times depending on the phases of their oscillations. This leads to an uncertainty in the arrival time of a particle. In particular, we study the effects of these oscillations on the neutrinos' arrival time. The arrival time uncertainty measured can be used to estimate the mass of a neutrino.

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Extension of Realisations for Low-Dimensional Lie Algebras

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We start with a realisation of a Lie algebra with the basis operators $L = \langle Q_i \rangle$, $Q_i = \zeta_{ik}\partial_{x_k} + \eta_{im}\partial_{y_m}$, where x_k are independent variables, and y_m are dependent variables. We distinguish these variables as further we are going to use our new realisations for construction of differential invariants.

We take additional variables F_k and study the extended action operators $\hat{Q}_i = Q_i + \lambda_{ijk}F_j\partial_{F_k}$ that form the same Lie algebra with the same structural constants. For a fixed realisation of any Lie algebra L we can classify all inequivalent extended action realisations for a finite number of additional variables. Such realisations allow e.g. to classify exhaustively relative differential invariants and invariant equations for the respective realisations of Lie algebras. They can be also applied to other problems in the symmetry analysis of differential equations. We classify extensions of realisations for inequivalent low-dimensional Lie algebras.

A recent development of spin CFT and level-rank duality

Shuichi Yokoyama

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Conformal Field Theory (CFT) which depends on spin structure is called fermionic or spin CFT. Such CFTs were studied abstractly in 90's, but explicit construction of a class of spin CFTs was not done until recently. A key technique of their explicit construction is gauging a discrete symmetry for a (parent) bosonic CFT coupling to "symmetry-protected topological phase factor" or 1-dimensional complex representation of the finite symmetry group.

A class of spin CFTs is important to understand in the sense not only to find a new class of CFTs but also to find a new duality. Indeed, a couple of (serious) issues have been known in the level-rank duality in 2d WZNW model or 3d Chern-Simons theory, and such issues were recently pointed out to be related to the poor understanding of a spin CFT and their resolution was also proposed by Hsin-Seiberg.

I would like to talk about this development from my personal viewpoint relating to my research of this topic.

References:

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Tetrahedron and 3D reflection equation from PBW bases of the nilpotent subalgebra of quantum superalgebras

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Abstract

We study transition matrices of PBW bases of the nilpotent subalgebra of quantum superalgebras associated with all possible Dynkin diagrams of type A and B in the case of rank 2 and 3, and examine relationships with three-dimensional (3D) integrability. We obtain new solutions to the Zamolodchikov tetrahedron equation via type A and the 3D reflection equation via type B, where the latter equation was proposed by Isaev and Kulish as a 3D analog of the reflection equation of Cherednik. As a by-product of our approach, the Bazhanov-Sergeev solution to the Zamolodchikov tetrahedron equation is characterized as the transition matrix for a particular case of type A, which clarifies an algebraic origin of it. Our work is inspired by the recent developments connecting transition matrices for quantum non-super algebras with intertwiners of irreducible representations of quantum coordinate rings. We also discuss the crystal limit of transition matrices, which gives a super analog of transition maps of Lusztig's parametrizations of the canonical basis.

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