Noncommutative spaces at finite resolution

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We extend the traditional framework of noncommutative geometry in order to deal with two types of approximation of metric spaces. On the one hand, we consider spectral truncations of geometric spaces, while on the other hand, we consider metric spaces up to finite resolution. In our approach, the traditional role played by C*-algebras is taken over by so-called operator systems. Essentially, this is the minimal structure required on a space of operators to be able to speak of positive elements, states, pure states, etc. We illustrate our methods in concrete examples obtained by spectral truncations of the circle and of metric spaces up to finite resolution. The former yield operator systems of finite-dimensional Toeplitz matrices, and the latter give suitable subspaces of the compact operators. We also analyze the cones of positive elements and the pure-state spaces for these operator systems, which turn out to possess a very rich structure.