

G4BetheHeitler5DModel, un modèle de physique 5D, polarisé, de la conversion d'un photon gamma en paire de leptons dans Geant4

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Séminaire LLR, 15 février 2021

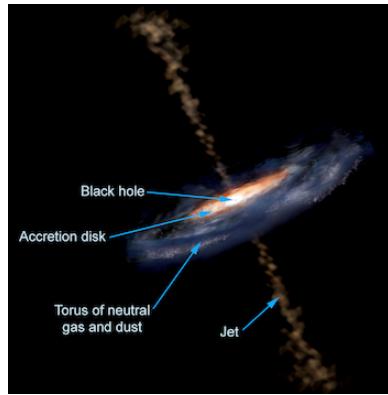


Plan de la présentation

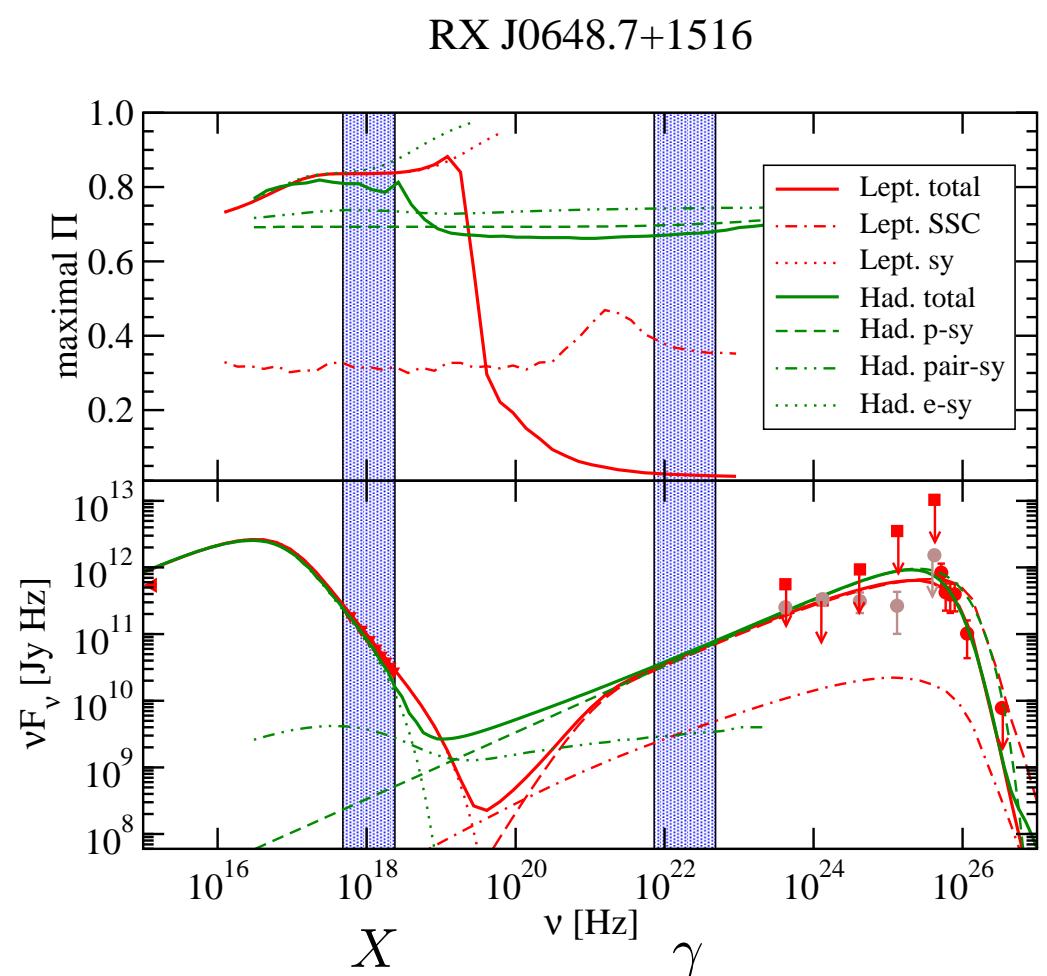
- μ introduction : projet HARPO (2010 - 2018)
 - motivation astrophysique; blazars; pulsars
 - détecteur : TPC à gaz
 - prise de données
 - résultats
- conversion de photons γ en paire e^+e^-
 - section efficace différentielle
- générateurs d'evts pré-existants
- générateurs 5D : 3 générations
- G4BetheHeitler5DModel, un modèle de physique 5D, polarisé, de la conversion d'un photon gamma en paire de leptons dans Geant4
- plus récemment
 - $\gamma \rightarrow \mu^+\mu^-$
 - spectre en masse invariante $e^+e^- \mu$ et asymétrie de polarisation en fn de μ

Deciphering emission mechanism in Blazars with γ -ray polarimetry

- Blazars: active galactic nuclei (AGN) with one jet pointing (almost) to us
leptonic synchrotron self-Compton (SSC) or **hadronic** (proton-synchrotron) ?

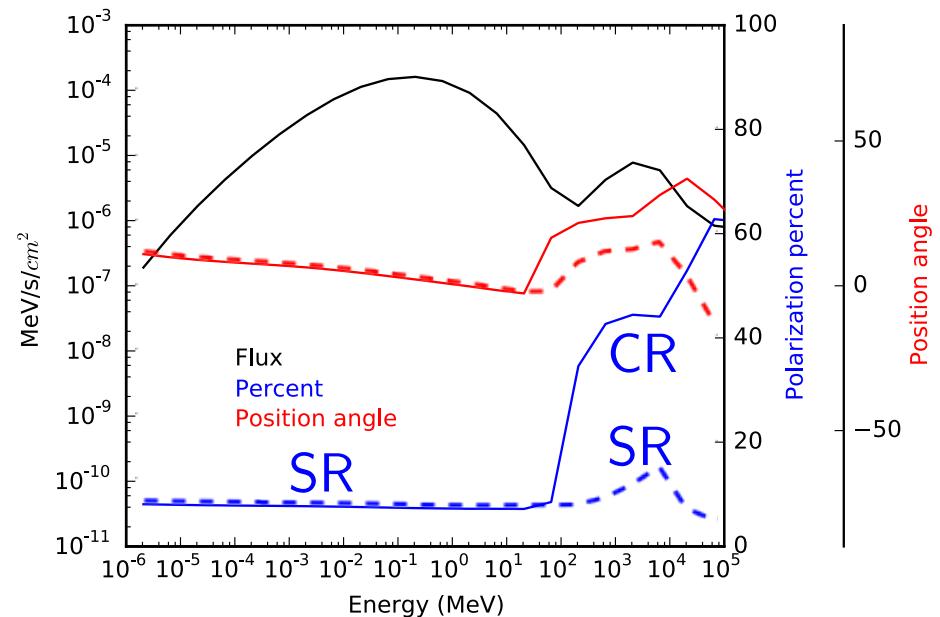
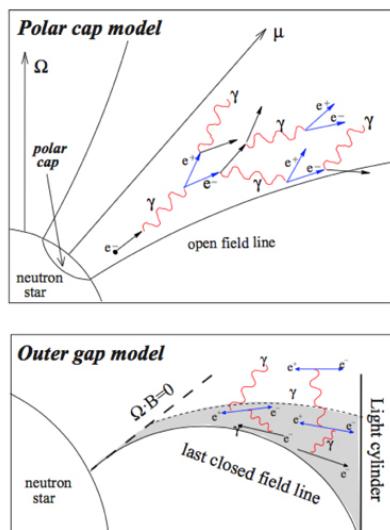
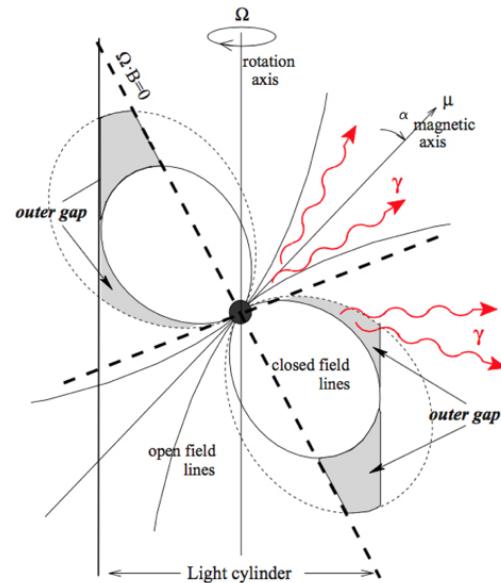


- high-frequency-peaked BL Lac
- X band: 2 -10 keV
- γ band: 30 - 200 MeV
- SED's indistinguishable, but
 - X-ray: $P_{\text{lept}} \approx P_{\text{hadr}}$
 - γ -ray: $P_{\text{lept}} \ll P_{\text{hadr}}$



Zhang and Böttcher, *Astrophys.J.* 774, 18 (2013)

Tagging the (curvature radiation CR – synchrotron radiation SR) transition in pulsars



Polar-cap model of Crab-like pulsar

- MeV component is SR from pairs
GeV component is either CR (solid line) or SR (dashed line)
- “Polarization of MeV and GeV emission is a powerful, independent diagnostic, capable of constraining both the location and mechanism of the radiation”.

Harding and Kalapotharakos, *Astrophys. J.* 840 73 (2017)

Other (hopeless at the moment) interesting goals

- GRB K. Toma, [arXiv:1308.5733 [astro-ph.HE]].
Lyutikov et al., *Astrophys. J.* **597** (2003), 998
Lundman et al., *Astrophys. J.* **856** (2018) 145
 - Search for Lorentz Invariance Violation, **LIV**, higher energy \Rightarrow higher sensitivity.
Kostelecky and Russell, *Rev. Mod. Phys.* 83 (2011) 11 (arXiv:0801.0287 [hep-ph] [v14] 2 Jan 2021)
F. W. Stecker, *Astropart. Phys.* 35 (2011) 95
 - Search for axion, Rubbia and Sakharov, *Astropart. Phys.* 29, 20 (2008)

All short of statistics (for conversions with pairs)

The HARPO (Hermetic ARgon POlarimeter) instrument project

- France: the detector

Denis Bernard, Philippe Bruel, Mickael Frotin, Yannick Geerebaert, Berrie Giebels, Philippe Gros, Deirdre Horan, Marc Louzir, Frédéric Magniette, Patrick Poilleux, Igor Semeniouk, Shaobo Wang ^a

^aLLR, Ecole Polytechnique and CNRS/IN2P3, France

David Attié, Pascal Baron, David Baudin, Denis Calvet, Paul Colas, Alain Delbart, Ryo Yonamine ^b

^bIRFU, CEA Saclay, France

Diego Götz ^{b,c}

^cAIM, CEA/DSM-CNRS-Université Paris Diderot, IRFU/SAp, CEA Saclay, France

- Japan: the beam.

S. Amano, T. Kotaka, S. Hashimoto, Y. Minamiyama, A. Takemoto, M. Yamaguchi,
S. Miyamoto^e

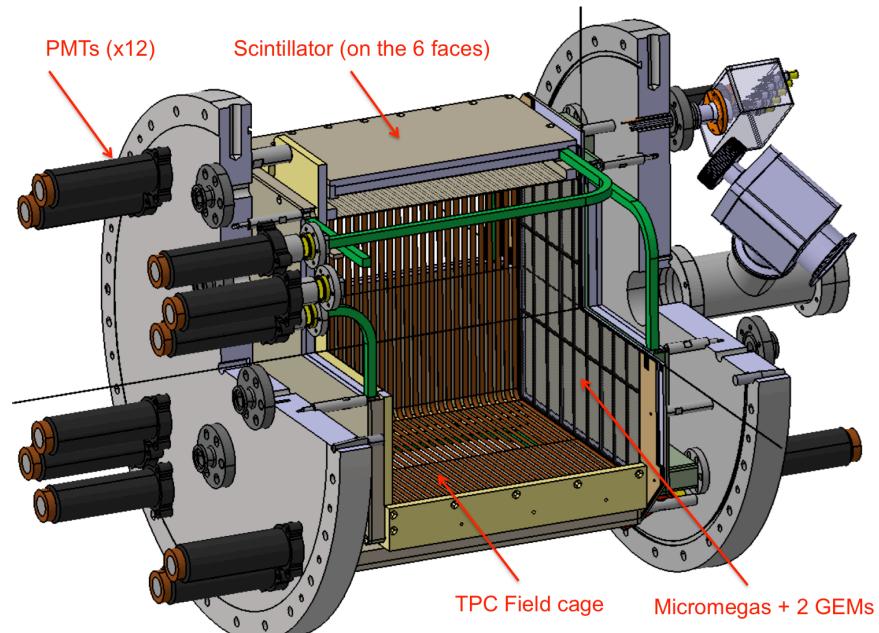
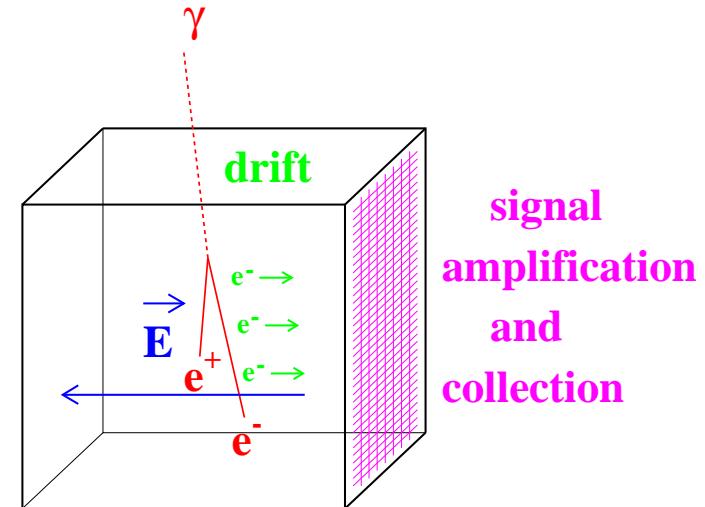
^e LASTI, University of Hyôgo, Japan

S. Daté, H. Ohkuma^f

^f JASRI/SPring8, Japan

HARPO: the Demonstrator

- Time Projection Chamber (TPC)
- $(30\text{ cm})^3$ cubic TPC
- Up to 5 bar.
- Micromegas + GEM gas amplification
- Collection on x, y strips, pitch 1 mm.
- AFTER chip readout, up to 100 MHz.
- Scintillator / WLS / PMT based trigger



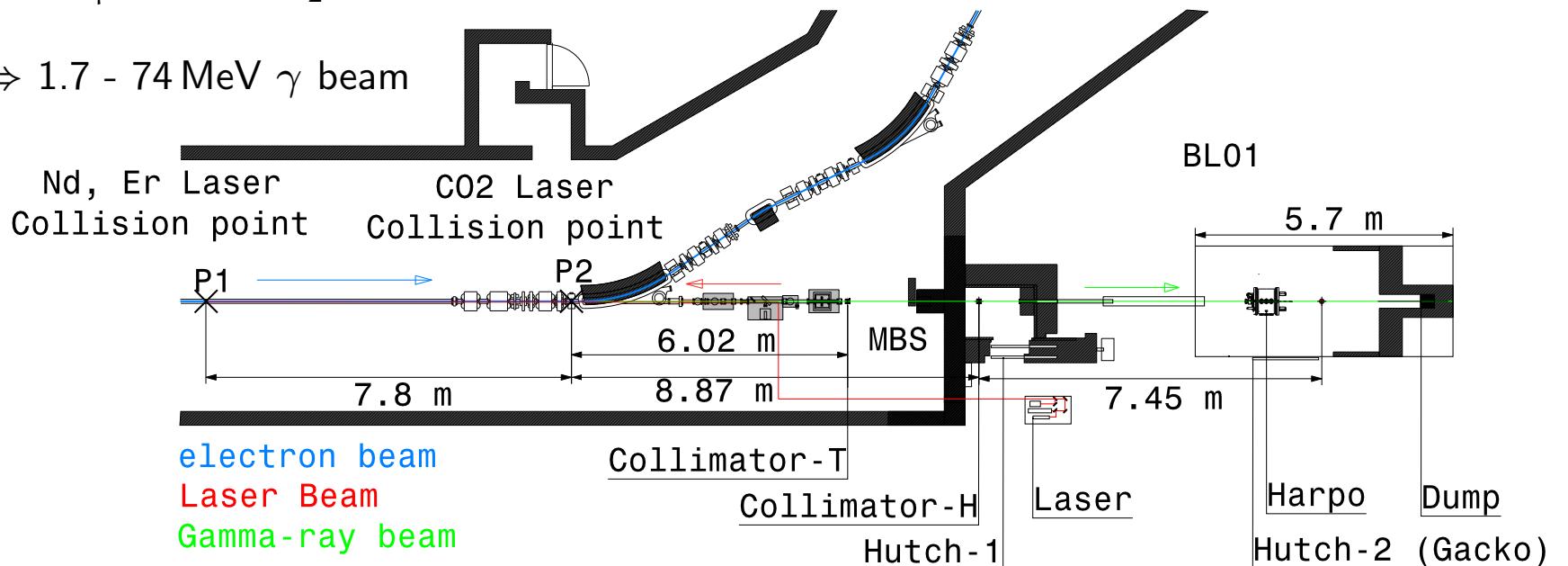
Nucl. Instrum. Meth. A 695 (2012) 71,

Nucl. Instrum. Meth. A 718 (2013) 395

Data Taking Nov. 2014 NewSUBARU, LASTI, Japan

- Linearly polarized γ beam from Laser inverse Compton scattering, e^- beam 0.6 – 1.5 GeV.
- 0.532 μm and 1.064 μm 20 kHz pulsed Nd:YVO₄ (2 ω and 1 ω),
1.540 μm 200 kHz pulsed Er (fibre) and
10.55 μm CW CO₂ lasers

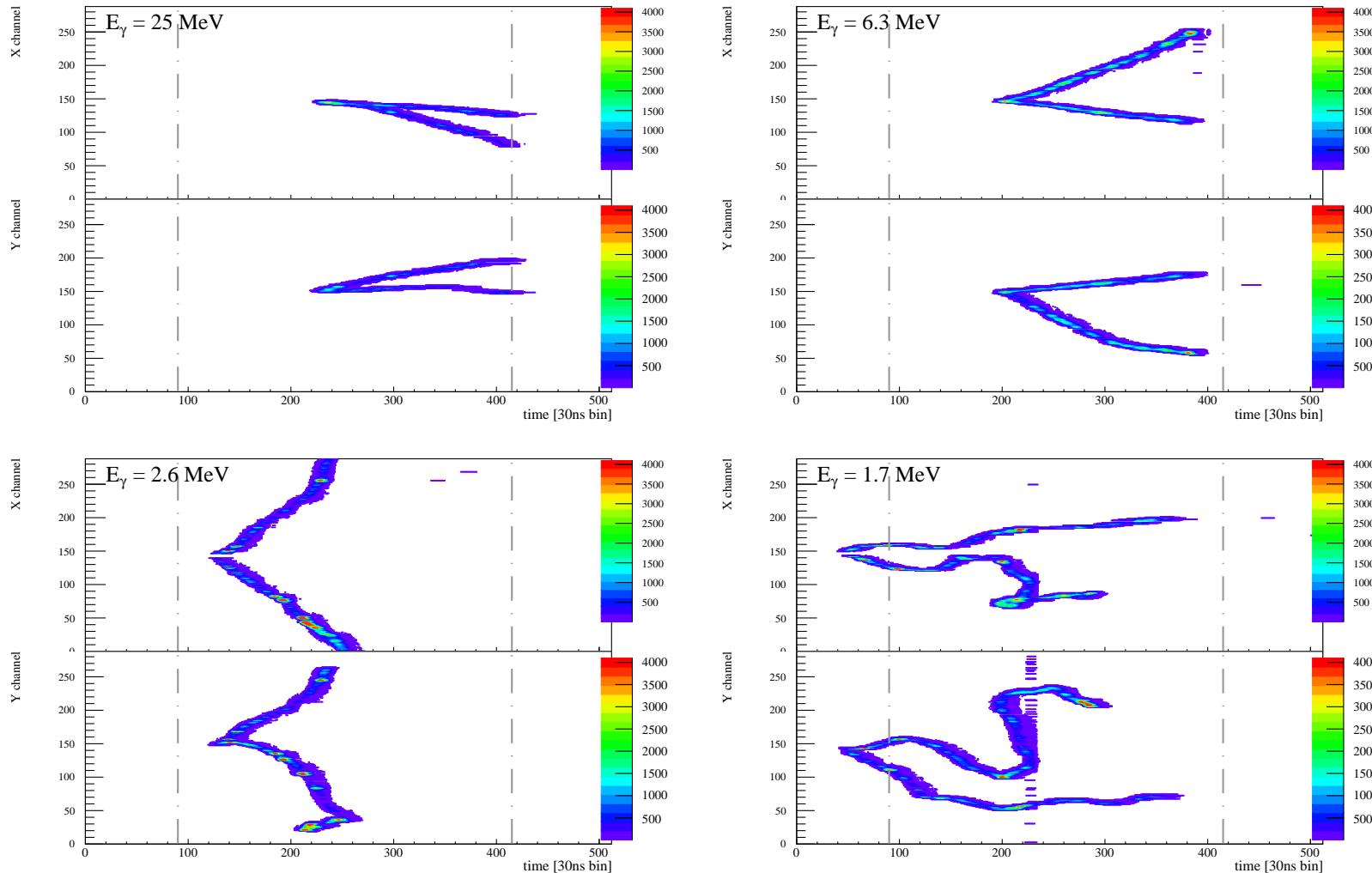
- \Rightarrow 1.7 - 74 MeV γ beam



- Monochromaticity by collimation on axis
- Fully polarized or random polarization beams ($P = 0$, $P = 1$)
- 2.1 bar Ar:isoC₄H₁₀ 95:5 (+ a 1-4 bar scan).

A. Delbart et al., ICRC2015, PoS(ICRC2015)1016

4 events

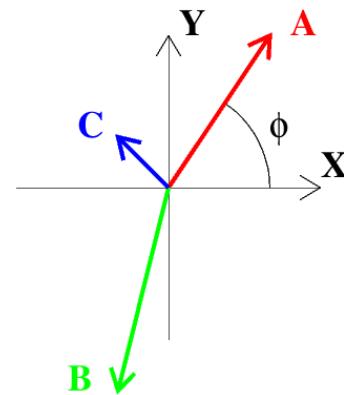
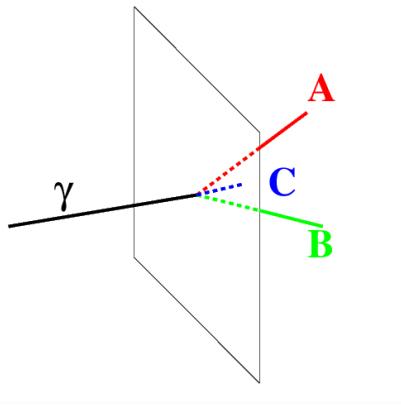


- Sample of γ -rays from the BL01 beam line at NewSUBARU (LASTI, Hyôgo U., Japan) converting to e^+e^- in the 2.1 bar Ar:Isobutane 95:5 gas of the HARPO TPC

Polarimetry with $\gamma \rightarrow e^+e^-$

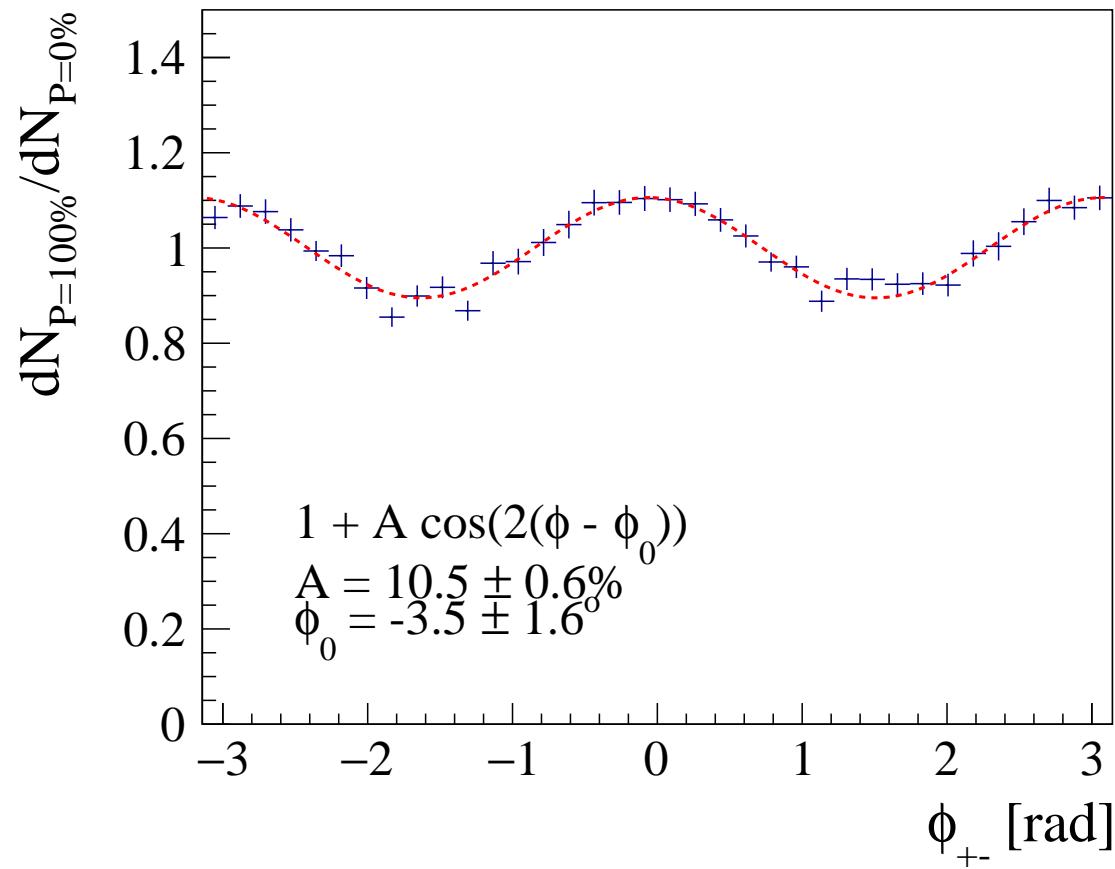
- $\frac{d\Gamma}{d\phi} \propto (1 + AP \cos [2(\phi - \phi_0)]),$

$$\sigma_P \approx \frac{1}{A} \sqrt{\frac{2}{N}},$$



- P source linear polarisation fraction
- A γ -ray conversion polarization asymmetry
- ϕ event azimuthal angle
- ϕ_0 source polarization angle.

Polarimetry: $(P = 1)/(P = 0)$ ratios, orientation averaged



11.8 MeV γ rays in 2.1 bar argon

P. Gros et al. Astroparticle Physics 97 (2018) 10

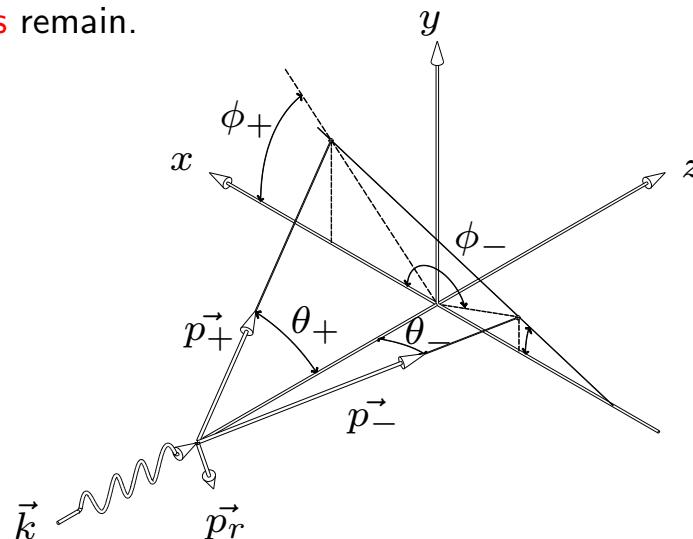
γ conversions to e^+e^- pairs

- Target, final state
 - nucleus (nuclear conversion) $\gamma Z \rightarrow e^+ e^- Z$ threshold $2mc^2$
 - electron (triplet conversion) $\gamma e^- \rightarrow e^+ e^- e^-$ threshold $4mc^2$

A three-particle final state, even when the trajectory of the recoiling target cannot be detected.

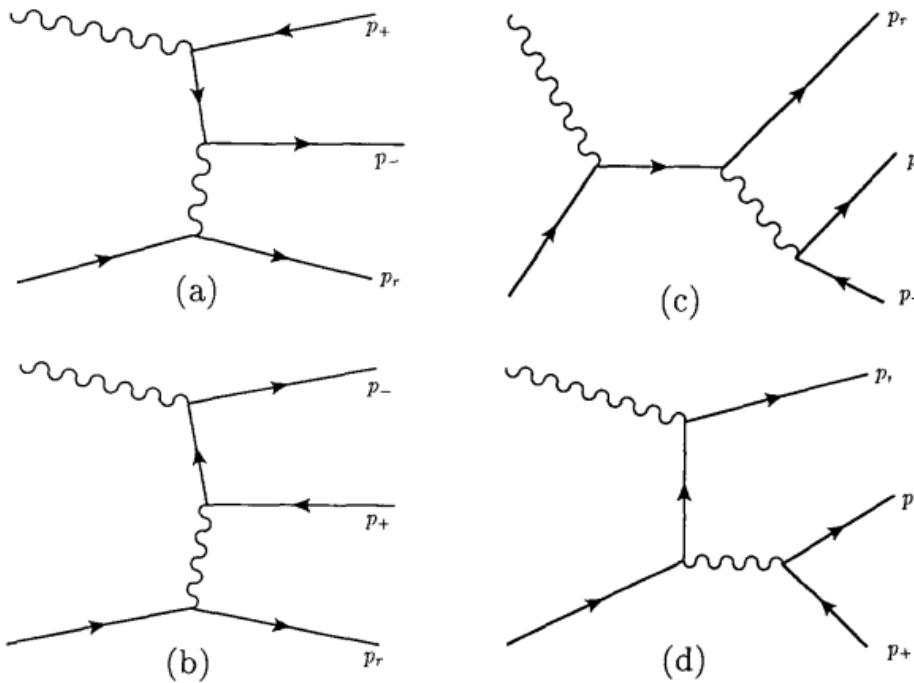
- A 5D phase space

- 3 particle final state, $4 - 1 = 3$ free parameters for each one,
- energy-momentum conservation fixes 4 of them.
- $3 \times 3 - 4 = 5$ variables remain.



- $+, -, r$ = positron, electron, recoil. ϕ azimuthal, θ polar angles.
- $\Omega \equiv (\phi_+, \phi_-, \theta_+, \theta_-, x_+ \equiv E_+/E_\gamma)$ "the Bethe-Heitler" variables

First Order Diagrams



Borsellino diagrams

$e - \gamma$ diagrams

Borsellino diagrams dominance:

- Nuclear conversion: ($M \gg m_e$)
- Triplet conversion: at high energy ($E \gg m_e$)

Bethe-Heitler differential cross-section; some properties

- Analytical “Bethe and Heitler” expressions:
 - $P = 0$, [Bethe and Heitler, Proc.Roy.Soc.Lond. A146 \(1934\) 83](#)
 - $P = 1$, [Berlin and Madansky, Phys. Rev. 78 \(1950\) 623](#) and [May, Phys. Rev. 84 \(1951\) 265.](#)
(misprint corrected in [The theory of photons and electrons](#), Jauch and Rohrlich, 1976.)
- $\Rightarrow d\sigma \propto (X_u + P X_p)$ (see eg. [Gros and Bernard Astroparticle Physics 88 \(2017\) 30](#))
- Borsellino diagrams only
- Only linear (not circular) polarization relevant to first Born order.
- Point-like target (nucleus size neglected)
- Electron-cloud field screening, $F(q^2)$ form factor
- Does not depend on target mass and nature (except form factor)
- LPM (Landau-Pomeranchuk-Migdal) suppression neglected $(E \gg 1 \text{ TeV})$
- Coulomb (final state lepton electrostatic interactions) corrections neglected
- Assumes recoil energy negligible $\Rightarrow E_- = E - E_+$

Bethe-Heitler differential cross section: divergences

- Non-polarised photons

Bethe and Heitler, Proc. R. Soc. Lond. A 146 (1934) 83

$$\begin{aligned} d\sigma = & \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} dE_+ d\Omega_+ d\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \\ & \left[\left(\frac{p_+ \sin \theta_+}{E_+ - p_+ \cos \theta_+} \right)^2 (4E_-^2 - q^2) + \left(\frac{p_- \sin \theta_-}{E_- - p_- \cos \theta_-} \right)^2 (4E_+^2 - q^2) + \right. \\ & \left. \frac{2p_+ p_- \sin \theta_+ \sin \theta_- \cos(\phi_+ - \phi_-)}{(E_- - p_- \cos \theta_-)(E_+ - p_+ \cos \theta_+)} (4E_+ E_- + q^2 - 2\omega^2) - 2\omega^2 \frac{(p_+ \sin \theta_+)^2 + (p_- \sin \theta_-)^2}{(E_+ - p_+ \cos \theta_+)(E_- - p_- \cos \theta_-)} \right] \end{aligned}$$

with: $|\vec{q}|^2 = |\vec{p}_+ + \vec{p}_- - \vec{k}|^2$.

- Divergences:

- For e^+ and e^- $\frac{1}{(E - p \cos \theta)}$ forward divergence at high energies
- $\frac{1}{q^4}$ small recoil divergence

Bethe-Heitler differential cross section: divergences

- Fully linearly polarised photons

Berlin and Madansky, Phys. Rev. 78 (1950) 623 and May, Phys. Rev. 84 (1951) 265.

(misprint corrected in The theory of photons and electrons, Jauch and Rohrlich, 1976.)

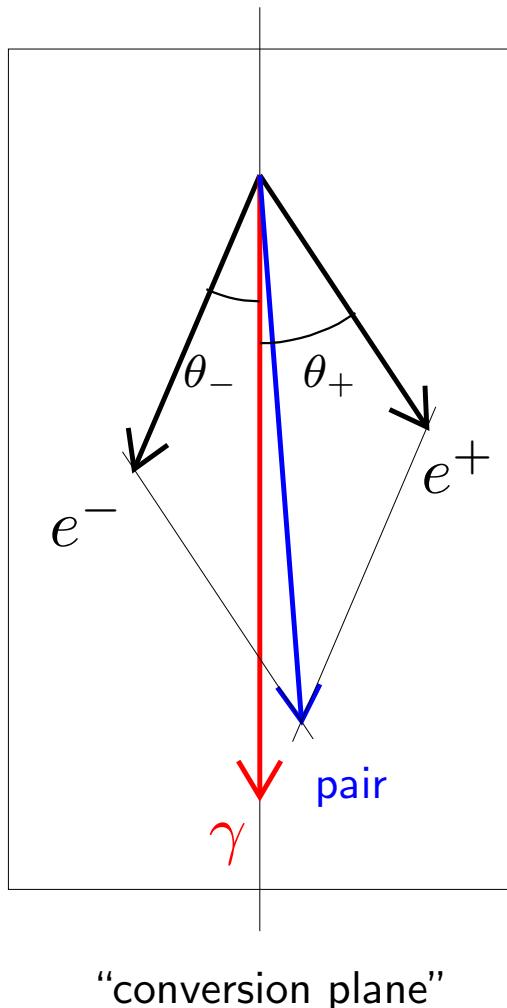
$$\begin{aligned} d\sigma = & \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} dE_+ d\Omega_+ d\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \left[\left(2E_+ \frac{p_- \sin \theta_- \cos \varphi_-}{E_- - p_- \cos \theta_-} + 2E_- \frac{p_+ \sin \theta_+ \cos \varphi_+}{E_+ - p_+ \cos \theta_+} \right)^2 \right. \\ & - q^2 \left(\frac{p_- \sin \theta_- \cos \varphi_-}{E_- - p_- \cos \theta_-} - \frac{p_+ \sin \theta_+ \cos \varphi_+}{E_+ - p_+ \cos \theta_+} \right)^2 \\ & \left. - E^2 \frac{(p_+ \sin \theta_+)^2 + (p_- \sin \theta_-)^2 + 2p_+ p_- \sin \theta_+ \sin \theta_- \cos(\varphi_+ - \varphi_-)}{(E_- - p_- \cos \theta_-)(E_+ - p_+ \cos \theta_+)} \right]. \end{aligned}$$

with: $|\vec{q}|^2 = |\vec{p}_+ + \vec{p}_- - \vec{k}|^2$.

- Divergences:

- For e^+ and e^- $\frac{1}{(E - p \cos \theta)}$ forward divergence at high energies
- $\frac{1}{q^4}$ small recoil divergence

Pre-existing event generators



- pdf = product of 1D pdf's
- No recoil explicitly generated ($\vec{p}_+ + \vec{p}_- = \vec{k}$)
 - \Rightarrow conversion wrongly generated in a plane that contains \vec{k}_γ
 - \Rightarrow no kick transverse to the plane
- e^+ and e^- polar angles generated independently
 - \Rightarrow energy-momentum not conserved !
 - \Rightarrow artificial kick in the plane (and wrong distribution)
- Attempts to verify polarized models failed
- Single-particle polar angle θ distribution, OK.
- "Energy share", x_+ distribution, OK. $x_+ \equiv E_+/E_\gamma$
- Geant4 Physics Reference Manual, release 10.4 (Dec. 2017, pdf)
Sects. 6.5 – 6.6. & 13.9

P. Gros and D. Bernard, Astroparticle Physics 88 (2017) 60

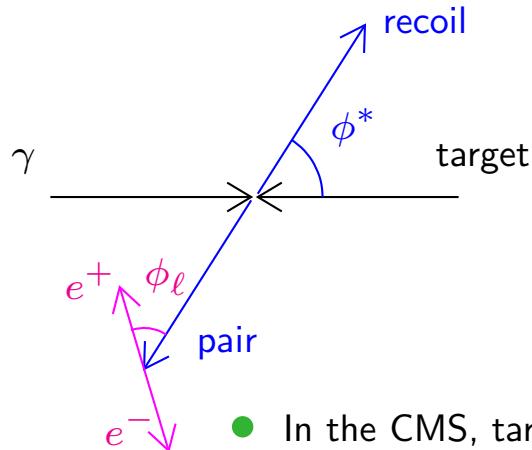
Sampling of the BH DCS: 3 Generations of Generators

- First attempt: [proceedings](#) of the 17th EGS Workshop, Aug 2010 KEK, Tsukuba, Japan
- HARPO: [2012 – 2018]: SPRING/BASES implementation of the VEGAS method.
[S. Kawabata, Comput. Phys. Commun. 88 \(1995\) 309.](#)
[G. P. Lepage, J. Comput. Phys. 27 \(1978\) 192.](#)
[Nucl. Instrum. Meth. A 729 \(2013\) 765](#)
[Astropart. Phys. 88 \(2017\) 30](#)
[Astropart. Phys. 88 \(2017\) 60](#)
[Astropart. Phys. 97 \(2018\) 10](#)
- [2018]: Geant4 - compatible
[Nucl. Instrum. Meth. A 899 \(2018\) 85](#)
[Nucl. Instrum. Meth., A 936 \(2019\) 290](#)

G4BetheHeitler5DModel: Sampling Method

- Perform each step in appropriate Lorentz frame

- Center-of-mass system (CMS) boost determined from photon energy E and target mass M .
- Five variables are taken at random, $(\ell: \text{lepton, that is, } e^+ \text{ or } e^-)$



variable	name	Lorentz frame
θ^*	target and pair polar angle	CMS
ϕ^*	target and pair azimuthal angle	CMS
μ	e^+e^- invariant mass	
θ_ℓ	electron and positron polar angle	pair frame
ϕ_ℓ	electron and positron azimuthal angle	pair frame

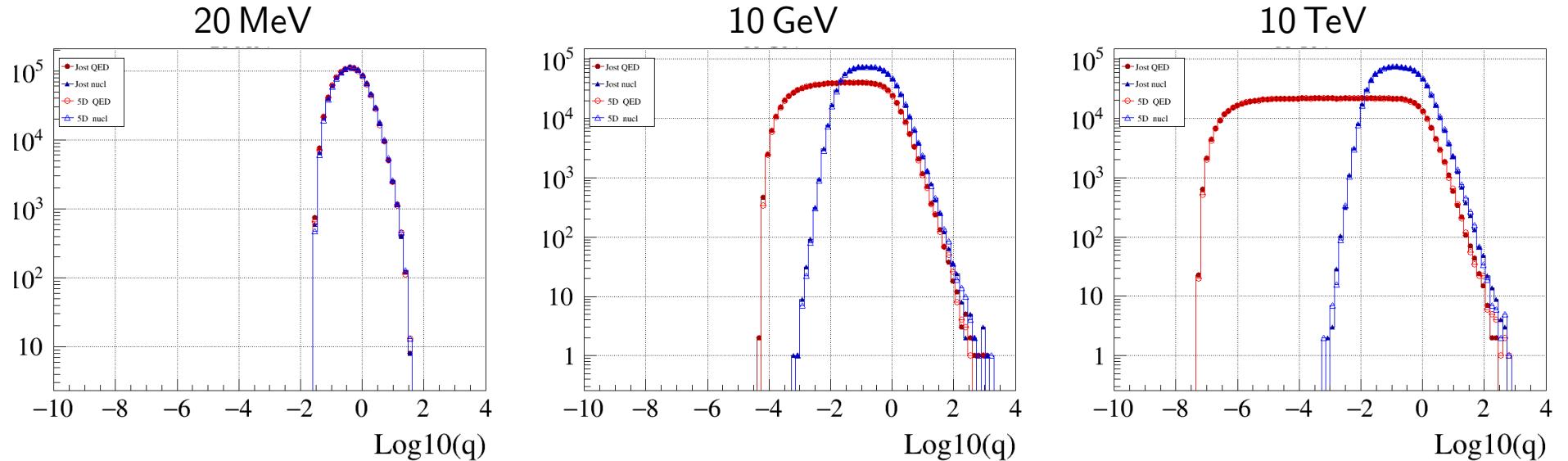
- In the CMS, target (mass M) and pair (mass μ) are back-to-back with opposite momenta.
- “Decay” of the pair to an electron and a positron performed in the pair Lorentz frame.
- The lepton 4-vectors are boosted “back” to the CMS.
- The three final particle 4-vectors are boosted “back” to the laboratory Lorentz frame.
- The Bethe-Heitler variables are obtained from the 4-vectors.
- The probability density function (pdf) is computed.

- Final-state phase space normalization for this set of cascade decays:

Review of Particle Physics (Particle Data Group) See eqs. (1)-(3) in Nucl. Instrum. Meth. A 899 (2018) 85

- In contrast to Bethe-Heitler we **do conserve energy momentum**, $E = E_- + E_+ + E_r$

Recoil momentum spectra



$\log_{10} (q/(\text{MeV}/c))$ distributions for γ -ray conversions on Argon

- Jost QED
- QED (isolated targets)
- Jost nucl
- Nuclear (same form-factor $F(q^2)$ for G4BetheHeitler5DModel and Jost)
- 5D QED
- △ 5D nucl

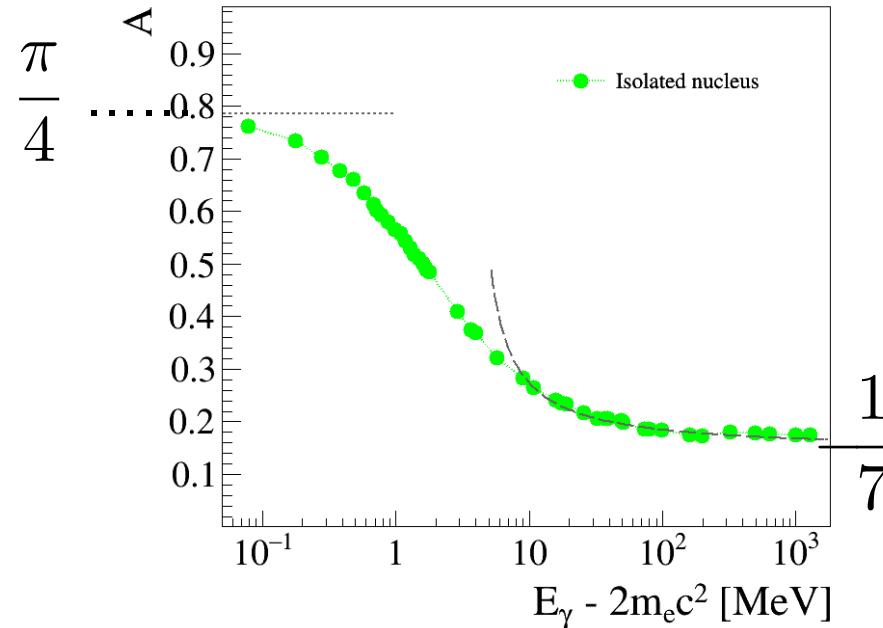
Normalization: same number of events.

High-energy asymptotic expression Jost, Phys. Rev. 80, 189 (1950).

Semeniuk and Bernard, 24th Geant4 Collaboration meeting, 23-27 Sept. 2019, JLab

Polarization asymmetry

Comparison with published asymptotic expressions



$$\phi \equiv (\phi_+ + \phi_-)/2,$$

- Low energy,

$$A = \frac{\pi}{4}.$$

Gros & Bernard, Astropart. Phys. 88 (2017) 30

- High energy,

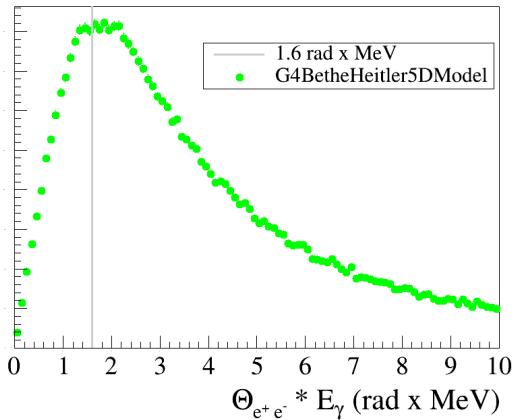
V. F. Boldyshev & Peresunko, Yad. Fiz. 14 (1971) 1027, (Sov.J.Nucl.Phys. 14 (1972) 576).

$$A \approx \frac{\frac{4}{9} \ln \frac{2E}{mc^2} - \frac{20}{28}}{\frac{9}{28} \ln \frac{2E}{mc^2} - \frac{218}{27}}$$

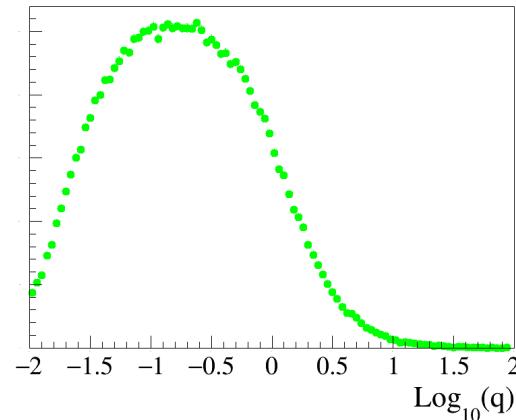
Implementation

- Provide final state: “SampleSecondaries”
- Inherit total cross section from G4BetheHeitlerModel
- Models flags provided via G4EmParameters flags and UI commands
 - Recoil particle (ion or electron): Nuclear / triplet / ($Z / 1$) natural mixture
 - Isolated targets (QED checks), charged targets in atoms (detector simulation)
- Linearly polarised / non polarised photons
- Since release 10.6, G4EmStandardPhysics_option4 uses 5D model for gamma conversion.

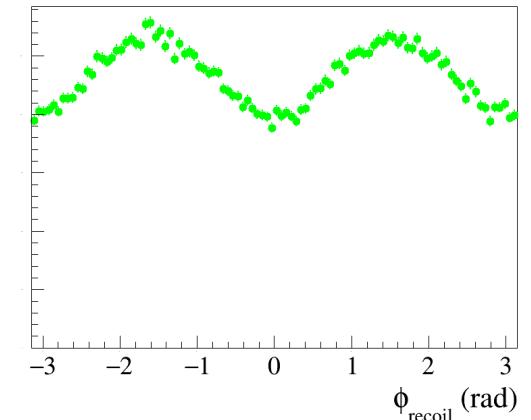
TestEm15



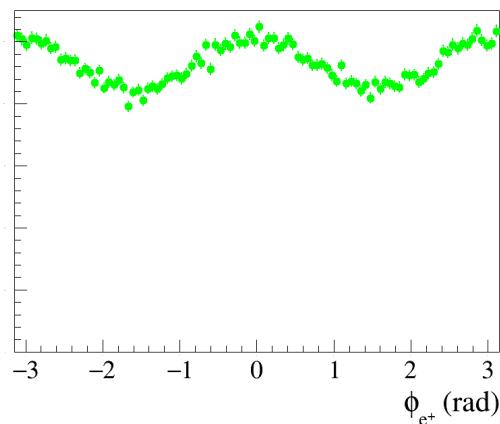
Opening angle



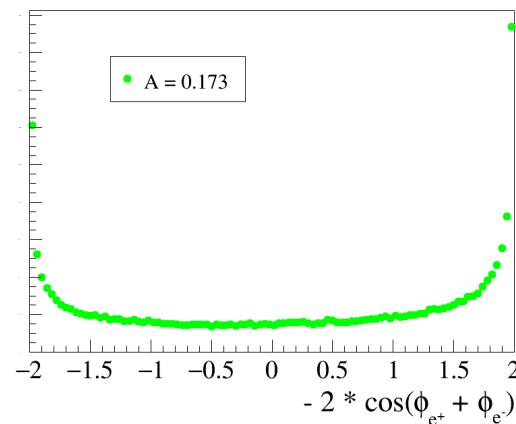
q (MeV/c) is recoil momentum



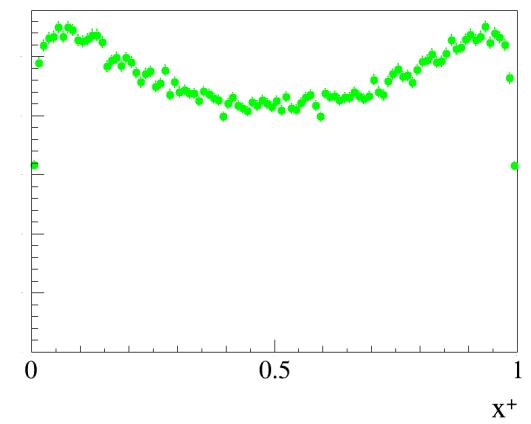
recoil preferentially \perp pol



e⁺ and e⁻ preferentially along pol



$$A = \langle -2 \cos 2\phi \rangle, \quad \phi = (\phi_+ + \phi_-)/2$$



1 GeV, argon

TestEm15/README.gamma

Documentation

- Geant4 Physics Reference Manual, release 10.7 (Dec. 2020, pdf) Sects. 6.5.4 & 13.10
- Talks / publications
 - Geant4 electromagnetic physics progress, Ivana Hrivnacova, CHEP 2019, November 2019, Adelaide link, EPJ Web Conf. **245** (2020), 02009
 - Progress of Geant4 electromagnetic physics developments and applications, Vladimir Ivantchenko, CHEP 2018, Sofia, July 2018 (link) EPJ Web Conf. **214** (2019), 02046
 - “C++ implementation of Bethe–Heitler, 5D, polarized, $\gamma \rightarrow e^+e^-$ pair conversion event generator,” I. Semeniouk and D. Bernard, Nucl. Instrum. Meth. A **936** (2019), 290
 - “A 5D, polarised, Bethe-Heitler event generator for $\gamma \rightarrow e^+e^-$ conversion,” D. Bernard, Nucl. Instrum. Meth. A **899** (2018), 85
 -

The gamma factory project @ CERN

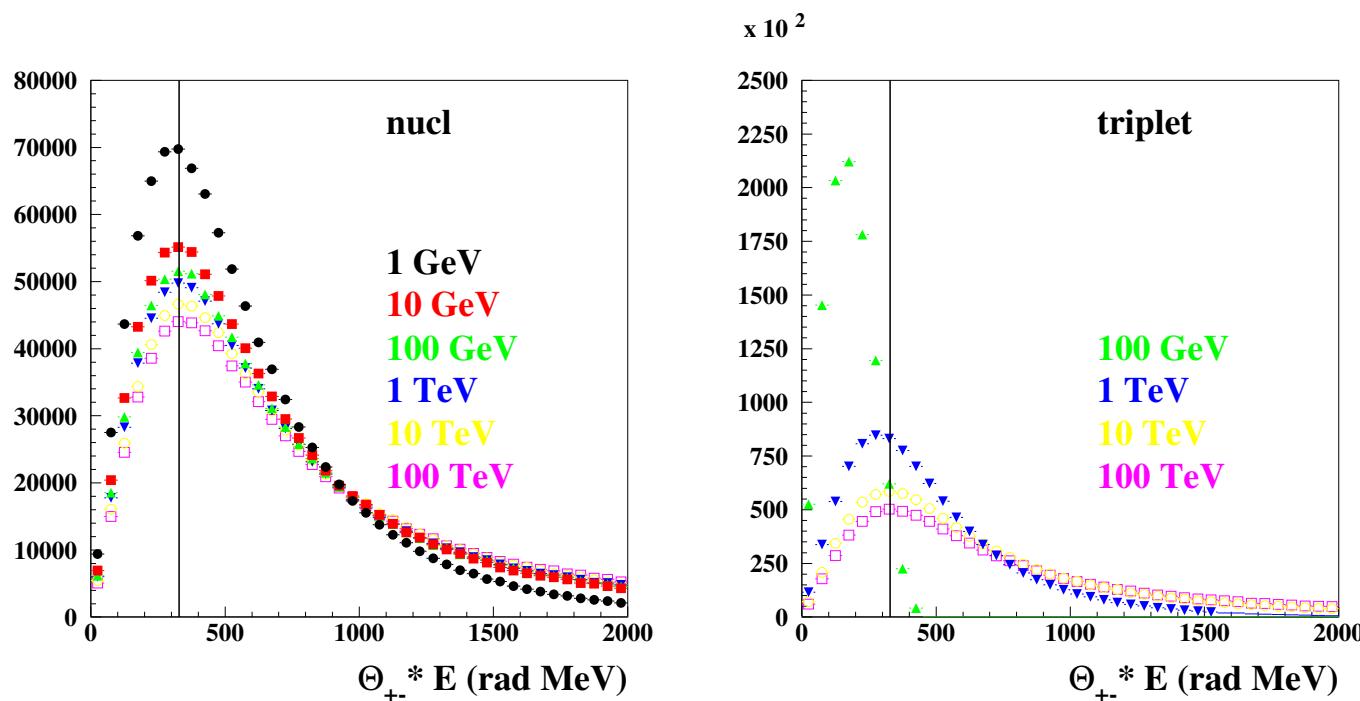
- $1 < E_\gamma < 400 \text{ MeV}$
- $10^{17} \gamma/\text{s}$
- 7 orders of magnitude larger than presently operating light-sources
- A powerful muon source by $\gamma \rightarrow \mu^+ \mu^-$ conversions \Rightarrow also neutrino source

M. Krasny *et al.*, 9th IPAC Vancouver 2018

- Geant4 G4GammaConversionToMuons $\gamma \rightarrow \mu^+ \mu^-$ uses high-energy approximations
Verified $E_\gamma > 10 \text{ GeV}$ H. Burkhardt *et al.*, CERN-SL-2002-016-AP, CLIC-NOTE-511.
- A Geant4 physics model for $\gamma \rightarrow \mu^+ \mu^-$, valid down to threshold was needed
V. Ivantchenko,. “Muon pair production Monte Carlo”, Gamma Factory meeting, CERN, 2019 (indico).

G4BetheHeitler5DModel extended to $\gamma \rightarrow \mu^+ \mu^-$ conversions

- Test model (fortran), arXiv:1910.12501
- Geant4 implementation (release 10.6)

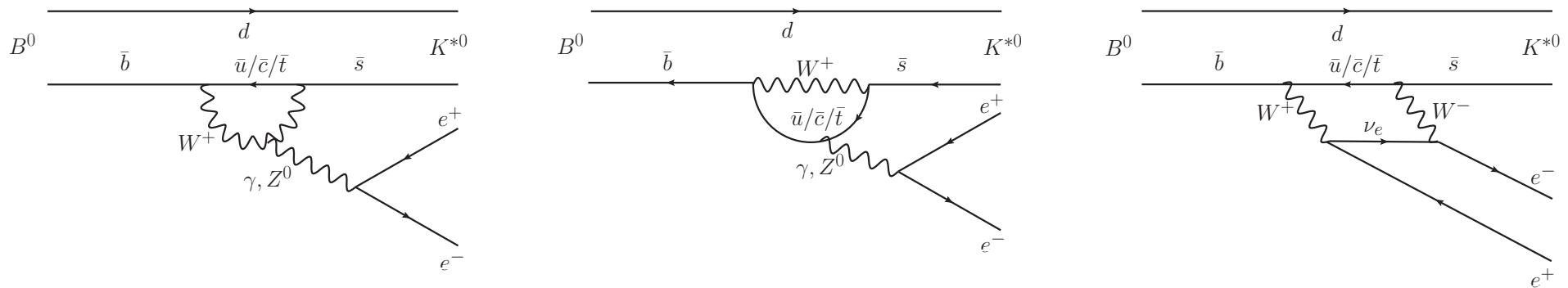


Most probable value $\hat{\theta}_{+-} = \frac{3.2 m_\mu c^2}{E} \approx \frac{338. \text{ MeV}}{E}$ H. Olsen, Phys. Rev. 131 (1963) 406.

LHCb analysis of low- μ $B \rightarrow K^* \ell^+ \ell^-$ decays

Aaij *et al.*, JHEP 04 (2015) 064

- SM : $b \rightarrow s\gamma$ decays produce left-handed photons to a high accuracy
⇒ polarimetry, sensitivity to BSM processes with right-handed couplings.



- But 3 diagrams contributing

J. Lefrançois and M. H. Schune, [LHCb-PUB-2009-008](#)

M. Borsato [CERN-THESIS-2015-219](#)

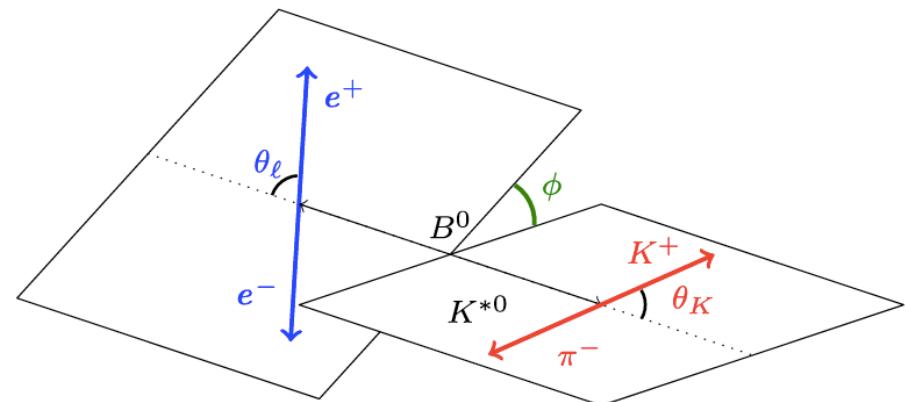
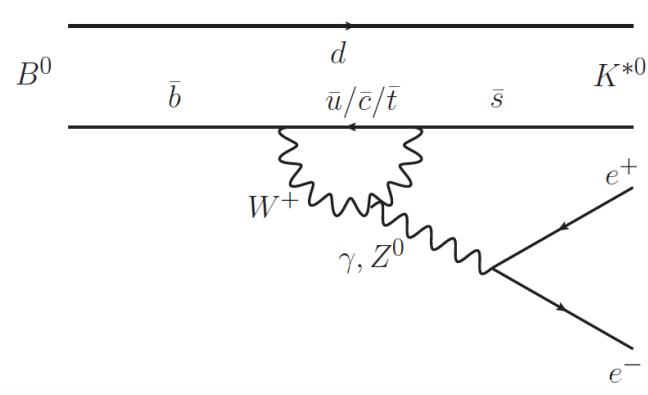
M. H. Schune, [4th Belle II Theory Interface Platform \(B2TiP\) Workshop, Pittsburgh, 23 - 25 May 2016](#)

E. Kou (Emi Kō) [CKM 2014, Vienna](#)

LHCb analysis of low- μ $B \rightarrow K^ \ell^+ \ell^-$ decays*

JHEP 04 (2015) 064

- Low μ : one diagram dominance
 - $e^+ e^-$ final state analyzed



- Transversity amplitude analysis
- Genuine $\gamma \rightarrow e^+ e^-$ background :
Geant4 simulation yielding μ spectrum incompatible with Bethe-Heitler
- See also **The Belle II Physics Book**, PTEP 2019 (2019) 123C01

Transversity amplitude analysis

R. Aaij *et al.* [LHCb], JHEP **12** (2020), 081

$$\frac{1}{d(\Gamma + \bar{\Gamma})/d\mu^2} \frac{d^4(\Gamma + \bar{\Gamma})}{d\mu^2 d\cos\theta_\ell d\cos\theta_K d\tilde{\phi}} = \frac{9}{16\pi} \left[\frac{3}{4}(1 - F_L) \sin^2\theta_K + F_L \cos^2\theta_K + \left(\frac{1}{4}(1 - F_L) \sin^2\theta_K - F_L \cos^2\theta_K \right) \cos 2\theta_\ell \frac{1}{2}(1 - F_L) A_T^{(2)} \sin^2\theta_K \sin^2\theta_\ell \cos 2\tilde{\phi} \right. \\ \left. (1 - F_L) A_T^{\text{Re}} \sin^2\theta_K \cos\theta_\ell + \frac{1}{2}(1 - F_L) A_T^{\text{Im}} \sin^2\theta_K \sin^2\theta_\ell \sin 2\tilde{\phi} \right].$$

$$F_L = \frac{|A_0|^2}{|A_0|^2 + |A_{||}|^2 + |A_{\perp}|^2} \quad A_T^{(2)} = \frac{|A_{\perp}|^2 - |A_{||}|^2}{|A_{\perp}|^2 + |A_{||}|^2} \quad A_T^{\text{Re}} = \frac{2\mathcal{R}e(A_{||L}A_{\perp L}^* + A_{||R}A_{\perp R}^*)}{|A_{||}|^2 + |A_{\perp}|^2} \quad A_T^{\text{Im}} = \frac{2\mathcal{I}m(A_{||L}A_{\perp L}^* + A_{||R}A_{\perp R}^*)}{|A_{||}|^2 + |A_{\perp}|^2},$$

where $|A_0|^2 = |A_{0L}|^2 + |A_{0R}|^2$, $|A_{\perp}|^2 = |A_{\perp L}|^2 + |A_{\perp R}|^2$ and $|A_{||}|^2 = |A_{||L}|^2 + |A_{||R}|^2$.

Amplitudes A_0 , $A_{||}$ and A_{\perp} correspond to different polarisation states of the K^{*0} in the decay.

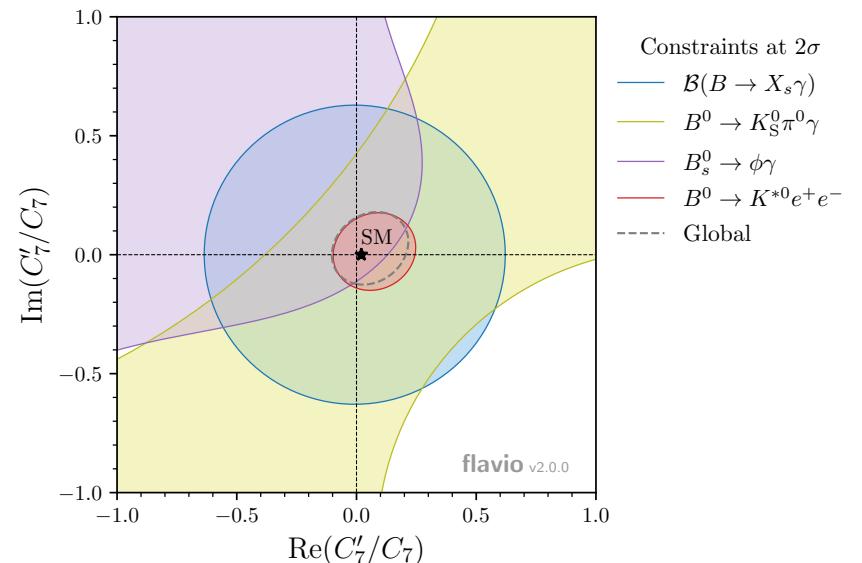
Labels L and R refer to the left and right chirality of the dielectron system.

$$A_T^{(2)}(\mu^2 \rightarrow 0) = \frac{2\mathcal{R}e(\mathcal{C}_7 \mathcal{C}_7'^*)}{|\mathcal{C}_7|^2 + |\mathcal{C}_7'|^2} \text{ and } A_T^{\text{Im}}(\mu^2 \rightarrow 0) = \frac{2\mathcal{I}m(\mathcal{C}_7 \mathcal{C}_7'^*)}{|\mathcal{C}_7|^2 + |\mathcal{C}_7'|^2}.$$

μ^2 from 0.0008 to 0.257 GeV^2/c^4

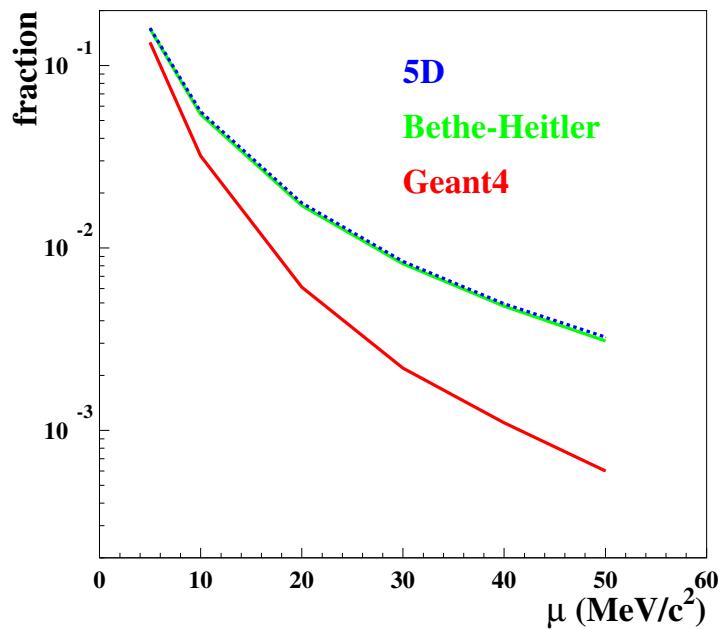
$$F_L = 0.044 \pm 0.026 \pm 0.014, \quad A_T^{\text{Re}} = -0.06 \pm 0.08 \pm 0.02, \\ A_T^{(2)} = +0.11 \pm 0.10 \pm 0.02, \quad A_T^{\text{Im}} = +0.02 \pm 0.10 \pm 0.01$$

$\mathcal{C}_7'/\mathcal{C}_7$ found compatible with zero.



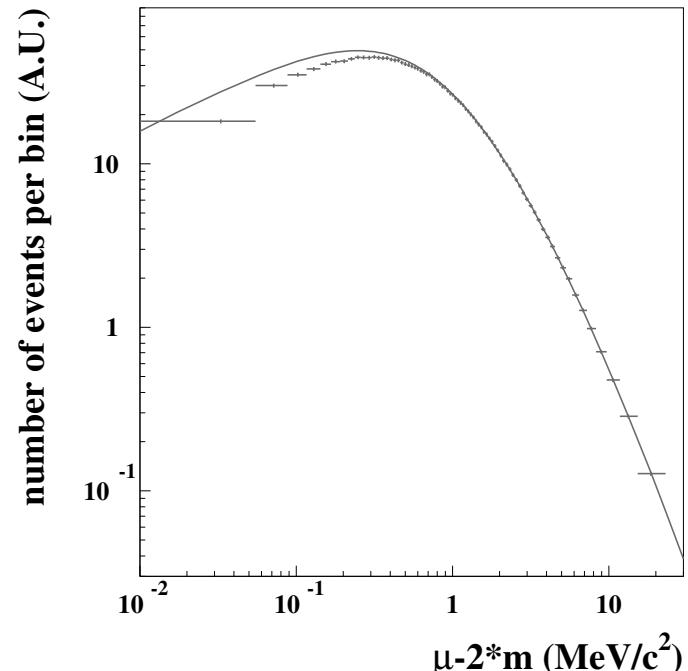
Dielectron Invariant Mass Spectra

40 GeV γ -ray conversions on Silicon



Fraction of events above μ

- M. Borsato [CERN-THESIS-2015-219](#) :
 - **Bethe-Heitler**
(A. Borsellino, [Phys. Rev. 89 \(1953\), 1023](#))
 - **Geant4**
- **5D** : G4BetheHeitler5DModel



μ spectrum, G4BetheHeitler5DModel
compared with A. Borsellino

D. Bernard, [arXiv:2102.00985 \[hep-ph\]](#)

$B \rightarrow K^* \gamma$ with converted γ

- Besoin d'un générateur exact de $A(\mu)$ M. Borsato CERN-THESIS-2015-219
- Polarized double-differential cross section, $\frac{d\sigma}{\sin \theta_\ell d\theta_\ell d\phi}(\beta)$,
G. C. Wick, Phys. Rev. 81, 467 (1951). (Weizsäcker-Williams approximation)

β and θ_ℓ velocity and polar angle of either of the leptons in the CMS

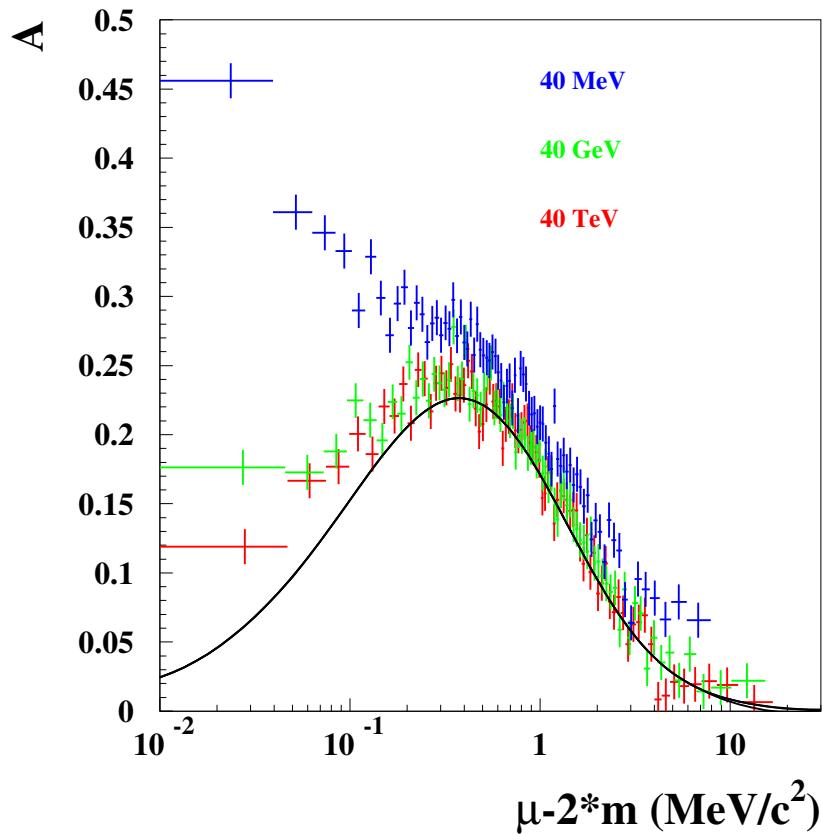
$$A = \frac{\beta(\beta^2 - 1) + (1 - \beta^4) \operatorname{arctanh} \beta}{\beta(\beta^2 - 2) + (3 - \beta^4) \operatorname{arctanh} \beta}.$$

D. Bernard, [arXiv:2102.00985 \[hep-ph\]](https://arxiv.org/abs/2102.00985)

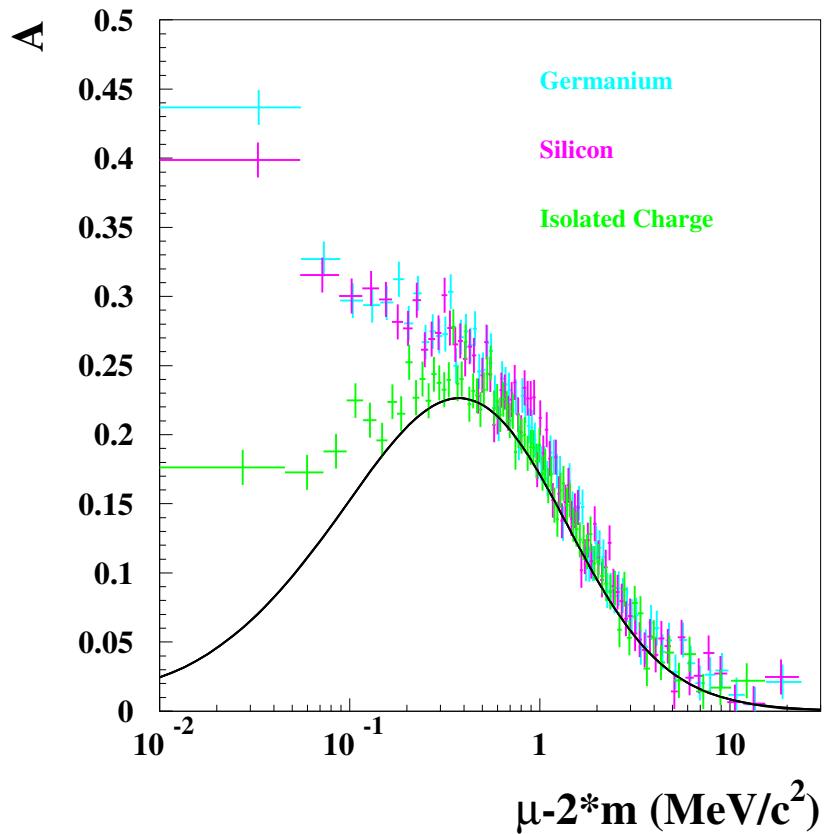
- (relation mass – velocity $\mu = 2m/\sqrt{1 - \beta^2}$)

$B \rightarrow K^*\gamma$ with converted γ

Isolated charge



40 GeV



Bin positions tuned to equalize statistics

D. Bernard, arXiv:2102.00985 [hep-ph]

Conclusion

G4BetheHeitler5DModel,
a ready to use, exact (5D), polarized, Bethe-Heitler
 $\gamma \rightarrow \ell^+ \ell^-$ event generator in Geant4

$$\ell \equiv e, \mu$$

A number of verifications wrt to published high-energy asymptotic expressions presented

$e^+ e^-$	$\mu^+ \mu^-$	
spectre $\log_{10}(q)$		Jost
$A(E)$		Gros & Bernard, Boldyshev & Peresunko
	$\hat{\theta}_{+-} \times E$	Olsen
frac($\mu > \mu_0$), spectre μ		Borsellino
$A(\mu)$		Wick / Bernard

Search for Axions

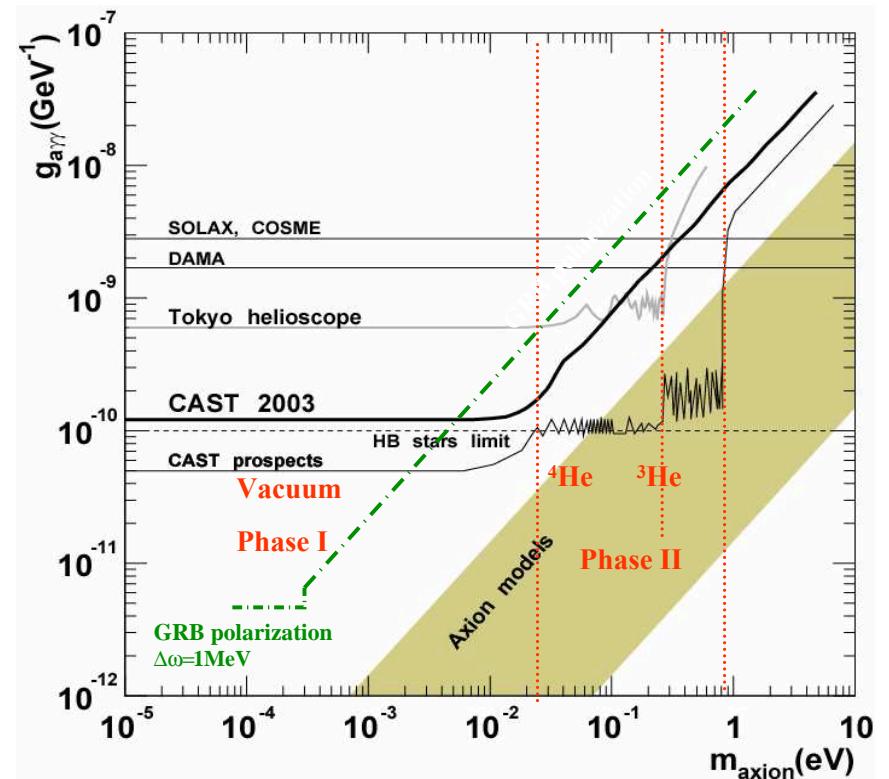
- Scalar field associated with $U(1)$ symmetry devised to solve the strong CP problem.
- Couples to 2γ through triangle anomaly.
- γ propagation through $B \Rightarrow$ Dichroism $\Rightarrow E$ dependant rotation of linear polarization \Rightarrow linear polarization dilution.

$$g_{a\gamma\gamma} \leq \pi \frac{m_a}{B \sqrt{\Delta\omega L_{GRB}}}$$

- Saturation over $L = 2\pi\omega/m_a^2 > L_{GRB}$ for $m_a \leq \sqrt{\frac{2\pi\omega}{L_{GRB}}}$

and the limit $g_{a\gamma\gamma}$ reaches a ω -independent constant.

A. Rubbia and A. S. Sakharov, Astropart. Phys. 29, 20 (2008)



LIV: Search for Lorentz Invariance Violation

- Particle (photon) dispersion relations modified in LIV effective field theories (EFT)
- Additional term to the QED Lagrangian parametrized by ξ/M , M Planck mass.
- ξ bounds:
 - time of flight from the Crab: $\Delta t = \xi(k_2 - k_1)D/M$, $\xi \leq \mathcal{O}(100)$.
 - birefringence $\Delta\theta = \xi(k_2^2 - k_1^2)D/2M$
LIV induced birefringence would blurr the linear polarization of GRB emission.
 $\xi \leq 3.4 \times 10^{-16}$ with IBIS on Integral (250 – 800 keV)
D. Götz, *et al.*, MNRAS 431 (2013) 3550
- Bound $\propto 1/k^2$!

Circular Polarization ?

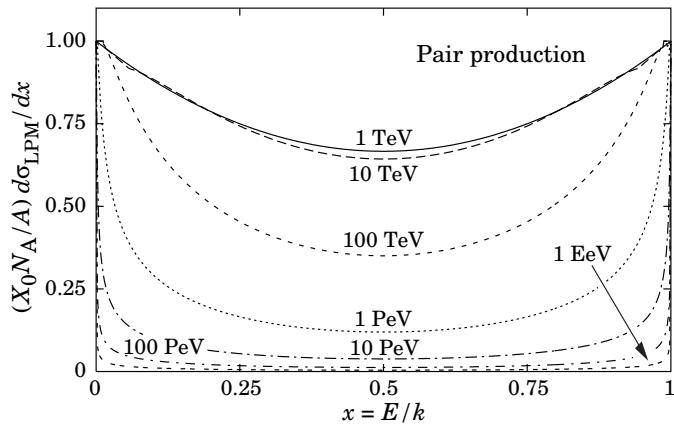
The “Bethe-Heitler” polarized differential cross section used here

- Involves photon **linear** polarization only
- Sums on the polarizations of the final leptons
- Uses the first term of the Born series

To measure the **circular** polarization of the final lepton, either

- Perform triplet conversion ($\gamma e^- \rightarrow e^+ e^- e^-$) on a tank of polarized electrons ?
[G.I. Gakh et al., Prob. Atomic Sci. Technol. 2012N1 \(2012\), 97](#) ?
- Analyze the polarization of the final leptons ?
[H. Olsen and L. C. Maximon, Phys. Rev. 114 \(1959\) 887.](#)
- Tackle the second order of the Born series ?
[H. Olsen and L. C. Maximon, Il Nuovo Cimento 24\(1962\) 186 ,](#) [H Kolbenstvedt, H Olsen Il Nuovo Cimento A 40 \(1965\) 13](#)

LPM



Landau-Pomeranchuk-Migdal

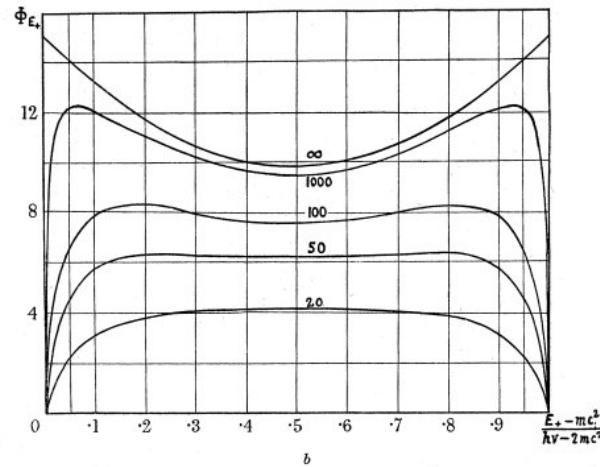
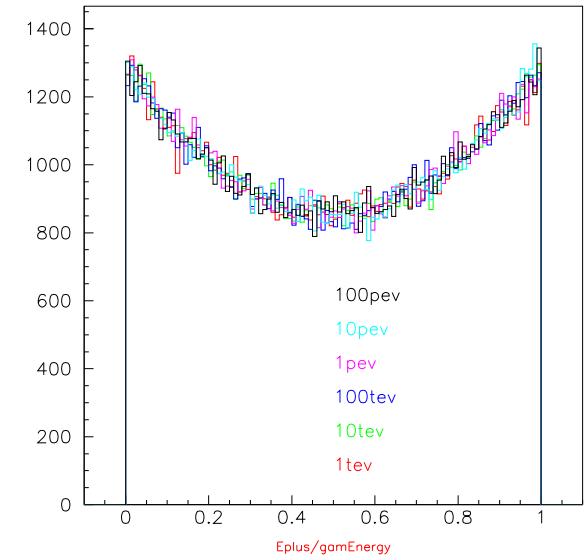


FIG. 5.—Energy distribution of pairs of positive and negative electrons. Φ_{E+} is the cross-section (units $Z^2 r_0^2 / 137$) for the creation of a positive electron with kinetic energy $E_+ - mc^2$. The numbers affixed to the curves refer to the energy of the light quantum $h\nu$ in units mc^2 . Fig. 5a is valid for any element (screening neglected), fig. 5b refers to lead.

Bethe and Heitler
Proc.Roy.Soc.Lond. A146 (1934) 83



5D

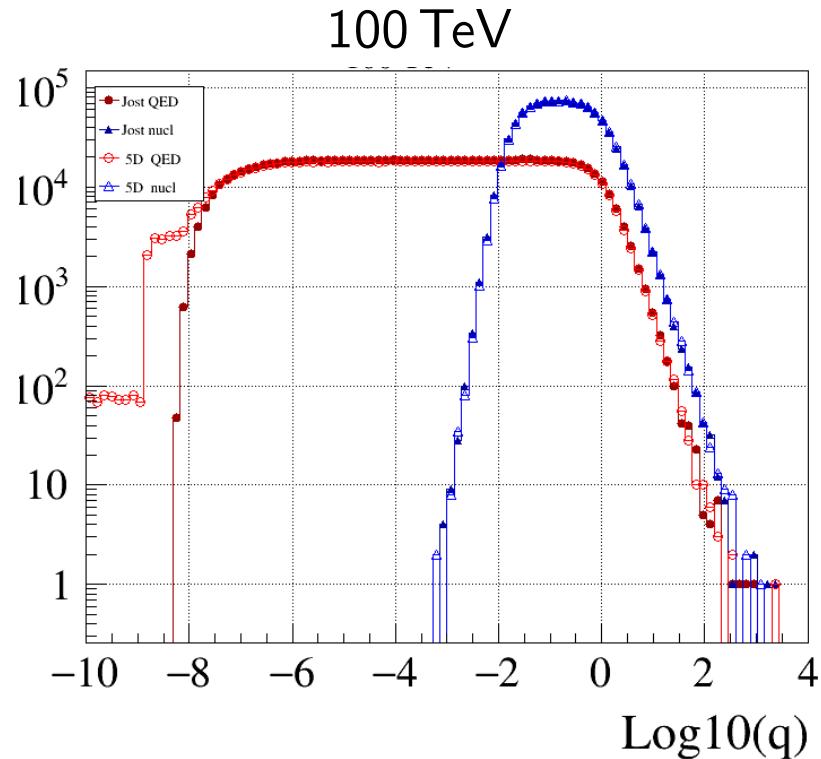
G4BetheHeitler5DModel: Validity Energy Range

- Technical: inability to sample meV/ c recoil events

—●— Jost QED
—★— Jost nucl
—○— 5D QED
—△— 5D nucl

High-energy asymptotic expression

Jost, Phys. Rev. 80, 189 (1950)



$\log_{10} (q/(\text{MeV}/c))$ distributions for γ -ray conversions on Argon

- Scientific: remember LPM differential cross section suppression not described