

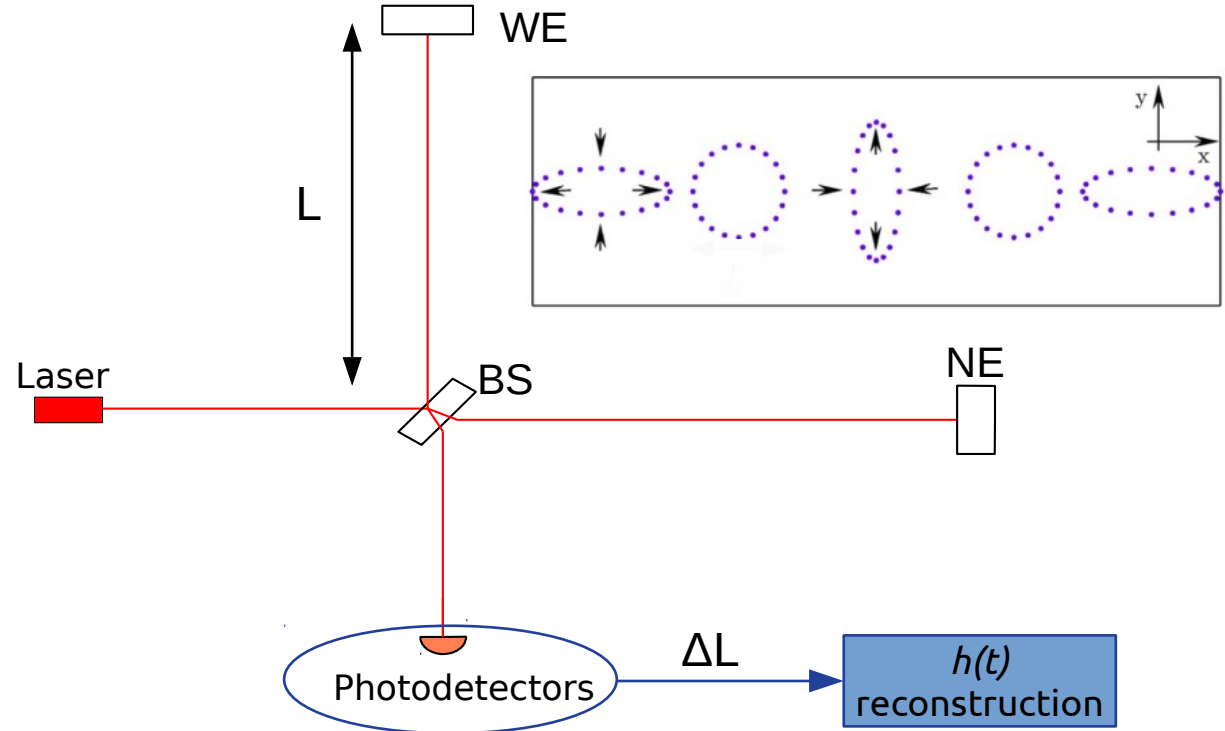
# Improving gravitational-wave detectors calibration for accurate and precise physics

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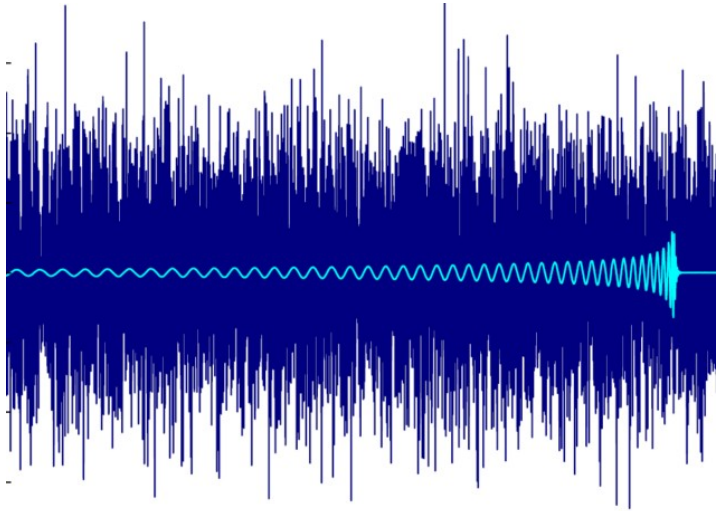
# Gravitational waves detection

Virgo detector, Cascina, Italy



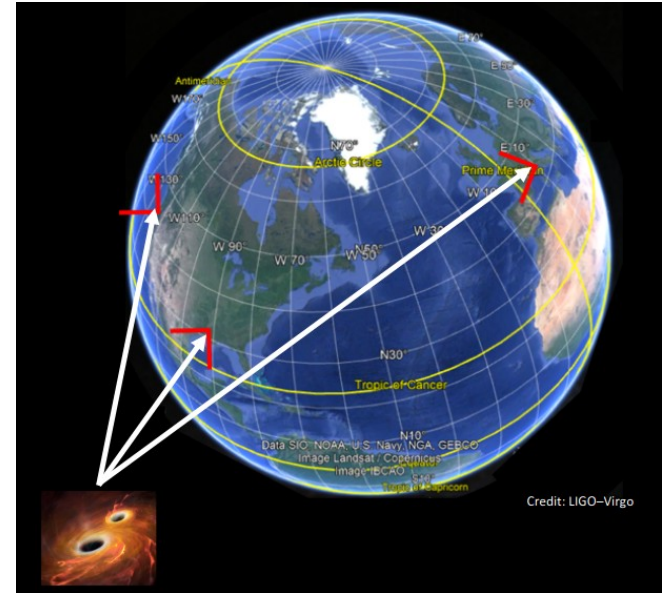
# Searching for gravitational waves

## Compact Binary Coalescence analysis: Matched filtering



- Following the GW signal in time and in frequency
- Data of detectors network need to be calibrated from  $\sim 20$  Hz to  $\sim 2$  kHz

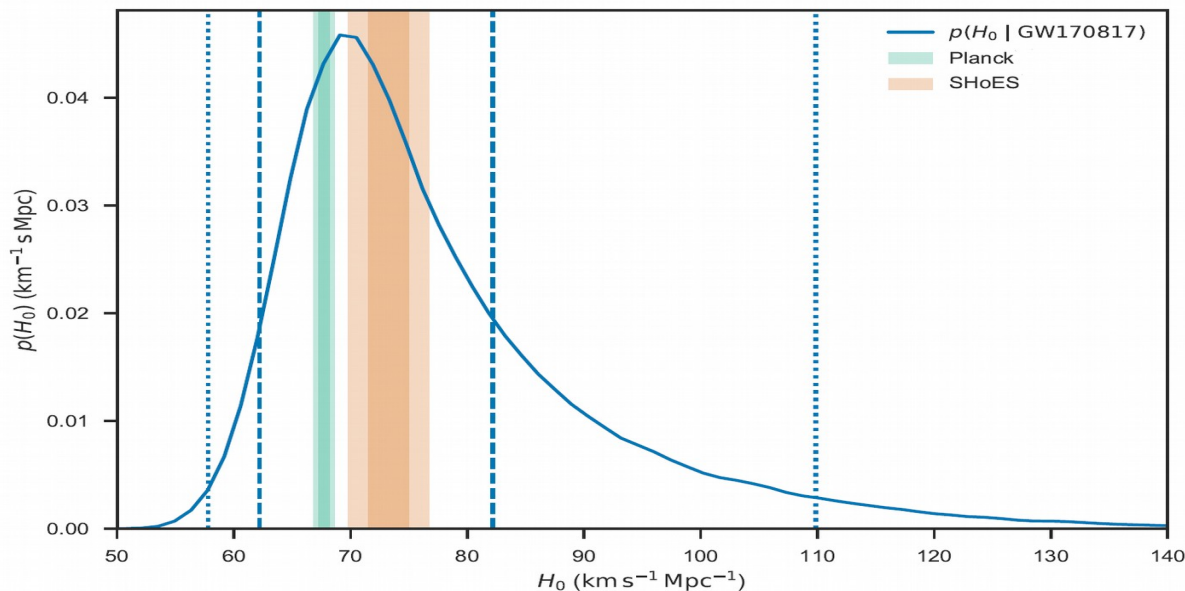
## Sky localization



- Absolute intercalibration of the detectors network in amplitude
- Absolute timing reference between the detectors

# Getting the Hubble constant right

Many results from GW detections, one example:  
Hubble constant with GW170817



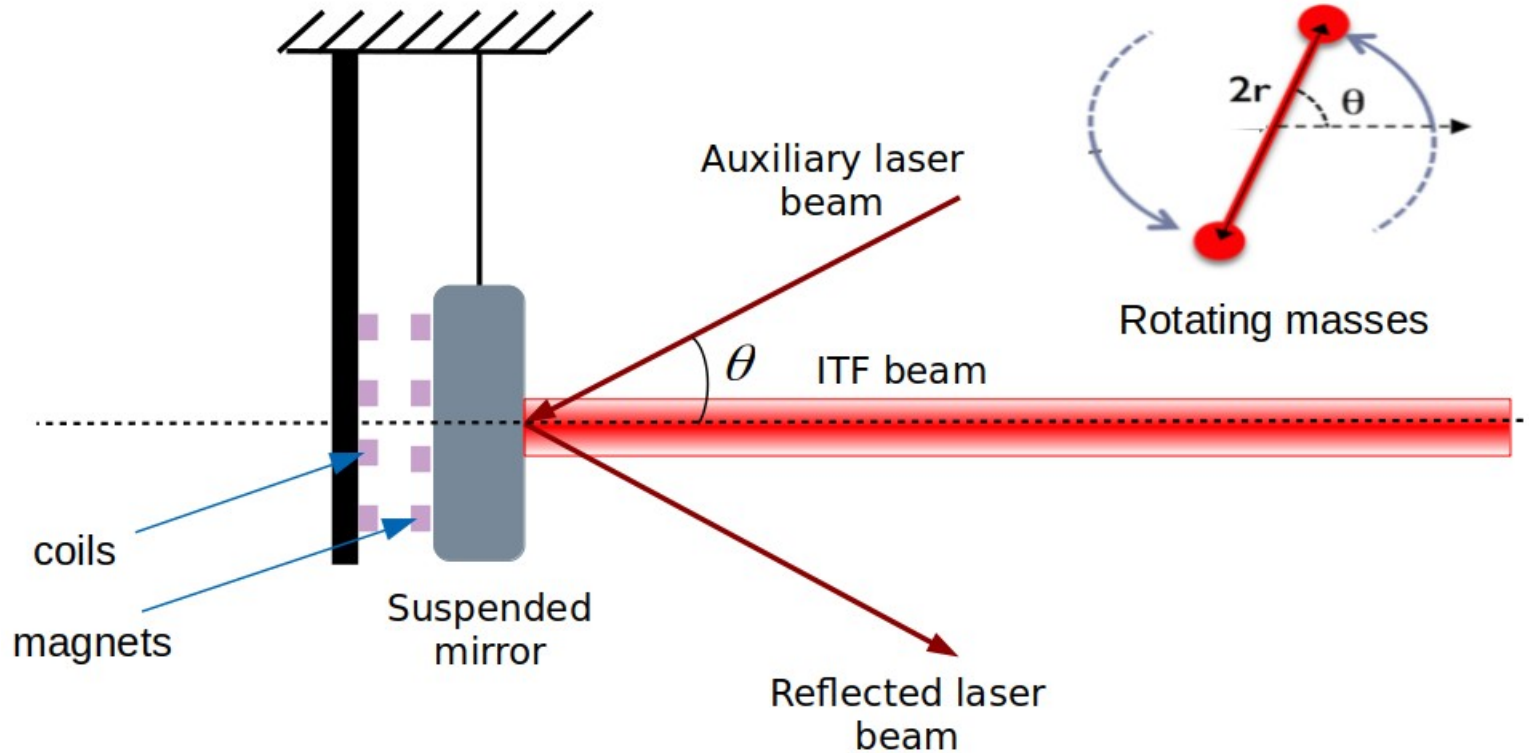
Abbott et al. 2017,  
*Nature*,  
[10.1038/nature24471](https://doi.org/10.1038/nature24471)

$$H_0 = \frac{cz}{d_L}, \quad z \ll 1$$

- Precise measurements of the luminosity distance:  $h \propto d_L^{-1}$
- Other part of my work: Classification of GW events to predict possible electromagnetic counterpart

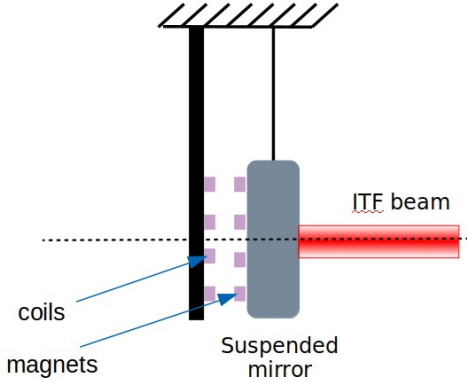
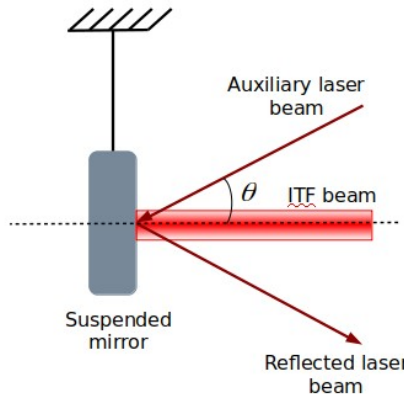
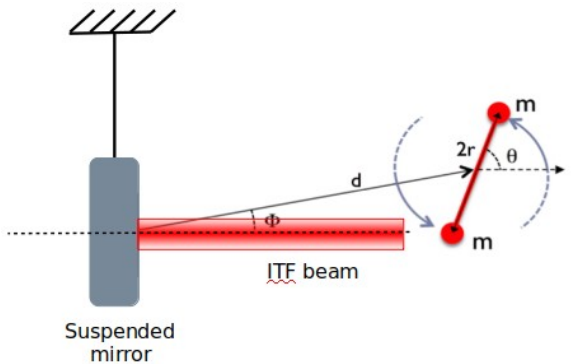
# Three types of actuators to move the mirrors

Calibration method: compare a known mirror motion with the reconstructed motion



# Three types of actuators to move the mirrors

Calibration method: compare a known mirror motion with the reconstructed motion

<b>Electromagnetic actuator (EM)</b>	<b>Photon calibrator (PCal)</b>	<b>Newtonian calibrator (NCal)</b>
 <p>Method: → Electromagnetic force</p> <p>Calibration reference: → ITF laser wavelength</p> <p>Actuation range: → ~10 Hz to ~1 kHz</p>	 <p>Method: → Radiation pressure</p> <p>Calibration reference: → <i>Absolute</i> laser power</p> <p>Calibration range: → ~10 Hz to ~10 kHz</p>	 <p>Method: → Variations of the local gravitational field</p> <p>Calibration reference: → Gravitational Constant <math>G</math></p> <p>Calibration range: → ~10 Hz to ~200 Hz (maybe more)</p>

# Photon Calibrator (PCal)

From laser power to force:

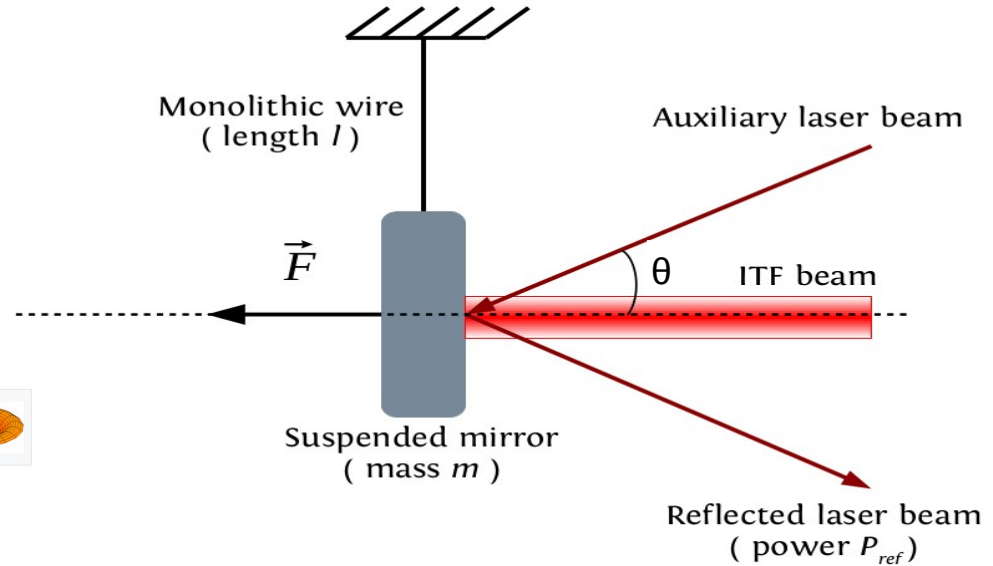
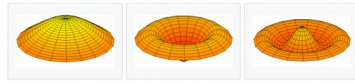
$$F(f) = \frac{2 \cos(\theta)}{c} P_{ref}(f)$$

From force to mirror motion:

$$x(f) = \left[ -\frac{1}{m(2\pi f)^2} + H(f) \right] \cdot F(f)$$

Simple pendulum

Elastic deformations

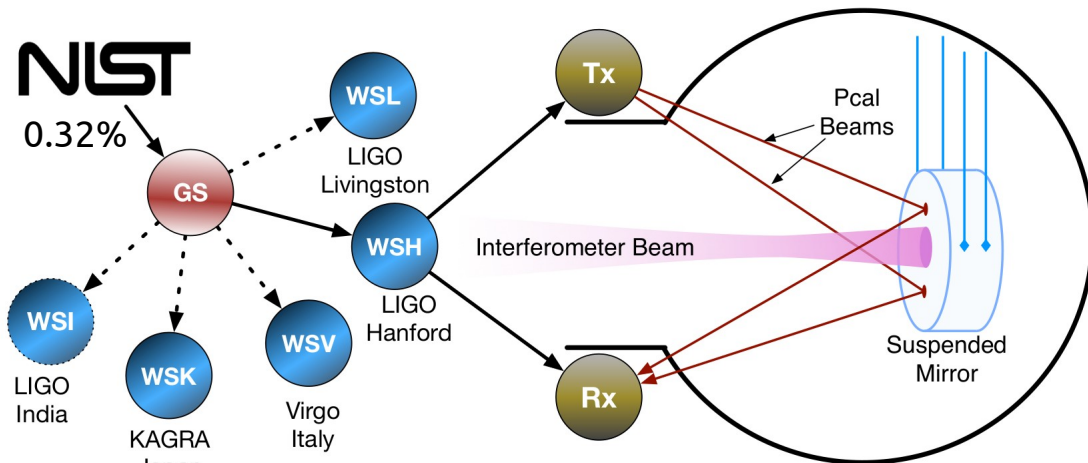


Power reflected by the mirror ( $\sim 2W$ ):

- Use photodiodes to read the laser power
- Photodiodes are calibrated with an integrating sphere (our reference photodetector)
- Calibration sensitive to temperature and humidity variations
- Online monitoring during O3

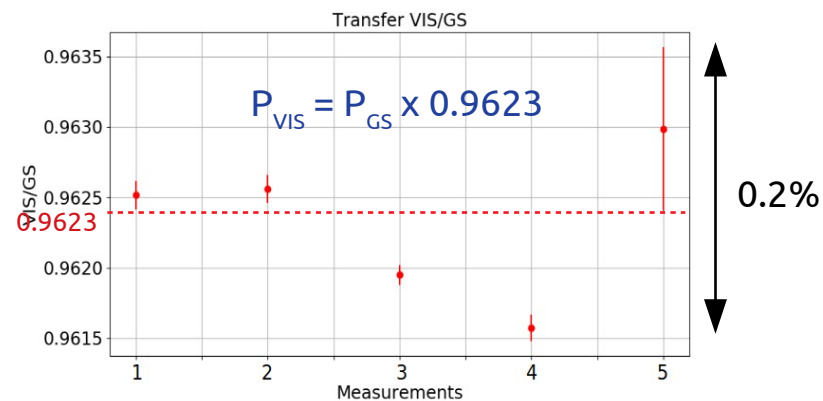
**Does Virgo photodetector measure the same laser power as LIGO photodetectors?**

# Worldwide intercalibration



Calibration transfer at LIGO Hanford  
between Virgo Integrating Sphere and  
Gold Standard

$P_{VIS}$  has to be corrected by +3.92%



Only stat. errors

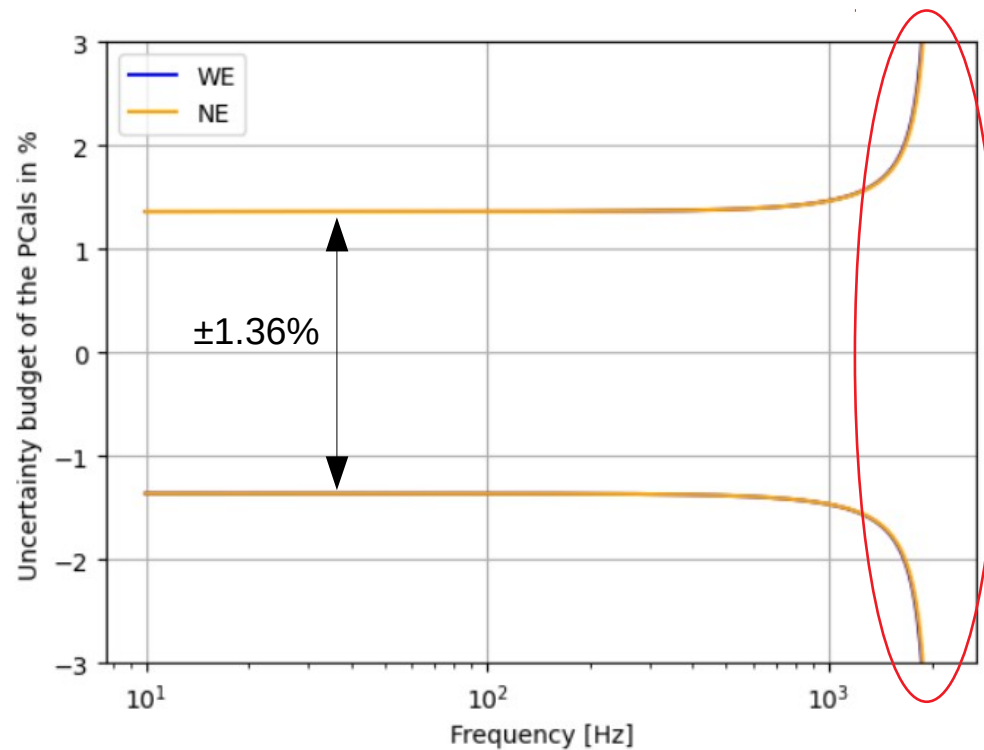


# Uncertainty on PCal-induced mirror motion

Estevez *et al.*, 2021, The Advanced Virgo Photon Calibrators, accepted in *Classical and Quantum Gravity*, DOI: [10.1088/1361-6382/abe2db](https://doi.org/10.1088/1361-6382/abe2db)

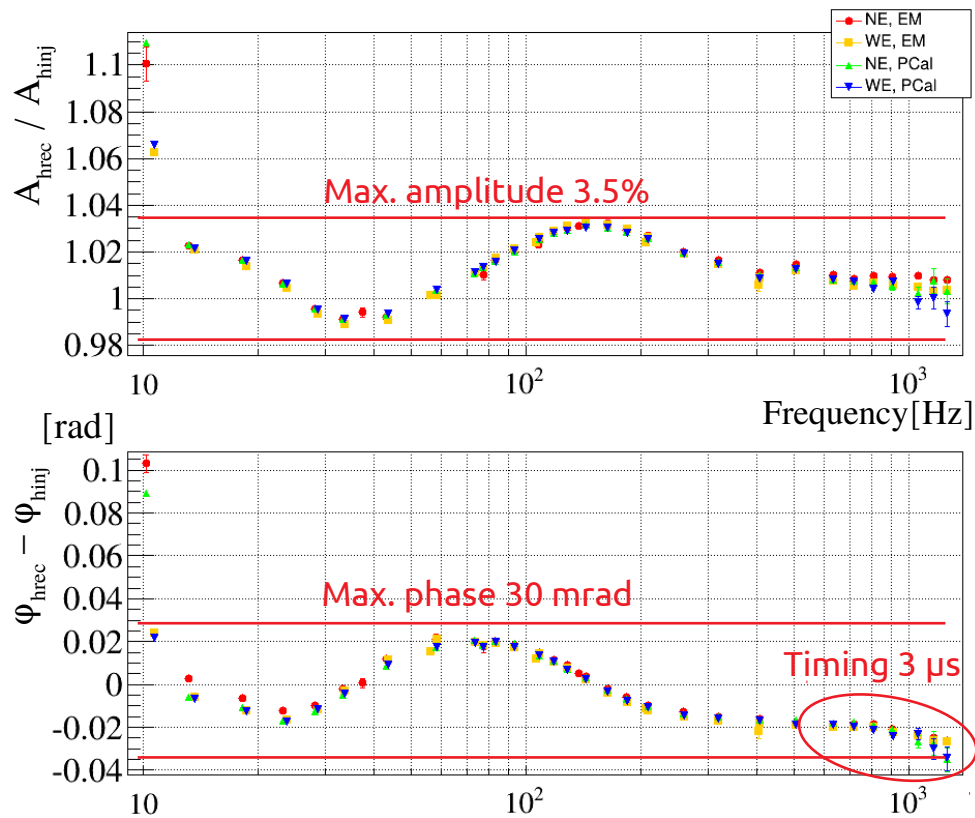
Parameter	$1\sigma$ uncertainty	
	O3a	O3b
Reflected laser power ( $P$ )	1.24%	1.24%
Geometrical parameters	0.20%	0.20%
Calibration stability (O3)	0.51%	0.61%
Total	1.36%	1.40%

From 10 Hz to 1 kHz



Elastic deformations effect

# Checking the reconstructed $h(t)$



Paper in preparation on behalf of Virgo collaboration

Adding uncertainty from calibration steps:

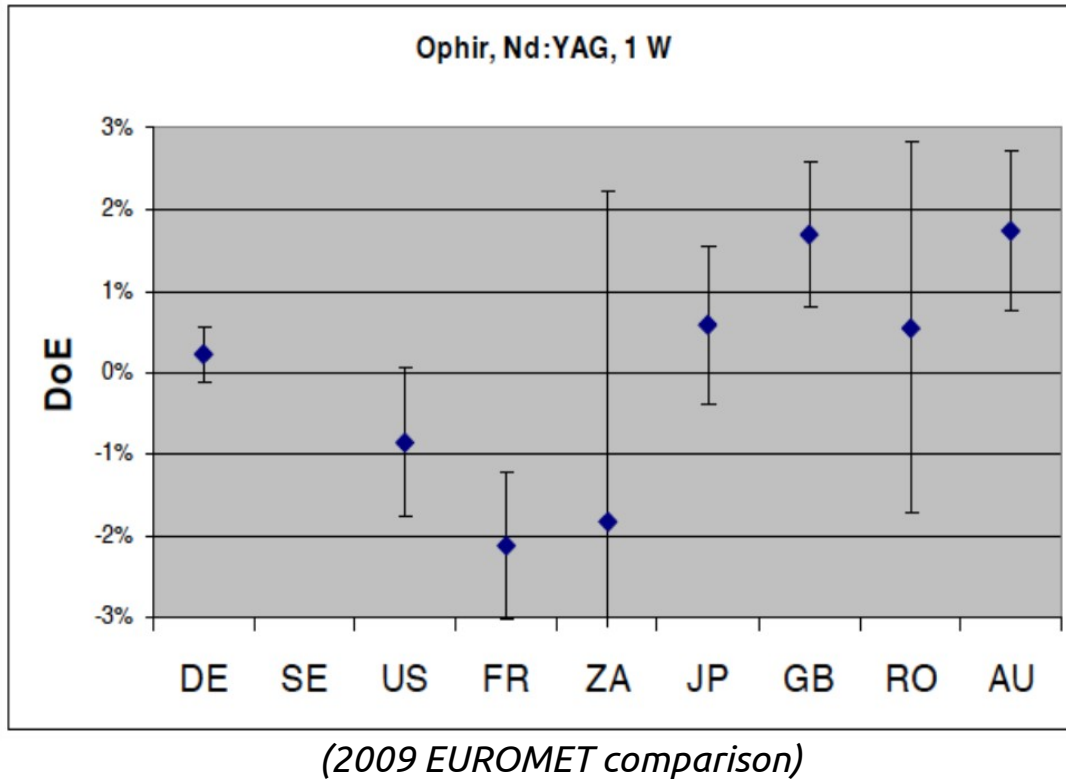
- Amplitude uncertainty  $\rightarrow \delta A = \pm 5\%$
- Phase uncertainty  $\rightarrow \delta \Phi = \pm 35 \text{ mrad}$
- Timing uncertainty  $\rightarrow \delta \tau = \pm 10 \mu\text{s}$

Uncertainties provided on online  $h(t)$  during O3 are good:

- SNR events in Virgo  $< 8$
- Uncertainty from the searches scales as  $(\text{SNR})^{-1}$

# Absolute laser power calibration?

1 Watt in France = 1 Watt in the US?



LIGO-Virgo reference:  
Gold Standard calibrated by NIST at  
the level of 0.32%...

Need another calibration method to  
check the absolute calibration  
→ Newtonian Calibrator (NCal)

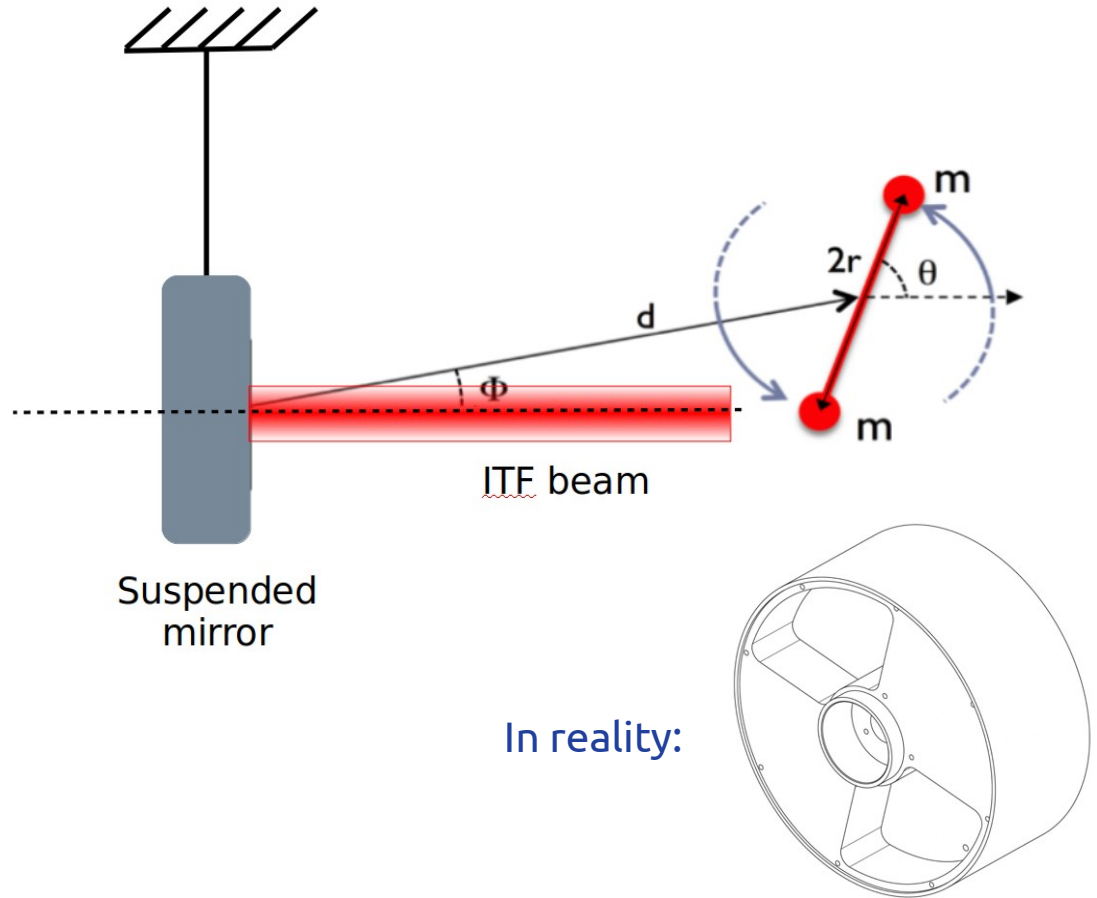
# Newtonian Calibrator (NCal)

Estevez et al., 2018, First tests of a Newtonian calibrator on an interferometric gravitational-wave detector,  
*Classical and Quantum Gravity* 35 235009  
DOI: [10.1088/1361-6382/aae95f](https://doi.org/10.1088/1361-6382/aae95f)

Point mass approximation:

$$F(\theta) = \frac{9GM_r m r^2}{2d^4} \cos(2\theta)$$

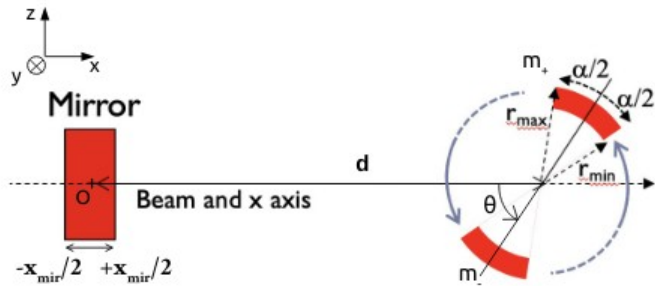
$$x(\theta, f_h) = \frac{F(\theta)}{m(2\pi f_h)^2} \quad f_h = 2f_{rotor}$$



# Improvements on the modeling

## Analytical model:

- Parametrization of the extended mirror and the extended rotor

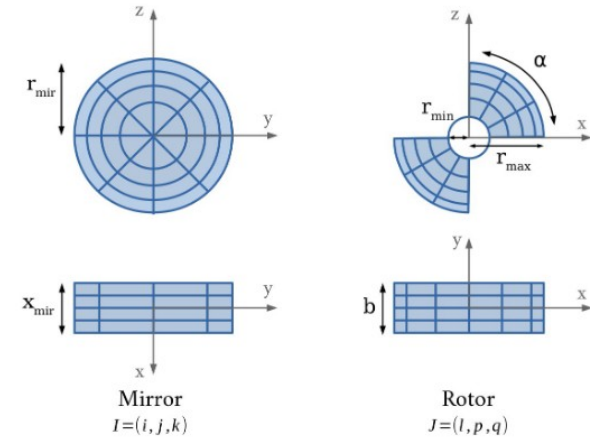


$$F_x \approx \frac{9G}{8d^4} \rho_{rot} b (r_{max}^4 - r_{min}^4) \sin(\alpha) \rho_{mir} \pi r_{mir}^2 x_{mir} \cos(\phi) \left[ \left( 1 + \frac{25}{54d^2} \frac{r_{max}^6 - r_{min}^6}{r_{max}^4 - r_{min}^4} \right. \right. \\ \left. \left. + \left( \frac{45}{8} \sin^2(\phi) - \frac{5}{2} \right) \left( \frac{r_{mir}}{d} \right)^2 \right. \right. \\ \left. \left. + \left( \frac{15}{8} \cos^2(\phi) - \frac{25}{24} \right) \left( \frac{x_{mir}}{d} \right)^2 \right. \right. \\ \left. \left. - \frac{25}{72} \left( \frac{b}{d} \right)^2 - \frac{35}{6} \left( \frac{z}{d} \right)^2 \right) \cos(2\theta) \right. \\ \left. + \frac{8}{3} \frac{z}{d} \sin(2\theta) \right]$$

- Difference of ~3% with the point mass force

## Numerical model:

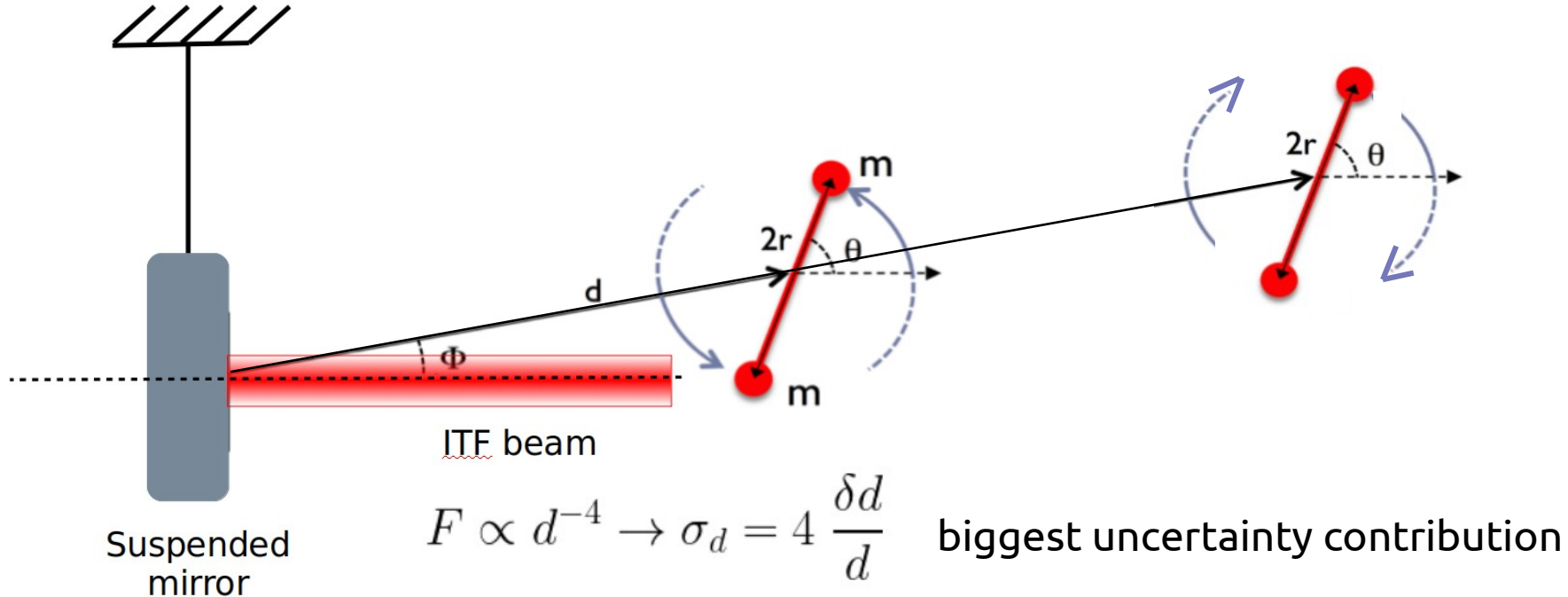
- FROMAGE: Finite element analysis of ROtating MAsses for Gravitational Effects



$$\vec{F} = G \sum_I \sum_J \frac{m_{mir,I} m_{rot,J}}{d_{I,J}^2} \vec{u}_{I,J}$$

Agreement on the NCal-induced mirror motion between both models better than 0.02%

# Reducing the uncertainty with two NCals



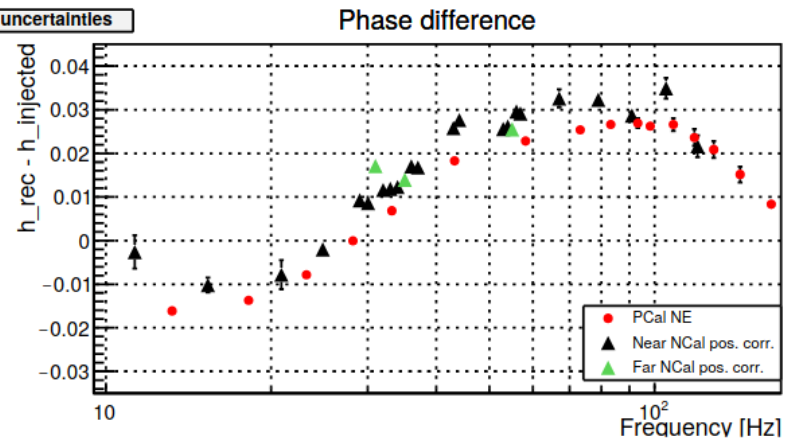
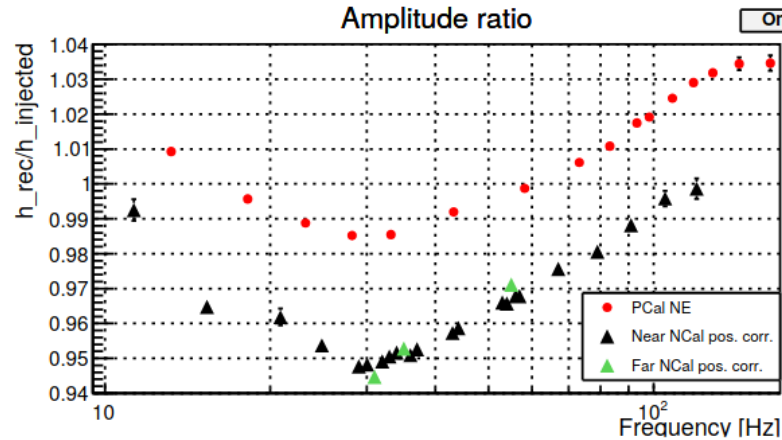
- No direct measurement of the distance (mirror in vacuum chamber)
  - Comparing the NCal signals amplitude allows to measure the distance
- No direct measurement of the vertical offset (w.r.t plane of the interferometer)
  - Comparing the NCal signals phase allows to measure the vertical offset

# NCal tests during O3

Estevez et al., 2021, Newtonian calibrator tests during the Virgo O3 data taking, accepted in *Classical and Quantum Gravity*  
DOI: [10.1088/1361-6382/abe2da](https://doi.org/10.1088/1361-6382/abe2da)

Parameter	uncertainty	formula	$h_{rec}/h_{inj}$ near [%]	$h_{rec}/h_{inj}$ far [%]
NCal to mirror distance $d$	6.4 mm	$4\delta d/d$	2.02	1.31
NCal to mirror angle $\Phi$	5.0/3.3 mrad	$\delta\Phi \sin \Phi$	0.28	0.19
NCal vertical position $z$	1.3 mm	$5/2(z/d)^2$	0.03	0.01
Rotor geometry	see table 1		0.53	0.53
Modeling method	see end of section 4		0.018	0.017
Mirror torque from NCal	see end of section 4		0.05	0.03
Total	quadratic sum		2.1	1.4

## Verification of $h(t)$ reconstruction:



Not incompatible with the PCal systematic uncertainty

# Toward a sub-percent calibration

- Which precision to reach?
  - Next observing runs will improve the GW events statistics: hope to reach 1% or less on  $h(t)$  systematic uncertainty
  - Third generations detectors (Einstein Telescope, Cosmic Explorer) events with SNR  $\sim 1000$ : calibration uncertainty smaller than  $1/\text{SNR} = 0.1\%$  for one event
- Photon Calibrators are the current reference calibration tools for the detectors network:
  - Intercalibration on a common “Gold Standard” calibrated by NIST
  - Measurement of laser power is not that simple, dependence on temperature, humidity etc...
- An independent method for relative and absolute calibration of GW detectors:
  - Newtonian Calibrators with “simpler” parameters to control (distance and geometry)
  - Difficult to check the reconstructed  $h(t)$  at high frequency (limitations on the rotor speed)
  - Calibrate the PCals at low frequency with the NCal signals and extend the calibration of the GW detectors at high frequency with the PCals
- Using astrophysical sources to make a relative calibration of the detectors network