

Rotating Inflation

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Work in progress with :
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Original motivations :

Homogeneity and Isotropy in Quantum Gravity,
What does it mean ?

- ▶ At the classical level, H&I on the fields : $\phi(t, \vec{x}) = \phi(t)$ → (Minkowski)
- ▶ At the quantum level, H&I on the wavefunctional : $\psi[\phi(R\vec{x} + \vec{a})] = \psi[\phi(\vec{x})]$ Background)

$$4 [\textcircled{red} \dots] = 4 [\dots \textcircled{red}]$$

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- ▶ What about GR ? $\psi[h_{ij}(x)]$

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We need a reference frame !
→ Solid, fluid ...

$$4 [\textcircled{red} \text{ grid }] = 4 [\text{grid } \textcircled{red}]$$

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- ▶ Minisuperspace, H&I in the spatial part : too simple... → **Bianchi models !**

Bianchi Type I, diagonal metric

- ▶ Bianchi type I in 2+1 : $ds^2 = -dt^2 + h_{ij}(t)dx^i dx^j$
- ▶ Symmetries of the spatial part : $GL(2)$

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- ▶ Initial conditions, in diagonal form : $h_{ij}(t_0) = \delta_{ij}$ $\dot{h}_{ij}(t_0) = \begin{pmatrix} A^2(t_0) & 0 \\ 0 & B^2(t_0) \end{pmatrix}$
- ▶ Equations of motion : $\ddot{h}_{ij}(t) = 16\pi(T_{ij} - h_{ij}T) + \dot{h}_{ik}h^{kl}\dot{h}_{lj} - \frac{1}{2}\dot{h}_{ij}h^{kl}\dot{h}_{kl}$

Bianchi Type I, diagonal metric

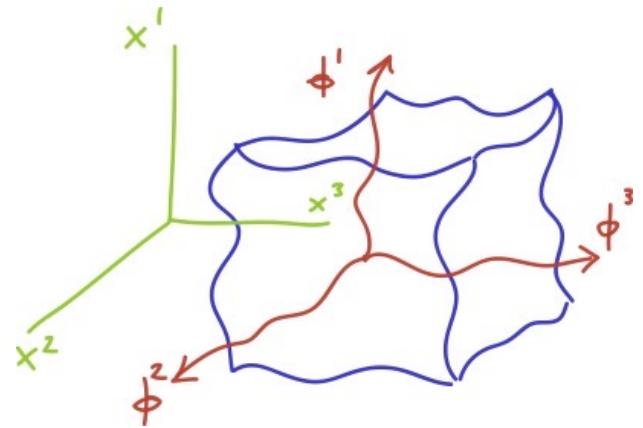
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 - ▶ Matter source, fluid : $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$
- $\ddot{h}_{ij}(t) = 8\pi(\rho - p)h_{ij} + \dot{h}_{ik}h^{kl}\dot{h}_{lj} - \frac{1}{2}\dot{h}_{ij}h^{kl}\dot{h}_{kl}$

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- $h_{ij} \ \dot{h}_{ij}$ will remain diagonal for all time t !
-
- No
anisotropic
stress !

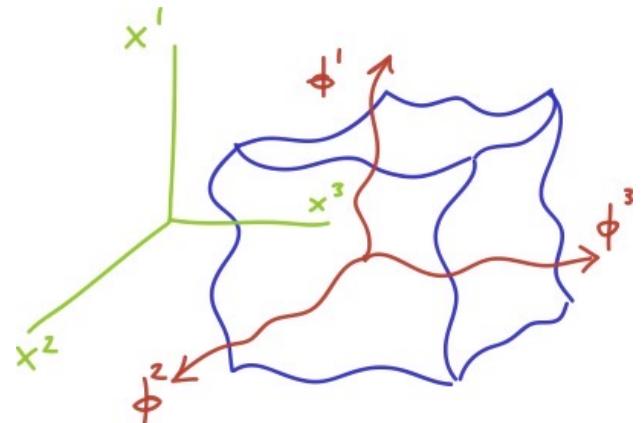
Solid

- ▶ Medium volume element : $\phi^I(\vec{x}, t)$
- ▶ Simple example of background configuration : $\langle \phi^I \rangle = \alpha x^I$



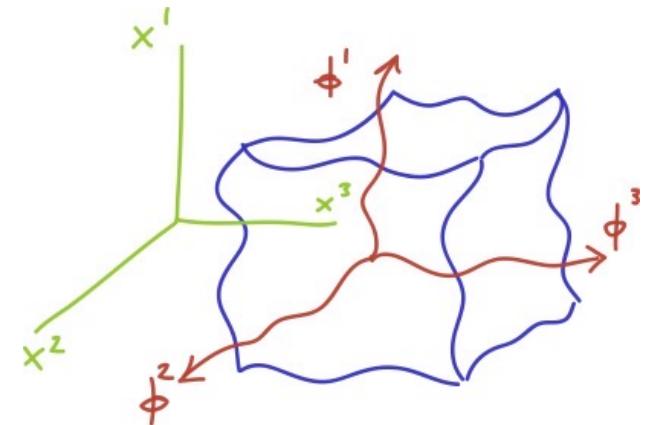
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 - Internal shift : $\phi^I \rightarrow \phi^I + a^I$, $a^I = \text{const}$,
 - Rotational symmetry : $\phi^I \rightarrow O^I_J \phi^J$, $O^I_J \in SO(3)$,



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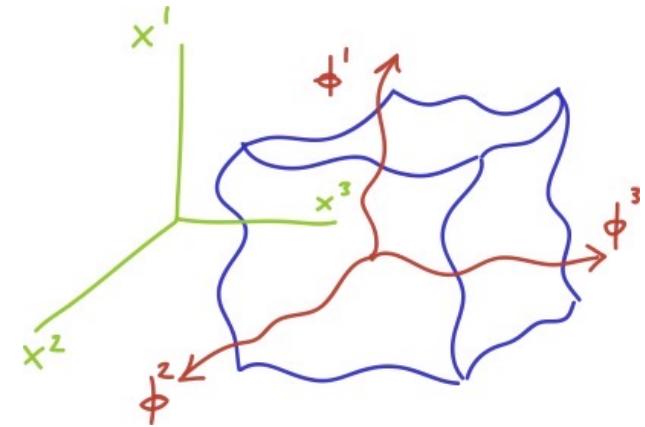
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- ▶ General covariance + shift symmetry : $B^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J$
 $+ SO(3) : \text{Tr } B = [B]$
- ▶ Most general Lagrangian for the solid : $L_{solid} = F([B], [B^2], [B^3])$



Solid inflation

- ▶ Medium volume element : $\phi^I(\vec{x}, t)$
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- ▶ Solid inflation : $\rho_\xi \sim e^{-2\epsilon N}$

→ Less effective in diluting anisotropies !



S. Endlich, A. Nicolis, J. Wang.2012
 "Solid Inflation"

N. Bartolo, S. Matarrese, M. Peloso, A. Ricciardone.2013 "anisotropy in solid inflation"

Bianchi + Solid in 2+1, rotating solutions !

- ▶ Simplest inflationary model: $L_{solid} = F([B], [B^2]) = -[B]^\epsilon$
- ▶ Unitary Gauge : $\phi^I = x^I \rightarrow B^{IJ} = h^{IJ}$

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- ▶ Parametrisation for the metric :

$$h_{ij} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} A^2 e^\xi & 0 \\ 0 & A^2 e^{-\xi} \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \rightarrow R^T \tilde{h} R$$

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- ▶ The resulting action : $S = \int dt \frac{A^2}{4N} \left(-4H^2 + \dot{\xi}^2 + \dot{\theta}^2 (\sinh \xi)^2 \right) - \Lambda \int dt N \frac{\cosh \xi^\epsilon}{A^{2(\epsilon-1)}}$

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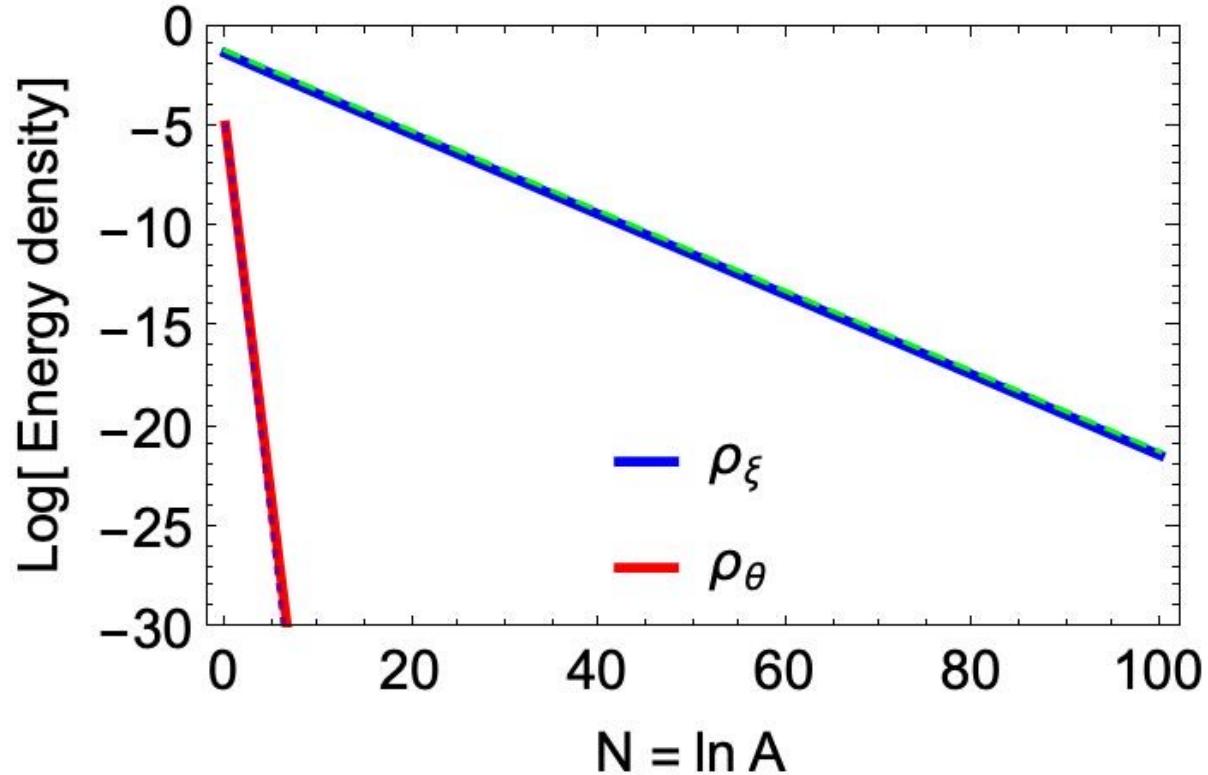
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Bianchi only : $\theta(t)$ fake !

Bianchi + solid : $\theta(t)$ real dynamical quantity !

Energy densities of ξ and θ :



$$\rho_\xi \sim e^{-2\epsilon N}$$

$$\rho_\theta \sim e^{-4N}$$

Conclusion and outlook

- ▶ Rotating cosmological models !
- ▶ Outlook : Quantum treatment ? Look for the case 3+1 ?

Thank you for your
attention !
