DHOST Inflation

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DHOST Theories and Scordatura perturbations

DHOST theories

- **Degenerate Higher-Order Scalar-Tensor** (DHOST) theories are scalar-tensor theories with one scalar degree of freedom depending on a scalar field, its gradient and also second derivatives, such that they don't lead to Ostrogradsky ghosts.
- They are the most general such theories which lead to second order equations of motion for the scalar field.
- Most studies of these theories have so far focussed on explaining the late time universe (dark energy).
- They generals the Beyond Horndeski theories, which are themselves a generalisation of Horndeski theories.
- Based on how many power of the second order derivatives are present, they can be quadratic, cubic, etc…Here we concentrate on quadratic theories.
- The results that I am presenting now look at the early universe, creating a viable inflationary model.

DHOST action

- Most general action involving up to second-order interaction in the scalar field

$$S = \int d^4x\sqrt{-g}\left[F_0(\phi, X) + F_1(\phi, X)\,\Box\,\phi + F_2(\phi, X)R + \sum_{i=1}^{5} A_i(\phi, X)L_i\right]$$

where
$$L_1 = \phi_{\nu\eta}\phi^{\nu\eta}$$
$$L_2 = (\Box\phi)^2$$
$$L_3 = \Box\,\phi\,\phi_\nu\phi^{\nu\eta}\phi_\eta$$
$$L_4 = \phi^\nu\phi_{\nu\eta}\phi^{\eta\lambda}\phi_\lambda$$
$$L_5 = (\phi_\nu\phi^{\nu\eta}\phi_\eta)^2$$

$$X = g^{\nu\eta}\phi_\nu\phi_\eta, \quad \phi_\nu \equiv \nabla_\nu\phi$$

- There are several degeneracy conditions in this action, coming from constraints of not having ghosts and from GW decay into DE:

$$S_{\rm DHOST} = \int d^4x\sqrt{-g}\left[F_0(X) + F_1(X)\,\Box\,\phi + F_2(X)R + \frac{6F_{2,X}^2}{F_2}\phi^\nu\phi_{\nu\eta}\phi^{\eta\lambda}\phi_\lambda\right]$$

Scordatura corrections

- We add small corrections to this action, the ***scordatura*** corrections, as one of the L_i terms. We choose L_2.

$$S_{\rm g} = S_{\rm DHOST} + S_{\rm S}$$

with

$$S_{\rm S} = \int d^4x\sqrt{-g}\left[-\frac{\alpha}{2}\frac{(\Box\phi)^2}{M_S^2}\right]$$

where **α** is the a small dimensionless parameter that breaks the degeneracy condition and M_S is a mass scale related to the strong coupling scale of the EFT

- We rewrite the action in terms of dimensionless coordinates and variables

$$\tilde{t} \equiv \Lambda t\,,\tilde{x}^i \equiv \Lambda x^i\,,$$
$$\phi \equiv M\varphi\,, X \equiv M^2\Lambda^2 \mathrm{x}\,, F_0 \equiv \Lambda^4 f_0\,, F_1 \equiv \frac{\Lambda^2}{M} f_1\,, F_2 \equiv \Lambda^2 f_2\,, H = \Lambda h\,.$$

- Models are low energy well beyond the Planck scale
- We consider $\Lambda \approx m_{\rm Pl}, M \ll m_{\rm Pl}$
- In order to have a consistent expansion in powers of X, we need $\mu_c \equiv \sqrt{M\Lambda} \ll m_{\rm Pl}$

DHOST background

- We start with an action

$$ds^2 = \Lambda^2\Big[-d\tilde{t}^2 + a(\tilde{t})^2\delta_{ij}d\tilde{x}^i d\tilde{x}^j\Big]$$

- We write the 00 and ii Einstein equations.
- We expect inflation to be driven by an energy density below μ_c, hence we impose $h \ll 1$.
- We define the following parameters, first order in the perturbations of f_i

$$\alpha_H \equiv -\mathrm{x}\frac{f_{2,\mathrm{x}}}{f_2}\,, \quad \alpha_B \equiv \frac{1}{2}\frac{\dot{\varphi}\,\mathrm{x}}{h_b}\frac{f_{1,\mathrm{x}}}{f_2} + \alpha_H\,, \quad \alpha_K \equiv -\frac{\mathrm{x}}{6h_b^2}\frac{f_{0,\mathrm{x}}}{f_2} + \alpha_H + \alpha_B$$

and

$$b \equiv \sqrt{f_2}\,a,\ h_b \equiv \frac{\dot{b}}{b} = h - \frac{\ddot{\varphi}}{\dot{\varphi}}\alpha_H$$

2nd order DHOST perturbations

- The line element for scalar perturbations is given by

$$ds^2 = \Lambda^2\Big(-(1+2A)d\tilde{t}^2 + 2\tilde{\partial}_i B d\tilde{t}d\tilde{x}^i + a^2(1+2\psi)\delta_{ij}d\tilde{x}^i d\tilde{x}^j\Big)$$

- The second order DHOST action can be written a

$$S^{(2)}_{\rm DHOST} \equiv \int d\tilde{t}d^3\tilde{k}\tilde{\mathscr{L}}^{(2)}_{\rm D}(\dot{\psi},\psi,\dot{A},A,B)$$

- We perform the change of variable $\zeta \equiv \psi + \alpha_H A$ to get

$$\tilde{\mathscr{L}}^{(2)}_{\rm D} = 2f_2\Big(-3a^3\dot{\zeta}^2 + 6a^3h_b(1+\alpha_B)\dot{\zeta}A - 2a\tilde{k}^2\dot{\zeta}B + a\tilde{k}^2\zeta^2$$
$$+2a(1+\alpha_H)\tilde{k}^2\zeta A - 3a^3h_b^2\beta_K A^2 + 2ah_b(1+\alpha_B)\tilde{k}^2 AB\Big)$$

with

$$S^{(2)}_{\rm DHOST} = \int d\tilde{t}d^3\tilde{k}M^4\tilde{\mathscr{L}}^{(2)}_{\rm D}(\dot{\zeta},\zeta,A,B)$$

where

$$\beta_K \equiv -\frac{\mathrm{x}^2}{3}\frac{f_{0,\mathrm{xx}}}{h_b^2 f_2} + (1-\alpha_H)(1+3\alpha_B) + \beta_B + \frac{(1+6\alpha_H-3\alpha_H^2)\alpha_K - 2(2-6\alpha_H+3\alpha_K)\beta_H}{1-3\alpha_H},$$

$$\beta_B \equiv \dot{\varphi}\,\mathrm{x}^2\frac{f_{1,\mathrm{xx}}}{h_b f_2}, \quad \beta_H \equiv \mathrm{x}^2\frac{f_{2,\mathrm{xx}}}{f_2}.$$

- We can treat A and B as Lagrange multipliers, solve for them and put the solutions back into the second order action to find

$$\tilde{\mathscr{L}}^{(2)}_{\rm D} = a^3 f_2 \left(\bar{\mathscr{A}}\, \dot{\zeta}^2 - \bar{\mathscr{B}}\, \frac{\tilde{k}^2}{a^2} \zeta^2 \right),$$

where

$$\bar{\mathscr{A}} = 6\left[1 - \frac{\beta_K}{(1+\alpha_B)^2}\right], \quad \bar{\mathscr{B}} = -2\left[1 - \frac{1}{af_2}\frac{d}{d\tilde{t}}\left(\frac{af_2}{h_b}\frac{1+\alpha_H}{1+\alpha_B}\right)\right]$$

- The equation of motion for ζ is

$$\ddot{\zeta} + \left(3h + \frac{d}{d\tilde{t}}\ln(f_2\bar{\mathscr{A}})\right)\dot{\zeta} + \left(\frac{\bar{c}_{\rm s}\tilde{k}}{a}\right)^2 \zeta = 0$$

where, $\bar{c}_{\rm s}^2 = \dfrac{\mathscr{B}}{\mathscr{A}}$

2nd order scordatura perturbations

- We can similarly expand the scordatura action at second order to get

$$\tilde{\mathscr{L}}_{\mathrm{S}}^{(2)} = \frac{1}{2}\Big(\bar{k}_{11}\dot{\zeta}^2 + \bar{k}_{22}\dot{A}^2 + 2\bar{k}_{12}\dot{\zeta}\dot{A} + 2\dot{\zeta}(\bar{n}_{12}A + \bar{n}_{13}\tilde{k}^2 B) + 2\bar{n}_{23}\tilde{k}^2\dot{A}B - \bar{m}_{11}\zeta^2 \\ -2\bar{m}_{12}\zeta A - \bar{m}_{22}A^2 - \bar{m}_{22\mathrm{s}}\tilde{k}^2 A^2 - 2\bar{m}_{23}\tilde{k}^2 AB + \bar{m}_{33}\tilde{k}^2 B^2 + \bar{m}_{33\mathrm{s}}\tilde{k}^4 B^2\Big)$$

- The total Lagrangian is

$$\tilde{\mathscr{L}}_{\mathrm{g}}^{(2)} = a^3 f_2\,\mathscr{K}\left[\dot{\zeta}^2 - \left(c_{\mathrm{s}}^2(\tilde{k})\frac{\tilde{k}^2}{a^2} + \alpha m^2\right)\zeta^2\right]$$

where, $c_{\mathrm{s}}^2(\tilde{k}) \equiv \bar{c}_{\mathrm{s}}^2 + \frac{\alpha}{2f_2}\left(\frac{\mathscr{B}_1}{\bar{\mathscr{A}}} - \bar{c}_{\mathrm{s}}^2\frac{\mathscr{A}_1}{\mathscr{A}} + \left(\frac{\tilde{k}}{a}\right)^2\frac{\mathscr{B}_2}{\bar{\mathscr{A}}}\right)$

and $m^2 \equiv \frac{1}{2f_2}\left(\frac{\mathscr{M}}{\bar{\mathscr{A}}} - \bar{c}_{\mathrm{s}}^2\frac{\mathscr{A}_2}{\bar{\mathscr{A}}}\right)$, $\mathscr{K} \equiv \bar{\mathscr{A}}\left(1 + \frac{\alpha}{2f_2}\left(\frac{\mathscr{A}_1}{\bar{\mathscr{A}}} + \frac{a^2}{\tilde{k}^2 + \alpha k_{\mathrm{IR}}^2}\frac{\mathscr{A}_2}{\bar{\mathscr{A}}}\right)\right)$

de Sitter solutions

- We consider an expanding universe with constant Hubble parameter. By choosing $\varphi(\tilde{t}) = \tilde{t}\,, \quad \mathrm{x} = -1$, the Friedmann equations become

$$f_0 + 6h_{\mathrm{dS}}^2 f_2 = 0$$
$$f_{0,\mathrm{x}} + 3h_{\mathrm{dS}}(4h_{\mathrm{dS}} f_{2,\mathrm{x}} - f_{1,\mathrm{x}}) = 0$$

- The functions f are now evaluated at x=-1 and hence they are constants.
- We get:

$$h_{\mathrm{dS}} = \sqrt{\frac{-f_0}{6f_2}}$$

$$\bar{c}_{\mathrm{s}}^2 = -\frac{(1+\alpha_B)(\alpha_B - \alpha_H)}{3(1+2\alpha_B+\alpha_B^2-\beta_K)}$$

Quantisation

- We consider the creation of primordial fluctuations from the Bunch-Davies vacuum
- The scale factor is

$$a(\eta) = -\frac{1}{h_{\rm dS}\eta}$$

- The second order action can be expressed in conformal time as

$$S_{\rm g}^{(2)} = \int d\eta d^3\tilde{k} z^2 \left[\zeta'^2 - a^2\Big(c_{\rm s}^2(\tilde{k})\frac{\tilde{k}^2}{a^2} + \alpha m^2\Big)\zeta^2\right]$$

where $z^2 = a^2 f_2 \mathscr{K}$ and hence

$$z = \frac{1}{h_{\rm dS}\eta}\sqrt{f_2\mathscr{A}}\sqrt{\left(1 + \frac{\alpha}{2f_2}\Big(\frac{\mathscr{A}_1}{\bar{\mathscr{A}}} + \frac{1}{h_{\rm dS}^2\eta^2}\frac{1}{\tilde{k}^2 + \alpha k_{\rm IR}^2}\frac{\mathscr{A}_2}{\bar{\mathscr{A}}}\right)}$$

- We define the Mukhanov-Sasaki variable $v = z\zeta$, which satisfies

$$v'' + \left[a^2\Big(c_{\rm s}^2(\tilde{k})\frac{\tilde{k}^2}{a^2} + \alpha m^2\Big) - \frac{z''}{z}\right]v = 0$$

where $\dfrac{z''}{z} = \dfrac{2}{\eta^2} + \dfrac{5\mathscr{A}_2}{2f_2\mathscr{A}h_{\rm dS}^2\eta^4(\tilde{k}^2 + \alpha k_{\rm IR}^2)}\alpha$

- We can write the equation for v as

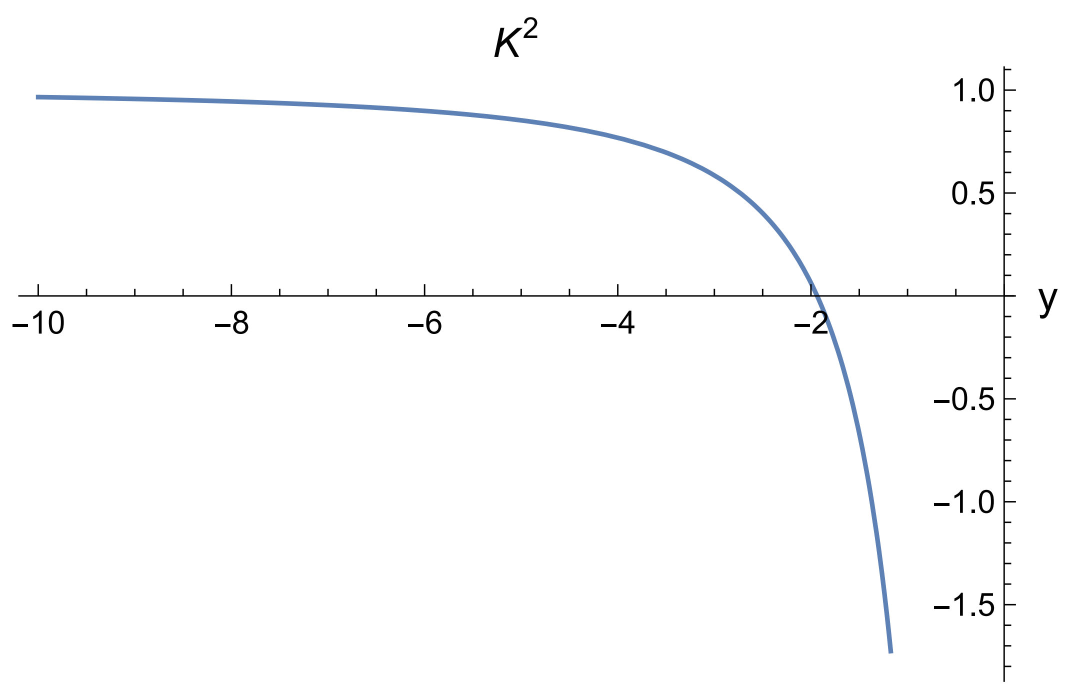
$$v''(y) + K^2(y)v(y) = 0$$

where

$$K^2(y) = \bar{c}_s^2 - \frac{2}{y^2} + \alpha\left[-\frac{b_1}{(y^4 + \alpha c_1 y^2)} + \frac{d_1}{y^2} + e_1 + f_1 y^2\right]$$

where b_1, c_1, d_1, e_1 and f_1 are constants.

- The equation $K^2(y)=0$ has a unique root in $(-\infty, 0)$

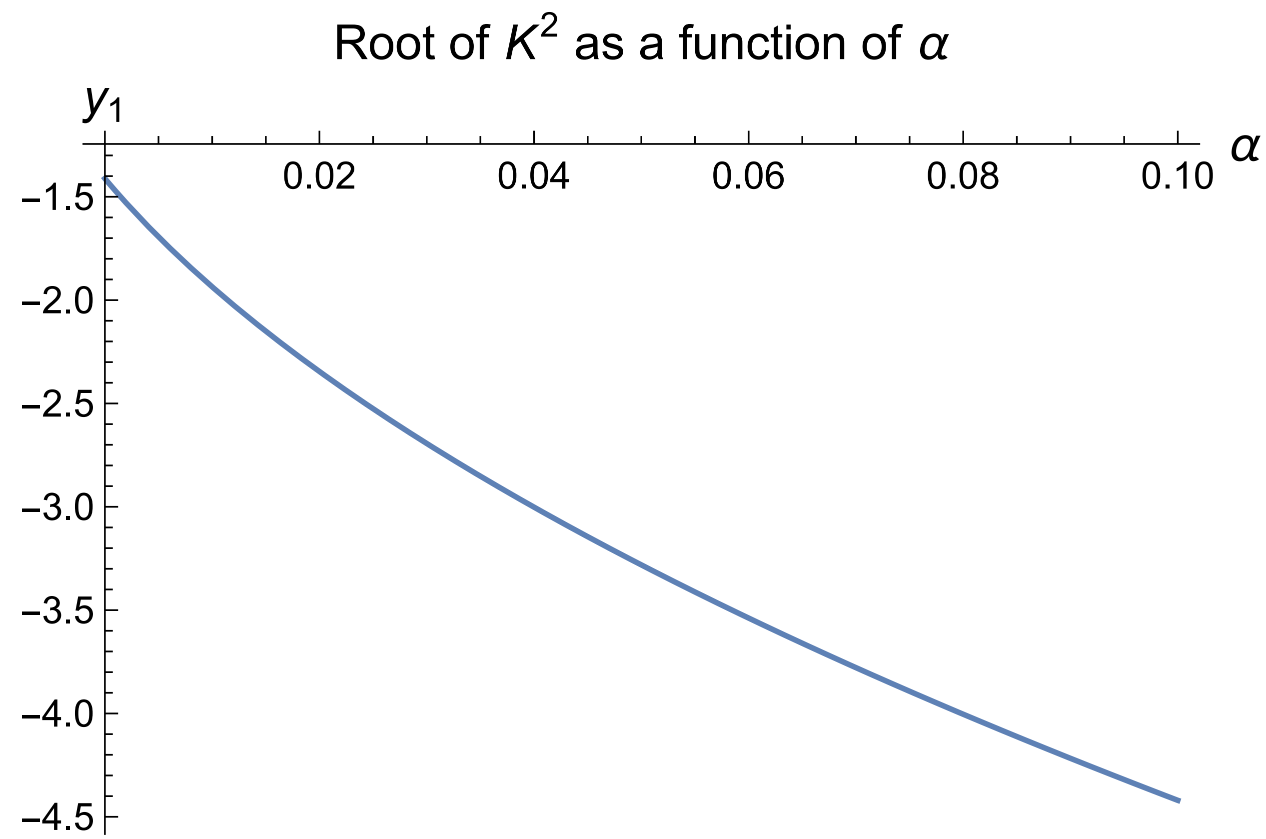


- In the absence of scordatura, there is an analytical solution,

$$v(\tilde{k}, y) = \frac{1}{\sqrt{2\bar{c}_s\tilde{k}}}\left(1 - \frac{i}{\bar{c}_s y}\right)\exp(-i\bar{c}_s y)$$

$$z^2(\tilde{k}, y) = \frac{6\tilde{k}^2 f_2}{h_{\mathrm{dS}}^2 y^2}\left(1 - \frac{\beta_K}{(1+\alpha_B)^2}\right)$$

- For $\alpha \neq 0$, we apply the improved WKB method [Weinberg]



- We define $\tilde{\phi}(y) = \int_y^{y_1} K(y')dy'$.
- Around the root y_1 of K^2, we can approximate the function as

$$K(y) = \beta_E \sqrt{y_1 - y}$$

where $\beta_E = \sqrt{-(K^2)'(y_1)}$

- Expansion is valid in interval, $y_1 - \delta_E \lesssim y \leq y_1$, where $\delta_E = \left| \frac{2(K^2)'(y_1)}{(K^2)''(y_1)} \right|$.
- In this interval $\tilde{\phi}(y) \simeq \frac{2\beta_E}{3}(y_1 - y)^{3/2}$.

$$\frac{d^2 v}{d\tilde{\phi}^2} + \frac{1}{3\tilde{\phi}}\frac{dv}{d\tilde{\phi}} + v = 0$$

$$v \propto A_1 \tilde{\phi}^{1/3} H_{1/3}^{(1)}(\tilde{\phi}) + A_2 \tilde{\phi}^{1/3} H_{1/3}^{(2)}(\tilde{\phi})$$

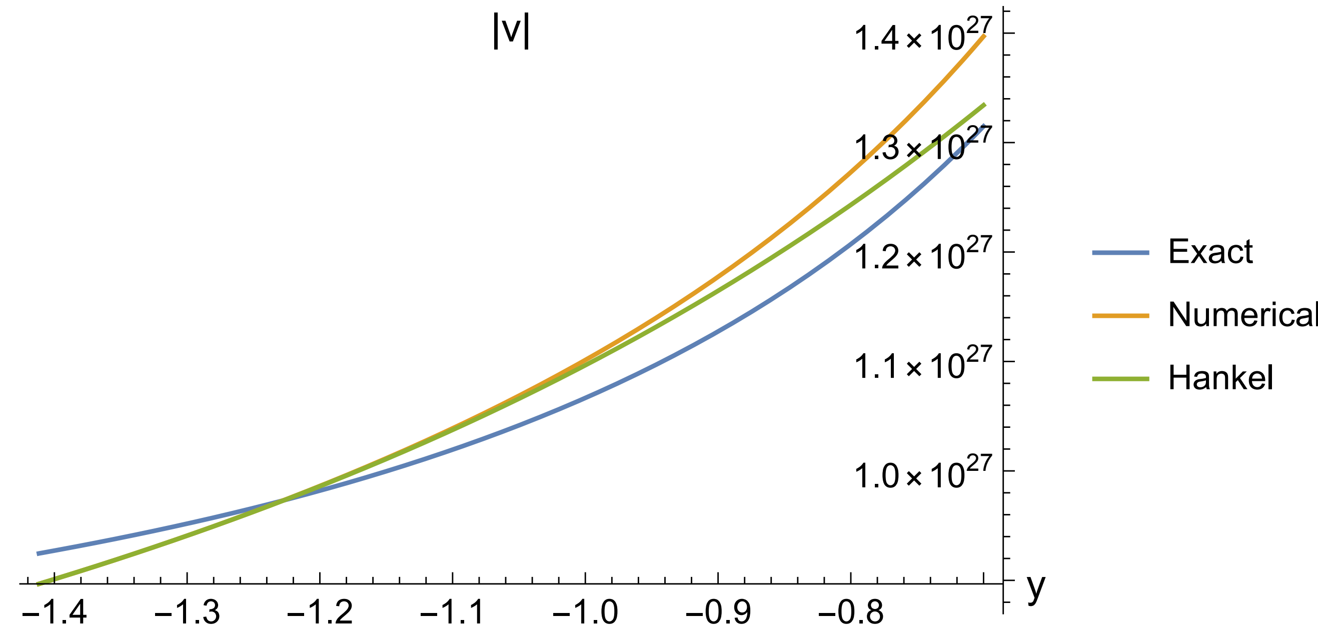
- Around $y_1 - \delta_E$ we can use the WKB approximation to find $v_{\mathrm{WKB}\pm} \propto \frac{1}{\sqrt{K(y)}} \exp(\pm i\tilde{\phi})$
Valid when $\left|K''/K'\right| \ll K, \; \left|K'/K\right| \ll K$
- From the canonical quantisation condition,

$$v_{\mathrm{WKB}} = \frac{1}{\sqrt{2\tilde{k}}} \frac{1}{\sqrt{K(y)}} \exp(i\tilde{\phi})$$

- By matching the two approximations, we find

$$v(y) = \frac{\sqrt{\pi}}{2\sqrt{\tilde{k}}} \left(\frac{2}{-3s_1}\right)^{1/6} \exp\left(\frac{5\pi}{12}i\right) \tilde{\phi}^{1/3}(y) H^{(1)}_{1/3}(\tilde{\phi}(y))$$

where $s_1 = \left.\frac{d(K^2(y))}{dy}\right|_{y=y_1}$.



- The power spectrum and is given by

$$\mathscr{P}_\zeta(\tilde{k}, y_1) = \frac{\tilde{k}^3}{2\pi^2}\left|\frac{v(y_1)}{z(\eta_1)}\right|^2 = \frac{y_1^2}{4\pi^3}\frac{\left(\frac{1}{3s_1}\right)^{1/3}\Gamma(1/3)^2}{\frac{1}{h_{\rm dS}^2} f_2 \mathscr{A}\left(1 + \frac{\alpha}{2f_2}\left(\frac{\mathscr{A}_1}{\bar{\mathscr{A}}} + \frac{1}{h_{\rm dS}^2 y_1^2}\frac{1}{1 + \alpha\frac{27\beta_K}{8f_2(1+\alpha_B)^2}}\frac{\mathscr{A}_2}{\bar{\mathscr{A}}}\right)\right)}$$

Independent of scale!

Shift-symmetry breaking perturbations

Models

- We start with the DHOST background and we consider a perturbation of the form

$$S_{\rm V} = -\int d^4x\sqrt{-g}\mu^4\left(\cos\frac{\phi}{f} - 1\right)$$

- If $\phi \ll f$, we can expand this action as

$$S_{\rm V} = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm phys}^2}{2}\phi^2 - \frac{\lambda_{\rm phys}}{4!}\phi^4\right]$$

where

$$m_{\rm phys}^2 = -\frac{\mu^4}{f^2} < 0,\ \lambda_{\rm phys} = \frac{\mu^4}{f^4}$$

- In reduced units we have

$$S_{\rm V} = \int d^4\tilde{x}\sqrt{-\tilde{g}}\left[-\frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4\right]$$

- Also

$$f = \frac{\sqrt{|m^2|}}{\sqrt{\lambda}}M, \mu = \frac{\sqrt{|m^2|}}{\lambda^{1/4}}\Lambda$$

and we have the constraint $\varphi \lesssim \dfrac{\sqrt{|m^2|}}{\sqrt{\lambda}}$

- We proceed as in the first part, to get a second order equation for

$$v'' + K^2(y,k)v = 0$$

- K^2 and z have complicated expressions (written in the paper).
- We still have $m^2, \lambda \ll 1$ and hence the turning point of K^2 still exists and we can use the same methods as before, and in particular the asymptotic matching.
- The power spectrum and its first three derivatives are

$$\mathscr{P}_\zeta(\tilde{k}, y_H) = \frac{\tilde{k}^3}{2\pi^2}\left|\frac{v(\tilde{k}, y_H, m^2, \lambda)}{z(\tilde{k}, y_H, m^2, \lambda)}\right|^2$$

$$n_s(\tilde{k}, y_H) = 1 + \frac{d\log(\mathscr{P}_\zeta(\tilde{k}, y_H))}{d\log(\tilde{k})}$$

$$\alpha_s(\tilde{k}, y_H) = \frac{dn_s(\tilde{k}, y_H)}{d\log(\tilde{k})}$$

$$\beta_s(\tilde{k}, y_H) = \frac{d\alpha_s(\tilde{k}, y_H)}{d\log(\tilde{k})}$$

Where y_H is the horizon position y_H=-1

Tensor perturbations

- In a similar fashion to scalars, we investigate the tensor perturbations in these models. We write the second order tensor perturbation action

$$S_2^{\text{tensor}} = \int d\eta d^3k \left[a^2 f_2 E'_{ij} E^{ij'} - a^2 f_2 k^2 E_{ij} E^{ij} + \frac{1}{24} E_{ij} E^{ij} a^4 (12m^2\varphi^2 + \lambda\varphi^4) \right]$$

- Using $\mu_T = z_T E$ and $z_T^2 = a^2 f_2$, we get the equation of motion for μ_T

$$\mu_T'' + \left[\tilde{k}^2 - \frac{1}{24 f_2} \frac{1}{h_{\text{ds}}^2 \eta^2} \left(12m^2 (c + \frac{1}{h_{\text{ds}}} \log(-h_{\text{ds}}\eta))^2 + \lambda (c + \frac{1}{h_{\text{ds}}} \log(-h_{\text{ds}}\eta))^4 \right) - \frac{2}{\eta^2} \right] \mu_T = 0$$

solved by

$$\mu_T(\tilde{k}, y) = \frac{1}{\sqrt{2\tilde{k}}} \left(1 - \frac{i}{y} \right) \exp(-iy)$$

$$z_T(\tilde{k}, y) = \frac{\tilde{k}^2 f_2}{h_{\text{ds}}^2 y^2}$$

- Hence the tensor power spectrum and the tensor to scalar ratio become (for the DHOST case)

$$\mathscr{P}_T(\tilde{k}, y) = \frac{\tilde{k}^3}{2\pi^2} \left| \frac{\mu_T}{z_T} \right|^2 = \frac{h_{\text{ds}}^2 y^2 \left(1 + \frac{1}{y^2} \right)}{4\pi^2 f_2}$$

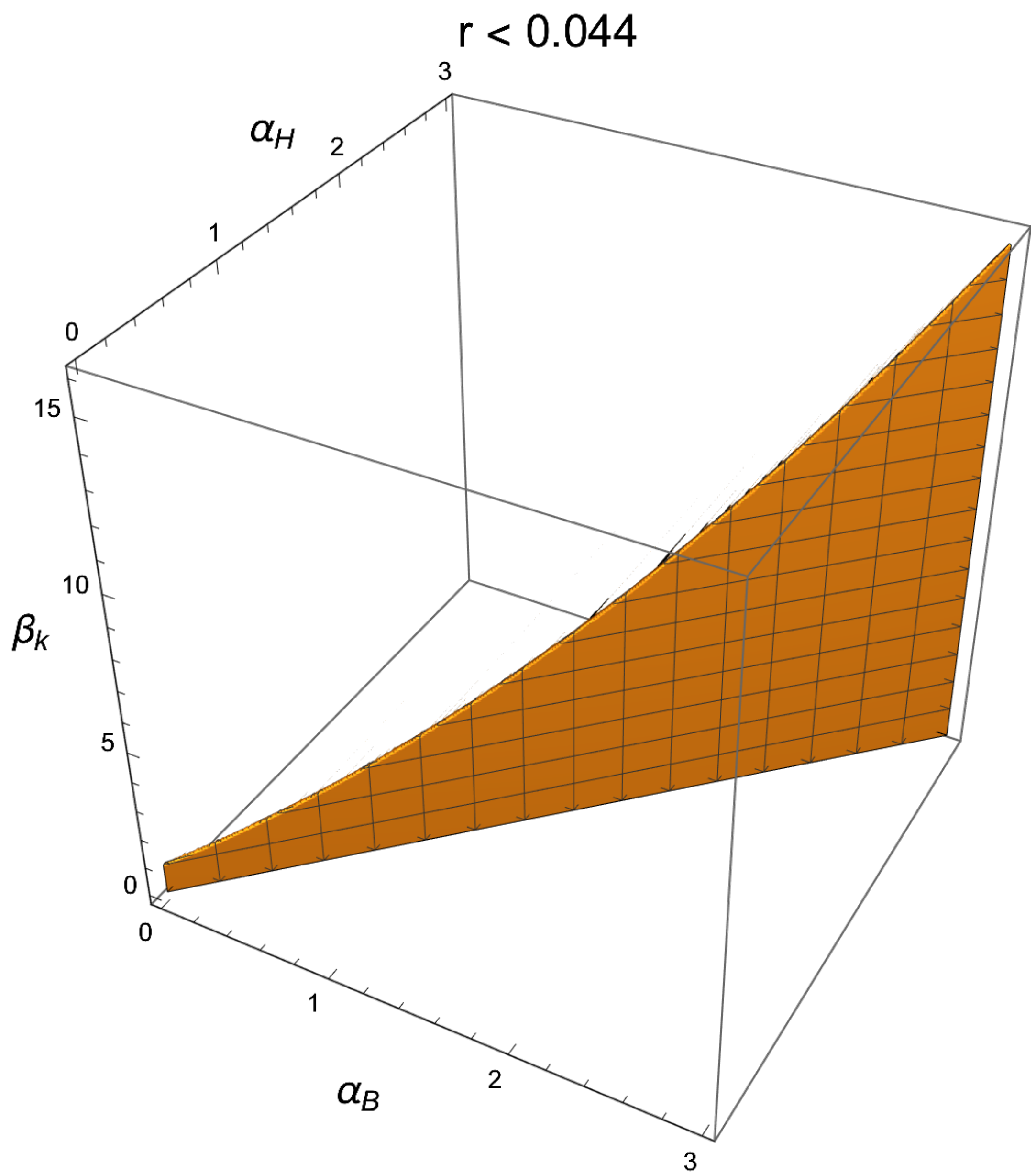
$$r_{\text{DHOST}}(\tilde{k}, y) = \frac{\mathscr{P}_T(\tilde{k}, y)}{\mathscr{P}_\zeta(\tilde{k}, y)} = 6\bar{c}_s \frac{1 + \frac{1}{y^2}}{1 + \frac{1}{\bar{c}_s^2 y^2}} \left(1 - \frac{\beta_K}{(1 + \alpha_B)^2} \right)$$

Numerical results

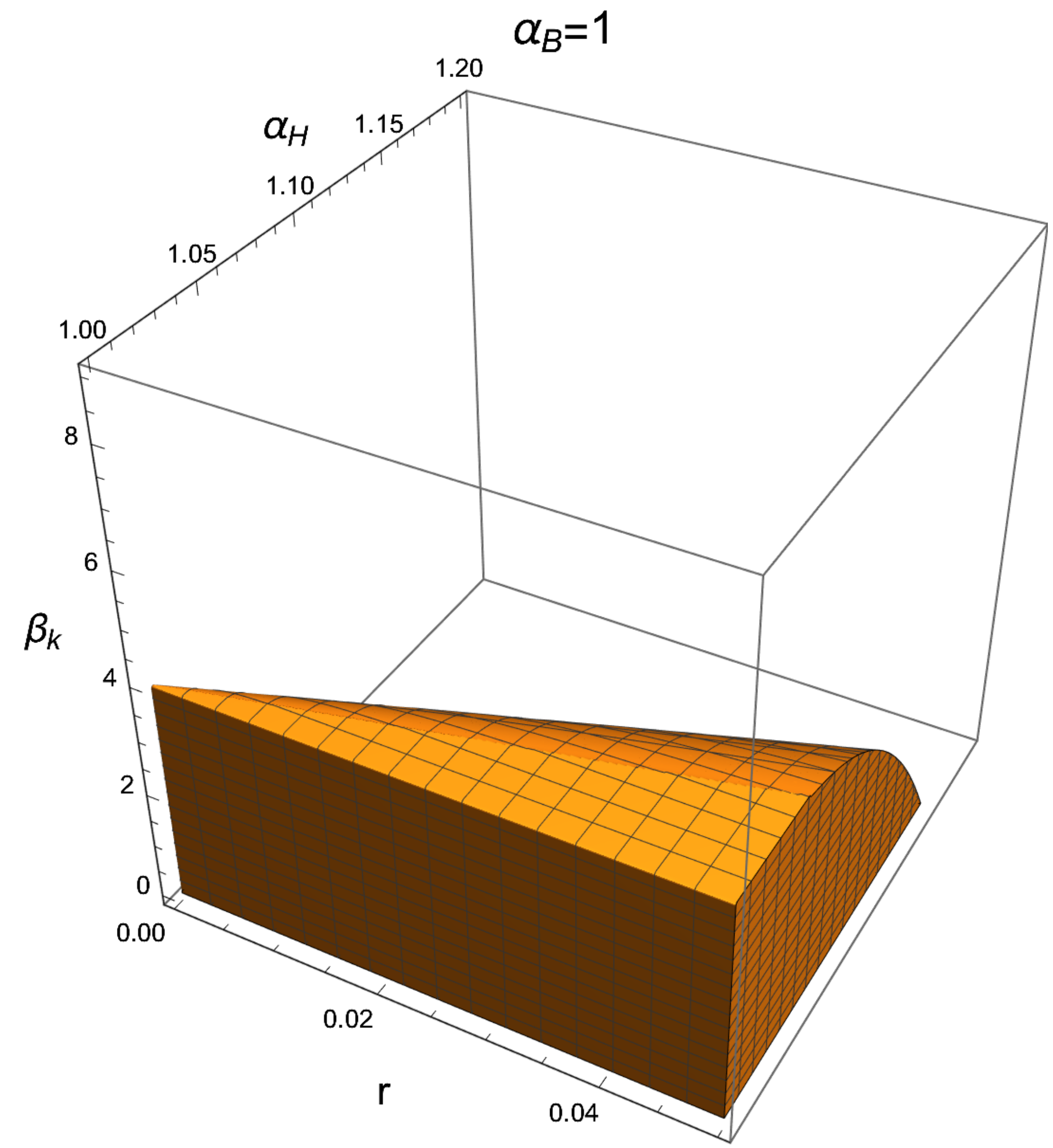
- Planck constraints on inflation (Planck 2018 data release)

$$\begin{aligned} n_s &= 0.9625 \pm 0.0048\,, \\ \alpha_s &= 0.002 \pm 0.010\,, \\ \beta_s &= 0.010 \pm 0.013\,, \\ \ln(10^{10} A_s) &= 3.044 \pm 0.014 \end{aligned}$$

  at $k_* = 0.05\,\mathrm{Mpc}^{-1}$.
- Planck + Bicep2/Keck constrained r < 0.044.
- In the future, LiteBird could lower this to 10^{-3}
- Observable scales correspond to $10^{-4}\ \mathrm{Mpc}^{-1} \lesssim k \lesssim 10^{-1}\ \mathrm{Mpc}^{-1}$.
- We fix $\Lambda = m_{\mathrm{Pl}}$ and hence $\tilde{k}_* = 2.62 \times 10^{-59}/h_{\mathrm{ds}}$.



Parameter space where the Planck CMB constraints are satisfied



Plot showing how the allowed space is narrowing as r is decreased

Model 1: r ~ 0.04

- Parameters

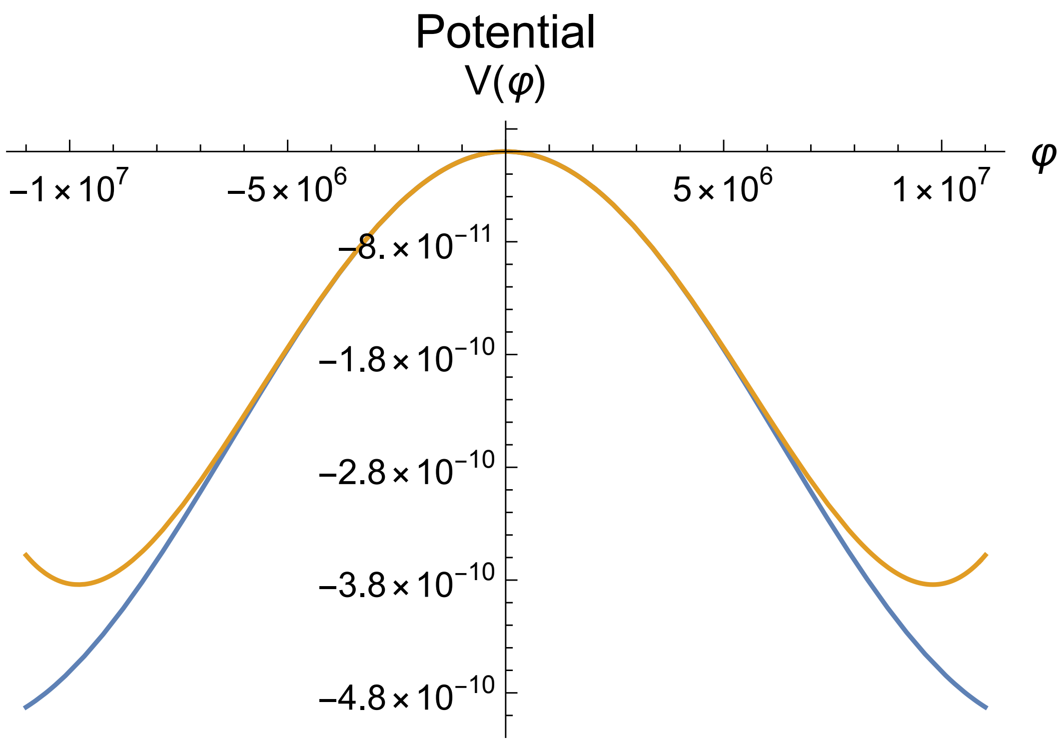
  $\alpha_B = 1, \alpha_H = 1.04, \beta_K = 3.97343$

  $r_{\mathrm{DHOST}} = 0.04$, $\bar{c}_s = 1.002$

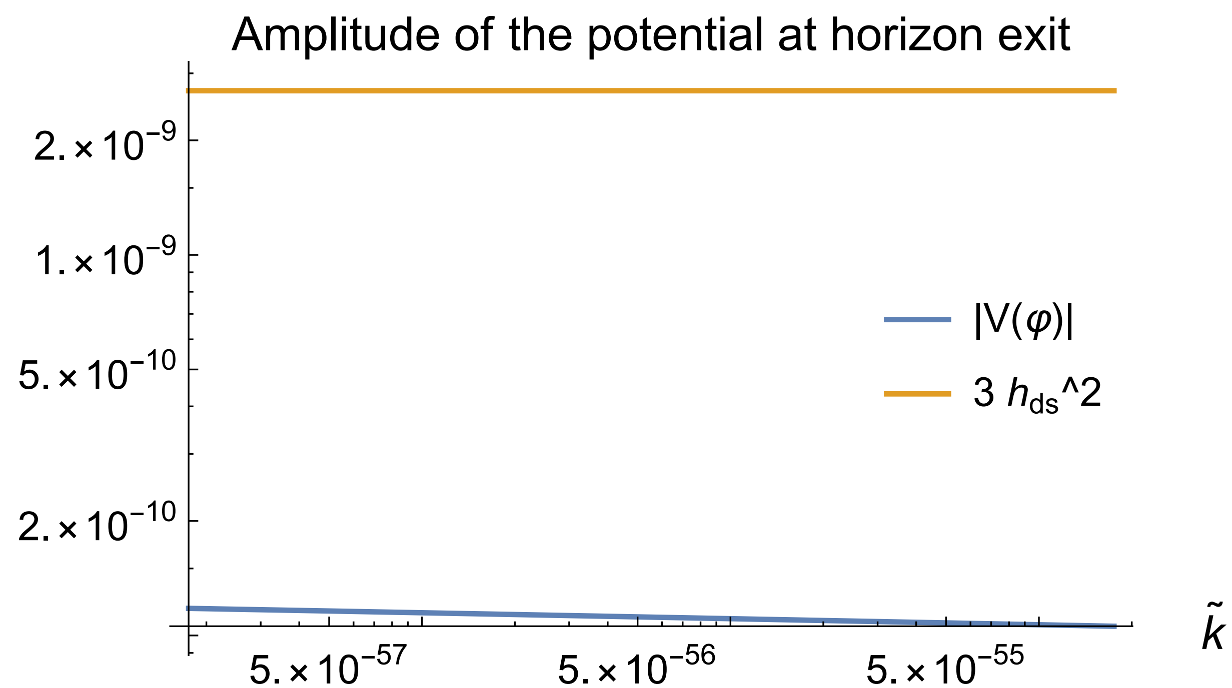
$$f_{2,\mathrm{x}} = 2.81, \qquad f_{1,\mathrm{x}} = -6.48 \times 10^{-6}, \qquad f_{0,\mathrm{x}} = -2.97 \times 10^{-8}$$

$$f_{2,\mathrm{xx}} = 2.7\beta_H, \qquad f_{1,\mathrm{xx}} = -8.1 \times 10^{-5}\beta_B, \qquad f_{0,\mathrm{xx}} = -2.97 \times 10^{-8}\left(\beta_B - 4\beta_H - 4.133\right)$$

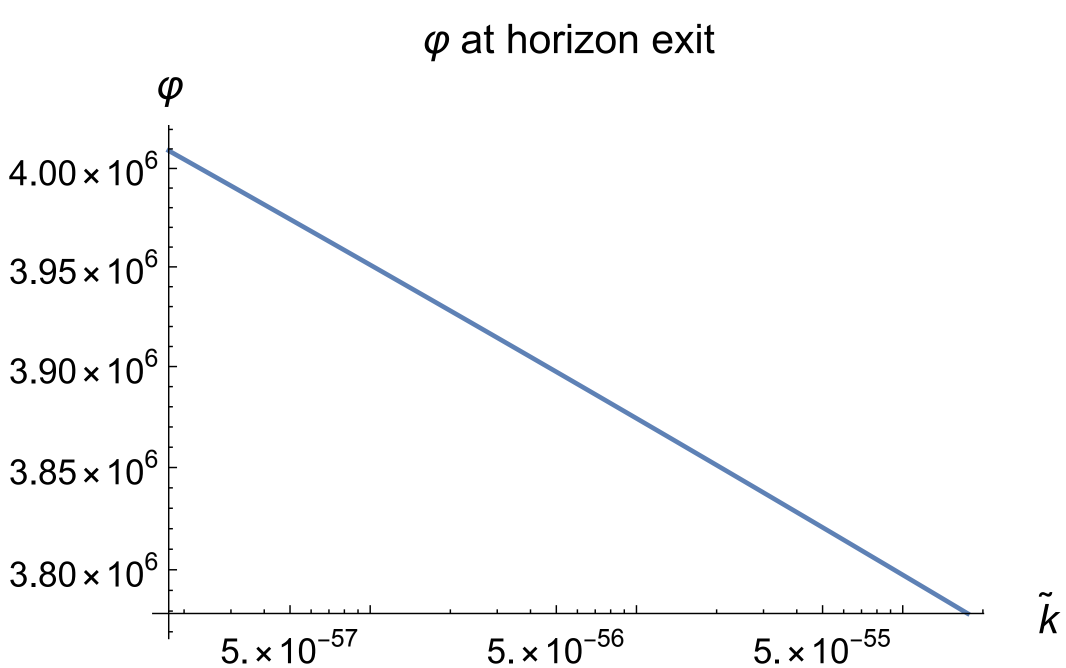
- $h_{\mathrm{dS}} = 3 \times 10^{-5}, f_2 = 2.7$
- Perturbations fixed at $m^2 = -1.6 \times 10^{-23}$ and $\lambda = 10^{-36}$.
- Parameters for the potential $f/M = 4 \times 10^6, \quad \mu/\Lambda = 0.004$.
- Results for inflation:
  - $A_s = 2.04 \times 10^{-9}$
  - $n_s = 0.966$
  - $\alpha_s = 0.00059$
  - $\beta_s = 0.000019$
  - $r = 0.0074$



Potential for the model considered

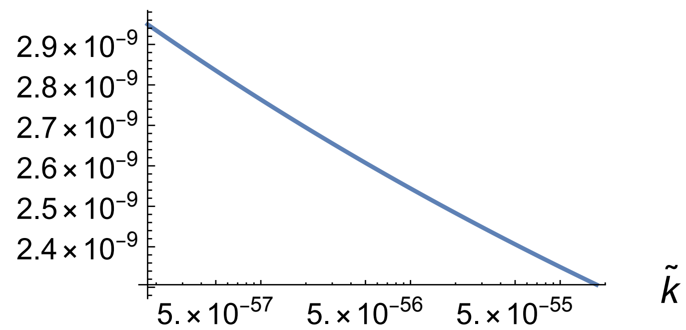


Amplitude of the potential at horizon exit is negligible compared to the background

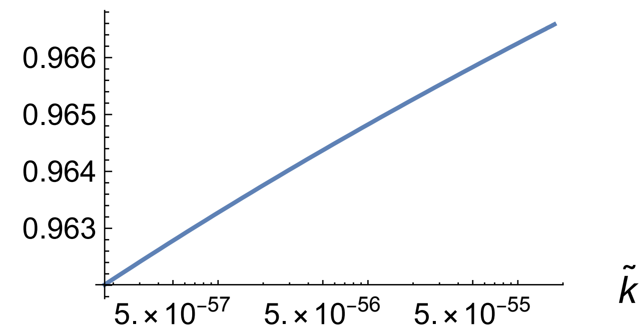


The scalar field at horizon exit showing that the condition of small perturbations is satisfied

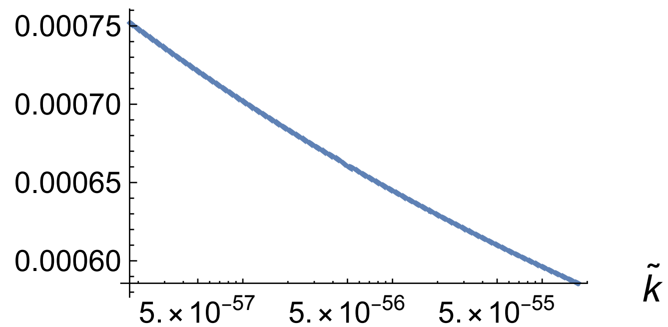
Scalar power spectrum



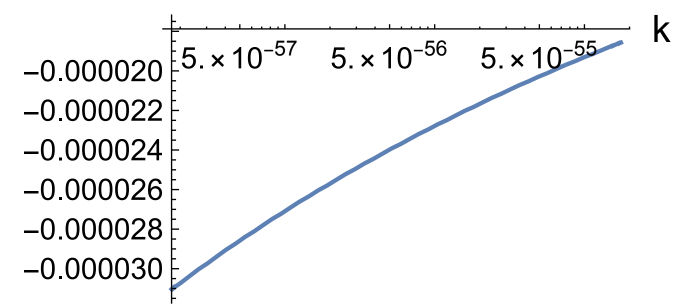
n_s



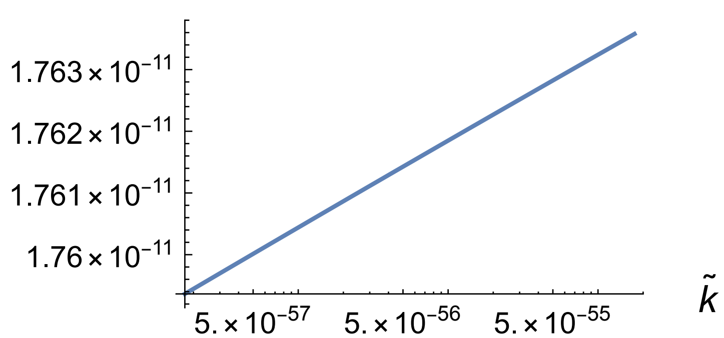
α_s



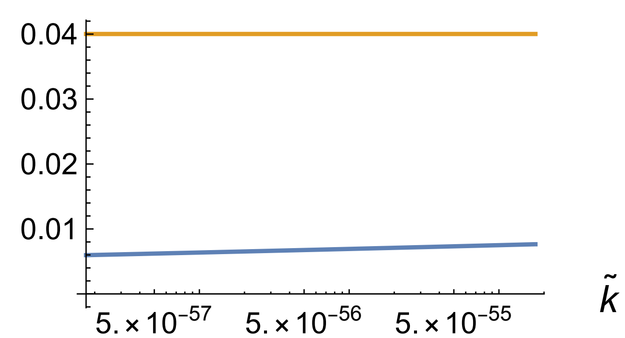
β_s



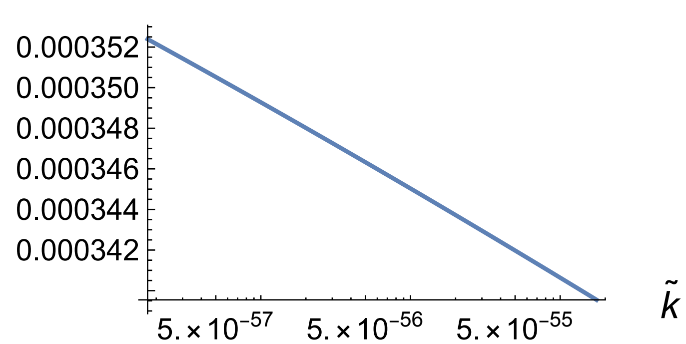
Tensor power spectrum



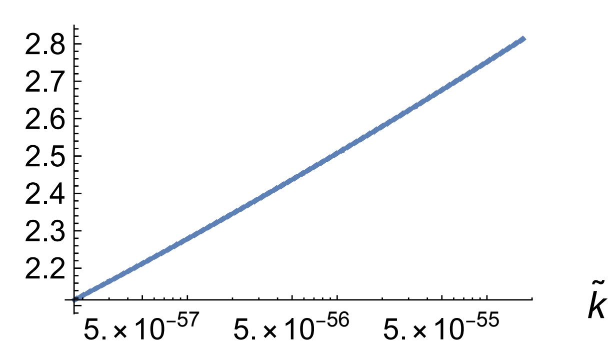
r



n_T



$r/(8\,|n_T|)$

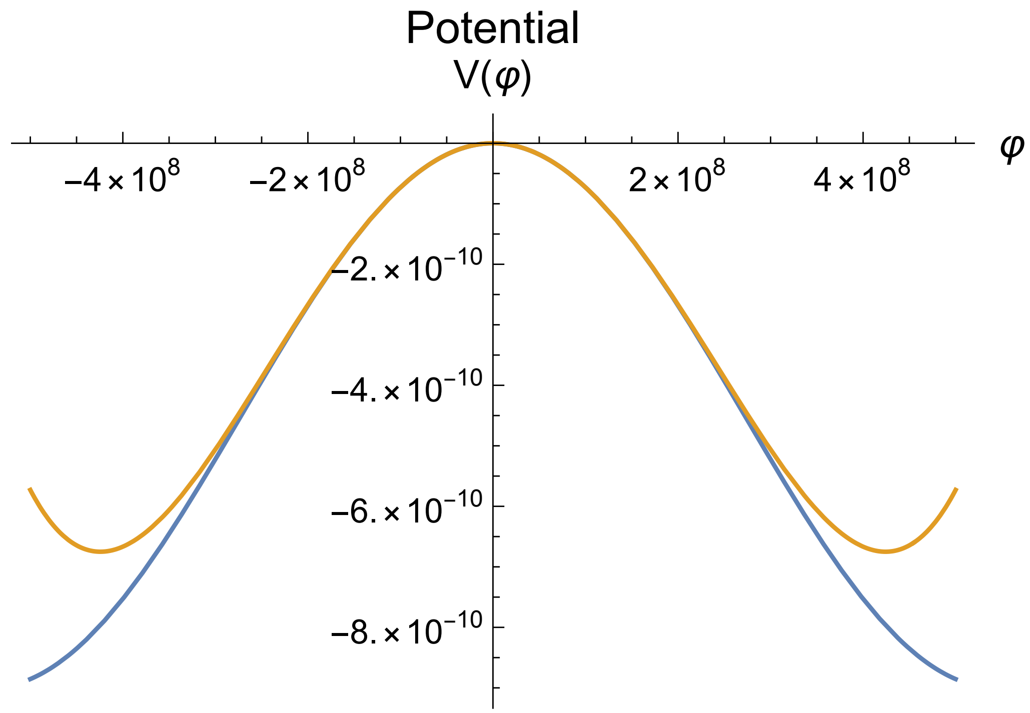


Model 2: r ~ 0.001

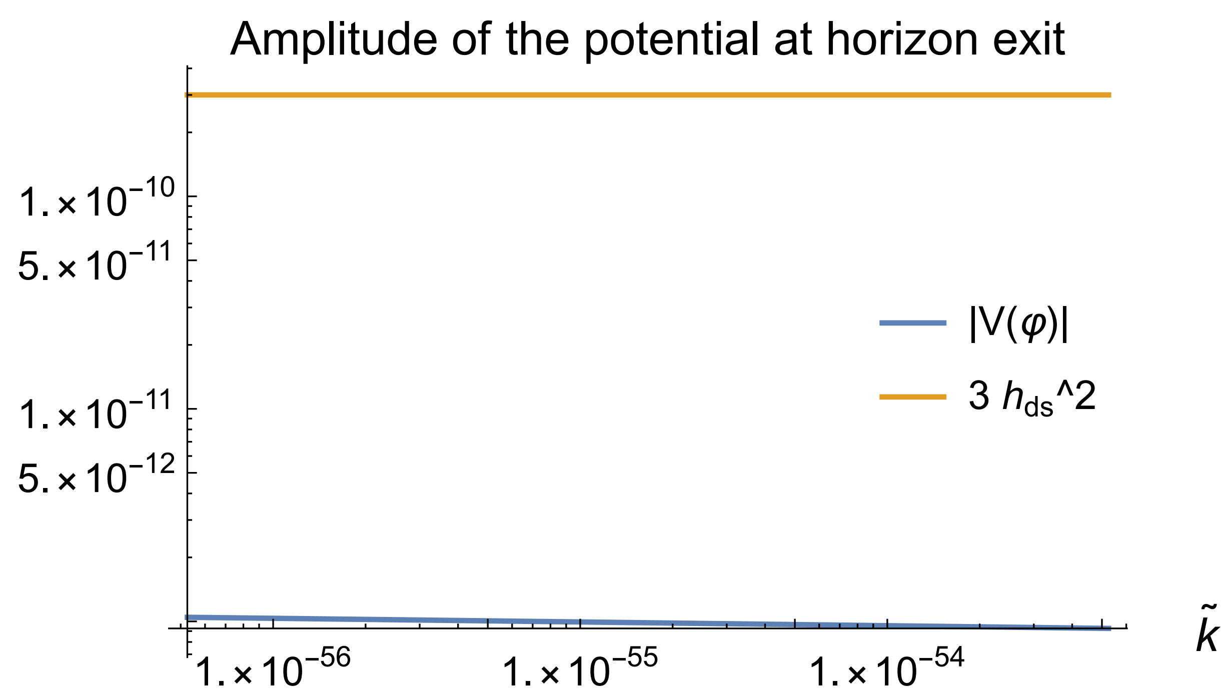
- Parameters
  $\alpha_B = 1, \alpha_H = 1.001, \beta_K = 3.9993$
  $r_{\rm DHOST} = 10^{-3}$, $\bar{c}_s = 0.976$

$$f_{2,\rm x} = 8.809, \qquad f_{1,\rm x} = -1.76 \times 10^{-7}, \qquad f_{0,\rm x} = -1.05 \times 10^{-9}$$
$$f_{2,\rm xx} = 8.8\beta_H, \qquad f_{1,\rm xx} = -0.000088\beta_B, \qquad f_{0,\rm xx} = 2.64 \times 10^{-9}\left(\beta_B - 4\beta_H - 4.0033\right)$$

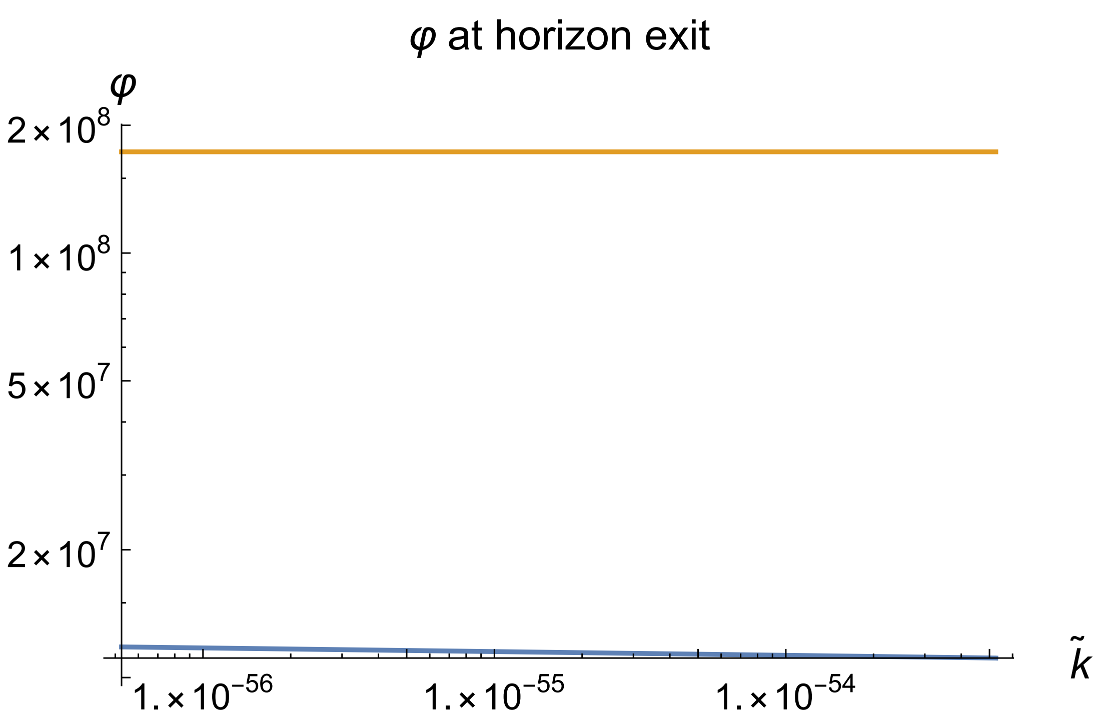
- $h_{\rm dS} = 10^{-5}, f_2 = 8.8$
- Perturbations fixed at $m^2 = -1.5 \times 10^{-26}$ and $\lambda = 5 \times 10^{-43}$.
- Parameters for the potential $f/M = 1.73 \times 10^8, \quad \mu/\Lambda = 0.0046$.
- Results for inflation:
  - $A_s = 2.76 \times 10^{-9}$
  - $n_s = 0.96716$
  - $\alpha_s = 0.00065$
  - $\beta_s = -0.000022$
  - $r = 3.9 \times 10^{-4}$



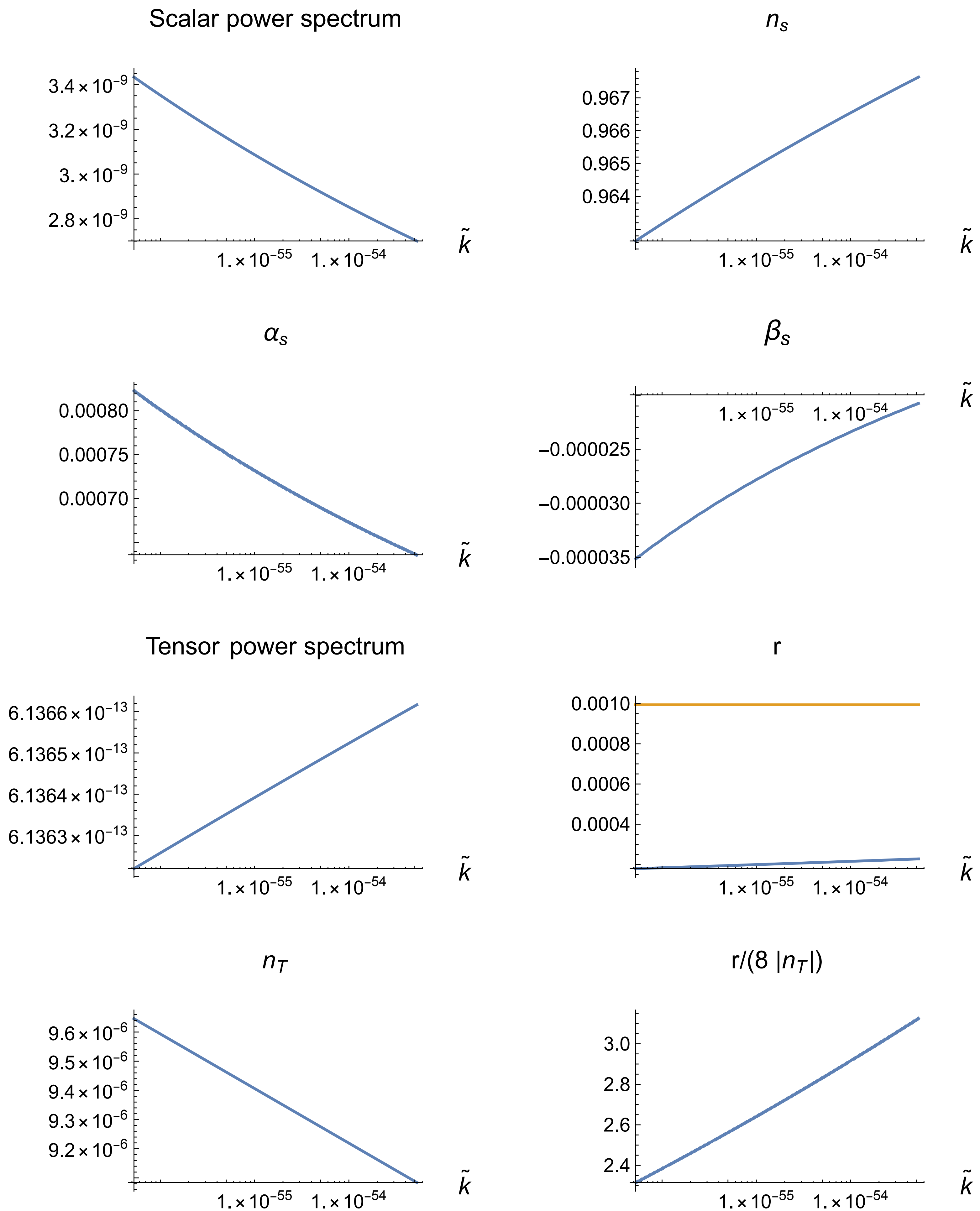
Potential for the model considered



Amplitude of the potential at horizon exit is negligible compared to the background



The scalar field at horizon exit showing that the condition of small perturbations is satisfied


Scalar power spectrum
3.4 × 10^-9
3.2 × 10^-9
3. × 10^-9
2.8 × 10^-9
n_s
0.967
0.966
0.965
0.964
α_s
0.00080
0.00075
0.00070
β_s
−0.000025
−0.000030
−0.000035
Tensor power spectrum
6.1366 × 10^-13
6.1365 × 10^-13
6.1364 × 10^-13
6.1363 × 10^-13
r
0.0010
0.0008
0.0006
0.0004
n_T
9.6 × 10^-6
9.5 × 10^-6
9.4 × 10^-6
9.3 × 10^-6
9.2 × 10^-6
$r/(8\,|n_T|)$
3.0
2.8
2.6
2.4
1. × 10^-55
1. × 10^-54
$\tilde{k}$

Non-Gaussianities

- In order to make an estimate of the level of non-Gaussianities produced by these models, we need to perturb the action to third order, where the most general action can be expressed as

$$S_3 = \int d\eta (\prod_{i=1}^{3} d^3\tilde{k}_i)\delta(\vec{\tilde{k}}_1 + \vec{\tilde{k}}_2 + \vec{\tilde{k}}_3)a^2(C_0\zeta(\tilde{k}_1)\zeta(\tilde{k}_2)\zeta(\tilde{k}_3) + C_1\zeta'(\tilde{k}_1)\zeta(\tilde{k}_2)\zeta(\tilde{k}_3)$$
$$+C_2\zeta'(\tilde{k}_1)\zeta'(\tilde{k}_2)\zeta(\tilde{k}_3) + C_3\zeta'(\tilde{k}_1)\zeta'(\tilde{k}_2)\zeta'(\tilde{k}_3))$$

- We aim to calculate

$$\langle 0 | \zeta(\tilde{k}_1)\zeta(\tilde{k}_2)\zeta(\tilde{k}_3) | 0 \rangle = -i \int d\eta \langle 0 | [\zeta(\tilde{k}_1)\zeta(\tilde{k}_2)\zeta(\tilde{k}_3), H_3] | 0 \rangle$$

Where H_3 is the interaction picture Hamiltonian given by

$$H_3 = -\int (\prod_{i=1}^{3} d^3\tilde{k}_i)\delta(\vec{\tilde{k}}_1 + \vec{\tilde{k}}_2 + \vec{\tilde{k}}_3)a^2(C_0\zeta(\tilde{k}_1)\zeta(\tilde{k}_2)\zeta(\tilde{k}_3) + C_1\zeta'(\tilde{k}_1)\zeta(\tilde{k}_2)\zeta(\tilde{k}_3)$$
$$+C_2\zeta'(\tilde{k}_1)\zeta'(\tilde{k}_2)\zeta(\tilde{k}_3) + C_3\zeta'(\tilde{k}_1)\zeta'(\tilde{k}_2)\zeta'(\tilde{k}_3))$$

- Full bispectrum can thus be determined for each of the 4 terms, e.g.

$$B_0(k_1, k_2, k_3, \eta_f) = -\,\mathrm{Re}\left[-2i\int_{-\infty(1-i\epsilon)}^{\eta_f} d\eta a C_0 u(k_1, \eta_f)u(k_2, \eta_f)u(k_3, \eta_f)u^*(k_1, \eta)u^*(k_2, \eta)u^*(k_3, \eta)\right] + 5\,\mathrm{perm}\,.$$

$$\eta_f = -\frac{1}{c_s \max(k_1, k_2, k_3)}$$

- We have 6 additional parameters which are not fixed: β_B, β_H, f_1, $f_{0,\mathrm{xxx}}$, $f_{1,\mathrm{xxx}}$, $f_{2,\mathrm{xxx}}$.
- Standard PNG shapes

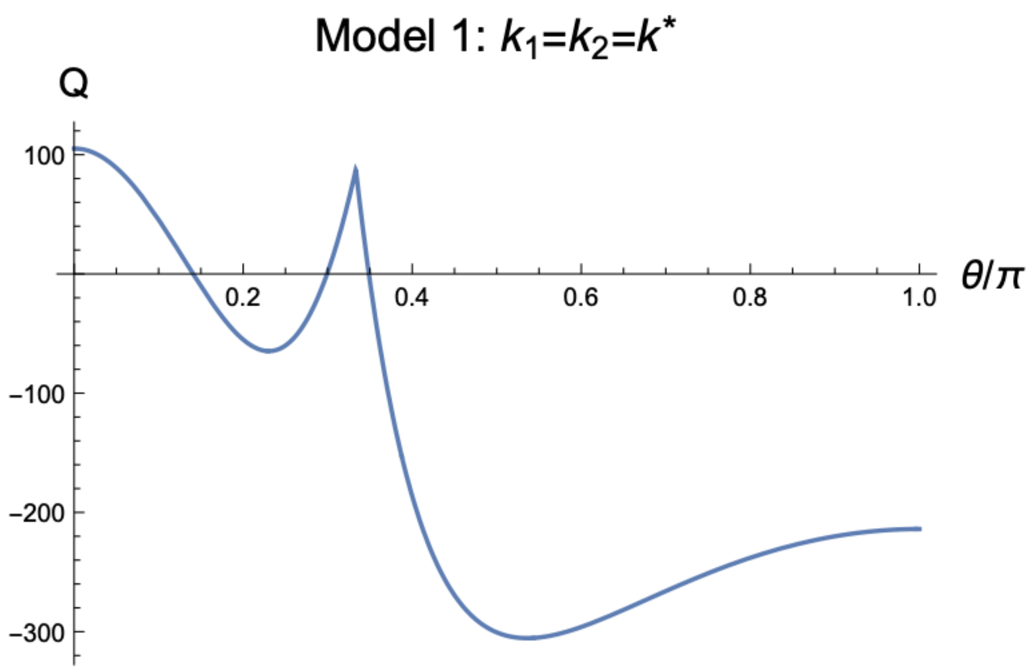
$$
\begin{aligned}
B_\Phi^{\mathrm{loc}}(k_1,k_2,k_3) &= 2\left[P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perms}\right] , \\
B_\Phi^{\mathrm{eq}}(k_1,k_2,k_3) &= 6\left\{-[P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perms}]\right. \\
&\quad \left. - 2[P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)]^{2/3} + [P_\Phi^{1/3}(k_1)P_\Phi^{2/3}(k_2)P_\Phi(k_3) + 5 \text{ perms}]\right\} \\
B_\Phi^{\mathrm{orth}}(k_1,k_2,k_3) &= 6\big[3(P_\Phi^{1/3}(k_1)P_\Phi^{2/3}(k_2)P_\Phi(k_3) + 5 \text{ perms}) \\
&\quad - 3\left[P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perms}\right] - 8(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3))^{2/3}\big] .
\end{aligned}
$$

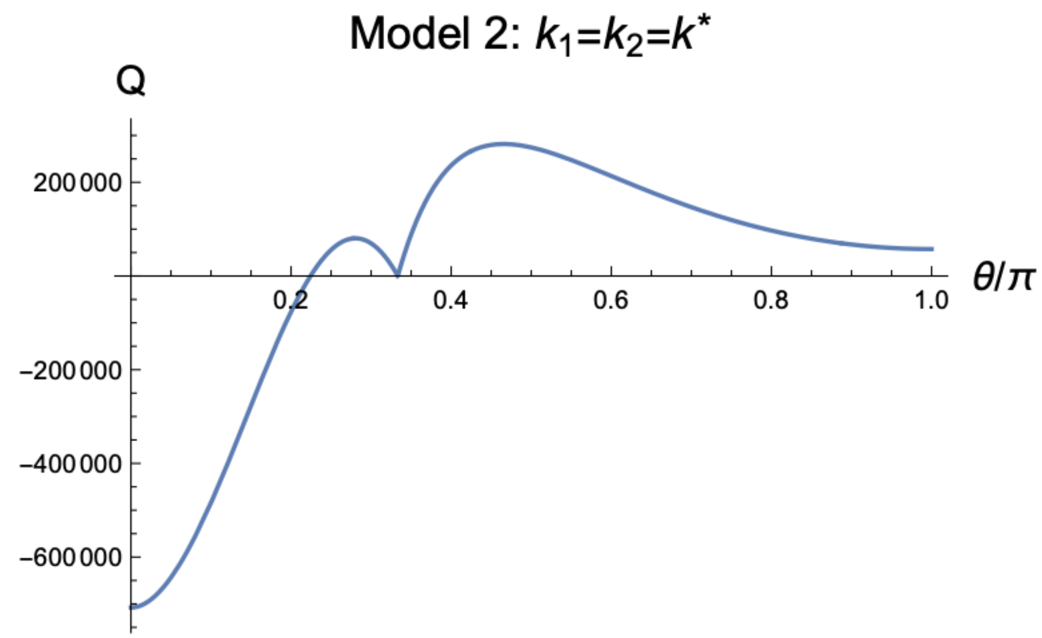
95% CL *Planck* constraints

$$
\begin{aligned}
-11.1 &< f_{\mathrm{NL}}^{\mathrm{local}} < 9.3 \\
-120 &< f_{\mathrm{NL}}^{\mathrm{equil}} < 68 \\
-86 &< f_{\mathrm{NL}}^{\mathrm{orth}} < 10
\end{aligned}
$$

- Shapes of DHOST bispectrum depend on the 6 parameters
- We can choose them such that the shape correlations between the DHOST bispectrum and all three standard shapes are small, and hence the *Planck* constraints are satisfied.
- However, the overall bispectrum remains large

- We plot the reduced bispectrum $Q(k_1, k_2, k_3) = \dfrac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$





for isosceles triangles with equal sides k*, in terms of the angle between them.

- Model 1 ($r = 0.04$): $\alpha_B = 1$, $\alpha_H = 1.04$ and $\beta_K = 3.97343$, $f_2 = 2.7$, $h_{ds} = 3 \times 10^{-5}$, $f_{0,xxx} = 2 \times 10^{-6}$, $f_{1,xxx} = -0.16$, $f_{2,xxx} = 150$, $f_1 = 0.0076$, $\beta_B = 0$, $\beta_H = 0$
- Model 2 ($r = 10^{-3}$): $\alpha_B = 1$, $\alpha_H = 1.001$, $\beta_K = 3.9993$, $h_{ds} = 10^{-5}$, $f_2 = 8.8$, $f_{0,xxx} = 7 \times 10^{-6}$, $f_{1,xxx} = 0.12$, $f_{2,xxx} = -2675$, $f_1 = -0.013$, $\beta_B = 0$, $\beta_H = 0.1$.
- Plots show that overall amplitude of the bispectrum is large - a careful comparison with data from *Planck* is required.

Field excursion

- Inflation must end !
- Number of e-foldings between the time when k* enters the horizon and the end of inflation

$$N_\star = \ln\left(\frac{a_{\rm end}H_{\rm end}}{k_\star}\right)$$

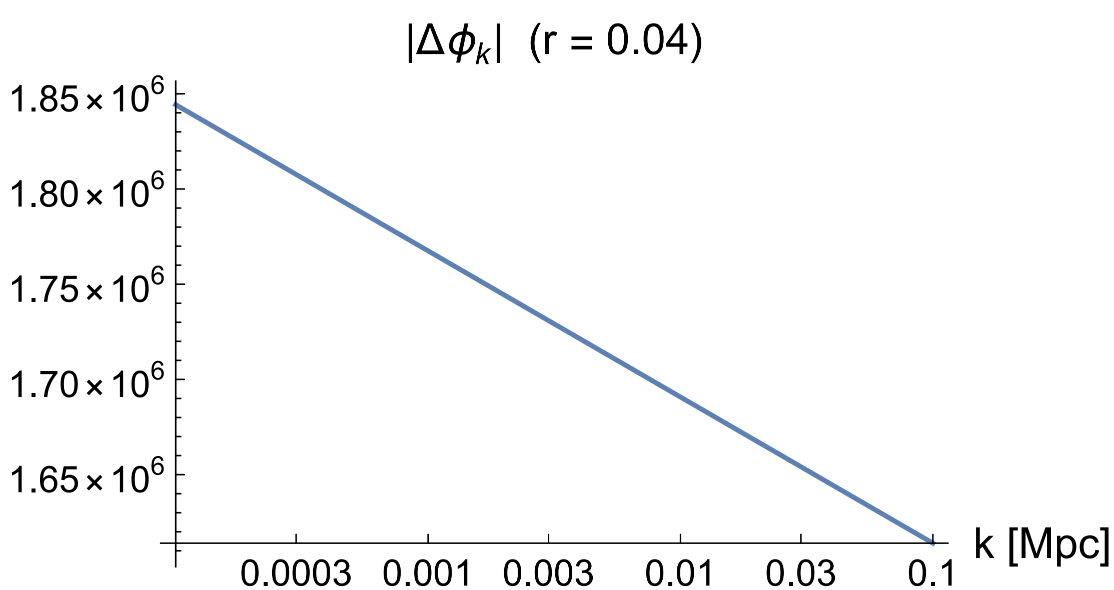
$$N_\star^{\rm model\,1} = 59.52$$
$$N_\star^{\rm model\,2} = 58.97$$

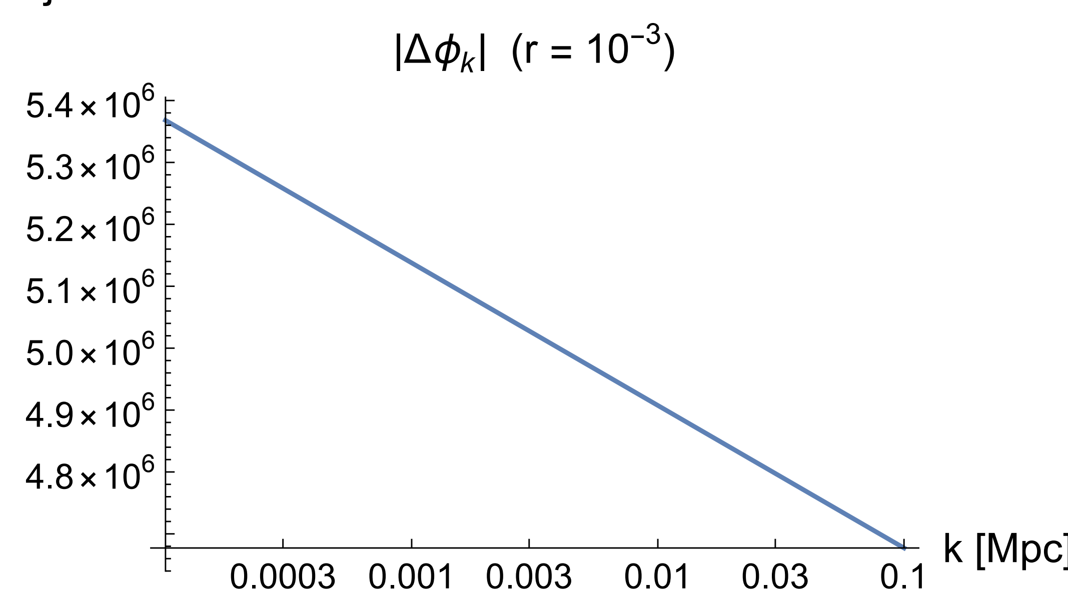
where $a_{\rm end} \simeq \left(\frac{H_0}{H_{end}}\right)^{1/2} = \left(\frac{H_0}{h_{\rm dS}m_{\rm Pl}}\right)^{1/2}$

- We define the **field excursion** as

$$\Delta\phi_k \equiv |\phi(t_{\rm end}) - \phi(t_k)|$$

- **Distance conjecture**: $\Delta\phi_k \ll l_{\rm Pl} = m_{\rm Pl}^{-1}$
- Distance conjecture is satisfied as long as $M \lesssim 10^{-6}m_{\rm Pl}$; as M is still a free parameter of the model, we can fix it such that the conjecture is satisfied





The trans-Planckian censorship conjecture

- **Trans-Planckian censorship conjecture**: the length scales observed today originate from modes that were larger than the Planck length during inflation
- In slow roll inflation, modes can become Trans-Planckinan unless all modes of length scale the Planck scale satisfy

$$\frac{a(t_{\text{end}})}{a_{\text{in}}} l_{\text{Pl}} < H^{-1}$$

- We satisfy the conjecture, we need

$$N_T = \ln \frac{a_{\text{end}}}{a_{\text{in}}} < -\ln(h_{\text{dS}})$$

- But $N_T^{\text{model 1}} < 10.41$

$$N_T^{\text{model 2}} < 11.51$$

- Hence, we would require a much lower number of e-foldings
- As there are no free parameters, h_{dS} is fixed, our models do not evade this issue.

Conclusions

- First look at DHOST theories in the early universe.
- Study of inflationary consequences of scordatura models, showing that they all produce scale invariant spectra in de Sitter universes.
- Analysis of shift-symmetry breaking perturbations to DHOST models.
- Built inflationary de Sitter models with $m^2\phi^2$ and $\lambda\phi^4$ interaction terms that yield nearly scale invariant power spectra.
- The parameters of these models can be tuned such that they are compatible with inflationary constraints on n_s, α_s and β_s and also to current and future constraints of the tensor-to-scalar ratio.
- The non-Gaussianities that they produce can be tuned to be small in the usual templates (local, equilateral and orthogonal), and they might be detected with *Planck* and future experiments.