The cosmological constant as a classical eigenvalue

Giovanni Manfredi

Centre National de la Recherche Scientifique

Institut de Physique et Chimie des Matériaux de Strasbourg

Motivation

- **Standard cosmological model: ΛCDM**
  - Concordant with observations, but…
  - Unobserved “dark” components make up ~95% of its content
  - Origin of dark energy (cosmological constant) is unclear
    - ➢ Linked to quantum vacuum energy?
    - ➢ Is it even a constant, or does it vary in space and time?
    - ➢ Is it a problem at all, or just one more gravitational constant on top of Newton’s *G*

- **Coasting cosmologies** (e.g. Dirac-Milne, see G. Chardin’s talk)
  - Unaccelerated expansion: $a(t) = t/t_0$
  - One gets almost everything out of the single parameter H_0

$$\begin{cases} t_0 = 1/H_0 \approx 14\,\mathrm{Gy}, \\ \Lambda = H_0^2/c^2 \approx 5 \times 10^{-53}\,\mathrm{m}^{-2}, \\ \rho_0 = H_0^2/(8\pi G) \approx 1.8\,\mathrm{protons/m^3}, \\ a_0 = cH_0 \approx 6.8 \times 10^{-10}\,\mathrm{ms}^{-2}, \end{cases}$$

$$H_0 = 70\,\mathrm{km\,s^{-1}/Mpc}$$

Milgrom’s acceleration

Left or right?

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Λ is a property of spacetime, which curves even in the absence of matter

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Λ is part of the energy-momentum tensor: it's a "substance" with peculiar properties

Λ as the eigenvalue of a (nonlinear) problem

$$\boxed{-G_{\mu\nu} + \frac{8\pi G}{c^4} T_{\mu\nu} = \Lambda g_{\mu\nu}}$$

$$\mathcal{G}(g_{\mu\nu}) = \Lambda g_{\mu\nu}$$

- No modifications of Einstein's field equations
  - No "new physics", only new interpretation
- **Eigenvalue is determined by boundary conditions** (as usual)

Scalar gravity model

$$\Delta\Phi = 4\pi G\rho,$$

Newtonian gravity

- We want to include self-gravity (as in GR)
- Energy density of the Newtonian gravitational field: $-|\nabla\Phi|^2/8\pi G,$
- Not enough to simply add it to Newton's equation
- Correct procedure yields:

$$\Delta\Phi = \frac{4\pi G}{c^2}\rho\Phi + \frac{|\nabla\Phi|^2}{2\Phi},$$

- With the cosmological constant included:

$$\boxed{-\Delta\Phi + \frac{4\pi G}{c^2}\rho_m\Phi + \frac{|\nabla\Phi|^2}{2\Phi} = \Lambda\,\Phi,}$$

- Already written as an eigenvalue problem.

Giulini, D.: *Consistently implementing the field self-energy in Newtonian gravity*. Phys. Lett. A **232**, 165 (1997)
Franklin, J.: *Self-consistent, self-coupled scalar gravity*. Am. J. Phys. **83**, 332 (2015)

Exact linearization

- The above scalar model can be linearized exactly by setting: $\Psi = \sqrt{\Phi}$,

$$-\Delta\Psi + \frac{2\pi G}{c^2}\rho_m \Psi = \frac{\Lambda}{2}\,\Psi.$$

- Identical to the stationary Schrödinger equation with eigenvalue Λ/2
- Just like in QM, the physically meaningful quantity is not Ψ, but rather $|\Psi|^2 = \Phi$
- First integral (energy):

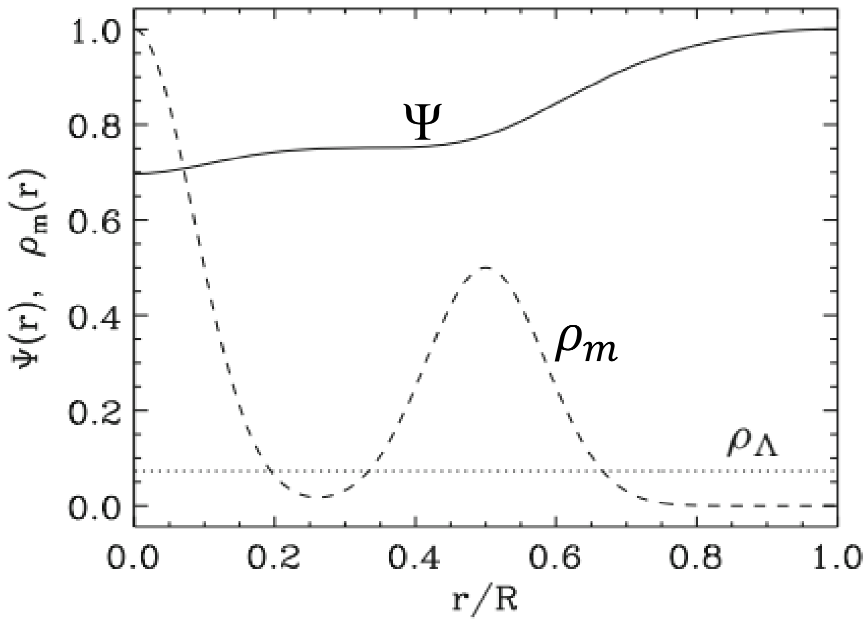
$$\frac{2\pi G}{c^2}\int_V \rho_m \Psi^2 d\mathbf{r} + \int_V |\nabla\Psi|^2 d\mathbf{r} - \cancel{\oint_S \Psi\nabla\Psi \cdot \mathbf{n}\, dS} = \frac{\Lambda}{2}\int_V \Psi^2 d\mathbf{r},$$

$$E_{matt} \quad + \quad E_{field} \quad = \quad E_{vac}$$

- **Boundary conditions**: $\Psi\ (|\mathbf{r}| = R) = c,$ (required for Newtonian limit)
  $\nabla\ \Psi\ (|\mathbf{r}| = R) = 0.$ (no force at $r = R$)
- Boundary conditions determine the value of Λ.
- We use: $R = R_0 = c/H_0$ (Hubble radius)

Numerical example

$$-\Delta\Psi + \frac{2\pi G}{c^2}\rho_m \Psi = \frac{\Lambda}{2}\,\Psi.$$



Λ term cancels on average the matter distribution

$$\rho_\Lambda \equiv \frac{c^2}{8\pi G}\,\Lambda$$

Fig. 1 Potential function $\Psi(r)$ normalized to c (solid lines), matter density $\rho_m(r)$ (dashed line) and vacuum density ρ_Λ (dotted line), as a function of the radius r normalized to a reference value R. Both $\rho_m(r)$ and ρ_Λ have been divided by the peak value $\rho_m(0)$.

$$E_m = 2.28,\ E_{\text{field}} = 0.36, \text{ and } E_\Lambda = 2.65.$$

Cosmological considerations: homogeneous universe

- Considering a **homogeneous universe** at large scales, with matter density ρ_m = const, an immediate solution of the field equation

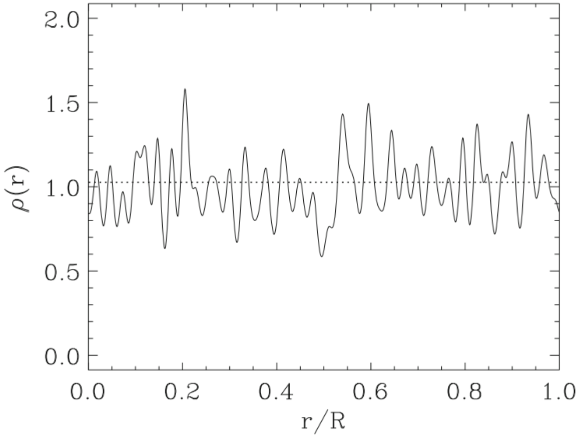
$$-\Delta\Psi + \frac{2\pi G}{c^2}\rho_m \Psi = \frac{\Lambda}{2}\Psi.$$

is

$$\boxed{\Psi = c, \quad \Lambda = \frac{4\pi G}{c^2}\rho_m, \quad \rho_\Lambda = \frac{\rho_m}{2}.}$$

- Taking $\rho_m \approx 1$ proton/m^3 yields the correct order of magnitude for the CC
- (Almost trivial, but nevertheless stems from interpreting Λ as an eigenvalue).
- For an **almost homogeneous distribution** with fluctuations: the vacuum term cancels on average the matter distribution (as long as gradients can be neglected)
- **Gravitationally empty universe** (coasting):

$$a(t) = t/t_0$$



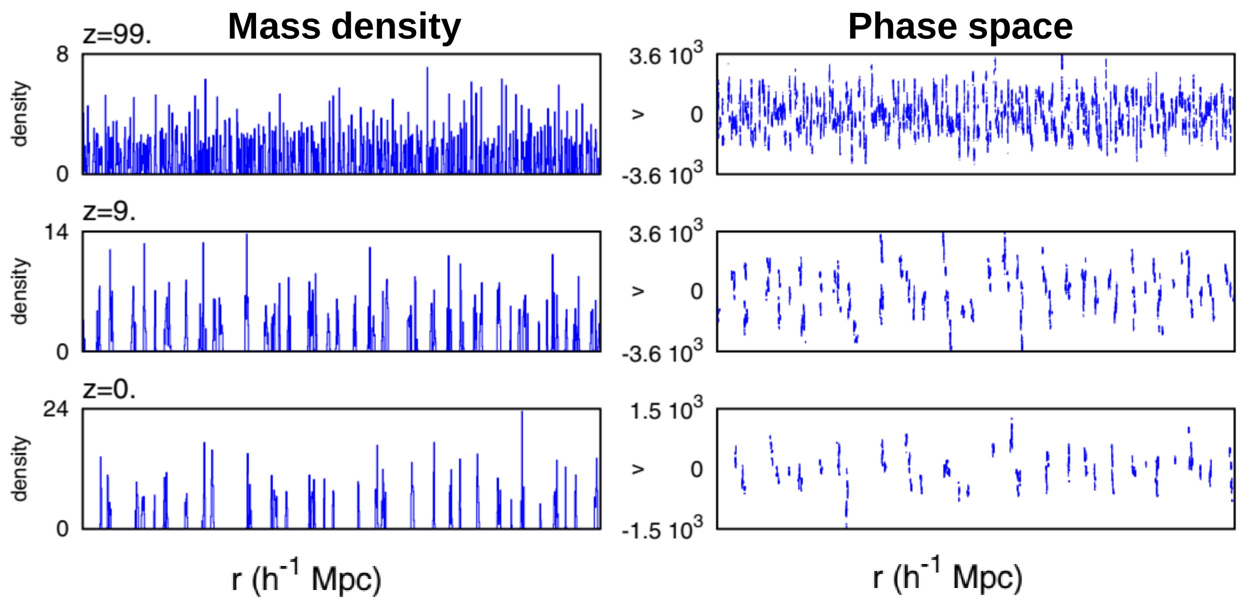
Structure formation

- The Newtonian limit of the scalar model is: $\Delta\Phi = 4\pi G\,(\rho_m - 2\rho_\Lambda).$
- In co-moving coordinates, the equation of motions are:

$$\frac{\mathrm{d}^2\hat{r}}{\mathrm{d}\hat{t}^2} + \dot{a}\,\frac{\mathrm{d}\hat{r}}{\mathrm{d}\hat{t}} = -\frac{1}{a}\,\frac{\partial\hat{\Phi}}{\partial\hat{r}}, \qquad a(t) = t/t_0$$

and the field equation is invariant: $\Delta_{\hat{r}}\hat{\Phi} = 4\pi G\,(\hat{\rho}_m - 2\rho_{\Lambda 0}).$

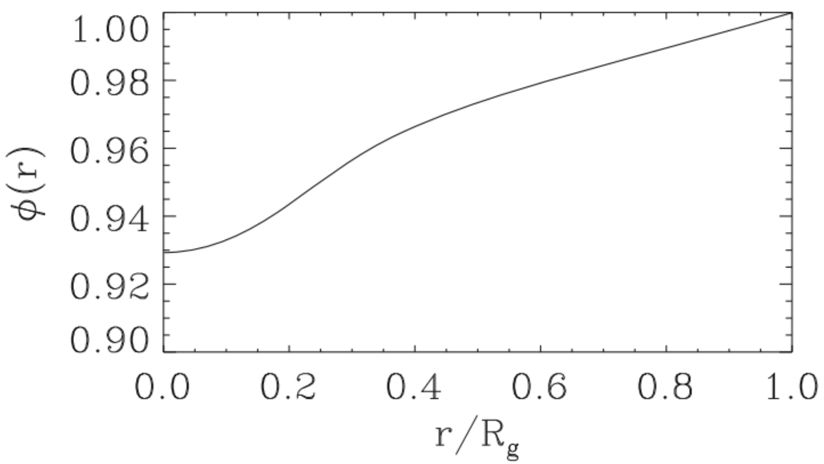
- These equations have been simulated with an N-body code in the context of the Dirac-Milne universe (G. Manfredi et al., PRD **98**, 023514 (2018))
- They show structure formation very similar to that of ΛCDM and occurring on similar timescales



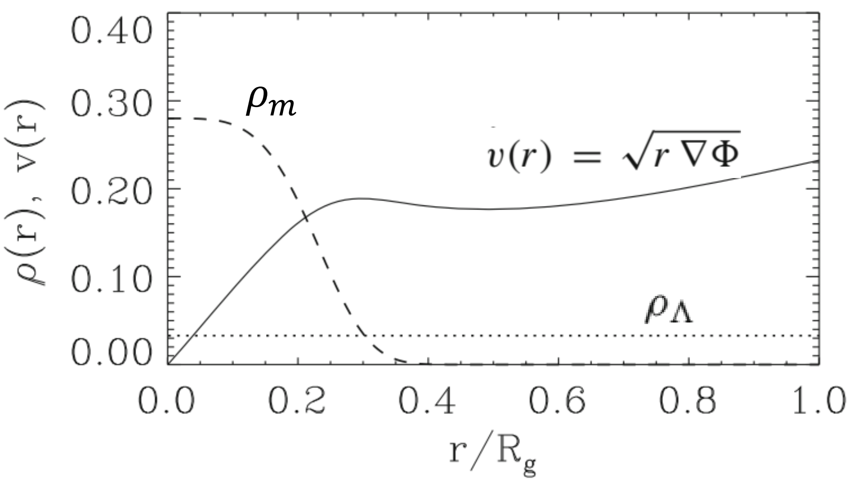
Galaxy rotation curves

- We solve the scalar field equation in the vicinity of a spherical "galaxy" of radius $R_g = 200\ \mathrm{kpc}$
- $R_g \ll R_0$= Hubble radius
- We can estimate the gradient of ψ as: $\nabla\psi(R_g) \approx c/R_0$

Potential [500^2 (km/s) ^{2}]



rotation velocity [500 km/s]



$$E_m = \frac{2\pi G}{c^2}\int_V \rho_m \Psi^2 d\mathbf{r} = 0.032$$

$$E_\Lambda = \frac{\Lambda}{2}\int_V \Psi^2 d\mathbf{r} = 0.136$$

Conclusion

- We proposed a new interpretation of the gravitational field equations as a **nonlinear eigenvalue problem**
- This conjecture relies on the following hypotheses:
  1. Any gravitational field equation that incorporates **self-gravity** can be cast mathematically in the form of a nonlinear eigenvalue problem;
  2. The cosmological constant Λ can be interpreted as the **smallest** ("ground state") **eigenvalue;**
  3. The value of Λ is determined by the **boundary conditions**;
  4. In a cosmological context, the b.c. are to be set on the **Hubble sphere** of radius R_0.
- **This approach**:
  - provides the correct order of magnitude for Λ;
  - is compatible with structure formation on a cosmological scale;
  - is compatible with the effects of Dark Matter on a local scale (galactic rotation curves).
- **Open problems**:
  - Can it work for full GR? How about exact linearization?
  - "Serious" comparison to observational data.

G. Manfredi, *Gen Relativ Gravit* **53**, 31 (2021). ArXiv:2102.09601