Non-ideal self-gravity and cosmology

the importance of correlations in the dynamics of the large-scale structures of the Universe

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ENS-Lyon, Exeter: G. Chabrier

- Goal: try to convince you that there is an incompatibility between statistical mechanics and the way we deal with gravity at large scales.
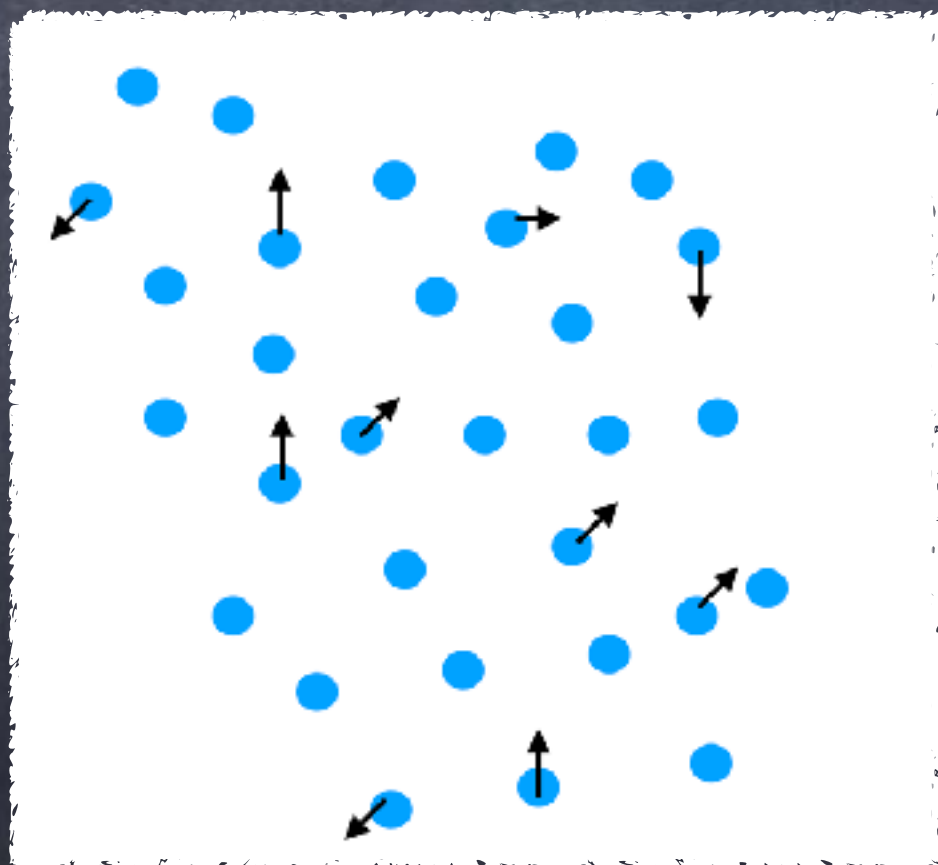
− The importance of correlations in statistical mechanics

$$P_1(\vec{r}), P_2(\vec{r}, \vec{r}'), P_3(\vec{r}, \vec{r}', \vec{r}''), \ldots$$

$$< ma > \; = \sum_i ma_i \qquad \textbf{Nbody} \qquad < H_{\text{int}} > \; = \sum_{i<j} \phi(r_{ij})$$
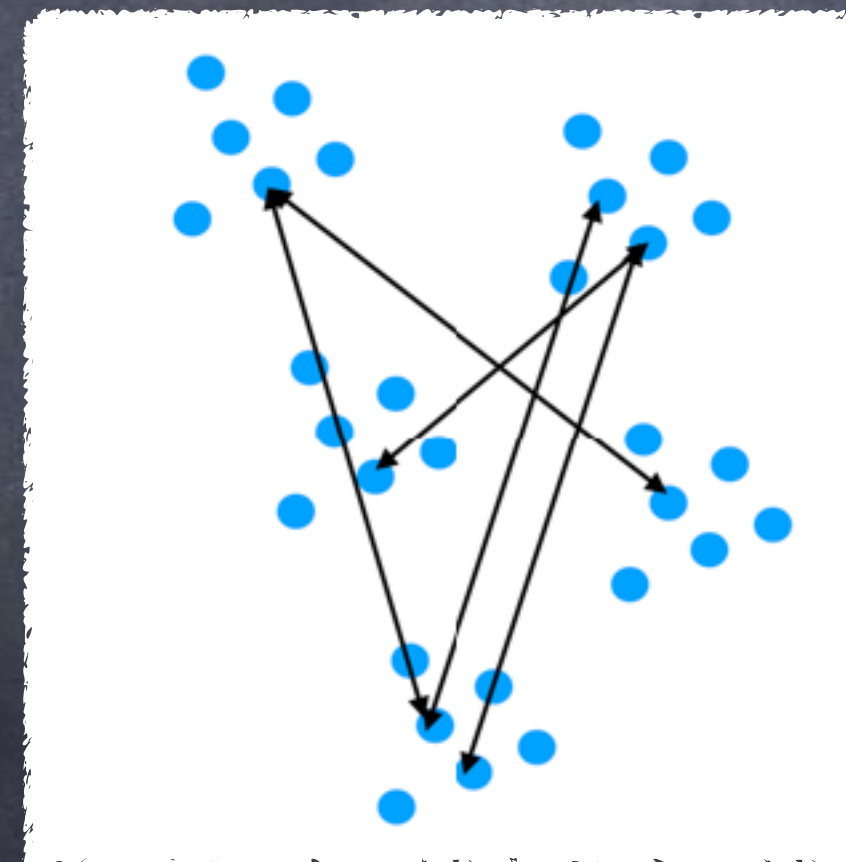
$$= N \int_V ma(\vec{r}) P_1(\vec{r}) dV \qquad \textbf{Fluid} \qquad = \frac{N(N-1)}{2} \int_{V,V'} P_2(\vec{r}, \vec{r}') \phi(|\vec{r} - \vec{r}'|) dV dV'$$

With e.g. for gravity

$$\phi(r) = \frac{Gm^2}{r}$$

$$\rho(\vec{r}) = N P_1(\vec{r}) \qquad\qquad \xi(\vec{r}, \vec{r}') = V^2 (P_2(\vec{r}, \vec{r}') - P_1(\vec{r}) P_1(\vec{r}'))$$

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– The importance of correlations in statistical mechanics

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With e.g. for gravity

$$\phi(r) = \frac{Gm^2}{r}$$

– For a homogeneous and isotropic fluid

$$P_1(\vec{r}) = 1/V, \quad \rho(\vec{r}) = N/V$$

$$\xi(\vec{r}, \vec{r}') = \xi(||\vec{r} - \vec{r}'||), \quad g(r) = 1 + \xi(r)$$

$\xi(r)$ used in cosmology to characterize the large scale structures of the universe see e.g. Peebles 1980

The importance of correlations in statistical mechanics

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$$\frac{< H_{\text{int}} >}{V} \approx 2\pi\rho^2 \int_0^R g(r)\phi(r)r^2 dr$$

- For an ideal fluid: $g(r) = 1, \quad \xi(r) = 0$ $P_2(\vec{r}, \vec{r}') = P_1(\vec{r})P_1(\vec{r}')$

The importance of correlations in statistical mechanics

$$< H_{\text{int}} > \approx \frac{N^2}{2} \int_{V,V'} P_2(\vec{r}, \vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV'$$

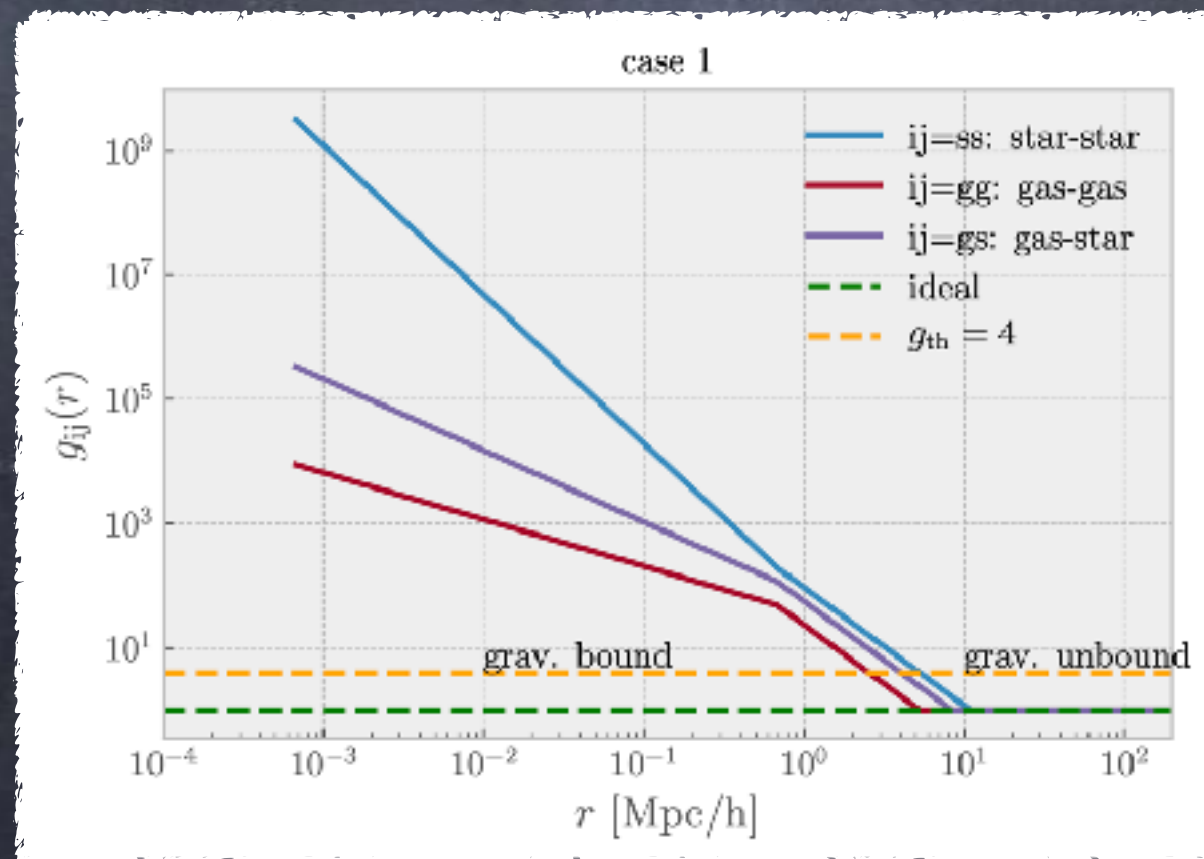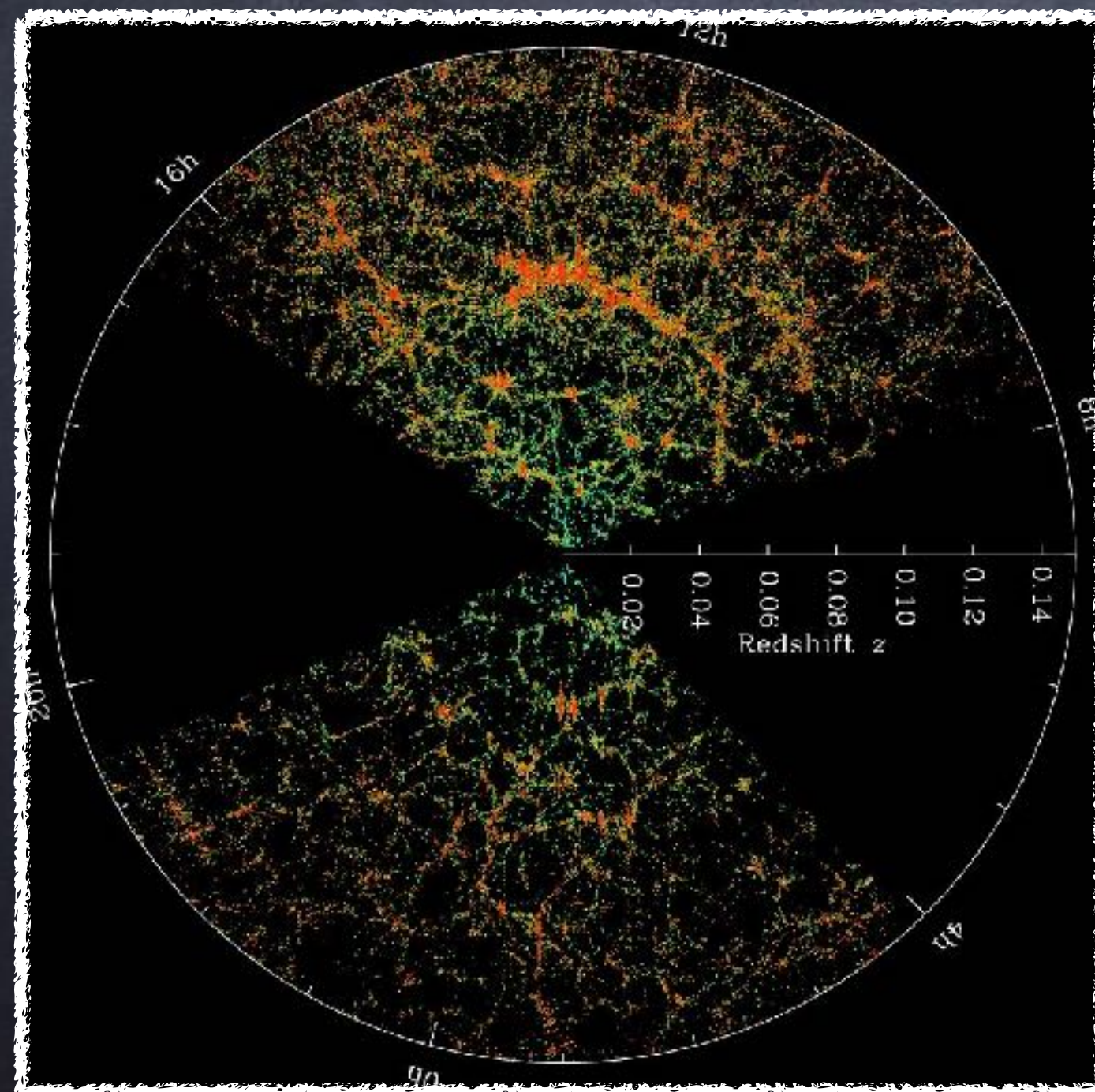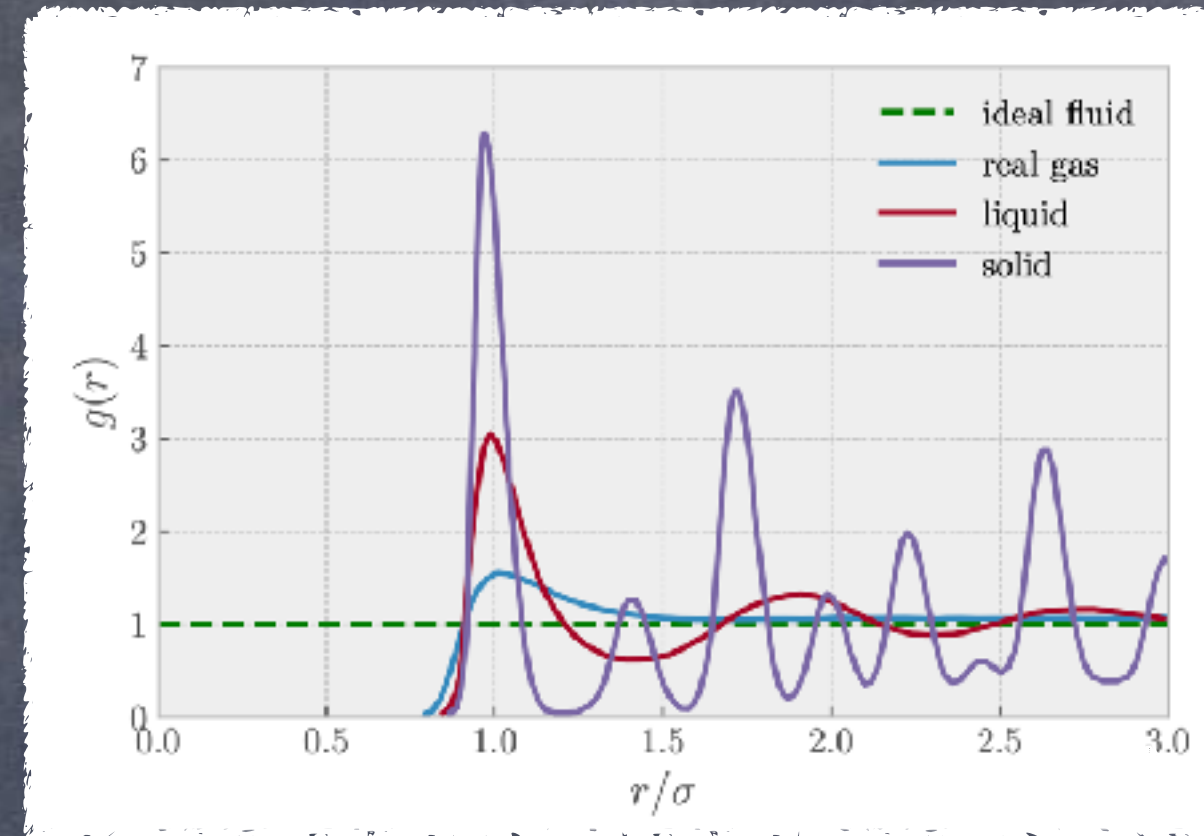Uncorrelated fluid at all scales

Correlated fluid at large scale
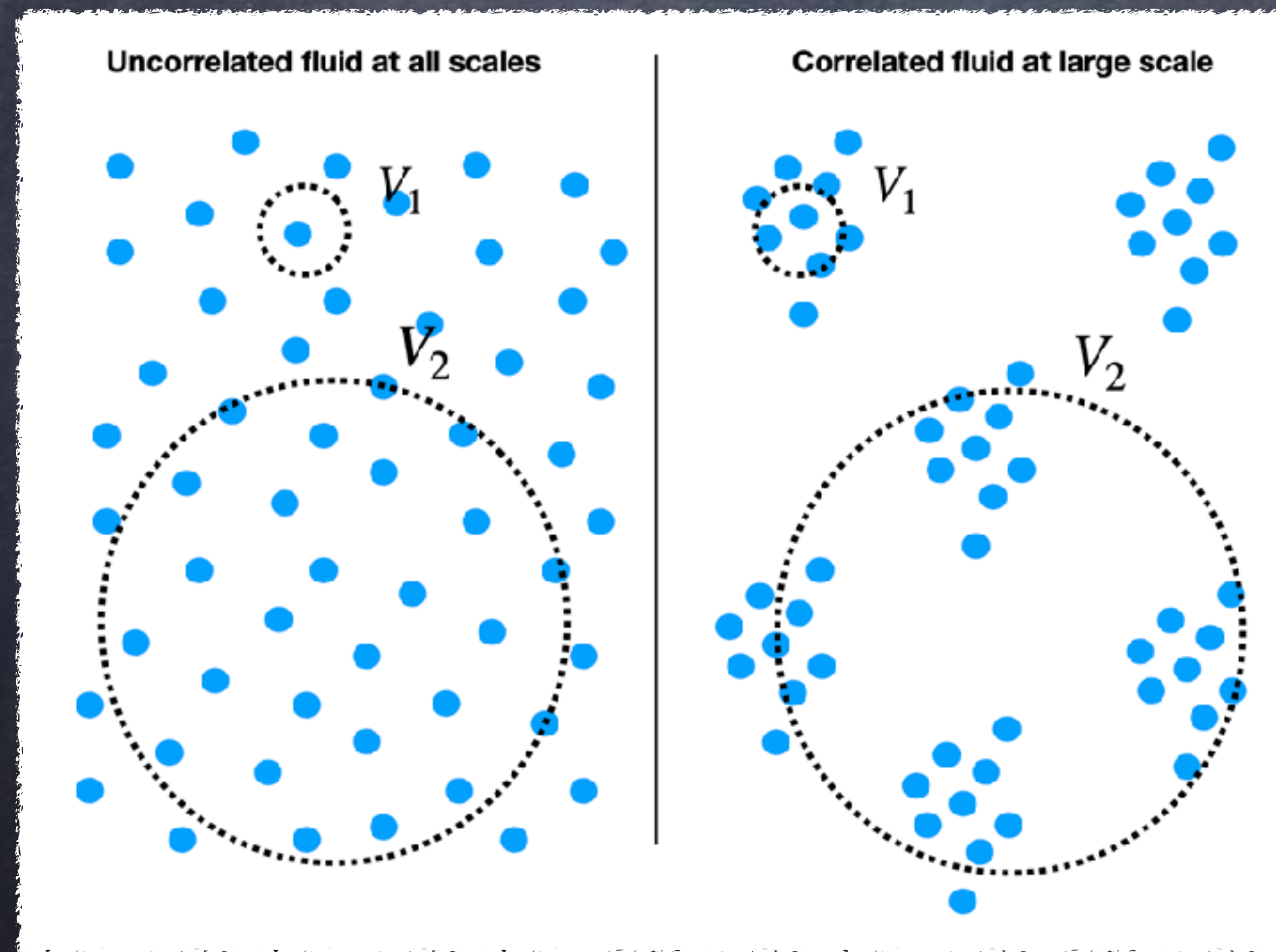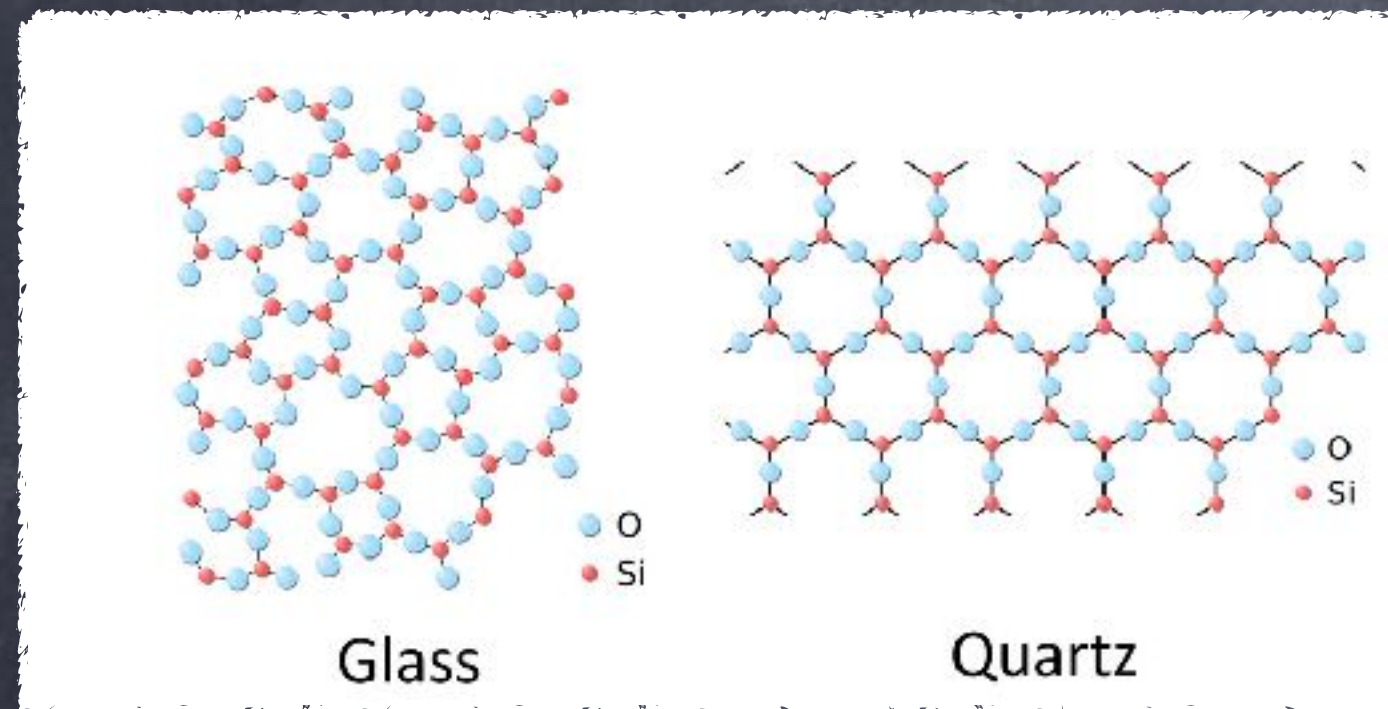
- With a control volume V1, the distribution is inhomogeneous and uncorrelated

$$\rho(\vec{x}) \neq \text{cst} \qquad P_2(\vec{r}, \vec{r}') = P_1(\vec{r})P_1(\vec{r}')$$

- With a control volume V2, the distribution is homogeneous and isotropic but correlated (non-ideal)

$$\rho(\vec{x}) = \text{cst} \qquad P_2(\vec{r}, \vec{r}') \neq P_1(\vec{r})P_1(\vec{r}')$$

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– Correlations in liquids/solids and the Universe

Glass Quartz

At large scale, the universe is homogeneous and isotropic but non-ideal (the most non-ideal fluid we will ever study)

- The importance of correlations in self gravitating astrophysical flows

- Virial theorem is most of the time applied in a very simplified way in astrophysics
- e.g. Zwicky 1933 need dark matter to explain velocity dispersion in the Coma Cluster

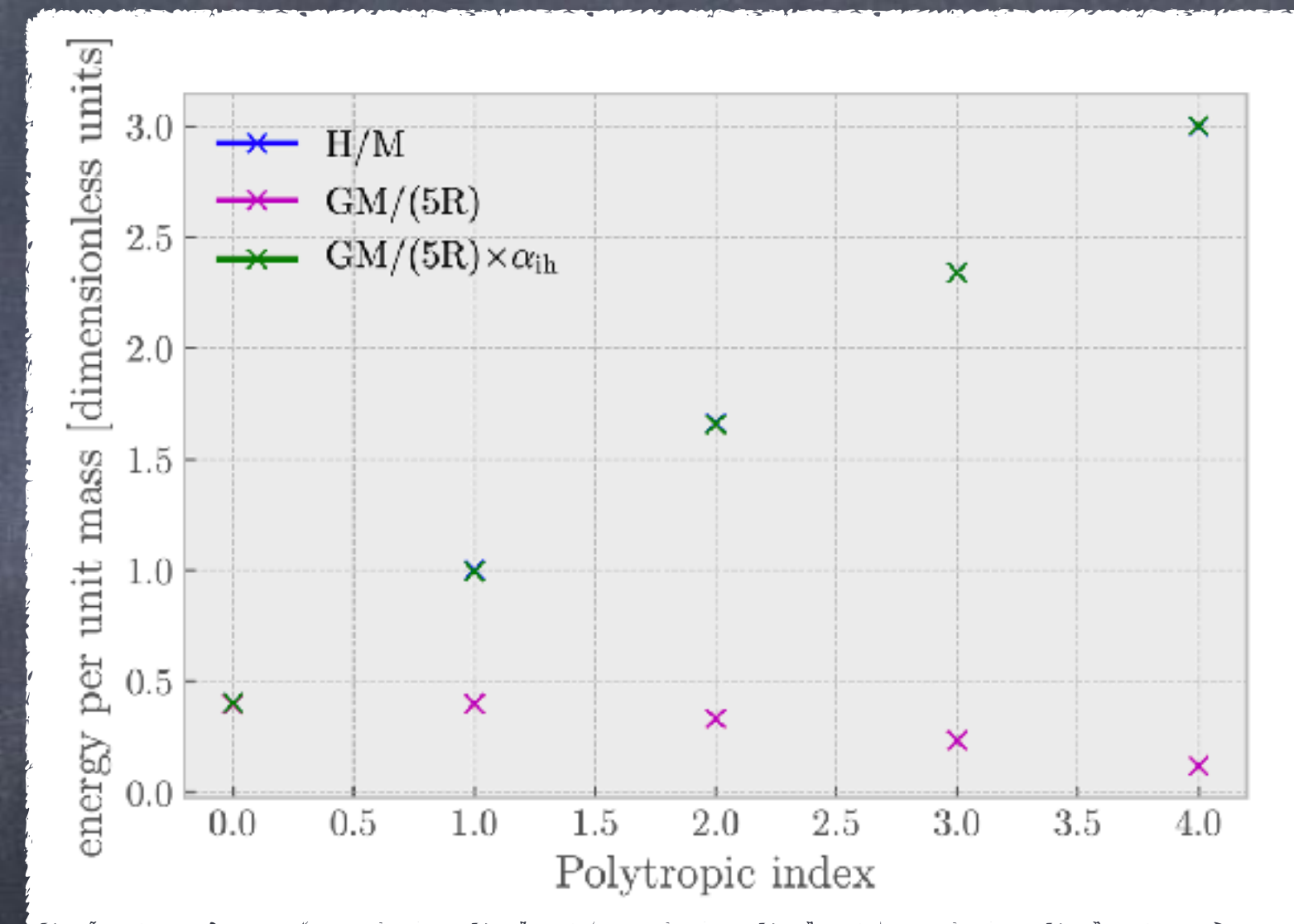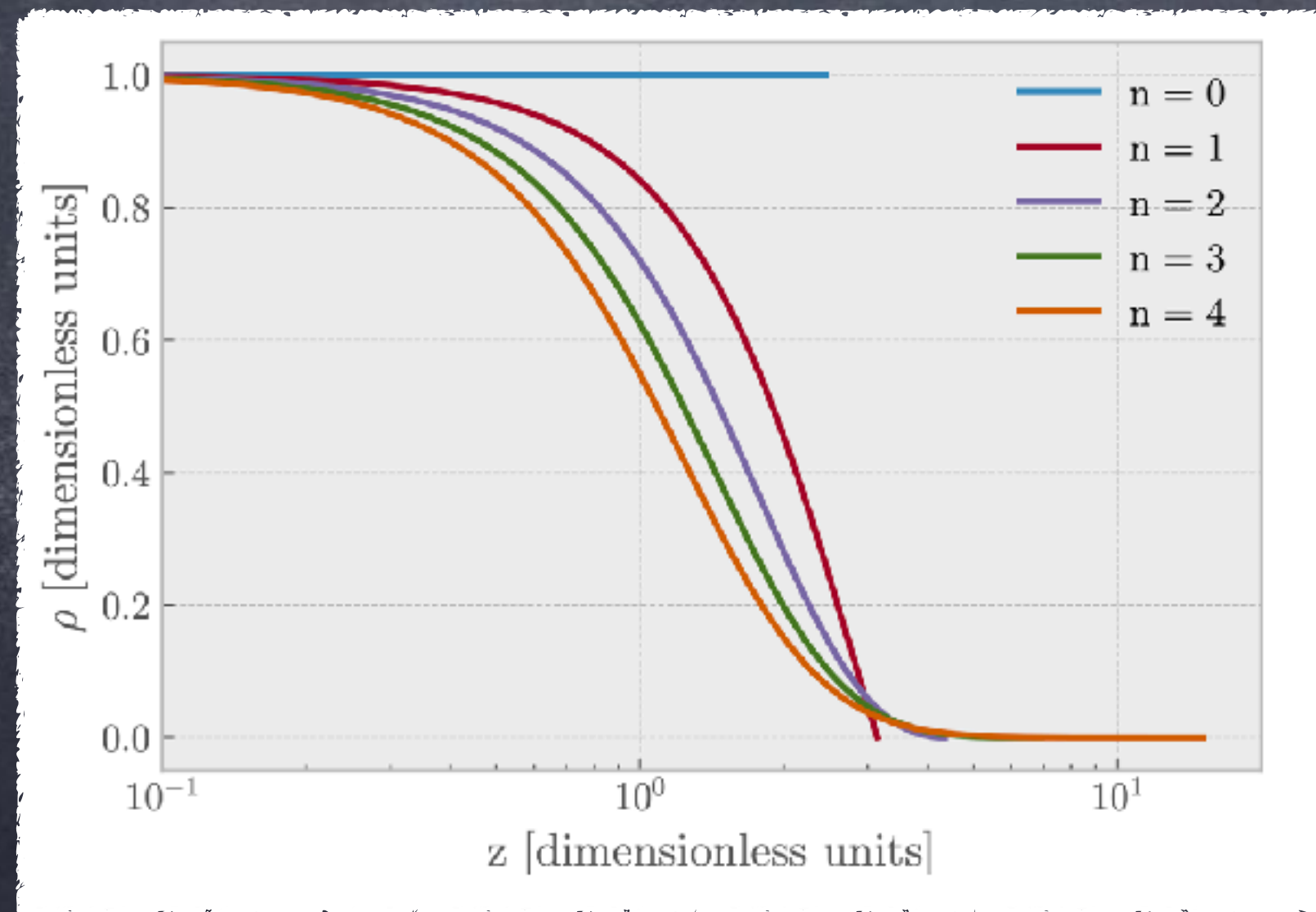$$\langle v \rangle^2 \approx \frac{GM}{R}$$

- But correlations matter for the Virial theorem

$$-2 < E_c > = < H_{\text{int}} > \approx \frac{N^2}{2} \int_{V,V} P_2(\vec{r}, \vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV'$$

− The importance of correlations in self gravitating astrophysical flows

− A semi-analytical example for finite size systems: the Lane-Emden equation, hydrostatic solution of self-gravitating polytropic stars P=K rho^gamma (n=1/(gamma-1))

25x

- Homogeneous: $H/M = GM/5R$
- Inhomogeneous: $H/M = GM/5R\alpha_{ih}$
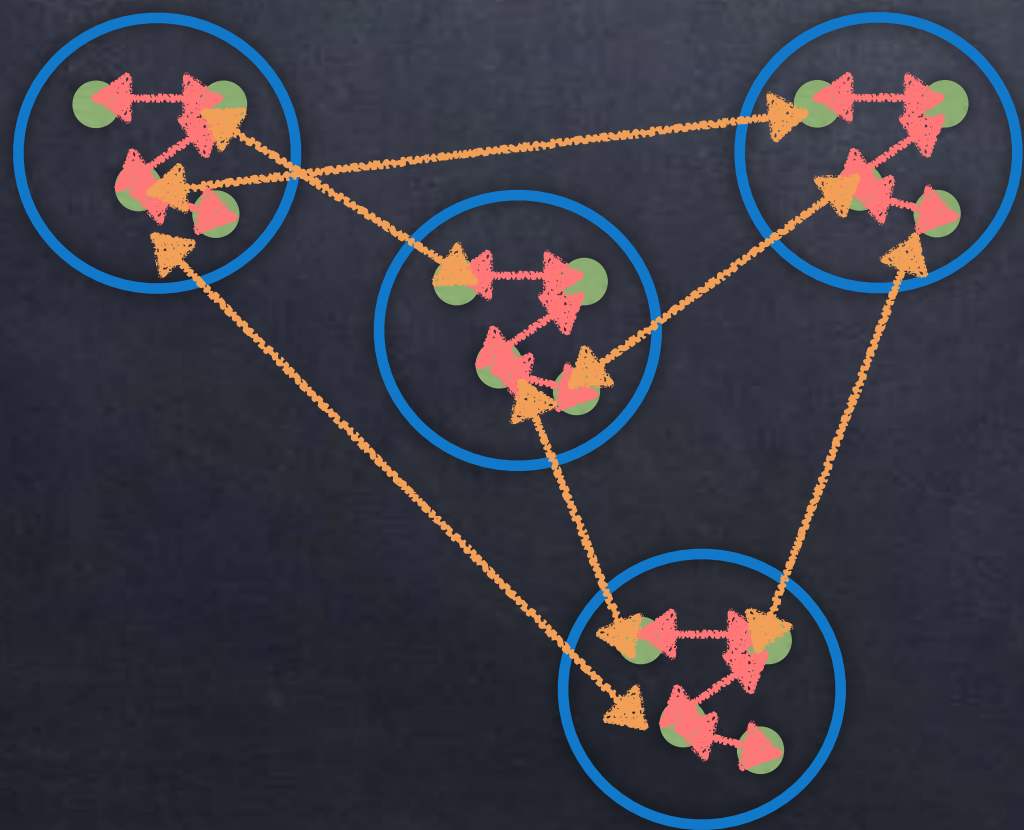
$$\alpha_{ih} = \frac{N^2 \iint_{V,V} P_2(\vec{r}, \vec{r'})\phi(|\vec{r} - \vec{r'}|)dVdV'}{N^2 \iint_{V,V} 1/(4\pi R^3/3)^2\phi(|\vec{r} - \vec{r'}|)dVdV'}$$

- **The importance of correlations in self gravitating astrophysical flows**

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$$-2 < E_c > = < H_{\text{int}} > \approx \frac{N^2}{2} \int_{V,V} P_2(\vec{r}, \vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV'$$

- Problem: for infinite systems, the gravitational interaction energy **does not converge in the thermodynamic limit** because the gravitational force is long range and always attractive

Fluid of polytropic stars: long-range interactions should not be accounted for the viral theorem in a star: they contribute to the star-cluster dynamic

- **The importance of correlations in self gravitating astrophysical flows**

  - But **correlations** matter for the **Virial theorem**

$$-2 < E_c > = < H_{\text{int}} > \approx \frac{N^2}{2} \int_{V,V} P_2(\vec{r}, \vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV'$$

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λ_0
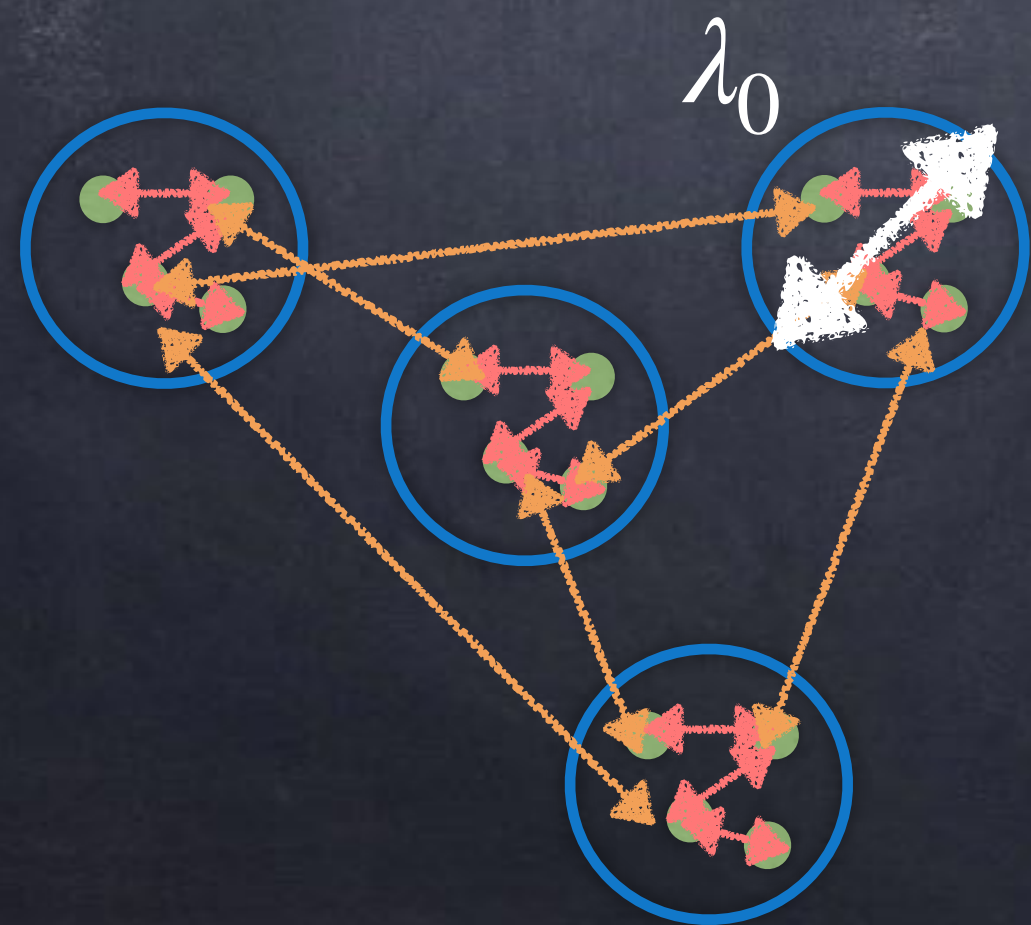
Decompose the force into a near-field and far-field component and use near-field for the virial theorem

$$\phi = \phi e^{-r/\lambda_0} + \phi(1 - e^{-r/\lambda_0})$$

- The importance of correlations in self gravitating astrophysical flows
  - Hierarchical viral theorem
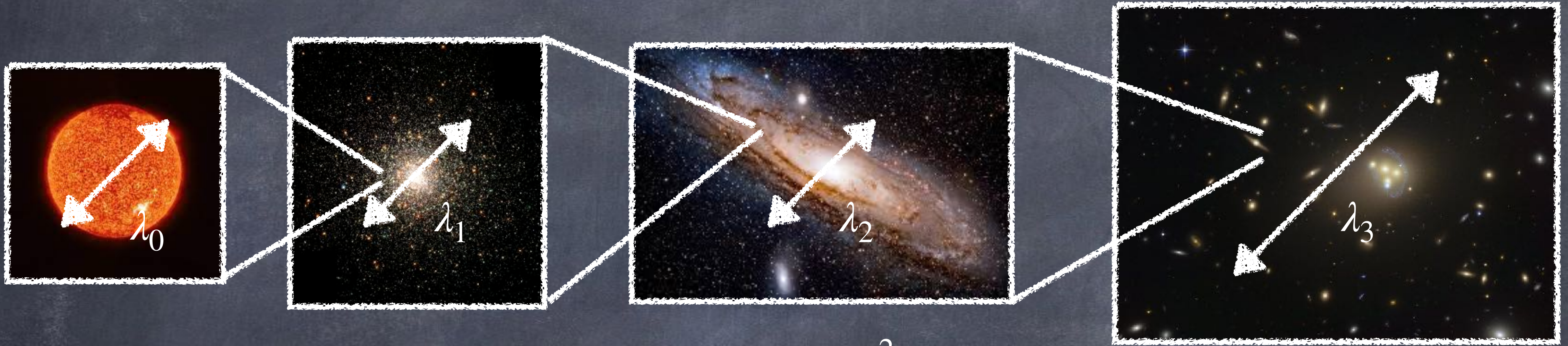
$\phi_0 = \phi e^{-r/\lambda_0}$

$\phi_1 = \phi(1 - e^{-r/\lambda_0})e^{-r/\lambda_1}$

$\phi_2 = \phi(1 - e^{-r/\lambda_0})(1 - e^{-r/\lambda_1})e^{-r/\lambda_2}$

$\phi_3 = \ldots$

$$-2 < E_{c,i} > \; \approx \frac{N^2}{2} \int_{V,V} P_2(\vec{r}, \vec{r}')\phi_i(|\vec{r} - \vec{r}'|)dVdV'$$

$$\langle v_i^2 \rangle \approx \frac{GM}{\lambda_i}\alpha_{\mathrm{ni}}$$

$$\alpha_{\mathrm{ni}} = \frac{\iint_{V,V} \phi_i(|\vec{r} - \vec{r}'|)P_2(\vec{r}, \vec{r}')dVdV'}{\iint_{V,V} \phi_i(|\vec{r} - \vec{r}'|)P_1(\vec{r})P_1(\vec{r}')dVdV'}$$

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– The importance of correlations in statistical mechanics

- We can define a mean interaction field only in the absence of correlations:

$$P_2(\vec{r}, \vec{r}') = P_1(\vec{r})P_1(\vec{r}')$$

$$
\begin{aligned}
< H_{\text{int}} > &\approx \frac{N^2}{2} \int_{V,V'} P_1(\vec{r})P_1(\vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV' \\
&\approx \frac{1}{2} \int_V \rho(\vec{r}) \int_{V'} \rho(\vec{r}')\phi(|\vec{r} - \vec{r}'|)dV'dV \\
&\approx \frac{1}{2} \int_V m\rho(\vec{r})\Phi(\vec{r})dV
\end{aligned}
$$

With e.g. for gravity $\nabla^2\Phi(\vec{r}) = 4\pi Gm\rho(\vec{r})$

- Using a mean field is an approximation of the N-Body problem
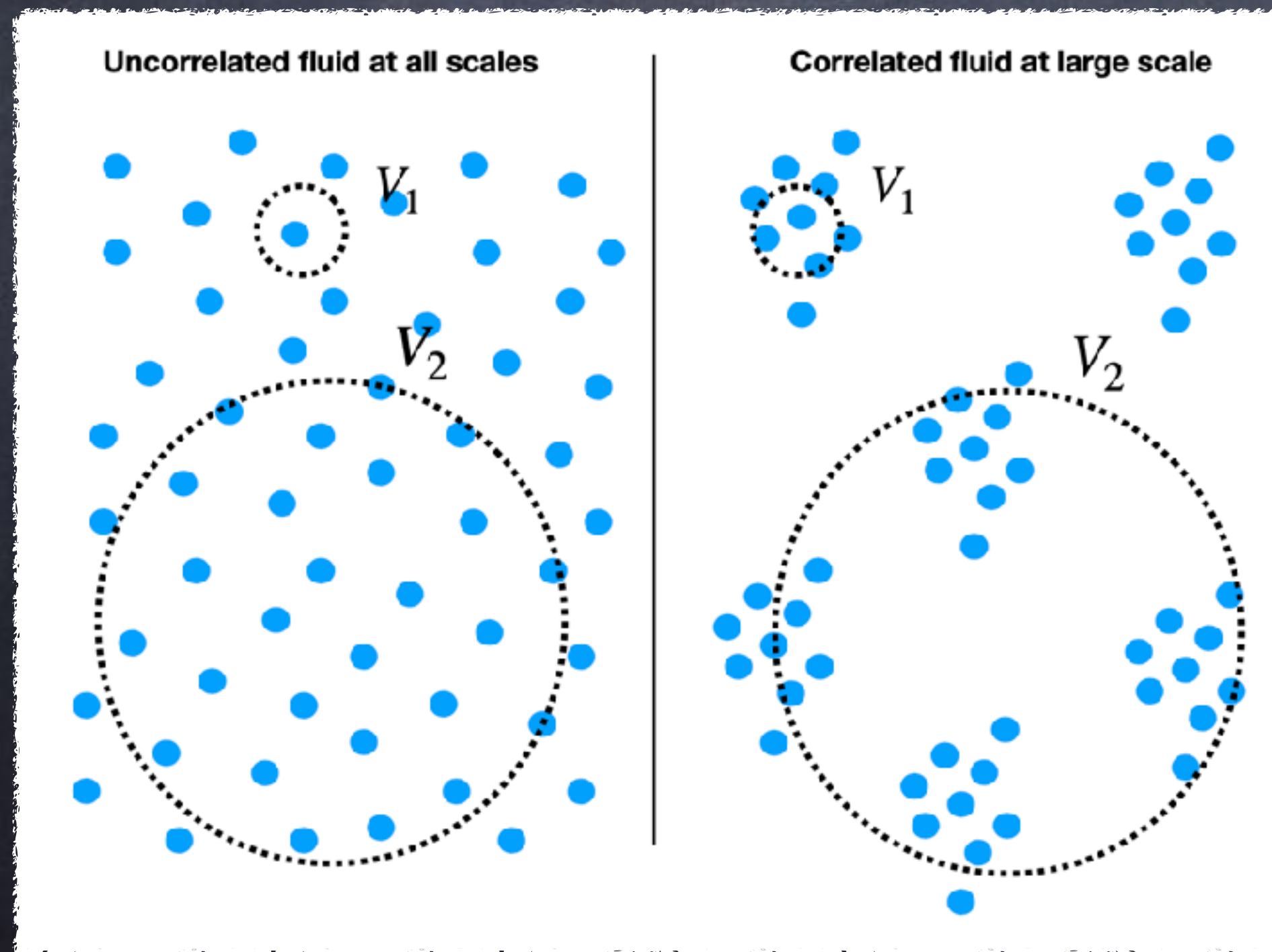
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Poisson/Einstein theory and correlations

$$< H_\text{int} > = \frac{N^2}{2} \int_{V,V'} P_2(\vec{r}, \vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV'$$

$$\neq \frac{N^2}{2} \int_{V,V'} P_1(\vec{r})P_1(\vec{r}')\phi(|\vec{r} - \vec{r}'|)dVdV'$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$\Delta\Phi \approx -4\pi GT_{00}/c^2$$

$$\langle H_\text{int}\rangle = -\frac{1}{2}\iint_{V,V} G\frac{\rho(\vec{r})T_{00}(\vec{r}')/c^2}{||\vec{r} - \vec{r}'||}dVdV'$$

Uncorrelated fluid at all scales | Correlated fluid at large scale

V_1 V_2 | V_1 V_2

- Poisson/Eisntein theories are valid for a inhomogeneous ideal fluid but should not be applied to a homogeneous correlated fluid as they ignore the correlations, i.e. they are mean-field theories
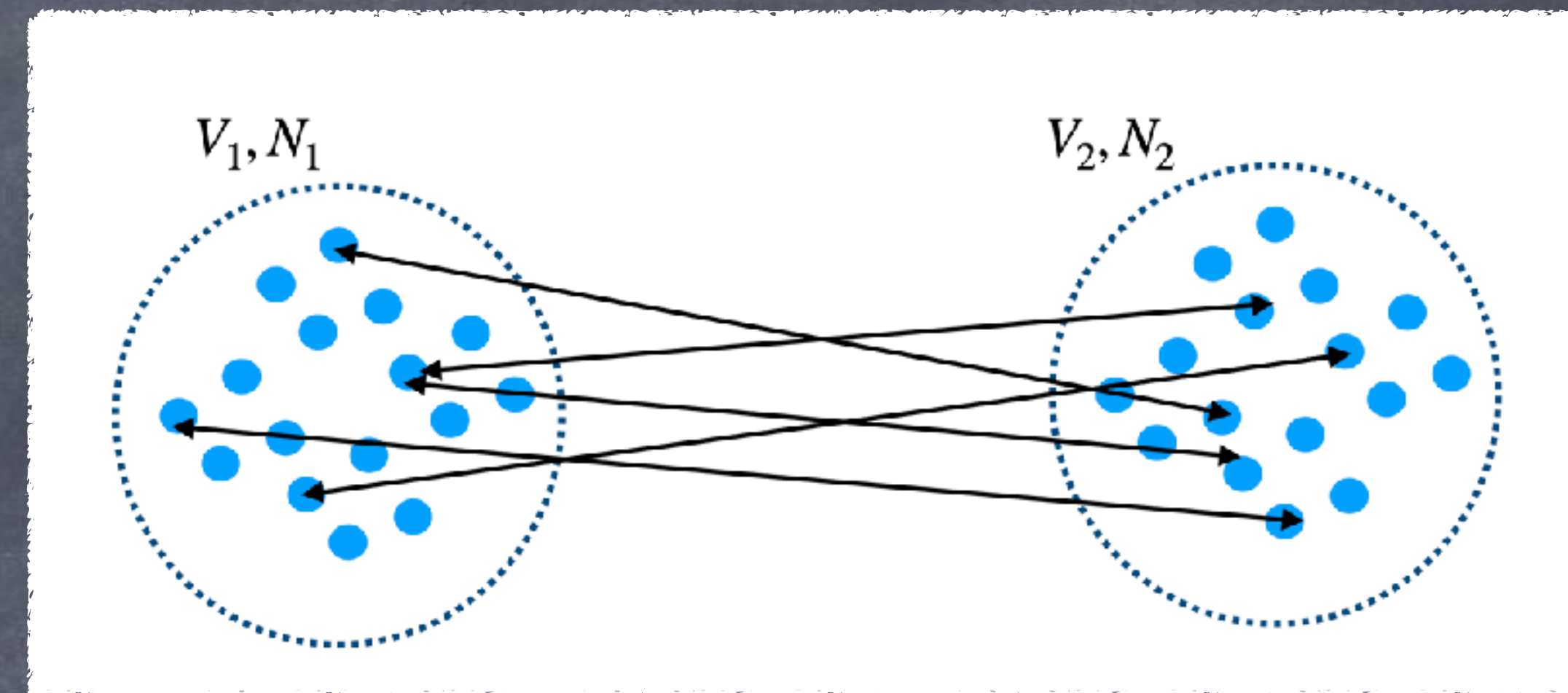
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– Statistical Newton laws for pressureless fluids

- Second law:

$$< ma >_1 \; = \; < f >_{2 \to 1}$$

$$\Sigma_i ma_{i,1} = \Sigma_{i,j} f_{j,2 \to i,1}$$

$$N_1 \int_{V_1} ma(\vec{r}) P_1(\vec{r}) dV_1 = N_1 N_2 \int_{V_1, V_2} P_2(\vec{r}, \vec{r}') f_{\vec{r}' \to \vec{r}} dV_1 dV_2$$

- First law, for an isolated fluid volume:

$$< ma >_1 \; = \; < f >_{1 \to 1} = 0$$

- Third law:

$$< ma >_2 \; = \; < f >_{1 \to 2} = - < f >_{2 \to 1}$$

- Using a mean field is an approximation with $P_2(\vec{r}, \vec{r}') = P_1(\vec{r}) P_1(\vec{r}')$

$$\rho ma \approx \rho m G \quad a \approx G \quad G = - \nabla \Phi$$

- Cosmological models

- Einstein equations of general relativity
- Assuming an homogeneous and isotropic fluid
- assuming an ideal fluid

-> Standard cosmological model

Cosmological models

- Einstein equations of general relativity
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$$g(r) \gg 1$$

-> Standard cosmological model

- Cosmological models

✓ - Einstein equations of general relativity ✗

✗ - Assuming an homogeneous and isotropic fluid

✓ - assuming an ideal fluid ✗ ✓

$$g(r) \gg 1$$ ✗

-> Standard cosmological model
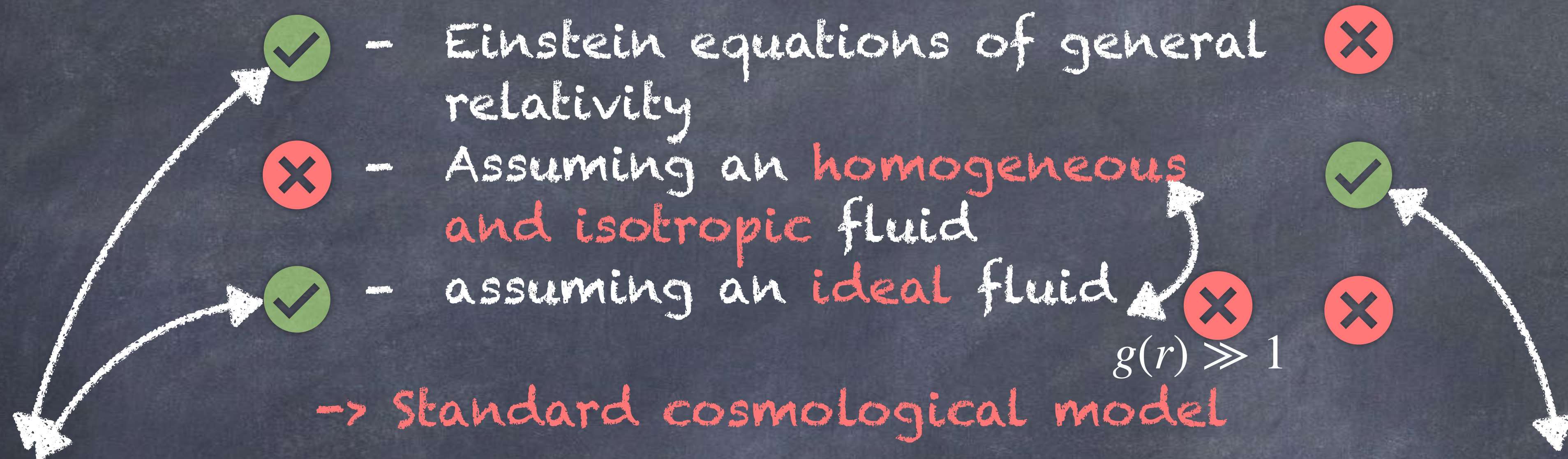
- With infinite resolution, the universe is inhomogeneous and ideal and Poisson/Einstein equations can be used -> inhomogeneous cosmology (e.g. Buchert et al.)

- At large scale, the universe is homogeneous and isotropic but non-ideal (the most non-ideal fluid we will ever study) and Poisson/Einstein equations cannot be used -> non-ideal cosmology

- non-ideal cosmology

- Starts from Newtonian demonstration of Friedmann equations and add correlations as in the Virial theorem

$$E = \langle E_{\text{kin}} \rangle + \langle H_{\text{int}} \rangle$$

$$E/M \approx \frac{1}{2}\dot{R}^2 - \frac{GM}{R}$$

$$\frac{\dot{R}^2}{R^2} - \frac{2E/M}{R^2} = \frac{8\pi G}{3c^2}\rho_b$$
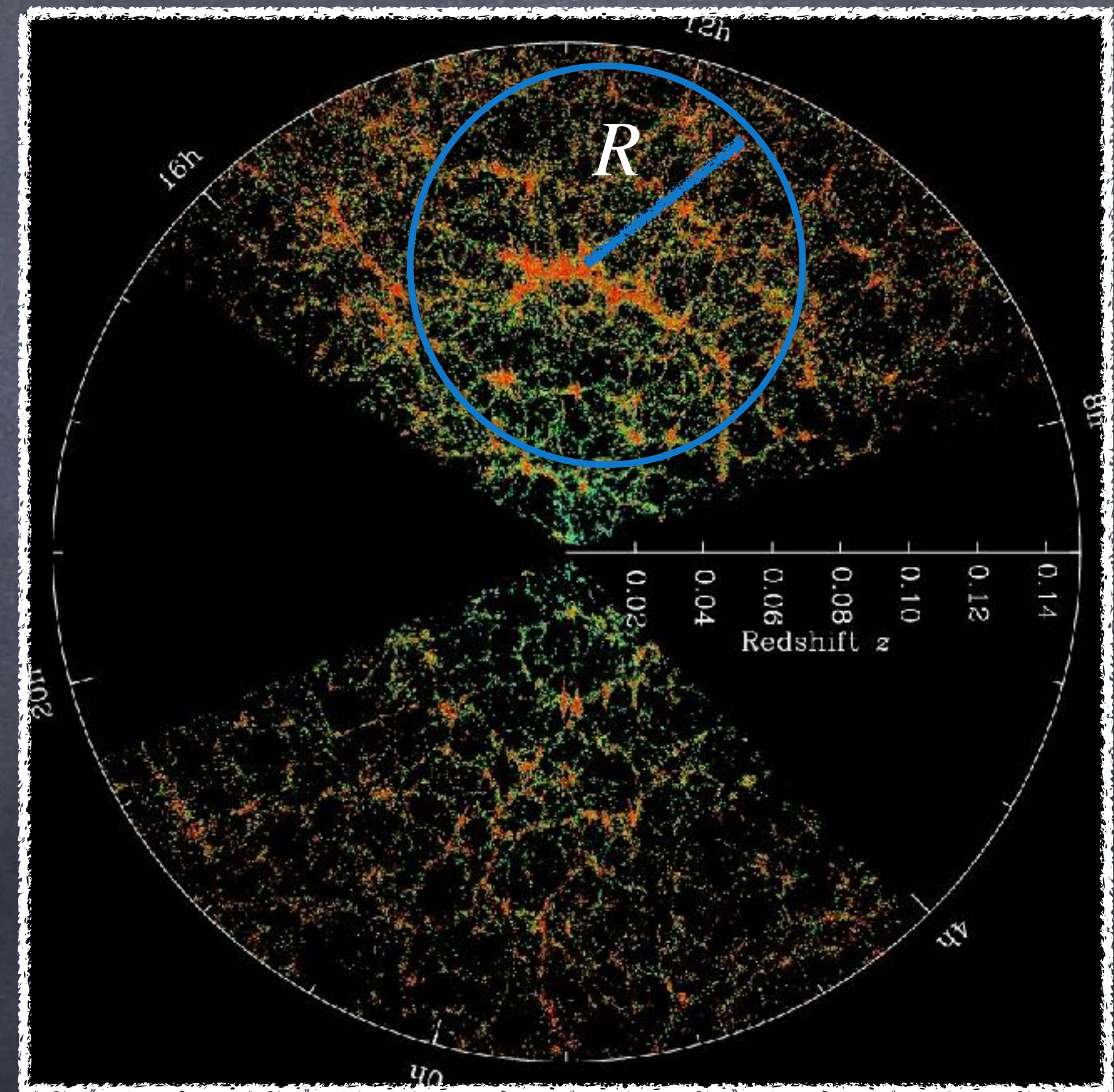
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$$H^2 = \frac{8\pi G}{3c^2}\rho_b\alpha_{\text{ni}}$$

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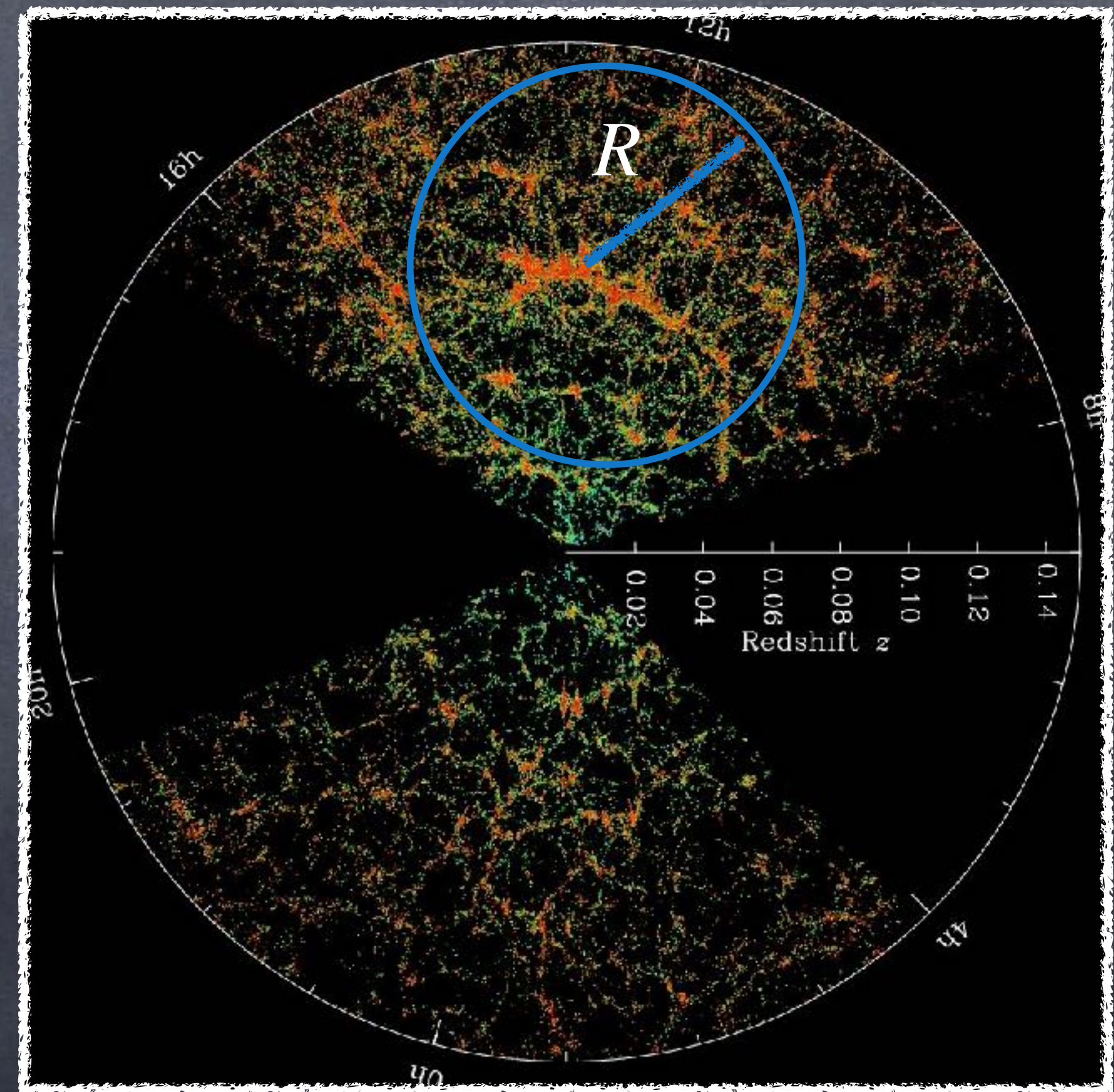- Starts from Newtonian demonstration of Friedmann equations and add correlations as in the Viral theorem

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- non-ideal cosmology

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$$H_{ni}^2 = \frac{8\pi G}{3c^2}\rho_b \alpha_{ni}$$

$$q_{ni} \approx \frac{1}{2} - \frac{\dot{\alpha}_{ni}}{2H_{ni}\alpha_{ni}}$$

$$\alpha_{ni} = \frac{\iint_{V,V} \phi_i(|\vec{r}-\vec{r}'|)P_2(\vec{r},\vec{r}')dVdV'}{\iint_{V,V} \phi_i(|\vec{r}-\vec{r}'|)P_1(\vec{r})P_1(\vec{r}')dVdV'}$$

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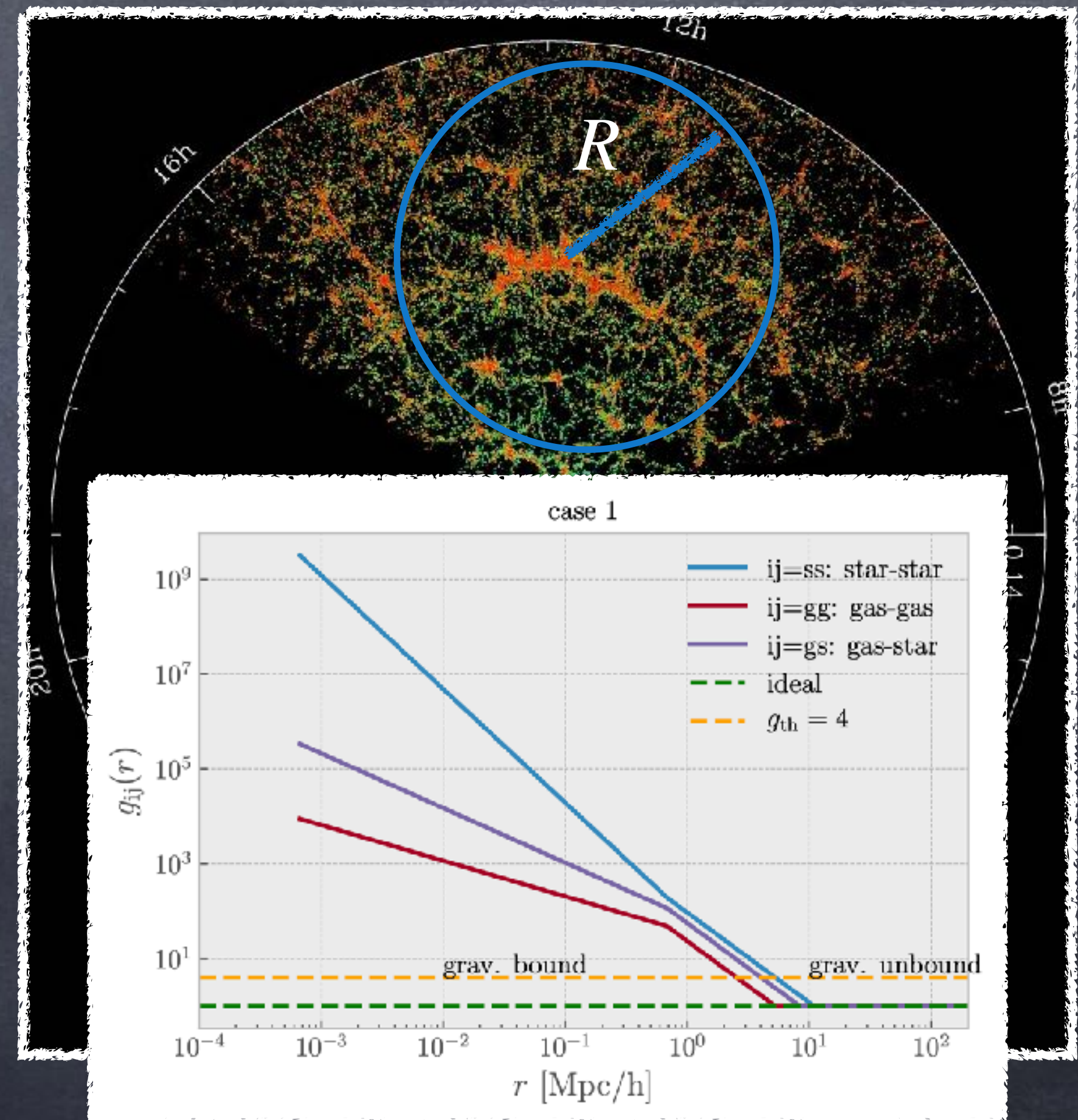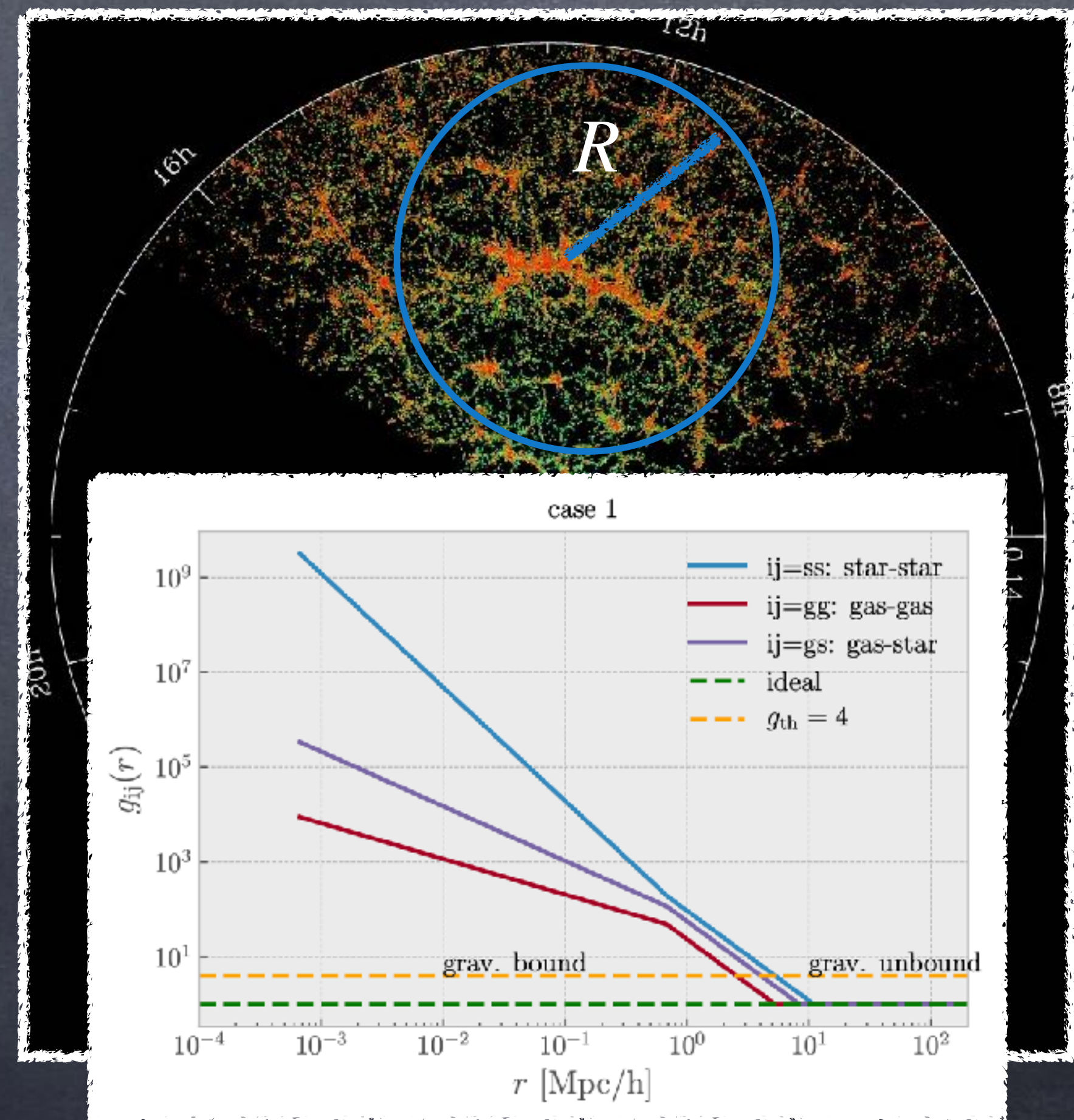- Starts from Newtonian demonstration of Friedmann equations and add correlations as in the Viral theorem

$$H_{\text{ni}}^2 = \frac{8\pi G}{3c^2}\rho_b \alpha_{\text{ni}}$$

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Assuming that alpha_ni as varied from 1 to 20 during the age of the Universe, we get an analytical expression for the acceleration:

$$q_{\text{ni}} \approx \frac{1}{2} - \frac{\ln(\rho_c/\rho_b)}{2t_u H_{\text{ni}}} \approx \frac{1}{2} - \frac{\ln(20)}{2t_u H_{\text{ni}}} \approx -1.06$$

Inferred q value from Type 1a supernovae:

$$q = -1.0 \pm 0.4$$

– Friedmann equations

Ideal LCDM: add dark matter/energy

$$H^2 = \frac{8\pi G}{3c^2}(\rho_b + \rho_{\text{CDM}}) + \frac{\Lambda c^2}{3}$$

$$q = -1 - \frac{\dot{H}}{H^2} \approx \frac{1}{2} - \frac{\Lambda c^2}{2H}$$

Non-ideal: add correlations

$$H_{\text{ni}}^2 = \frac{8\pi G}{3c^2}\rho_b \alpha_{\text{ni}}$$

$$q_{\text{ni}} \approx \frac{1}{2} - \frac{\dot{\alpha}_{\text{ni}}}{2H_{\text{ni}}\alpha_{\text{ni}}}$$

$$\alpha_{\text{ni}} = \frac{\iint_{V,V} \phi(|\vec{r}-\vec{r}'|)P_2(\vec{r},\vec{r}')dVdV'}{\iint_{V,V} \phi(|\vec{r}-\vec{r}'|)P_1(\vec{r})P_1(\vec{r}')dVdV'}$$

The baryon energy density account for ~5% of the observed expansion

Estimations of alpha_ni lie between 5 and 20 potentially explaining the observed expansion

Assuming that alpha_ni as varied from 1 to 20 during the age of the Universe, we get an analytical expression for the acceleration:

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– Conclusions

– We propose to account for correlations in astrophysical self-gravitating flows by using a non-ideal viral theorem, non-ideal Navier-Stockes equations and non-ideal Friedmann equations

– This is not a modification of newton law of gravitation. This does not contradict Poisson/Einstein theories, they remain valid if one can use them taking into account all inhomogeneities down to the scale at which we can assume the fluid to be ideal. They, however, cannot be used for a correlated (non-ideal) fluid

– The universe at large scale is homogeneous and isotropic but non-ideal, the most non-ideal fluid we will ever study: the standard model of cosmology should be revised to properly take into account correlations, even when using lambda and CDM.

Conclusions

- The strength of this approach is that the non-ideal effects are linked to the correlation function that can easily be constrained in astrophysics by observations, or numerical simulations at different scale

- This is still a very simple model, a lot needs to be done to properly address other observable constrains (CMB, grav. Lensing, etc)

– Non-ideal self-gravitating hydrodynamics

Non-ideal equation of state: impact of the gravitational interaction on transport properties of a fluid

$$P = \rho k_B T - \frac{2\pi}{3}\rho^2 \int_0^R g(r)\frac{d\phi(r)}{dr}r^3 dr$$

$$\rho e = \frac{\rho k_B T}{\gamma - 1} + 2\pi\rho^2 \int_0^R g(r)\phi(r)r^2 dr$$

« Virial theorem »
 – virial equilibrium: P=0
 – Expansion P>0
 – Gravitational collapse P<0

Ideal hypothesis ok if

$$\rho k_B T \gg \frac{2\pi}{3}\rho^2 \int_0^R g(r)\frac{d\phi(r)}{dr}r^3 dr$$

$R \ll \lambda_J$ If the Jeans length is resolved

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$$\rho e = \frac{\rho k_B T}{\gamma - 1} + 2\pi \rho^2 \int_0^R g(r) \phi(r) r^2 dr$$

Decompose the force into:
- a near-field component accounted for in the EOS by statistical mechanics
- A far-field component as an external force in the mean field approximation

$$\phi = \phi_{\text{int}} + \phi_{\text{ext}}$$

$$\phi = \phi e^{-r/\Delta x} + \phi(1 - e^{-r/\Delta x})$$

– Non-ideal self-gravitating hydrodynamics

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u}\right) = 0,$$

$$\frac{\partial \rho_m \vec{u}}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u} \otimes \vec{u} + \sigma\right) = \vec{F}_{\text{ext}},$$

$$\frac{\partial \rho_m E}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u} E + \sigma \cdot \vec{u}\right) = \vec{F}_{\text{ext}} \cdot \vec{u},$$

$$\vec{F}_{\text{ext}}(\vec{x}) = -\rho(\vec{x})\vec{\nabla}_{\vec{x}}(\Phi_{\text{ext}}(\vec{x})),$$

$$\Phi_{\text{ext}}(\vec{x}) = \int_{V_{\text{sim}}} \phi_{\text{ext}}(|\vec{x} - \vec{x}'|)\rho(\vec{x}')dV_{\text{sim}},$$

$$\text{with} \quad \phi_{\text{ext}}(r) = -\frac{Gm^2}{r}(1 - e^{-r/\Delta x})$$

$$E = e + \frac{1}{2}\vec{u}^2,$$

$$\sigma_{ij} = -\left(P + \left(\frac{2}{3}\eta - \chi\right)\sum_k \partial_k u_k\right)\delta_{ij} + \eta\left(\partial_i u_j + \partial_j u_i\right),$$

$$P = \rho k_B T - \frac{2\pi\rho^2}{3}\int_0^\infty g(r)\frac{d\phi_{\text{int}}(r)}{dr}r^3 dr$$

$$\rho_m e = \frac{\rho k_B T}{\gamma - 1} + 2\pi\rho^2 \int_0^\infty g(r)\phi_{\text{int}}(r)r^2 dr$$

$$\eta = \frac{1}{2}\rho_m D + \frac{\pi}{15D}\rho^2 \int_0^\infty \psi_2(r)g(r)\frac{d\phi_{\text{int}}(r)}{dr}r^3 dr$$

$$\chi = \frac{1}{3}\rho_m D + \frac{\pi}{9D}\rho^2 \int_0^\infty \psi_0(r)g(r)\frac{d\phi_{\text{int}}(r)}{dr}r^3 dr$$
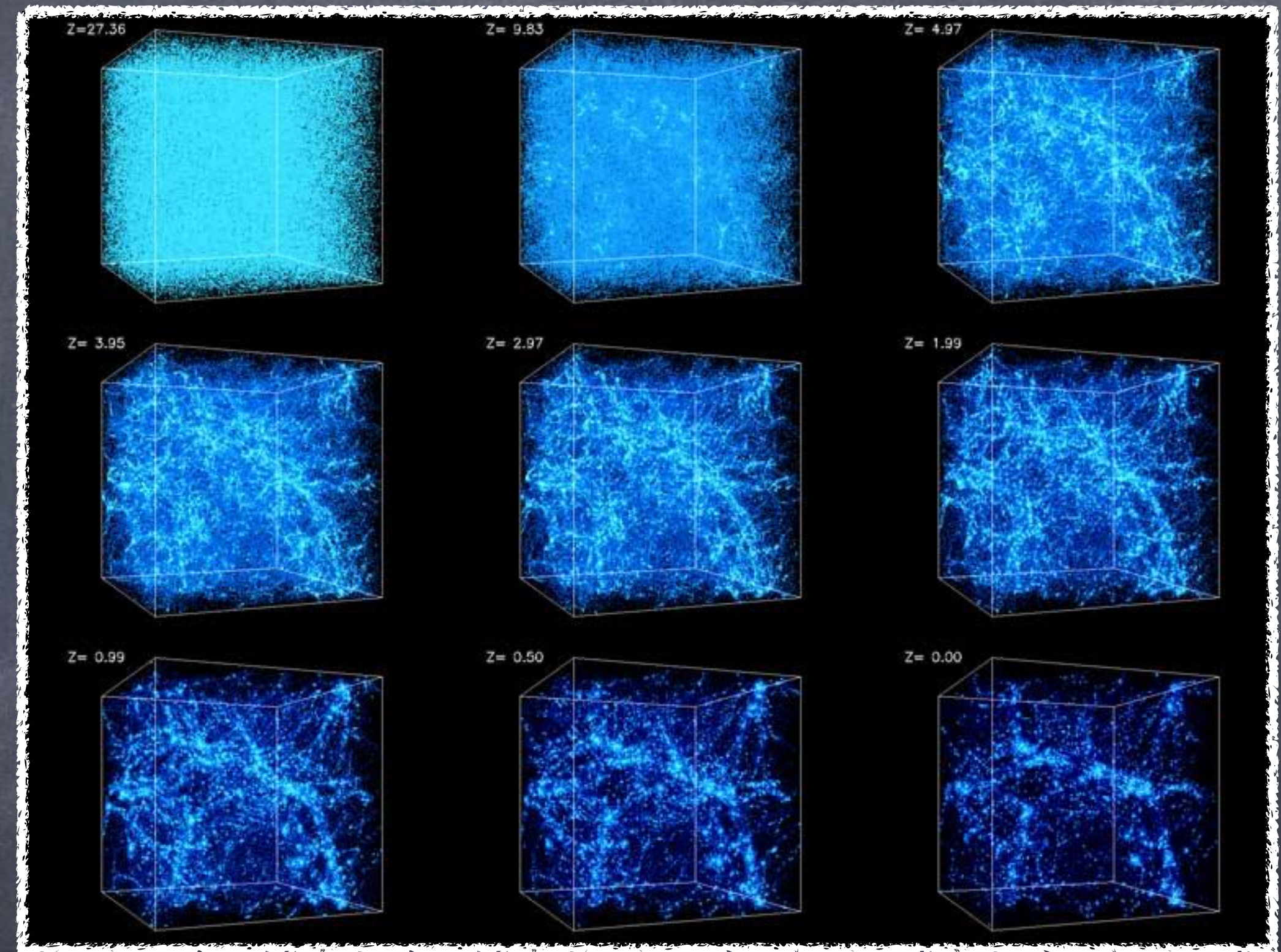
$$\text{with} \quad \phi_{\text{int}}(r) = -\frac{Gm^2}{r}e^{-r/\Delta x},$$

– Non-ideal self-gravitating hydrodynamics

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u}\right) = 0,$$

$$\frac{\partial \rho_m \vec{u}}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u} \otimes \vec{u} + \sigma\right) = \vec{F}_{\text{ext}},$$

$$\frac{\partial \rho_m E}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u} E + \sigma \cdot \vec{u}\right) = \vec{F}_{\text{ext}} \cdot \vec{u},$$

Formation of the large scale structures
The need of **cold** dark matter, i.e. a collisionless fluid with P=0

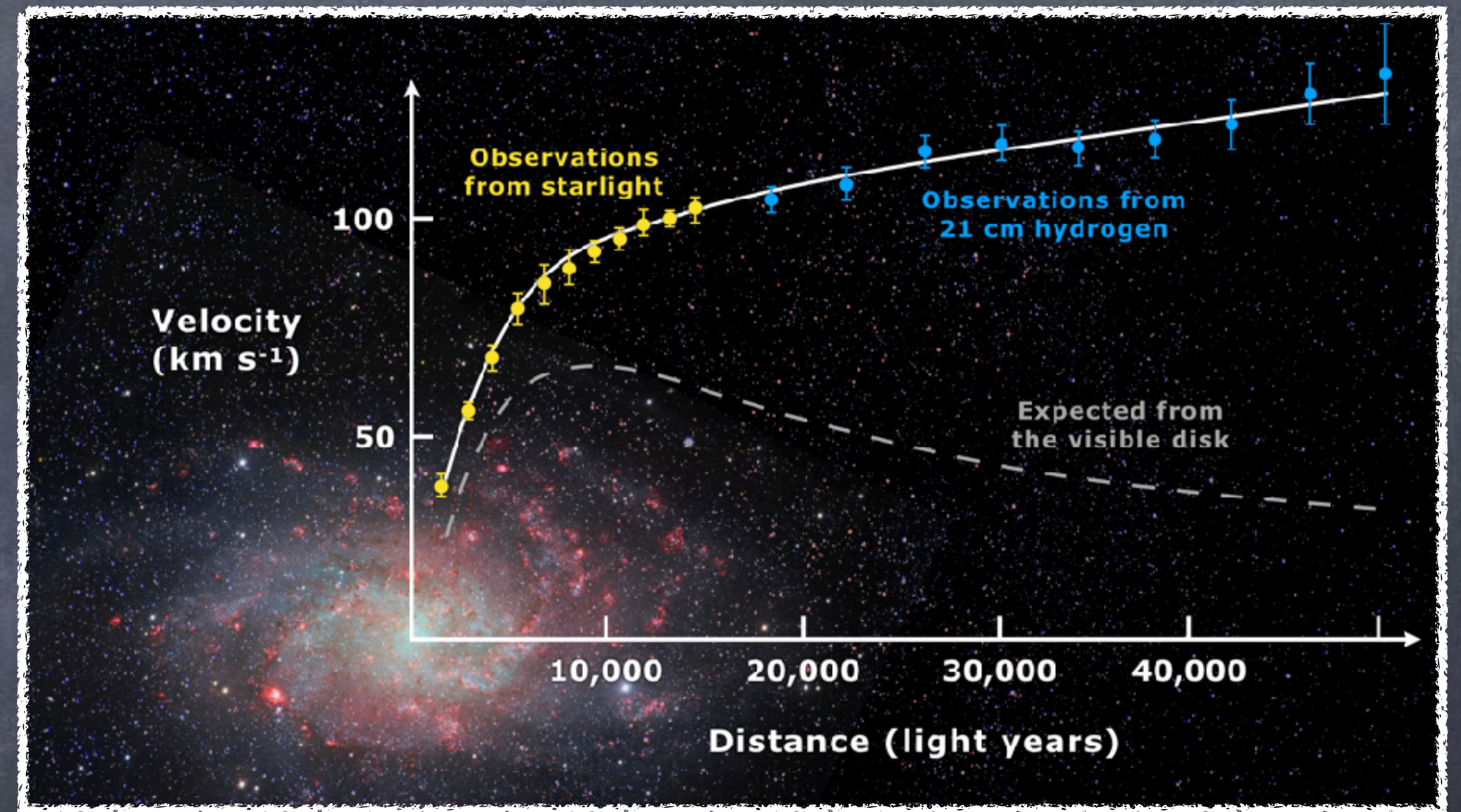$$P = \rho k_B T - \frac{2\pi\rho^2}{3}\int_0^\infty g(r)\frac{d\phi_{\text{int}}(r)}{dr}r^3 dr$$

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– Non-ideal self-gravitating hydrodynamics

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Rubin & Ford 1970

Rotation curve of galaxies

− Non-ideal self-gravitating hydrodynamics

$$\frac{\partial \rho_m}{\partial t} + \overrightarrow{\nabla}\left(\rho_m \overrightarrow{u}\right) = 0,$$

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$0^{\text{th}}, r - \text{component}:$
$$\frac{-(u_\theta^0)^2}{r} = \frac{1}{\rho_m^0} F_{\text{ext,r}}^0$$

$0^{\text{th}}, \theta - \text{component}:$
$$0 = 0,$$

$0^{\text{th}}, r - \text{component}:$
$$\frac{-(u_\theta^0)^2}{r} = -\frac{1}{\rho_m^0}\frac{\partial P^0}{\partial r} + \frac{1}{\rho_m^0} F_{\text{ext,r}}^0$$

$0^{\text{th}}, \theta - \text{component}:$
$$0 = \eta^0 \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_\theta^0}{\partial r}\right) - \frac{u_\theta^0}{r^2}\right).$$

Rotation curve of galaxies
A phase transition to a viscous fluid?
Taylor-Couette flow

$$\eta = \frac{1}{2}\rho_m D + \frac{\pi}{15D}\rho^2 \int_0^\infty \psi_2(r) g(r) \frac{d\phi_{\text{int}}(r)}{dr} r^3 dr$$

$$\text{with } \phi_{\text{int}}(r) = -\frac{Gm^2}{r} e^{-r/\Delta x},$$

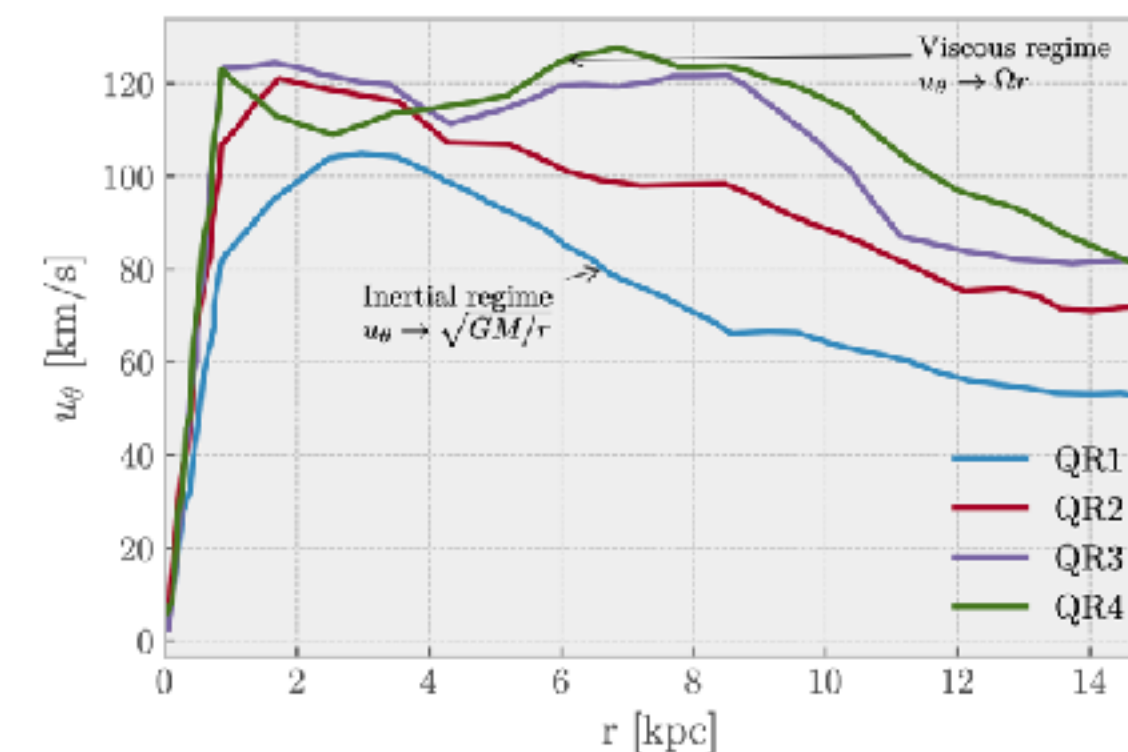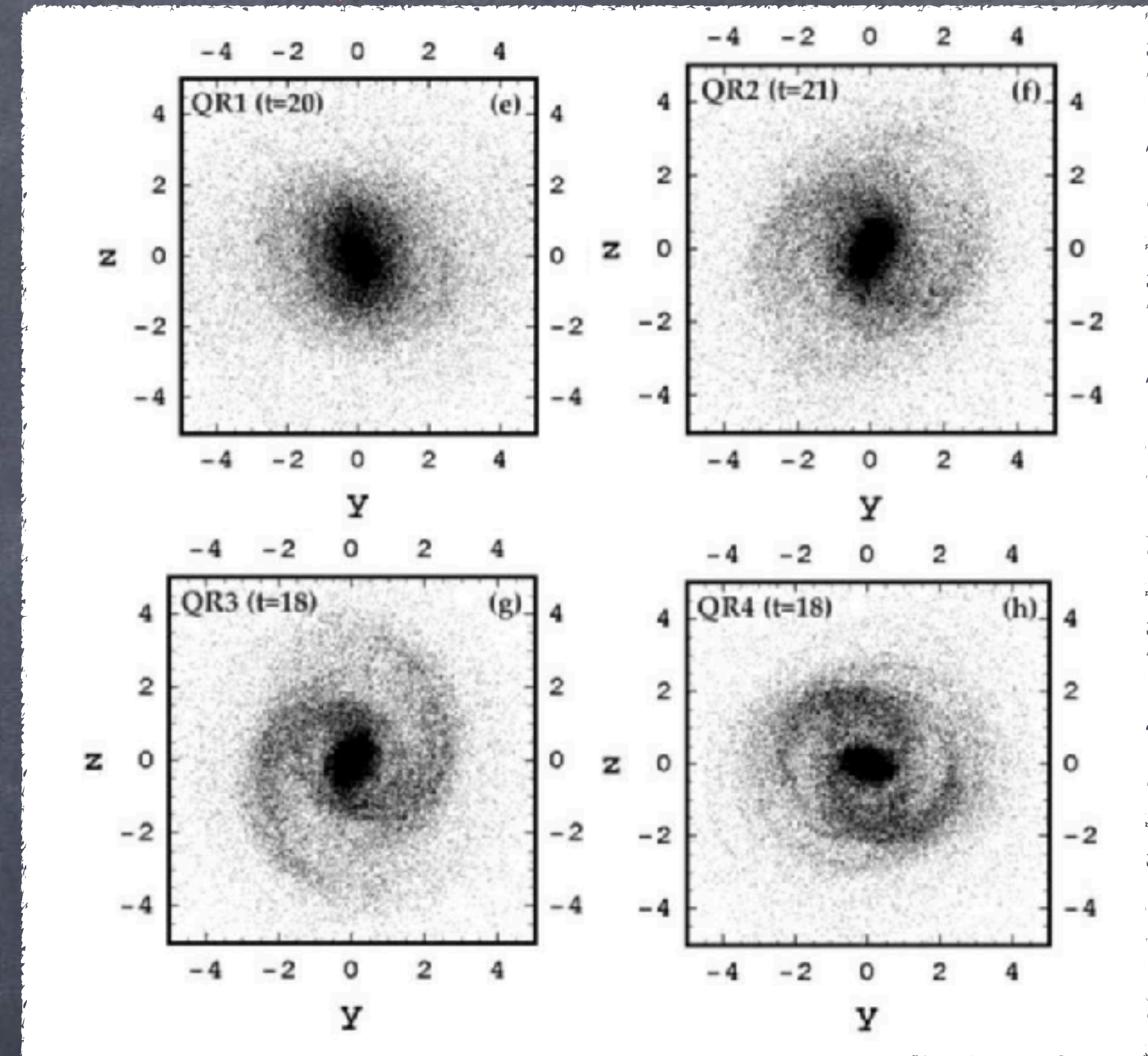# – Non-ideal self-gravitating hydrodynamics

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}\left(\rho_m \vec{u}\right) = 0,$$

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Rotation curve of galaxies
A phase transition to a viscous fluid?
Impulsion transfer by interactions

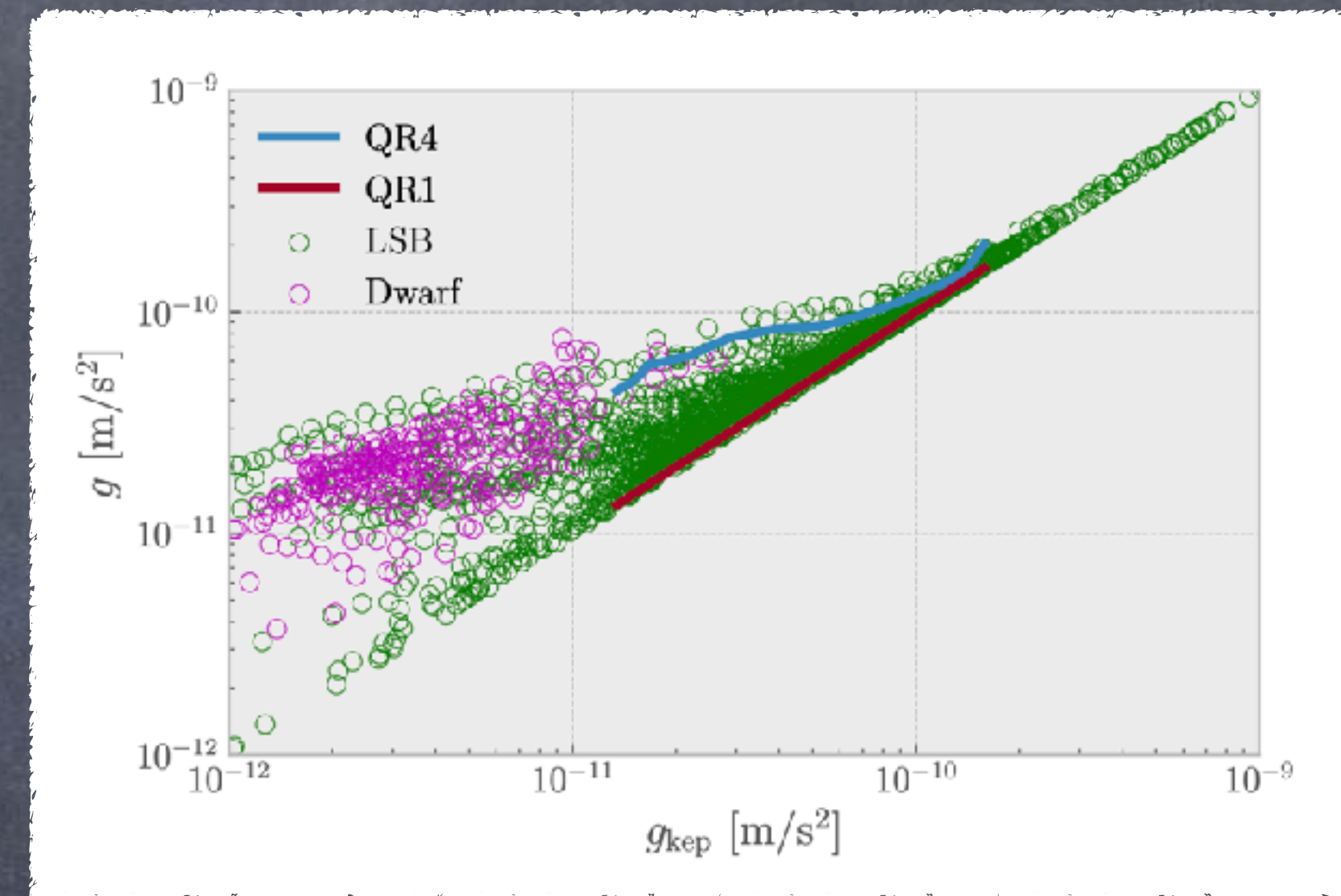Voglis et al. (2006),
Harsoula &
Kalapotharakos (2009)

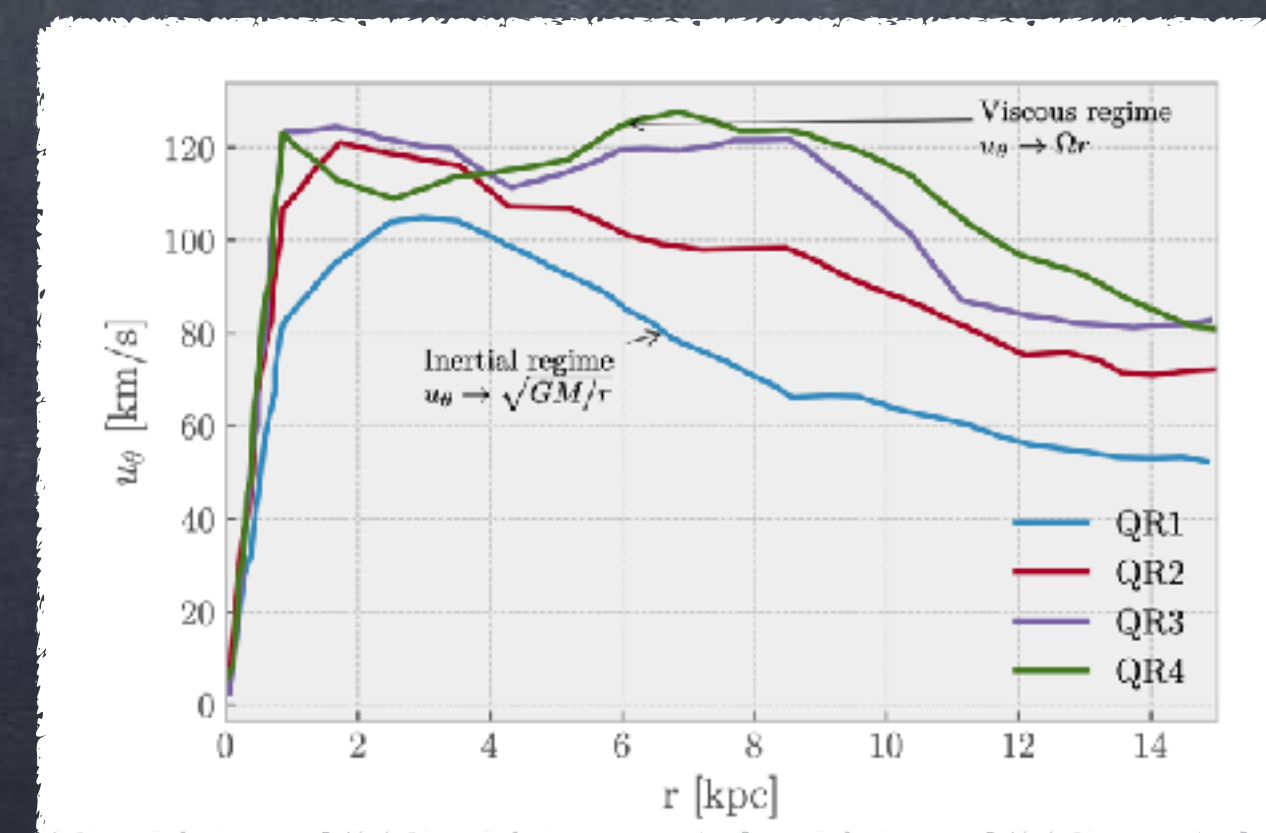# – Non-ideal self-gravitating hydrodynamics

$$\frac{\partial \rho_m}{\partial t} + \overrightarrow{\nabla}\left(\rho_m \overrightarrow{u}\right) = 0,$$

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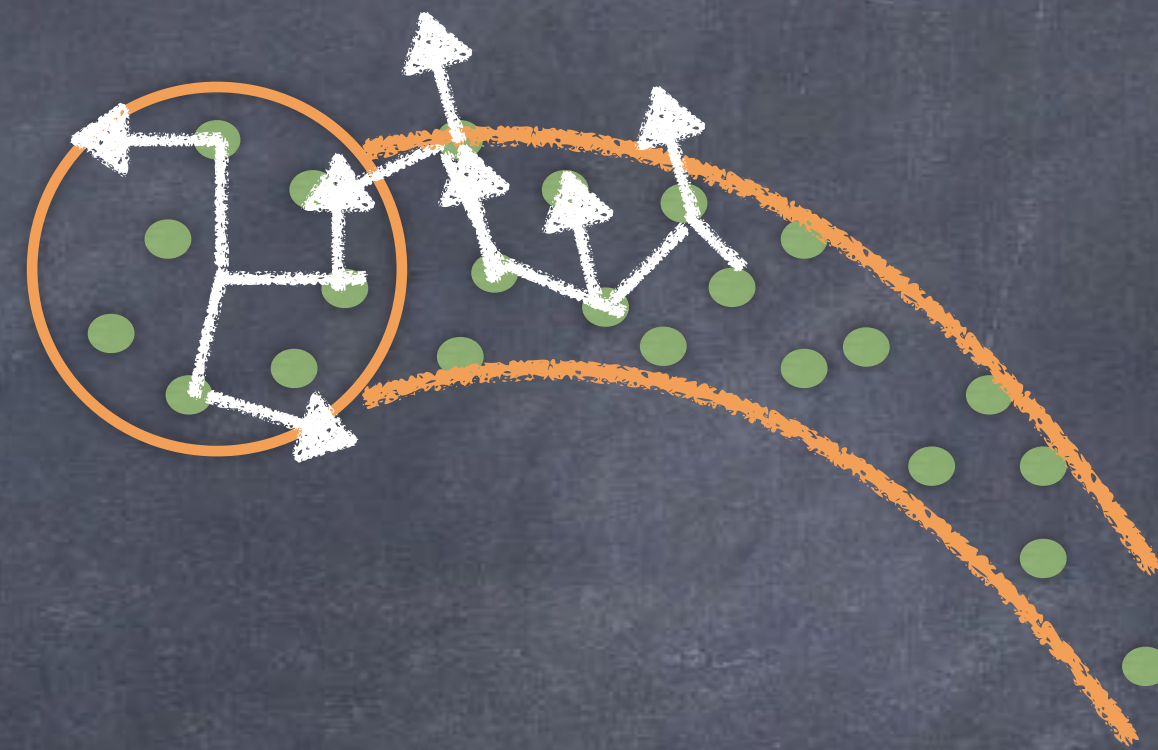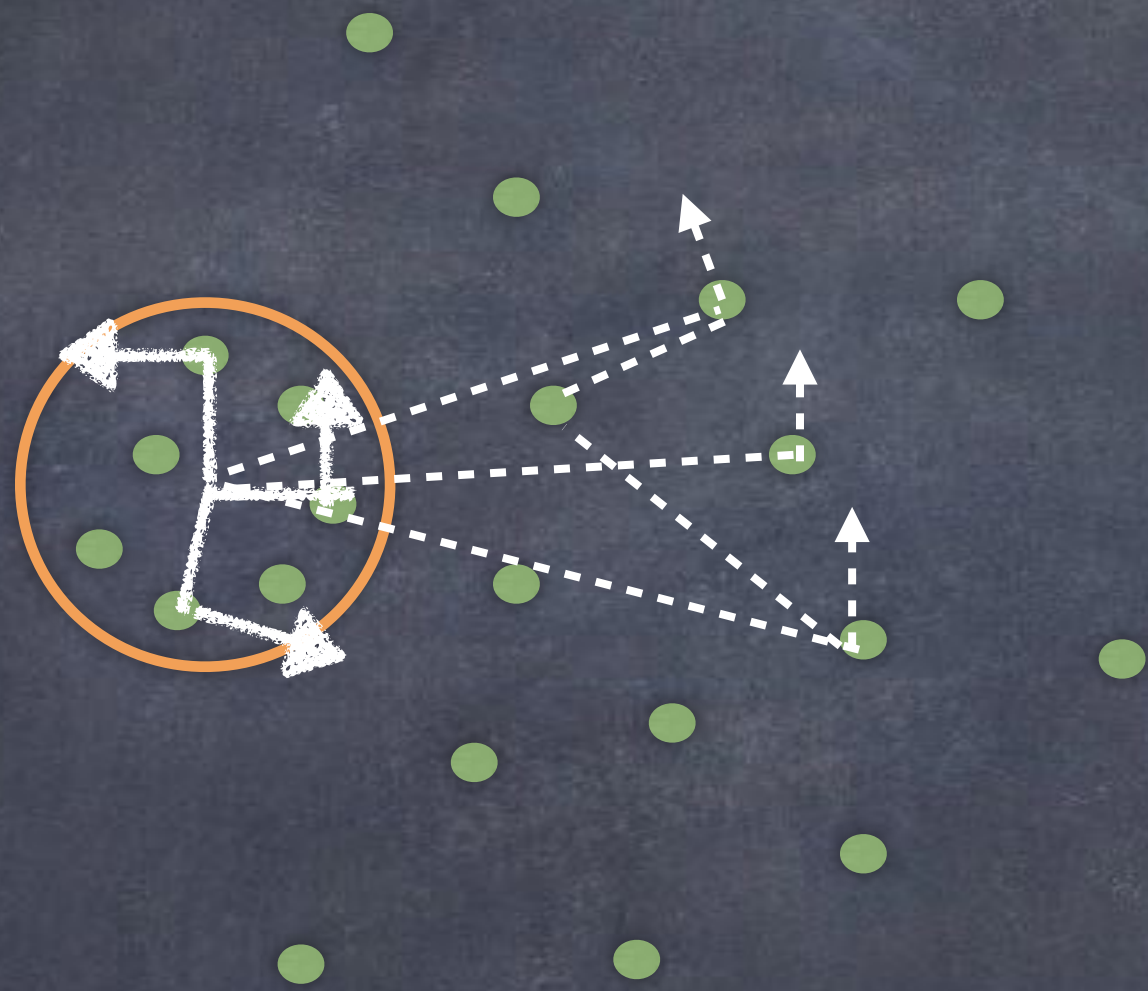

Rotation curve of galaxies
A phase transition to a viscous fluid?

Voglis et al. (2006),
Harsoula &
Kalapotharakos (2009)

– Non-ideal self-gravitating hydrodynamics

Rotation curve of galaxies
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