# From effective interactions to the Giant Monopole Resonance in atomic nuclei

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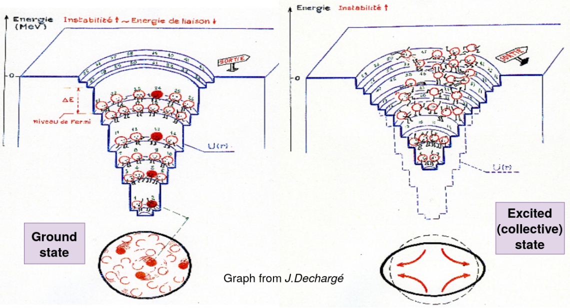
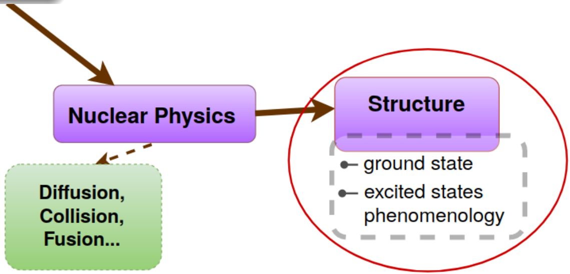
# Nuclear physics as a real challenge!

## Nuclei:

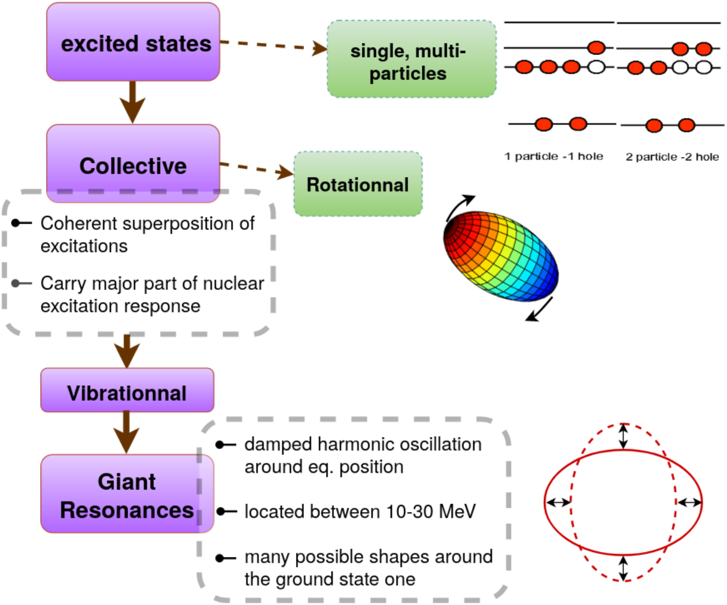
- composed of neutrons + protons (Coulomb interaction treated exactly)
- Quantum system
- Too few nucleons for an accurate stat. treatment
- Self bound system
- Self-consistent potential




$$H\Psi = E\Psi$$



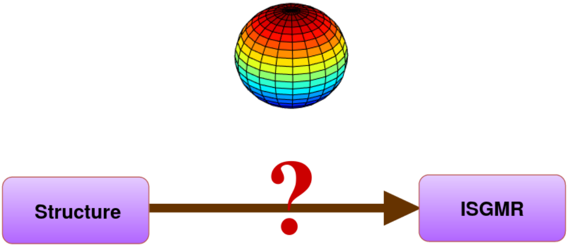
# The path to Giant Resonances



# The "breathing mode"

Many possible GR shapes with p, n, spins $\uparrow$, spins $\downarrow$ combinations

$\rightarrow$ we focus on the IsoScalar Giant Monopole Resonance



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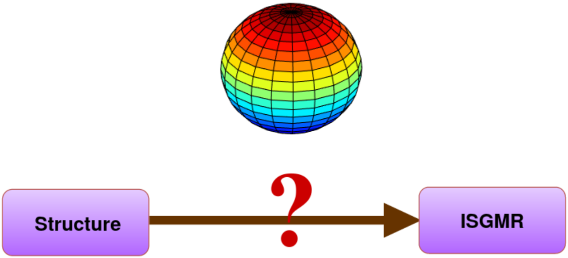

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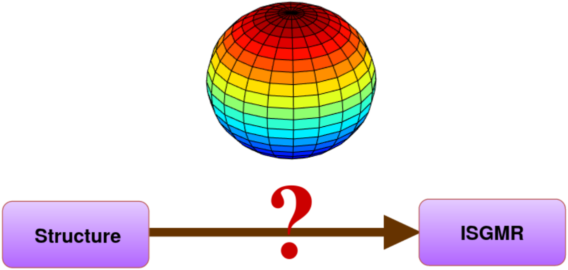

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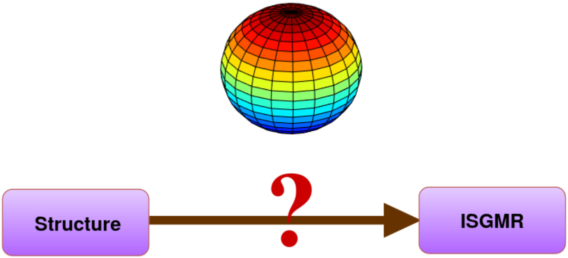

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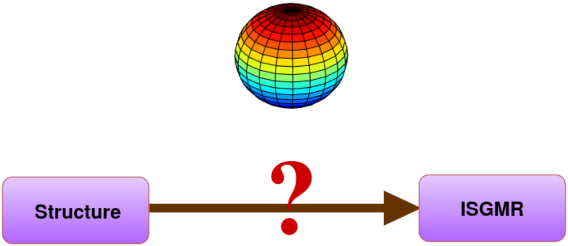

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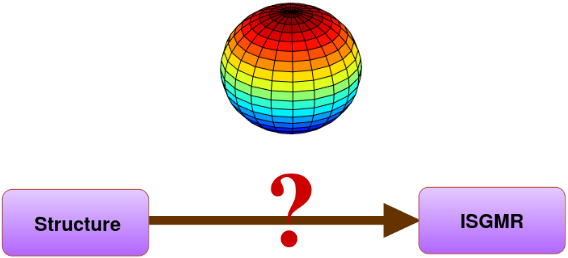

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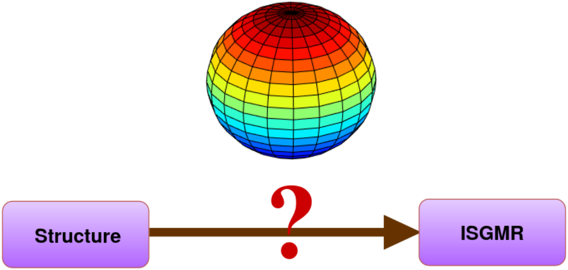

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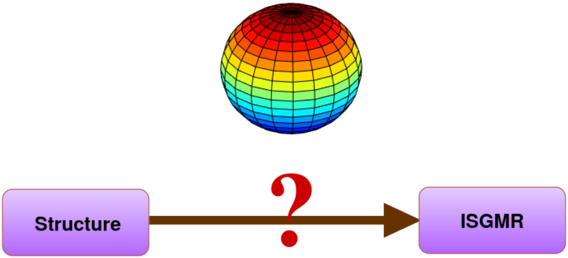

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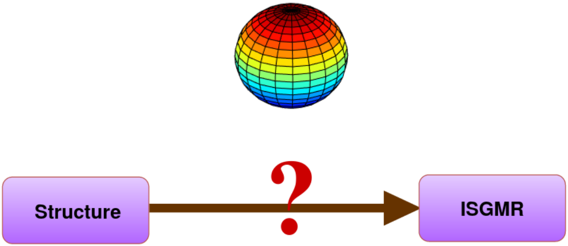
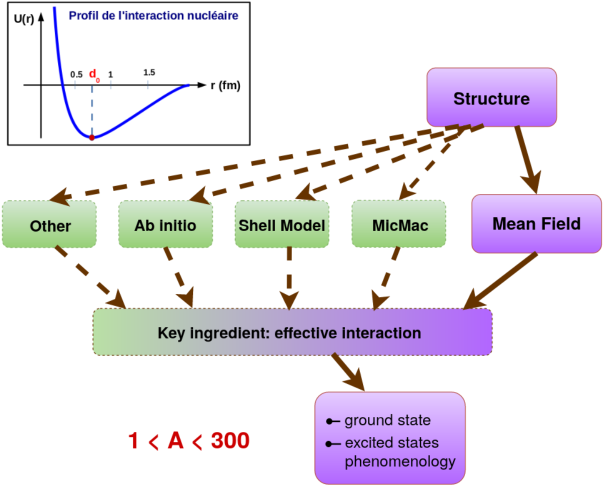


# Main effective models

→ Nuclear interaction in medium = residual QCD interaction ⇒ No mean to know the "true" nuclear pot. so far ! Then...




**1 < A < 300**

if ++ applications, effective models: 👍

# Effective interactions

Can be relativistic or not...

...Involve 2-body terms, 3-body terms...

... with zero-range (Skyrme), a finite range (Gogny, M3Y)...

**...generally composed of:**

- Central terms
  → including effective N-body term(s) if not explicit
- Spin-orbit term
- Tensor term

$$
\begin{aligned}
V_{12} \quad = \quad & \sum_{j=1}^{2}(W_j + B_j P_\sigma + H_j P_\tau + M_j P_\sigma P_\tau) e^{\frac{-(\vec{r}_1 - \vec{r}_2)^2}{\mu_j^2}} \\
& + t_3(1 + x_0 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho^\alpha(\frac{\vec{r}_1 + \vec{r}_2}{2}) \\
& + i W_{LS} \overleftarrow{\nabla}_{12}\delta(\vec{r}_1 - \vec{r}_2) \wedge \vec{\nabla}_{12}(\vec{\sigma}_1 + \vec{\sigma}_2)
\end{aligned}
$$

Effective phenomenological interactions → free parameters

One must constrain them for 1 given analytical form!

# Constraining effective interaction: Infinite Nuclear Matter + Nuclei

## INM :=

- infinite ideal medium related to the EoS at saturation point $\rho_0$
- Homogeneous $\rho = \rho_0$ and isotropic
- No more surface nor Coulomb effects
- Provide constrains: Binding energy, compressibility, effective mass...

**INM** introduction motivated by the core nucleonic matter of nuclei

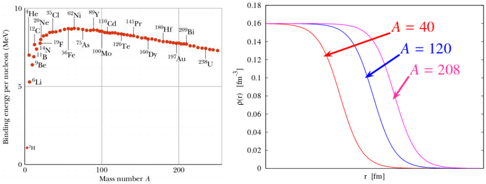
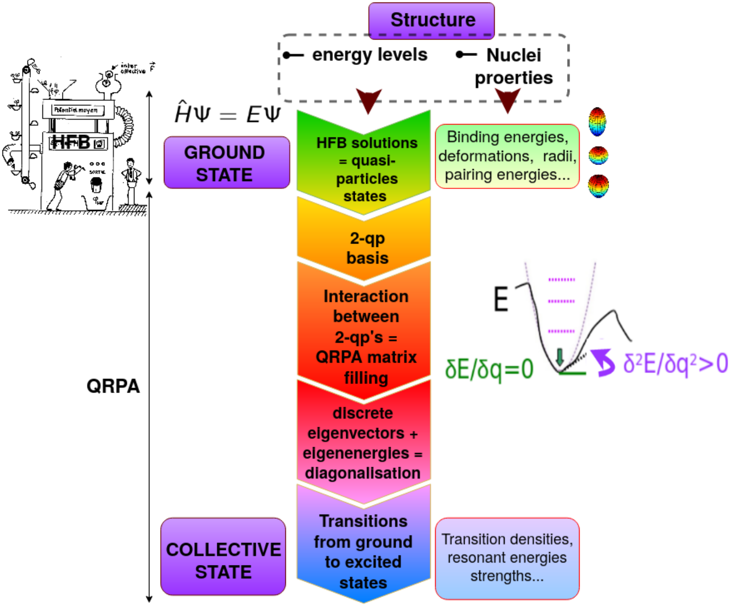



Figure: schemes from *K. Bennaceur* classes

## Nuclei

$\rightarrow$ Constitute additional constrains: charge radii, pairing energies...

# Constructing theoretical treatment



# PhD motivation: understanding differences between theory and experiment

- $^{208}Pb$ ISGMR nicely reproduced by a wide scope of theoretical models
  ... But not Sn isotopes
  ... And then not Cd isotopes

How do we position ourselves ? What do we reproduce ?

| Nucleus | main peak exp. energy (MeV) | main peak th. energy *(MeV)* | *main peak % EWSR* th. | $m_1/m_0$ exp. (MeV) | $m_1/m_0$ th. (MeV) |
|---|---|---|---|---|---|
| $^{56}$Ni | $19.1 \pm 0.5$ | 20.82 | 97.52 | - | 20.94 |
| $^{58}$Ni | $18.43 \pm 0.15$ | 20.10 | 74.54 | $19.20^{+0.44}_{-0.19}$ | 20.76 |
| $^{60}$Ni | $17.62 \pm 0.15$ | 19.41 | 59.88 | $18.04^{+0.35}_{-0.23}$ | 20.44 |
| $^{90}$Zr | $16.55 \pm 0.08$ | 18.21 | 73.10 | $18.13 \pm 0.09$ | 18.42 |
| $^{92}$Zr | $16.12 \pm 0.04$ | 17.22 | 37.90 | $18.05 \pm 0.05$ | 18.23 |
| $^{112}$Sn | $16.1 \pm 0.1$ | 17.04 | 77.27 | $16.2 \pm 0.1$ | 17.03 |
| $^{204}$Pb | $13.8 \pm 0.1$ | 14.0 | 91.9 | - | 14.1 |
| $^{206}$Pb | $13.8 \pm 0.1$ | 14.0 | 85.8 | - | 14.1 |
| $^{208}$Pb | $13.7 \pm 0.1$ | 13.8 | 84.0 | - | 14.1 |

Table: $E0$ experimental and theoretical ISGMR main peak values and related quantities

# Transition operator and reaction kinematics

- Standard operator in the long wavelength limit
  = small momentum transfer q approx
  ⇒ q neglected

- Complete (electromagnetic) operator
  ⇒ q accounted for

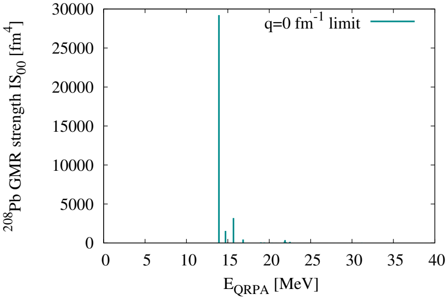



Figure: *IS response in $^{208}Pb$*

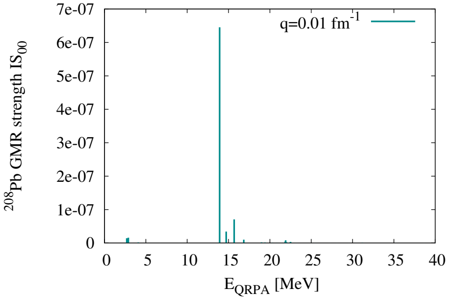



Figure: *IS response evolution with q in $^{208}Pb$*

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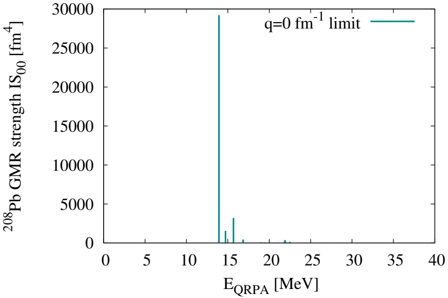


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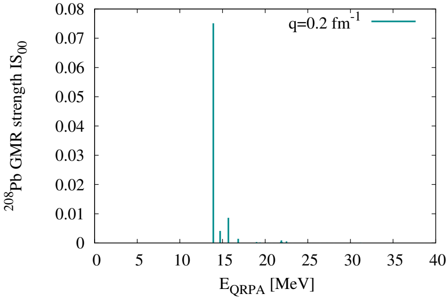


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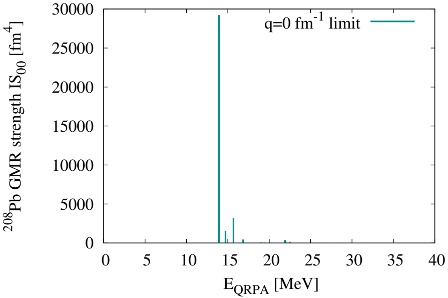


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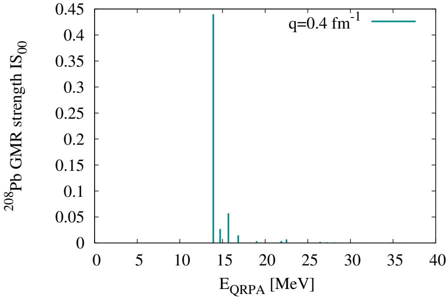


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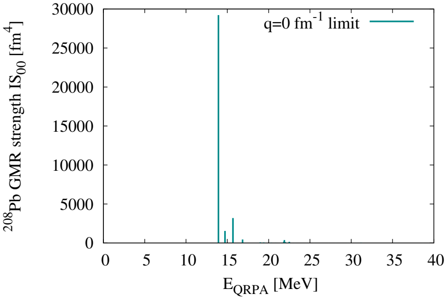


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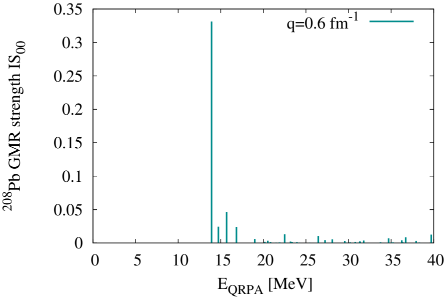


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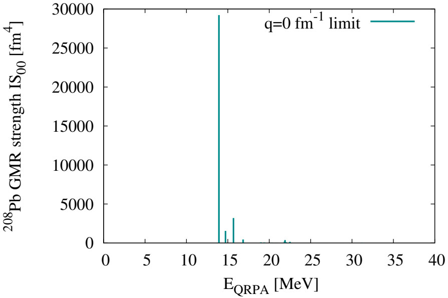


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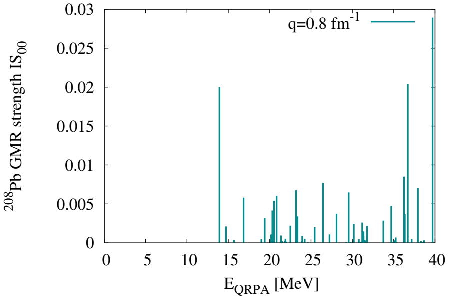
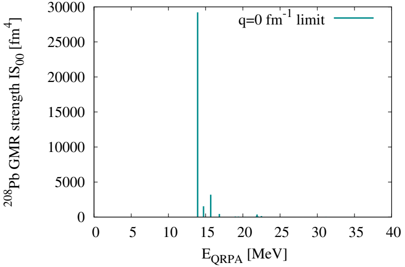



Figure: *IS response evolution with q in* $^{208}Pb$

# Transition operator and reaction kinematics

- the variable q depends on the the reaction kinematics


Figure: Inelastic scattering process considered

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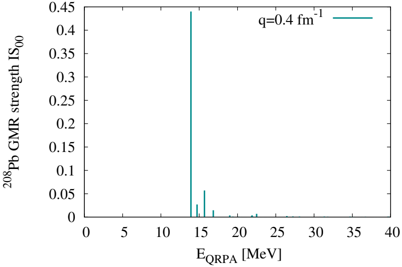


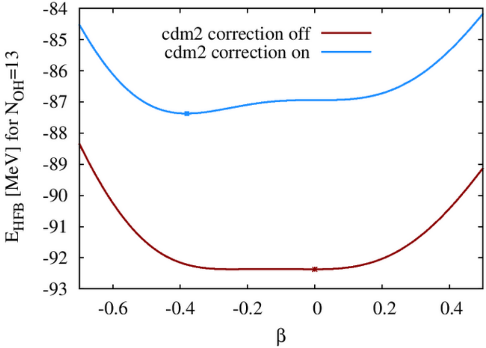

Figure: *IS response evolution with q in* $^{208}Pb$

**Best suiting operator but...**

... Does not explain the theory-experiment gap

# 2-body center-of-mass correction and influence in $^{12}C$

Mean field ⇒ defined position set in space ⇒ translation symetry broken ⇒ spurious contribution to the total Energy




**Reexamination of isoscalar giant resonances in $^{12}C$ and $^{93}Nb$ through $^{6}Li$ scattering**

J. C. Zamora,[1] C. Sullivan,[2,3,4] R. G. T. Zegers,[2,3,4] N. Aoi,[5] L. Batail,[6] D. Bazin,[2,4] M. Carpenter,[7] J. J. Carroll,[8] I. Deloncle,[9,6] Y. D. Fang,[5] H. Fujita,[5] U. Garg,[10] G. Gey,[5] C. J. Guess,[11,*] M. N. Harakeh,[5,12] T. H. Hoang,[5] E. Hudson,[11] N. Ichige,[13] E. Ideguchi,[5] A. Inoue,[5] J. Isaak,[5,14] C. Iwamoto,[15] C. Kacir,[11] N. Kobayashi,[5] T. Koike,[13] M. Kumar Raju,[5] S. Lipschutz,[2,3,4] M. Liu,[16] P. von Neumann-Cosel,[14] S. Noji,[2,3] H. J. Ong,[5,†] S. Péru,[6] J. Pereira,[2,3] J. Schmitt,[2,3,4] A. Tamii,[5] R. Titus,[2,3,4] V. Werner,[14] Y. Yamamoto,[5] X. Zhou,[16] and S. Zhu[7]

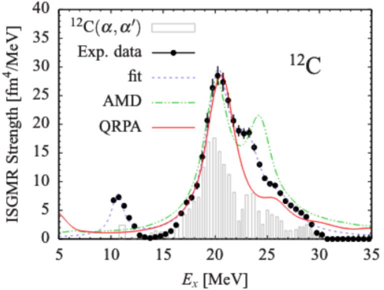
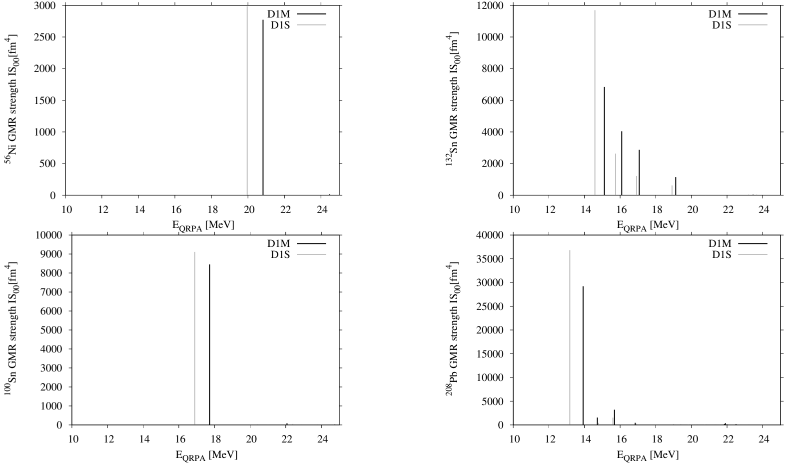



FIG. 5. ISGMR strength of $^{12}C$. The data are compared with results from a $^{12}C(\alpha,\alpha')$ experiment reported in Ref. [25]. The dashed line is a sum of multiple Lorentzian functions fitted to the data (see Table II). The dash-dotted line corresponds to a theoretical calculation based on antisymmetrized molecular dynamics extracted from Ref. [45]. The solid red line is a QRPA calculation (shifted by a constant factor $\Delta = 1$ MeV) as described in the text.

# Influence of the interaction used: D1S vs D1M




## strong interaction influence

- D1S $\neq$ from D1M by quasi-constant $\Delta E$
- D1M: shift ($\approx$ 1.5 MeV) for ALL nuclei but Pb isotopes (used for fitting)
- D1S: shift ($\approx$ 0.6 MeV) for ALL nuclei

# Conclusions

**Main studies:**

- Use of a complete operator: more rigorous but not sufficient
- Cdm2 corrections: shifting resonant states to lower energies but not enough
- Good reproduction of global ISGMR shape with D1M and D1S but shifts

# What to do now?

$$V_{12} = \sum_{j=1}^{2,3?}(W_j + B_j P_\sigma + H_j P_\tau + M_j P_\sigma P_\tau) e^{\frac{-(\vec{r}_1 - \vec{r}_2)^2}{\mu_j^2}}$$

$$+ t_3(1 + x_0 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho^\alpha(\frac{\vec{r}_1 + \vec{r}_2}{2})$$

$$+ i W_{LS} \overleftarrow{\nabla}_{12}\delta(\vec{r}_1 - \vec{r}_2) \wedge \vec{\nabla}_{12}(\vec{\sigma}_1 + \vec{\sigma}_2)$$

**Perspectives: focus on the interaction**

→ Need for a new set of parameters ?
→ Need for a new analytical expression ?

## Thank you for your attention !

# La correction du centre de masse à 2 corps

Champ moyen $\Rightarrow$ localisation noyau dans l'espace $\Rightarrow$ brisure de la symétrie de translation $\Rightarrow$ contribution spurieuse à l'énergie etc

Il faut enlever cette vilaine contribution !

Imposer la nullité du moment linéaire total:

$$\mathbf{P} = \sum_i \mathbf{p}_i = \mathbf{0} \tag{1}$$

Revient à soustraire l'énergie associée

$$\frac{\mathbf{P}^2}{2mA} = \frac{1}{2mA}[\sum_i \mathbf{p}_i^2 + \sum_{i \neq j} \mathbf{p}_i . \mathbf{p}_j] \tag{2}$$

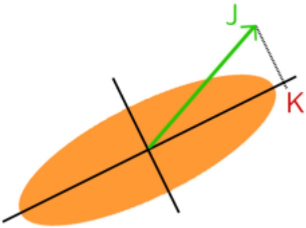
- ✾ On regarde en parallèle l'impact du centre de masse à 2 corps (cdm2) sur les orbitales HFB
- ✾ + Programmation en cours de la correction du terme cdm2 dans les champs QRPA

# Formalisme de l'état fondamental: approximation Hartree-Fock-Bogoliubov

- approximation HFB: champ moyen + appariement (niveau tres proches en energie)
- Développement des fonctions tests sur une base d'un OH axial. Passage de la base OH à la base excitations particule-trou (ph) et *vice verse*:

$$\begin{pmatrix} c \\ c^+ \end{pmatrix} = \mathcal{B} \begin{pmatrix} \eta \\ \eta^+ \end{pmatrix} = \begin{pmatrix} U & V* \\ V & U* \end{pmatrix} \begin{pmatrix} \eta \\ \eta^+ \end{pmatrix} \quad (3)$$

- noyau sphérique $\Rightarrow$ J bon nombre quantique VS noyau déformé $\Rightarrow$ la dégénerescence selon $J_Z \equiv K$ est levée,K nouveau bon nombre quantique



# Formalisme des états excités: l'approximation QRPA

- Quasi-particule: ni une particule ni un trou mais une combinaison des 2

$$\theta_n^+ = \frac{1}{2}\sum_{ij}(X_n^{ij}\eta_i^+\eta_j^+ - Y_n^{ij}\eta_j\eta_i) \quad (4)$$

- Approximation harmonique: états excités construits dans un potentiel harmonique autour de l'énergie HFB minimum

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \omega_n \begin{pmatrix} X_n \\ Y_n \end{pmatrix} \quad (5)$$

- où $\omega_n$ sont les énergies QRPA des $|n>$ états excités construits sur le vide QRPA:

$$\theta_n^+|\tilde{0}>=|n> \quad (6)$$