Applications of the renormalisation group equations :

From Grand Unification theories to epidemiology

COT Corentin

Institut de Physique des 2 Infinis de Lyon
Université Claude Bernard de Lyon

PhD Day- 27 January 2021

Outline

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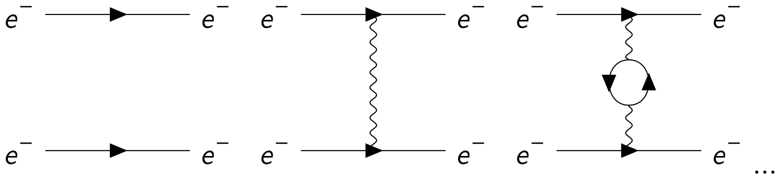
Quantum Field Theory

- Description of nature mixing field theory, quantum description and special relativity
- Quantities $\mathcal{O}$ can be computed by looking at Feynman diagrams



- Problem : loops diagrams gives infinities → Renormalisation

Scale invariance of the scaling function

- The theory of renormalisation implies that it exists a scale-invariant function β such that for any coupling $\mathcal{O}$:

$$\frac{\partial \mathcal{O}}{\partial \mathrm{Ln}\mu} = \beta(\mathcal{O}) \tag{1}$$

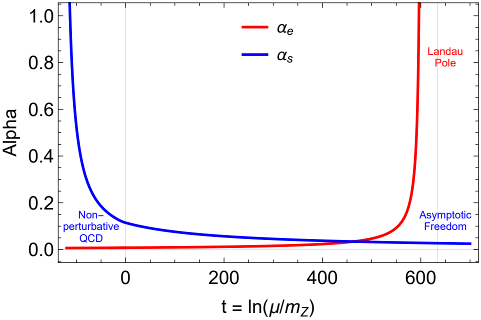
- Famous equations for QED and QCD at one-loop :

$$\frac{d\alpha_e}{dLn(\mu)} = \frac{2}{3\pi}\alpha_e^2 \tag{2}$$

$$16\pi^2 \frac{d\alpha_s}{dLn(\mu)} = \left(\frac{2}{3}n_f - 11\right)\alpha_s^2 \tag{3}$$

Running of the coupling constants

- Solving the equations for $\alpha_e(m_Z) = \frac{1}{127.5}$ and $\alpha_s(m_Z) = 0.115$, we find :

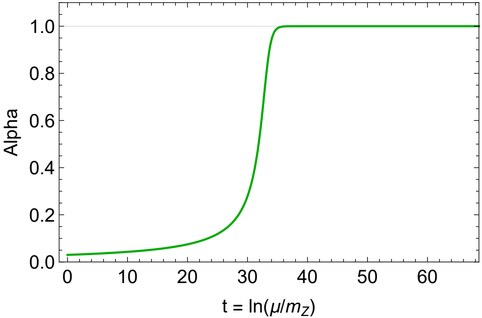


- Two Landau poles : one in UV for α_e one in IR for α_s → QED and QCD are effective theories

Fixed points

- $\beta(\alpha_*) = 0 \rightarrow \alpha_*$ fixed point
- Fixed points are the sign of a scale-invariant theory.

$$\frac{d\alpha}{dt} = a\alpha - b\alpha^2 = a\alpha\left(1 - \frac{b}{a}\alpha^2\right) \rightarrow \alpha_* = 0 \text{ or } \frac{a}{b} \qquad (4)$$

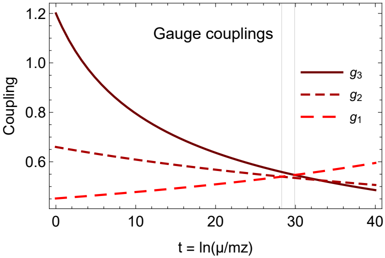


Outline

The Standard Model

$$G_{SM} = SU(3)_c \times SU(2)_W \times U(1)_Y$$

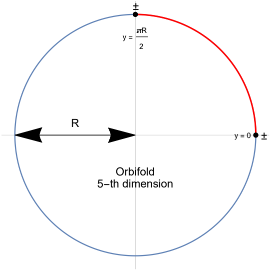
- 3 couplings associated to the 3 gauge groups : g_1, g_2 and g_3.



- Coupling constants cross each other at different scales.
- Classical unification = Intersection point.
- Problem : Couplings unify at 2 different scales.

5-th dimension

- Extra-dimensions cannot be seen as they are compactified $\rightarrow$ Papersheet rolled on itself.



- Orbifold $S^1/\left(Z_2 \times Z_2'\right)$ of radius $R \rightarrow$ 2 parity symmetries at points $y = 0$ and $y = \frac{\pi R}{2}$.
- 5D dimension fields $\xrightarrow{Fourier}$ Sum of 4D fields with mass $\frac{2n}{R}$.
- Only zero-modes (so $(+,+)$ fields) can be seen at the SM energy scale.

$SU(5)$ Unification

- The Symmetry group $SU(5)$ contains $SU(3) \times SU(2) \times U(1)$.
- Fermion content :

$$\phi_5 = \begin{pmatrix} H \\ \phi_h \end{pmatrix}, \quad \psi_{5_{L/R}} = \begin{pmatrix} b \\ L^c \end{pmatrix}_{L/R}, \quad \psi_{\overline{5}_{L/R}} = \begin{pmatrix} B^c \\ l \end{pmatrix}_{L/R}, \tag{5}$$

$$\psi_{10_{L/R}} = \frac{1}{\sqrt{2}} \begin{pmatrix} T^c & q \\ & \mathcal{T}^c \end{pmatrix}_{L/R}, \quad \psi_{\overline{10}_{L/R}} = \frac{1}{\sqrt{2}} \begin{pmatrix} t & Q^c \\ & \tau \end{pmatrix}_{L/R}, \tag{6}$$

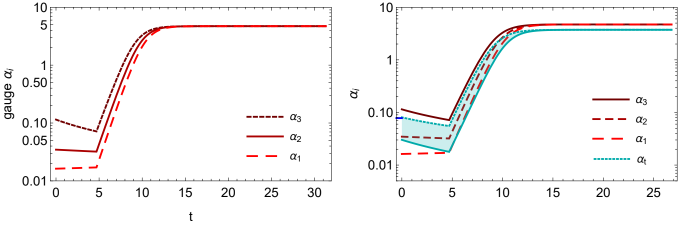
- Small letters are SM fields and capital letters are new fields called Indalo ("creation" in Zulu) fields (noted ) with the same SM quantum numbers as their associated SM field.

Renormalisation Group Equation for aGUT

- Renormalisation group equation for the one-loop factor gauge coupling constants :

$$2\pi \frac{d\alpha_i}{dt} = b_i^{SM} \alpha_i^2 + \left(m_Z R e^t - 1 \right) b_5 \alpha_i^2 \tag{7}$$

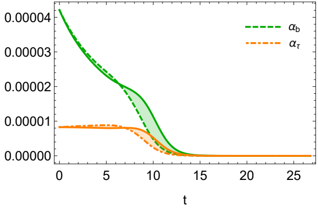
with $b_i^{SM} = (\frac{41}{10}, -\frac{19}{6}, -7)$ and $b_5 = -\frac{52}{3} + \frac{16}{3} n_g$.



- All couplings constants are going to the same non-zero UV fixed point $(-\frac{2\pi}{b_5})$ asymptotically. (If $n_g < 3$)
- Scale of Unification $\neq$ Scale of compactification

Advantages of the model

- Bottom and tau yukawa can be added by considering them localised on the brane $y = \frac{\pi R}{2}$.

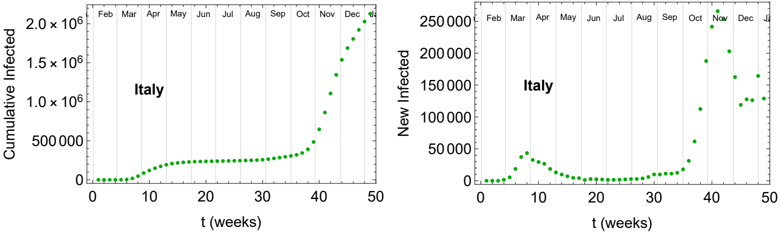


- No proton decay.
- Dark Matter candidate (the lightest -field)
- Baryogenesis and Leptogenesis.
- $SU(5)$ Simplest model $\rightarrow$ More complete models as $SO(10)$.

Outline

Covid-19 pandemics raging over the globe

- In the end of 2019, SARS-Cov2 virus appeared, creating pandemics across the World in 2020.
- Number of infected cases behaving like waves



- Fixed points behaviour

The framework basis

- The main equation :

$$\frac{dI}{dt} = \gamma I(t)\left(1 - \frac{I(t)}{a}\right) \quad (8)$$

- Two parameters needed to reproduce a wave : γ the "infection rate per time unit" and a the total number of cases.

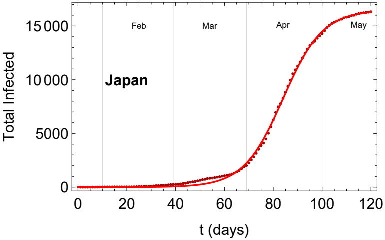


Figure: Logistic function for Japan with $\gamma = 0.123\ day^{-1}$ and $a = 16479$.

Interactions between countries

- Basis equation for each country + interaction term

$$\frac{dI_i}{dt} = \gamma_i I_i(t)\left(1 - \frac{I_i}{a_i}\right) + \sum_j k_{ij}\left(I_j(t) - I_i(t)\right) \qquad (9)$$

- k_{ij} represent the number of traveller from country j to country i per time unit (of order 1,000 to 10,000 per week).

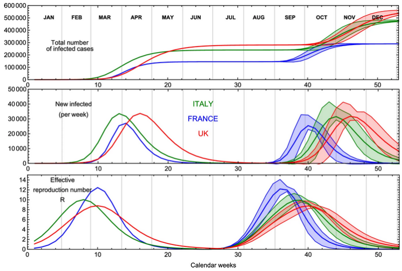


Figure: https://www.nature.com/articles/s41598-020-72611-5

Endemic phase and multiwave pattern

$$\frac{dI}{dt} = \gamma I(t)\left[\left(1-\frac{I(t)}{a}\right)^2-\delta_0\right]^{p_0}\prod_{\rho=1}^{w}\left[\left(1-\zeta_\rho\frac{I(t)}{a}\right)^2-\delta_\rho\right]^{p_\rho}$$

- Constant new number of cases $=$ complex fixed point δ.
- p_ρ $=$ steepness of wave ρ, ζ_ρ $=$ total number of cases ratio between wave 1 and wave ρ.

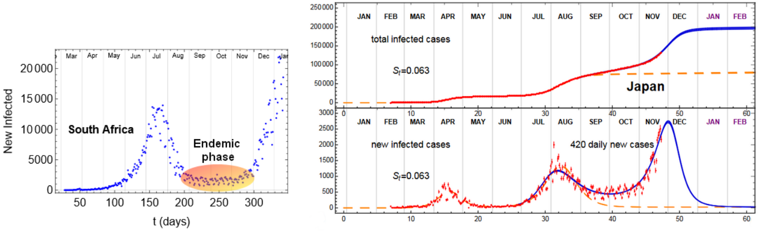


Figure: https://arxiv.org/abs/2011.12846

eRG formalism achievements

Using the eRG approach, we were able to achieve :

- Prediction of the timing of the second wave in Europe.
- Using Google and Apple mobility data, we were able to see a 2-4 weeks effect of the Social Distancing measures.
- More precise predictions on the third wave in various countries around the world using complex fixed point framework.
- Understand the geographic spread of the virus in the US by using the interaction between US division and flight data.
- Link the eRG formalism with other epidemiological models from lattice and percolation models to compartmental models in a coherent way.

Outline

Conclusion

Renormalisation Group Equations permitted me to :

- Develop a new Grand Unification scheme getting rid of proton decay, having a DM candidate and reproducing the baryogenesis/leptogenesis.
- Study the spread of the Covid-19 pandemic using fixed points to simulate the future of an ongoing pandemic wave and to understand the effect on the spreading of social distancing, vaccination, interaction between countries...

Thanks

Thank you for your attention !
And special thanks to Lucrezia, Shahram and all the "Equipe Séminaire" for organizing this conference !
Let's go for the questions !

Outline

Publications

Asymptotic Grand Unification

Giacomo Cacciapaglia, Alan S. Cornell, Corentin Cot and Aldo Deandrea.
Minimal SU(5) Asymptotic Grand Unification.
arXiv

Epidemic Renormalisation Group

Cacciapaglia, G., Cot, C. Sannino, F.
Second wave COVID-19 pandemics in Europe: a temporal playbook.
Sci Rep 10, 15514 (2020).

Giacomo Cacciapaglia, Corentin Cot and Francesco Sannino.
Mining Google and Apple mobility data: Temporal Anatomy for COVID-19 Social Distancing.
Sci Rep (2021).

Giacomo Cacciapaglia, Corentin Cot, Francesco Sannino.
Multiwave pandemic dynamics explained: How to tame the next wave of infectious diseases.
arXiv

Giacomo Cacciapaglia, Corentin Cot, Anna Sigridur Islind, María Óskarsdóttir and Francesco Sannino.
You better watch out: US COVID-19 wave dynamics versus vaccination strategy.
arXiv

Giacomo Cacciapaglia, Corentin Cot, Michele Della Morte, Stefan Hohenegger, Francesco Sannino, Shahram Vatani.
The field theoretical ABC of epidemic dynamics.

Appendix A : Fields quantum numbers

Multiplets	Fields	L	B	Q	Q_3
$\psi_{\overline{5}}$	B_R^c	1/2	1/6	1/3	0
	$\boxed{\tau_L}$	1	0	-1	-1
	$\boxed{\nu_L}$	1	0	0	1
ψ_5	$\boxed{b_R}$	0	1/3	-1/3	0
	$\mathcal{T}_L^c$	-1/2	1/2	1	1
	$\mathcal{N}_L^c$	-1/2	1/2	0	-1
ψ_{10}	T_R^c	1/2	1/6	-2/3	0
	$\mathcal{T}_R^c$	-1/2	1/2	1	0
	$\boxed{t_L}$	0	1/3	2/3	1
	$\boxed{b_L}$	0	1/3	-1/3	-1
$\psi_{\overline{10}}$	$\boxed{t_R}$	0	1/3	2/3	0
	$\boxed{\tau_R}$	1	0	-1	0
	T_L^c	1/2	1/6	-2/3	-1
	B_L^c	1/2	1/6	1/3	1
ϕ_5	H	1/2	-1/6	-1/3	0
	$\boxed{\phi^+}$	0	0	1	1
	$\boxed{\phi_0}$	0	0	0	-1
A_X	X	1/2	-1/6	-4/3	-1
	Y	1/2	-1/6	-1/3	1

Figure: Quantum numbers for the fields considered in the model.

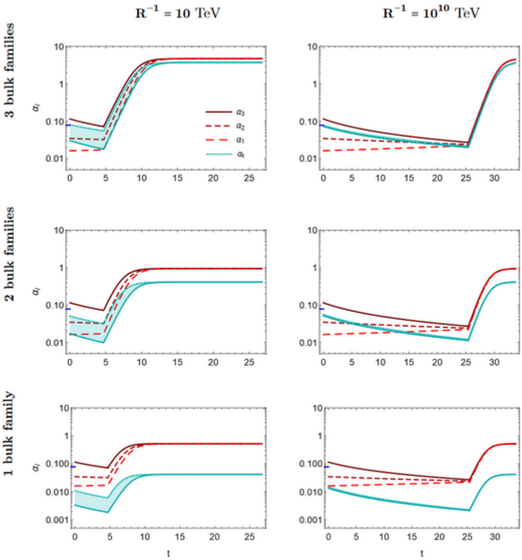
Appendix B : $SU(5)$ aGUT equation

$$2\pi\frac{d\alpha_t}{dt} = 2\pi\frac{d\alpha_t}{dt}\bigg|_{\mathrm{SM}} + (S(t)-1)\left[15\alpha_t^2 + 9\left(\alpha_b + \alpha_\tau\right)\alpha_t - \frac{66}{5}\alpha_5\alpha_t\right].$$

$$2\pi\frac{d\alpha_b}{dt} = 2\pi\frac{d\alpha_b}{dt}\bigg|_{\mathrm{SM}} + (S(t)-1)\left[11\alpha_b^2 + \frac{27}{2}\alpha_t\alpha_b + 8\alpha_\tau\alpha_b - \frac{57}{5}\alpha_5\alpha_b\right].$$

$$2\pi\frac{d\alpha_\tau}{dt} = 2\pi\frac{d\alpha_\tau}{dt}\bigg|_{\mathrm{SM}} + (S(t)-1)\left[11\alpha_\tau^2 + \frac{27}{2}\alpha_t\alpha_\tau + 8\alpha_b\alpha_\tau - \frac{57}{5}\alpha_5\alpha_\tau\right].$$

Appendix C : Running for differents n_g and R



Appendix D : Relic Density plots

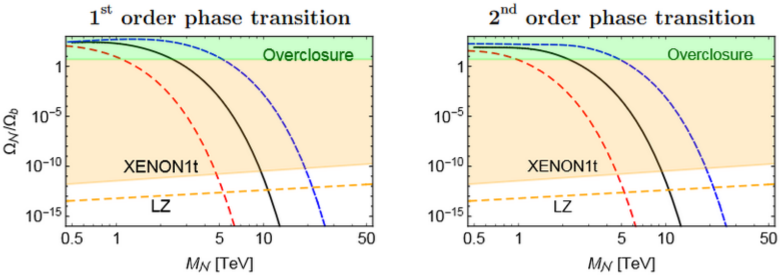


Figure: Mass density of the DM candidate ($\mathcal{N}$) divided by the baryon density for 1^{st} order (left panel) and 2^{nd} order (right panel) phase transition. Values of the relic density as a function of the mass for $T^{=v_{SM}}$ (solid black), $v_{SM}/2$ (dashed red) and $2v_{SM}$ (dashed blue). The green shaded region is excluded by the over-closure of the Universe, while the orange shaded one is excluded by XENON1t. The dashed orange indicates the projected exclusion from LUX-ZEPLIN.

Appendix E : Process for the second wave

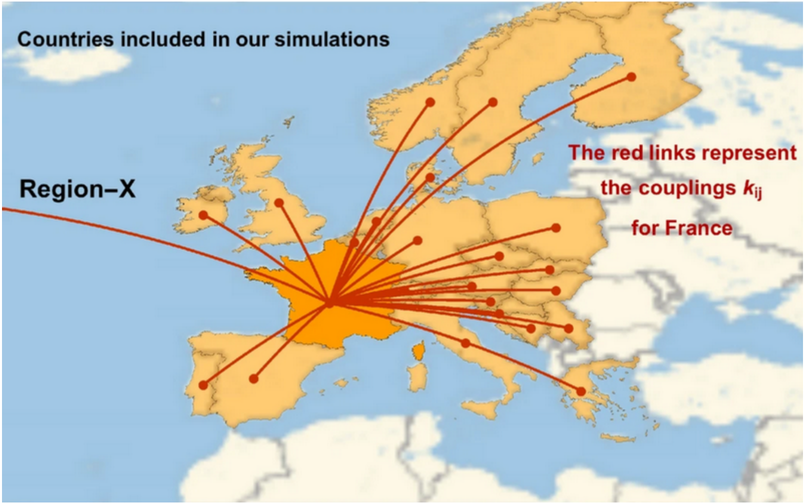


Figure: Couplings considered for the second-wave simulation in Europe.

Appendix F : Process for the second wave

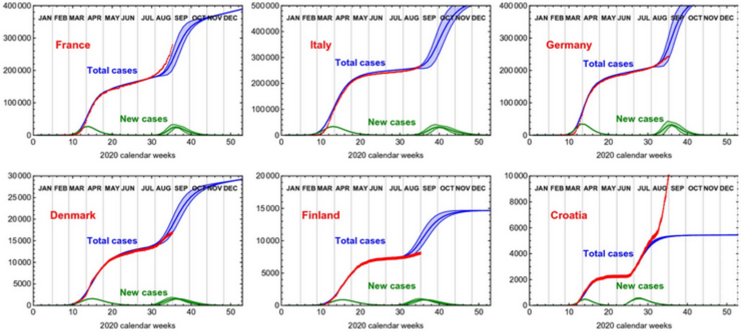


Figure: Epidemiological data (red), adjourned to the 30th of August, for six sample countries compared to an updated simulation. For all countries, except Croatia, the second wave from case (e) simulation is anticipated by 4 weeks, thus in agreement with the results of case (a). The bands are generated by varying the infection rates i within 10%.

Appendix G : Mobility in Europe and US

- Mobility data from Google (Residential and Workplaces %) and Apple (Walking and Driving %)

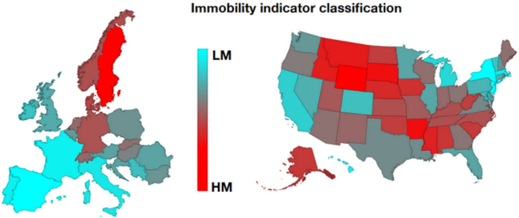


Figure: https://www.researchsquare.com/article/rs-76030/v1

- Start of SD measures = 20% workingplaces reduction
- 2-4 weeks effect of the SD measures

Appendix G' : Mobility in Europe and US

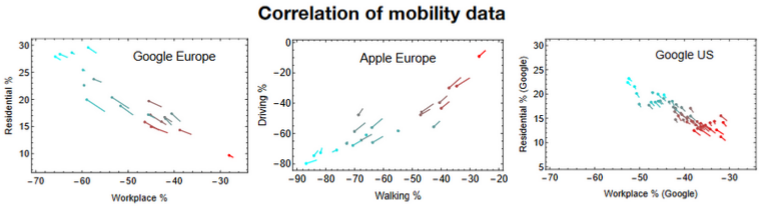


Figure: Tadpole-like plots showing correlations between the four mobility reduction categories: Residential and Workplaces from Google, Driving and Walking from Apple. The head of the tadpoles correspond to the average over 6 weeks after social distancing begins, while the tail indicates a 8 week average.

Appendix H : Social distancing effect time shift

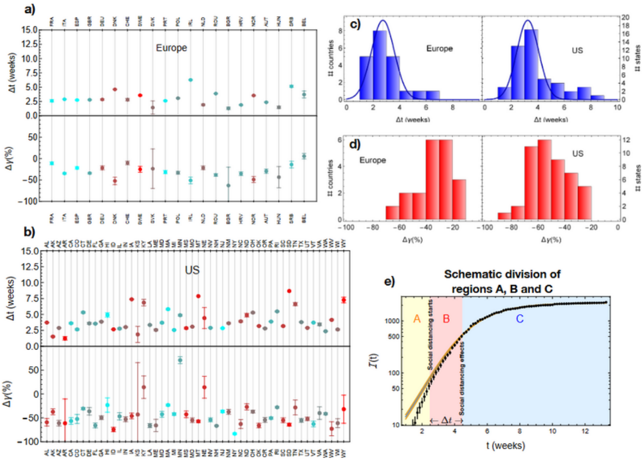


Figure: Δt and $\Delta\gamma$ for the countries and states studied.

Appendix I : Geographical uniformity indicator

$$\chi^2(t) = \frac{1}{n_r}\sum_{i=1}^{n_r}\left(\frac{I_i^I(t)}{< I^I(t) >} - 1\right)^2 \qquad (10)$$

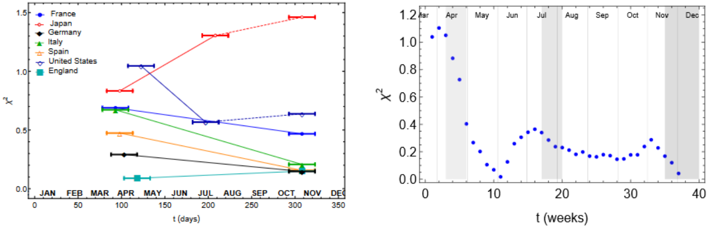


Figure: Geographical uniformity indicator for countries where regional data is available.

Appendix J : Third wave simulation

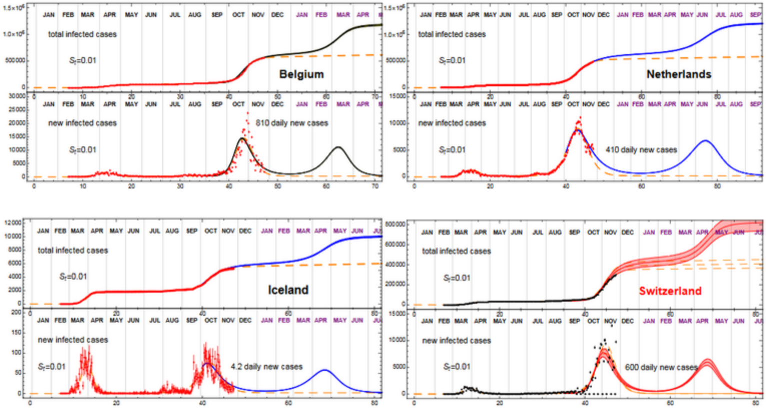


Figure: Simulation of the 3rd wave in European countries using CeRG framework.

Appendix K : US flight kappa matrix

k_{ij} values (1st wave fits)									
Division code	NE	MA	SA	ESC	WSC	ENC	WNC	M	P
MA	0.72	0	0.0014	0.00075	0.0017	0.0023	0.0005	0.002	0.0053
First wave k_{ij} values (Flight data, from April 1st to May 31st)									
Division code	NE	MA	SA	ESC	WSC	ENC	WNC	M	P
NE	0	0.72	0.0045	0.00088	0.00087	0.0024	0.00052	0.00067	0.00091
MA	0.72	0	0.019	0.0056	0.0041	0.012	0.0025	0.0031	0.0059
SA	0.0043	0.018	0	0.0085	0.013	0.019	0.0057	0.0050	0.0067
ESC	0.00092	0.0053	0.0093	0	0.0051	0.0068	0.0023	0.0035	0.0065
WSC	0.00095	0.0038	0.014	0.0055	0	0.0092	0.0054	0.011	0.010
ENC	0.0025	0.012	0.018	0.0063	0.0086	0	0.0082	0.0079	0.0099
WNC	0.00038	0.0022	0.0056	0.0019	0.0046	0.0070	0	0.0055	0.0027
M	0.00050	0.0020	0.0042	0.0026	0.011	0.0072	0.0043	0	0.028
P	0.00084	0.0055	0.0063	0.0050	0.010	0.0092	0.0033	0.030	0
Second wave k_{ij} values (Flight data, from September 1st to October 31st)									
Division code	NE	MA	SA	ESC	WSC	ENC	WNC	M	P
Region-X	0.0066	0.028	0.029	0.013	0.019	0.027	0.014	0.03	0.03
NE	0	0.72	0.0028	0.00046	0.00031	0.0015	0.00026	0.00041	0.00082
MA	0.72	0.	0.011	0.002	0.0017	0.0064	0.0013	0.0021	0.0029
SA	0.0026	0.011	0	0.005	0.005	0.0096	0.003	0.0033	0.0035
ESC	0.00041	0.0019	0.0051	0	0.0019	0.0028	0.00087	0.0012	0.0015
WSC	0.00028	0.0015	0.0049	0.0018	0	0.0028	0.0016	0.004	0.0034
ENC	0.0014	0.0062	0.0089	0.0028	0.003	0	0.0039	0.0043	0.0045
WNC	0.00024	0.0013	0.0028	0.0009	0.0017	0.0038	0	0.0028	0.0016
M	0.00032	0.0017	0.0029	0.0011	0.0054	0.004	0.0026	0	0.014
P	0.00074	0.0028	0.0032	0.0014	0.0046	0.0041	0.0018	0.015	0

Figure: Kappa matrix based on flight data between the 9 divisions of US.

Appendix K : US division simulation

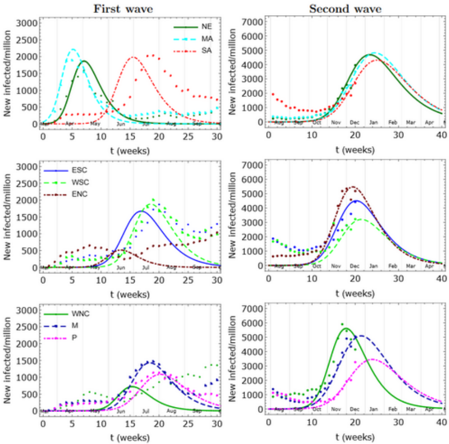


Figure: Number of infected for the 9 US division and the simulation using flight kappa matrix.