Soft gluon contributions to Higgs production at LHC beyond two loops

V. Ravindran

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- Introduction
- Factorisation of soft and collinear contributions
- Resummation
- Soft-plus-Virtual at N^3LO_{SpV} for total cross section, rapidity distribution
- Conclusions

In collaboration with

W.L. van Neerven, Jack Smith

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- We are in the process of discovering Exotic and New Physics beyond the Standard Model

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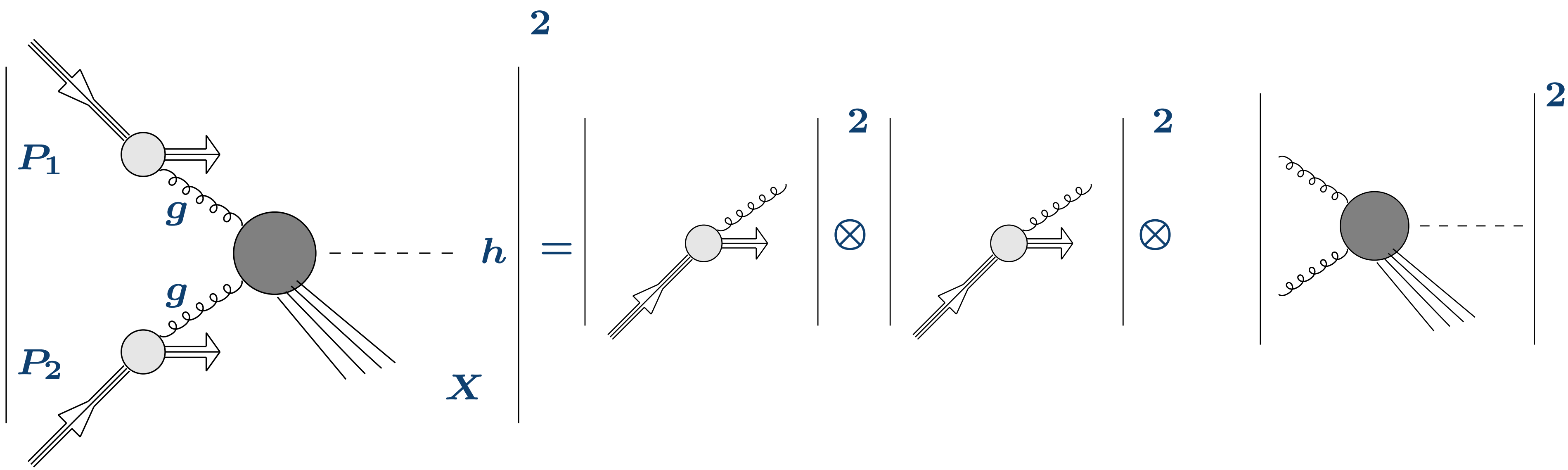

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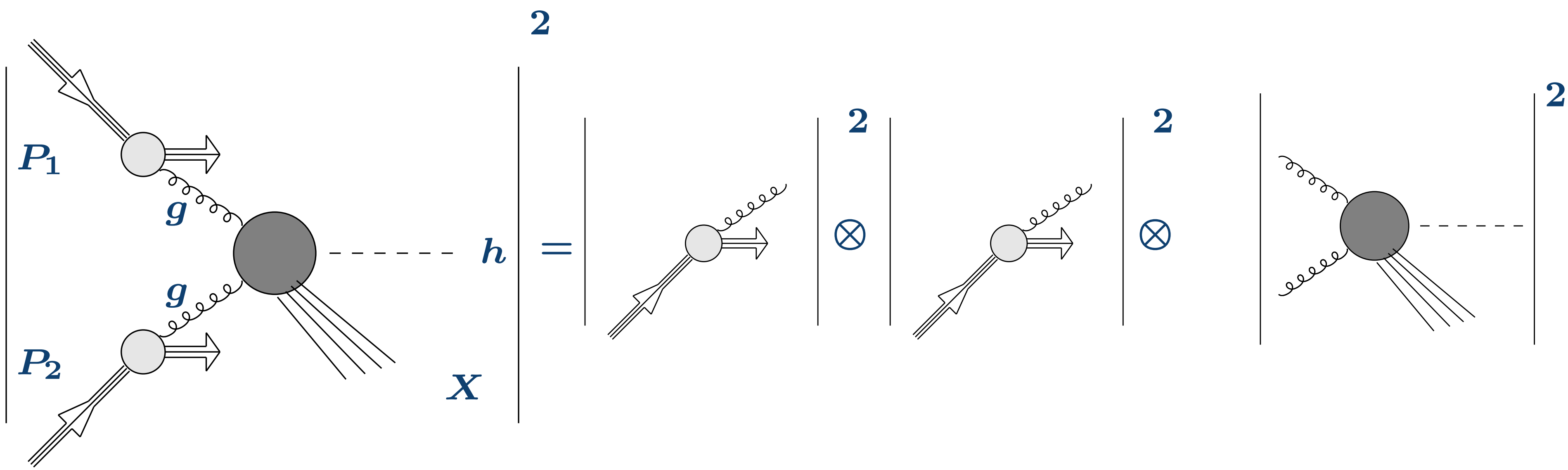

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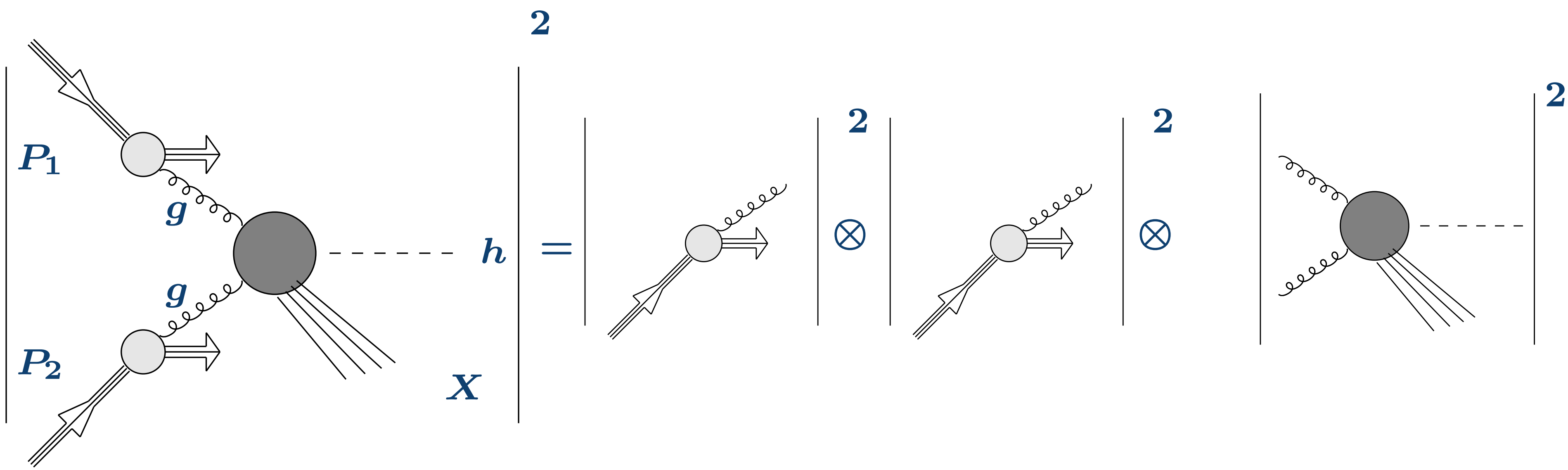

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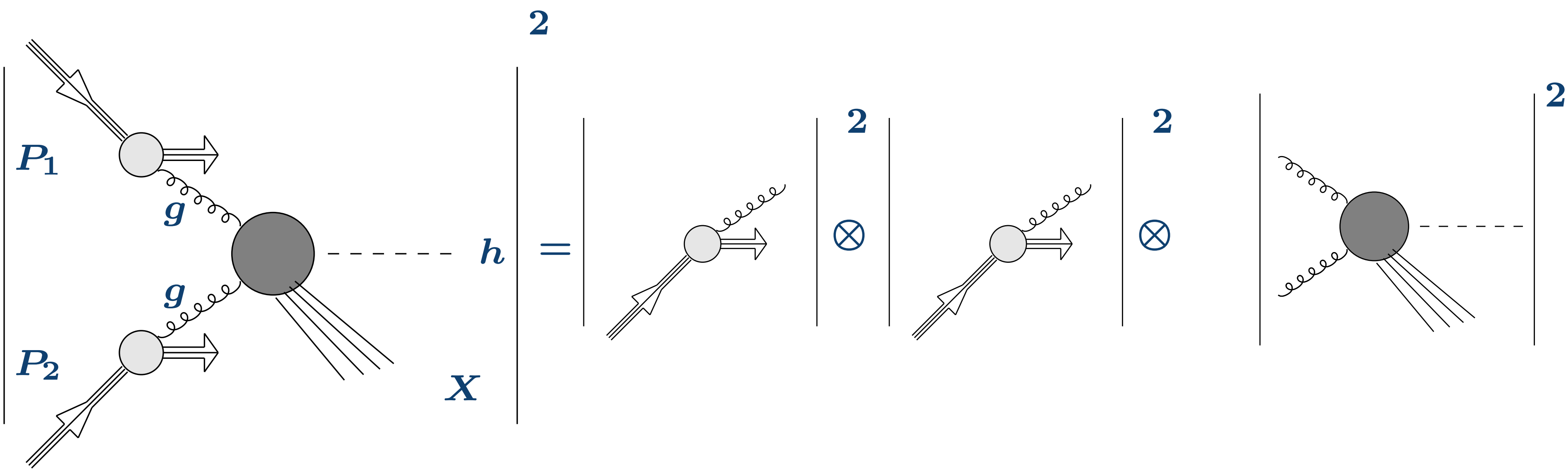

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$$f_1(x) \otimes \cdots \otimes f_n(x) = \int_0^1 dx_1 \cdots \int_0^1 dx_n\, f_1(x_1) \cdots f_n(x_n)\, \delta(x - x_1 x_2 \cdots x_n)$$

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- Renormalised version of parton model:

$$\boxed{2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \quad f_a\left(\tau, \mu_F\right) \quad \otimes \quad f_b\left(\tau, \mu_F\right) \quad \otimes \quad 2\hat{s}\, d\hat{\sigma}^{ab}\left(\tau, m_h^2, \mu_F\right),}$$

where $$d\hat{\sigma}_{ab}\left(\tau, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{(i)}_{ab}\left(\tau, m_h^2, \mu_F, \mu_R\right)$$

Higgs production at LHC

Georgi, Spira et al,Dawson, Harlander, Kilgore, Anastasiou, Melnikov, Smith, van Neerven, VR

$$m_H/2 < \mu_F = \mu_R < 2m_H$$

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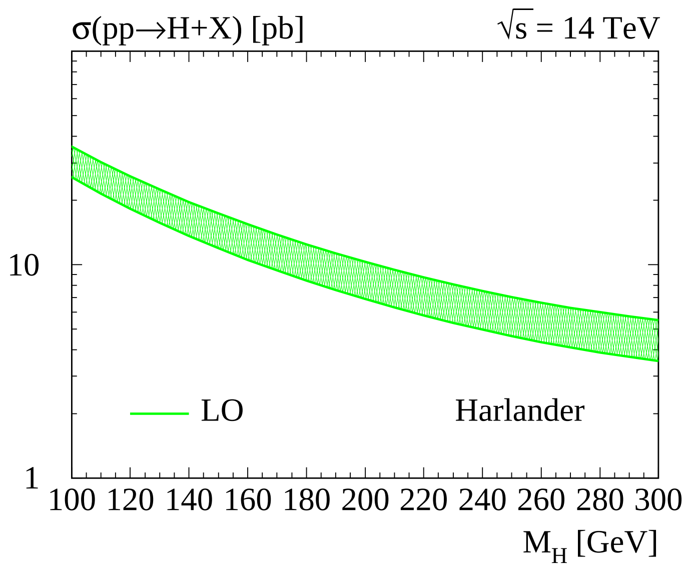

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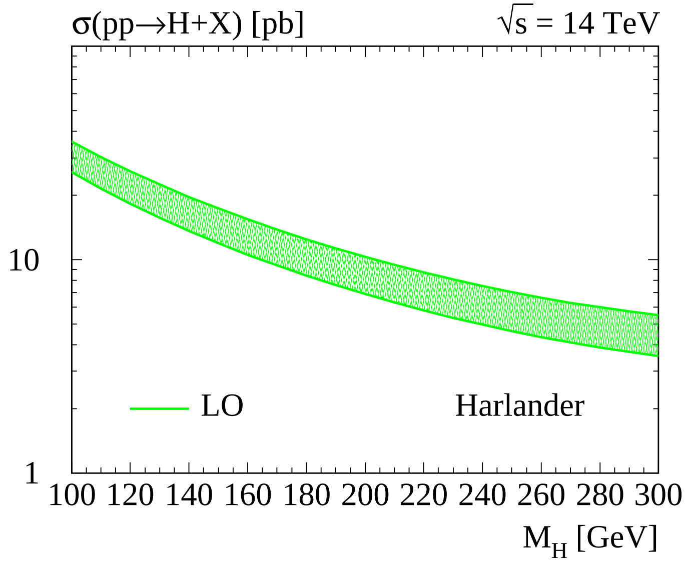

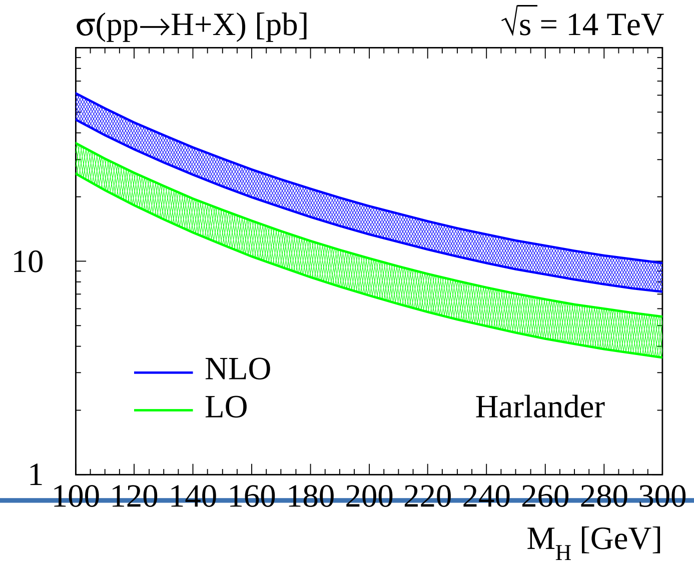

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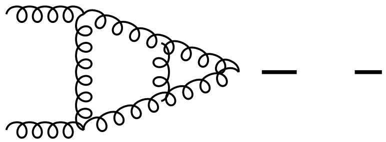

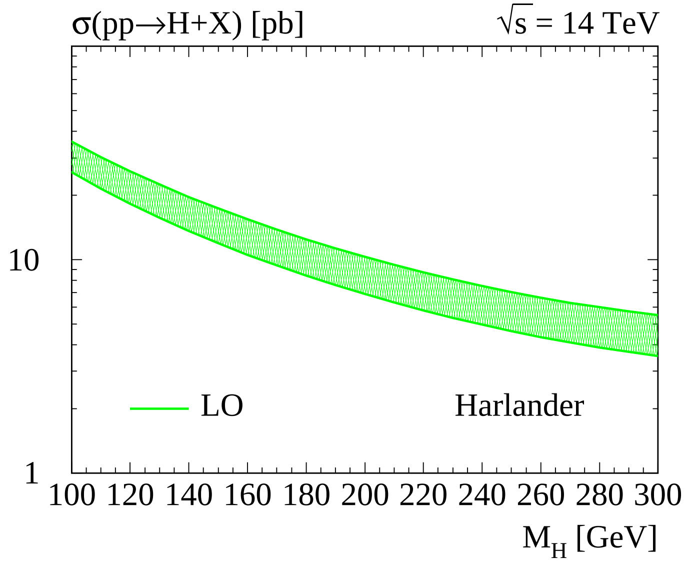

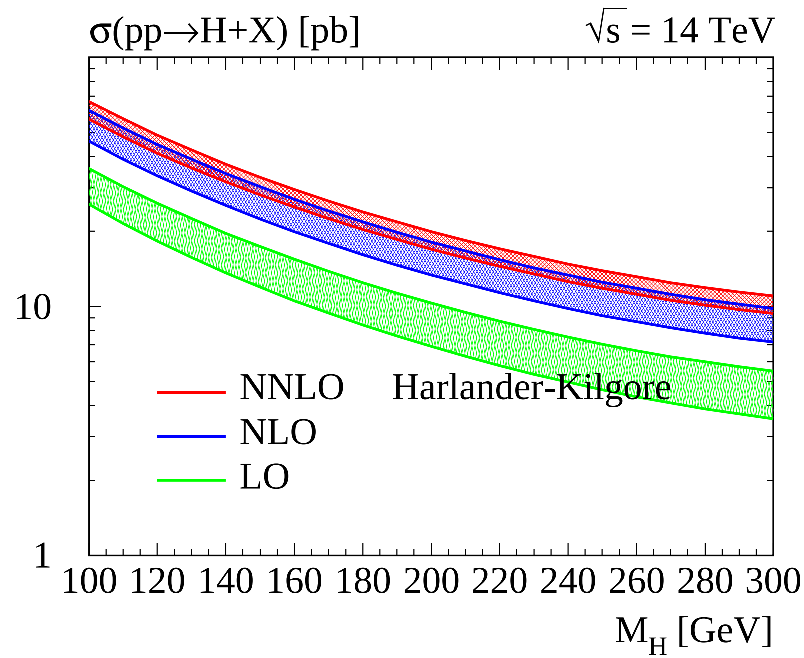

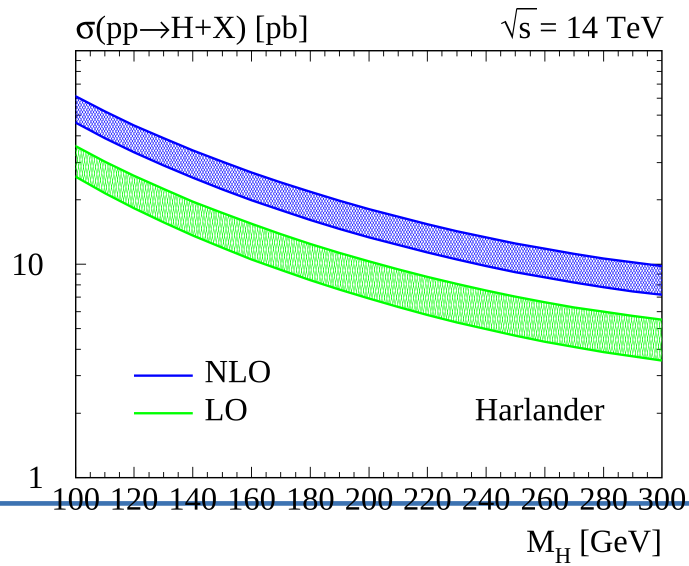

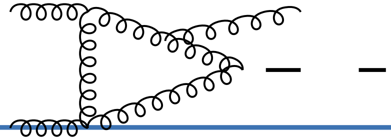

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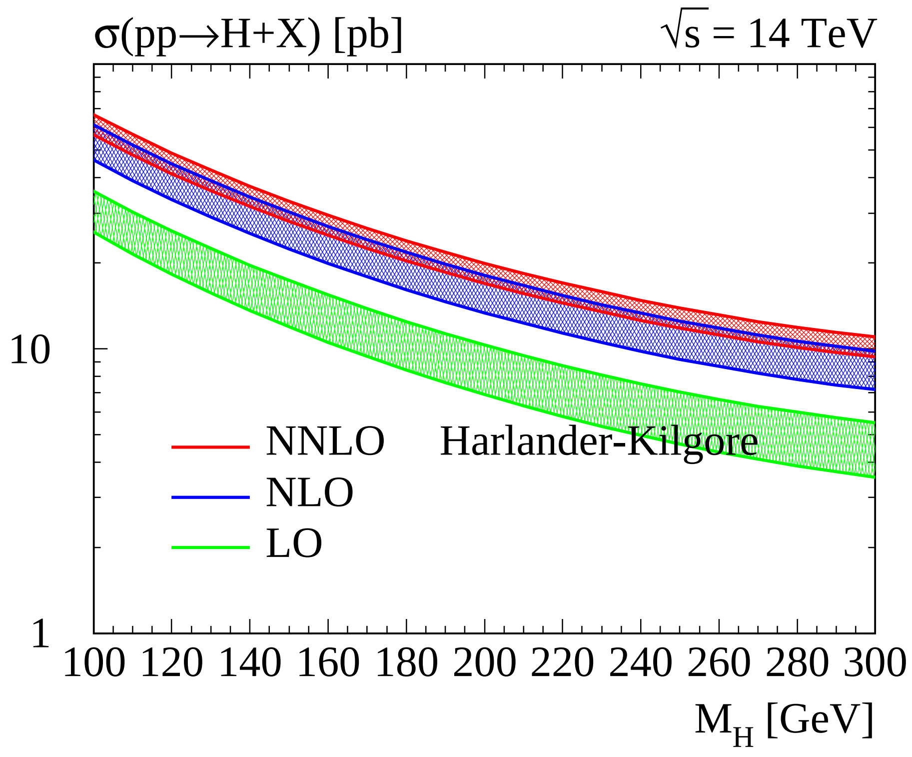

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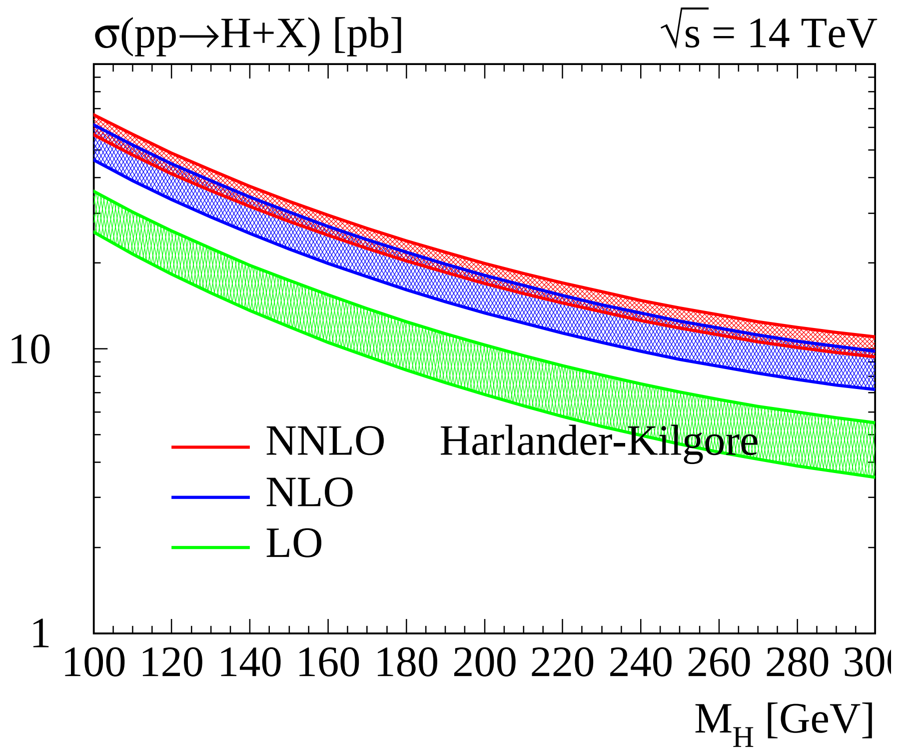

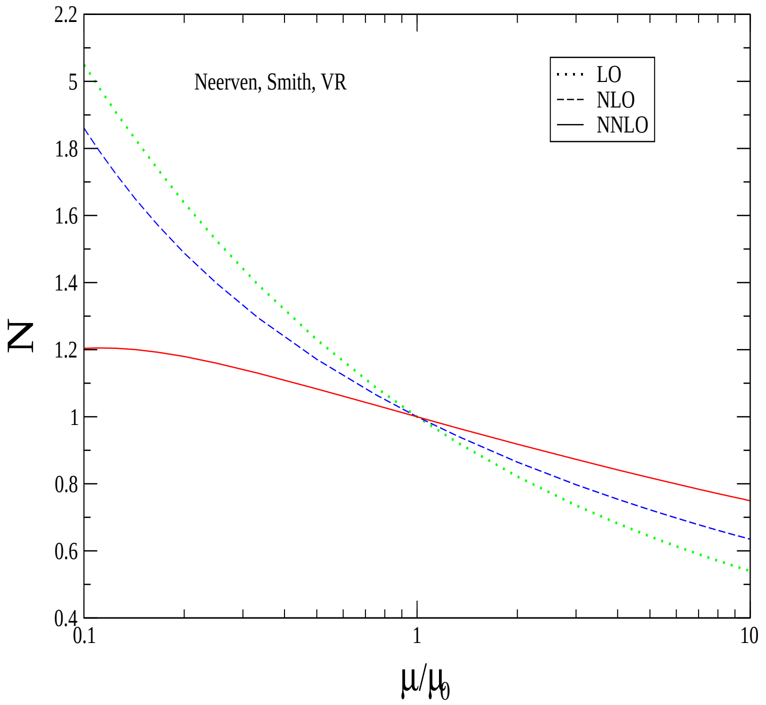

What is next?

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"Going Beyond NNLO"

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- To check perturbative stability
  - Technically non-trivial task
    - Attempt to compute them for the dominant soft gluon contributions

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Catani, Harlander, Kilgore

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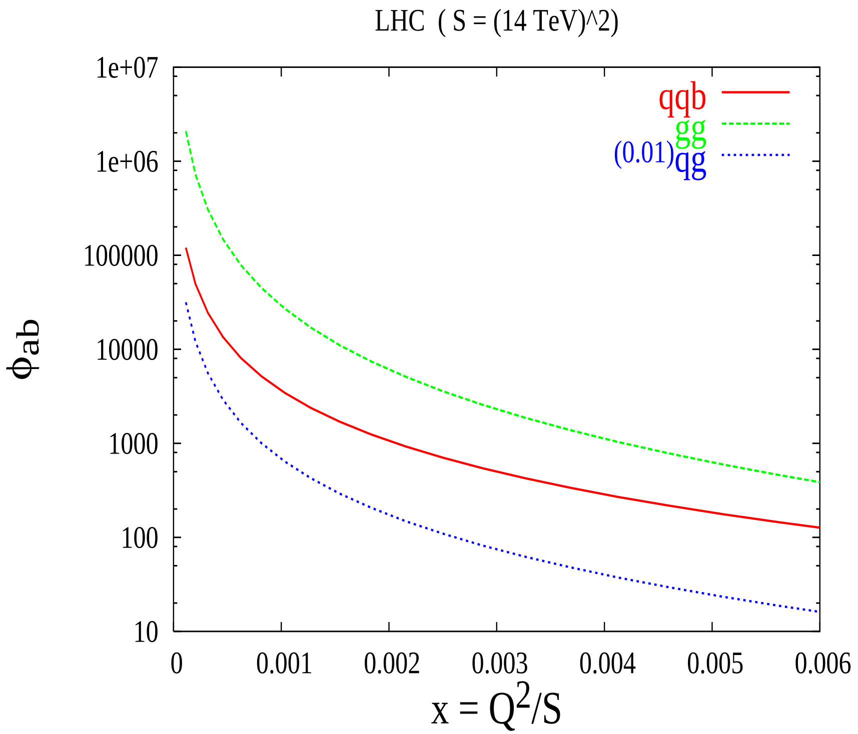

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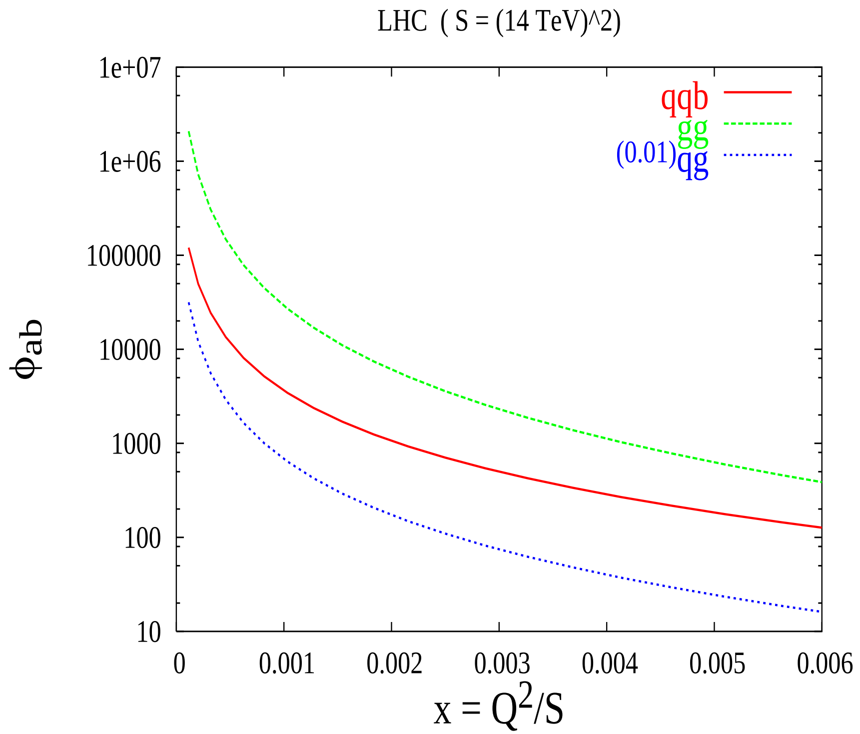

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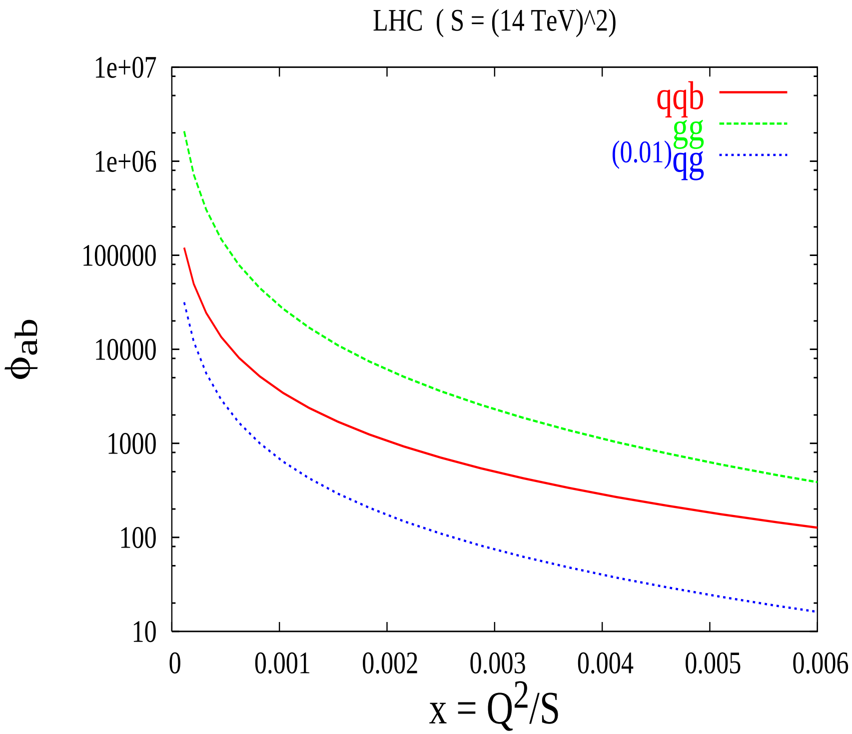

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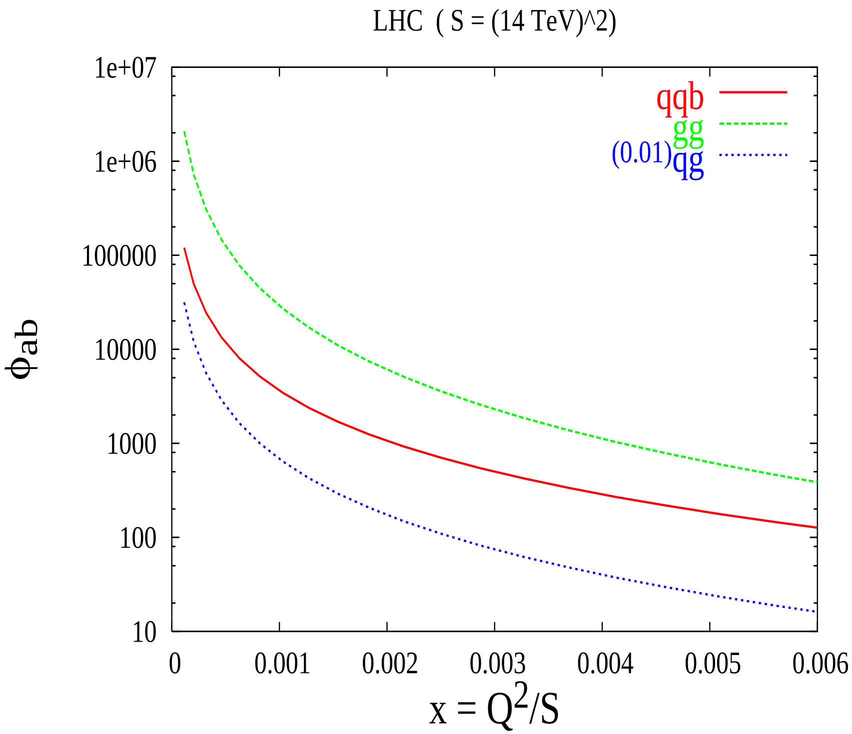

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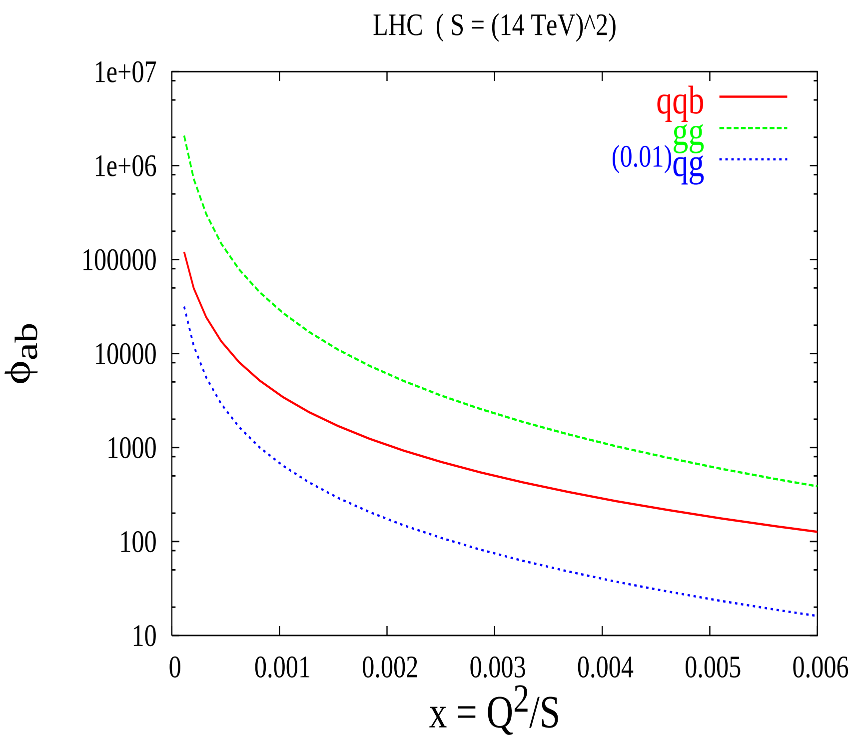

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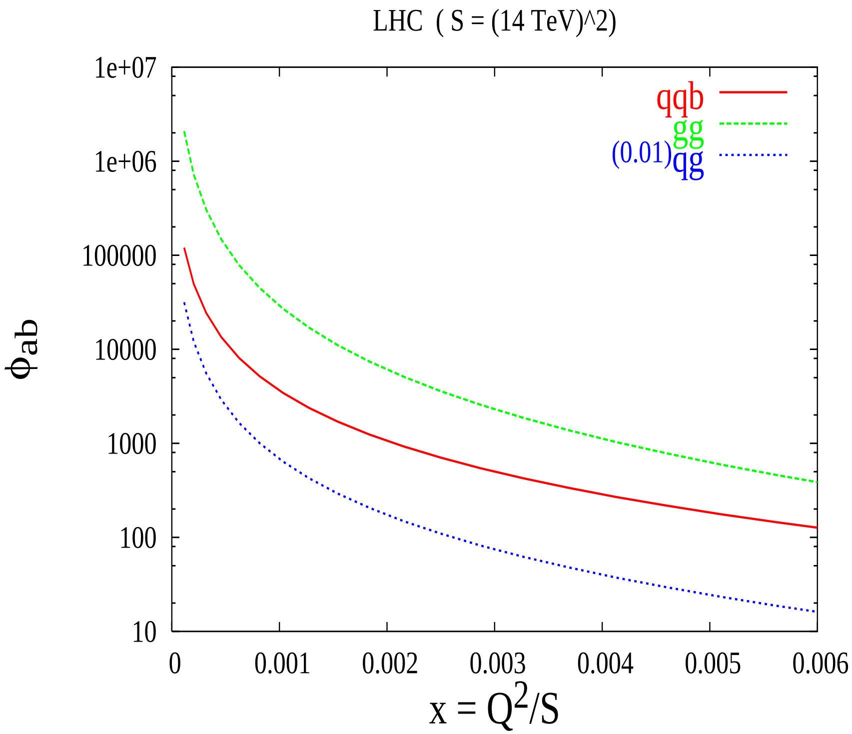

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- Expand the partonic cross section around $x = \tau$.

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  OR

  Extract from "Form factors and DGLAP kernels" using

  1) Factorisation theorem 2) Renormalisation Group Invariance

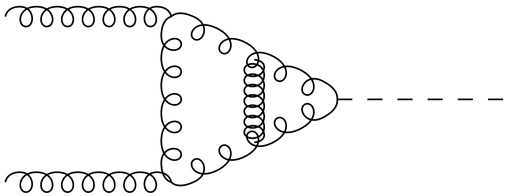
  3) Sudakov Resummation

Soft gluon cancellation

Soft gluon cancellation

V. Ravindran

$\sigma^V_{ab}(z, Q^2, \varepsilon_s, \varepsilon_c)$ - Virtual soft gluon ($|F^I|^2$)



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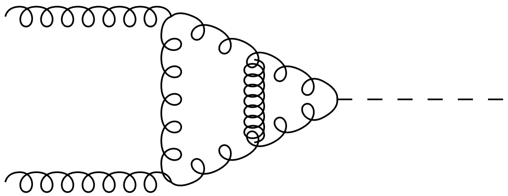

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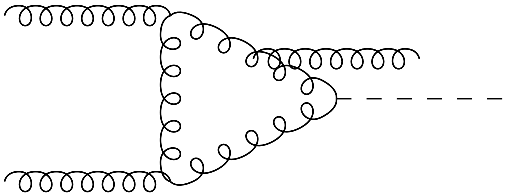
$\sigma^R_{ab}(z, Q^2, \varepsilon_s, \varepsilon_c)$ - Real soft gluon (Φ^I_{ab})



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Soft gluon cancellation

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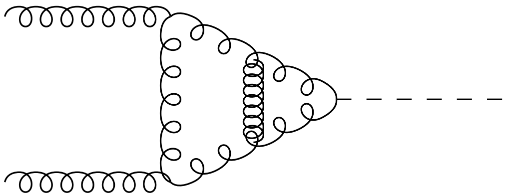

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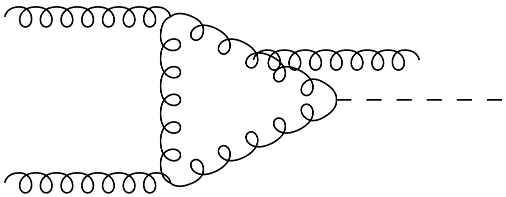

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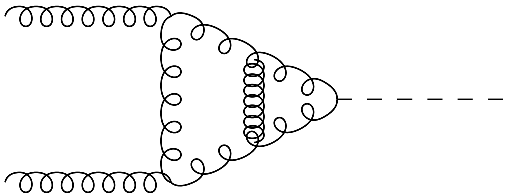

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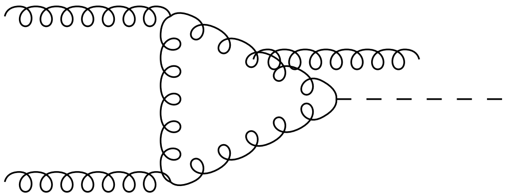

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Only collinear singularities remain

Factorisation of Collinear partons

Due to the massless partons, collinear singularities appear in

- the phase space of the real emission processes
- loop integrals of the virtual corrections

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The diagonal terms of the splitting functions $P^{(i)}(z)$ have the following structure

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$\boxed{A_i^I}$ - cusp anomalous dimension

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$$\Delta(z,Q^2) = \delta(1-z) + \alpha_s\Bigg[a_{11}\delta(1-z) + \frac{a_{12}}{(1-z)_+} + a_{13}\left(\frac{\ln(1-z)}{1-z}\right)_+$$

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Drop all the regular functions.

Soft plus Virtual at N^3LO and beyond

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Using "factorisation" of UV, Soft and Collinear:

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$$+$$

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$A^I(a_s(\mu_R^2))$ are finite, called cusp anomalous dimensions.

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$Vogt, Vermaseren, Moch, VR$

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$$\cdots\cdot \quad \cdots\cdot \quad \cdots\cdot \quad \cdots\cdot$$

$$\cdots\cdot \quad \cdots\cdot \quad \cdots\cdot \quad \cdots..$$

A^I are maximally non-abelian

$$A_i^g = \frac{C_A}{C_F} A_i^q \qquad i = 1, 2, 3.$$

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$$\hat{\mathcal{L}}_F^{I,(2)} = \frac{1}{\varepsilon^3}\Big(\beta_0 A_1^I\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_2^I - \beta_0 G_1^I(\varepsilon)\Big) + \frac{1}{2\varepsilon}G_2^I(\varepsilon)$$

$$\cdots\cdot \quad \cdots\cdot \quad \cdots\cdot \quad \cdots\cdot$$

$$\cdots\cdot \quad \cdots\cdot \quad \cdots\cdot \quad \cdots\cdot$$

A^I are maximally non-abelian

$$A_i^g = \frac{C_A}{C_F} A_i^q \qquad i = 1, 2, 3.$$

Every order in $\hat{a}_s$, all the poles except the lowest one can be predicted

New observation

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$VR, Smith, van\ Neerven$

Two loop results for $\hat{\boldsymbol{F}}^{\boldsymbol{q}}$ and $\hat{\boldsymbol{F}}^{\boldsymbol{g}}$ in $\boldsymbol{SU(N)}$ solves the single pole problem:

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Two loop results for $\hat{F}^q$ and $\hat{F}^g$ in $SU(N)$ solves the single pole problem: G^Is have interesting structure:

$$G_1^I(\varepsilon) = 2(B_1^I - \gamma_1^I) + f_1^I + \sum_{k=1}^{\infty} \varepsilon^k {g_1}^{I,k}$$

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B_i^I are collinear anomalous dimension. The new constants "f_1^I and f_2^I" satisfy

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Even the single pole can be predicted.

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This completes the understanding of all the poles of the form factors.

New observation(contd)

VR

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The single poles $G_i^I(\varepsilon)$ has the structure:

$$\frac{1}{\varepsilon}\left[2(B_i^I-\gamma_i^I)+f_i^I+\sum_j C_i^j(g_j,\beta_i,\varepsilon)\right]$$

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Operator renormalisation constant satisfies the RG:

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^I(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \left(\gamma_i^I\right)$$

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After operator renormalisation, the poles of the form factors will have

$$A_i^I \qquad \textbf{and} \qquad 2B_i^I + f_i^I$$

Drell-Yan and Wilson loops

G.P. Korchemsky, A.V. Belitzky

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$$\boldsymbol{Q^2 \frac{d\sigma_{\rm DY}(z,Q^2)}{dQ^2} = \sigma^{(0)}_{\rm DY}\mathcal{H}_{\rm DY}(Q^2)\widetilde{W}_{\rm DY}(z)}$$

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via the Fourier transform, $\widetilde{W}_{\mathrm{DY}}(z)$, of the vacuum average of the Wilson loop

$$\widetilde{W}_{\mathrm{DY}}(z) = \frac{Q}{2} \int_{-\infty}^{\infty} \frac{dy_0}{2\pi} e^{iy_0\omega(z)} W_{\mathrm{DY}}(y_0),$$

$$where \qquad W_{\mathrm{DY}}(y_0) = \frac{1}{N_c} \langle 0| \mathrm{Tr} \mathcal{T} \mathcal{P} \exp \left(ig \oint_{C_{\mathrm{DY}}} dx_\mu B_\mu(x) \right) |0\rangle,$$

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Renormalisation group equation:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(a_s)\frac{\partial}{\partial a_s}\right)\ln W_{\rm DY}(y) = 2\Gamma_{\rm cusp}(a_s)\ \ \ln L(y) + \Gamma_{\rm DY}(a_s),$$

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Soft anomalous dimension $\Gamma_{\rm DY}$ is now known upto two loop level:

$$\Gamma_{\rm DY}(g) = \sum_{i=1} a_s^i\Gamma^{(i)}_{\rm DY} \qquad\qquad i=1,2$$

We find

$$\boxed{\Gamma^{(i)}_{\rm DY} = f_i^q \qquad\qquad i=1,2}$$

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Catani, Aybat, Dixon, Sterman, Magnea

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- The peculiar combination $\boxed{2(B^I - \gamma^I) + f^I}$ with universal f^I appears in $G^I(\varepsilon)$.

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- $\mathcal{N} = 4$ Yang-Mills theory is conformal ($C_A = 2C_F = N_c$), one of the simplest field theories.

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- In the strong coupling using AdS/CFT, Alday has shown recently that it is indeed to true

$$G_I(\lambda) = 2B_I(\lambda) + c_I \frac{\sqrt{\lambda}}{2\pi}\Big(-1 - 2\gamma_E + 5\log(2) + 2\log(\pi) - \log(\lambda)\Big)$$

Solution to (Soft)Sudakov Equation

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$$q^2 \frac{d}{dq^2} \Phi^I \left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) \quad = \quad \frac{1}{2}\left[\overline{K}^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) + \overline{G}^I \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right)\right]$$

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- $\overline{K}^I$ contains all the infra-red poles
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$$\Phi^I \left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\Phi}^{I,(i)}(z, \varepsilon)$$

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Most general solution:

$$\begin{aligned}
\Phi^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon) &= \Phi^I(\hat{a}_s, q^2(1-z)^{2m}, \mu^2, \varepsilon) \\
&= \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^{2m}}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\, m\, \varepsilon}{2(1-z)}\right) \hat{\phi}^{\,I,(i)}(\varepsilon)
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\end{aligned}$$

- All the poles are known upto three loop
- All the poles and finite terms are known upto two loop level

Threshold Resummation

- Alternate derivation for the threshold resummation formula in z space for both DY and DIS:

$$
\begin{aligned}
\Phi^{I}_{P}(\hat{a}_s, q^2, \mu^2, z, \varepsilon) \quad = \quad & \Bigg(\frac{m}{1-z} \Bigg\{ \int_{\mu_R^2}^{q^2(1-z)^{2m}\delta_P} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right) \\
& + \overline{G}^{I}_{P}\left(a_s\left(q^2(1-z)^{2m}\delta_P\right), \varepsilon\right) \Bigg\} \Bigg)_+ \\
& + \delta(1-z) \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2\delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\phi}^{I,(i)}_{P}(\varepsilon) \\
& + \left(\frac{m}{1-z}\right)_+ \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \overline{K}^{\,I,(i)}(\varepsilon)
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& \left.\left. + \overline{G}^{I}_{P}\left(a_s\left(q^2(1-z)^{2m}\delta_P\right), \varepsilon\right) \right\} \right)_+ \\
& + \delta(1-z) \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2 \delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\phi}_P^{I,(i)}(\varepsilon) \\
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- The threshold exponents D^I_i for DY and B^I_i for DIS are related to $\overline{G}^I_P(\varepsilon = 0)$.
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Threshold Resummation

- Alternate derivation for the threshold resummation formula in z space for both DY and DIS:

$$\Phi_P^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \left(\frac{m}{1-z} \left\{ \int_{\mu_R^2}^{q^2(1-z)^{2m}\delta_P} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right) + \overline{G}_P^I\left(a_s\left(q^2(1-z)^{2m}\delta_P\right), \varepsilon\right) \right\} \right)_+ + \delta(1-z) \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2\delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \hat{\phi}_P^{I,(i)}(\varepsilon) + \left(\frac{m}{1-z}\right)_+ \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \overline{K}^{I,(i)}(\varepsilon)$$

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- Expansion of $\mathcal{C}e^{\left(2\Phi_P^I\right)}$ leads to soft part of the cross section.
- Soft part of Wilson Coeffient of $F_2(x,Q^2)$ structure functions upto "four loops" can be reproduced (Moch,Vogt,Vermaseren)

Soft plus Virtual part at N^3LO_{pSV} for Higgs Production

Moch, Vogt, V. Ravindran

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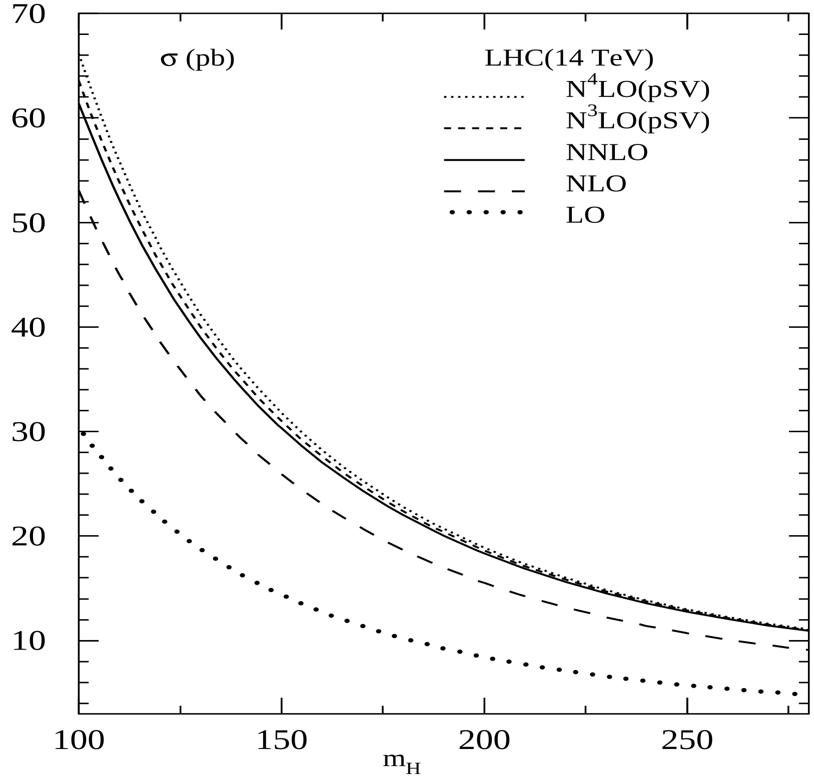

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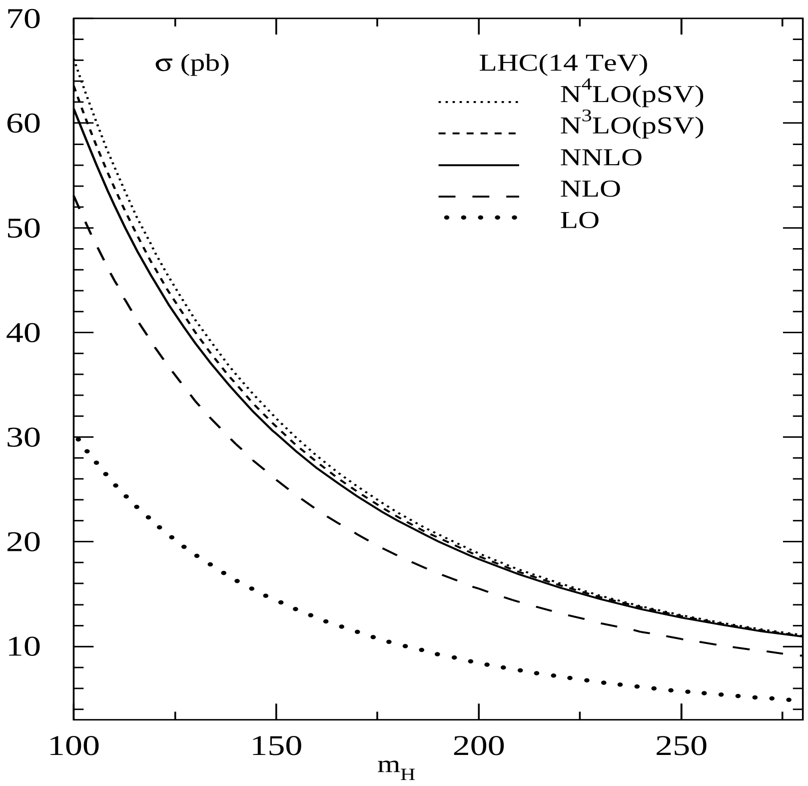

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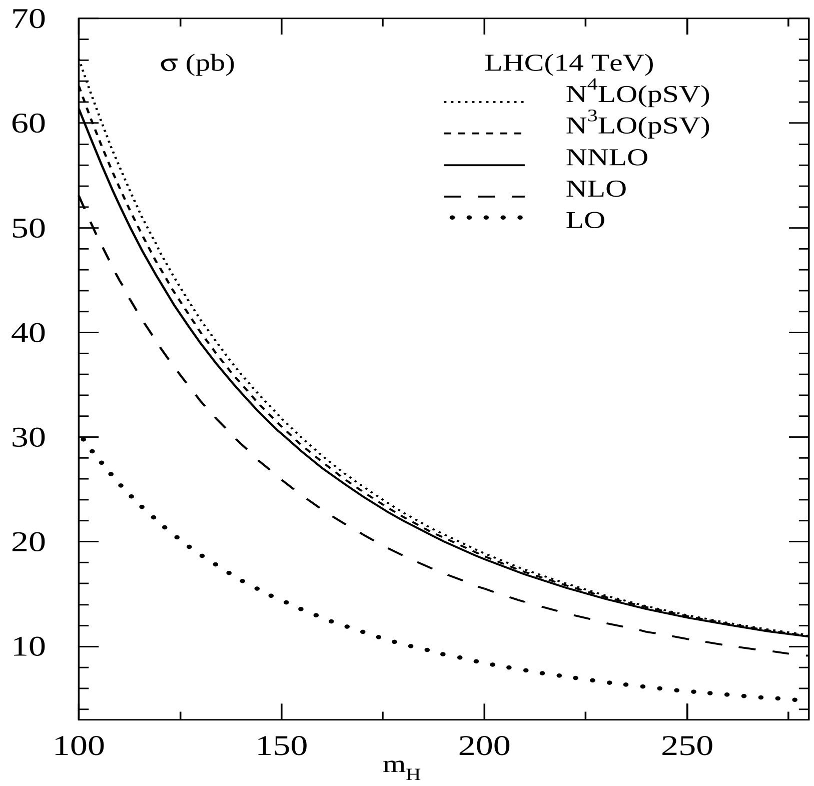

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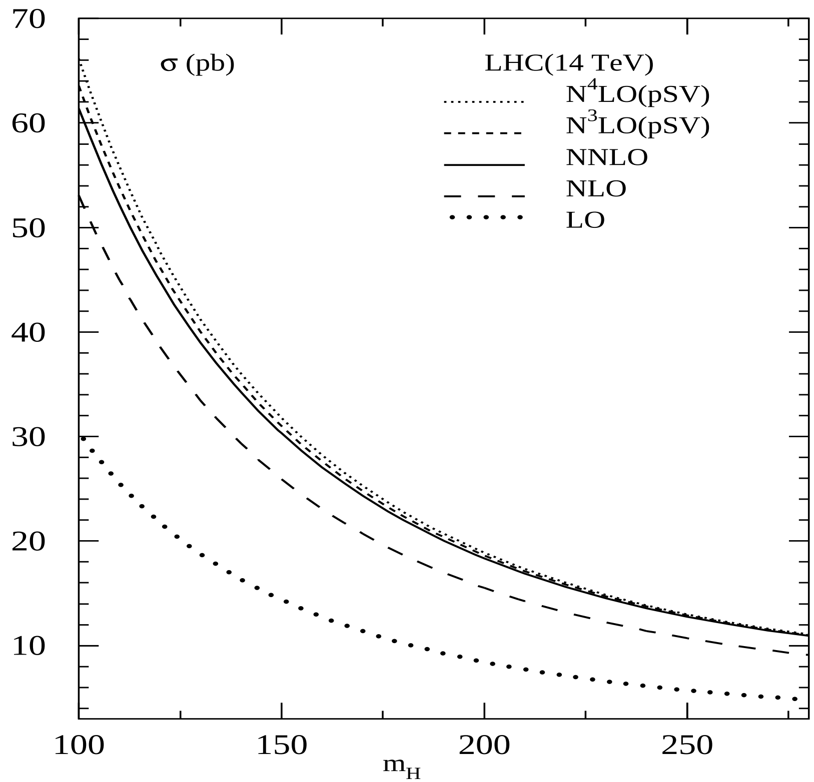

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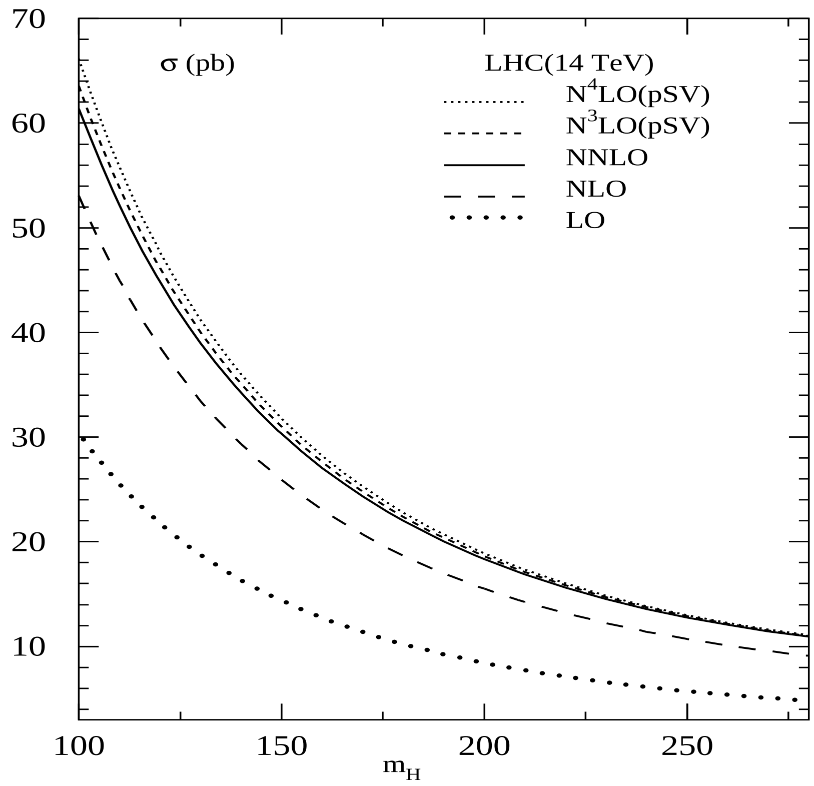

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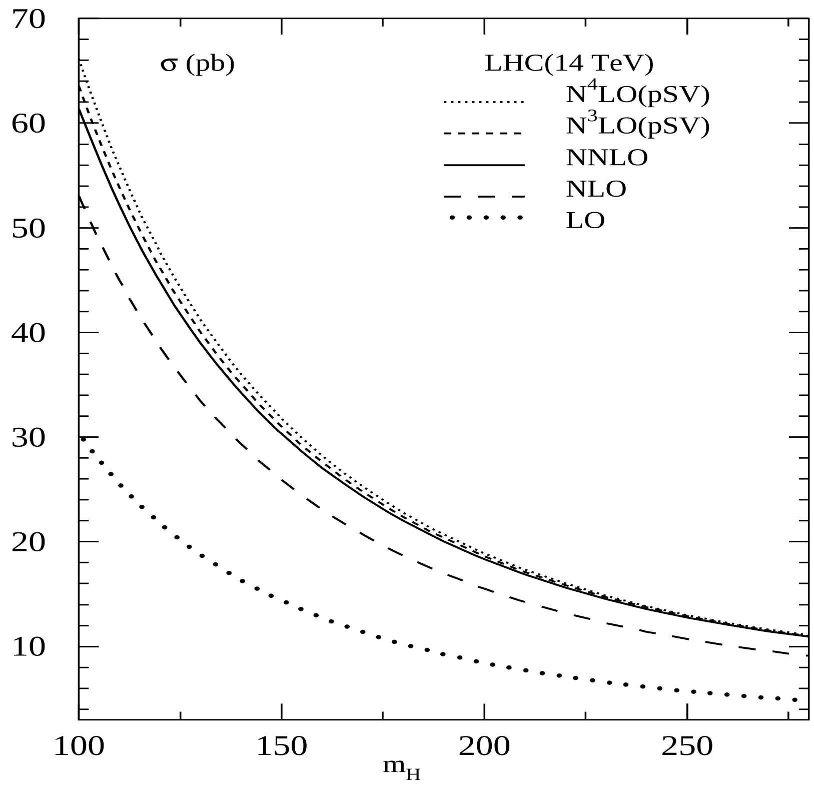

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Scale variation at N^3LO_{pSV} for Higgs production

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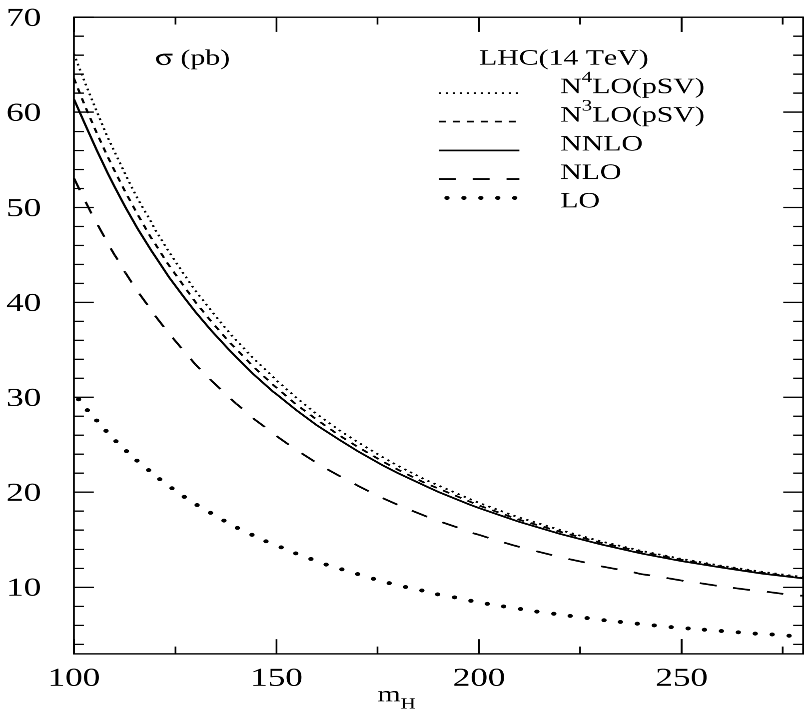

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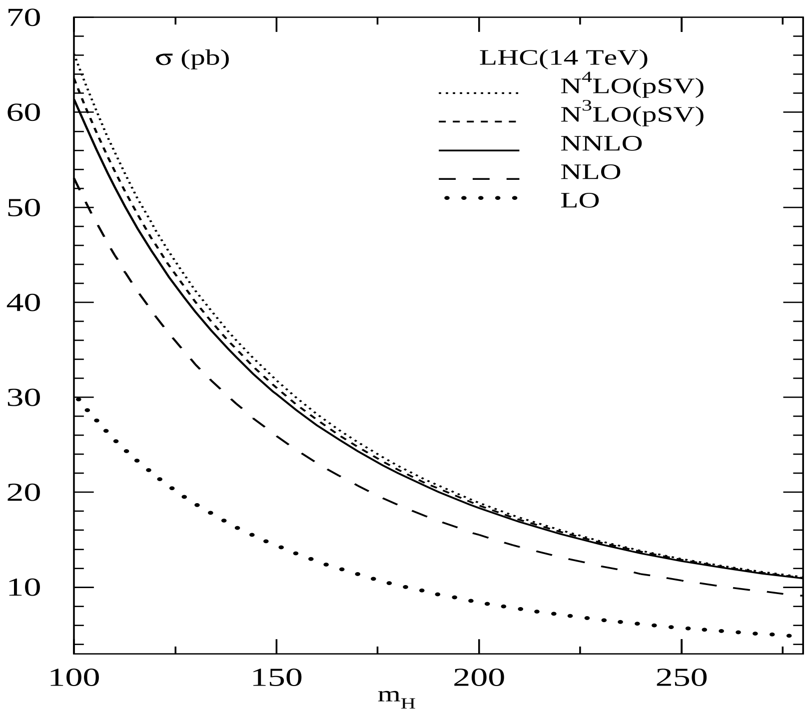

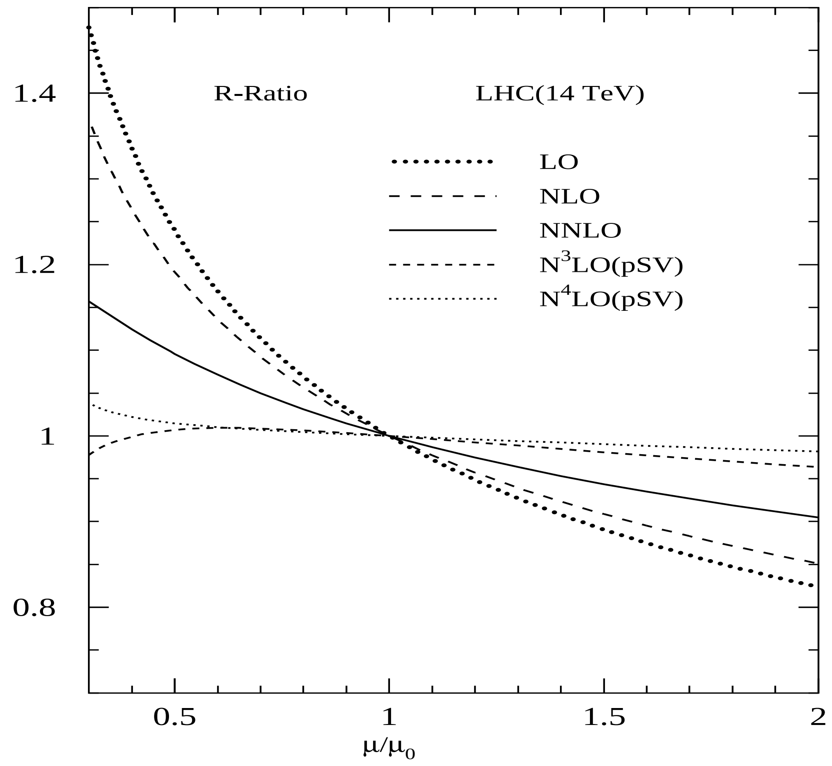

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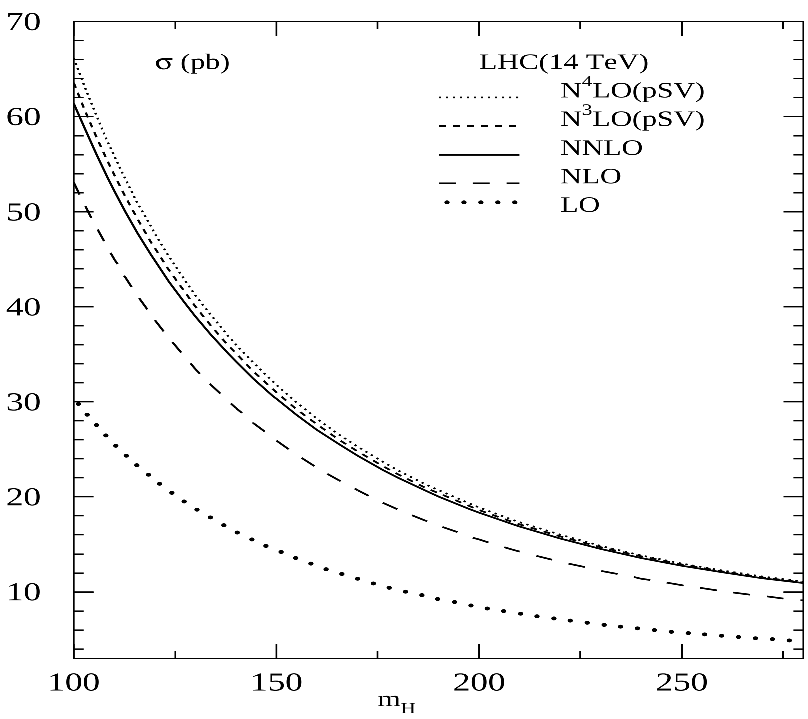

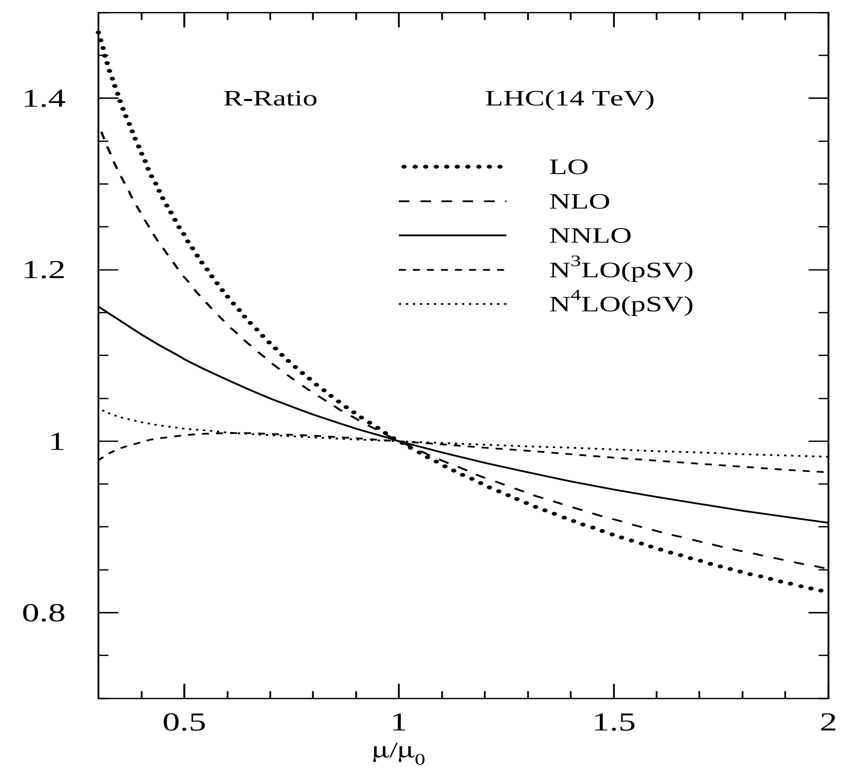

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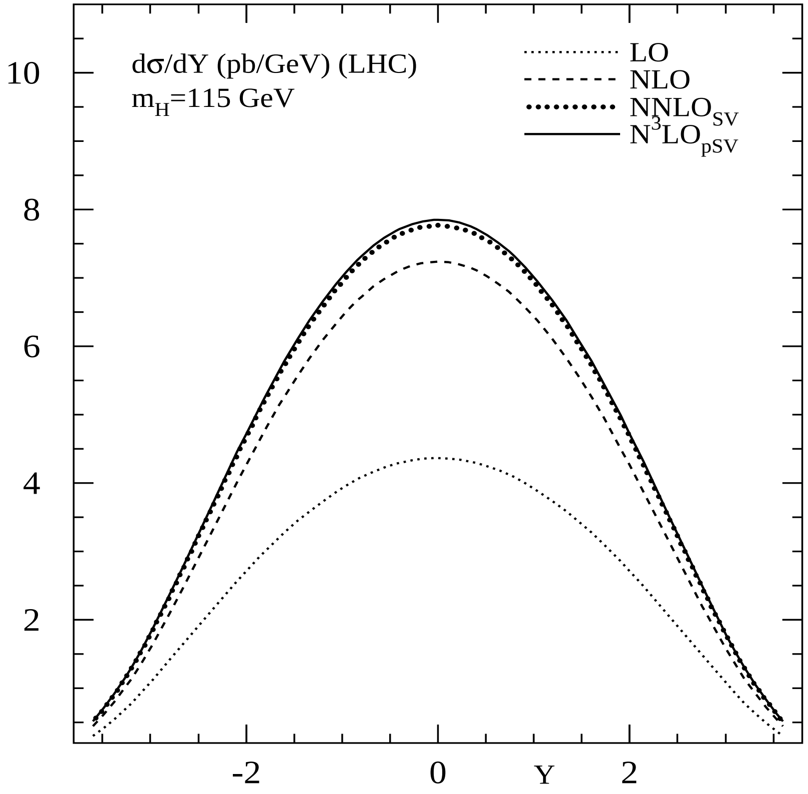

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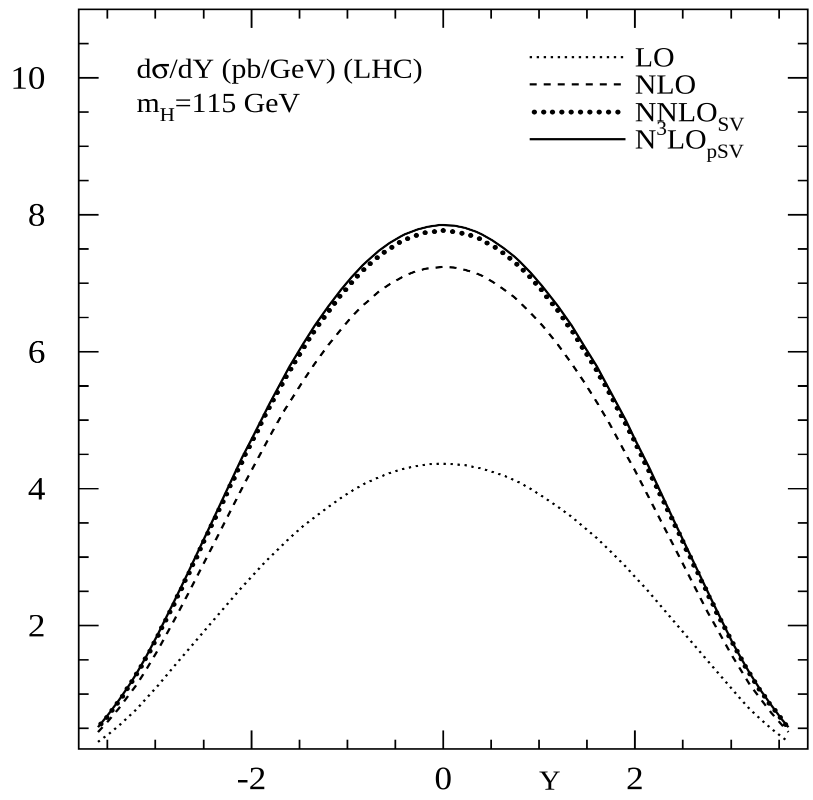

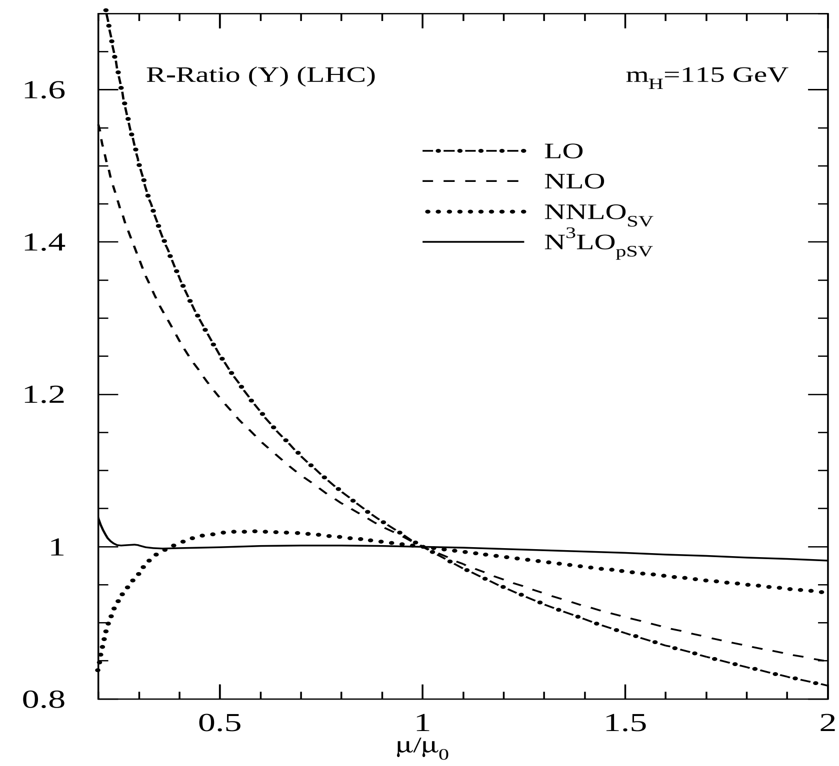

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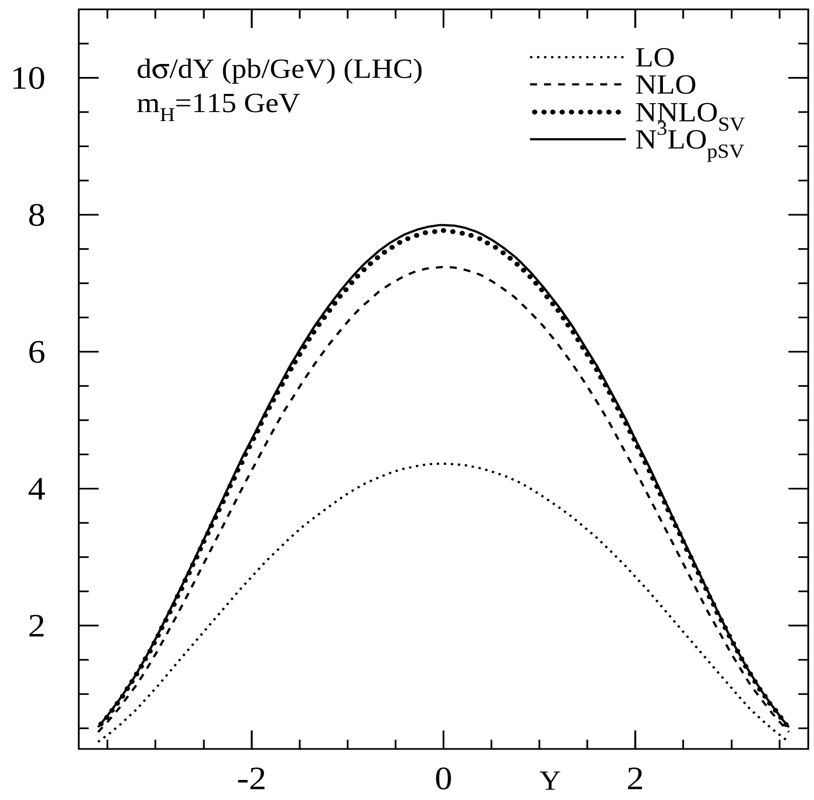

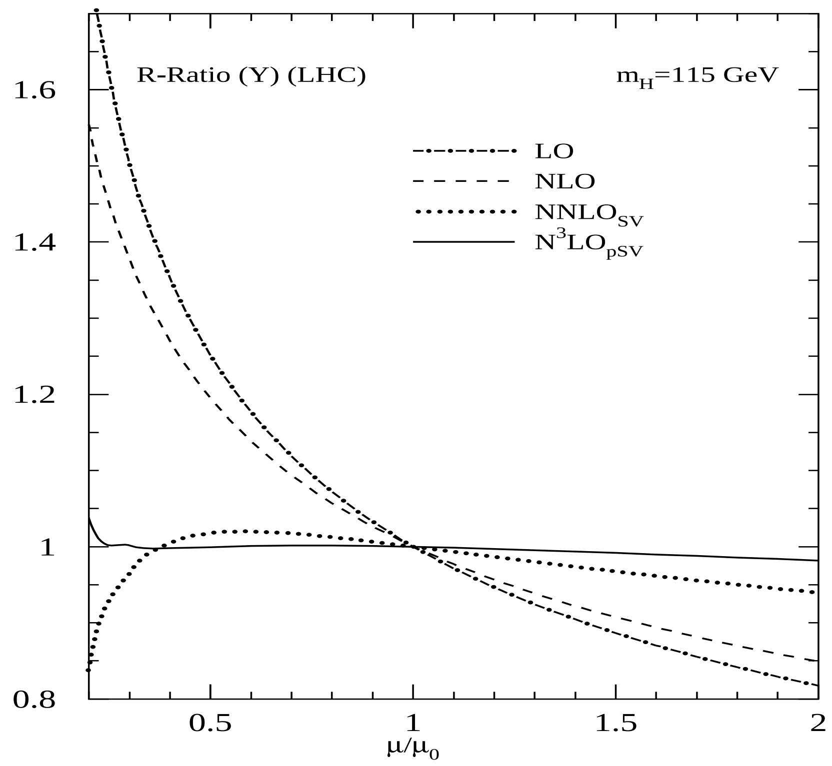

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- Soft plus virtual (**SpV**) to order N^3LO stabilises the perturbative prediction against scale variations.