Chaotic diffusion of fundamental frequencies of the Solar System

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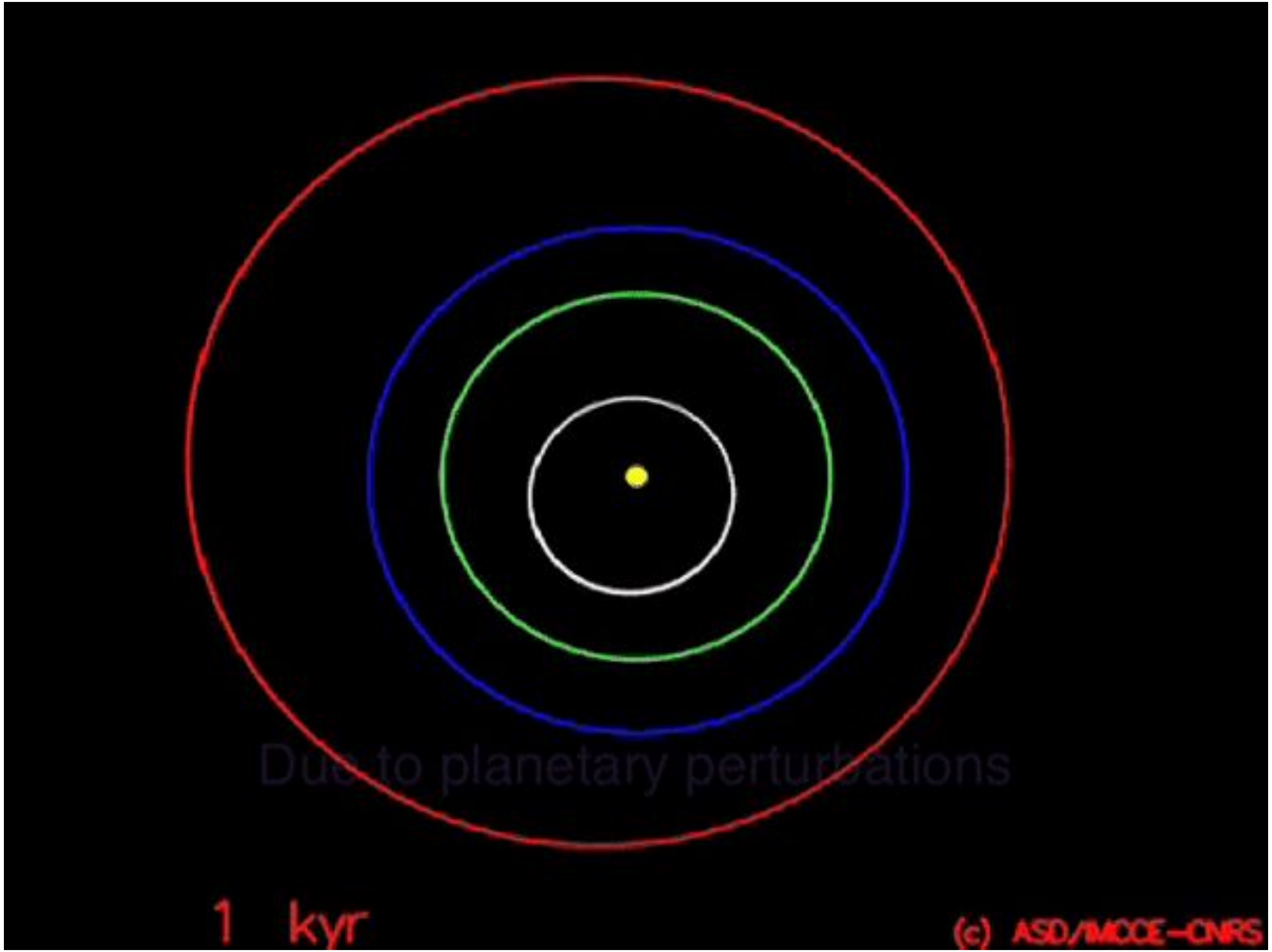
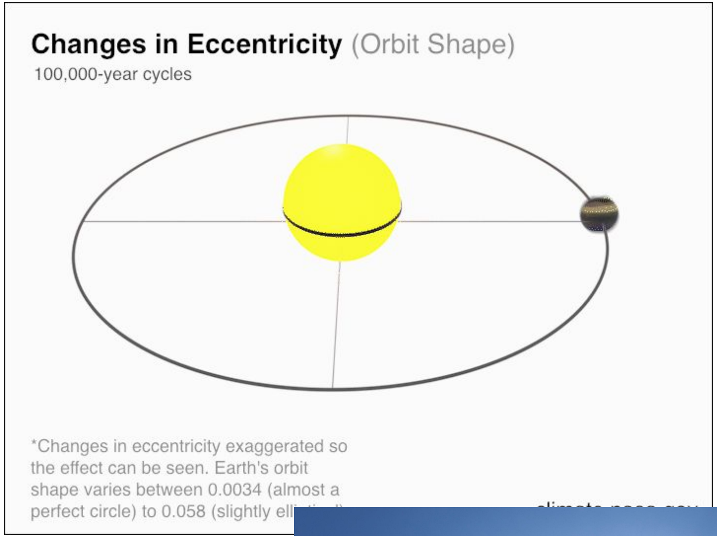
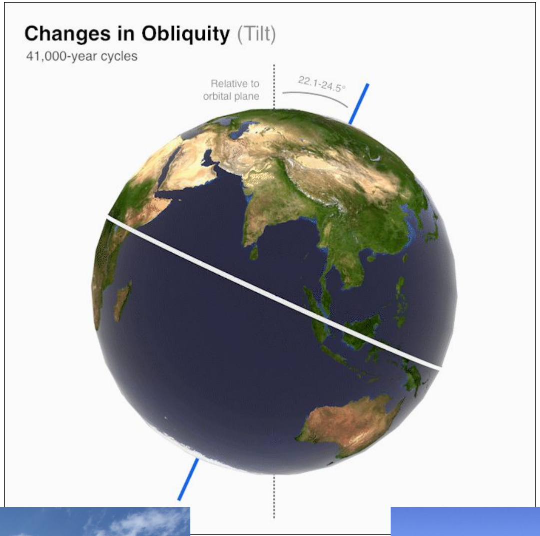
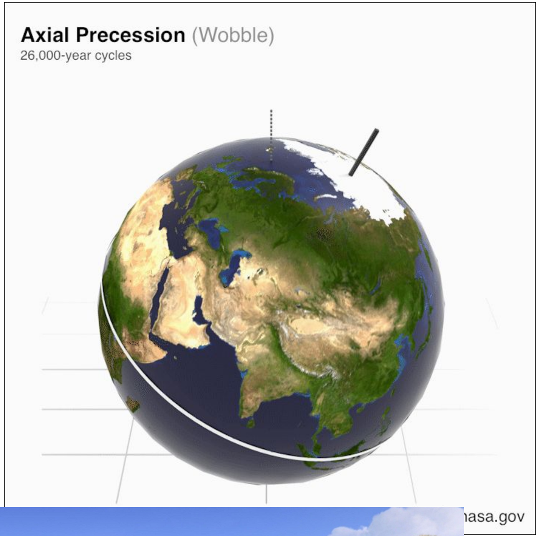

Due to planetary perturbations
1 kyr
(c) ASD/IMCCE-CNRS

Milankovitch cycles

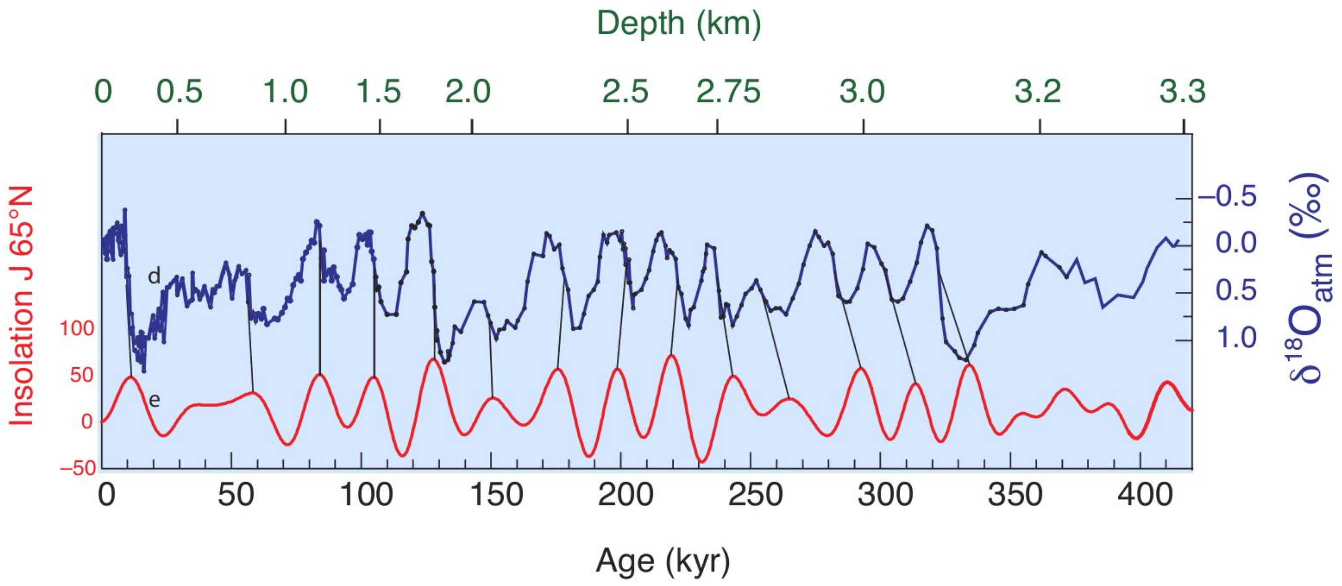


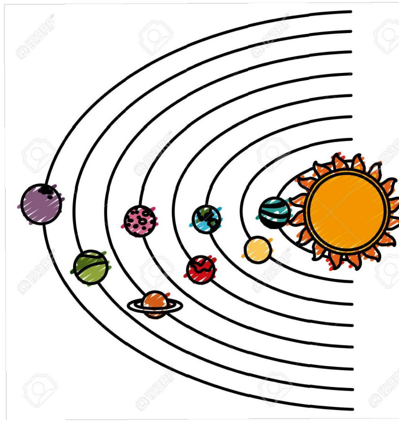




Milankovitch cycles:

Ice Archives





Petit, Jouzel, Raynaud et al, Nature, 1999

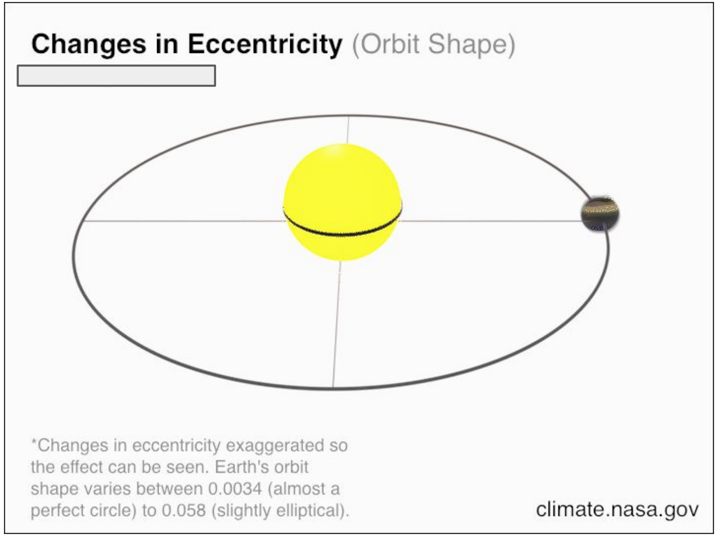
Fundamental Frequencies

Orbital Variation of the Earth can be approximated by a quasiperiodic series of combinations of frequencies:

- Eccentricity Frequencies $(g_i)_{i=1,8}$
- Obliquity Frequencies $(s_i)_{i=1,8}$

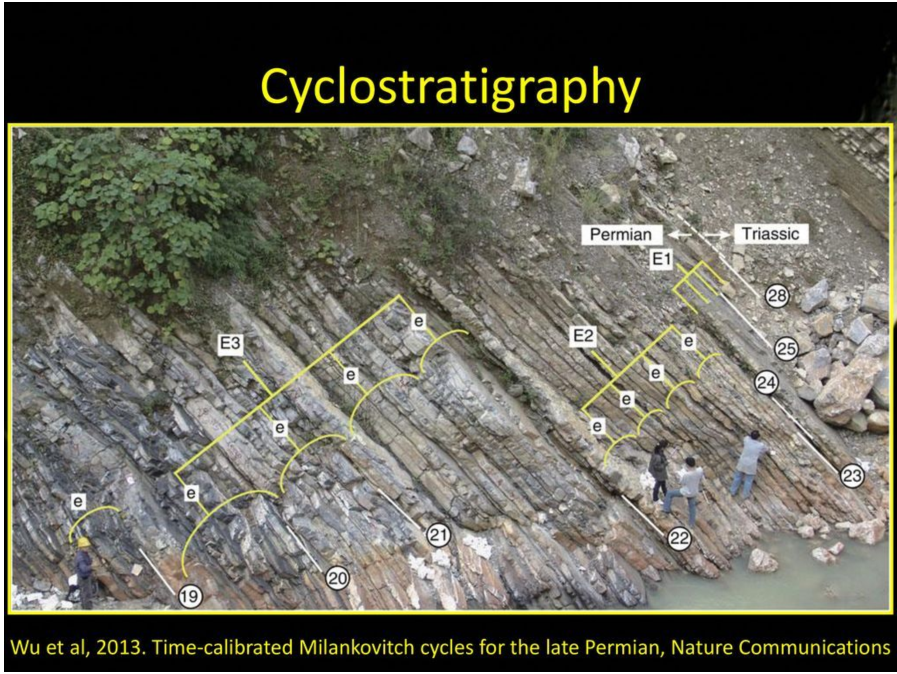
Example: Eccentricity Cycles of the Earth:

- 405 kyr: $g_2 - g_5$
- 100 kyr: $g_4 - g_5$, $g_3 - g_2$, ..



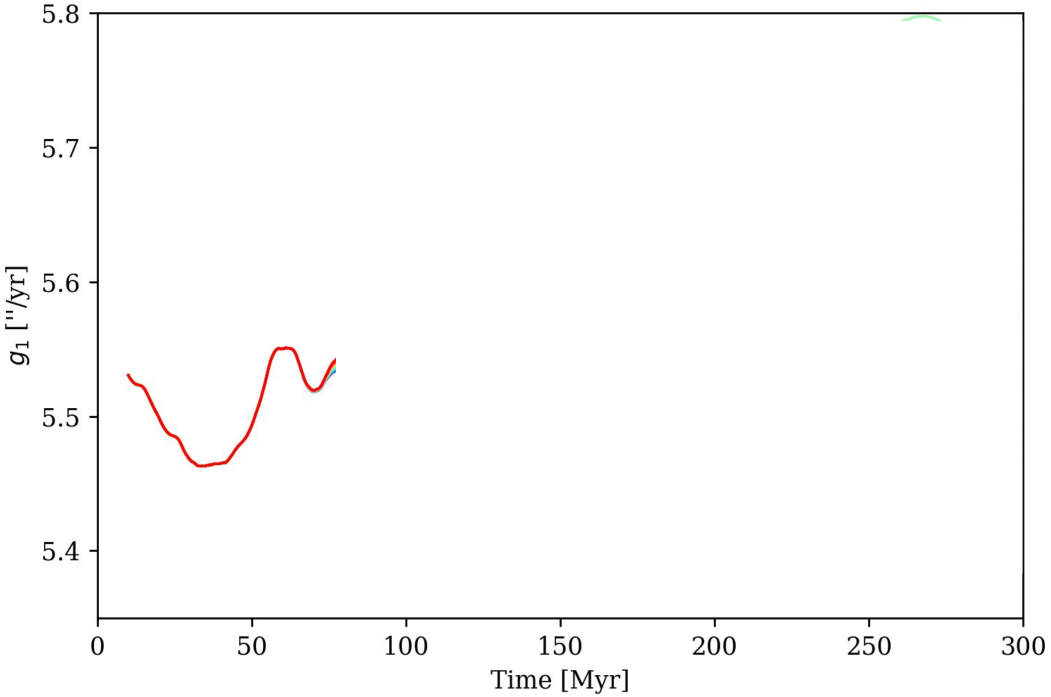
Milankovitch cycles: Cyclostratigraphy

Kuiper et al. 2008 Science



Celestial Mechanics

- The Solar System is chaotic
  with Lyapunov exponent
  ~ 5 Myr^{-1} (Laskar 1989)
- Fundamental Frequencies change over time
- Fundamental Frequencies can only be precisely determined up to 60 Myr (Laskar et al. 2011)

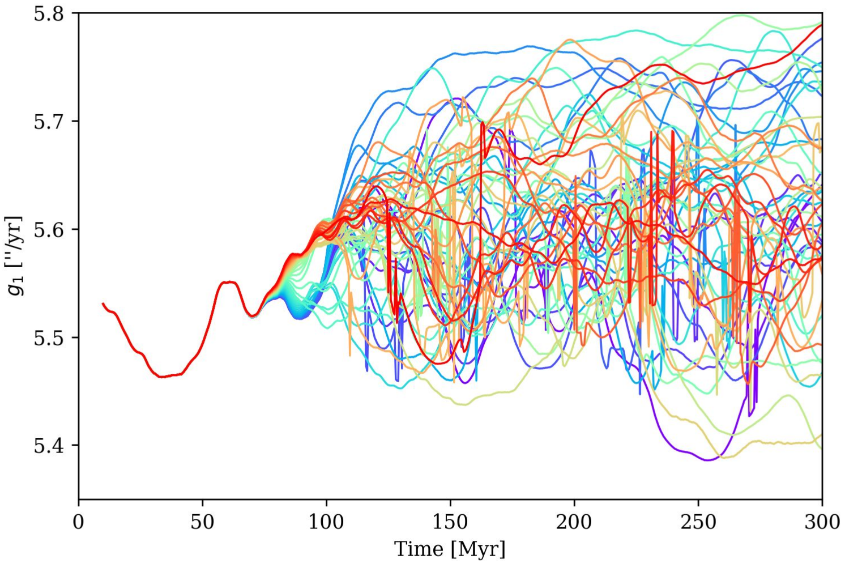


Numerical Implementation

Goal: the PDFs of the Fundamental Frequencies

- The averaged model of the solar System (Laskar 1990)
    - => Longer time step => 2000 times faster than the complete equations

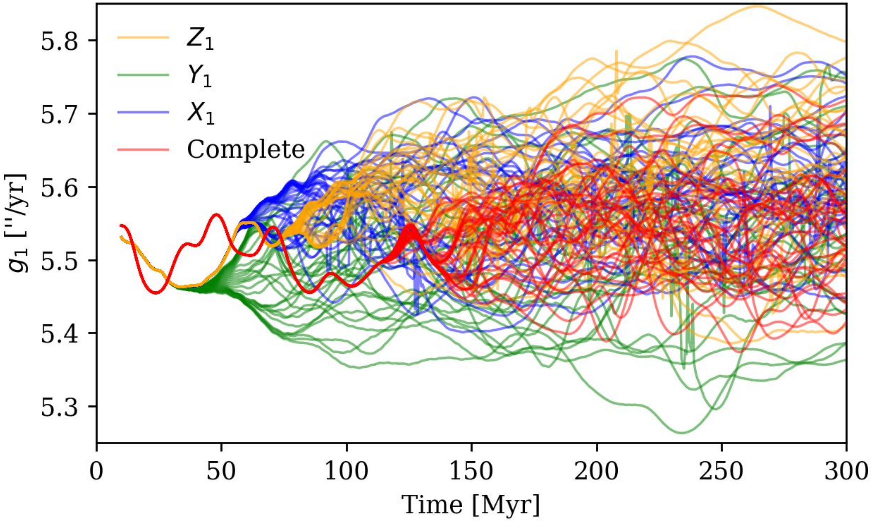
- PDF is estimated by Kernel Density Estimator
- The PDF uncertainty is estimated by Moving Block Bootstrap (Kuffner et al. 2019)
    - The resampling unit a block of solutions

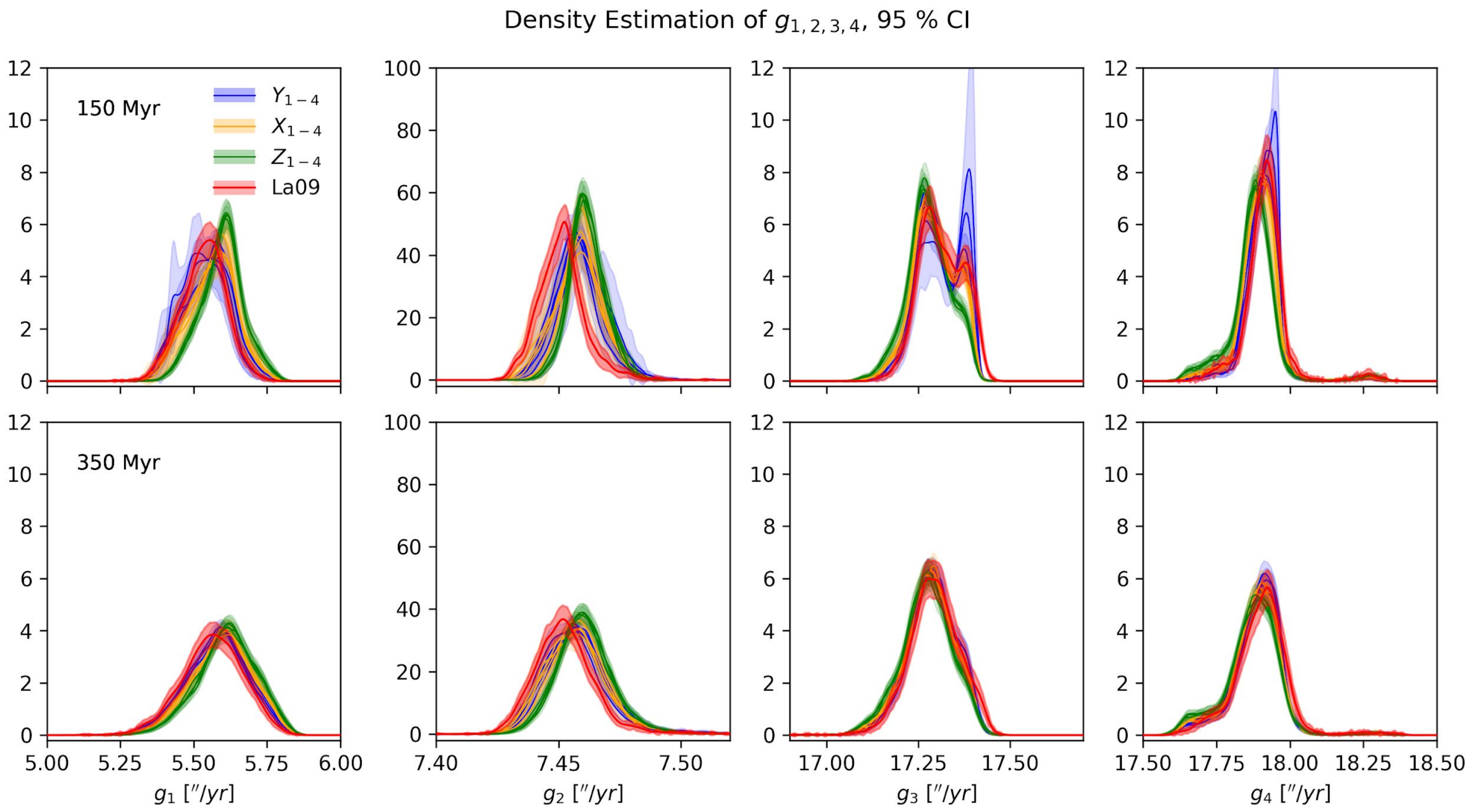


Goal: the PDFs of the Fundamental Frequencies

Benchmark:

- Consistency and Robustness (with respect to sampling of IC)
  - Response: Sample 12 sets of IC differently: $X_{1\text{-}4}$, $Y_{1\text{-}4}$, $Z_{1\text{-}4}$ generate 10,000 solutions each, and compare!
- Accuracy (with respect to the complete model)
  - Response: Compare with the solutions from the complete models (2500 complete solutions integrated in the future) (Laskar and Gastineau 2009 Nature)



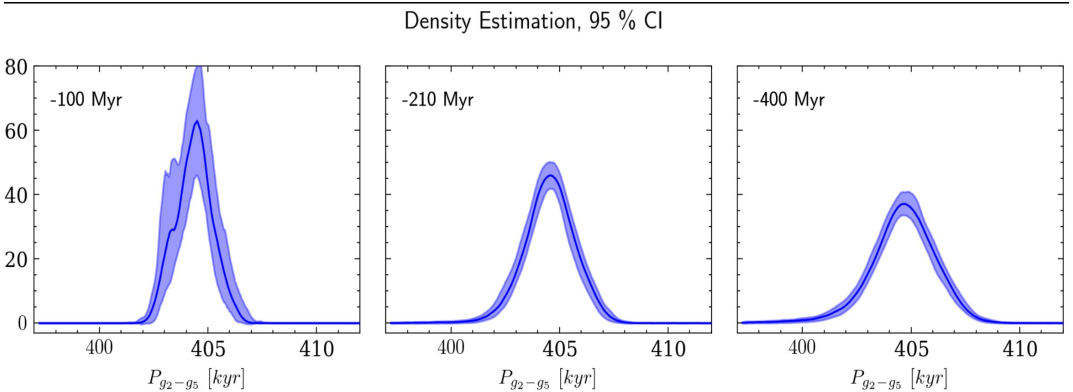


Density Estimation of $g_{1,2,3,4}$, 95 % CI
150 Myr
350 Myr
Y_{1-4}
X_{1-4}
Z_{1-4}
La09
g_1 [″/yr]
g_2 [″/yr]
g_3 [″/yr]
g_4 [″/yr]

Geological Application

The uncertainty of any astronomical signal is quantified!

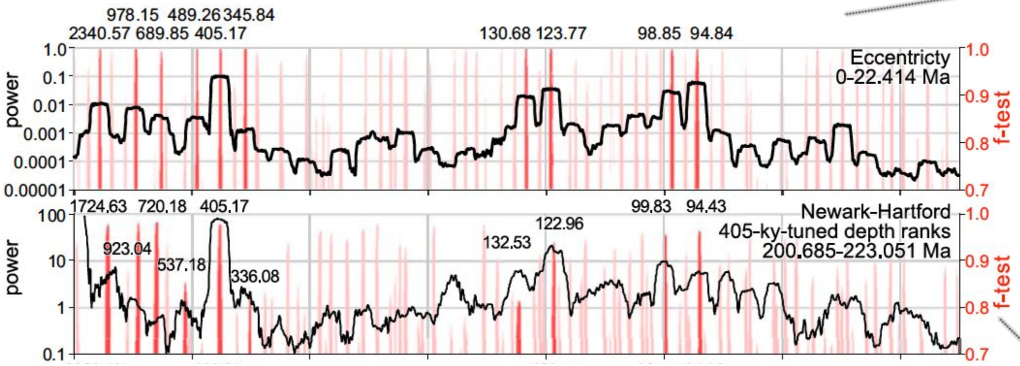
Kuiper et al. 2008 Science

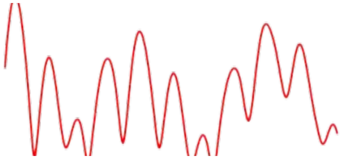


Next step

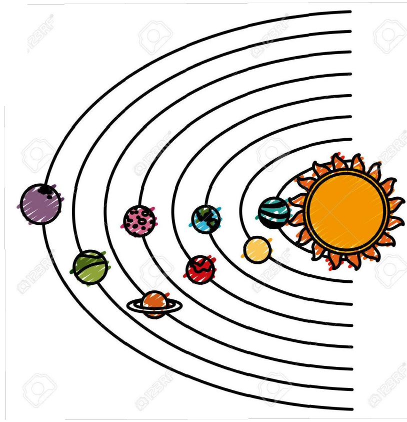
- A systematic formalism to combine the uncertainty from the geological records and the uncertainty from the astronomical frequencies:
  - Bayesian approach (Meyers & Malinverno 2018)

Geological Application: Newark-Hartford data



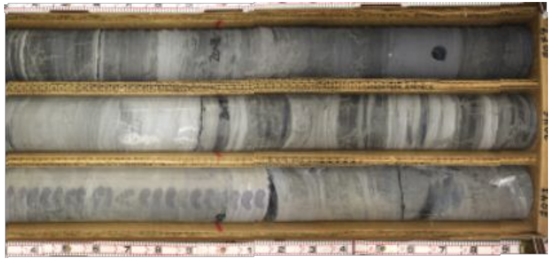


Eccentricity

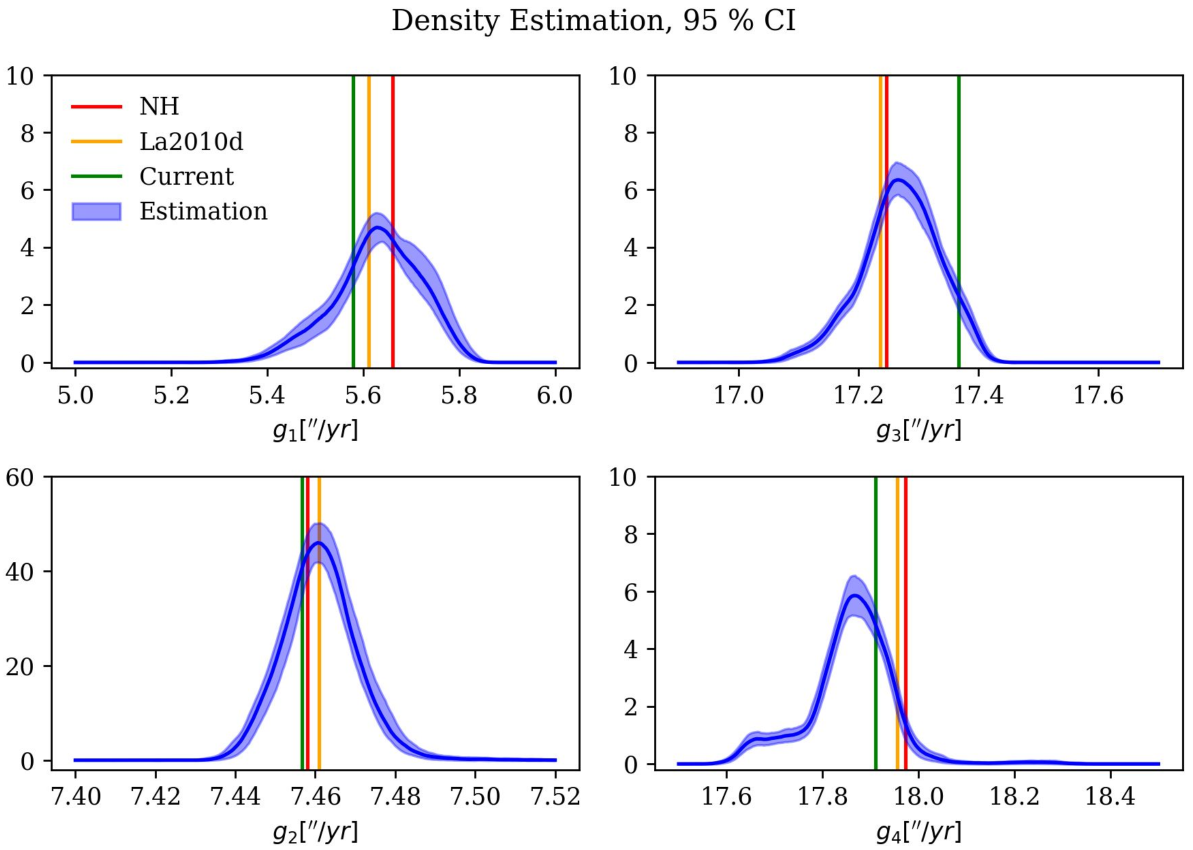


Olsen, Laskar et al. 2019
PNAS

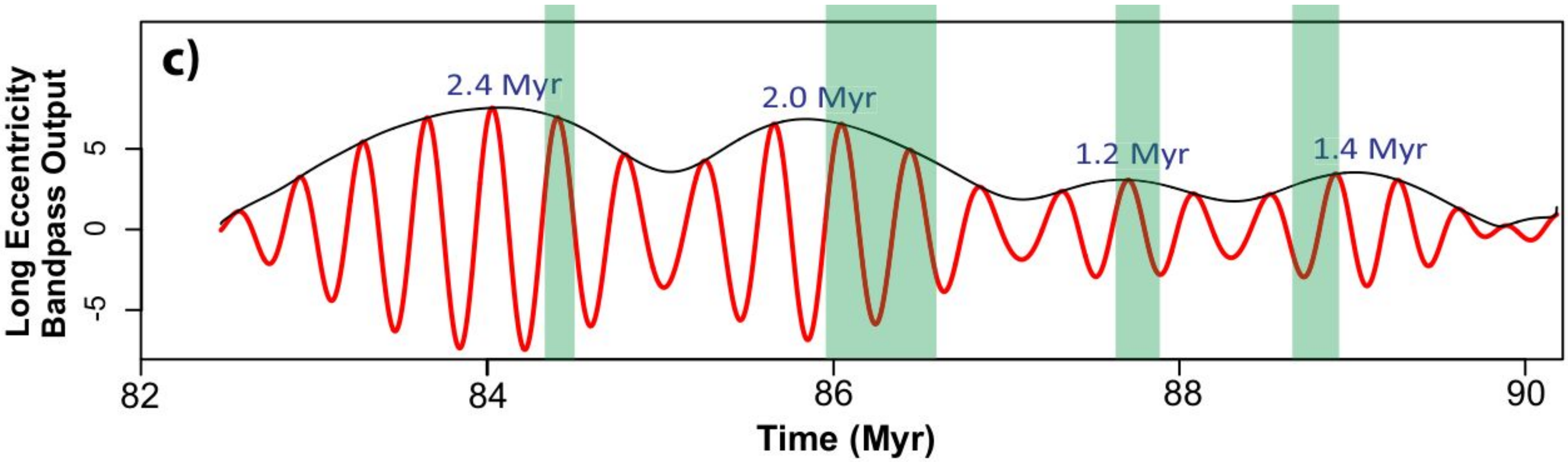
Newark-Hartford data
210 Myr ago



Geological Application: Newark-Hartford data



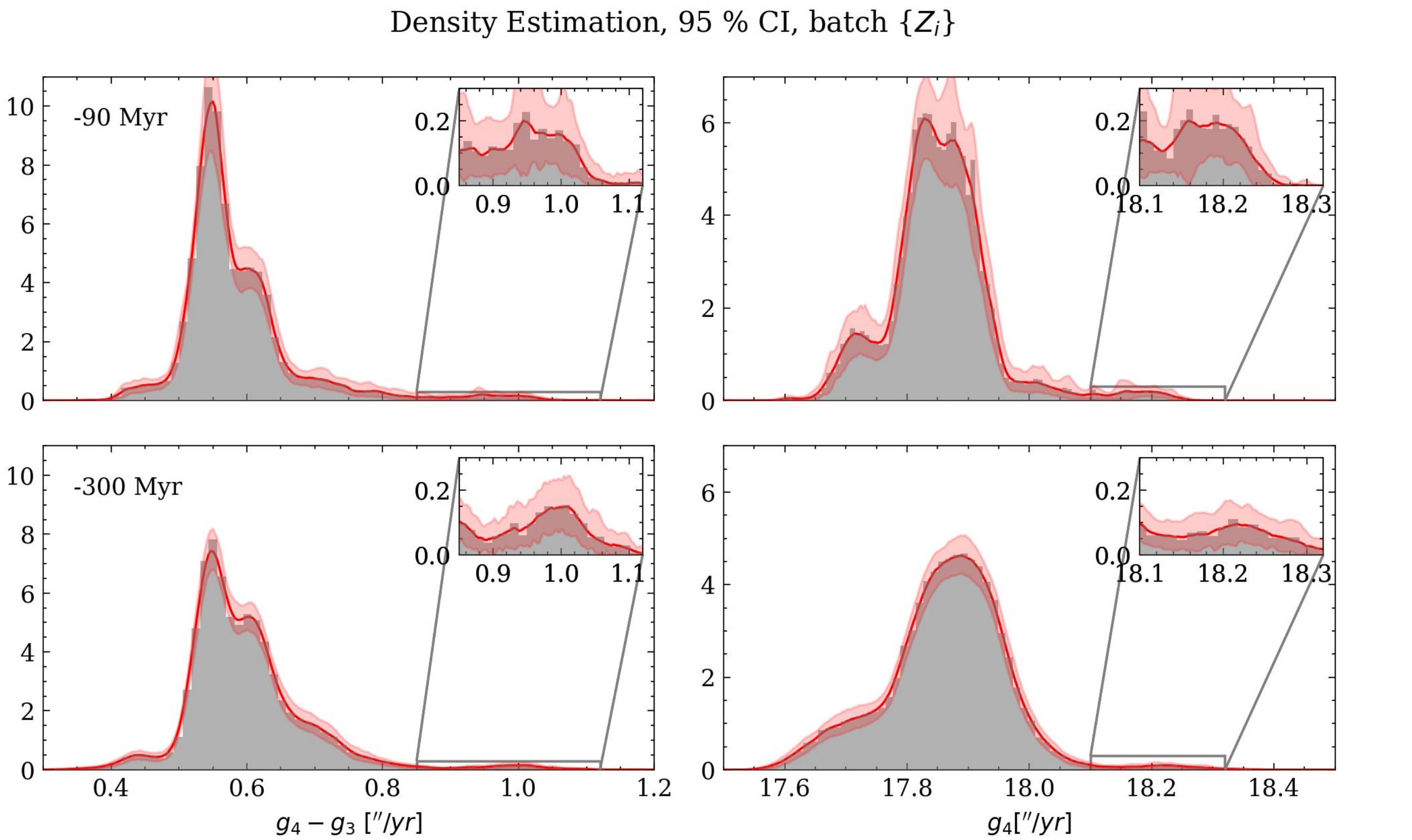
Geological Application: Libsack core



Bandpass filtered Libsack signal with window of [2.6 “/yr, 4.53 “/yr]

Ma et al. 2017, Nature

Geological Application: Libsack core



Geological Application: Libsack core