IMCCE l'Observatoire de Paris | PSL SORBONNE UNIVERSITÉ AstroGeo

On Ice Ages and the Earth's Dynamical Ellipticity

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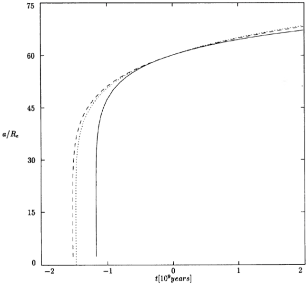
General Framework

In the context of planetary tidal and rotational evolution

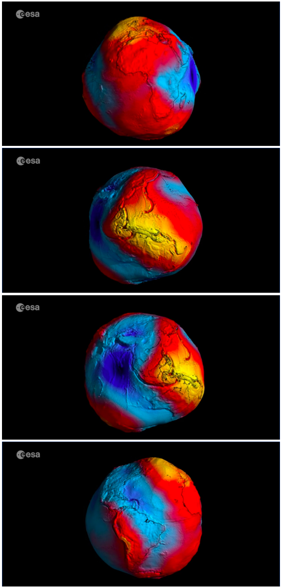
We seek to model long term dissipative effects under various geophysical aspects.

- Tidal dissipation and the "Time-Scale Problem" of Lunar age.
- Geophysical processes: plate tectonics, mantle convection, and surface loading.



Touma & Wisdom (1994)

The Earth's dynamical Ellipticity

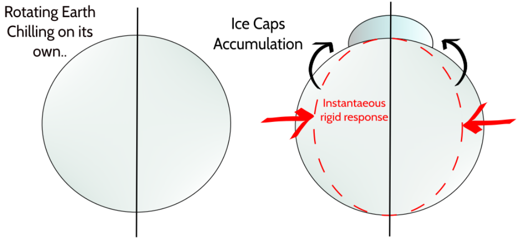


- Dynamical Ellipticity:

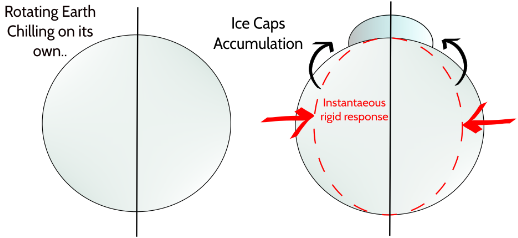
$$H = \frac{C - (A+B)/2}{C}$$

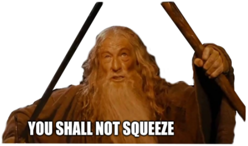
- Present observations $\rightarrow$ 0.5% excess.
- History is hardly constrained!

Variation Due to Ice Ages

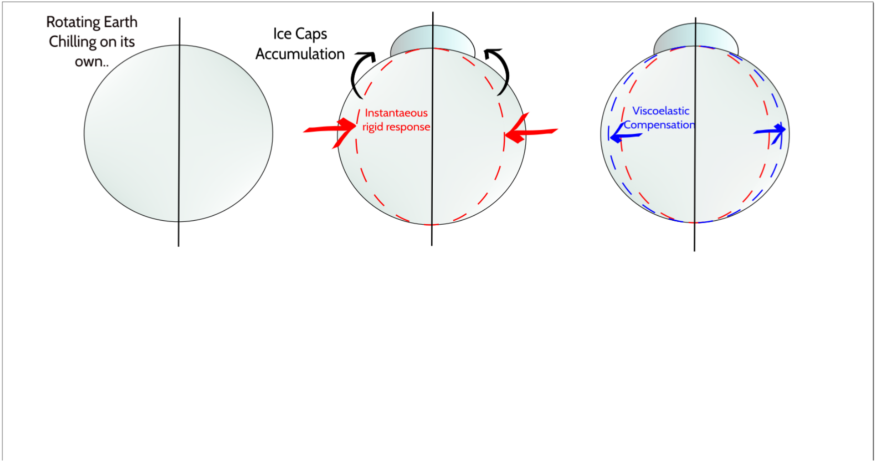


Variation Due to Ice Ages

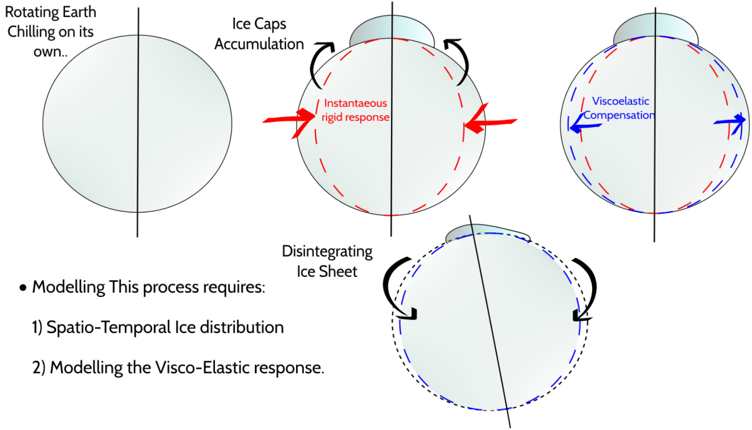




Variation Due to Ice Ages



Variation Due to Ice Ages

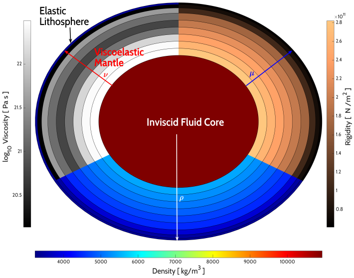


- Modelling This process requires:

  1) Spatio-Temporal Ice distribution

  2) Modelling the Visco-Elastic response.

Love Numbers and the VE Response



- Momentum Conservation
- Poisson's Equation
+
Continuity Equation
++
Rheology Constraint

Deformation Differential System:

Solved by Propagator Matrix Method

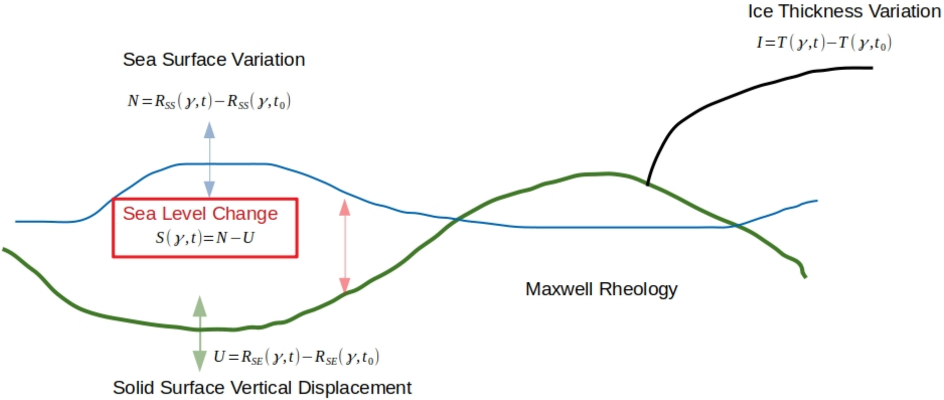
Calculate normal Modes using the

Residue Theorem

Love Numbers:

$$x_l^L(t) = x_l^{L,e}\delta(t) + \sum_{i=1}^{N} x_{l,i}^{L} e^{s_{l,i}t}$$

Utilizing the Sea level Equation



Sea Level Equation in Simplest Form

Because both vertical displacement and sea surface variations are function of sea level itself (Farrel & Clark 1976)

$$\boxed{S} = \frac{\rho_i}{\gamma}\boxed{G_s}\otimes_i\boxed{I} + \frac{\rho_w}{\gamma}\boxed{G_s}\otimes_o\boxed{S} - \frac{m_i}{\rho_w A_0} - \frac{\rho_i}{\gamma}\overline{\boxed{G_s}\otimes_i\boxed{I}} - \frac{\rho_w}{\gamma}\overline{\boxed{G_s}\otimes_o\boxed{S}} \tag{1}$$

$\boxed{S}$ =sea level change

m_i = ice mass variation

ρ_i, ρ_w = ice and water densities

A_0 = area of oceans

$\boxed{G_s}$ = sea level Green function

$\otimes_i, \otimes_o$ = 3D convolutions

$\boxed{I}$ = ice thickness variation

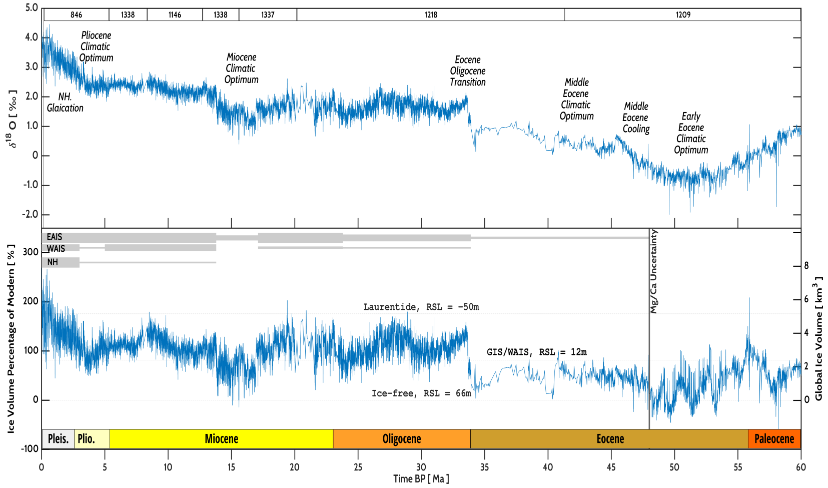
$\overline{(x)}$ = ocean averaging

$$\mathcal{G}_{lm} = \frac{3}{\rho^E}\sum_n^N \frac{\Delta\mathcal{L}_{lm,n}}{2l+1}\left(1 + k_l^{L,e} + \sum_{i=1}^M \frac{k_{li}^L}{s_{li}}(e^{s_{li}(t-t_n)} - 1)\right)H(t - t_n) \tag{2}$$

$$H = \frac{C - (A+B)/2}{C} \tag{3}$$

$$\delta H(t) \approx 3\delta J_l(t) = -\frac{1}{a}\sqrt{2l+1}\,\mathcal{G}_{l0}(t) \tag{4}$$

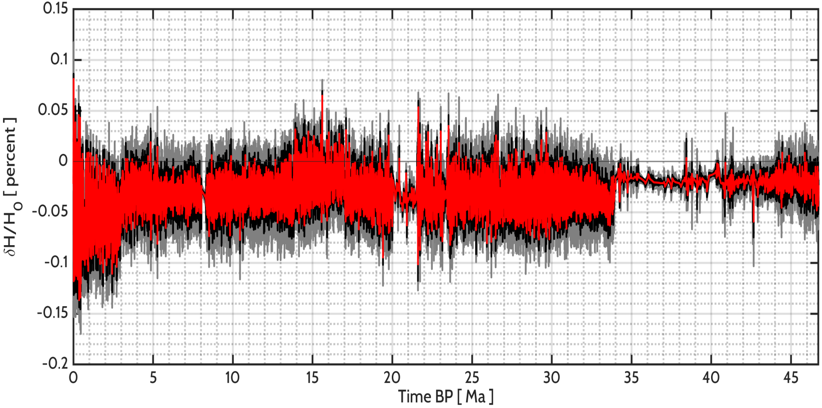
Ice History



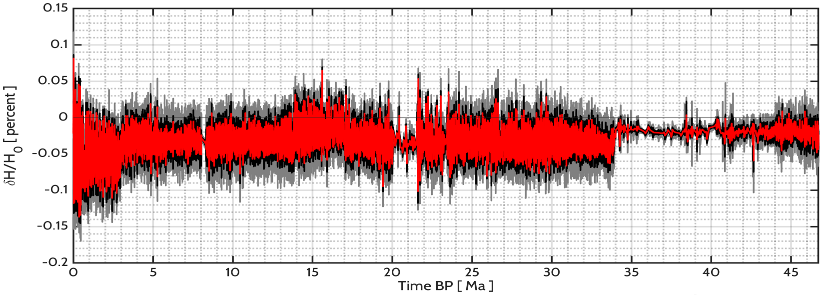
846
1338
1146
1338
1337
1218
1209
5.0
4.0
3.0
2.0
1.0
0
-1.0
-2.0
δ^{18} O [‰]
Pliocene Climatic Optimum
NH. Glaication
Miocene Climatic Optimum
Eocene Oligocene Transition
Middle Eocene Climatic Optimum
Middle Eocene Cooling
Early Eocene Climatic Optimum
EAIS
WAIS
NH
Mg/Ca Uncertainty
Laurentide, RSL = -50m
GIS/WAIS, RSL = 12m
Ice-free, RSL = 66m
300
200
100
0
-100
Ice Volume Percentage of Modern [%]
8
6
4
2
0
Global Ice Volume [km^3]
Pleis.
Plio.
Miocene
Oligocene
Eocene
Paleocene
0
5
10
15
20
25
30
35
40
45
50
55
60
Time BP [Ma]

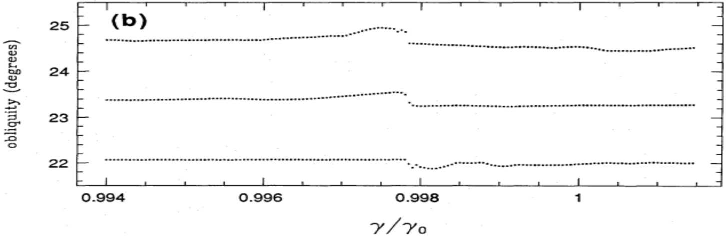
Evolution of Dynamical Ellipticity:

$s_6 - g_6 + g_5$



Evolution of Dynamical Ellipticity:
Ice Sensitivity





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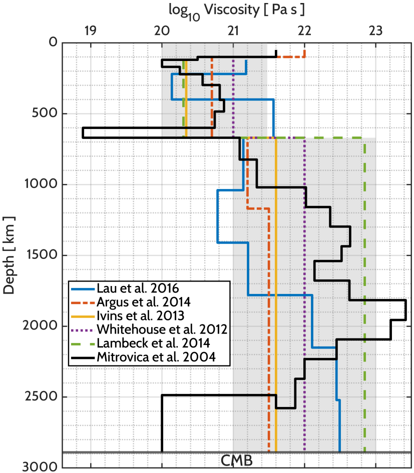
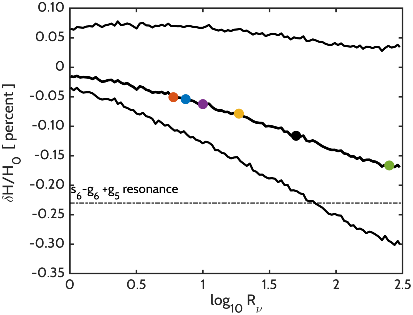
Evolution of Dynamical Ellipticity:
Viscosity Sensitivity





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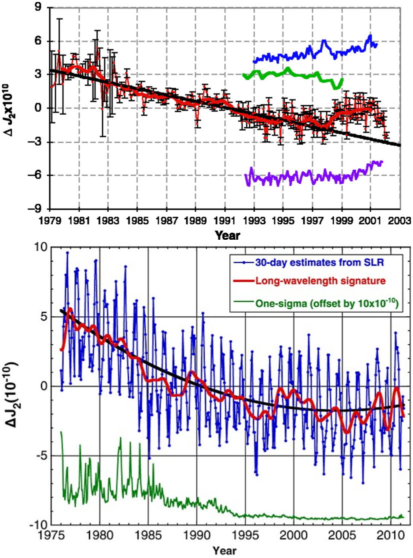
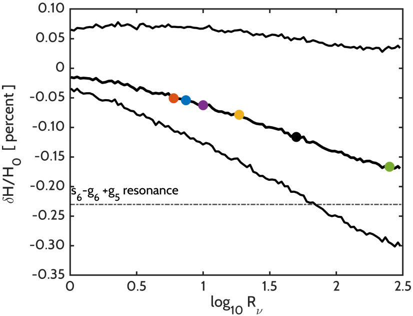
Evolution of Dynamical Ellipticity:
Constraining by $\dot{J}_2$ SLR Observations





Thank you for your attention!