# Imagerie par interférométrie optique depuis l'Espace

**Hiyam DEBARY**
***Doctorante 2ème année***
***ONERA, Département DOTA/HRA***

**Directeurs de thèse :**
**Vincent Michau (DOTA/HRA)**
**Laurent Mugnier (DOTA/HRA)**
**Encadrant Airbus Defence & Space:**
**Sebastien Lopez**

AIRBUS DEFENCE & SPACE

ONERA THE FRENCH AEROSPACE LAB

# Context [1/2]

Earth/space observation

☹ Expensive
☹ Voluminous

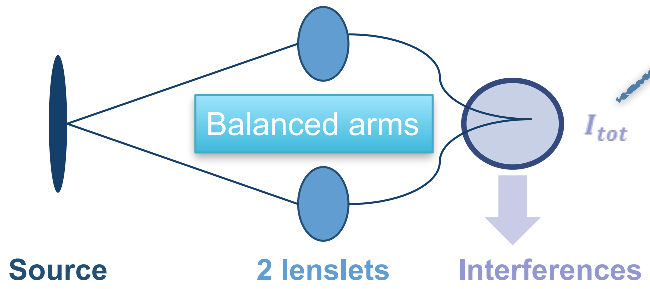
**SPIDER (Segmented Planar Imaging Detector for ElectroOptical Reconnaissance) concept**

☺ Compact structure
☺ Less expensive
☺ Can be embarked on nano-satellites

# Context [2/2]

## Optical interferometry

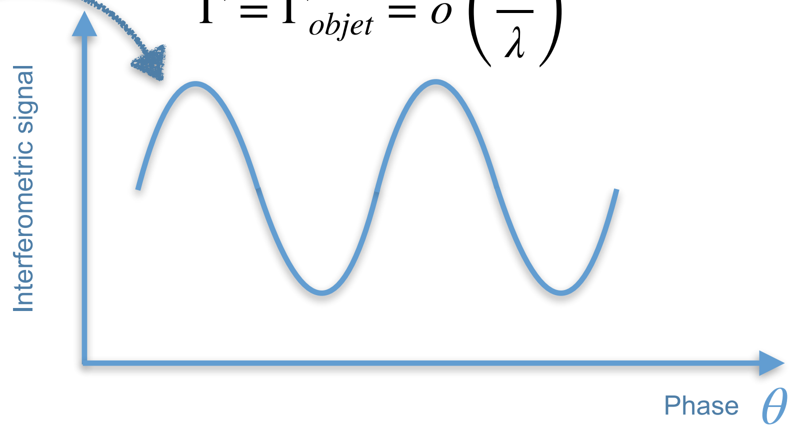
- **Relies on the interferometry principle**




$$\theta = \theta_{object}$$

**Zernike-Van Cittert theorem**

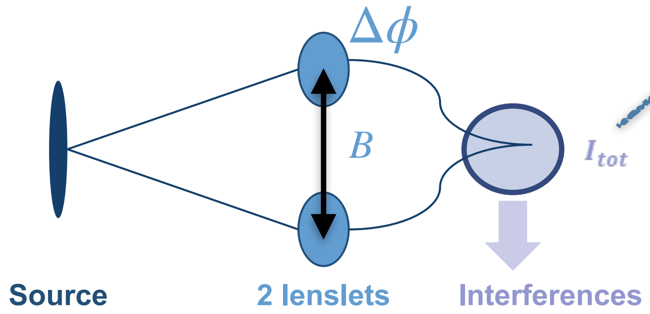
$$\Gamma = \Gamma_{objet} = \tilde{o}\left(\frac{B}{\lambda}\right)$$



# Context [2/2]

## Optical interferometry

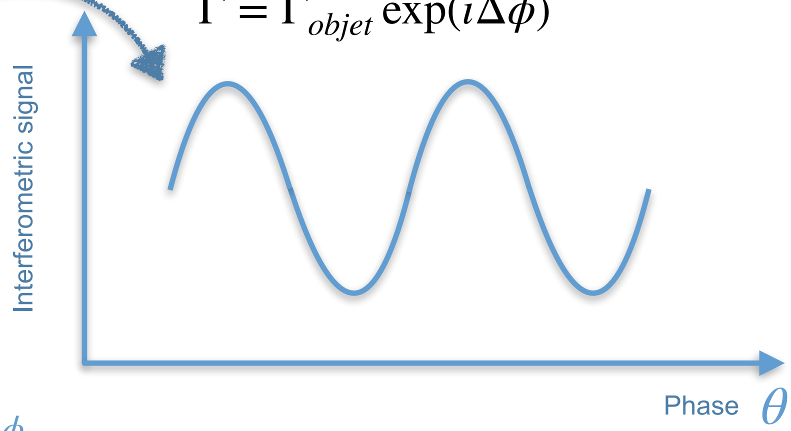
- **Relies on the interferometry principle**




$$\theta = \theta_{object} + \Delta\phi$$

**Zernike-Van Cittert theorem**

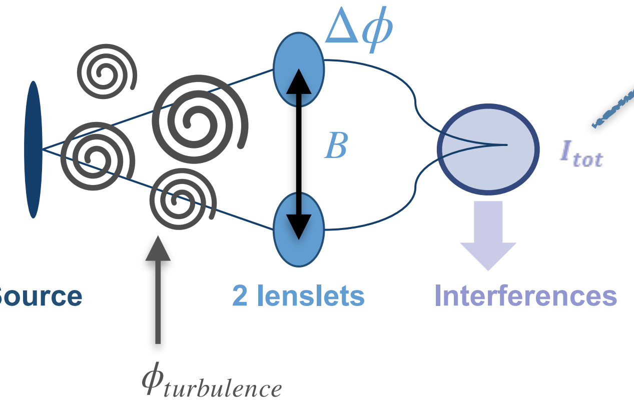
$$\Gamma = \Gamma_{objet} \exp(i\Delta\phi)$$



# Context [2/2]

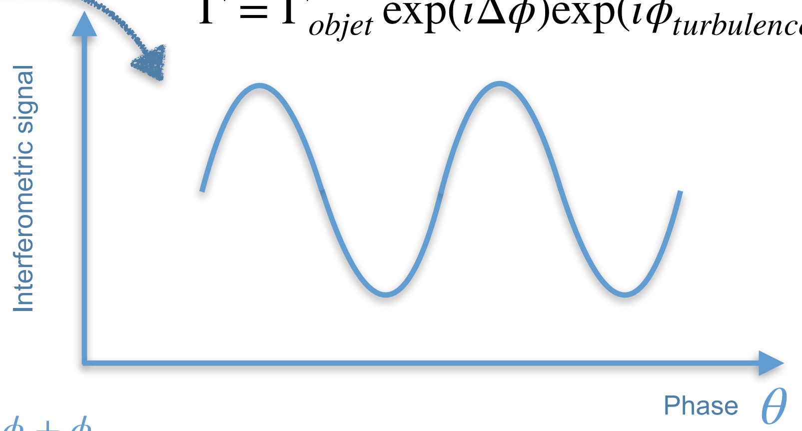
## Optical interferometry

- Relies on the interferometry principle




**Zernike-Van Cittert theorem**

$$\Gamma = \Gamma_{objet} \exp(i\Delta\phi)\exp(i\phi_{turbulence})$$




$$\theta = \theta_{object} + \Delta\phi + \phi_{turbulence}$$

**Detected signal** ⇨ **Complex visibility** ⇨ **Image through ZVC, with noise taken into account**

**PhD aims:**

- Performance analysis of the SPIDER concept
- Improvement suggestions

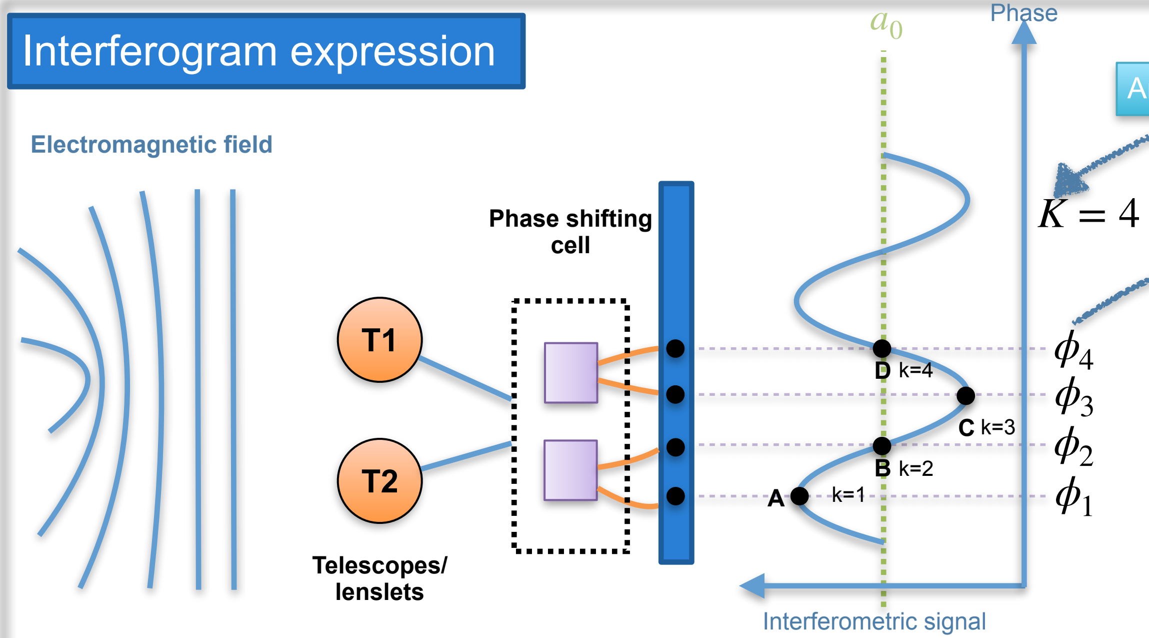
# Outline

- Context
- Observational model
- Noise propagation
- Method summarization
- Conclusion & perspectives

# Observation model

## Measurement of complex visibilities - Noise propagation

### Interferogram expression




$$i_k = a_0 + a_1 \cos \phi_k + a_2 \sin \phi_k$$

Signal

$\Re(\Gamma)$ $\Im(\Gamma)$

### Observable of interest : complex visibility

$$\Gamma = a_1 + i a_2$$

### Matricial formalism of the observation model

$$\begin{bmatrix} i_1 \\ \vdots \\ i_K \end{bmatrix} = \begin{bmatrix} 1 & \cos \phi_1 & \sin \phi_1 \\ \vdots & \vdots & \vdots \\ 1 & \cos \phi_K & \sin \phi_K \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} + b$$

Measures

Observables

Noise (photon or detector)

Reconstruction matrix ***D*** built either with:
- Maximum likelihood method (ML)
- Or least squares method (LS)
- Varying K

# Noise propagation [1/5]

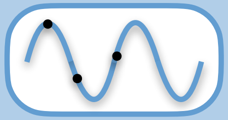
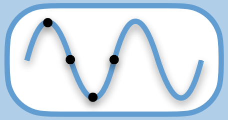
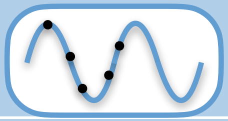
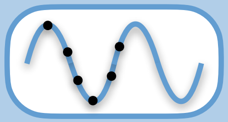
## Analytical formulas - Complex visibility variance

**Noise covariance matrix**

$$\mathbb{E} = DC_bD^T = \begin{bmatrix} \sigma^2_{a_0} & \sigma^2_{a_0,a_1} & \sigma^2_{a_0,a_2} \\ \sigma^2_{a_1,a_0} & \sigma^2_{a_1} & \sigma^2_{a_1,a_2} \\ \sigma^2_{a_2,a_0} & \sigma^2_{a_2,a_1} & \sigma^2_{a_2} \end{bmatrix}$$

$$\sigma^2_\Gamma = \sigma^2_{a_1} + \sigma^2_{a_2}$$

Complex visibility variance

| Modulation values | Photon noise | | Detector noise | |
|---|---|---|---|---|
| | **Maximum likelihood** | **Least squares** | **Maximum likelihood** | **Least squares** |
|  $K=3$ | $4a_0$ | $4a_0$ | $12\sigma_d^2$ | |
|  $K=4$ | $a_0(4-\gamma^2)$ | | $16\sigma_d^2$ | |
|  $K=5$ | $\frac{8a_0(2-\gamma^2)}{4-\gamma^2}$ | | $20\sigma_d^2$ | |
|  $K=6$ | $\frac{a_0(16-12\gamma^2+\gamma^4)}{4-2\gamma^2}$ | | $24\sigma_d^2$ | |

**Validation of expressions through Monte Carlo simulations**

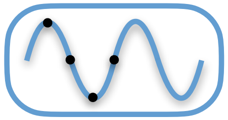
# Noise propagation [2/5]

## Application to GRAVITY (VLT)

Simulation hypotheses of the analytical estimation:
- Visibility : 90%
- Constant flux of 100 photons
- **Photon noise only**

**Complex visibility error**

K=4 + LS (GRAVITY) vs ML



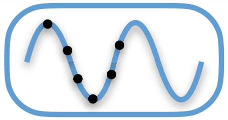
**Least squares**

$$\sigma_\Gamma^2 = 4a_0 = 400$$

**Maximum likelihood** +

$$\sigma_\Gamma^2 = a_0(4-\gamma^2) = 300$$

Improvement suggestion: K=6 + ML



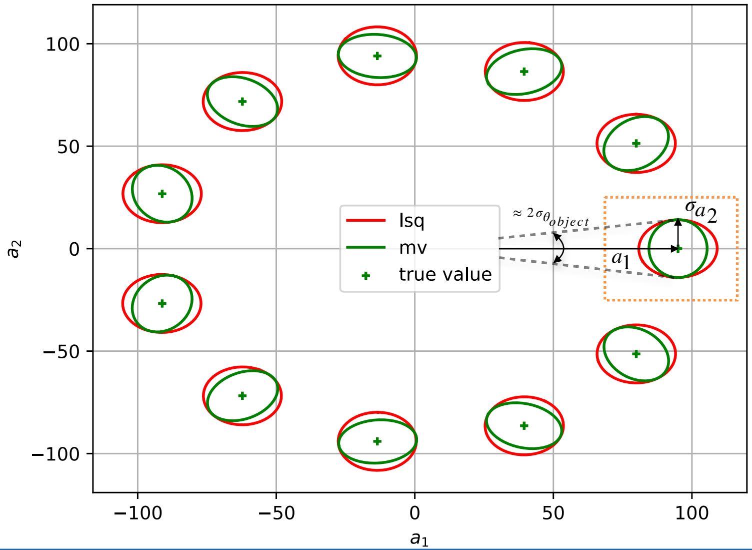
**Maximum likelihood** ++

$$\sigma_\Gamma^2 = \frac{5}{2}a_0 = 250$$

**To go farther… Error on phase estimation : fringe sensor**

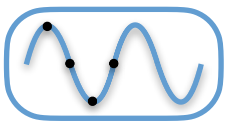
**Half-axes of the ellipses represent the eigenvalues of the truncated covariance matrix**

$$\begin{cases} \theta_{object} = 0 \\ \gamma = 1 \end{cases} \Longrightarrow \sigma^2_{\theta_{object}} \approx \frac{\sigma^2_{a_2}}{a_1^2}$$




**Phase error**

K=4 + LS (GRAVITY) vs ML



**Least squares**

$$\sigma^2_{\theta_{object}} = \frac{2}{a_0}$$

=

**Maximum likelihood**

$$\sigma^2_{\theta_{object}} = \frac{2}{a_0}$$

# Noise propagation [3/5]

## Application to the fringe sensor of GRAVITY (VLT) - Phase variance

| Modulation values | Photon noise | | Detector noise | |
|---|---|---|---|---|
| | Maximum likelihood | Least squares | Maximum likelihood | Least squares |
| $K = 3$ | $\frac{1}{a_0}$ | | $\frac{6\sigma_d^2}{a_0^2}$ | |
| $K = 4$ | $\frac{2}{a_0}$ | $\frac{2}{a_0}$ | $\frac{8\sigma_d^2}{a_0^2}$ | |
| $K = 5$ | $\frac{1}{a_0}$ | | $\frac{10\sigma_d^2}{a_0^2}$ | |
| $K = 6$ | $\frac{3}{2a_0}$ | | $\frac{12\sigma_d^2}{a_0^2}$ | |

**Reminder:** $$\sigma^2_{\theta_{object}} = \frac{\sigma^2_{a_1} a_2^2 + \sigma^2_{a_2} a_1^2 - 2 a_1 a_2 \sigma^2_{a_1,a_2}}{(a_1^2 + a_2^2)^2}$$

# Noise propagation [4/5]

## Improving the choice of the object phase with 100% visibility - Phase variance

| Modulation values | Photon noise (maximum likelihood) $\gamma = 1$ | | | | | |
|---|---|---|---|---|---|---|
| | Derivative solutions | | | | | |
| Extremums | $0$ | $\pi/3$ | $2\pi/3$ | | | |
| $K = 3$ | $\frac{1}{a_0}$ | $\frac{3}{a_0}$ | $\frac{1}{a_0}$ | | | |
| Extremums | $0$ | $\pi/4$ | $\pi/2$ | $3\pi/4$ | | |
| $K = 4$ | $\frac{2}{a_0}$ | $\frac{1}{a_0}$ | $\frac{2}{a_0}$ | $\frac{1}{a_0}$ | | |
| Extremums | $0$ | $2\arctan\left(\sqrt{5}\sqrt{-2\sqrt{5}+5/5}\right)$ | $2\arctan\left(\sqrt{-2\sqrt{5}+5}\right)$ | $2\arctan\left(\sqrt{5}\sqrt{2\sqrt{5}+5/5}\right)$ | $4\pi/5$ | |
| $K = 5$ | $\frac{1}{a_0}$ | $\frac{5}{3a_0}$ | $\frac{1}{a_0}$ | $\frac{5}{3a_0}$ | $\frac{1}{a_0}$ | |
| Extremums | $0$ | $\pi/6$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $2\arctan\left(\sqrt{3}+2\right)$ |
| $K = 6$ | $\frac{3}{2a_0}$ | $\frac{1}{a_0}$ | $\frac{3}{2a_0}$ | $\frac{1}{a_0}$ | $\frac{3}{2a_0}$ | $\frac{1}{a_0}$ |

**Reminder:** $$\sigma^2_{\theta_{object}} = \frac{\sigma^2_{a_1} a_2^2 + \sigma^2_{a_2} a_1^2 - 2 a_1 a_2 \sigma^2_{a_1,a_2}}{(a_1^2 + a_2^2)^2}$$

# Noise propagation [4/5]

## Improving the choice of the object phase with any visibility - Phase variance

| Modulation values | Photon noise (maximum likelihood) $\forall \gamma$ | | | | | |
|---|---|---|---|---|---|---|
| | Derivative solutions | | | | | |
| Extremums | $0$ | $\pi/3$ | $2\pi/3$ | | | |
| $K = 3$ | $\frac{2-\gamma}{\gamma^2 a_0}$ | $\frac{2+\gamma}{\gamma^2 a_0}$ | $\frac{2-\gamma}{\gamma^2 a_0}$ | | | |
| Extremums | $0$ | $\pi/4$ | $\pi/2$ | $3\pi/4$ | | |
| $K = 4$ | $\frac{2}{\gamma^2 a_0}$ | $\frac{2}{\gamma^2 a_0} - \frac{1}{a_0}$ | $\frac{2}{\gamma^2 a_0}$ | $\frac{2}{\gamma^2 a_0} - \frac{1}{a_0}$ | | |
| Extremums | $0$ | $4\pi/5$ | $2 \arctan\left(\sqrt{5}\sqrt{-2\sqrt{5}+5/5}\right)$ | $2 \arctan\left(\sqrt{5}\sqrt{2\sqrt{5}+5/5}\right)$ | $2 \arctan\left(\sqrt{-2\sqrt{5}+5}\right)$ | |
| $K = 5$ | $\frac{4-2\gamma-\gamma^2}{\gamma^2 a_0(2-\gamma)}$ | $\frac{4-2\gamma-\gamma^2}{\gamma^2 a_0(2-\gamma)}$ | $\frac{-4-2\gamma+\gamma^2}{\gamma^2 a_0(-2-\gamma)}$ | $\frac{-4-2\gamma+\gamma^2}{\gamma^2 a_0(-2-\gamma)}$ | $\frac{4-2\gamma-\gamma^2}{\gamma^2 a_0(2-\gamma)}$ | |
| Extremums | $0$ | $\pi/6$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $2 \arctan\left(\sqrt{3}+2\right)$ |
| $K = 6$ | $\frac{2-\frac{\gamma^2}{2}}{\gamma^2 a_0}$ | $\frac{4-3\gamma^2}{\gamma^2 a_0(2-\gamma^2)}$ | $\frac{2-\frac{\gamma^2}{2}}{\gamma^2 a_0}$ | $\frac{4-3\gamma^2}{\gamma^2 a_0(2-\gamma^2)}$ | $\frac{2-\frac{\gamma^2}{2}}{\gamma^2 a_0}$ | $\frac{4-3\gamma^2}{\gamma^2 a_0(2-\gamma^2)}$ |

**Reminder:**

$$\sigma^2_{\theta_{object}} = \frac{\sigma^2_{a_1} a_2^2 + \sigma^2_{a_2} a_1^2 - 2 a_1 a_2 \sigma^2_{a_1,a_2}}{(a_1^2 + a_2^2)^2}$$

# Method summarization

| | | | | | | |
|---|---|---|---|---|---|---|
| **Variable of interest** | Phase | OR | Complex visibility | | | |
| **Estimation method** | Maximum likelihood | OR | Least squares | | | |
| **Modulation method** | K = 3 | OR | K = 4 | OR | K = 5 | OR K = 6 |
| **Noise type** | Photon noise | AND/OR | Detector noise | | | |

# Outline


- Context
- Observational model
- Noise propagation
- Method summarization
- Conclusion & perspectives




AIRBUS DEFENCE & SPACE

ONERA THE FRENCH AEROSPACE LAB

# Conclusion & perspectives

**Goals reached:**

- Development of an inverse problem approach, aiming to recover variables of interest from measurements through LS/ML approaches:
  - Complex visibility (SPIDER)
  - Phase variance (GRAVITY) (approached formula)
- Solid comparison between both estimation methods with different modulation values (K=3, 4, 5, 6)

**Perspectives:**

- Presentation of the results in a conference (VLTI summer school / SFO)
- Work on Earth observation:
  - Estimation of performance
  - Comparison of different combination architectures
  - According to the performance criterion of the mission: optimize SPIDER

## Thank you for your attention.