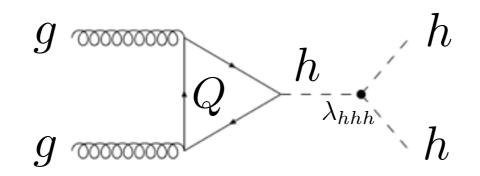
## 2nd FCC-France Workshop / January 20-21 2021

### HIGGS BOSON PAIR PRODUCTION AT N<sup>3</sup>LO QCD



### HUA-SHENG SHAO

Work with L.-B. Chen, H.T. Lí and J. Wang (1909.06808, 1912.13001)







2ND FCC-FRANCE WORKSHOP 20 JANUARY 2021



Tuesday, January 19, 21



### Undoubtably important to measure Higgs self couplings

Unique way to understand the Higgs potential

EW symmetry breaking



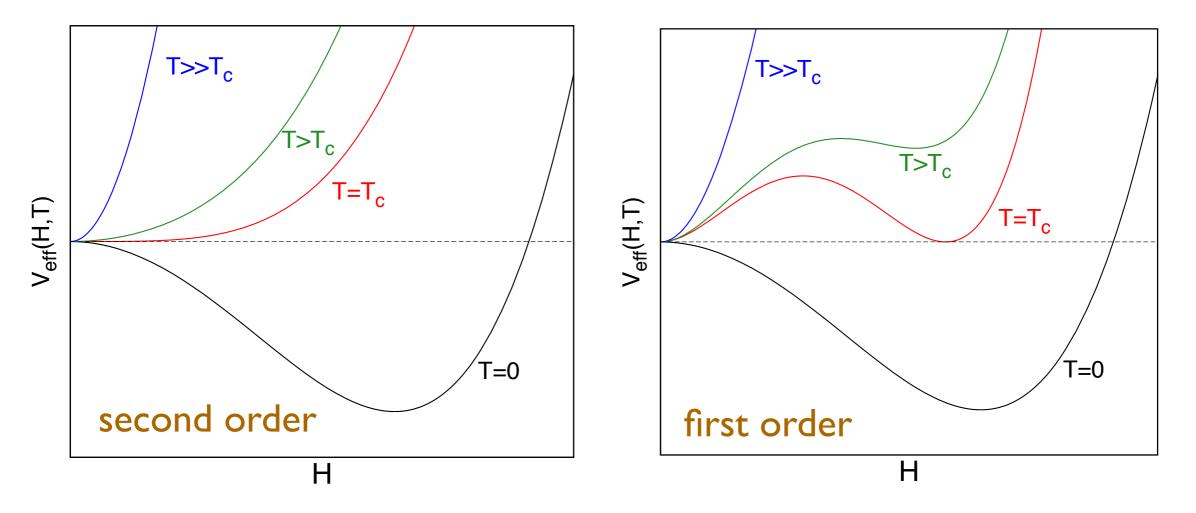
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- Unique way to understand the Higgs potential
- · Phase transition in the (very) early Universe

SM: no phase transition (crossover)

Kajantie et al. (1996); Csikor et al. (1998)

=> EW symmetry breaking





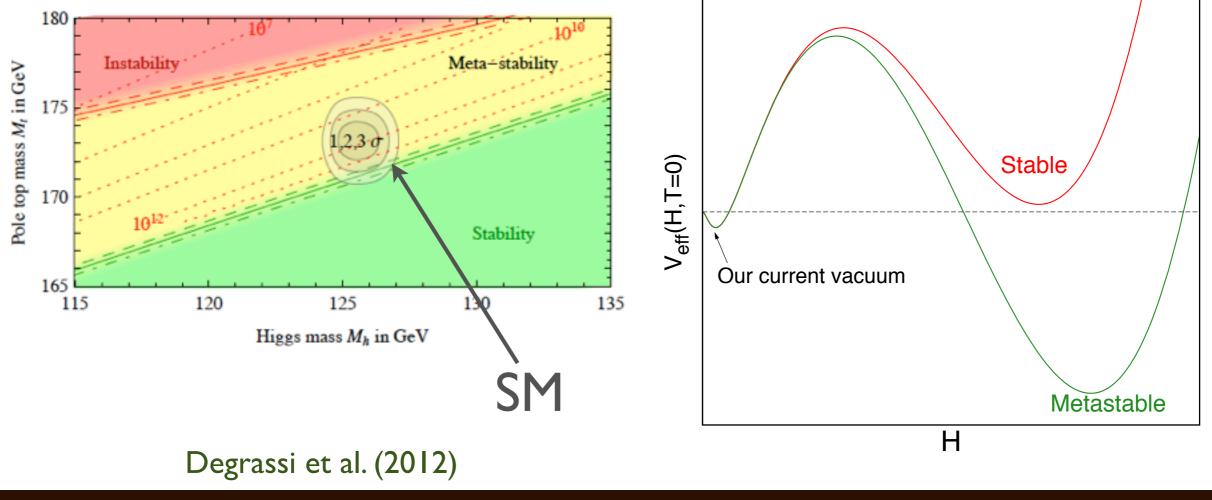
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Tuesday, January 19, 21



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## • Trilinear coupling as first target to be measured precisely How much precision is needed for BSM purposes ?

Raman Sundrun BSM Wishlist (Snowmass21 EF meeting)Strong first order PTGeneral EW PT distinguishSM crossover $\delta\lambda_{hhh} \gtrsim 10\%$  $\delta\lambda_{hhh} \ll 1\%$  $\delta\lambda_{hhh} \sim 0\%$ 

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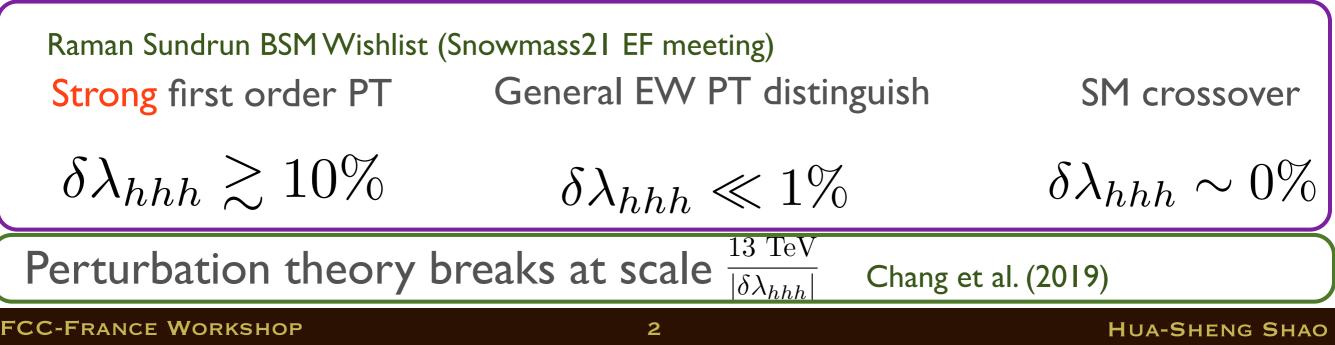
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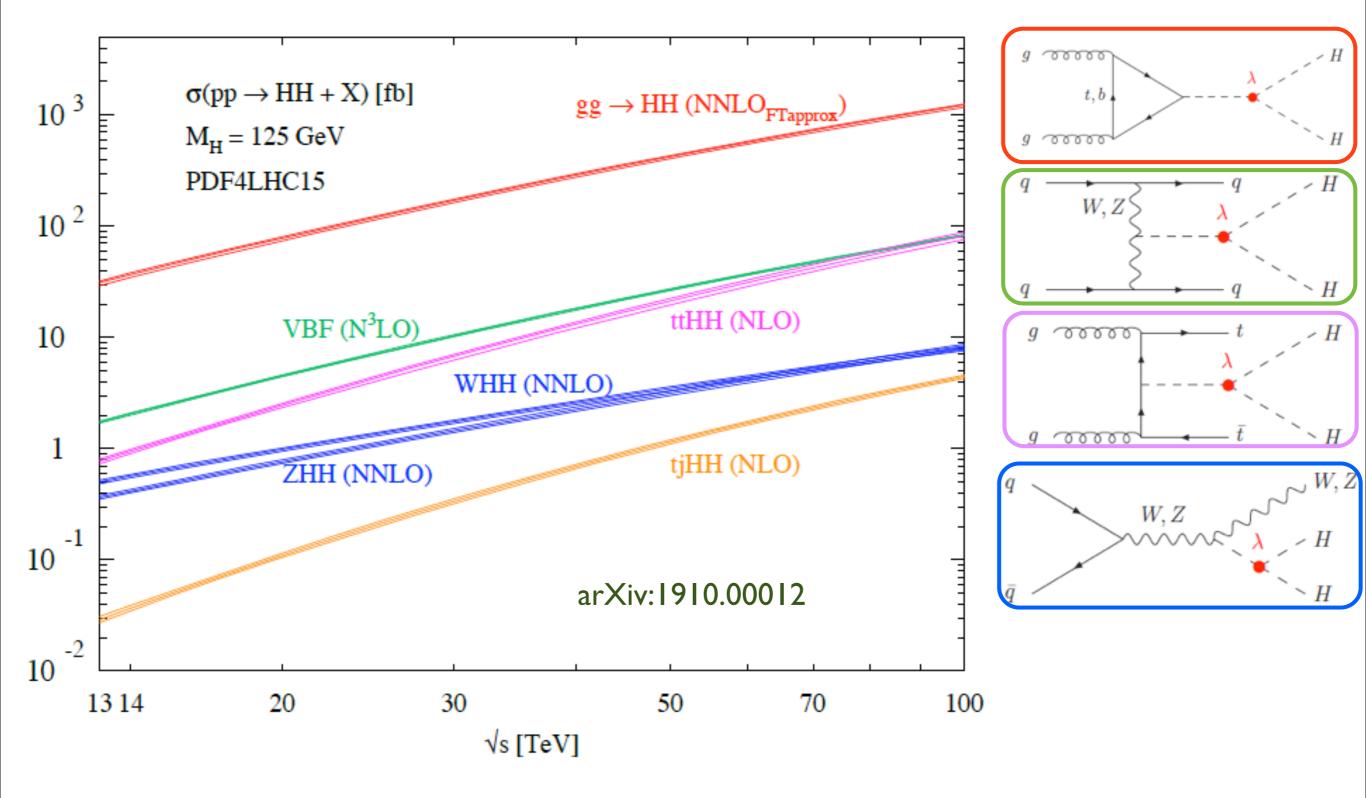
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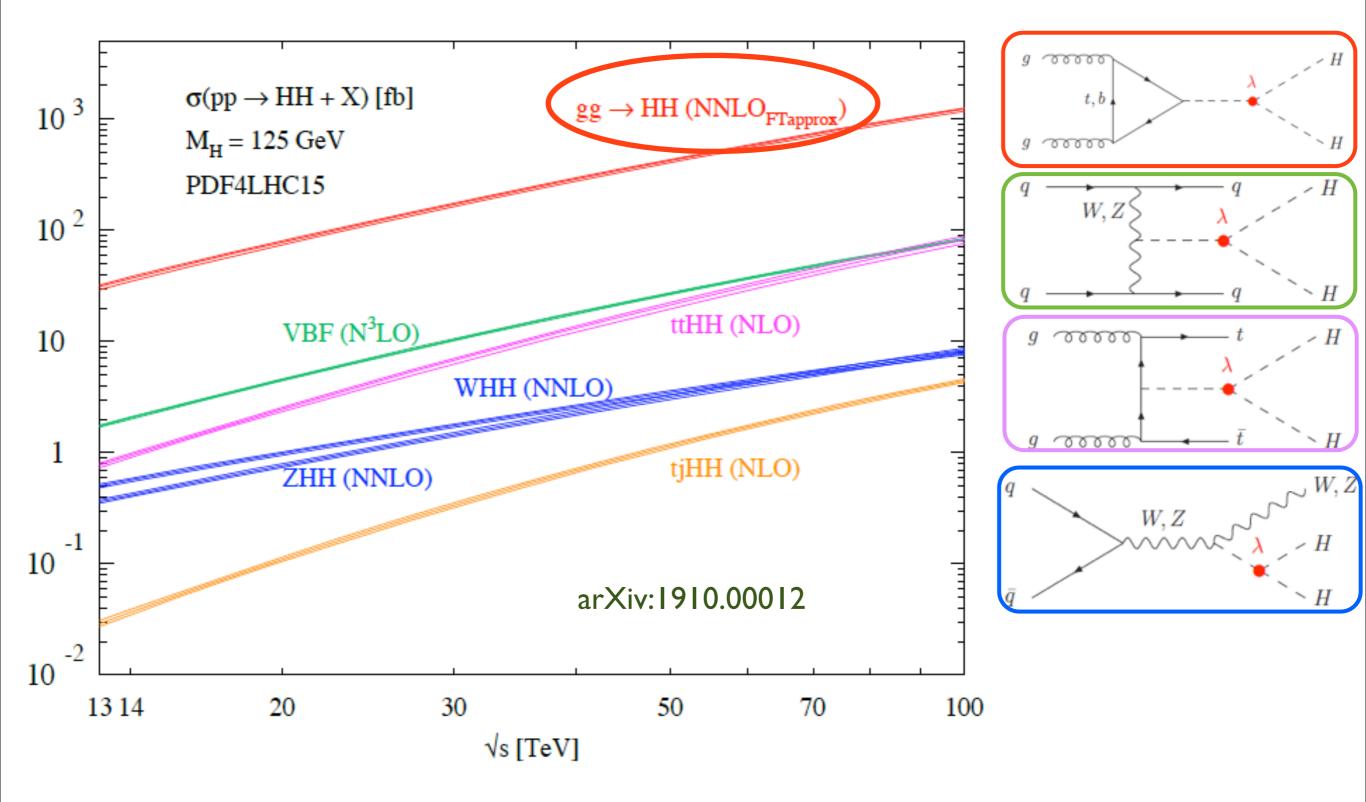
## **HIGGS BOSON PAIR PRODUCTION**

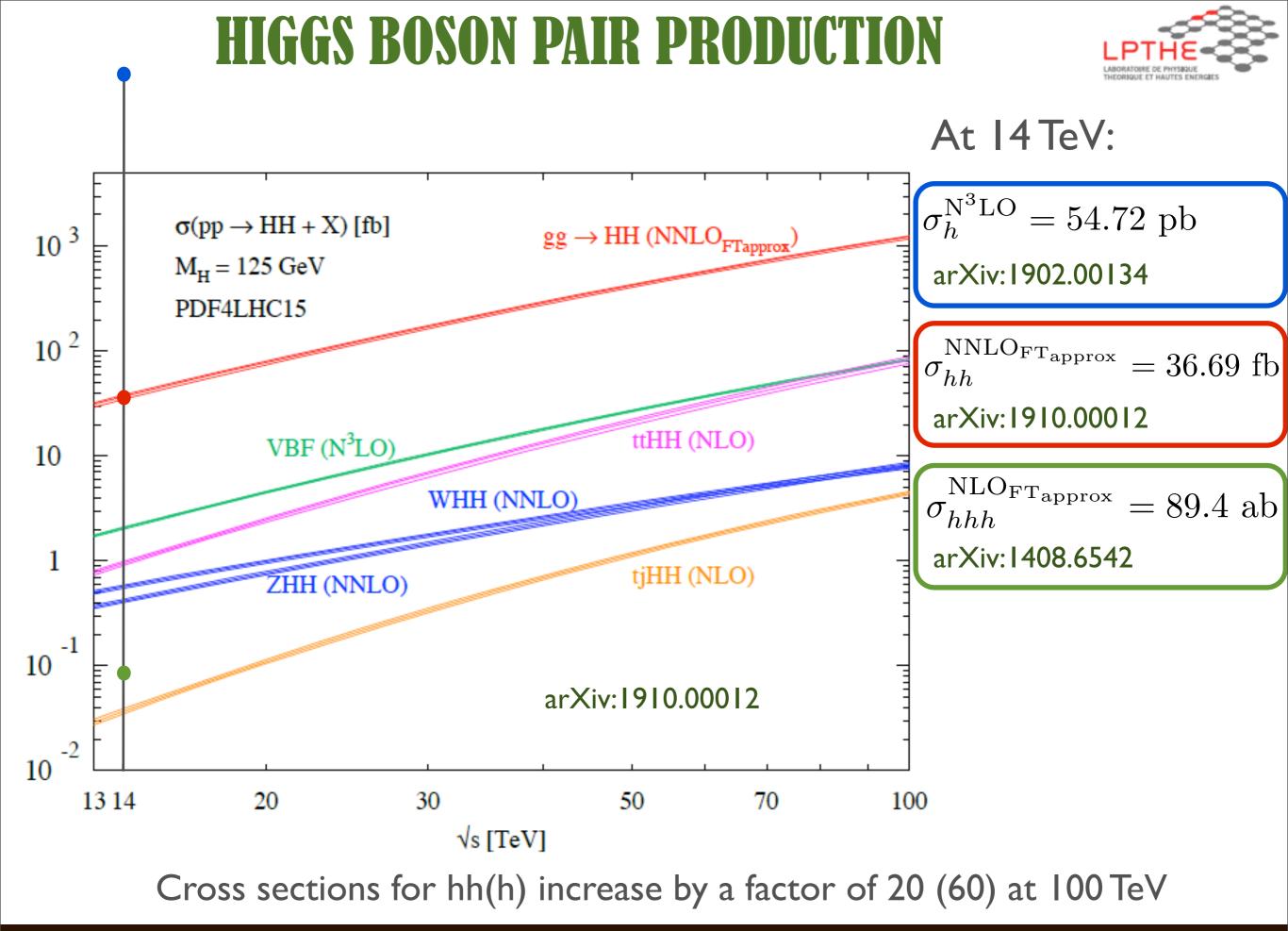




## **HIGGS BOSON PAIR PRODUCTION**

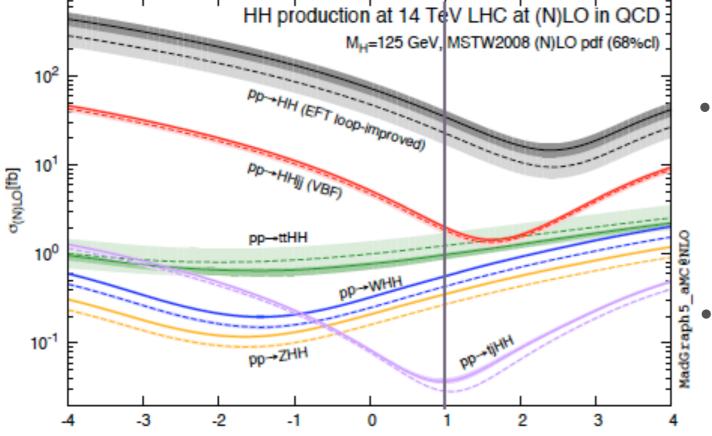






## **PROBING HIGGS SELF COUPLING**





arXiv:1401.7340

 The self-coupling value can be extracted by measuring the cross sections.

### However:

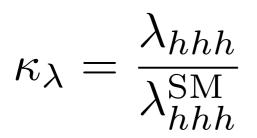
- Interpretations of these bounds in terms of BSM always need additional assumptions on how the SM has been deformed.
  - The most commonly assumption is only changing the value of  $\lambda_{hhh}$ , which leads to (differential) cross section variations

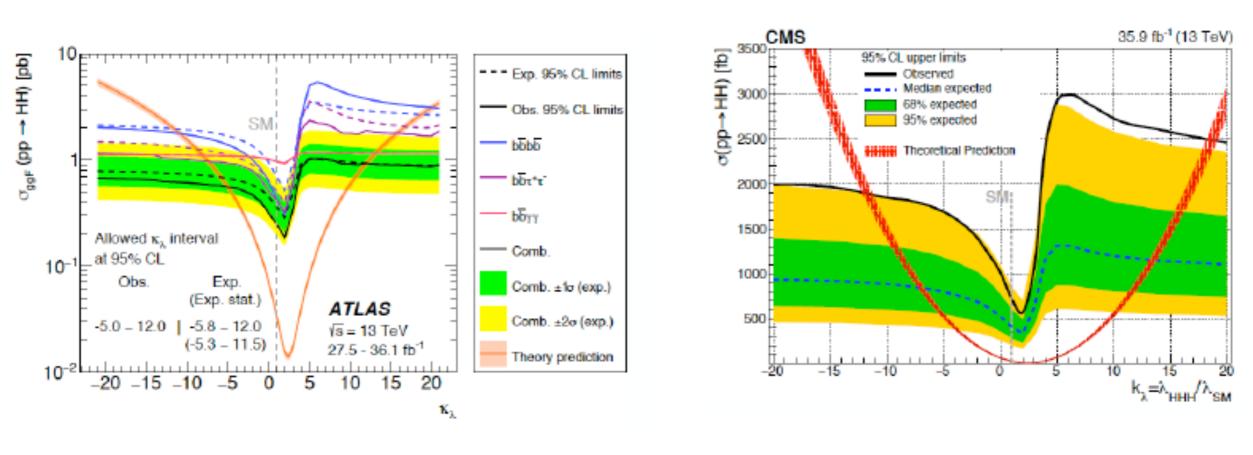
$$\sigma = \sigma_{\rm SM} \left[ 1 + (\kappa_{\lambda} - 1)A_1 + (\kappa_{\lambda}^2 - 1)A_2 \right]$$

Tuesday, January 19, 21

## THE MEASUREMENTS





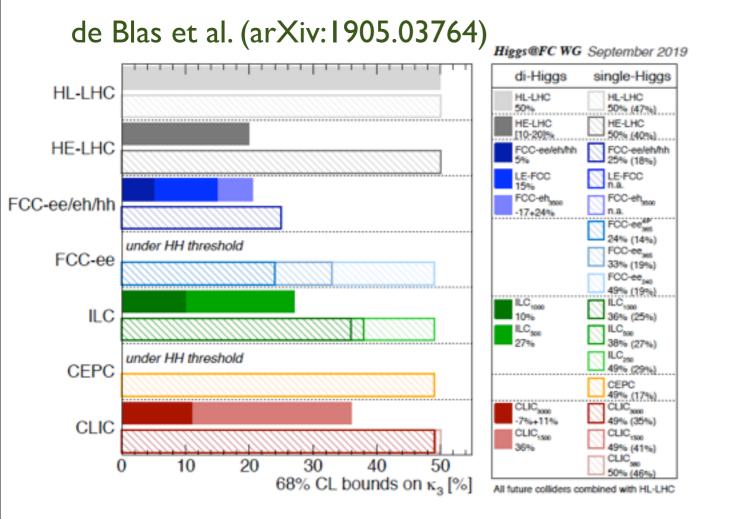


 $-5.0 < \kappa_{\lambda} < 12 \qquad \qquad -11.8 < \kappa_{\lambda} < 18.8$ 

- Strong shape effects with κ<sub>λ</sub> variations
- Soft spectra for  $\kappa_{\lambda} \approx 5 \rightarrow$  difficult to constrain anomalous positive values

## **A QUICK INTRODUCTION**





### Caterina Vernieri (Snowmass21EF meeting)

| collider             | single-H | HH  | combined |
|----------------------|----------|-----|----------|
| HL-LHC               | 100-200% | 50% | 50%      |
| CEPC <sub>240</sub>  | 49%      | -   | 49%      |
| ILC <sub>250</sub>   | 49%      | -   | 49%      |
| ILC <sub>500</sub>   | 38%      | 27% | 22%      |
| ILC <sub>1000</sub>  | 36%      | 10% | 10%      |
| CLIC <sub>380</sub>  | 50%      | -   | 50%      |
| CLIC <sub>1500</sub> | 49%      | 36% | 29%      |
| CLIC <sub>3000</sub> | 49%      | 9%  | 9%       |
| FCC-ee               | 33%      | -   | 33%      |
| FCC-ee (4 IPs)       | 24%      | -   | 24%      |
| HE-LHC               | -        | 15% | 15%      |
| FCC-hh               | -        | 5%  | 5%       |
|                      |          |     |          |

50% accuracy (HL-LHC): sensitive to BSM with the largest new physics effects

20% accuracy (future e+e-): discovery of SM-like  $\lambda_{hhh}$ 

5% accuracy (FCC-hh): sensitive to BSM loop corrections

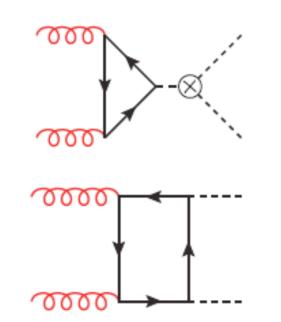
—— Ultimate precision machine !

Mangano et al. (arXiv:2004.03505)

Tuesday, January 19, 21



- Full top-quark mass dependence
  - Leading order (LO) is a loop-induced process

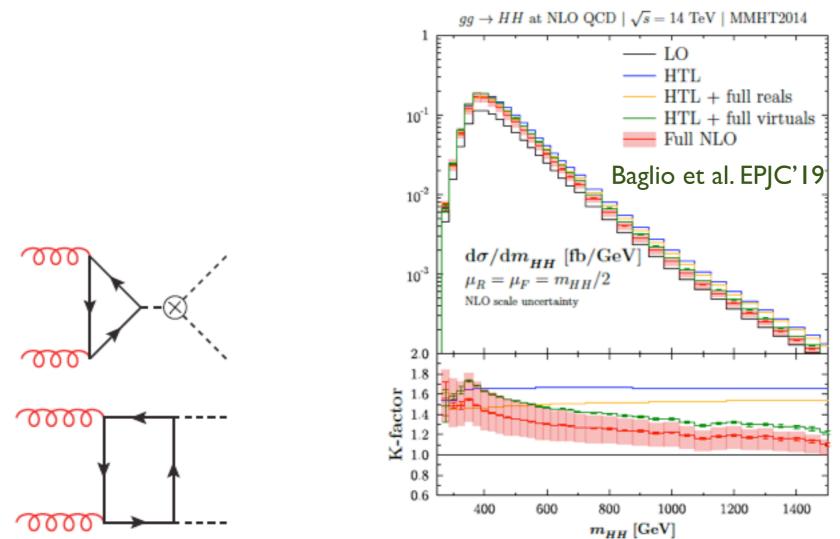




### Full top-quark mass dependence

- Leading order (LO) is a loop-induced process
- Next-to-leading order (NLO) was computed numerically

Borowka et al. PRL'16, JHEP'16; Baglio et al. EPJC'19, JHEP'20



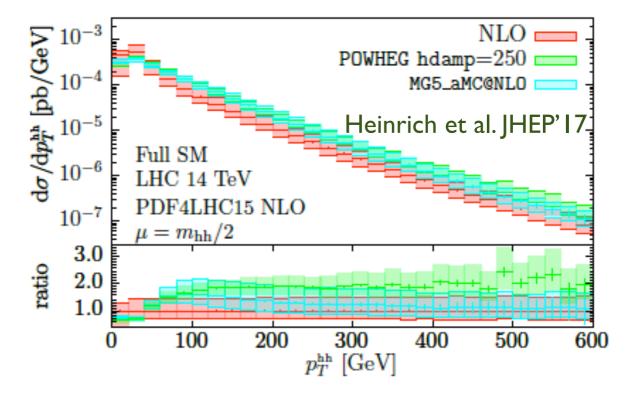
Reasonable approximations to extend I/m<sub>t</sub> result (rescaled exact Born, include exact real radiation) can fail the true K factor significantly.

virtual is so crucial, which is remaining to be understood

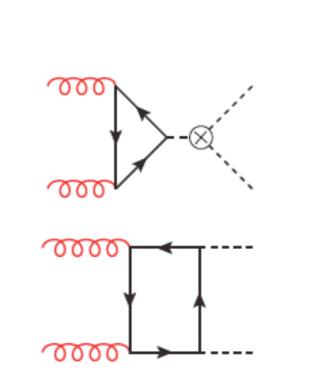


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    - Heinrich et al. JHEP' I 7, JHEP' I 9; Jones, Kuttimalai JHEP' I 8



Matching scheme dependence starts to be significant at large  $p_T^{hh}$ 



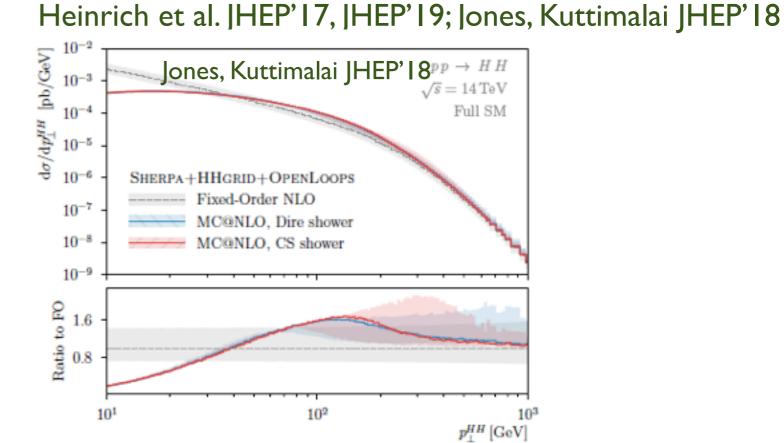


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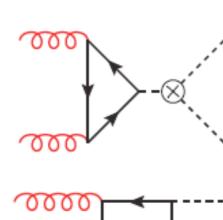
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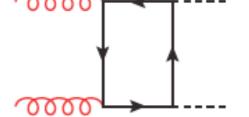
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Shower scale uncertainty is also significant at large  $p_T^{hh}$ 







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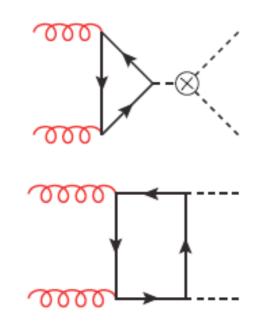
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... even after matching to parton showers (i.e. NLO+PS)

Heinrich et al. JHEP'17, JHEP'19; Jones, Kuttimalai JHEP'18

• Scale unc. (>10%)

| Energy | 13 TeV                         | 14 TeV                                       | 27 TeV                         | 100 TeV                   |
|--------|--------------------------------|--|--------------------------------|---------------------------|
| NLO    | $27.78^{+13.8\%}_{-12.8\%}$ fb | 32.88 <sup>+13.5%</sup> <sub>-12.5%</sub> fb | $127.7^{+11.5\%}_{-10.4\%}$ fb | 1147 <sup>+10.7%</sup> fb |





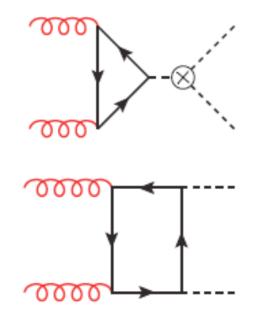
HUA-SHENG SHAO

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- Scale unc. (>10%)
- ... and large top-quark mass scheme dependence



$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=300 \text{ GeV}} = 0.02978(7)^{+6\%}_{-34\%} \text{ fb/GeV},$$
  
$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=400 \text{ GeV}} = 0.1609(4)^{+0\%}_{-13\%} \text{ fb/GeV},$$
  
$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=600 \text{ GeV}} = 0.03204(9)^{+0\%}_{-30\%} \text{ fb/GeV},$$
  
$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=1200 \text{ GeV}} = 0.000435(4)^{+0\%}_{-35\%} \text{ fb/GeV}$$

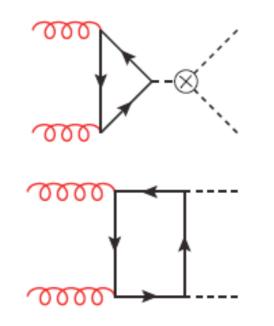


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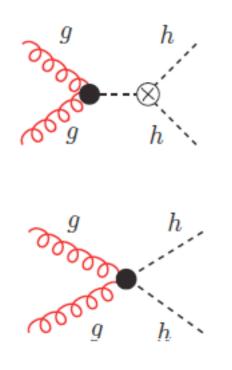
- Scale unc. (>10%)
- ... and large top-quark mass scheme dependence
- A lot of analytical approximations (well-motivated to deepen understanding) Grigo et al. NPB'13, NPB'15; Degrassi EPJC'16;, Davies et al. JHEP'18, JHEP'19; Bonciani et al. PRL'18; Xu and Yang JHEP'19; Davies and Steinhauser (1909.01361)





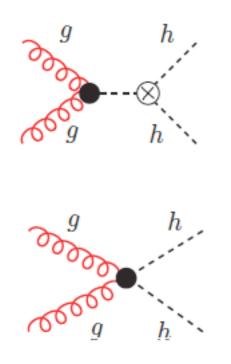
Infinite top-quark mass limit

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\ \mu\nu} \left( C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right)$$





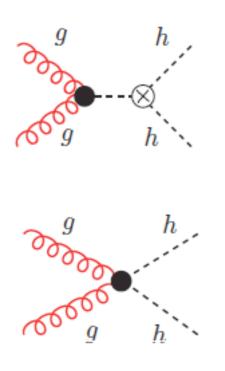
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  - The Wilson coefficients  $C_h$  and  $C_{hh}$  are known up to 4 loops Schroder and Steinhauser JHEP'06; Baikov et al. PRL'17; Spira JHEP'16; Gerlach et al. JHEP'18





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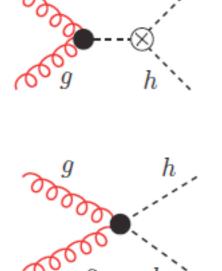
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- Technically, it is much easier to achieve high precision
  - NLO was 20 years old Dawson PRD'98
  - NNLO was known as well Florian and Mazzitelli PLB'13,PRL'13; Grigo et al. NPB'14; Florian et al. JHEP'16
  - Threshold resummation Shao et al. JHEP'13; Florian and Mazzitelli JHEP'15, JHEP'18
  - NLO<sub>FTapprox</sub>: NLO plus full top quark mass in Born and real Frederix et al. PLB'14; Maltoni et al. JHEP'14
  - Combine NNLO with full top-quark mass NLO Grazzini et al. JHEP'18





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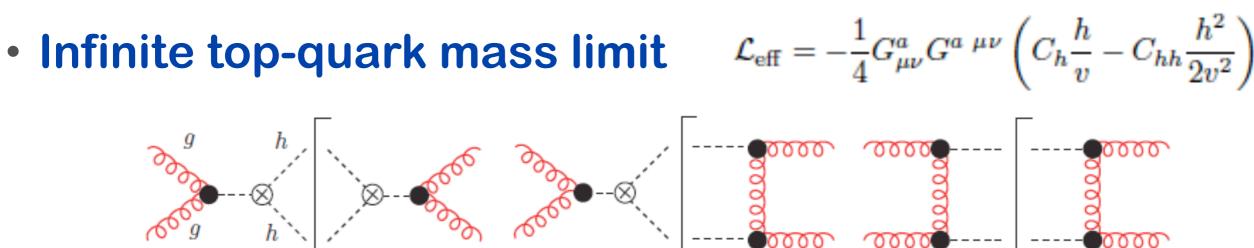


Our aim is to push the calculation to  $N^{3}LO$  !





(a)



(c)

N<sup>3</sup>LO NNLO LO NLO  $\mathcal{O}(\alpha_s^5)$  $(\alpha_s^4)$  $\operatorname{total}$ O  $\alpha_s^3$  ${\mathcal O}$  $\mathcal{O}(\alpha_s^2)$  $\alpha_s^4$  $\alpha_s^5$  $(\alpha_s^2)$  $\alpha_s^3$ O  $\mathcal{O}$ O O  $\mathbf{a}$  $\alpha_s^4$  $\alpha_s^5$  $\left(\alpha_{s}^{3}\right)$ O b О 0 O  $(\alpha_s^4)$ ъ 0  ${\mathcal O}$ 0  $\alpha_s$  $\mathbf{c}$ 

Chen, Li, HSS, Wang (PLB'20, JHEP'20)

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}$$

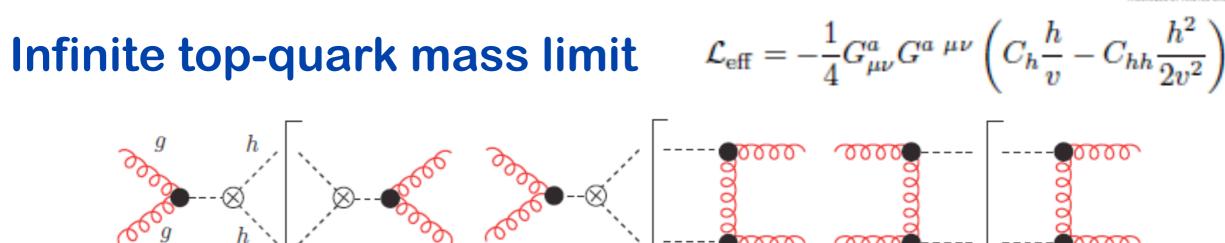
#### **FCC-FRANCE WORKSHOP**

(b)





(a)



(c)

|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| total | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| с     | 0                         | 0                         | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |

Chen, Li, HSS, Wang (PLB'20, JHEP'20)

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}$$

class-a: same topology as ggH

$$\begin{aligned} \frac{d\sigma_{hh}^a}{dm_{hh}} &= f_{h \to hh} \left( \frac{C_{hh}}{C_h} - \frac{6\lambda v^2}{m_{hh}^2 - m_h^2} \right)^2 \times \sigma_h(m_h \to m_{hh}) \\ f_{h \to hh} &= \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{16\pi^2 v^2} \end{aligned}$$

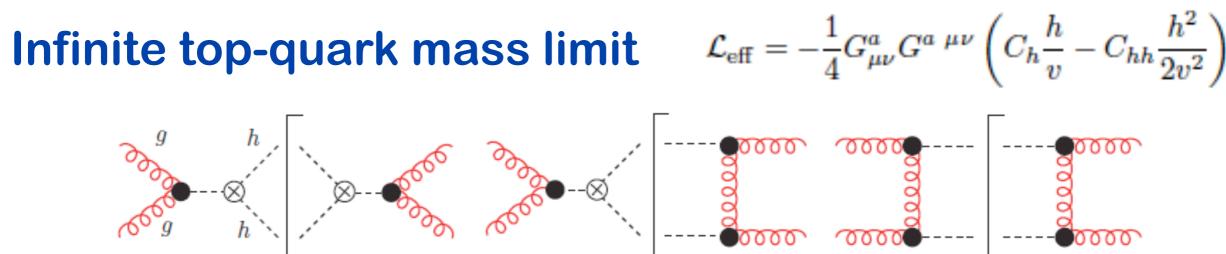
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 $\boldsymbol{a}$ 



(c)

|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
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| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
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$$f_{h \to hh} = \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{16\pi^2 v^2}$$

From iHixs2 Dulat et al. CPC'18

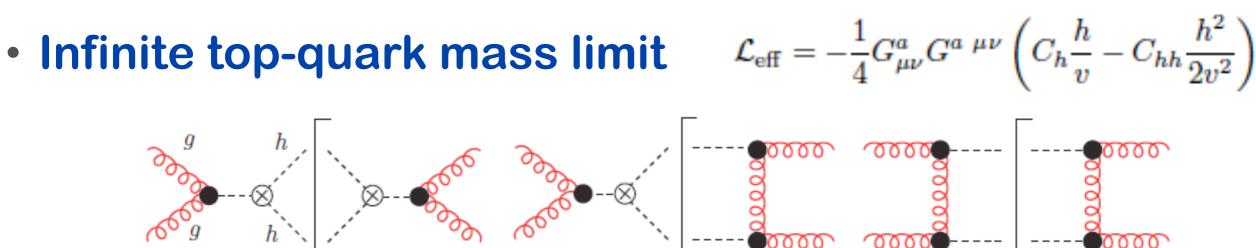
#### **FCC-FRANCE WORKSHOP**

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(a)



(c)

|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
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| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
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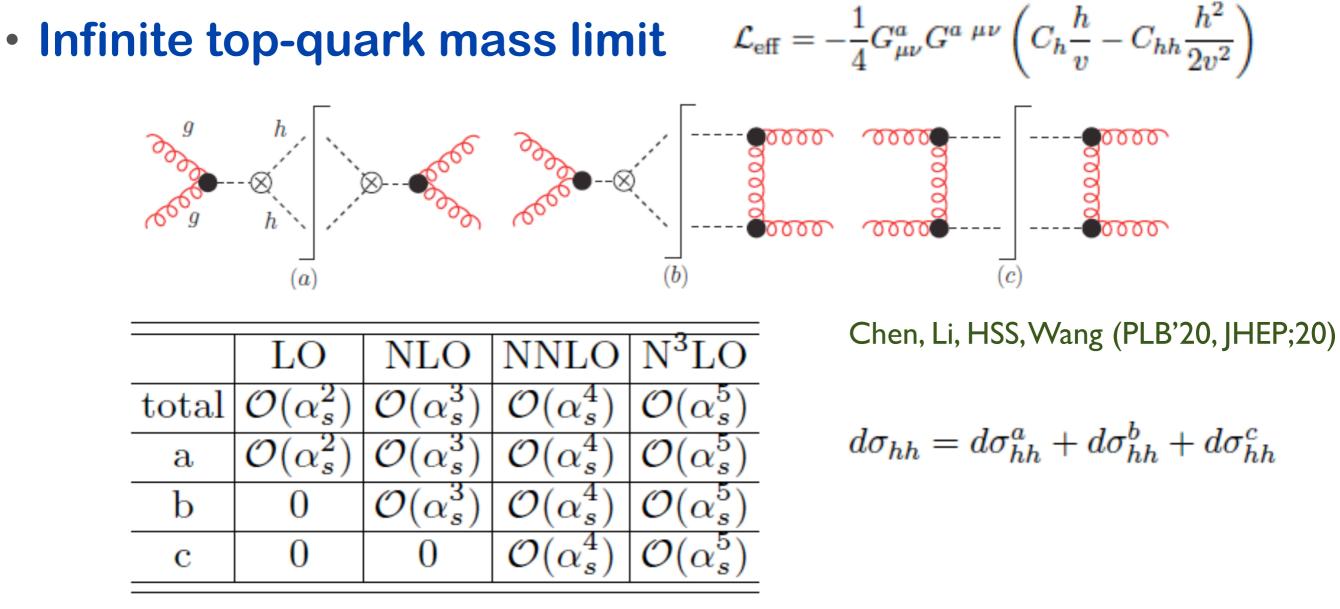
Chen, Li, HSS, Wang (PLB'20, JHEP;20)

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}$$

#### **FCC-FRANCE WORKSHOP**

(b)

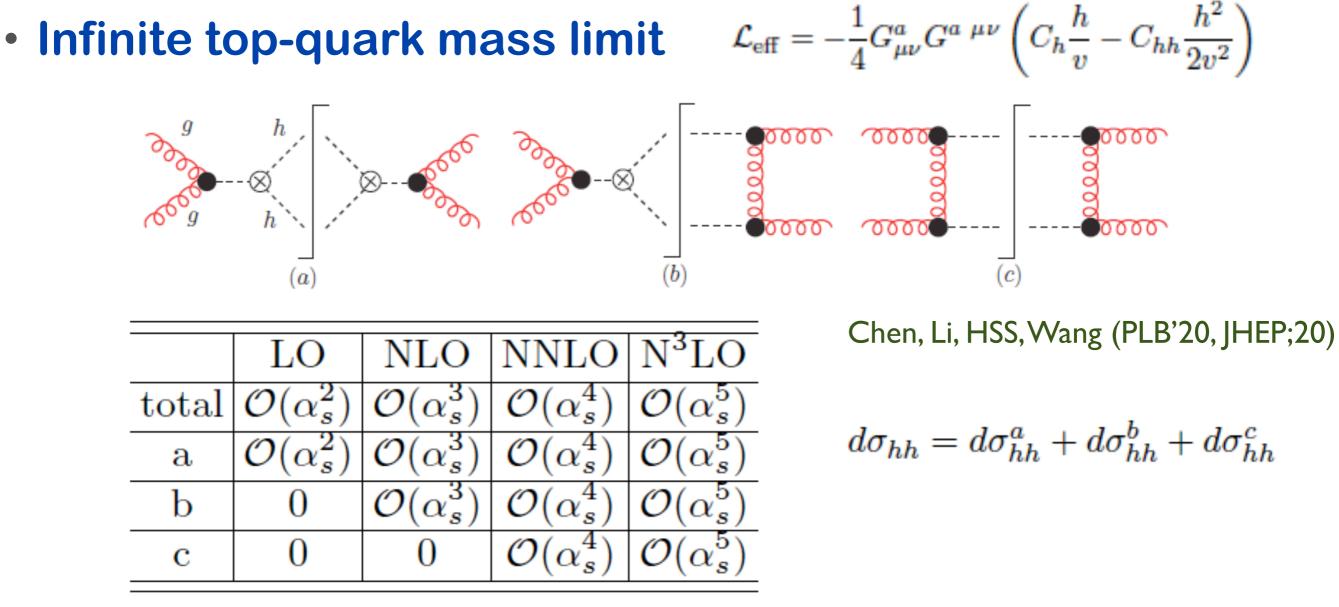




• class-b: need NNLO as its as<sup>2</sup> is zero (qT subtraction, Catani & Grazzini PRL'07)

$$d\sigma_{hh}^{b} = d\sigma_{hh}^{b} \Big|_{p_{T}^{hh} < p_{T}^{\text{veto}}} + d\sigma_{hh}^{b} \Big|_{p_{T}^{hh} > p_{T}^{\text{veto}}}$$





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$$d\sigma_{hh}^{b} = \left. d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} + d\sigma_{hh}^{b} \right|_{p_{T}^{hh} > p_{T}^{\text{veto}}}$$
SCET: 
$$d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} = \mathcal{H} \otimes \mathcal{B}_{1} \otimes \mathcal{B}_{2} + \mathcal{O}\left(\left(\frac{p_{T}^{\text{veto}}}{Q}\right)^{2}\right)$$





NLO

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 $\alpha_s^3$ 

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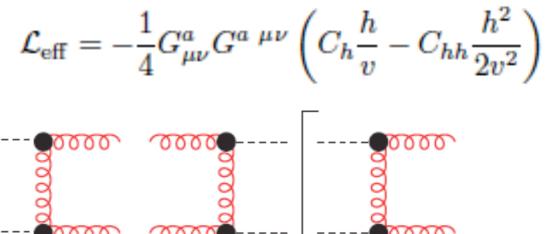
 $\mathcal{O}$ 

total

 $\mathbf{a}$ 

b

 $\mathbf{c}$ 



(c)



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 $(\alpha_s^{\mathfrak{b}})$ 

 $\alpha_s$ 

 $\mathcal{O}($ 

О

O

$$\begin{aligned} d\sigma_{hh}^{b} &= \left. d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} + d\sigma_{hh}^{b} \right|_{p_{T}^{hh} > p_{T}^{\text{veto}}} & \mathcal{H} \\ \text{SCET:} \left. d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} = \mathcal{H} \otimes \mathcal{B}_{1} \otimes \mathcal{B}_{2} + \mathcal{O} \left( \left( \frac{p_{T}^{\text{veto}}}{Q} \right)^{2} \right) & \text{Bane} \\ \text{new} \end{aligned}$$

NNLO

О

O

 $(\alpha_s^4)$ 

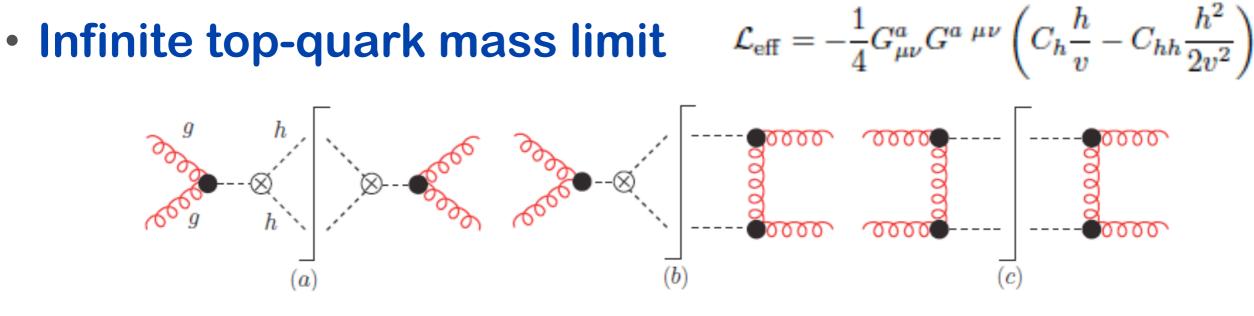
 $\alpha_s^4$ 

 $\alpha_s^4$ 

 $\left(\alpha_{s}^{4}\right)$ 

*H* hard function
two-loop amplitude
Banerjee et al., JHEP'18
new one-loop amplitude





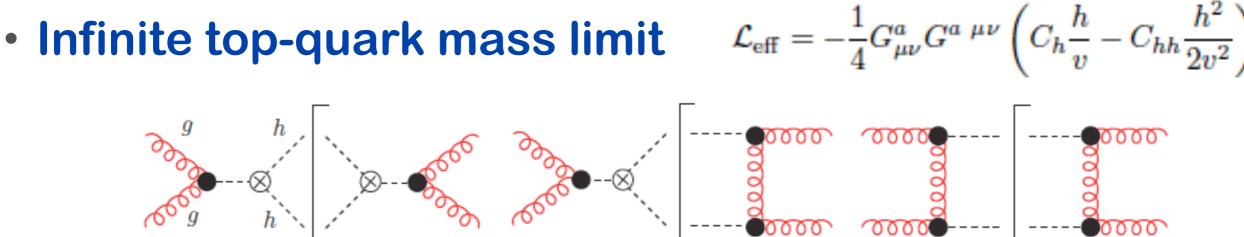
|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| total | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| с     | 0                         | 0                         | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |

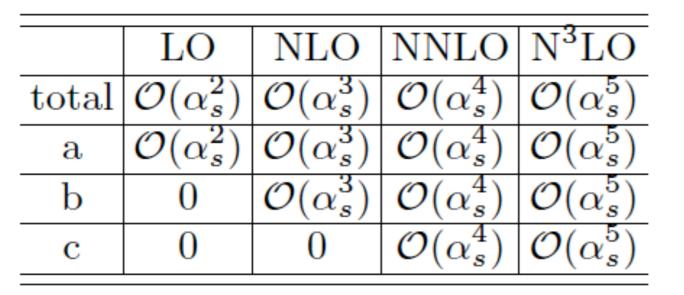
Chen, Li, HSS, Wang (PLB'20, JHEP;20)

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}$$

• class-b: need NNLO as its as<sup>2</sup> is zero (q<sub>T</sub> subtraction, Catani & Grazzini PRL'07)







Chen, Li, HSS, Wang (PLB'20, JHEP;20)

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}$$

(c)

• class-b: need NNLO as its as<sup>2</sup> is zero (qT subtraction, Catani & Grazzini PRL'07)

$$d\sigma^b_{hh} = d\sigma^b_{hh} \Big|_{p_T^{hh} < p_T^{\text{veto}}} + \left. d\sigma^b_{hh} \right|_{p_T^{hh} > p_T^{\text{veto}}}$$

$$\mathbf{MG5\_aMC:} d\sigma_{hh}^{b,\mathrm{NNLO}} \Big|_{p_T^{hh} > p_T^{\mathrm{veto}}} = d\sigma_{hh+j}^{b,\mathrm{NLO}}$$

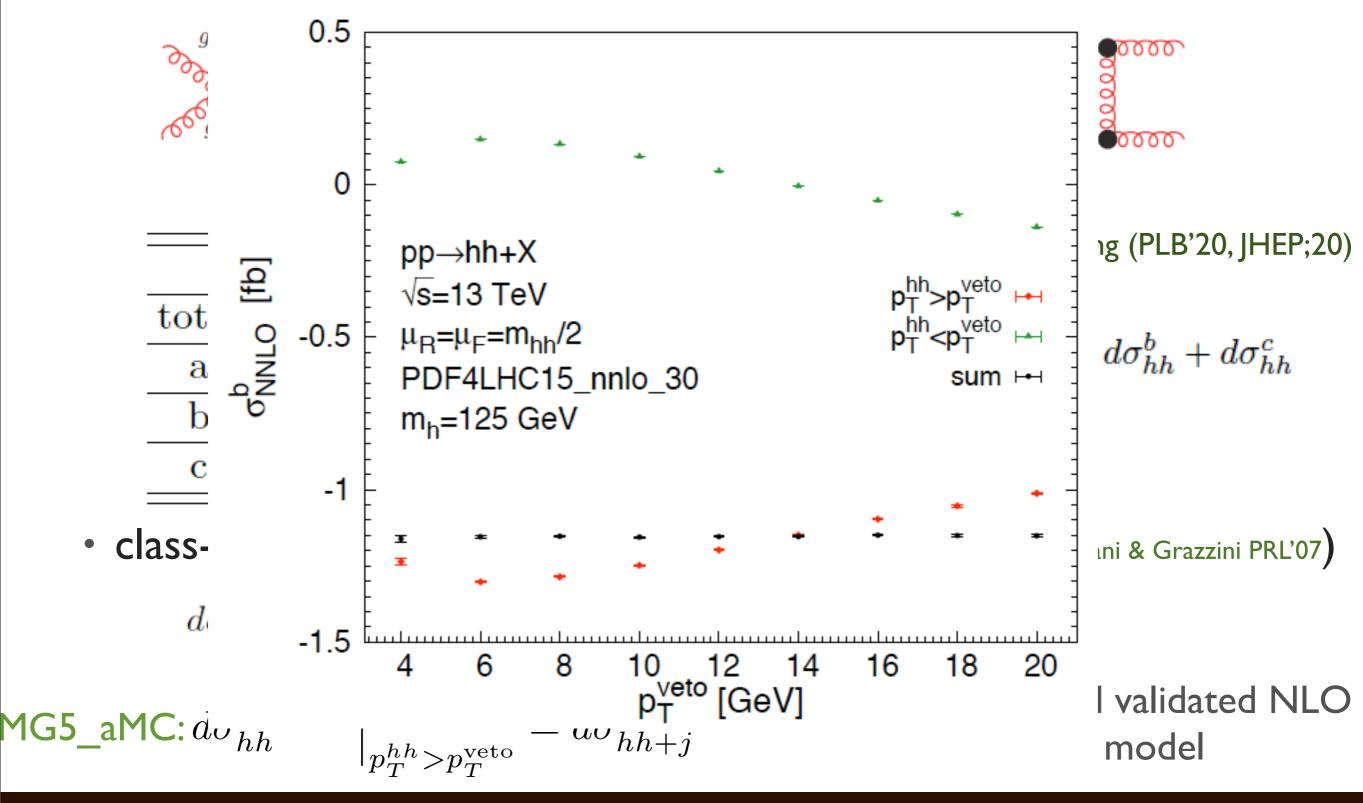
New and validated NLO model

HUA-SHENG SHAO

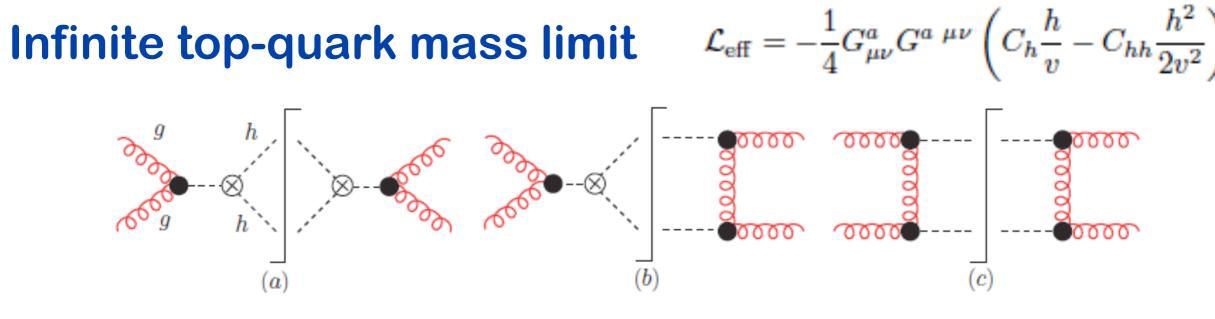












|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| total | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| с     | 0                         | 0                         | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |

Chen, Li, HSS, Wang (PLB'20, JHEP'20)

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}$$

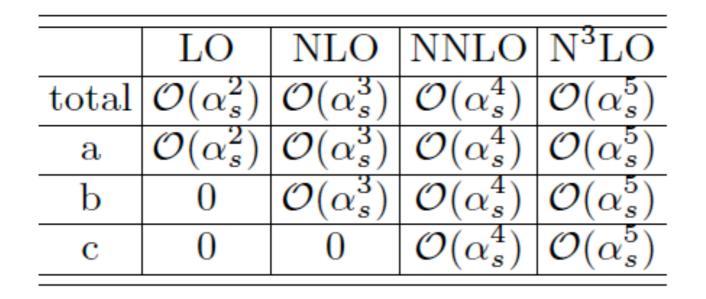
class-c: need NLO (full fledged)

#### **FCC-FRANCE WORKSHOP**



- Infinite top-quark mass limit  $\mathcal{L}_{eff} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} C_{hh}\frac{h^2}{2v^2}\right)$ 
  - A lot of cross checks

Chen, Li, HSS, Wang (PLB'20, JHEP'20)



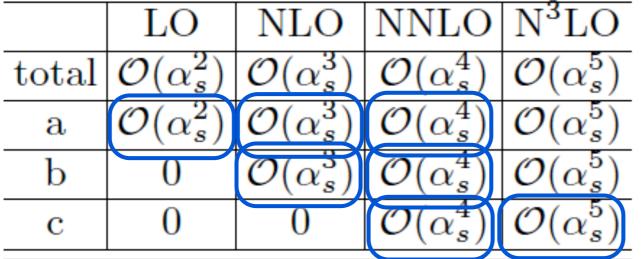
- - A lot of cross checks

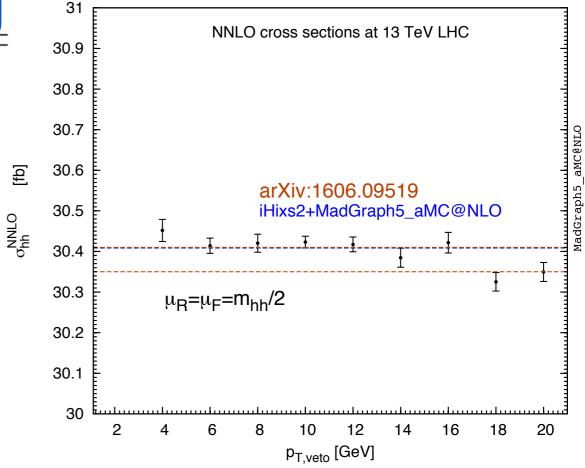




Chen, Li, HSS, Wang (PLB'20, JHEP'20)

At least two independent calculations





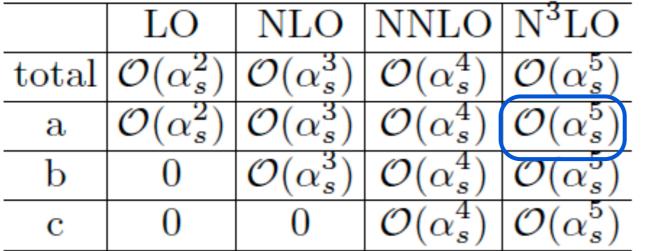
- Infinite top-quark mass limit
  - A lot of cross checks

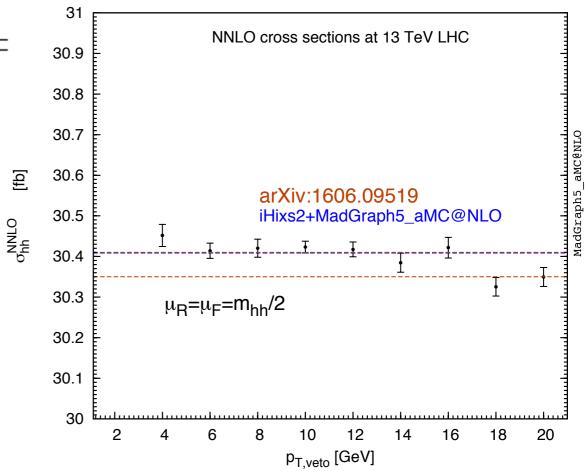


$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\ \mu\nu} \left( C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right)$$

Chen, Li, HSS, Wang (PLB'20, JHEP'20)

 $\sqrt{}$  At least two independent calculations  $\sqrt{}$  Orthogonal check with NNLO ggHH







A lot of cross checks

|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| total | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| с     | 0                         | 0                         | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
|       |                           |                           |                           |                           |

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^a_{\mu\nu} G^a^{\mu\nu} \left( C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right)$$

Chen, Li, HSS, Wang (PLB'20, JHEP'20)

 $\sqrt{}$  At least two independent calculations  $\sqrt{}$  Orthogonal check with NNLO ggHH  $\sqrt{}$  Check piece-by-piece

Tuesday, January 19, 21



A lot of cross checks

|       | LO                        | NLO                       | NNLO                      | N <sup>3</sup> LO         |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| total | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| a     | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| b     | 0                         | $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |
| с     | 0                         | 0                         | $\mathcal{O}(\alpha_s^4)$ | $\mathcal{O}(\alpha_s^5)$ |



$$\mathcal{L}_{\rm eff} = -\frac{1}{4} G^a_{\mu\nu} G^{a\ \mu\nu} \left( C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right)$$

Chen, Li, HSS, Wang (PLB'20, JHEP'20)

 $\sqrt{}$  At least two independent calculations  $\sqrt{}$  Orthogonal check with NNLO ggHH  $\sqrt{}$  Check piece-by-piece

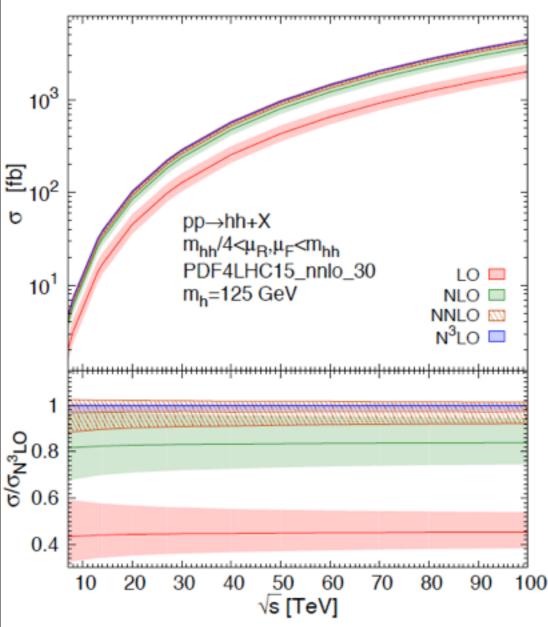
|       | NLO                     | NNLO                    | N <sup>3</sup> LO       |
|-------|-------------------------|-------------------------|-------------------------|
| order | $\mathcal{O}(lpha_s^3)$ | $\mathcal{O}(lpha_s^4)$ | $\mathcal{O}(lpha_s^5)$ |
|       | IHIXS2                  | IHIXS2                  | IHIXS2                  |
| a     | $q_T$ -subtraction      | $q_T$ -subtraction      |                         |
|       | MG5_AMC                 |                         |                         |
| b     | -                       | $q_T$ -subtraction      | $q_T$ -subtraction      |
| D     |                         | MG5_AMC                 |                         |
| 0     |                         |                         | $q_T$ -subtraction      |
| с     | -                       | -                       | MG5_AMC                 |

Tuesday, January 19, 21



### Infinite top-quark mass limit

• N<sup>3</sup>LO cross sections



| i | in unit of fb     |                            | Chen, Li, HSS, Wang (PLB'20) |                            |                           |  |
|---|-------------------|----------------------------|------------------------------|----------------------------|---------------------------|--|
|   | $\sqrt{s}$ order  | $13 { m TeV}$              | $14 { m TeV}$                | $27 { m TeV}$              | $100 { m TeV}$            |  |
| ſ | LO                | $13.80^{+31\%}_{-22\%}$    | $17.06^{+31\%}_{-22\%}$      | $98.22^{+26\%}_{-19\%}$    | $2015^{+19\%}_{-15\%}$    |  |
|   | NLO               | $25.81^{+18\%}_{-15\%}$    | $31.89^{+18\%}_{-15\%}$      | $183.0^{+16\%}_{-14\%}$    | $3724_{-11\%}^{+13\%}$    |  |
|   | NNLO              | $30.41^{+5.3\%}_{-7.8\%}$  | $37.55^{+5.2\%}_{-7.6\%}$    | $214.2^{+4.8\%}_{-6.7\%}$  | $4322_{-5.3\%}^{+4.2\%}$  |  |
|   | N <sup>3</sup> LO | $31.31^{+0.66\%}_{-2.8\%}$ | $38.65^{+0.65\%}_{-2.7\%}$   | $220.2^{+0.53\%}_{-2.4\%}$ | $4438^{+0.51\%}_{-1.8\%}$ |  |

- Scale unc. is significantly reduced !
- PDF unc. > scale unc. now !
- Good (asymptotic) perturbative convergence !



### Infinite top-quark mass limit

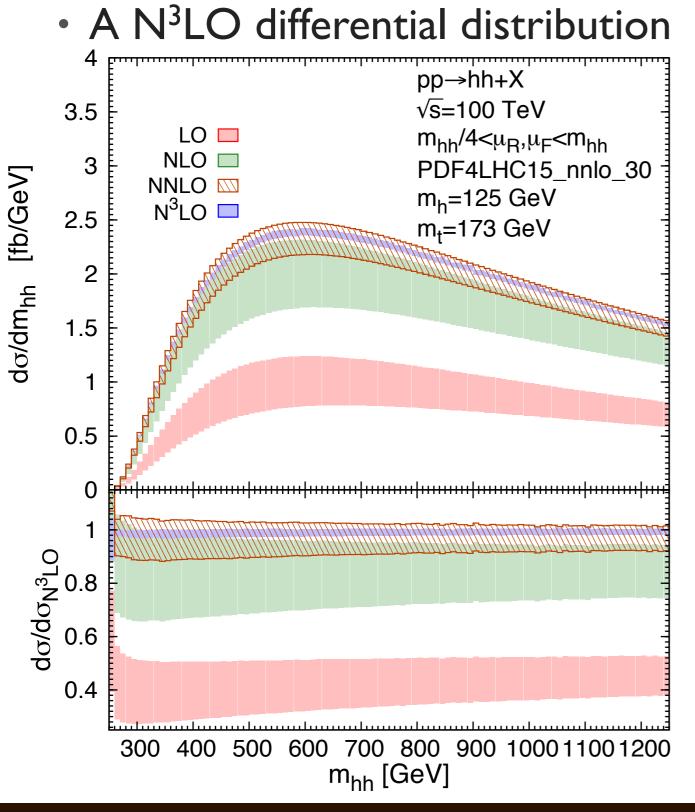
• N<sup>3</sup>LO cross sections

15 \_\_\_\_\_ pp→hh+X LO 🗖 √s=100 TeV NLO 🗆 NNLO 🔤  $m_{hh}/4 < \mu_R, \mu_F < m_{hh}$ 10 N<sup>3</sup>LO PDF4LHC15\_nnlo\_30 σ [pb] m<sub>h</sub>=125 GeV m<sub>t</sub>=173 GeV 5 0 1 8.0<sup>CD</sup> α/α<sup>N</sup>3<sup>CD</sup> 0.4 6 8 Chen, Li, HSS, Wang (JHEP'20)

- Shapes change from LO to NLO and from NLO to NNLO
- The shape variation from NNLO to N<sup>3</sup>LO is quite invisible



### Infinite top-quark mass limit



Chen, Li, HSS, Wang (JHEP'20)

- Scale unc. is significantly reduced !
- Very good (asymptotic) perturbative convergence !
- N<sup>3</sup>LO/NNLO is quite flat

Tuesday, January 19, 21

## **TOP QUARK MASS APPROXIMATIONS**



assuming we have  $\begin{cases} N^{k}LO \\ N^{l}IO \end{cases}$ 

Chen, Li, HSS, Wang (JHEP'20)

infinite top-quark mass limit full top-quark mass dependence

•  $\mathbf{N}^{k}\mathbf{LO}\oplus\mathbf{N}^{l}\mathbf{LO}_{m_{t}}$ 

 $d\sigma^{\mathbf{N^kLO} \oplus \mathbf{N^lLO}_{m_t}} = d\sigma_{m_t}^{\mathbf{N^lLO}} + \left( d\sigma_{m_t=\infty}^{\mathbf{N^kLO}} - d\sigma_{m_t=\infty}^{\mathbf{N^lLO}} \right)$  missing top mass in correction

The • N<sup>k</sup>LO $\otimes$ N<sup>l</sup>LO<sub>m</sub>, best  $d\sigma^{N^{k}LO\otimes N^{l}LO_{m_{t}}} = d\sigma_{m_{t}}^{N^{l}LO} \frac{d\sigma_{m_{t}=0}^{N^{k}LO}}{d\sigma_{m_{t}=0}^{N^{l}LO}}$ 

Same K factor for mass correction

•  $\mathbf{N}^{k}\mathbf{LO}_{\mathbf{B}-\mathbf{i}}\oplus\mathbf{N}^{l}\mathbf{LO}_{m_{t}}$ 

$$d\sigma^{\mathbf{N^{k}LO}_{\mathbf{B}-i}\oplus\mathbf{N^{l}LO}_{\mathbf{m_{t}}}} = d\sigma_{m_{t}}^{\mathbf{N^{l}LO}} + \Delta\sigma_{m_{t}=\infty}^{k,l} \frac{d\sigma_{m_{t}}^{\mathbf{LO}}}{d\sigma_{m_{t}=\infty}^{\mathbf{LO}}}$$

Born mass improved for correction

#### **FCC-FRANCE WORKSHOP**



k > l

## **TOP QUARK MASS RESULTS**



- Top-quark mass dependent results NLO<sub>mt</sub> from Powheg, arXiv:1903.08137
  - N<sup>3</sup>LO cross sections

Chen, Li, HSS, Wang (JHEP'20)

in unit of fb

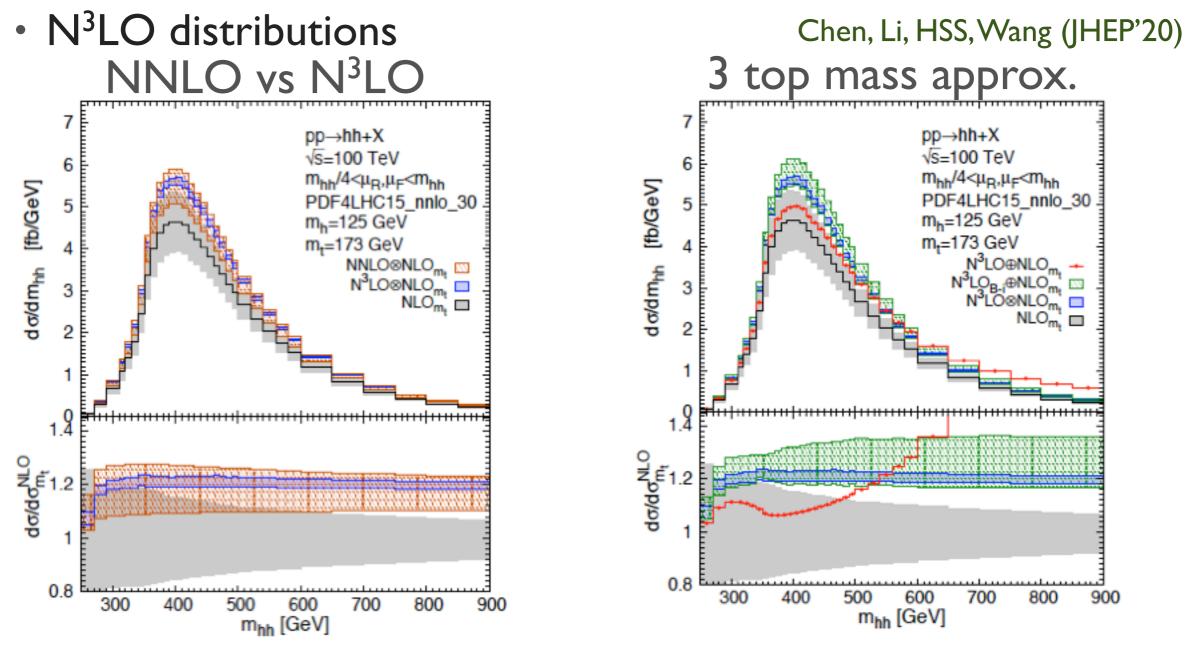
| $\sqrt{s}$                         | 13 TeV                     | 14  TeV                          | 27  TeV                    | 100  TeV                  |
|------------------------------------|----------------------------|----------------------------------|----------------------------|---------------------------|
| $NLO_{m_t}$                        | $27.56^{+14\%}_{-13\%}$    | $32.64^{+14\%}_{-12\%}$          | $126.2^{+12\%}_{-10\%}$    | $1119^{+13\%}_{-13\%}$    |
| $NNLO \oplus NLO_{m_t}$            | $32.16^{+5.9\%}_{-5.9\%}$  | $38.29^{+5.6\%}_{-5.5\%}$        | $157.3^{+3.0\%}_{-4.7\%}$  | $1717^{+5.8\%}_{-12\%}$   |
| $NNLO_{B-i} \oplus NLO_{m_t}$      | $33.08^{+5.0\%}_{-4.9\%}$  | $39.16\substack{+4.9\%\\-5.0\%}$ | $150.8^{+4.6\%}_{-5.7\%}$  | $1330^{+4.0\%}_{-7.2\%}$  |
| $NNLO \otimes NLO_{m_t}$           | $32.47^{+5.3\%}_{-7.8\%}$  | $38.42^{+5.2\%}_{-7.6\%}$        | $147.6^{+4.8\%}_{-6.7\%}$  | $1298^{+4.2\%}_{-5.3\%}$  |
| $N^{3}LO \oplus NLO_{m_{t}}$       | $33.06^{+2.1\%}_{-2.9\%}$  | $39.40^{+1.7\%}_{-2.8\%}$        | $163.3^{+4.0\%}_{-8.3\%}$  | $1833^{+14\%}_{-20\%}$    |
| $N^{3}LO_{B-i} \oplus NLO_{m_{t}}$ | $34.17^{+1.9\%}_{-4.6\%}$  | $40.44^{+1.9\%}_{-4.7\%}$        | $155.5^{+2.3\%}_{-5.0\%}$  | $1372^{+2.8\%}_{-5.0\%}$  |
| $N^3LO\otimes NLO_{m_t}$           | $33.43^{+0.66\%}_{-2.8\%}$ | $39.56^{+0.64\%}_{-2.7\%}$       | $151.7^{+0.53\%}_{-2.4\%}$ | $1333^{+0.51\%}_{-1.8\%}$ |

- N<sup>3</sup>LO enhances NNLO by 3% but enhances NLO by 20%
- N<sup>3</sup>LO reduces scale unc. to 3%
- The missing top quark mass uncer. at N<sup>3</sup>LO is around 5%
- The top mass scheme uncer. is unknown (not expected to be improved)

## **TOP QUARK MASS RESULTS**



Top-quark mass dependent results NLO<sub>mt</sub> from Powheg, arXiv:1903.08137



- Scale is again significantly reduced from NNLO to N<sup>3</sup>LO
- Missing top-quark mass effect at large m<sub>hh</sub> is very bad (red)





- We have carried out N<sup>3</sup>LO calculations for Higgs pair production in the gluon fusion channel with the infinite top-quark mass limit.
- The scale uncertainty is significantly reduced to be below 2% at 100 TeV. PDF param. uncertainty is bigger than scale uncertainty.
- The asymptotic perturbative convergence in the process shows pretty good at N<sup>3</sup>LO.





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- Remaining (theory) challenges:
  - How to improve the big top-quark mass scheme dependence seen at NLO ?
  - How to further improve the finite top quark mass corrections ?
  - Other theoretical uncertainties (e.g. EW corr., parameterical errors) ?





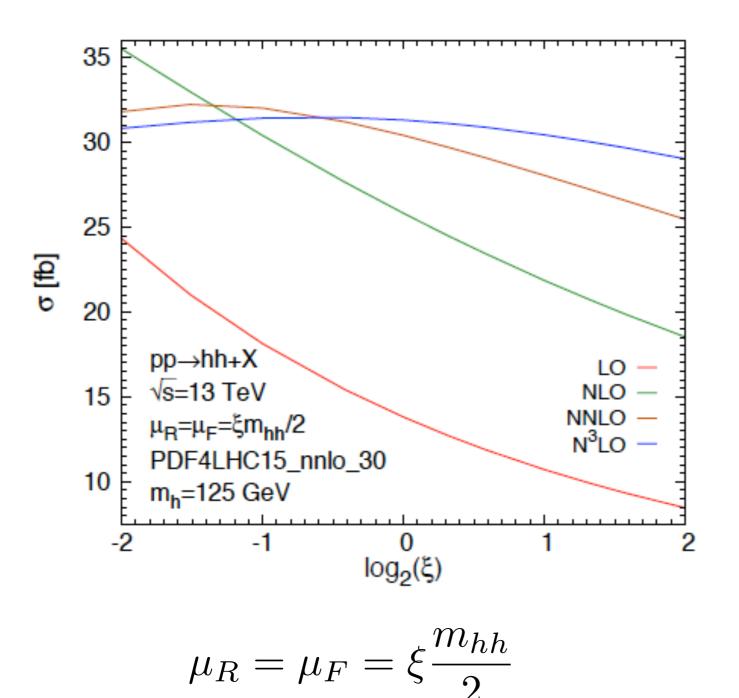
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### **Thank you for your attention !**

## BACKUP



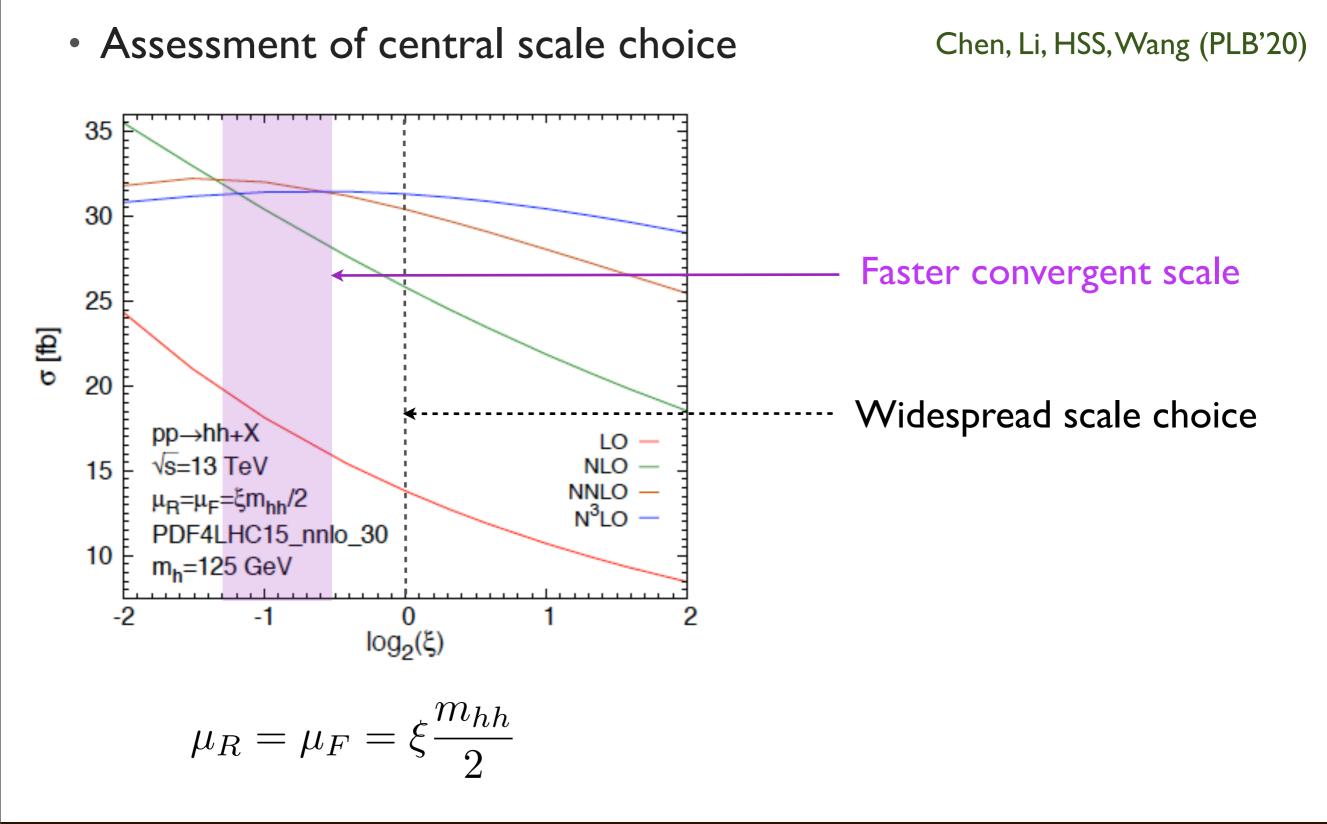
- Infinite top-quark mass limit
  - Assessment of central scale choice



Chen, Li, HSS, Wang (PLB'20)



Infinite top-quark mass limit

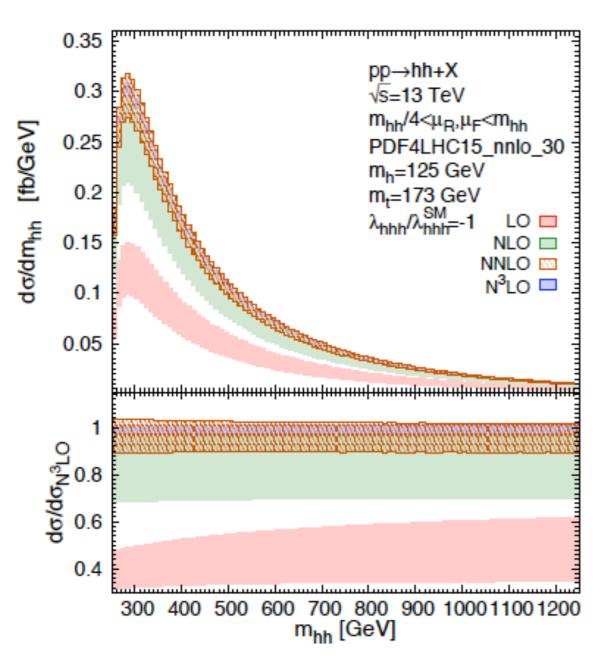




### Infinite top-quark mass limit

• A N<sup>3</sup>LO differential distribution

Chen, Li, HSS, Wang (JHEP'20)



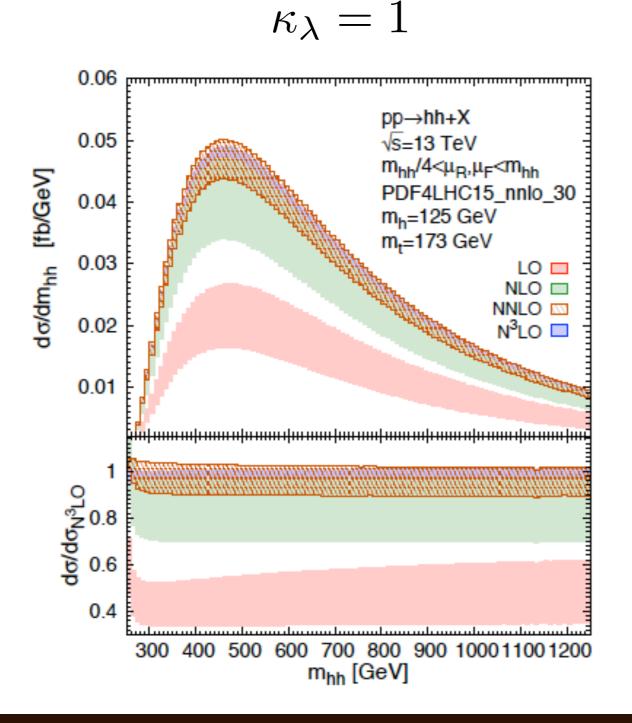
#### $\kappa_{\lambda} = -1$



### Infinite top-quark mass limit

• A N<sup>3</sup>LO differential distribution

Chen, Li, HSS, Wang (JHEP'20)





HUA-SHENG SHAO

### Infinite top-quark mass limit

• A N<sup>3</sup>LO differential distribution

10<sup>-1</sup> [A9] 10<sup>-2</sup> μμ 10<sup>-3</sup> 10<sup>-4</sup> 10<sup>-2</sup> pp→hh+X LO 🗖 √s=13 TeV NLO 🗆  $m_{hh}/4 < \mu_R, \mu_F < m_{hh}$ NNLO 📖 N<sup>3</sup>LO PDF4LHC15\_nnlo\_30 10<sup>-4</sup> m<sub>h</sub>=125 GeV m<sub>t</sub>=173 GeV  $\lambda_{hhh}^{i}/\lambda_{hhh}^{SM}=3$ 10<sup>-5</sup> 1.2 dσ/dσ<sub>N3</sub>LO 1 0.8 0.6 0.4 600 700 800 900 100011001200 300 400 500 m<sub>hh</sub> [GeV]

#### $\kappa_{\lambda} = 3$

Chen, Li, HSS, Wang (JHEP'20)



Chen, Li, HSS, Wang (JHEP'20)

### Infinite top-quark mass limit

• A N<sup>3</sup>LO differential distribution

10<sup>1</sup> pp→hh+X √s=13 TeV 10<sup>0</sup> m<sub>hh</sub>/4<µ<sub>R</sub>,µ<sub>F</sub><m<sub>hh</sub> LO 🗖 dq/qµµµ [10<sup>-1</sup> 10<sup>-2</sup> 10<sup>-3</sup> PDF4LHC15\_nnlo\_30 NLO 🔲 m<sub>h</sub>=125 GeV NNLO 🖾 N<sup>3</sup>LO 🗖 m<sub>t</sub>=173 GeV , hhh/λhhh=5 10<sup>-3</sup> 10<sup>-4</sup> 10<sup>-5</sup> 1.2 0.0 0.0 0.6 0.4 400 500 600 700 800 900 1000 1100 1200 300 m<sub>hh</sub> [GeV]

 $\kappa_{\lambda} = 5$ 

#### FCC-FRANCE WORKSHOP

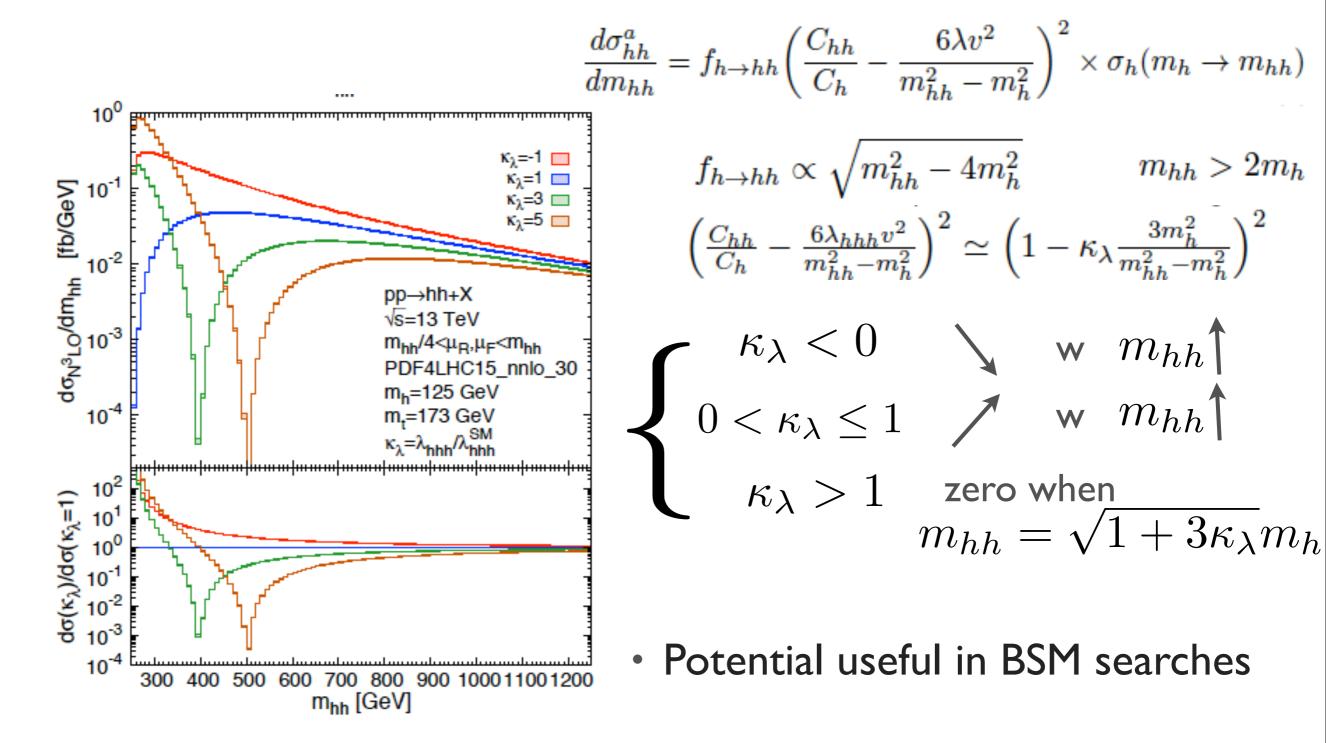
HUA-SHENG SHAO



### Infinite top-quark mass limit

• A N<sup>3</sup>LO differential distribution

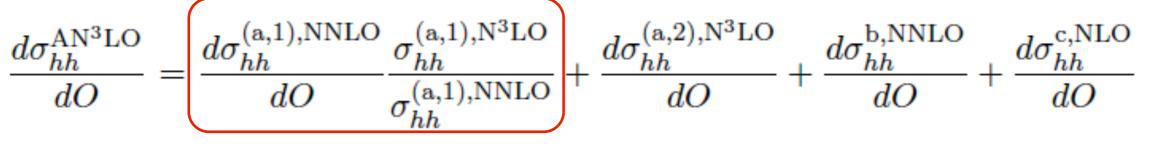
Chen, Li, HSS, Wang (JHEP'20)





### Infinite top-quark mass limit

Other approximated N<sup>3</sup>LO differential distributions



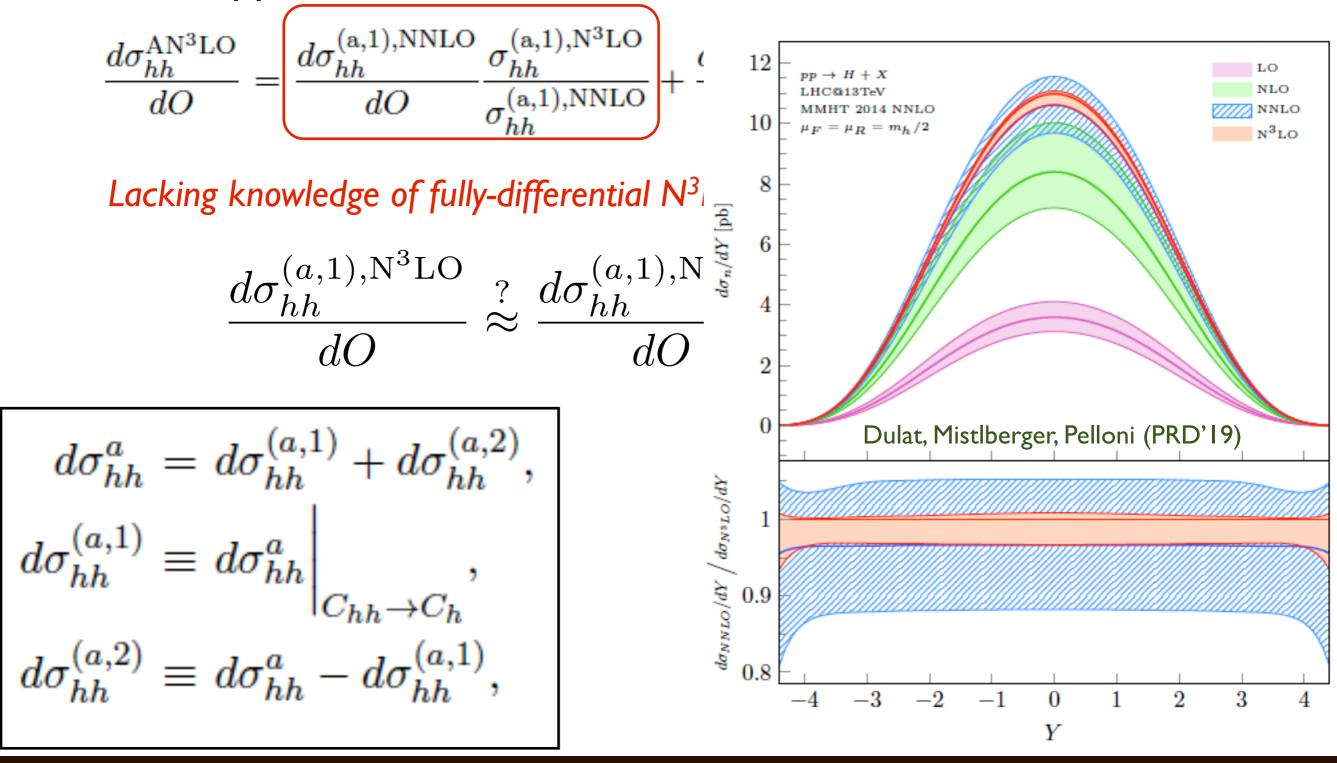
Lacking knowledge of fully-differential N<sup>3</sup>LO ggH

$$\frac{d\sigma_{hh}^{(a,1),\mathrm{N}^{3}\mathrm{LO}}}{dO} \approx \frac{d\sigma_{hh}^{(a,1),\mathrm{NNLO}}}{dO} \frac{\sigma_{hh}^{(a,1),\mathrm{NNLO}}}{\sigma_{hh}^{(a,1),\mathrm{NNLO}}}$$
$$\frac{d\sigma_{hh}^{a} = d\sigma_{hh}^{(a,1)} + d\sigma_{hh}^{(a,2)}}{\sigma_{hh}^{(a,1),\mathrm{NNLO}}}$$
$$\frac{d\sigma_{hh}^{(a,1)} \equiv d\sigma_{hh}^{a}\Big|_{C_{hh}\to C_{h}}}{\sigma_{hh}^{(a,2)} \equiv d\sigma_{hh}^{a} - d\sigma_{hh}^{(a,1)}},$$



### Infinite top-quark mass limit

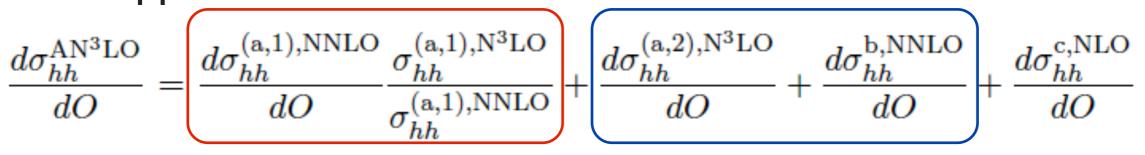
Other approximated N<sup>3</sup>LO differential distributions





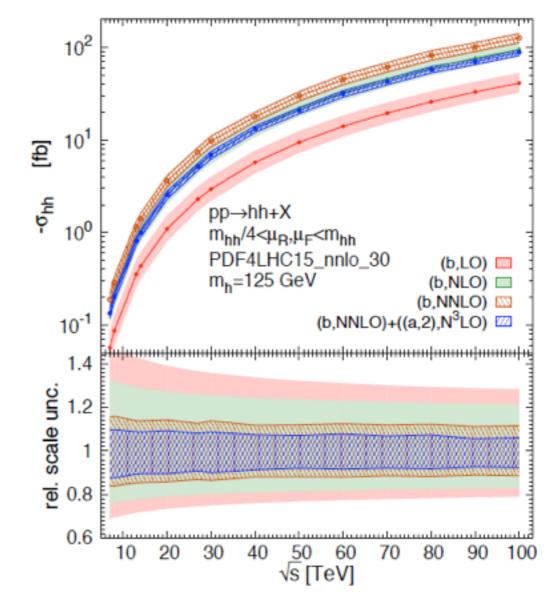
### Infinite top-quark mass limit

Other approximated N<sup>3</sup>LO differential distributions



Non-trivial scale cancellation emerges ... due to operator mixing Zoller (JHEP'16)

$$\begin{aligned} d\sigma^a_{hh} &= d\sigma^{(a,1)}_{hh} + d\sigma^{(a,2)}_{hh}, \\ d\sigma^{(a,1)}_{hh} &\equiv d\sigma^a_{hh} \bigg|_{C_{hh} \to C_h}, \\ d\sigma^{(a,2)}_{hh} &\equiv d\sigma^a_{hh} - d\sigma^{(a,1)}_{hh}, \end{aligned}$$

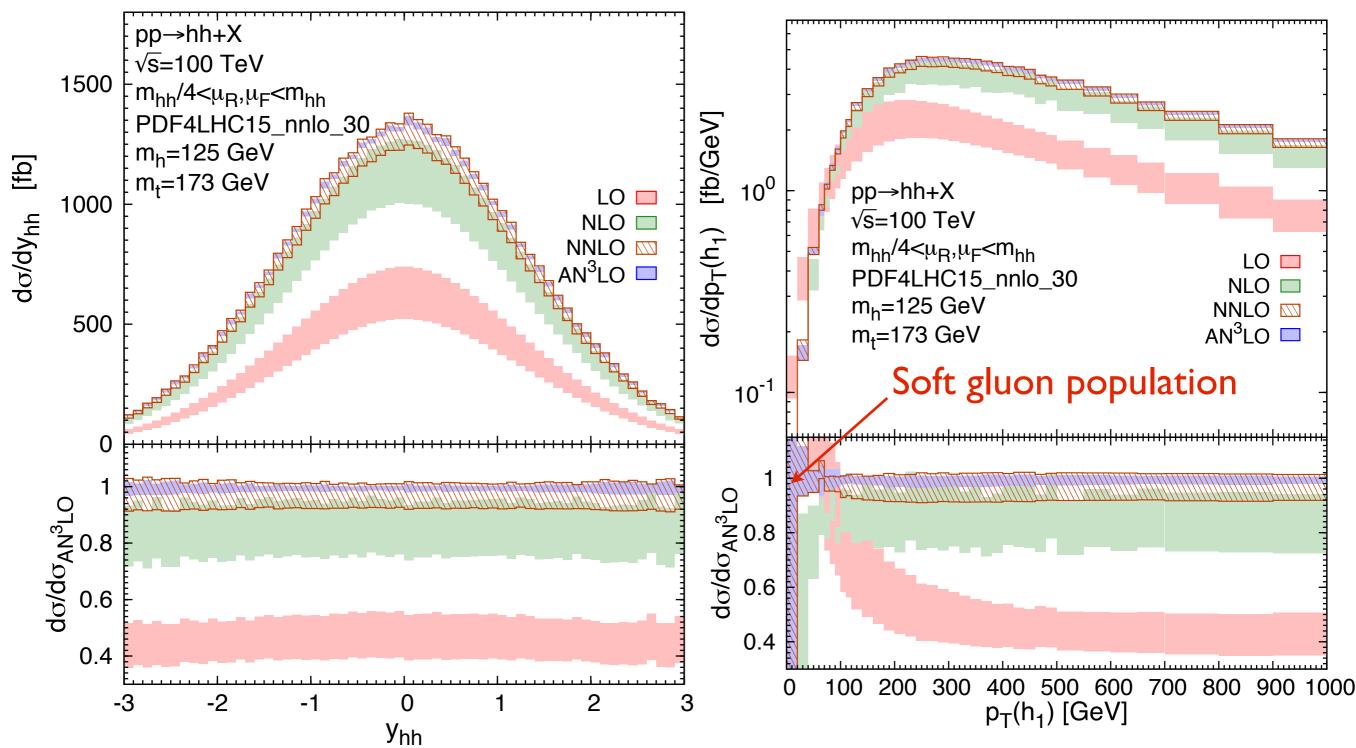




### Infinite top-quark mass limit

• Two showcases

Chen, Li, HSS, Wang (JHEP'20)



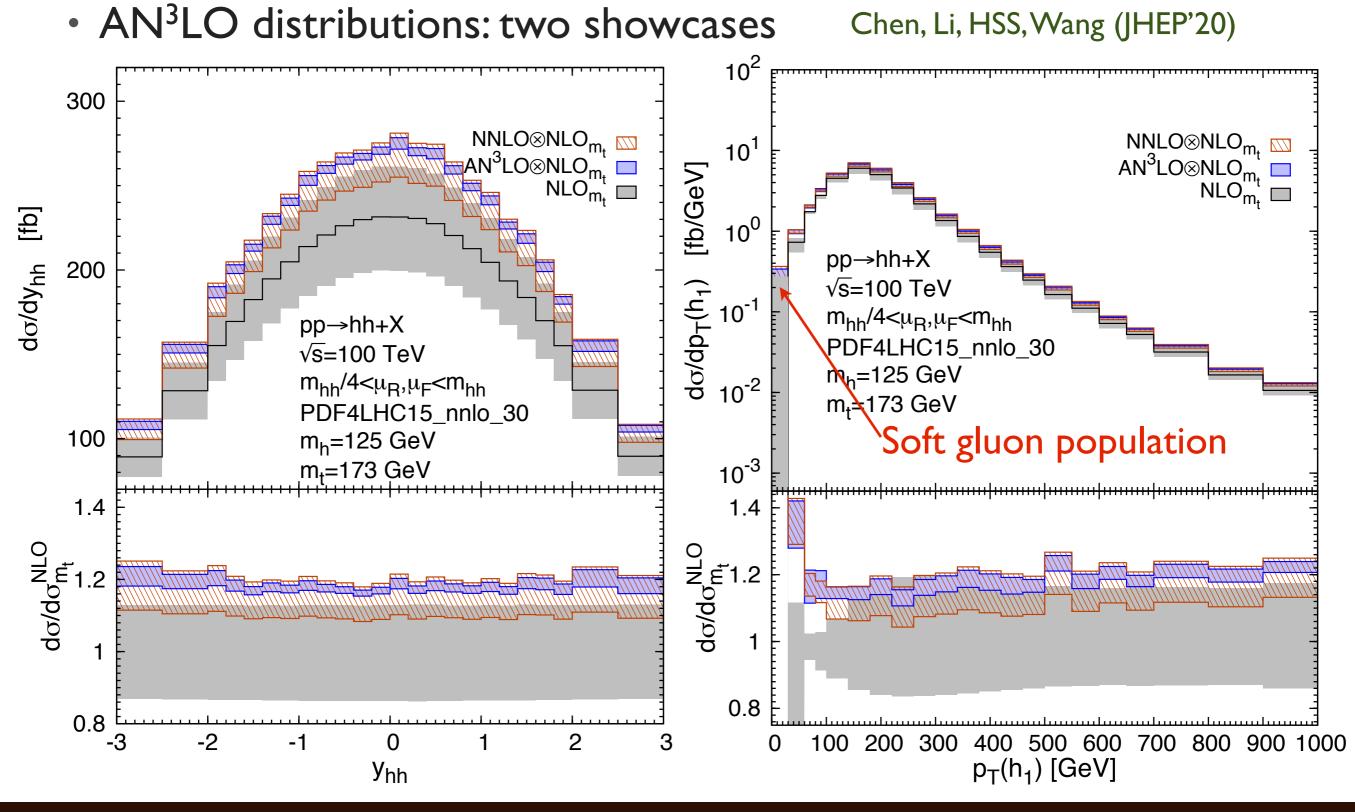
#### **FCC-FRANCE WORKSHOP**

Tuesday, January 19, 21

## **TOP QUARK MASS RESULTS**



• Top-quark mass dependent results NLO<sub>mt</sub> from Powheg, arXiv:1903.08137



#### **FCC-FRANCE WORKSHOP**

Tuesday, January 19, 21