Gravitational radiation from MHD turbulence in the early universe

GdR Gravitational Waves. Cosmology Meeting
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ARP *et al., Geophys. Astrophys. Fluid Dyn.* **114**, 130. arXiv:1807.05479 (2020)

ARP *et al., Phys. Rev. D* **102**, 083512. arXiv:1903.08585 (2020)

A. Neronov, ARP, C. Caprini, D. Semikoz., *Phys. Rev. D Lett.* arXiv:2009.14174 (2021)

T. Kahniashvili, A. Brandenburg, G. Gogoberidze, S. Mandal, ARP, *Phys. Rev. Res.* arXiv:2011.0556 (2021)

Overview

1. Introduction and Motivation
2. Magnetohydrodynamics
3. Gravitational waves
4. Numerical Results

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
  - **Electroweak phase transition** ~ 100 GeV
  - **Quantum chromodynamic (QCD) phase transition** ~ 100 MeV
  - Inflation

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
- Magnetohydrodynamic (MHD) sources of GWs:
    - Hydrodynamic turbulence from phase transition bubbles nucleation
    - Primordial magnetic fields

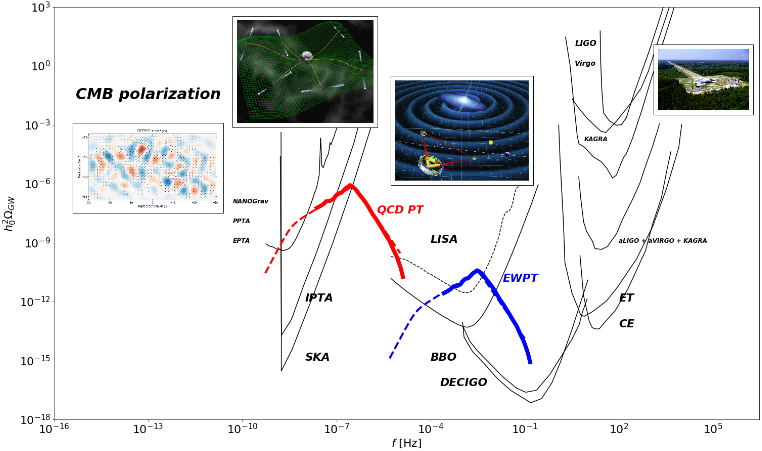
Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
- Magnetohydrodynamic (MHD) sources of GWs
- GW radiation as a probe of early universe physics
  - Nature of cosmological phase transitions (BSM)
  - Matter-anti-matter asymmetry (helicity)
  - Indirect detection of primordial magnetic fields

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
- Magnetohydrodynamic (MHD) sources of GWs
- GW radiation as a probe of early universe physics
- Possibility of GWs detection with
    - **Space-based GW detector LISA**
    - **Pulsar Timing Arrays (PTA)**
    - *B*-mode of CMB polarization

Gravitational Spectrum



Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during phase transitions and inflation
- Magnetohydrodynamic (MHD) sources of GWs
- GW radiation as a probe of early universe physics
- Possibility of detecting GWs
- Numerical simulations using Pencil Code to solve:
  - Relativistic MHD equations
  - Gravitational waves equation

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD

- Quark-gluon plasma (above QCD phase transition)
- Charge-neutral, electrically conducting fluid
- Relativistic magnetohydrodynamic (MHD) equations
- Ultrarelativistic equation of state

$$p = \rho c^2/3$$

- Friedmann–Lemaître–Robertson–Walker model

$$g_{\mu\nu} = \mathrm{diag}\{-1, a^2, a^2, a^2\}$$

Stress–energy tensor

Contributions to the stress-energy tensor

$$T^{\mu\nu} = \left(p/c^2 + \rho\right) U^\mu U^\nu + p g^{\mu\nu} + F^{\mu\gamma} F^{\nu}{}_{\gamma} - \frac{1}{4} g^{\mu\nu} F_{\lambda\gamma} F^{\lambda\gamma},$$

- From fluid motions
  $T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p\delta_{ij}$
  Relativistic equation of state:
  $p = \rho c^2/3$
- From magnetic fields:
  $T_{ij} = -B_i B_j + \delta_{ij} B^2/2$

- 4–velocity $U^\mu = \gamma(c, u^i)$
- 4–potential $A^\mu = (\phi/c, A^i)$
- 4–current $J^\mu = (c\rho_{\rm e}, J^i)$
- Faraday tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

MHD equations

Conservation laws

$$T^{\mu\nu}{}_{;\nu} = 0$$

Relativistic MHD equations are reduced to[1]

MHD equations

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho\right) + \frac{1}{\rho c^2}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right]$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{3}\mathbf{u}\left(\nabla \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \ln \rho\right) - \frac{\boldsymbol{u}}{\rho c^2}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta J^2\right] - \frac{1}{4}c^2 \nabla \ln \rho + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho}\nabla \cdot (\rho \nu \boldsymbol{S})$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,_{\rm phys}}, u_i$, and conformal time t.

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[1] A. Brandenburg, K. Enqvist, and P. Olesen, *Phys. Rev. D* **54**, 1291 (1996)

Gravitational waves equation

GWs equation for an expanding flat Universe

- Assumptions: isotropic and homogeneous Universe
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric $\gamma_{ij} = a^2 \delta_{ij}$
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}^{\rm phys} \right)$$

- GWs equation is[2] $$\left(\partial_t^2 - \cancel{\frac{a''}{a}} - c^2 \nabla^2 \right) h_{ij} = \frac{16 \pi G}{\mathbf{a} c^2} T_{ij}^{\rm TT}$$
    - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\rm phys}$
    - Comoving spatial coordinates $\nabla = a \nabla^{\rm phys}$
    - Conformal time $dt = a \, dt^{\rm phys}$
    - Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\rm phys}$
    - Radiation-dominated epoch such that $a'' = 0$

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[2] L. P. Grishchuk, *Sov. Phys. JETP*, 40, 409-415 (1974)

Normalized GW equation[3]

$$\left(\partial_t^2 - \nabla^2\right) h_{ij} = 6\, T_{ij}^{\rm TT}/t$$

Properties

- All variables are normalized and non-dimensional
- Conformal time is normalized with t_*
- Comoving coordinates are normalized with c/H_*
- Stress-energy tensor is normalized with $\mathcal{E}_{\rm rad}^* = 3H_*^2 c^2/(8\pi G)$
- Scale factor is $a_* = 1$, such that $a = t$

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[3] A. Roper Pol *et al., Geophys. Astrophys. Fluid Dyn.* **114**, 130. arXiv:1807.05479 (2020)

Gravitational waves equation

Properties

- Tensor-mode perturbations are gauge invariant
- h_{ij} has only two degrees of freedom: h^{+}, $h^{\times}$
- The metric tensor is traceless and transverse (TT gauge)

Energy density and amplitude

- Energy density $\Omega_{\rm GW}$ is proportional to $\dot{h}_+ \dot{h}_+^* + \dot{h}_\times \dot{h}_\times^*$ and normalized with the critical energy density at present time
- The characteristic strain h_c is proportional to $h_+ h_+^* + h_\times h_\times^*$

Numerical results for decaying MHD turbulence[4]

Initial conditions

- Fully helical stochastic magnetic field
- Batchelor spectrum, i.e., $E_{\rm M} \propto k^4$ for small k
- Kolmogorov spectrum for inertial range, i.e., $E_{\rm M} \propto k^{-5/3}$
- Total energy density at t_* is $\sim 10\%$ to the radiation energy density
- Spectral peak at $k_{\rm M} = 100 \cdot 2\pi$, normalized with $k_H = H/c$

Numerical parameters

- 1152^3 mesh gridpoints
- 1152 processors
- Wall-clock time of runs is $\sim 1 - 5$ days

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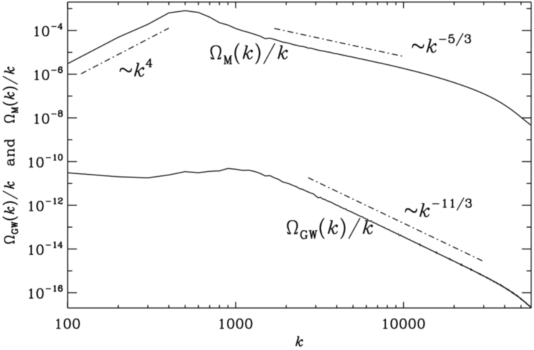
[4] A. Brandenburg, *et al. Phys. Rev. D* **96**, 123528 (2017),
A. Roper Pol, *et al. Phys. Rev. D* **102**, 083512 (2020)

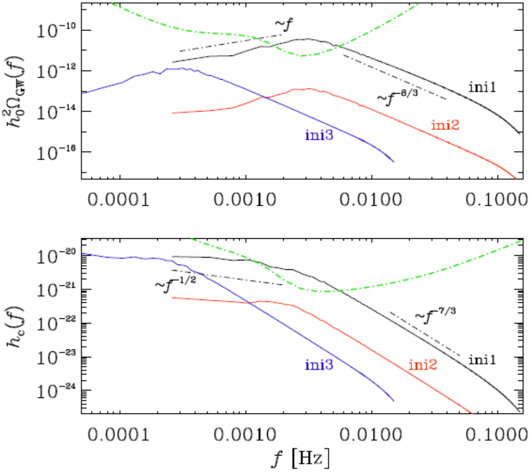
Numerical results for decaying MHD turbulence



Comments

- **Novel k^0 scaling in the subinertial range**
- k^2 **is expected for larger scales**
- Further investigation on the k^0 development

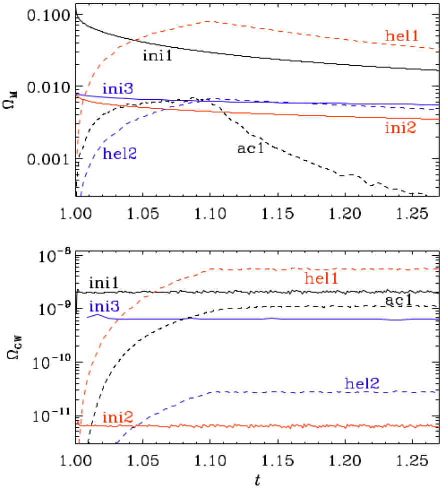
Numerical results for decaying MHD turbulence (EWPT)



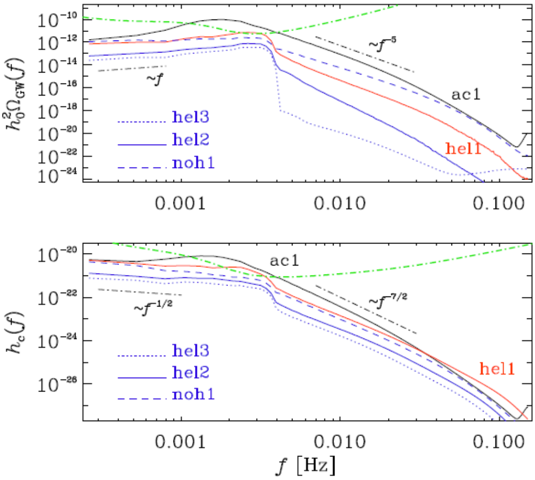
Run	$\mathcal{E}_0, \mathcal{F}_0$	η	$\Omega_i^{\rm max}$	$\Omega_{\rm GW}^{\rm sat}$	i	hel	$t_{\rm max}$	N
ini1	—	5e-6	1.16e-01	2.05e-09	M	y	1.00	100
ini2	—	5e-8	7.62e-03	6.38e-12	M	y	1.00	100
ini3	—	5e-7	7.62e-03	6.36e-10	M	y	1.00	10

Initial given field vs generated field

- Time evolution of energy density



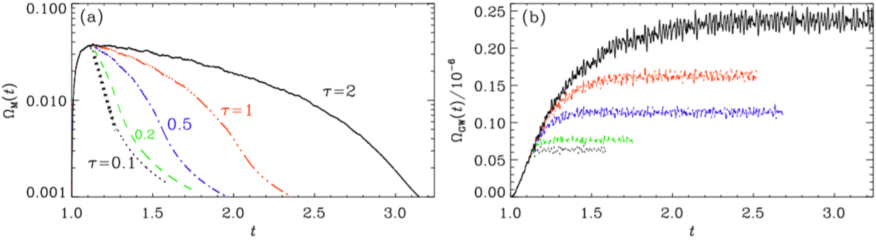
Forced turbulence (built-up primordial magnetic fields and hydrodynamic turbulence)



Run	$\mathcal{E}_0, \mathcal{F}_0$	η	$\Omega_i^{\max}$	$\Omega_{\rm GW}^{\rm sat}$	i	hel	$t_{\max}$	N
hel1	1.4e-3	5e-7	2.17e-02	4.43e-09	M	y	1.10	100
hel2	8.0e-4	5e-7	7.18e-03	4.67e-10	M	y	1.10	100
hel3	2.0e-3	5e-7	4.62e-03	2.09e-10	M	y	1.01	100
hel4	1.0e-4	2e-6	5.49e-03	1.10e-11	M	y	1.01	1000
noh1	1.4e-3	5e-7	1.44e-02	3.10e-09	M	n	1.10	100
noh2	8.0e-4	2e-6	4.86e-03	3.46e-10	M	n	1.10	100
ac1	3.0	2e-5	1.33e-02	5.66e-08	K	n	1.10	100
ac2	3.0	5e-5	1.00e-02	3.52e-08	K	n	1.10	100
ac3	1.0	5e-6	2.87e-03	2.75e-09	K	n	1.10	100

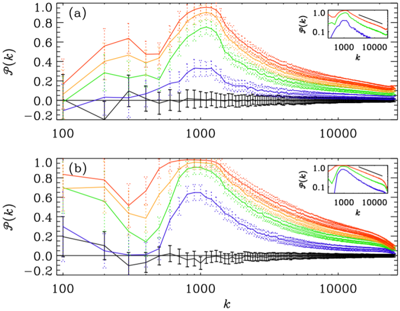
Extended pumping of energy

- The GW energy density becomes 'stationary' shortly after the source energy density reaches its maximum
- If we extend the pumping of energy, the difference is small ($\sim$ 4 times for $\tau = 2$, $\tau > 0.5$ is highly unrealistic)



Polarization degree (magnetic vs kinetic)[5]

- Helical magnetic fields induce circularly polarized GWs (same sign)
- Previous analytical predictions fail due to:
  - (i) Assumed stationary turbulence
  - (ii) Neglecting dynamical effects



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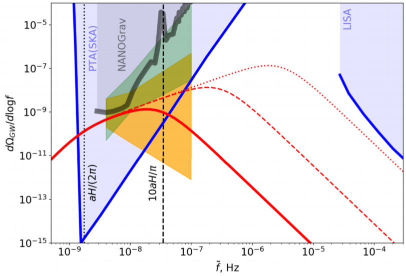
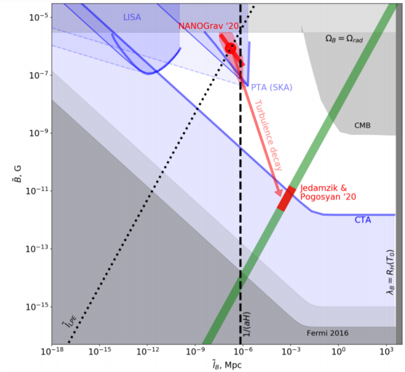
[5] T. Kahniashvili, A. Brandenburg, A. Kosowsky, S. Mandal, A. Roper Pol *Phys. Rev. Res.* arXiv:2009.14174 (2021)

NANOGrav observation QCD phase transition[6]





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[6] A. Neronov, A. Roper Pol, C. Caprini, D. Semikoz *Phys. Rev. D Lett.* arXiv:2009.14174 (2021)

Conclusions

- Depending on the mechanism of turbulence generation and/or the initial energy density and characteristic scale, the GW signal from the EWPT could be detected by LISA.
- GW equation is normalized such that it can be easily scaled for different times within the radiation-dominated epoch.
- Novel f spectrum obtained for GWs in high frequencies range vs f^3 obtained from analytical estimates (above horizon scales)
- Bubble nucleation and magnetogenesis physics can be coupled to our equations for more realistic production analysis
- Potential detection by NANOGrav if magnetic scale is near horizon
- Information on large-scale relic magnetic fields with cosmological origin
- Detection of GW spectrum can provide with *clean* information from the epoch of generation
- Polarization degree can provide information on magnetic helicity of the seed field, and whether it is kinetic or magnetic

The End
Thank You!

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