Unitarity Test in Neutrino Oscillations

Hiroshi Nunokawa
Pontificia Universidade Catolica do Rio de Janeiro, PUC-Rio, Brazil and IJCLab, Orsay France (sabbatical visit)

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Outline

- **Introduction**
- **Oscillation probabilities without unitarity**
- **Current status**
- **Future Prospect**
- **Summary**

Review + past/on-gong works in collab. with
H. Minakata and C. S. Fong/A. Cabrera

Introduction

- Currently almost all the neutrino data are explained very well by the standard 3 neutrino flavor scheme (**ν**SM) without New Physics beyond neutrino mass and mixing
  (Some exceptions: LSND/MiniBooNE, Reactor/Ga anomaly, etc)

- Most of the oscillation parameters $(\Delta m^2_{ij}\ ,\ \sin^2\theta_{ij})$ are measured with precision at the level of a few percent, or we are in an era of precise measurement

- More precice measurements will be performed by coming new experiments such as JUNO, DUNE, Hyper-K, ORCA, PINGU, etc, whose goal includes the determination of neutrino mass ordering and CP phase

Introduction

- In order to be sure about the measurements of the mass ordering, CP phase, θ_{23} octant, etc, it is crucial to test the standard 3 flavor neutrino paradigm (**ν**SM)

- Any deviation from **ν**SM implies New Physics Beyond SM or new discovery

- Test of Unitarity would be one way to go because it can be done in a relatively model indepent way and several New Physics scenarios imply (or induce effectively as in the case of NSI) Violation of Unitarity to some extent

Introduction

For 3 flavor of mixed neutrinos

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = U\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

mixing matrix U is unitary if

$$UU^\dagger = U^\dagger U = 1$$

9 indep. eqs.

$$|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 = 1 \quad (\alpha = e, \mu, \tau)$$

or

$$|U_{ei}|^2 + |U_{\mu i}|^2 + |U_{\tau i}|^2 = 1 \quad (i = 1, 2, 3)$$

normalisation (3 eqs.)

$$U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2} + U_{\alpha 3}U^*_{\beta 3} = 0 \quad (\alpha, \beta = e, \mu, \tau, \alpha \neq \beta)$$

or

$$U_{ei}U^*_{ej} + U_{\mu i}U^*_{\mu j} + U_{\tau i}U^*_{\tau j} = 0 \quad (i, j = 1, 2, 3, i \neq j)$$

closure of unitarity triangle (6**R** eqs.)

Introduction

Where the Non-Unitarity (NU) effect can manifest?

See e.g. Antusch et al, JHEP10, 084 (2006)

- **Decay Processes**

  W decay

  Invisible Z decay

  Universality test

  Rare charged lepton decay

- **Neutrino Oscillation**

  Disappearance mode

  Appearance mode

In this talk we focus on NU effect for oscillation

Introduction

(Incomplete) List of relevant references on Non Unitarity Effects for Neutrinos

Antusch et al, JHEP10, 084 (2006)

Qian et al, arXiv:1308.5700 [hep-ex]

Goswami and Ota, PRD78, 033012 (2008)

Escrihuela et al, PRD92, 053009 (2015)

Parke and Ross-Lonergan, PRD93, 113009 (2016)

Ge et al, PRD95, 033005 (2017)

Escrihuela et al, New J. Phys. 19, 093005 (2017)

Fong et al, JHEP02, 114 (2017)

Fong et al, JHEP02, 015 (2019)

Ellis et al, JHEP12, 068 (2020)

Hu et al, JHEP12, 124 (2021)

there are many more works!

How to parametrise the Non-Unitary mixing matrix?

How many free parameters (for oscillation) we have for U if Unitarity is not assumed?

18 - 3 - 2 = 13 free parameters

3 phase can be removed by redefinition of charged lepton fields

2 Majorana phases

by relaxing normalisation (3) and closure (6) conditions

or 4 + 9 = 13 free parameters

Non-Unitarity U can be parametrised, e.g., as,

$$U = \begin{bmatrix} |U_{e1}| & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}|e^{i\phi_{e3}} \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}|e^{i\phi_{\tau2}} & |U_{\tau3}|e^{i\phi_{\tau3}} \end{bmatrix}$$

Another possible parameterisation

Miranda et al, PRL117, 061804 (2016)

α_{ii} : real

α_{ij} $(i \neq j)$: complex

$$U = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} U_0$$

$\alpha_{ii} \sim 1, \; |\alpha_{ij}| (i \neq j) \ll 1$

non-unitary if the triangular α matrix is not $\mathbb{1}$

standard 3x3 unitary matrix

$\alpha_{ij} \neq \delta_{ij}$ implies unitarity violation

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = \alpha_{11}^2$$
$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = \alpha_{22}^2 + |\alpha_{21}|^2$$
$$|U_{\tau 1}|^2 + |U_{\tau 2}|^2 + |U_{\tau 3}|^2 = \alpha_{33}^2 + |\alpha_{32}|^2 + |\alpha_{31}|^2$$

normalisation

$$U_{e1}^* U_{\mu 1} + U_{e2}^* U_{\mu 2} + U_{e3}^* U_{\mu 3} = \alpha_{11}\alpha_{21}$$
$$U_{e1}^* U_{\tau 1} + U_{e2}^* U_{\tau 2} + U_{e3}^* U_{\tau 3} = \alpha_{11}\alpha_{32}$$
$$U_{\mu 1}^* U_{\tau 1} + U_{\mu 2}^* U_{\tau 2} + U_{\mu 3}^* U_{\tau 3} = \alpha_{21}^*\alpha_{31} + \alpha_{22}^*\alpha_{32}$$

closure (of the unitarity triangle)

How to compute probability if U is non-unitary?

I will follow the following 2 works

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A framework for testing leptonic unitarity by neutrino oscillation experiments

Chee Sheng Fong,[a] **Hisakazu Minakata**[b] **and Hiroshi Nunokawa**[c]

[a]*Instituto de Física, Universidade de São Paulo, C.P. 66.318, 05315-970 São Paulo, Brazil*
[b]*Department of Physics, Yachay Tech, San Miguel de Urcuquí, 100119 Ecuador*
[c]*Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, C.P. 38097, 22451-900, Rio de Janeiro, Brazil*

E-mail: fong@if.usp.br, hminakata@yachaytech.edu.ec, nunokawa@puc-rio.br

ABSTRACT: If leptonic unitarity is violated by new physics at an energy scale much lower than the electroweak scale, which we call low-scale unitarity violation, it has different characteristic features from those expected in unitarity violation at high-energy scales. They include maintaining flavor universality and absence of zero-distance flavor transition. We present a framework for testing such unitarity violation at low energies by neutrino oscillation experiments. Starting from the unitary 3 active plus N (arbitrary positive integer) sterile neutrino model we show that by restricting the active-sterile and sterile-sterile neutrino mass squared differences to $\gtrsim 0.1\,\mathrm{eV}^2$ the oscillation probability in the $(3+N)$ model becomes insensitive to details of the sterile sector, providing a nearly model-independent framework for testing low-scale unitarity violation. Yet, the presence of the sterile sector leaves trace as a constant probability leaking term, which distinguishes low-scale unitarity violation from the high-scale one. The non-unitary mixing matrix in the active neutrino subspace is common for the both cases. We analyze how severely the unitarity violation can be constrained in ν_e-row by taking a JUNO-like setting to simulate medium baseline reactor experiments. Possible modification of the features of the $(3+N)$ model due to matter effect is discussed to first order in the matter potential.

KEYWORDS: Beyond Standard Model, Neutrino Physics

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Non-unitary evolution of neutrinos in matter and the leptonic unitarity test

Chee Sheng Fong,[a,1] **Hisakazu Minakata**[b,c] **and Hiroshi Nunokawa**[d]

[a]*Instituto de Física, Universidade de São Paulo, C.P. 66.318, 05315-970 São Paulo, Brazil*
[b]*Instituto Física Teórica, UAM/CSIC, Calle Nicolás Cabrera 13-15, Cantoblanco E-28049 Madrid, Spain*
[c]*Research Center for Cosmic Neutrinos, Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan*
[d]*Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, C.P. 38097, 22451-900, Rio de Janeiro, Brazil*

E-mail: fong@if.usp.br, hisakazu.minakata@gmail.com, nunokawa@puc-rio.br

ABSTRACT: We present a comprehensive study of the three-active plus N sterile neutrino model as a framework for constraining leptonic unitarity violation induced at energy scales much lower than the electroweak scale. We formulate a perturbation theory with expansion in small unitarity violating matrix element W while keeping (non-W suppressed) matter effect to all orders. We show that under the same condition of sterile state masses $0.1\,\mathrm{eV}^2 \lesssim m_J^2 \lesssim (1\text{–}10)\,\mathrm{GeV}^2$ as in vacuum, assuming typical accelerator based long-baseline neutrino oscillation experiment, one can derive a very simple form of the oscillation probability which consists only of zeroth-order terms with the unique exception of probability leaking term $\mathcal{C}_{\alpha\beta}$ of $\mathcal{O}(W^4)$. We argue, based on our explicit computation to fourth-order in W, that all the other terms are negligibly small after taking into account the suppression due to the mass condition for sterile states, rendering the oscillation probability *sterile-sector model independent*. Then, we identify a limited energy region in which this suppression is evaded and the effects of order W^2 corrections may be observable. Its detection would provide another way, in addition to detecting $\mathcal{C}_{\alpha\beta}$, to distinguish between low-scale and high-scale unitarity violation. We also solve analytically the zeroth-order system in matter with uniform density to provide a basis for numerical evaluation of non-unitary neutrino evolution.

KEYWORDS: Beyond Standard Model, Neutrino Physics

ARXIV EPRINT: 1712.02798

[1]Now at: Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Santo André, 09210-580 SP, Brazil; sheng.fong@ufabc.edu.br.

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As a possible origin of non-unitarity, let us assume that there is(are) extra (beynd 3) neutrino state(s) which must be sterile

let us assume *N* (arbitrary) extra states which are heavy (more than ~O(0.1) eV) but "light" enough to participate in oscillation

$$i\frac{d}{dt}\nu = H\nu$$

ignoring the matter effect

$$H = \mathbf{U}\begin{bmatrix} \Delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{3+N} \end{bmatrix}\mathbf{U}^\dagger$$

(3+N)x(3+N)

3x3

$$\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

$$\Delta_i \equiv \frac{m_i^2}{2E} \quad (i = 1, 2, 3), \qquad \Delta_J \equiv \frac{m_J^2}{2E} \quad (J = 4, \cdots, 3+N)$$

Let us assume that oscillation driven by extra heavier neutrino states are averaged out

then we have equation for vacuum oscillation similar to what we naively expect apart from the "leaking" term

$$P(\nu_\alpha \to \nu_\beta) = \mathcal{C}_{\alpha\beta} + \left| \sum_{k=1}^{3} U^*_{\alpha k} U_{\beta k} \right|^2 - 4 \sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \sin^2 \frac{\Delta m^2_{ij}}{4E} L$$

$$-2 \sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \sin \frac{\Delta m^2_{ij}}{2E} L,$$

note that U is non-unitary

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=4}^{3+N} |W_{\alpha J}|^2 |W_{\beta J}|^2,$$

we call this "leaking" term

If violation of unitarity is small (if U is close to unitary matrix), W matrix should be small, therefore, $\mathcal{C}_{\alpha\beta}$ is excpected to be quite small

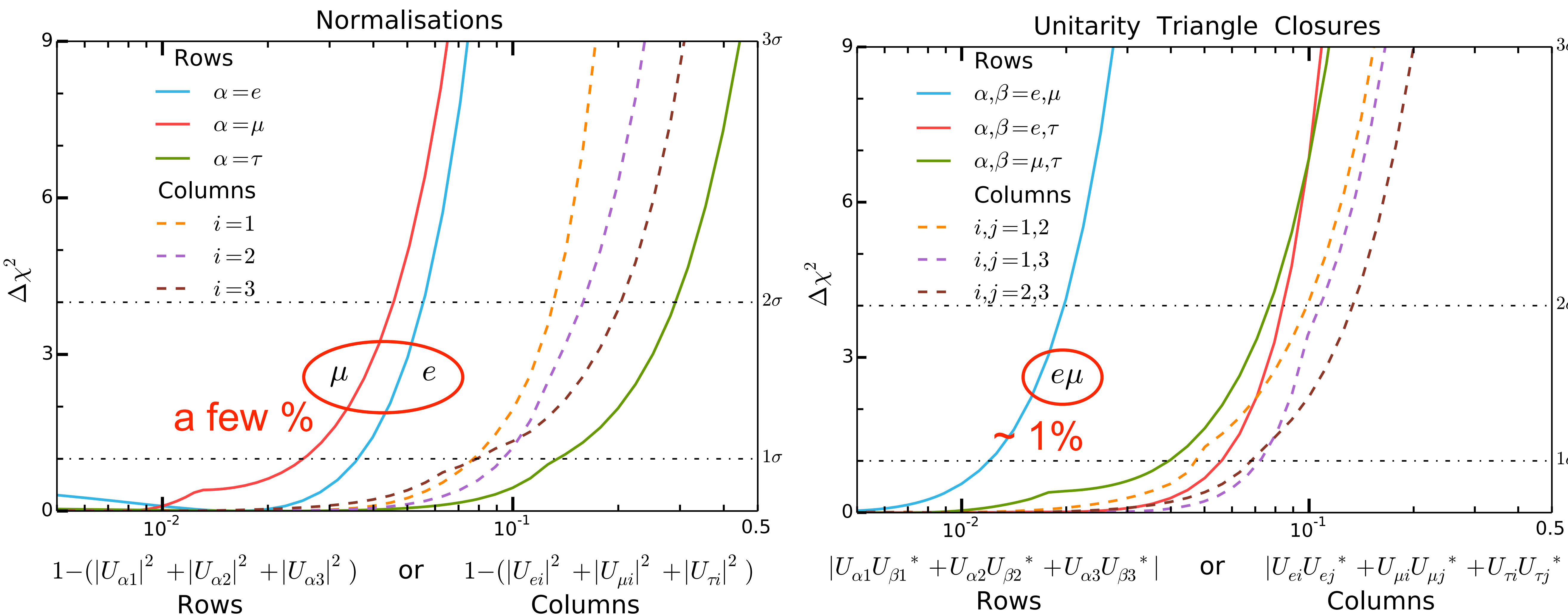
if W < O(0.1), then $\mathcal{C}_{\alpha\beta} \leq O(10^{-4})$

most of the previous works consider the eq. w/o $\mathcal{C}_{\alpha\beta}$ term though in practice we can ignore it at 1st approximation

we also developed a formalism to take matter effect into account, see Fong et al, JHEP02, 015 (2019)

Ignoring the leaking term, our oscillation formulas are the same in several papers, for example, the one by Parke and Ross-Lonergan who derived NU bounds using neutrino data around 2016

Current (around ~2016) limits on Non-Unitarity



normalisation bounds are limited by flux uncertainty (see later discussion)

Parke and Ross-Lonergan, PRD93, 113009 (2016)

Most Updated Constraints by Ellis et al, JHEP12, 068 (2020)

Based on data coming from solar neutrinos, KamLAND, Daya Bay, OPERA, T2K, NOvA, MINOS/MINOS+, short baseline accelerator experiments (KARMEN, NOMAD, CHORUS)

(1) Normalisation

$$N_\alpha \equiv |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 \quad (\alpha = e, \mu, \tau)$$

$$N_k \equiv |U_{ek}|^2 + |U_{\mu k}|^2 + |U_{\tau k}|^2 \quad (k = 1, 2, 3)$$

	Best-fit (current)	3σ (current)
N_e	1.00	[0.94, 1.05]
N_μ	0.99	[0.96, 1.04]
N_τ	1.12.	[0.32, 1.82]
N_1	1.01	[0.84, 1.22]
N_2	1.05	[0.75, 1.27]
N_3	1.05	[0.67, 1.40]

~ a few % (1 σ)

Consistent with 1

Most Updated Constraints

by Ellis et al, JHEP12, 068 (2020)

(2) Closure

$$t_{kl} \equiv U^*_{ek} U_{el} + U^*_{\mu k} U_{\mu l} + U^*_{\tau k} U_{\tau l}$$

$$t_{\alpha\beta} \equiv U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} + U^*_{\alpha 3} U_{\beta 3} \quad (\alpha \neq \beta,\ \alpha, \beta = e, \mu, \tau)$$

	Current 3σ Upper Limit
$\lvert t_{e\mu} \rvert$	3.2×10^{-2}
$\lvert t_{e\tau} \rvert$	1.3×10^{-1}
$\lvert t_{\mu\tau} \rvert$	1.6×10^{-2}
$\lvert t_{12} \rvert$	2.5×10^{-1}
$\lvert t_{13} \rvert$	3.2×10^{-1}
$\lvert t_{23} \rvert$	3.3×10^{-1}

~ 1% (1 σ)

Consistent with 0

Future Expectation by Ellis et al, JHEP12, 068 (2020)

Based on additional data coming from IceCube, JUNO, DUNE, T2HK

(1) Normalisation

$$N_\alpha \equiv |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 \quad (\alpha = e, \mu, \tau)$$
$$N_k \equiv |U_{ek}|^2 + |U_{\mu k}|^2 + |U_{\tau k}|^2 \quad (k = 1, 2, 3)$$

	Best-fit (current)	3σ (current)	3σ (future)
N_e	1.00	[0.94, 1.05]	[0.97, 1.03]
N_μ	0.99	[0.96, 1.04]	[0.96, 1.03]
N_τ	1.12.	[0.32, 1.82]	[0.79, 1.23]
N_1	1.01	[0.84, 1.22]	[0.89, 1.12]
N_2	1.05	[0.75, 1.27]	[0.92, 1.10]
N_3	1.05	[0.67, 1.40]	[0.90, 1.10]

improvement is not so dramatic apart from tau sector which is not so good anyway

Future Expectations

by Ellis et al, JHEP12, 068 (2020)

(2) Closure

$$t_{kl} \equiv U^*_{ek} U_{el} + U^*_{\mu k} U_{\mu l} + U^*_{\tau k} U_{\tau l}$$

$$t_{\alpha\beta} \equiv U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} + U^*_{\alpha 3} U_{\beta 3} \quad (\alpha \neq \beta,\ \alpha, \beta = e, \mu, \tau)$$

	Current 3σ Upper Limit	Future 3σ Upper Limit
$\lvert t_{e\mu}\rvert$	3.2×10^{-2}	2.5×10^{-2}
$\lvert t_{e\tau}\rvert$	1.3×10^{-1}	No Improvement
$\lvert t_{\mu\tau}\rvert$	1.6×10^{-2}	No Improvement
$\lvert t_{12}\rvert$	2.5×10^{-1}	1.0×10^{-1}
$\lvert t_{13}\rvert$	3.2×10^{-1}	1.2×10^{-1}
$\lvert t_{23}\rvert$	3.3×10^{-1}	1.1×10^{-1}

improvement is again not so dramatic

Summary of the current/future constraints studied by Parke-Ross-Lonergan and Ellis et al.

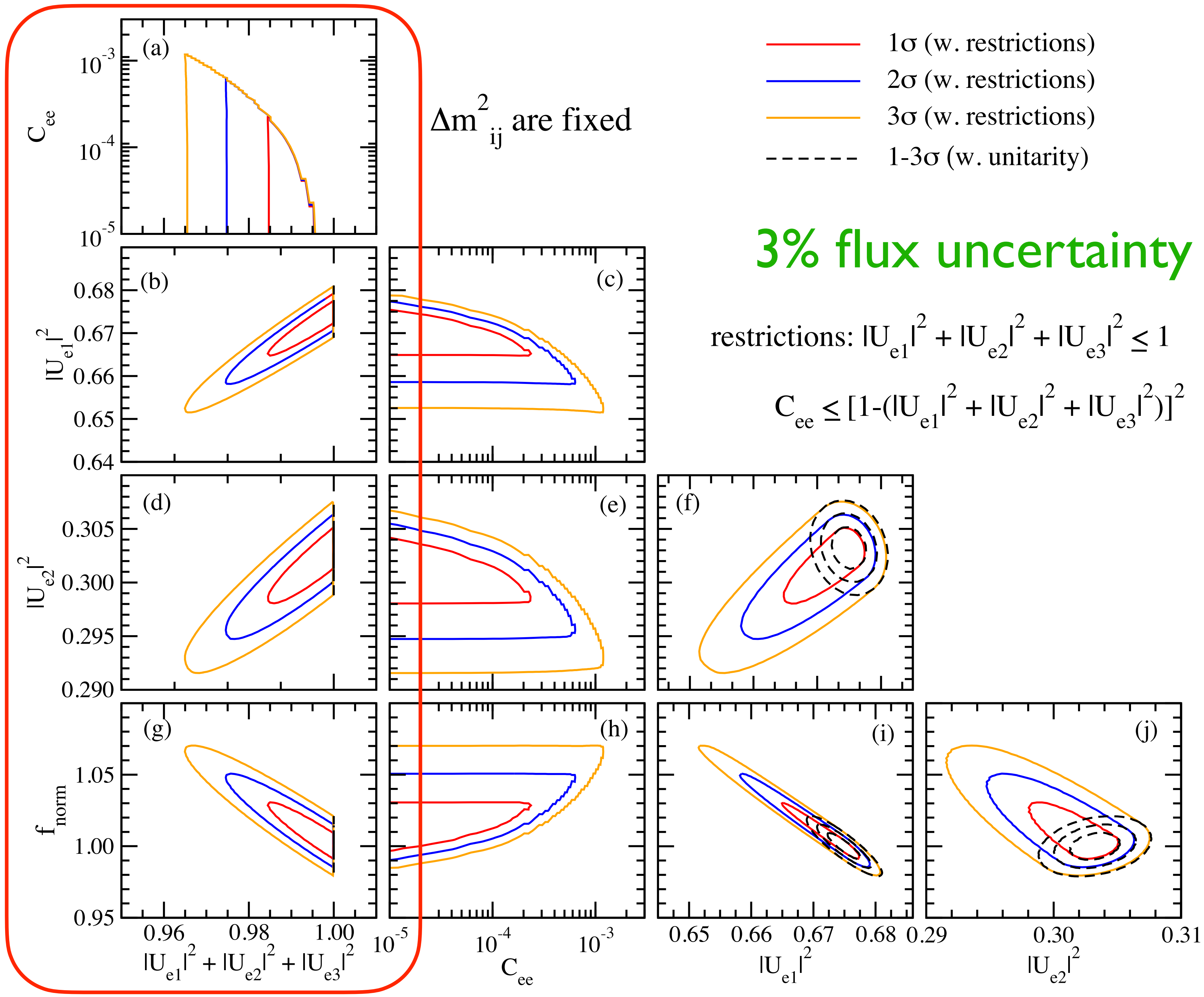
Their results seem to be consistent with each other if we take into account some differences regarading assumptions and used data set

normalisation NU parameters for e/μ rows can be constrained ~1% level (1σ)

closure (of unitarity triangle) parameters for e/μ rows can also be constrained ~1% level (1σ)

Let us compare with our analysis done only considering JUNO

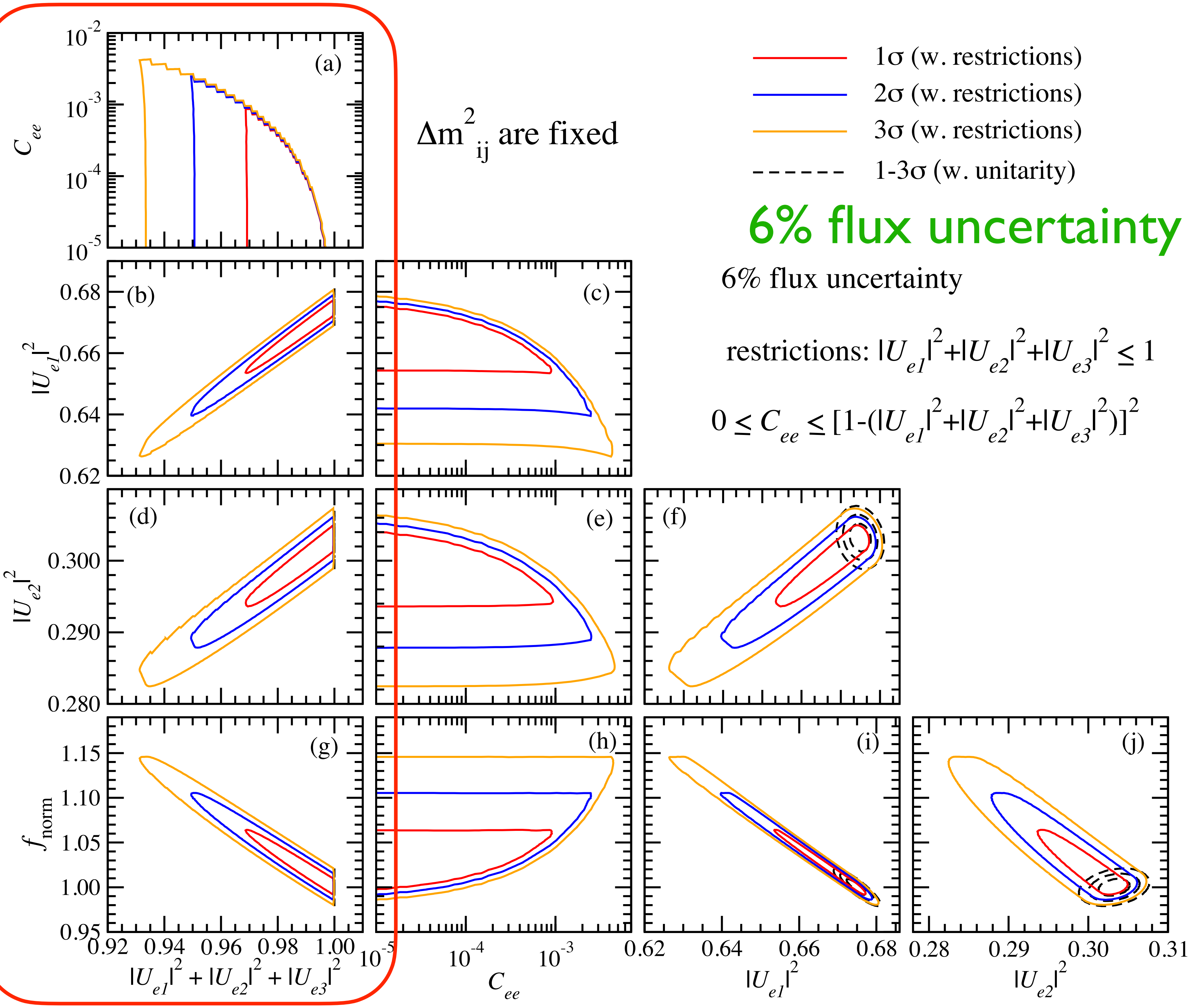
Expected sensitivity of JUNO for the unitarity (normalisation) test



$$1 - (|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2) < 1.5\%$$

Fong et al, JHEP02, 114 (2017)

Expected sensitivity of JUNO for the unitarity (normalisation) test



6% flux uncertainty is assumed

$$1-(|U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2) < 3\%$$

Fong et al, JHEP02, 114 (2017)

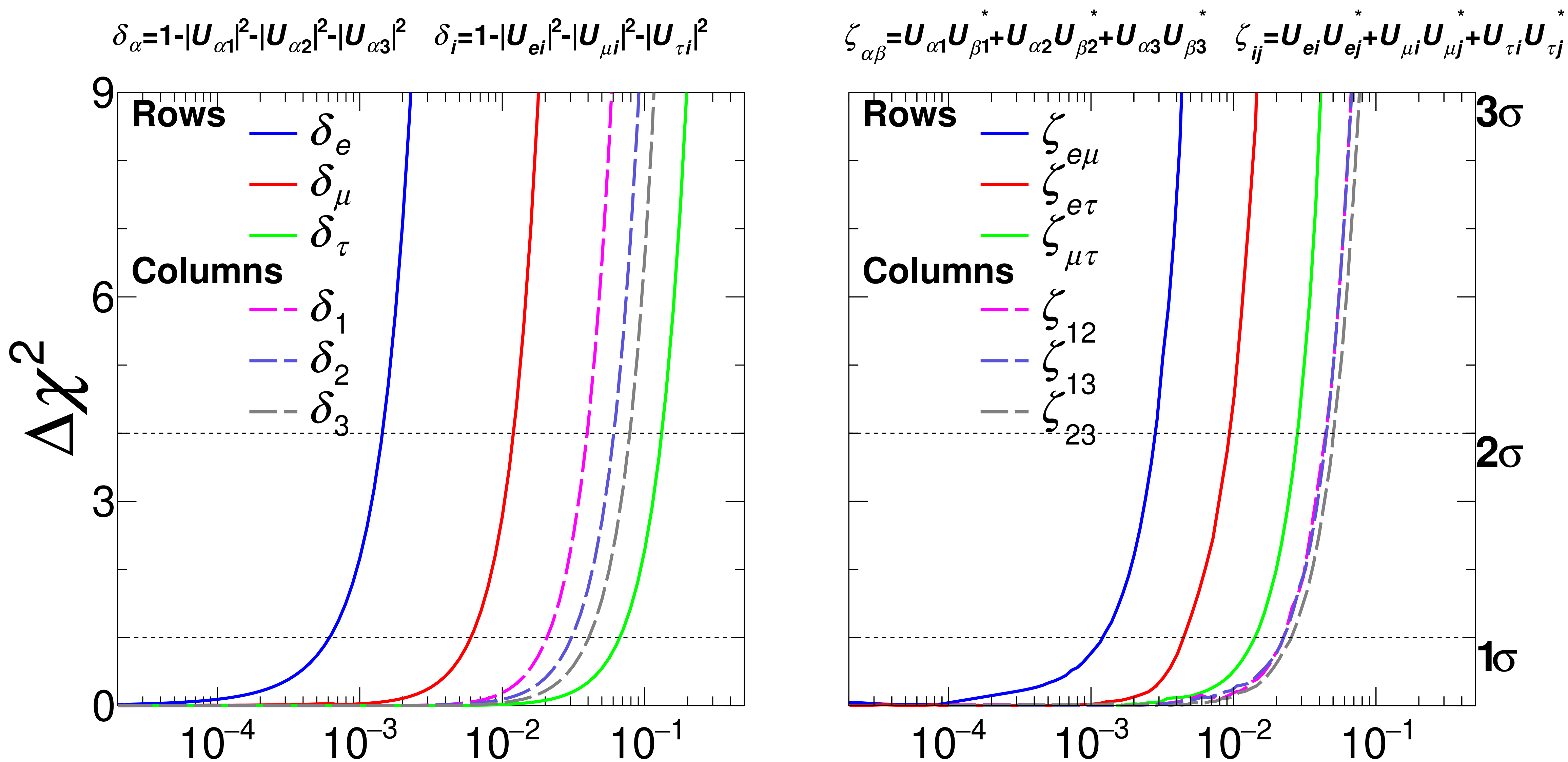
Our results seem to be consistent with Ellis et al

Non-Unitarity studies (in partucluar, normalisation NU effect), are dominated by “absolute” (not relative) FLUX knowledge

Not too surprising since most neutrino oscillation experiments use multi-detector to cancel this effect

Most Updated Constraints (2) by Hu et al, JHEP01, 124 (2021)

done based on the current neutrino data
which obtained stronger bounds beyond our expectation



~1 order of magnitude better than results by other works!

Due to different assumptions?

Can we improve more the NU bounds?

A. Cabrera and H. Nunokawa, work in progress

Let us adopt the alpha parameterisation and consider only electron row for now

non-unitary   unitary

$$V = \begin{bmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{bmatrix} \equiv \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_{11} U_{e1} & \alpha_{11} U_{e2} & \alpha_{11} U_{e3} \\ \alpha_{21} U_{e1} + \alpha_{22} U_{\mu 1} & \alpha_{21} U_{e2} + \alpha_{22} U_{\mu 2} & \alpha_{21} U_{e3} + \alpha_{22} U_{\mu 3} \\ \alpha_{31} U_{e1} + \alpha_{32} U_{\mu 1} + \alpha_{33} U_{\tau 1} & \alpha_{31} U_{e2} + \alpha_{32} U_{\mu 2} + \alpha_{33} U_{\tau 2} & \alpha_{31} U_{e3} + \alpha_{32} U_{\mu 3} + \alpha_{33} U_{\tau 3} \end{bmatrix}$$

$$|V_{e1}|^2 + |V_{e2}|^2 + |V_{e3}|^2 = \alpha_{11}^2 (|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2) = \alpha_{11}^2$$

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) = \alpha_{11}^4 \left[\left(\sum_{j=1}^{3} |U_{ej}|^2 \right)^2 - 4 \sum_{k>j}^{3} |U_{ej}|^2 |U_{ek}|^2 \sin^2 \left(\frac{\Delta m_{kj}^2}{4E} L \right) \right]$$

$$= \alpha_{11}^4 \left[1 - 4|U_{e2}|^2 |U_{e1}|^2 \sin^2 \left(\frac{\Delta m_{21}^2}{4E} L \right) - 4|U_{e3}|^2 |U_{e1}|^2 \sin^2 \left(\frac{\Delta m_{31}^2}{4E} L \right) - 4|U_{e3}|^2 |U_{e2}|^2 \sin^2 \left(\frac{\Delta m_{32}^2}{4E} L \right) \right]$$

$$= \alpha_{11}^4 P_{ee}^{(\mathrm{std})}$$ non-unitary effect manifests only as a overall normalisation

Let us focus on the NU normalisation parameter α_{11}

Since the number of events $\propto \phi_\nu \, \sigma_\nu \, \alpha_{11}^4 P_{ee}^{\rm std}$

error (uncertainty) for NU parameter is given by

$$\frac{\delta(\alpha_{11}^4)}{\alpha_{11}^4} \sim \delta(\alpha_{11}^4) \sim 1 - \alpha_{11}^4 \sim \left[\left(\frac{\delta \phi_\nu}{\phi_\nu} \right)^2 + \left(\frac{\delta \sigma_\nu}{\sigma_\nu} \right)^2 + \left(\frac{\delta P_{ee}^{(\rm std)}}{P_{ee}^{(\rm std)}} \right)^2 \right]^{1/2}$$

in the limit of large statistics

Even if we know cross section and $P_{ee}^{(\rm std)}$ very well

NU normalisation bound is limited by flux uncertainty as

$$1 - \alpha_{11}^2 = 1 - (|V_{e1}|^2 + |V_{e2}|^2 + |V_{e3}|^2) \lesssim \frac{1}{2} \frac{\delta \phi_\nu}{\phi_\nu}$$

For 3 (6)% flux uncertainty, unitarity would be at most constrained to be ~1.5 (3)% level

Fong et al, JHEP02, 114 (2017)

Looks consistent with our results and that by Parke and Ellis et al.

For the purpose of NU test, the problem of the uncertainty of the source neutrino flux seems to be rather complicated

Because, no matter how accurately the neutrino flux is measured by observing neutrinos, once we assume the possible presence of Non-Unitary effect, the flux can not be determined separately from the NU effect

Namely, we can not distinguish the following situations

number of events $\propto \alpha_{11}^4 \phi_\nu = \alpha_{11}'^4 \phi_\nu'$

where $\alpha_{11}'^4 = \alpha_{11}^4 (1+\epsilon)$ and $\phi_\nu' = \phi_\nu/(1+\epsilon)$

If we want to test NU (if α_{11} is 1 or not) we need to use some neutrino source whose flux is known/determined not by using neutrino!

a good example: ILL reactor neutrino flux was determined by the measured beta spectrum

Summary

● Testing unitarity is one way to go in order to verify the standard 3 flavor neutrino paradigm

● If unitarity is violated, it is a new discovery of BSM physics

● Current bounds on normalisation/closure which involve only electron and/or muon neutrino is ~1% level (see however, Hu et al work which obtained ~0.1% bound), but the normalisation bounds depend on the assumption of the flux uncertainty

● We need a new strategy using different neutrino source whose flux is well known and/or new detector (LiquidO?) which is currently under investigation, so stay tuned!

Backup

Oscillation Probabilities with matter effect

Assuming Non-Unitary effect (or W) is small, we can perform a perturbation theory

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2$$

$$- 2 \sum_{j \neq k} \mathrm{Re} \left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin^2 \frac{(h_k - h_j)x}{2}$$

$$- \sum_{j \neq k} \mathrm{Im} \left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin(h_k - h_j)x,$$

X: matrix which diagonalize zeroth order (in W) Hamiltonian

h_i (i=1,2,3): energy eigenvalue of zeroth order neutrino states in matter

we can compute also numerically by solving

$$i \frac{d}{dx} \nu_i = \sum_j \left(\mathbf{\Delta_a} + U^\dagger A U \right)_{ij} \nu_j .$$

vacuum Hamiltonian

matter Hamiltonian

See Fong et al, JHEP02, 015 (2019) for more details

Cauchy-Schwartz Inequaities

$$\left|\sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^{*}\right|^2 \leq \left(1 - \sum_{j=1}^{3} |U_{\alpha j}|^2\right) \left(1 - \sum_{j=1}^{3} |U_{\beta j}|^2\right)$$

for $\alpha, \beta = e, \mu, \tau \ (\alpha \neq \beta)$

$$\left|\sum_{\alpha=e}^{\tau} U_{\alpha i} U_{\alpha j}^{*}\right|^2 \leq \left(1 - \sum_{\alpha=e}^{\tau} |U_{\alpha i}|^2\right) \left(1 - \sum_{\alpha=e}^{\tau} |U_{\alpha j}|^2\right)$$

for $i, j = 1, 2, 3 \ (i \neq j)$

Neutrino Evolution in Matter with Non-Unitarity case

$$H = \mathbf{U}\begin{bmatrix} \Delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{3+N} \end{bmatrix}\mathbf{U}^\dagger$$

(3+N)x(3+N)

3x3

$$\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

$$\Delta_i \equiv \frac{m_i^2}{2E} \quad (i = 1, 2, 3), \qquad \Delta_J \equiv \frac{m_J^2}{2E} \quad (J = 4, \cdots, 3+N)$$

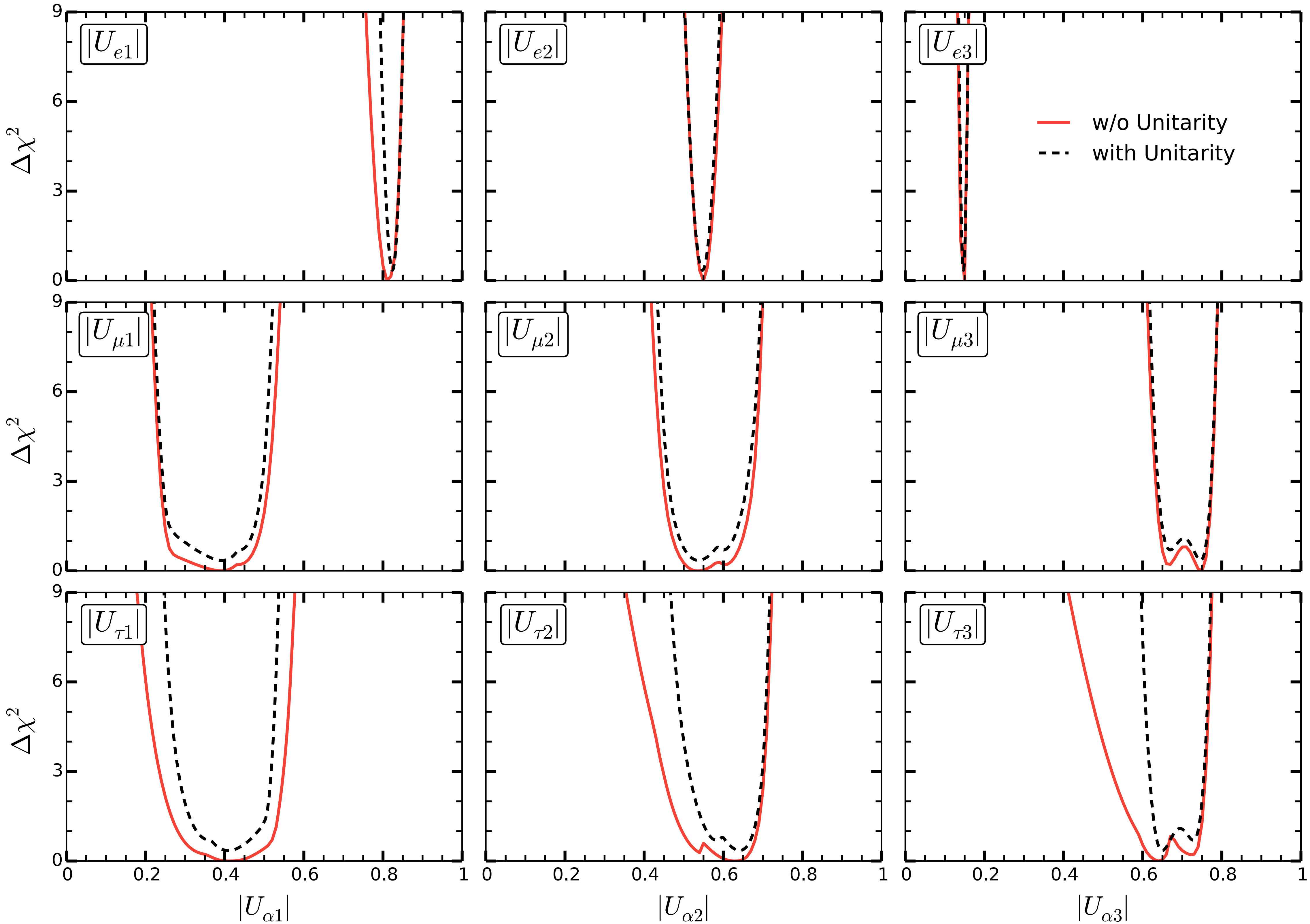
$$i\frac{d}{dx}\begin{bmatrix} \tilde{\nu}_i \\ \tilde{\nu}_J \end{bmatrix} = \begin{bmatrix} \mathbf{\Delta_a} + U^\dagger A U & U^\dagger A W \\ W^\dagger A U & \mathbf{\Delta_s} + W^\dagger A W \end{bmatrix}\begin{bmatrix} \tilde{\nu}_i \\ \tilde{\nu}_J \end{bmatrix} \qquad \tilde{\nu}_z = (\mathbf{U}^\dagger)_{z\zeta}\nu_\zeta$$

For zeroth order in W,

$$i\frac{d}{dx}\nu_i = \sum_j \left(\mathbf{\Delta_a} + U^\dagger A U\right)_{ij} \nu_j$$

Current limits on Unitarity

Parke and Ross-Lonergan, PRD93, 113009 (2016)

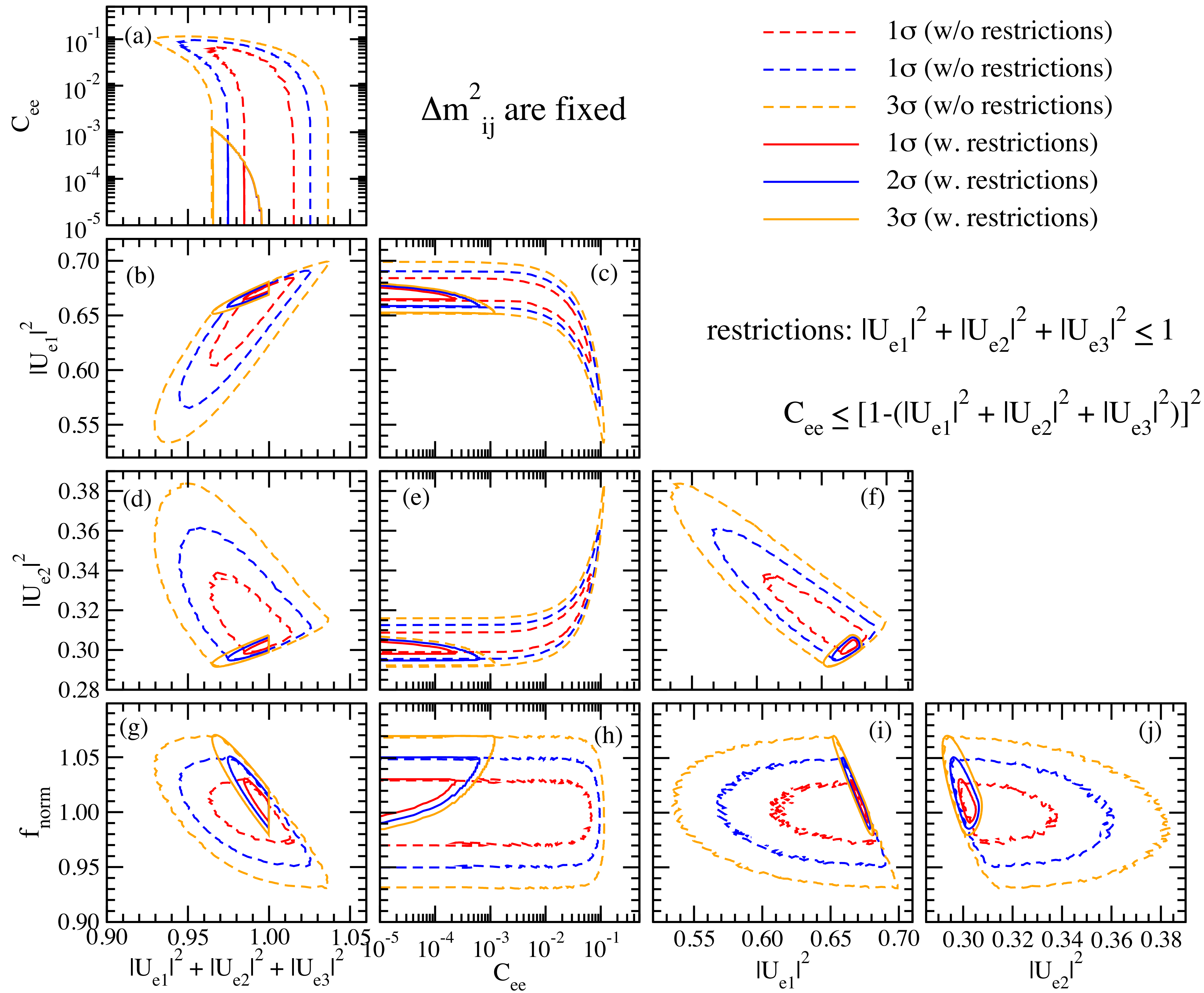


Current limits on Unitarity

Parke and Ross-Lonergan, PRD93, 113009 (2016)

they used exactly the same formulas without $\mathcal{C}_{\alpha\beta}$ term

$$|U|_{3\sigma}^{\substack{\textbf{w/o Unitarity}\\ \text{(with Unitarity)}}} = \begin{pmatrix} \underset{(0.79\to 0.85)}{0.76 \to 0.85} & \underset{(0.50\to 0.59)}{0.50 \to 0.60} & \underset{(0.14\to 0.16)}{0.13 \to 0.16} \\ \underset{(0.22\to 0.52)}{0.21 \to 0.54} & \underset{(0.43\to 0.70)}{0.42 \to 0.70} & \underset{(0.62\to 0.79)}{0.61 \to 0.79} \\ \underset{(0.24\to 0.54)}{0.18 \to 0.58} & \underset{(0.47\to 0.72)}{0.38 \to 0.72} & \underset{(0.60\to 0.77)}{(0.40 \to 0.78)} \end{pmatrix}.$$



Fong et al, JHEP02, 114 (2017)

Computation of the evetn number

$$\frac{dN(E_{\rm vis})}{dE_{\rm vis}} = n_p t_{\rm exp} \int_{m_e}^{\infty} E_e \int_{E_{\rm min}}^{\infty} dE \sum_{i={\rm reac,geo}-\nu} \frac{d\phi_i(E)}{dE} \epsilon_{\rm det}(E_e) \frac{d\sigma(E_\nu, E_e)}{dE_e}$$

$$\times P_i(\bar{\nu}_e \to \bar{\nu}_e; L_i, E) R(E_e, E_{\rm vis})$$

spectrum: Mueller et al PRC83, 054615 (2011)

cross section: Strumia & Vissani PLB564, 42 (2003)

$$R(E_e, E_{\rm vis}) \equiv \frac{1}{\sqrt{2\pi}\sigma(E_e)} \exp\left[-\frac{1}{2}\left(\frac{E_e + m_e - E_{\rm vis}}{\sigma(E_e)}\right)^2\right]$$

For LPMT we assume

$$\frac{\sigma(E_e)}{(E_e + m_e)} = \frac{3\%}{\sqrt{(E_e + m_e)/{\rm MeV}}}$$

For SPMT we assume

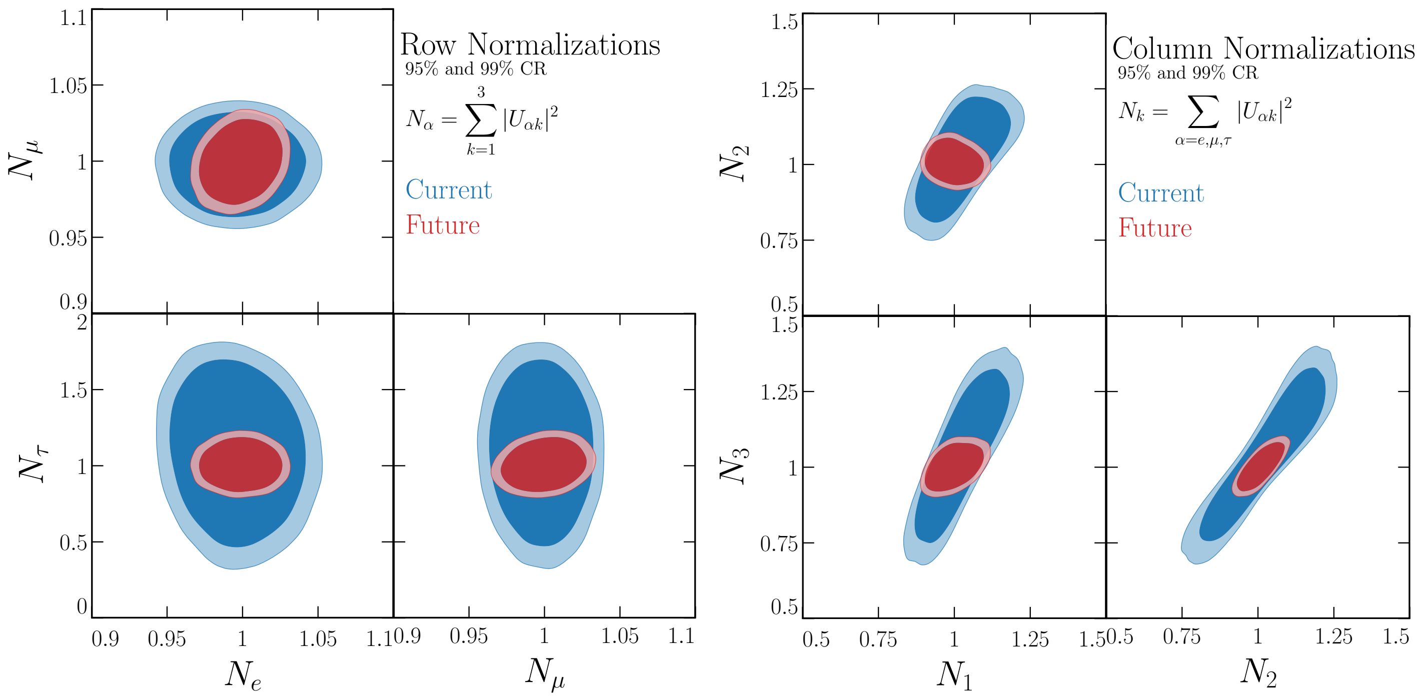
$$\frac{\sigma(E_e)}{(E_e + m_e)} = \frac{X}{\sqrt{(E_e + m_e)/{\rm MeV}}}$$

For simplicity, we assume 100% detection effiency

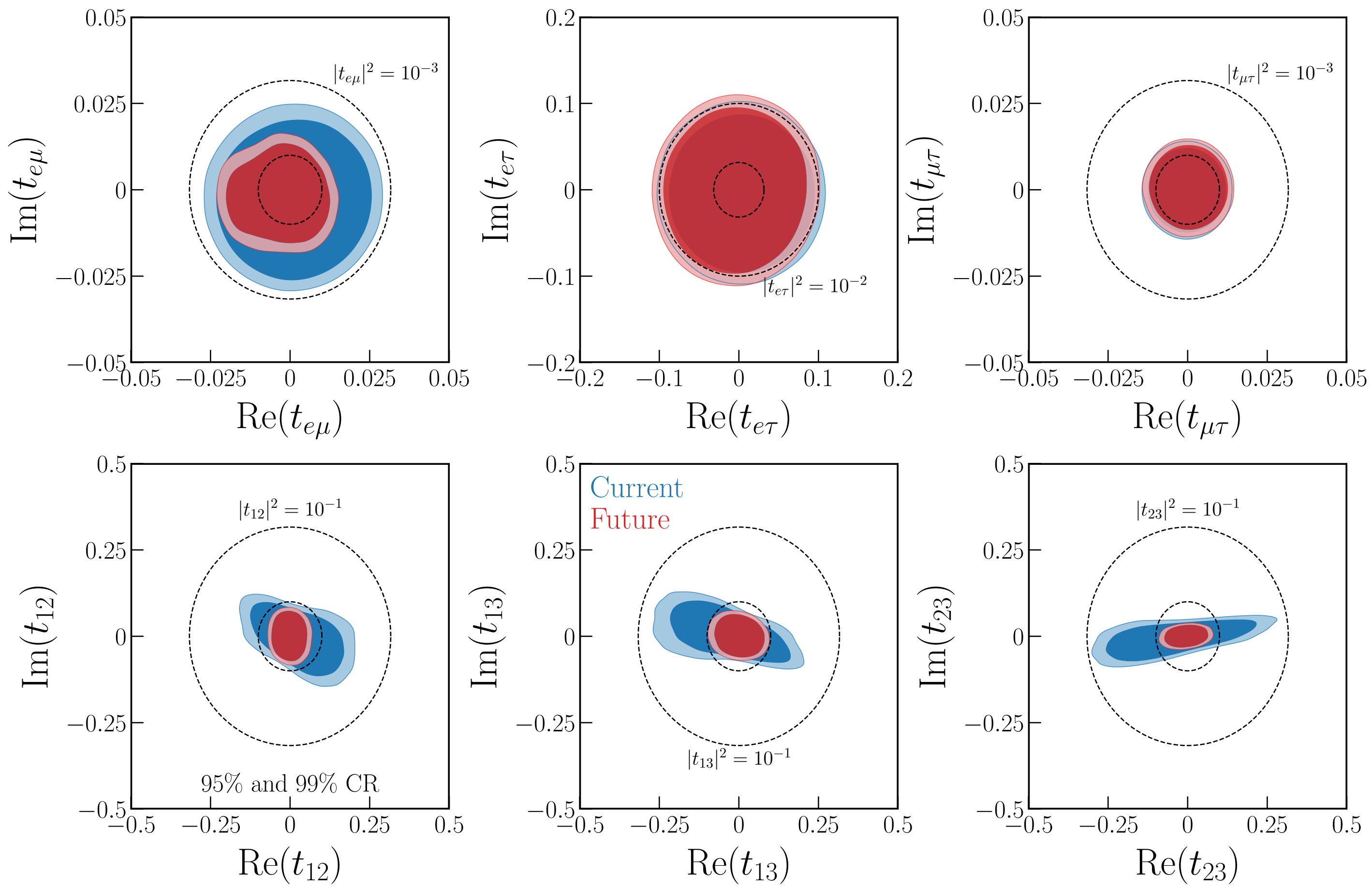
$$\epsilon_{\rm det} = 100\%$$

Fong et al, JHEP02, 114 (2017)

Current vs Future by Ellis et al, JHEP12, 068 (2020)



Normalisation



Closure

	Current 3σ Upper Limit	Future 3σ Upper Limit
$\lvert t_{e\mu}\rvert$	8.2×10^{-1}	3.0×10^{-2}
$\lvert t_{e\tau}\rvert$	1.0×10^{0}	8.1×10^{-1}
$\lvert t_{\mu\tau}\rvert$	6.8×10^{-1}	3.2×10^{-1}
$\lvert t_{12}\rvert$	8.2×10^{-1}	5.5×10^{-1}
$\lvert t_{13}\rvert$	1.1×10^{0}	5.2×10^{-1}
$\lvert t_{23}\rvert$	8.7×10^{-1}	2.7×10^{-1}

above results do not reflect the results below

Search	90% CL Limit	Angle Constrained	Unitarity Constraint
KARMEN $\overline{\nu}_\mu \to \overline{\nu}_e$ [85]	1.8×10^{-3}	$\sin^2(2\theta_{\mu e})$	$\lvert t_{\mu e}\rvert^2$
NOMAD $\nu_\mu \to \nu_e$ [86]	1.4×10^{-3}	$\sin^2(2\theta_{\mu e})$	$\lvert t_{\mu e}\rvert^2$
NOMAD $\nu_e \to \nu_\tau$ [87]	1.5×10^{-2}	$\sin^2(2\theta_{e\tau})$	$\lvert t_{e\tau}\rvert^2$
NOMAD $\nu_\mu \to \nu_\tau$ [87]	3.3×10^{-4}	$\sin^2(2\theta_{\mu\tau})$	$\lvert t_{\mu\tau}\rvert^2$
CHORUS $\nu_\mu \to \nu_\tau$ [88]	4.4×10^{-4}	$\sin^2(2\theta_{\mu\tau})$	$\lvert t_{\mu\tau}\rvert^2$
MINOS/MINOS+ [89] $\nu_\mu \to \nu_\mu$	2.5×10^{-2}	$\sin^2(2\theta_{\mu\mu})$	N_μ^2

by Ellis et al, JHEP12, 068 (2020)

Summary of the constraints from decays (90% CL)

$$|NN^\dagger| \approx \begin{pmatrix} 1.002 \pm 0.005 & < 7.2 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.2 \cdot 10^{-5} & 1.003 \pm 0.005 & < 1.3 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.3 \cdot 10^{-2} & 1.003 \pm 0.005 \end{pmatrix} .$$

$$|N^\dagger N| \approx \begin{pmatrix} 1.00 \pm 0.032 & < \ 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix} .$$

Antusch et al, JHEP10, 084 (2006)

$UU^\dagger = 1$ and $U^\dagger U = 1$

are mathematically equivalent

$$UU^\dagger = 1 \rightarrow U(U^\dagger U) = U \times 1$$
$$\rightarrow U^{-1}U(U^\dagger U) = U^{-1}U$$
$$\rightarrow U^\dagger U = 1$$

Test of Unitarity

check if $UU^\dagger = 1$