

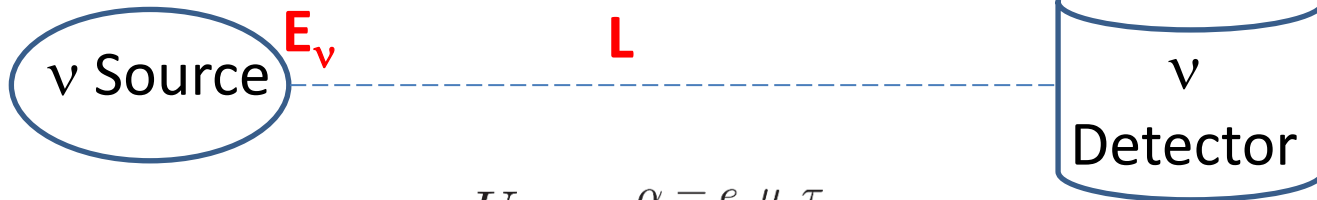
# Nuclear models for long-baseline neutrino oscillation experiments and neutrinoless double beta decay : synergies

Marco Martini

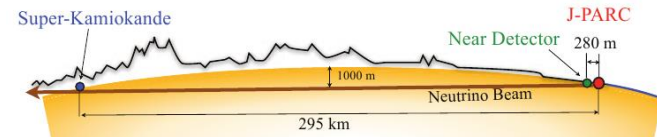


# Neutrino oscillation experiments

$$\nu_\alpha \rightarrow \nu_\beta$$



$$\nu_\alpha = U_{\alpha i} \nu_i \quad \begin{matrix} \alpha = e, \mu, \tau \\ i = 1, 2, 3 \end{matrix}$$



Number of detected events

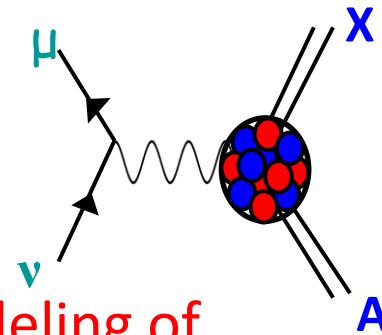
$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \rightarrow \nu_\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu_\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

Reconstructed  $\nu$  energy



## Modern accelerator-based neutrino oscillation experiments:

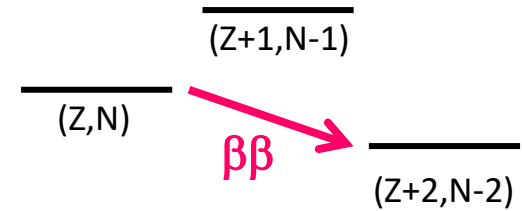
- Nuclear targets (C, O, Ar, Fe...)
- Neutrino energy is reconstructed



the knowledge and the modeling of neutrino-nucleus cross sections is crucial

# Neutrinoless $\beta\beta$ decay

Double beta decay occurs between even-even nuclei  
bypassing the intermediate, more massive, odd-odd nucleus

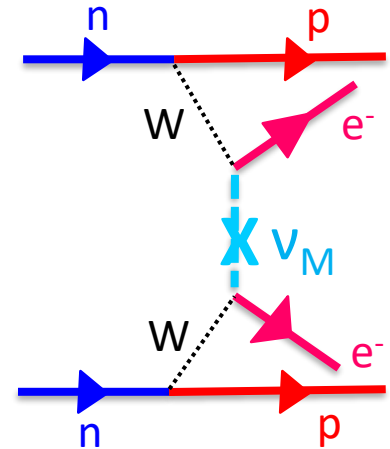


$$A(Z, N) \rightarrow A(Z + 2, N - 2) + 2e^- + 2\bar{\nu}_e \quad 2\nu\beta\beta \text{ decay already observed}$$

If neutrinos are their own antiparticles (Majorana particles) one can also observe:

$$A(Z, N) \rightarrow A(Z + 2, N - 2) + 2e^- \quad 0\nu\beta\beta \text{ decay}$$

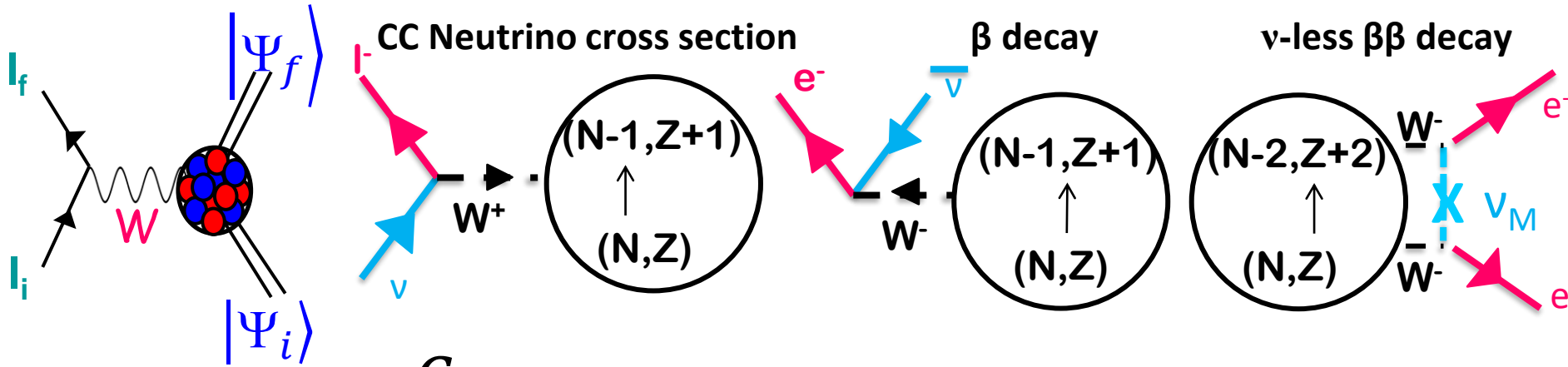
Important experimental program



$$\left[ \tau_{1/2}^{0\nu} \right]^{-1} = G_{0\nu} \left| M_{0\nu} \right|^2 |\langle m_{\nu} \rangle|^2$$

- The rate of any kind of  $\beta\beta$  decay depends on **nuclear matrix elements** and decisions about which and how much material to use in the experiments rely on our ability to calculate them accurately
- If  $0\nu\beta\beta$  decay is actually observed, the **nuclear matrix elements** will play a key role in extracting information about neutrino masses

# Generalities



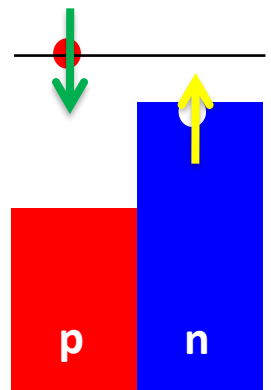
$$\mathcal{L}_W = \frac{G}{\sqrt{2}} l^\mu h_\mu \quad d\Gamma, d\sigma \propto L_{\mu\nu} W^{\mu\nu}$$

$$d\Gamma, d\sigma \sim (\text{leptonic factors}) \underbrace{|\langle \Psi_f | \hat{O} | \Psi_i \rangle|^2}_{\text{Nuclear matrix elements}}$$

Example of nuclear excitation operator:  
Gamow Teller

$$\hat{O}_{GT} = \sum_{i=1}^A \vec{\sigma}(i) \tau_-(i)$$

$\Delta S=1 \quad \Delta T=1$



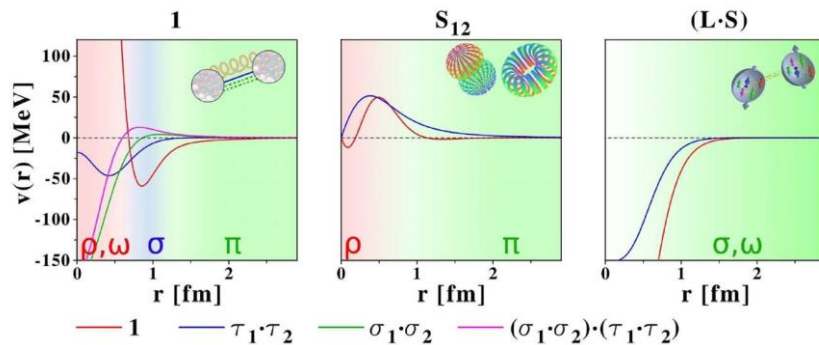
Nuclear matrix elements

$$d\Gamma, d\sigma \sim (\text{leptonic factors}) \left| \langle \Psi_f | \hat{O} | \Psi_i \rangle \right|^2$$

- If we experimentally know the nuclear matrix elements: very good!
- Otherwise we need nuclear models

## Two main “problems” of nuclear physics

- The nuclear interaction
- The nucleus is a many body system



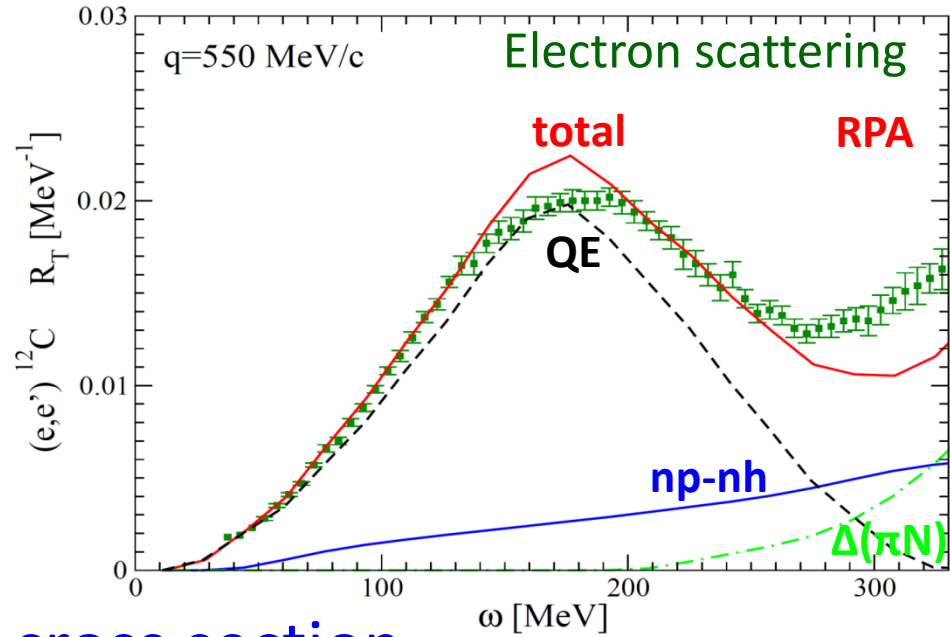
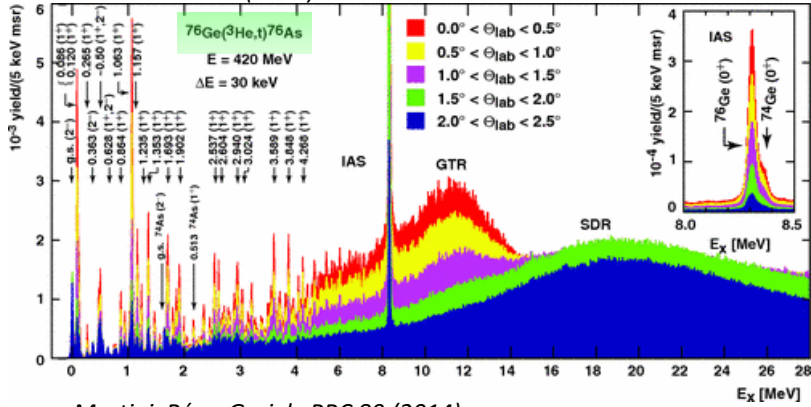
$$H = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i < j=1}^A V_{ij} + \sum_{i < j < k=1}^A V_{ijk} + \dots$$

Different Nuclear forces, Different Many-Body methods, Different degrees of freedom (quarks, mesons, nucleons) depending on the domains one is interested in

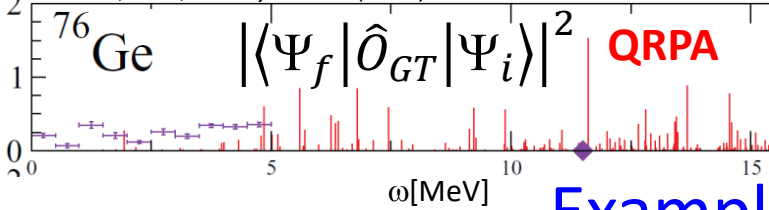
## Calculating Matrix Elements is hard

# Examples of experimental and theoretical matrix elements

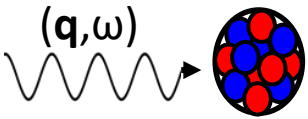
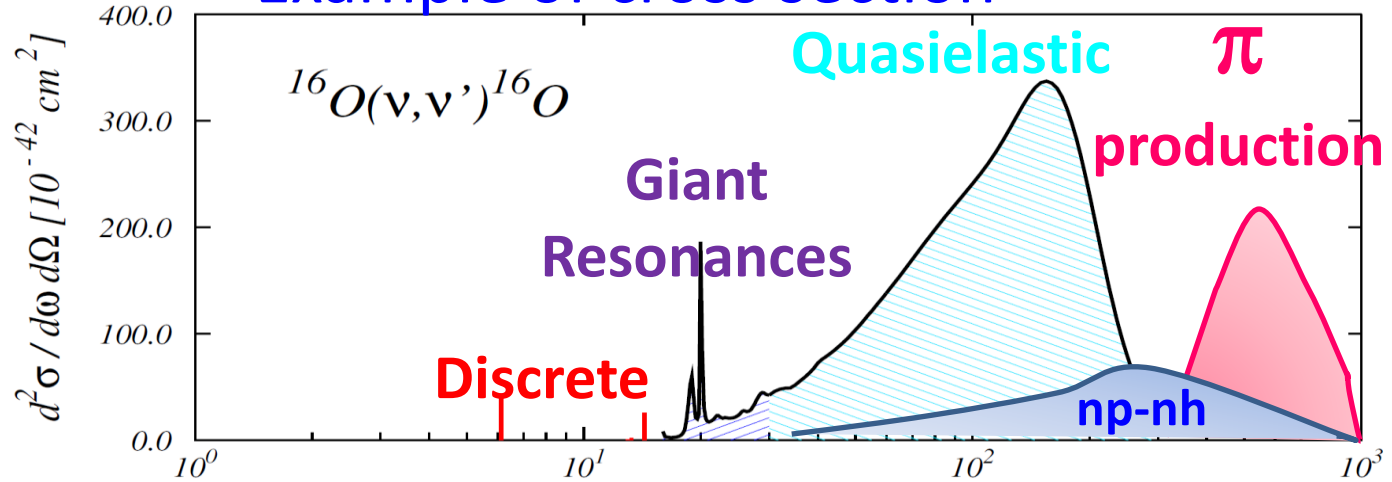
Thies et al. PRC 86 (2012)



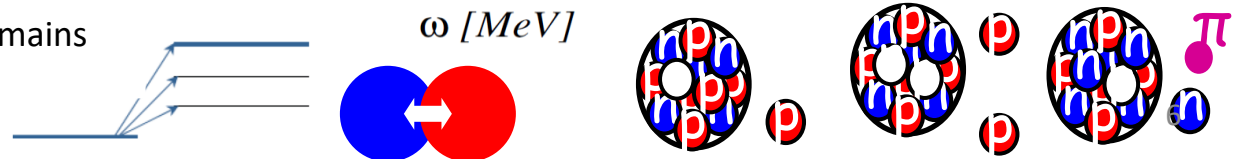
Martini, Péru, Goriely PRC 89 (2014)



## Example of cross section



- Different energy and momentum domains
- Different Nuclear models
- Different Nuclei



# Prelude: Two “Axial” puzzles

# I) The $M_A$ puzzle in the CC Quasielastic $\nu$ cross section on Carbon

$$\frac{\partial^2 \sigma}{\partial \Omega \partial \epsilon'} = \frac{G_F^2 \cos^2 \theta_c k' \epsilon' \cos^2 \frac{\theta}{2}}{2 \pi^2} \left[ \frac{(q^2 - \omega^2)^2}{q^4} G_E^2 R_\tau + \frac{\omega^2}{q^2} G_A^2 R_{\sigma\tau(L)} + 2 \left( \tan^2 \frac{\theta}{2} + \frac{q^2 - \omega^2}{2q^2} \right) \left( G_M^2 \frac{\omega^2}{q^2} + G_A^2 \right) R_{\sigma\tau(T)} \pm 2 \frac{\epsilon + \epsilon'}{M_N} \tan^2 \frac{\theta}{2} G_A G_M R_{\sigma\tau(T)} \right]$$

Nuclear dynamics  $\rightarrow$  Nuclear Response Functions  $R(q, \omega) \leftrightarrow$  Nuclear Matrix elements

Nucleon properties  $\rightarrow$  Form factors: Electric  $G_E$ , Magnetic  $G_M$ , Axial  $G_A$

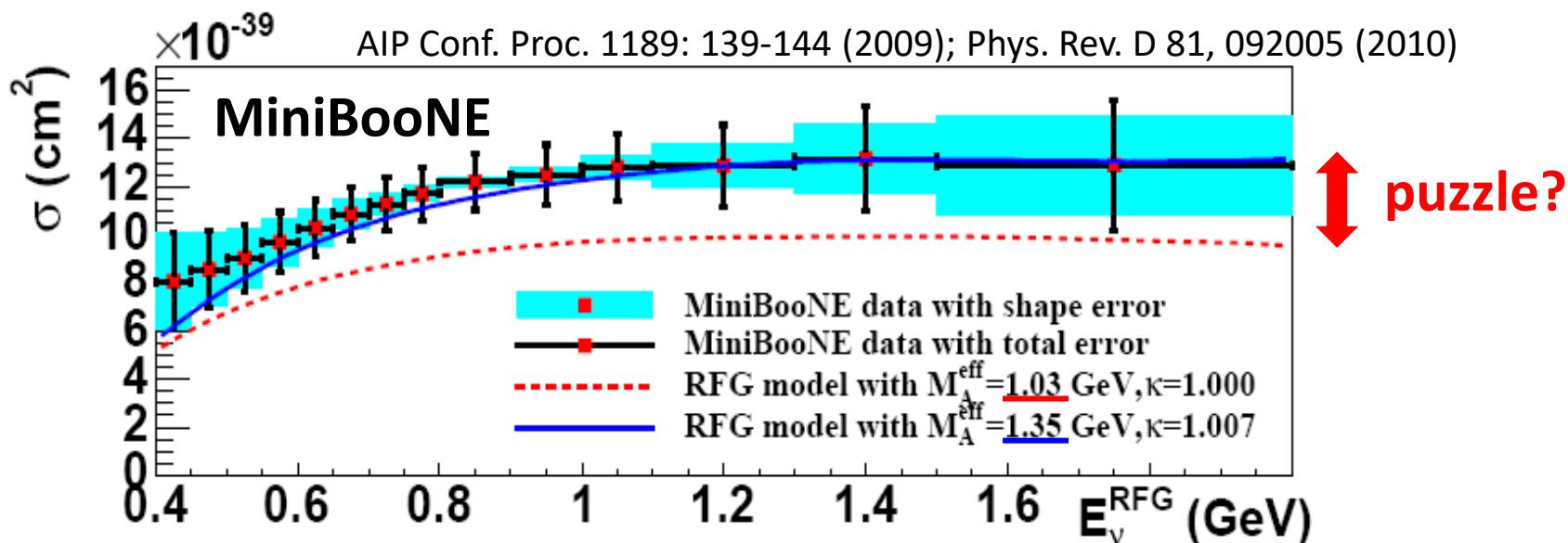
Axial form factor:

$$G_A(Q^2) = g_A (1 + Q^2 / M_A^2)^{-2}$$

$g_A = 1.26$   
from neutron  $\beta$  decay

$$M_A = (1.026 \pm 0.021) \text{ GeV} / c^2$$

from  $\nu$ -deuterium CCQE and  
from  $\pi$  electroproduction

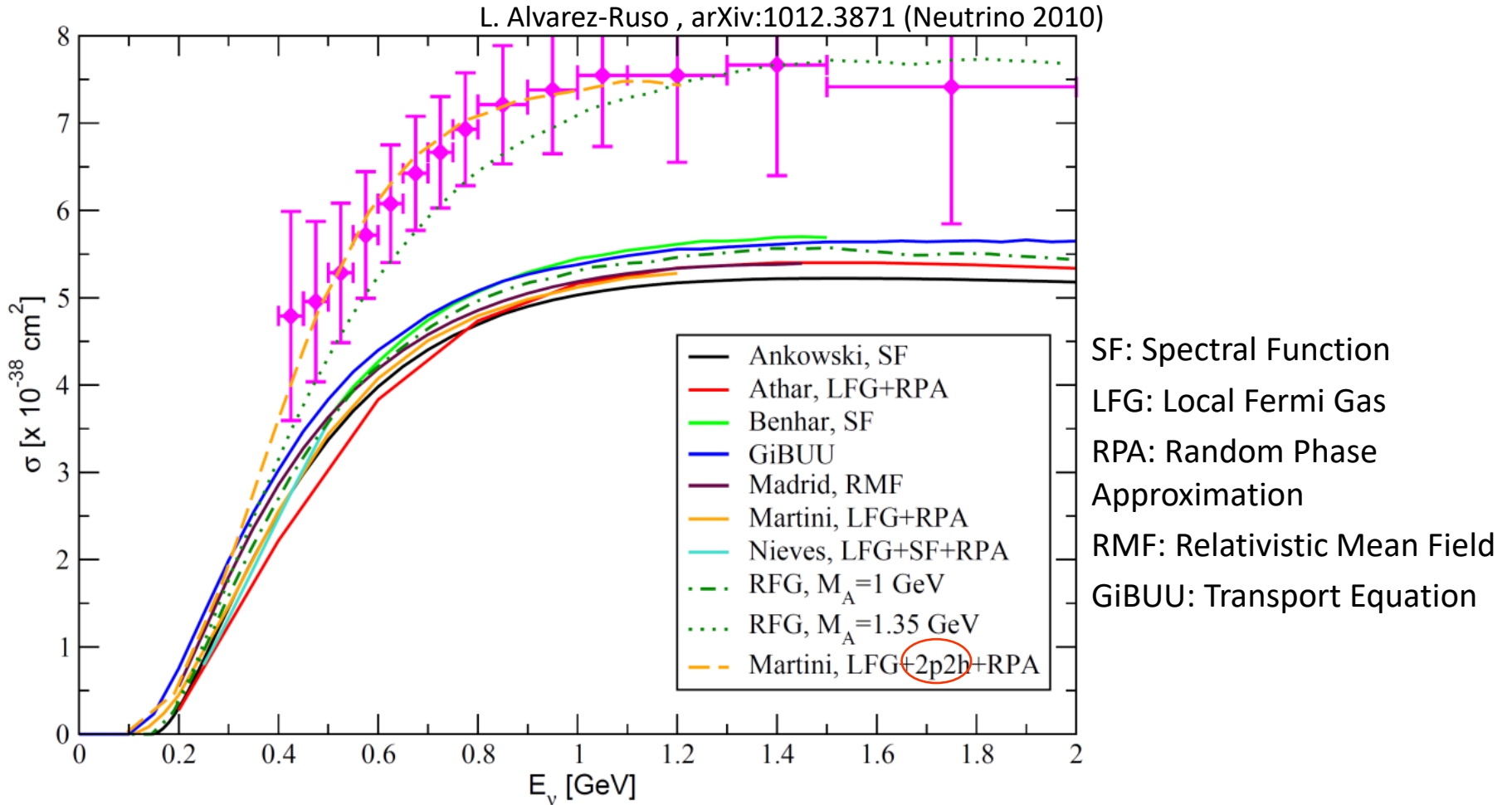


Comparison with a prediction based on RFG using  $M_A = 1.03$  GeV reveals a discrepancy

In the Relativistic Fermi Gas (RFG) model an axial mass of 1.35 GeV is needed to account for data



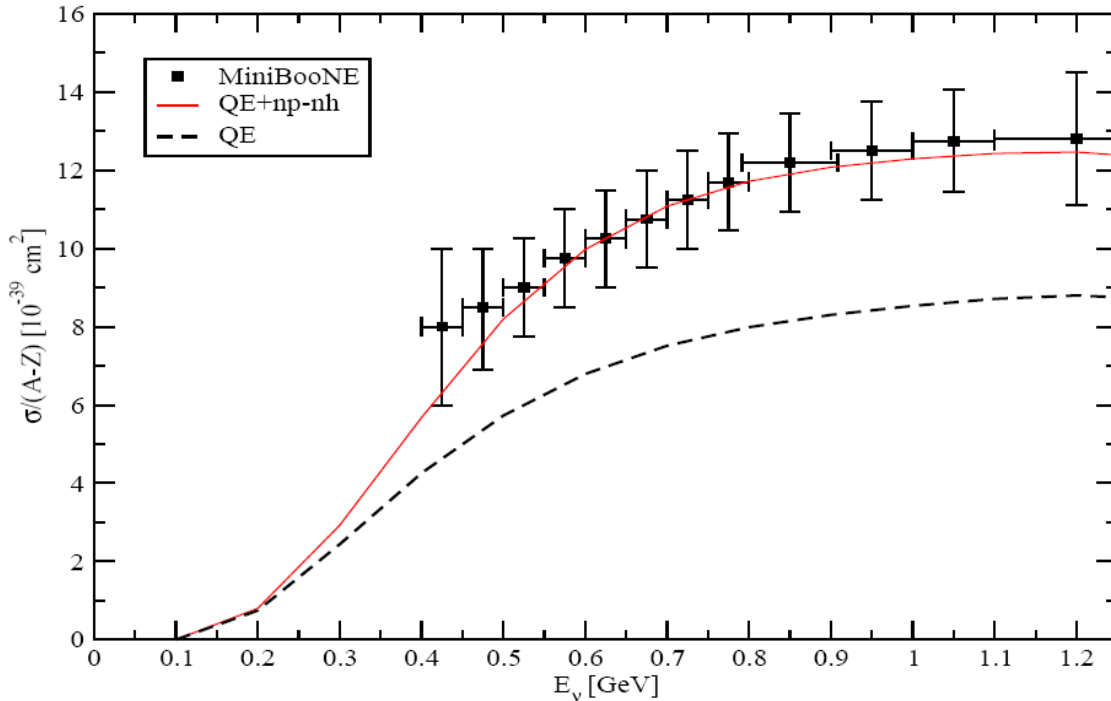
# Comparison of different theoretical models for Quasielastic



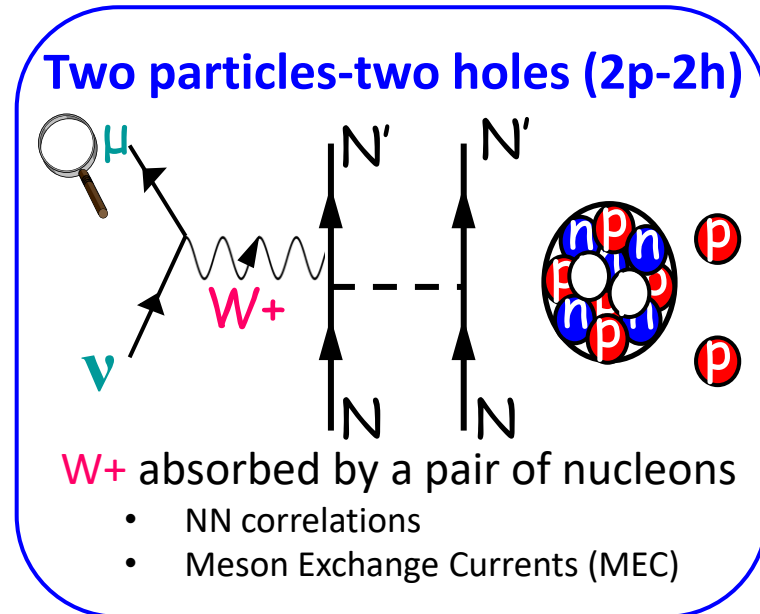
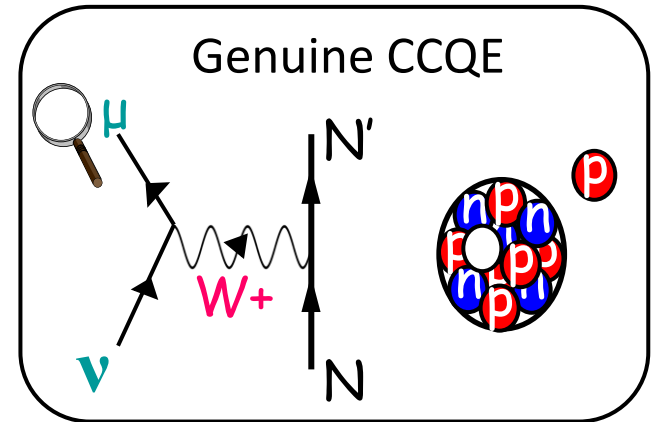
**puzzle??**

# An explanation of this puzzle

Inclusion of the multinucleon emission channel  
( $np-nh = 2p-2h + 3p-3h$ )



**CCQE-like** = Genuine CCQE +  $np-nh$

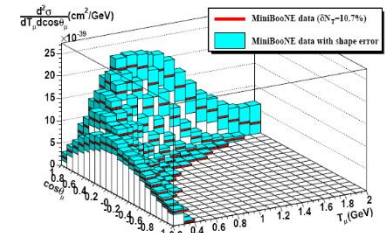
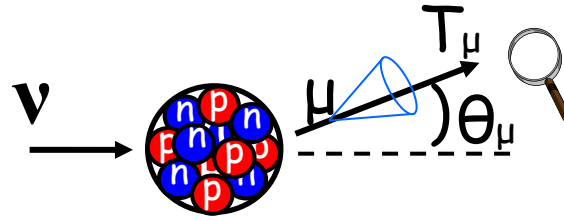


*M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009)*

**Agreement with MiniBooNE without increasing  $M_A$**

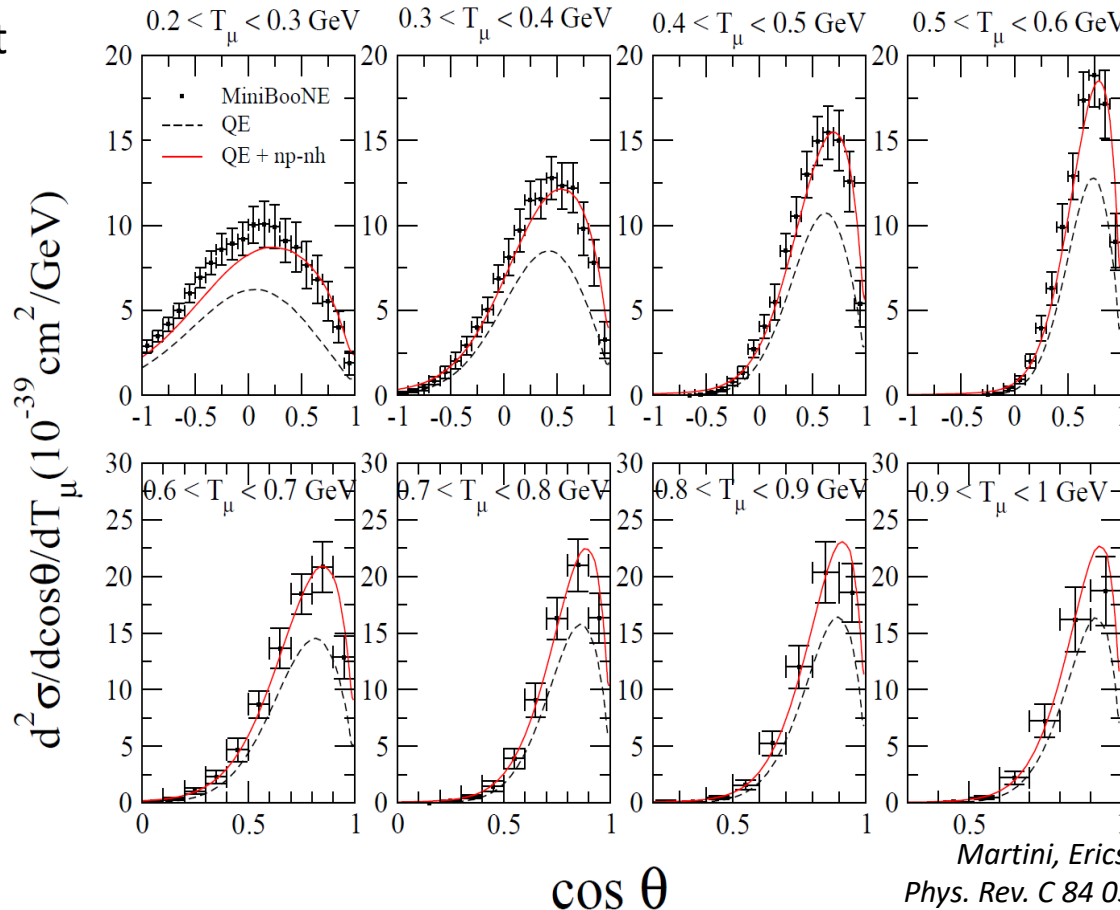
# CCQE-like flux-integrated double differential cross section

$$\frac{d^2\sigma}{dE_\mu d\cos\theta} = \int dE_\nu \left[ \frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu} \Phi(E_\nu)$$



MiniBooNE, *Phys. Rev. D* 81, 092005 (2010)

- Less model dependent than  $\sigma(E_\nu)$
- Flux dependent

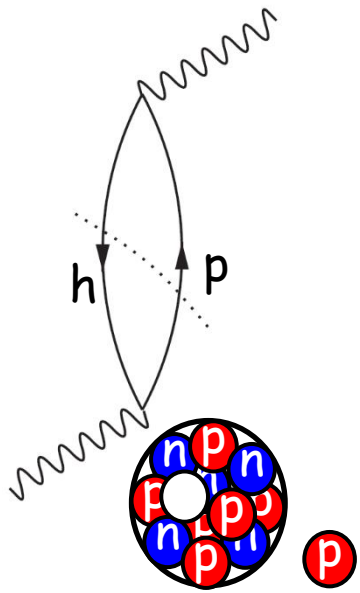


Martini, Ericson, Chanfray, *Phys. Rev. C* 84 055502 (2011)

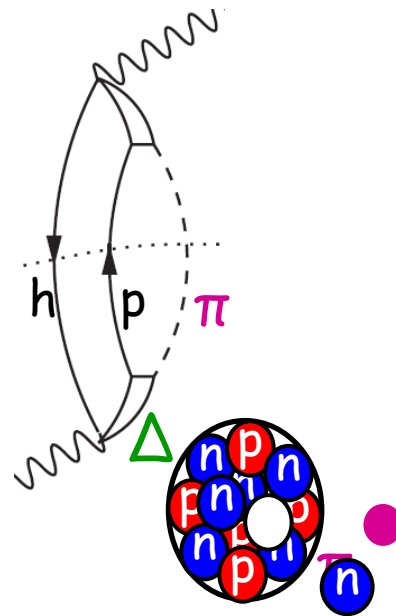
- Good agreement with data without increasing  $M_A$  by including NN correlations and MEC 11
- Similar conclusions by other approaches including the same contributions (discussed later)

# Sketch of the model: nuclear response function in RPA

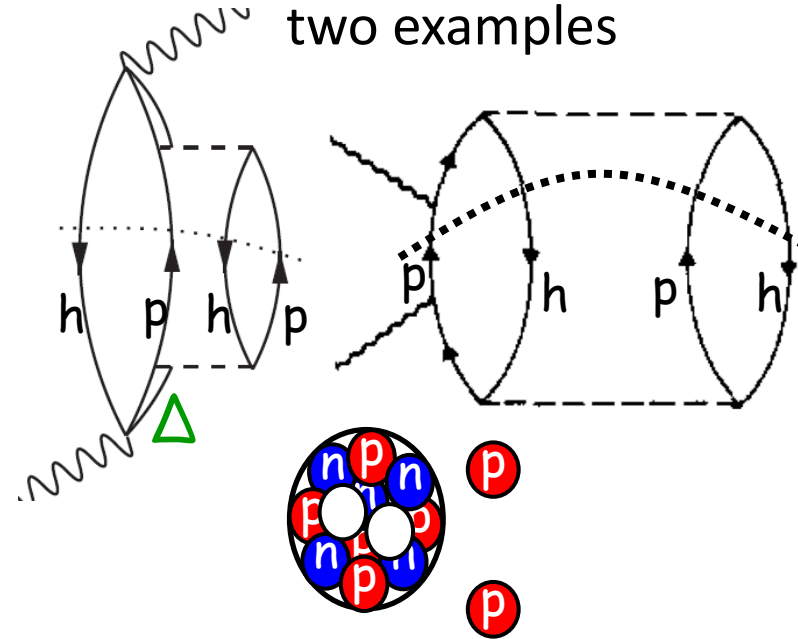
1p-1h QE



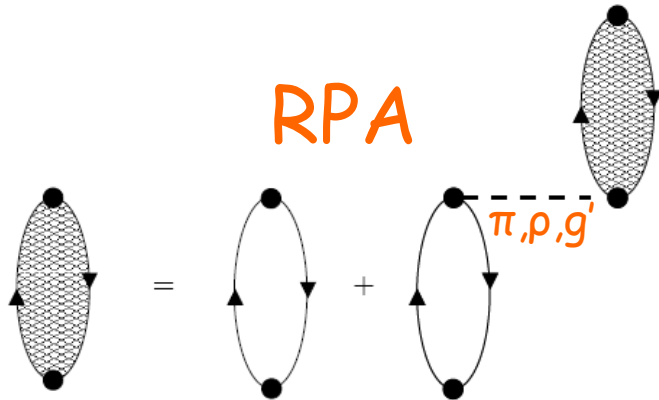
1p-1h  $1\pi$  production



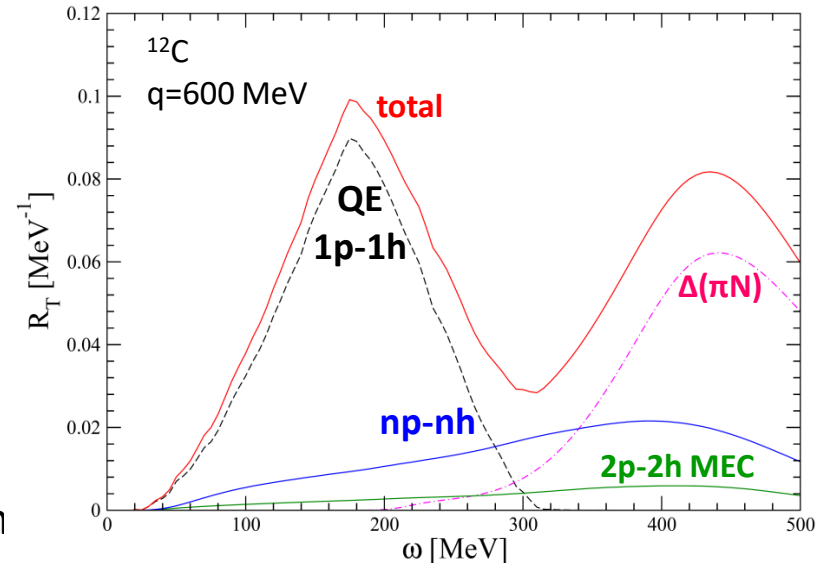
np-nh:  
two examples



RPA



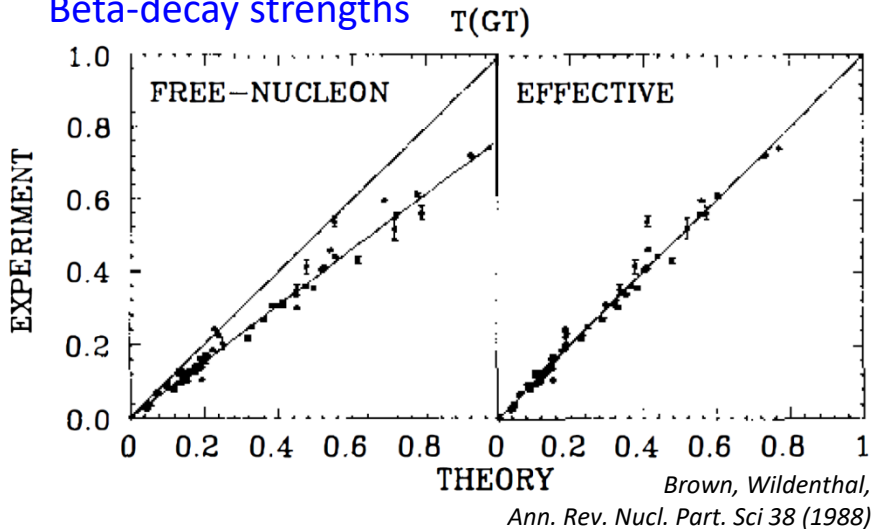
- Unified description of several channels
- Phenomenological many-body approach



# “Axial” puzzle II) The quenching of $g_A$

**Fifty-year old problem:** Theoretical over-prediction of GT Matrix elements and related observables [single-beta rates, (p,n)-nucleus cross sections,  $2\nu$  double-beta rates]

## Beta-decay strengths



$$[\tau_{1/2}^{\beta}]^{-1} \propto g_A^2 |M_{GT}^{\beta}|^2$$

Typical practice: “Renormalize”  $g_A$  to get correct results

$$g_A = 1.26 \quad \rightsquigarrow \quad g_A^{\text{eff}} \sim 1$$

N.B. if  $g_A$  is renormalized in  $0\nu\beta\beta$  decay  
 $\Rightarrow$  experiments are in trouble

$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu} |M_{0\nu}|^2 |\langle m_{\nu} \rangle|^2$$

$$M_{0\nu} = g_A^2 M^{(0\nu)} \quad \text{rates go as } (g_A)^4$$

## Several suggestions about the sources of the quenching of the GT strength:

- Nucleons are effective degrees of freedom; non nucleonic degrees of freedom (e.g.  $\Delta$ ) are omitted
- Two-body currents contributions
- Short range correlations between nucleons
- Truncation of the many-nucleon Hilbert space

**No consensus until very recent ab initio results**

# Parenthesis: Nuclear Interaction and Many Body Physics

Nuclear forces are usually associated to a family of Many-Body methods and they do not work well (or at all) if applied outside their domain

## Two different approaches

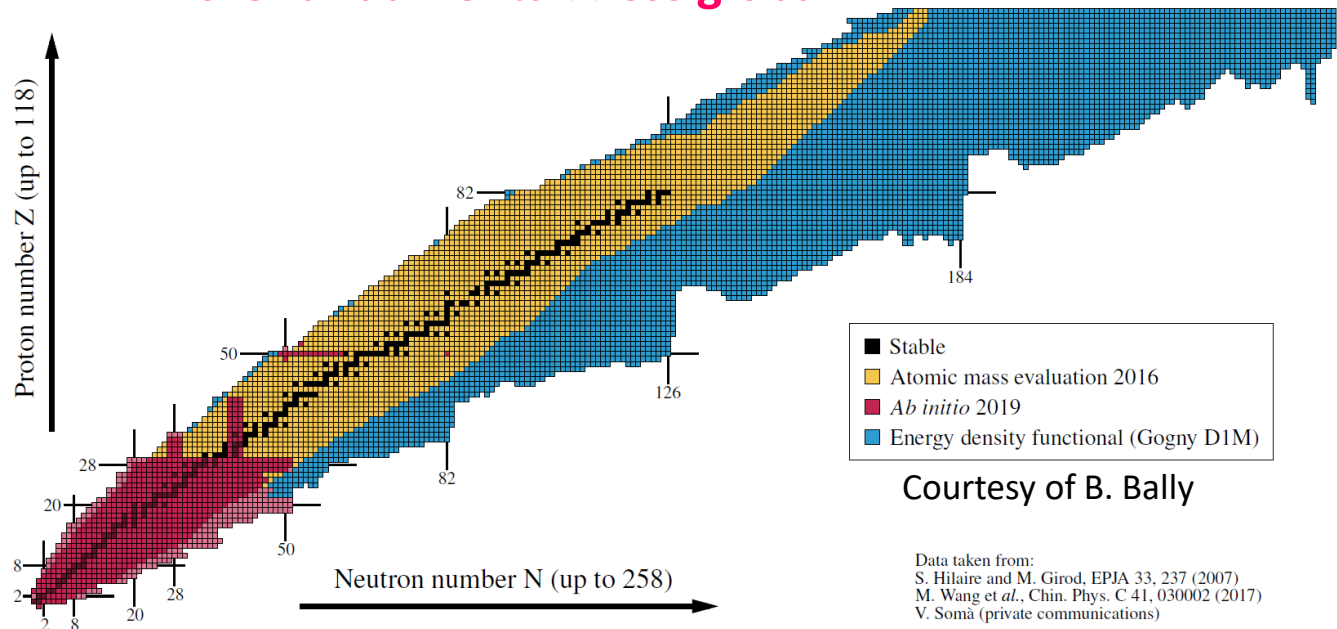
### 1 Realistic (more fundamental):

- Postulate analytical form of the nucleon-nucleon force “*in the vacuum*” and adjust to nucleon scattering data, deuteron (and very light nuclei)
- Solve the many-body problem with this bare nucleon-nucleon interaction (**ab initio methods**)

### 2. Effective (more phenomenological):

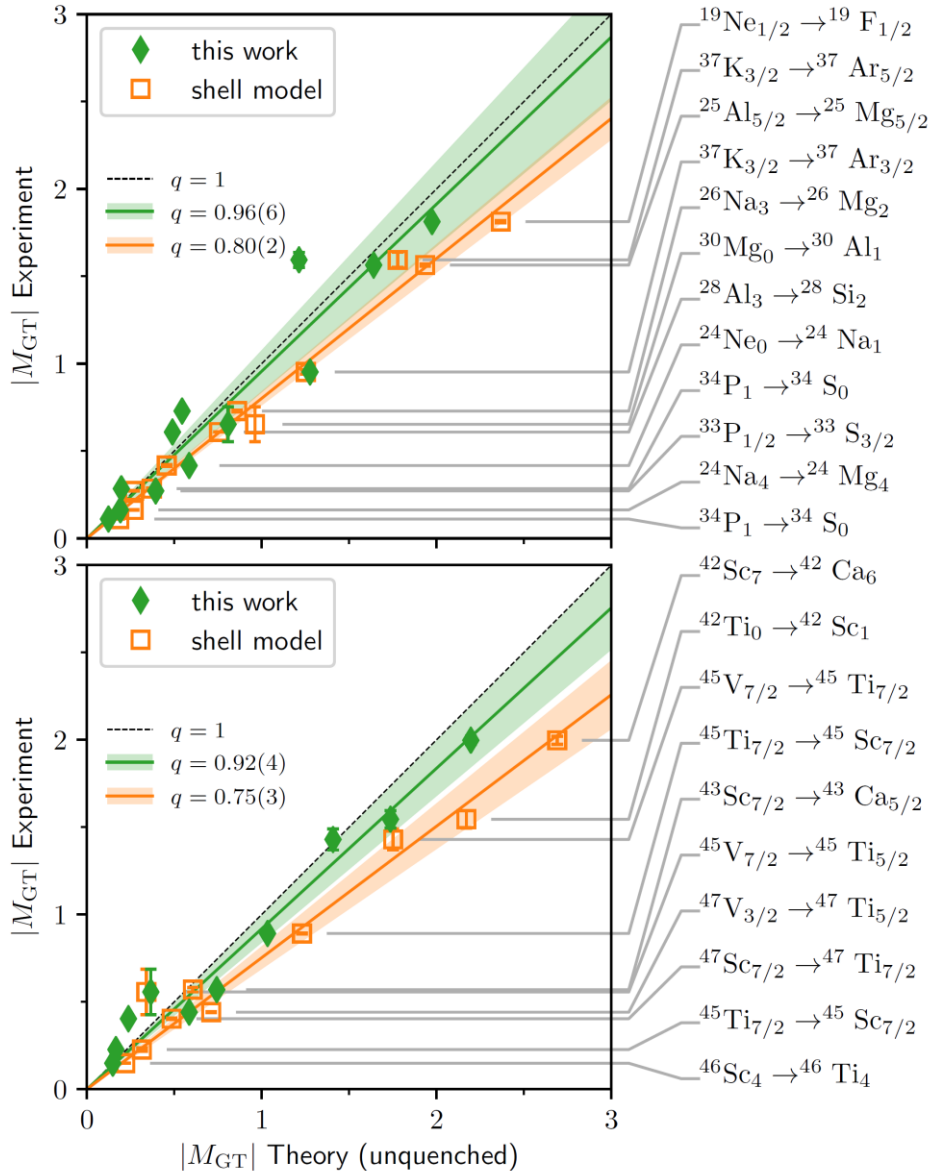
- Choose a Many-Body methods, postulate analytical form of the force “*in the nuclear medium*” and adjust to some nuclear properties (nuclear matter and nuclei)
- Use the same Many-Body method and this « in medium » nucleon – nucleon interaction to study other nuclei and other nuclear properties

More fundamental...less global

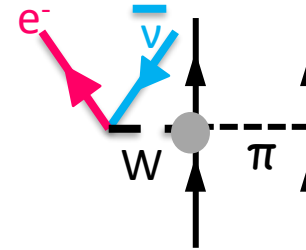


# The solution of the “ $g_A$ quenching” puzzle

Gysbers et al. Nature Phys. 15 428 (2019)



Ab initio calculations using chiral EFT including **two-body Meson-Exchange Currents** and additional **nuclear correlations** do not need any “quenching” to reproduce the  $\beta$  decay matrix elements



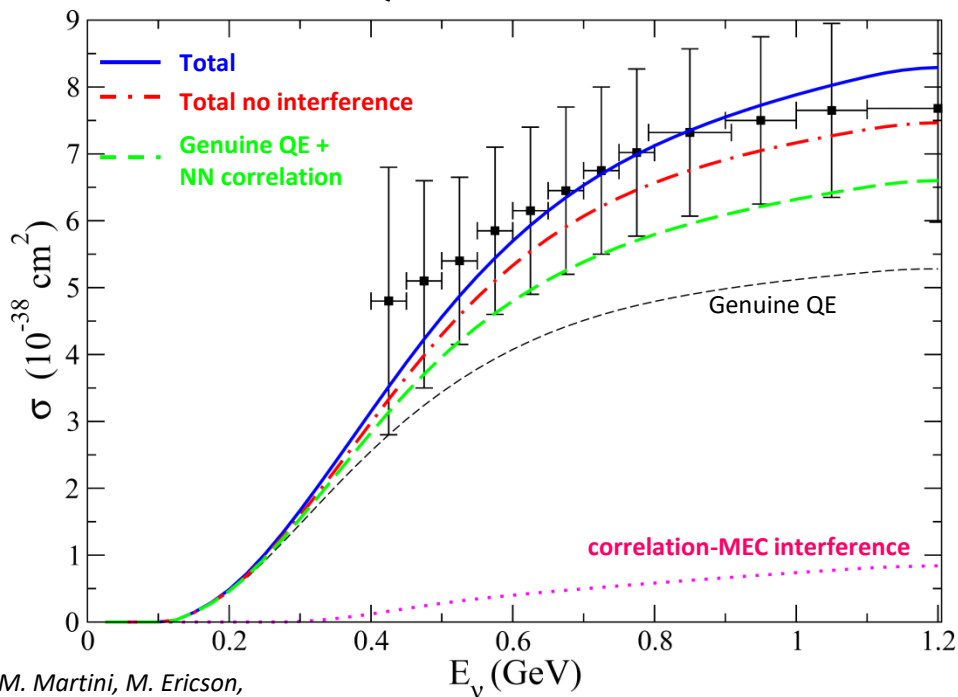
## Chiral Effective Field Theory

- low energy approach to QCD, nuclear structure energies
- Consistent treatment of nuclear many-body forces and electroweak currents
- Long-range physics given explicitly by pion-exchanges
- Short-range physics: contact interactions to fit

# Same solution for the two “axial” puzzles

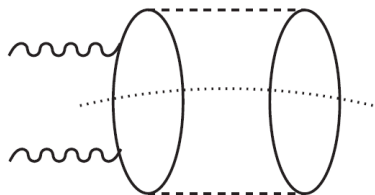
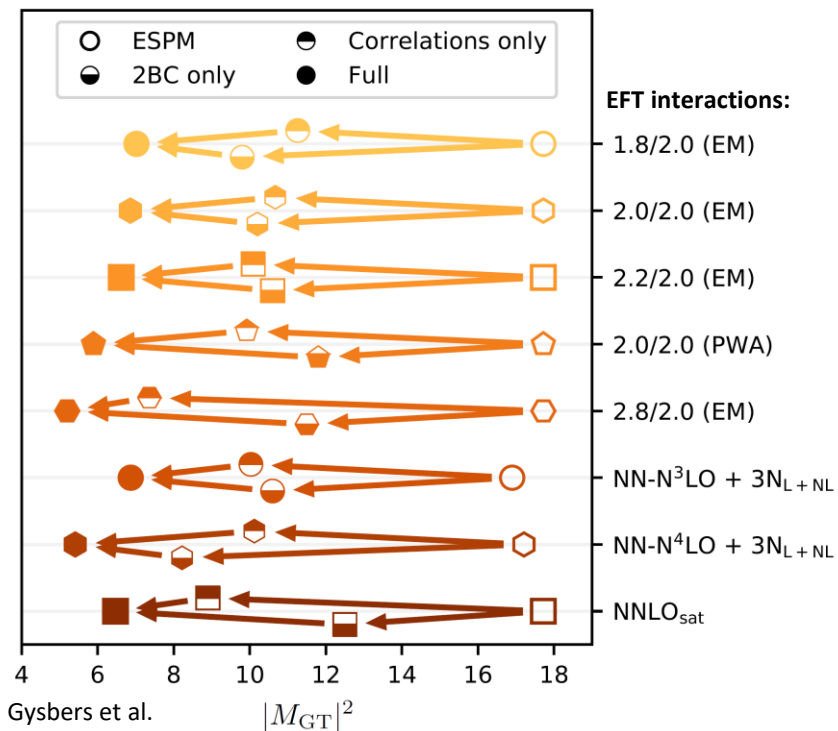
Relatively similar, complementary and important impact of nuclear correlations and 2-body (MEC) currents

Neutrino CCQE-like cross section on  $^{12}\text{C}$

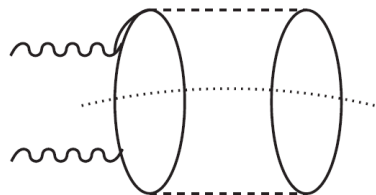


M. Martini, M. Ericson,  
G. Chanfray, J. Marteau

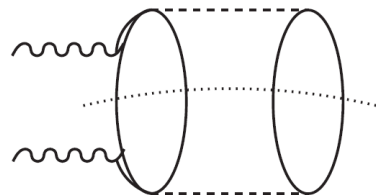
$|M_{GT}|^2$  for the  $\beta$  decay of  $^{100}\text{Sn}$



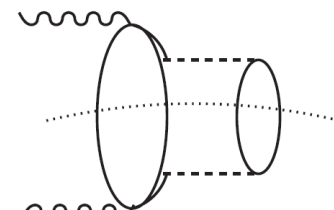
NN correlations



NN-MEC Interference

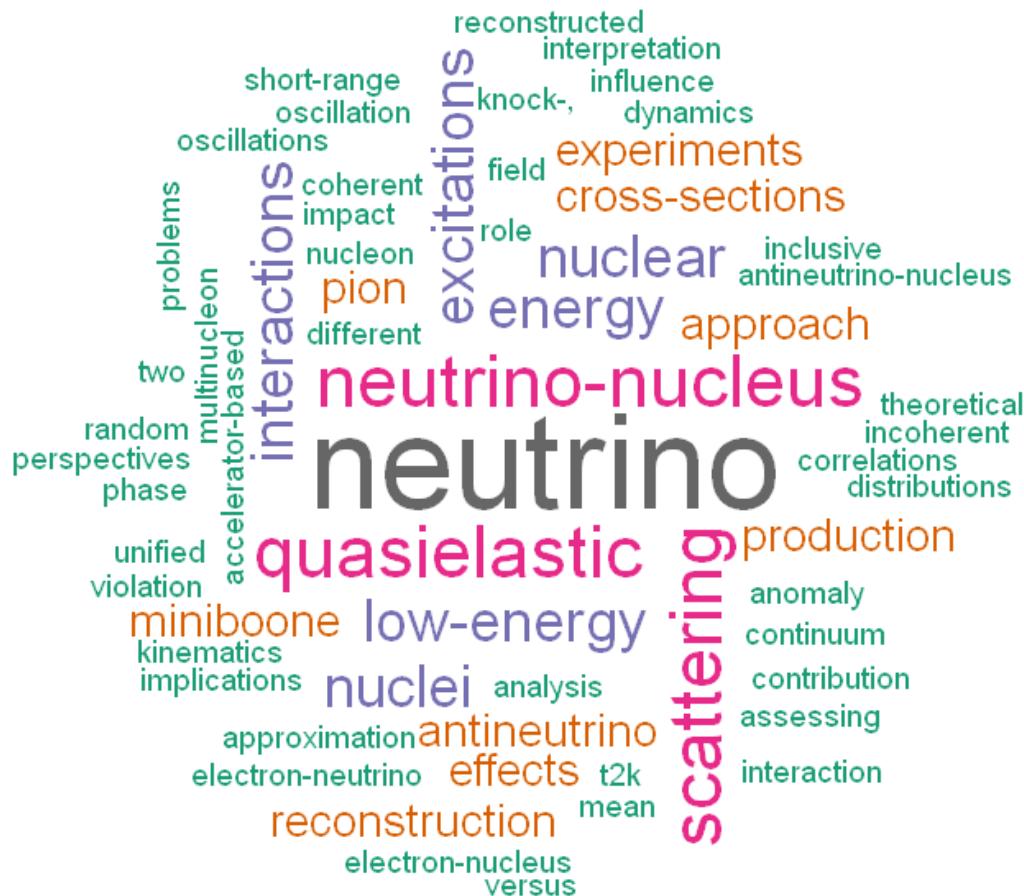


$\Delta$ -MEC



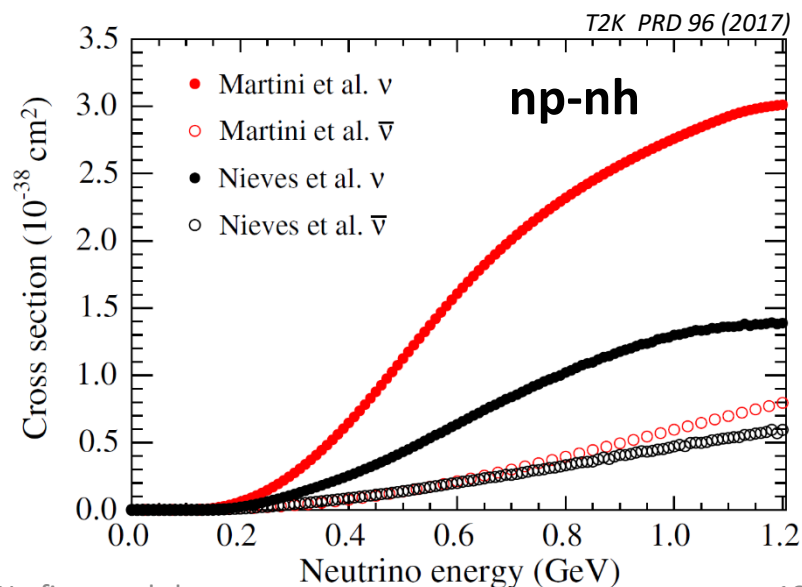
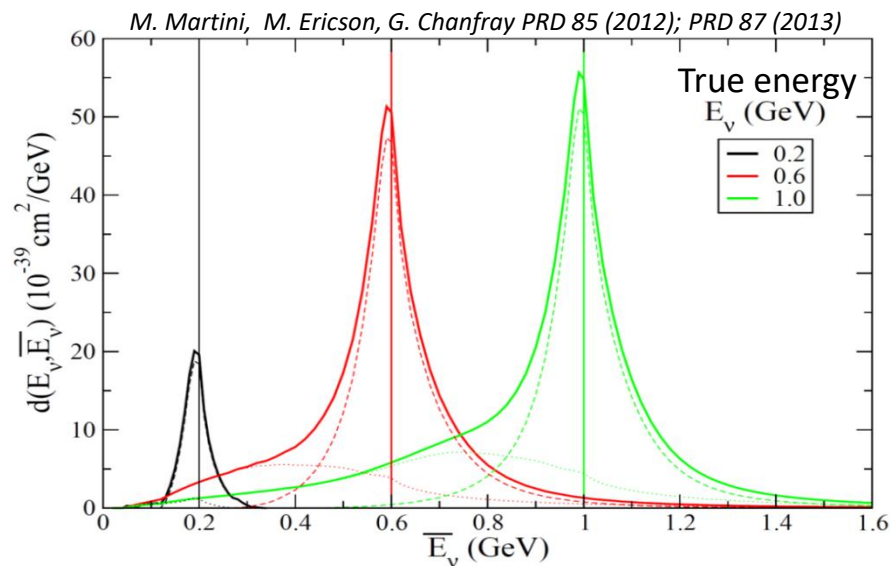


# Further details on nuclear models for LBL oscillation experiments



# The np-nh channel (nuclear correlations and 2-body currents)

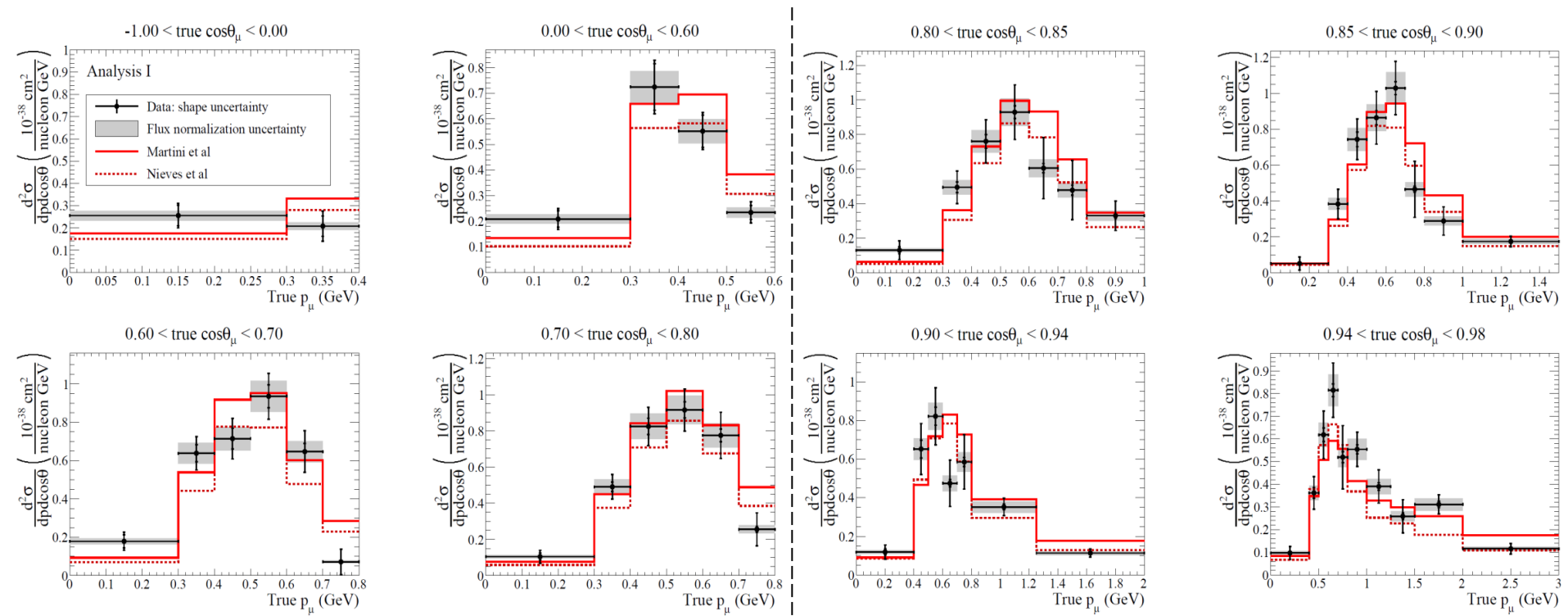
- A lot of interest in these last 10 years
- Explanation of the axial mass puzzle
- It was not included in the generators used for the analyses of  $\nu$  cross sections and oscillations experiments
- The effort to include this np-nh channel in several Monte Carlo is in progress
- One of the most important sources of systematic errors in oscillation experiments since it **affects the neutrino energy reconstruction**
- Several theoretical calculations agree on its crucial role but there are differences on the results obtained for this channel



# A T2K measurement more model independent: $d^2\sigma$ $CC0\pi$

$CC0\pi = CCQE$ -like without subtraction of  $\pi$  absorption background

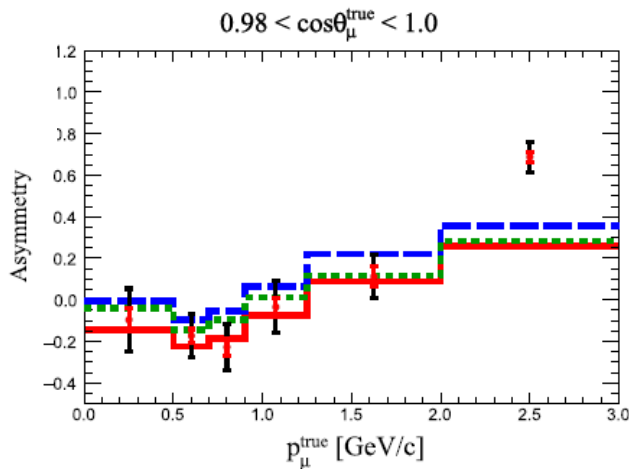
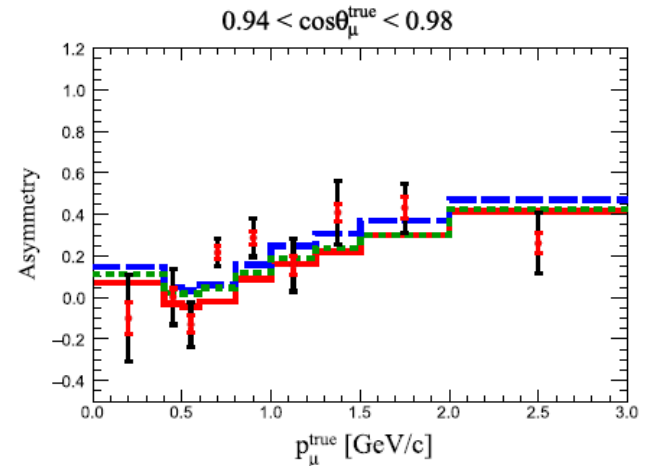
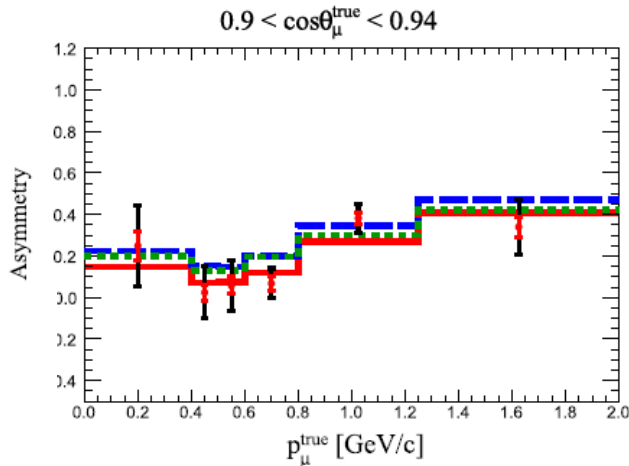
T2K collaboration: Abe et al. Phys. Rev. D 93 11012 (2016)








The two theoretical models including np-nh are compatible with data at this level of experimental accuracy

First combined measurement of the muon neutrino and antineutrino charged-current cross section without pions in the final state at T2K

$$\frac{\nu - \bar{\nu}}{\nu + \bar{\nu}}$$



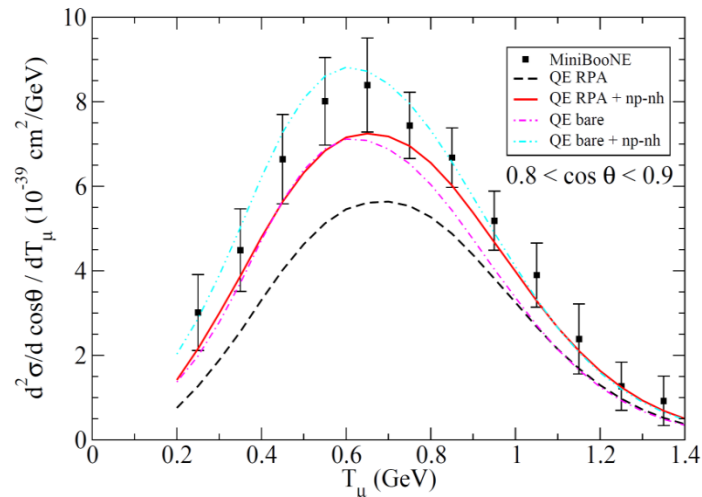
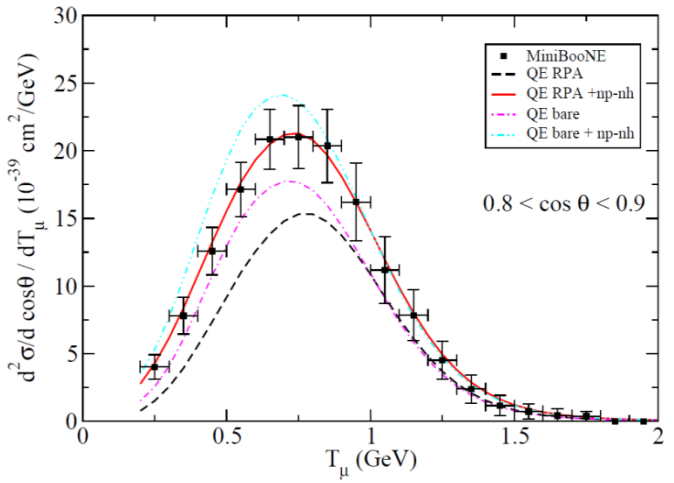
-  Total Uncertainty (stat+syst)
-  Systematic Uncertainty
-  NEUT LFG+2p2h  $\chi^2 = 150.5(147.8)/58$
-  Martini et al.  $\chi^2 = 93.9(131.2)/48$
-  SuSAv2  $\chi^2 = 152.6(146.3)/58$

**[Maria Barbaro talk]**

# V

Martini et al.

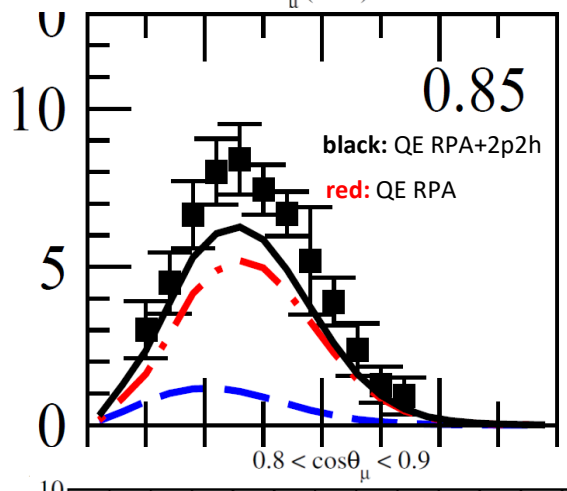
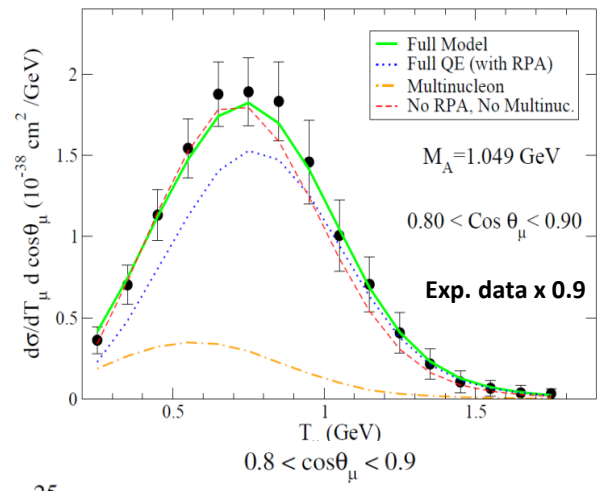
PRC 84 (2011)



PRC 87 (2013)

Nieves et al.

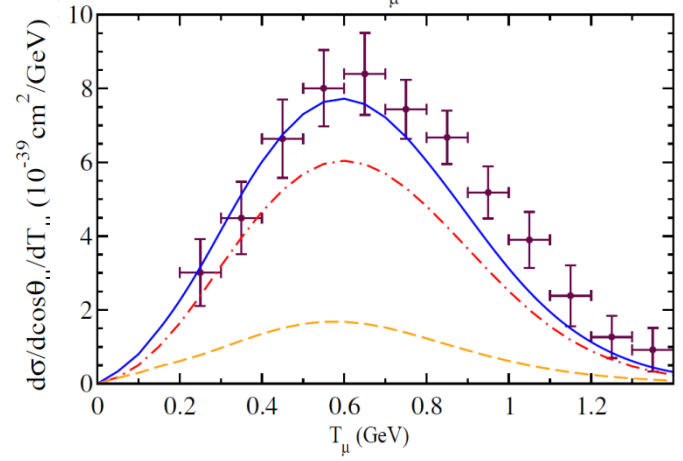
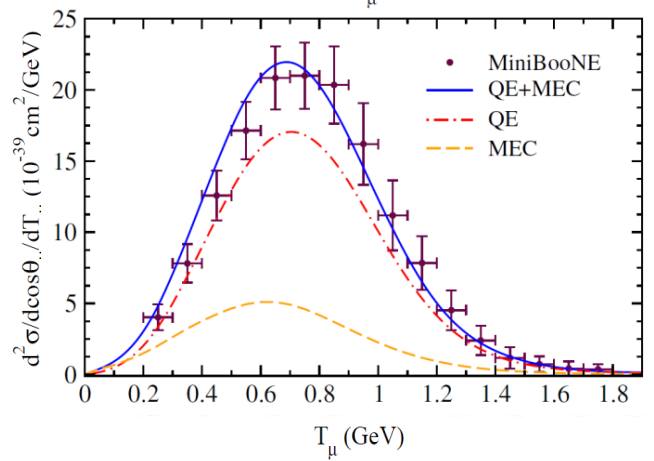
PLB 707 (2012)



PLB 721 (2013)

SuSAv2

PRD 94 (2016)



PRD 94 (2016)

# Main difficulties in the np-nh sector

$$W_{2p-2h}^{\mu\nu}(\mathbf{q}, \omega) = \frac{V}{(2\pi)^9} \int d^3 p'_1 d^3 p'_2 d^3 h_1 d^3 h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \theta(p'_2 - k_F) \theta(p'_1 - k_F) \theta(k_F - h_1) \theta(k_F - h_2) \\ \underbrace{\langle 0 | J^\mu | \mathbf{h}_1 \mathbf{h}_2 \mathbf{p}'_1 \mathbf{p}'_2 \rangle \langle \mathbf{h}_1 \mathbf{h}_2 \mathbf{p}'_1 \mathbf{p}'_2 | J^\nu | 0 \rangle}_{\text{matrix elements}} \delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q})$$

- 7-dimensional integrals  $\int d^3 h_1 d^3 h_2 d\theta'_1$  of thousands of terms
- Huge number of diagrams and terms
- Divergences (angular distribution; NN correlations contributions)
- Calculations for all the kinematics compatible with the experimental neutrino flux

## Computing very demanding

Hence different approximations by different groups:

- choice of subset of diagrams and terms;
- different prescriptions to regularize the divergences;
- reduce the dimension of the integrals

(7D --> 2D if non relativistic; 7D --> 1D if  $h_1 = h_2 = 0$ )

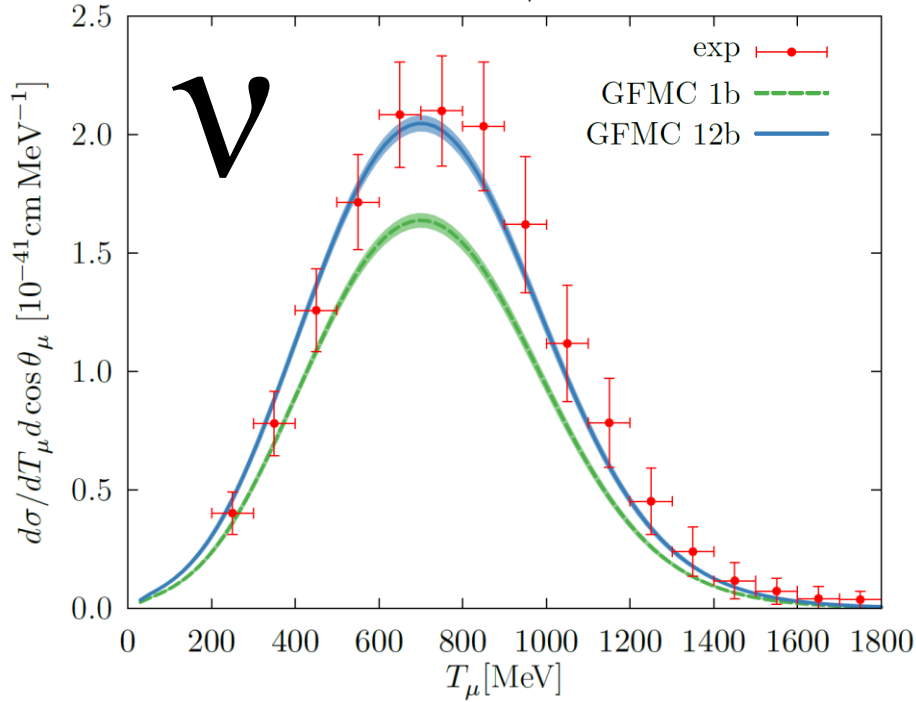
⇒ Different final results by different groups

⇒ Different final results for  $\nu$  and anti $\nu$

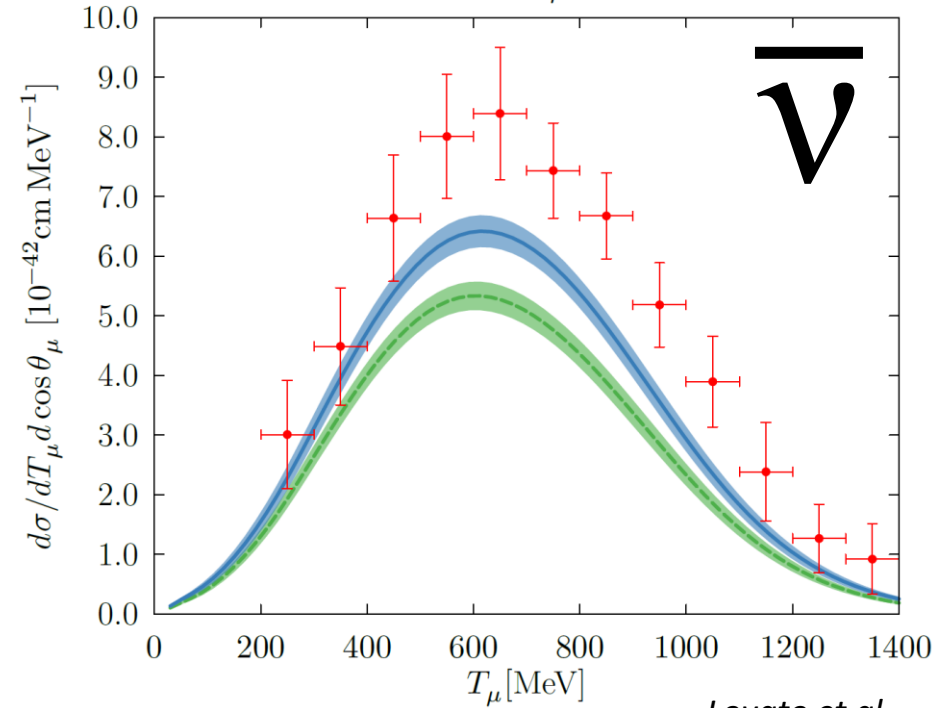
# First ab-initio calculation of flux-integrated double differential cross sections

Ab initio: state-of-the-art description of nuclear ground state and correlations

$0.8 < \cos \theta_\mu < 0.9$



$0.8 < \cos \theta_\mu < 0.9$



The important role of two-body currents is confirmed

Lovato et al.  
*Phys.Rev.X* 10 (2020)

Details of the approach (Green's Function Monte Carlo):

- Nuclear interaction in the **Hamiltonian**:  $V_{ij}$  Argonne AV18;  $V_{ijk}$  Illinois IL7
- Electroweak **Currents**: non relativistic expansion;  $\pi$  and  $\rho$  Meson Exchange Currents

Limitations:

- Static limit for  $\Delta$  MEC
- It cannot describe the  $\Delta$  peak region; no mechanisms for real pion production

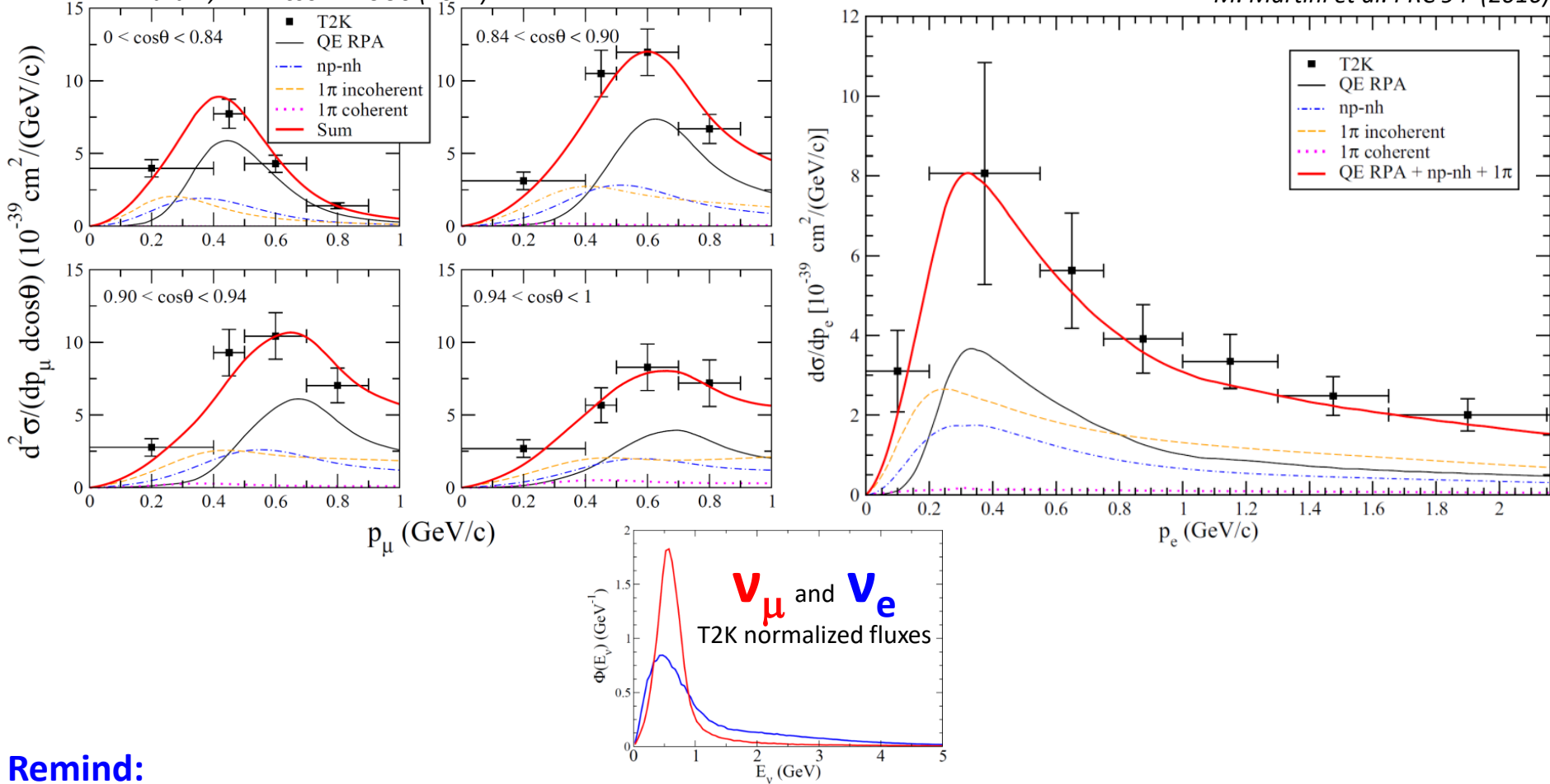
# Different contributions in CC inclusive T2K flux-integrated differential cross sections

$\nu_\mu$

$\nu_e$

M. Martini, M. Ericson PRC 90 (2014)

M. Martini et al. PRC 94 (2016)



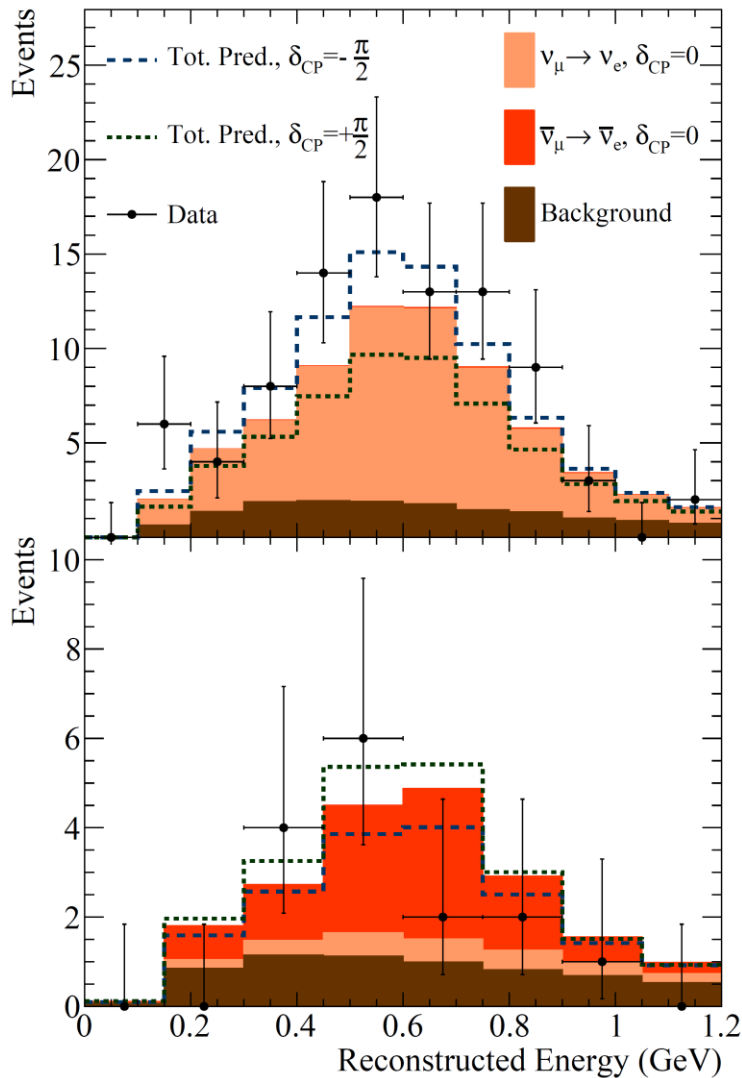
## Remind:

A precise and simultaneous knowledge of  $\nu_\mu$ ,  $\nu_e$ ,  $\bar{\nu}_\mu$ ,  $\bar{\nu}_e$  cross sections is important in connection to the oscillation experiments aiming at the search for CP violation in the lepton sector

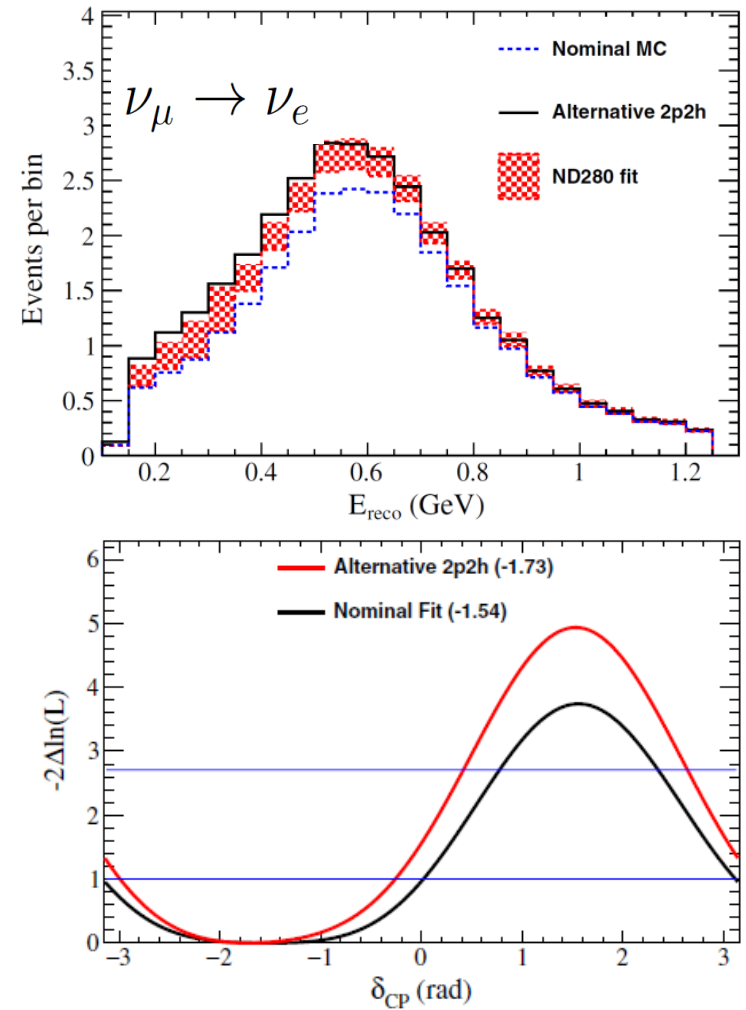


# $\nu_\mu \rightarrow \nu_e$ T2K results and studies on $\delta_{CP}$

T2K Nature (2020)



Example of impact of 2p-2h differences on oscillation analyses



T2K PRD 96 (2017)

More recent: 2101.03779

# Neutrino-nucleus cross sections: Present studies and perspectives

- Exclusive cross sections and hadronic variables

Many models used up to now to compare with the neutrino flux-integrated differential cross sections are not directly applicable for exclusive studies. More nuclear response functions contribute.

- Generalization of models to asymmetric nuclei (in particular Argon)

Differences between neutron and proton densities and energies should be taken into account

Some results for genuine QE and  $1\pi$  production  
Very few results for 2-body current contributions

[Maria Barbaro talk]

- From meson(s) production to Deep-Inelastic Scattering

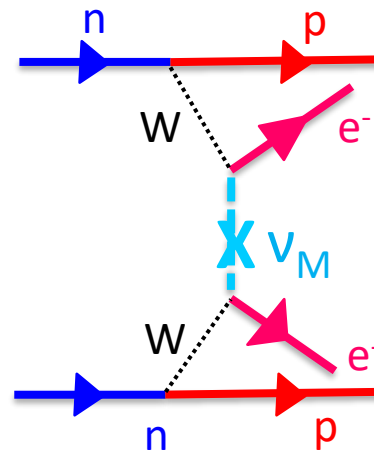
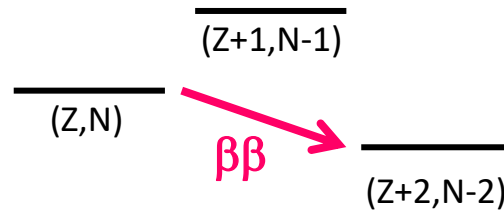
Recent Review: *M. Sajjad Athar and J. Morfin, J.Phys.G 48 (2021) 3*

- Low-energy neutrino scattering

Giant Resonances, discrete excitations, CEvNS

[Vishvas Pandey talk]

# Nuclear matrix elements for neutrinoless Double-Beta decay



# $0\nu\beta\beta$ nuclear matrix elements

$$M_{(0\nu)} = M_{(0\nu)}^{\text{GT}} - \frac{g_V^2}{g_A^2} M_{(0\nu)}^{\text{F}} + \dots$$

$$M_{(0\nu)}^{\text{GT}} = \langle f | \sum_{a,b} H(r_{ab}, \bar{E}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle$$

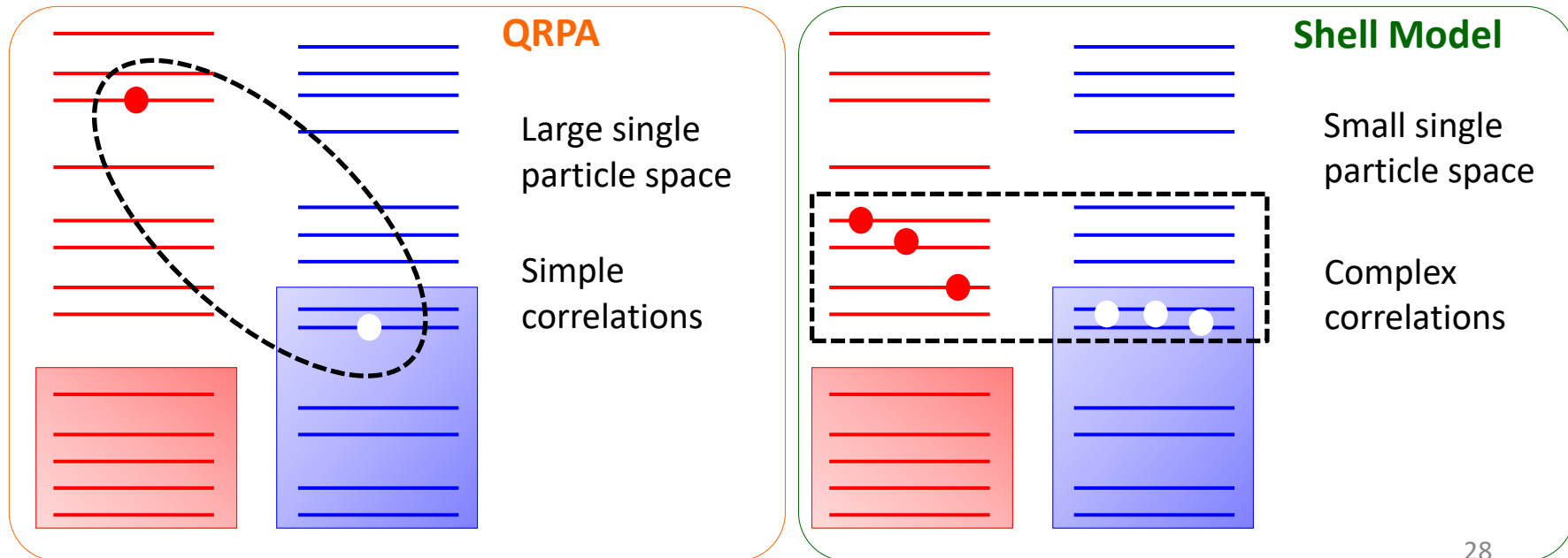
$$M_{(0\nu)}^{\text{F}} = \langle f | \sum_{a,b} H(r_{ab}, \bar{E}) \tau_a^+ \tau_b^+ | i \rangle$$

## Calculating Matrix Elements is hard

- Most of relevant nuclei are heavy ( $A > 75$ ) and complicated
- Never measured
- Structure of initial and final nuclear ground states quite different  $\Rightarrow$  matrix element small and sensitive

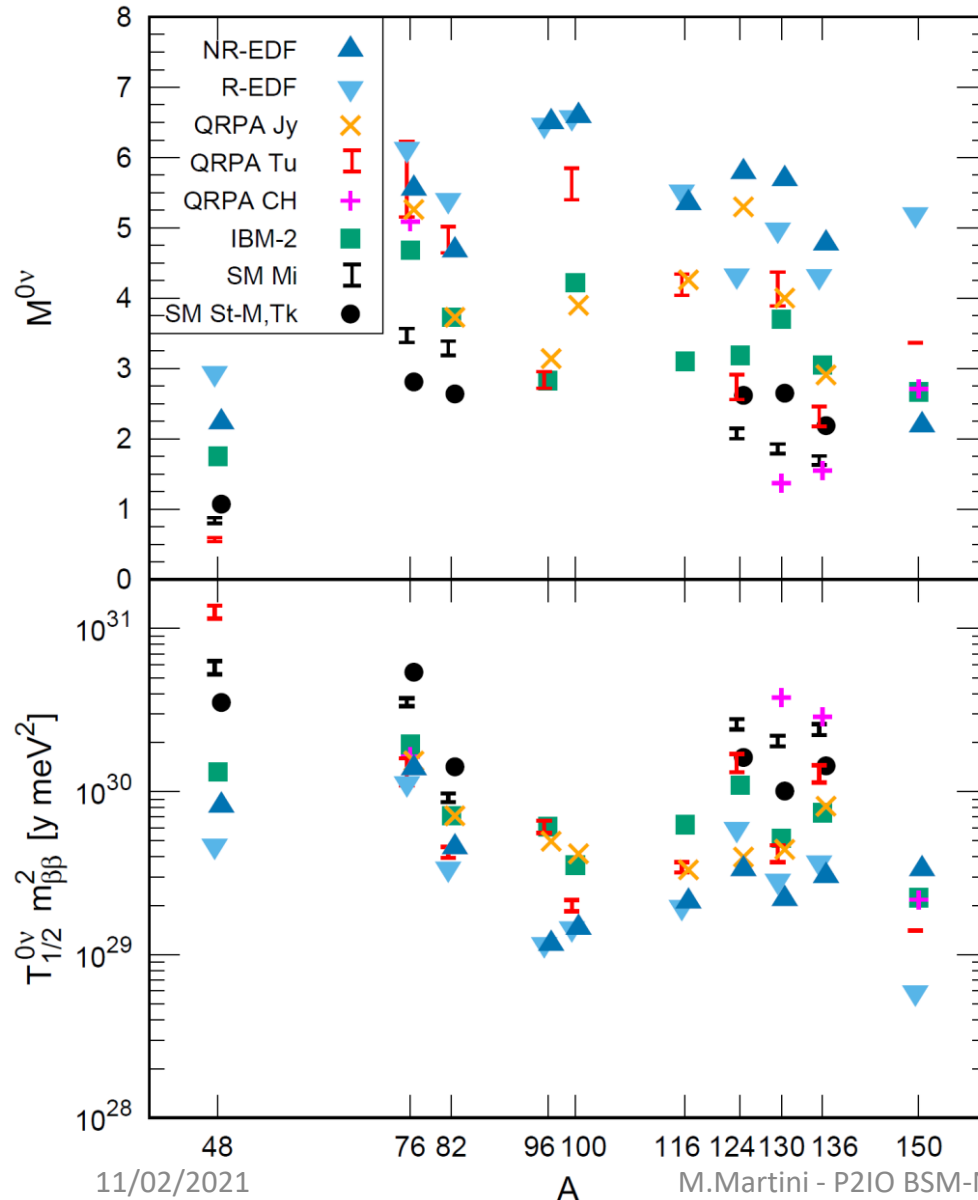
## Different models, different approximations, different NN interaction

Pictorial representation of two between the most used phenomenological models :



# $0\nu\beta\beta$ nuclear matrix elements calculations

J. Engel and J. Menéndez, *Rept.Prog.Phys.* 80 (2017)



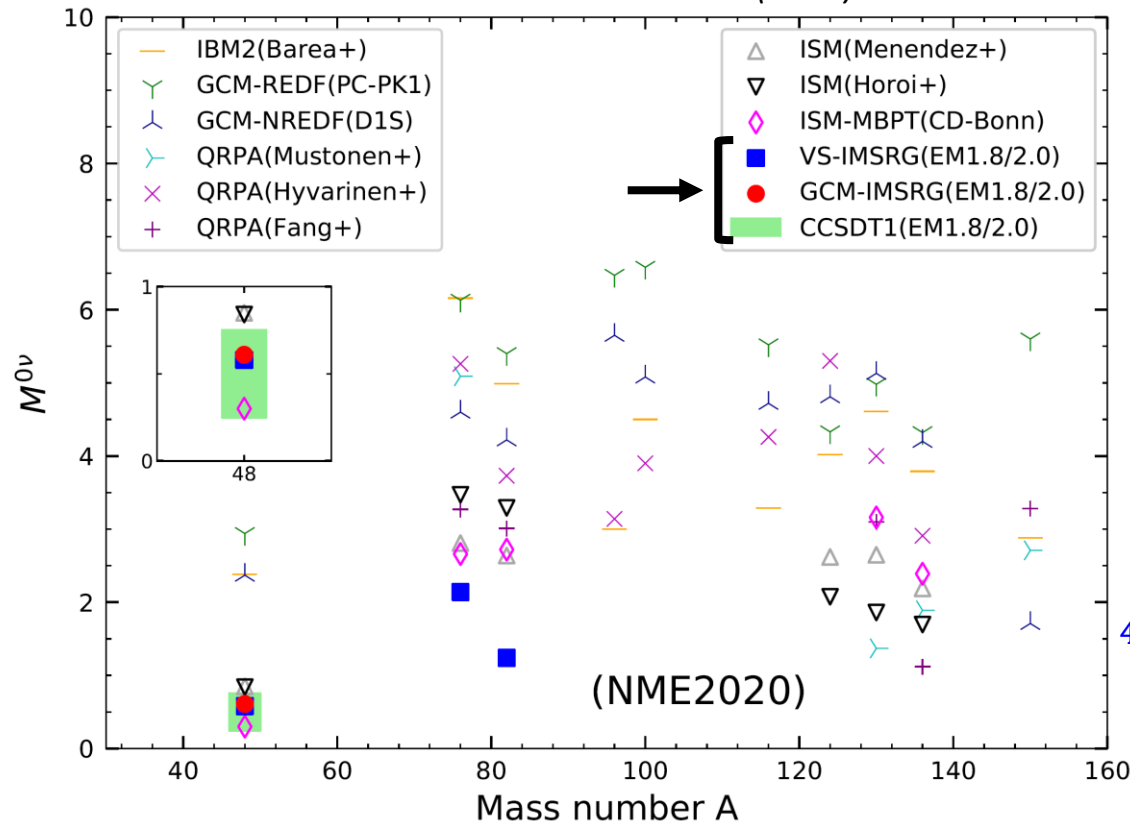
- Large spread
- Factors 2 or 3 of difference between different approaches  $\Leftrightarrow$  factor 10 for  $T_{1/2}^{0\nu}$
- EDFs: large NME  
QRPA: wider range  
SM: small NME
- Uncertainty can't be quantified.

Recently it started an effort of the community to go to some more first principles approaches:  
**ab initio methods**

# Recent ab initio calculations of $0\nu\beta\beta$ matrix elements using chiral EFT

Complete nuclear correlations included in ab initio calculations

*J.M. Yao Science Bulletin 66 (2021) 1*



● Generator Coordinate Method –  
In Medium Similarity Renormalization Group  
 $^{48}\text{Ca}$  *J.M. Yao et al. PRL 124 (2020)*

■ Coupled Cluster with  
Singles Doubles and Triples excitations  
 $^{48}\text{Ca}$  *S. J. Novario et al. arxiv: 2008.09696*

■ Valence Space –  
In Medium Similarity Renormalization Group  
 $^{48}\text{Ca} \ ^{76}\text{Ge} \ ^{82}\text{Se}$  *A. Belley et al. PRL 126 (2021)*

**Good News:** agreement between the different ab initio calculations for Calcium-48

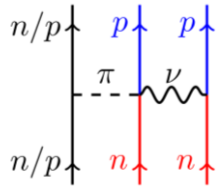
**Bad News:** NME 25-45% smaller than phenomenological shell model  
If confirmed, not good for experiment

# Two-body currents and new contact operator in $0\nu\beta\beta$ using chiral EFT

Chiral interactions describe low-energy, low-momentum physics (e.g.  $\beta$  and  $2\nu\beta\beta$  decays)

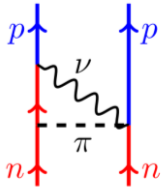
How do they work at large energy and momenta? (e.g.  $0\nu\beta\beta$   $\omega \approx$  few MeV,  $q \approx 10^2$  MeV, or  $\nu$  scattering)

## Two-body currents



- Estimations of this contributions suggest that two-body currents are smaller in  $0\nu\beta\beta$  than in  $\beta$  decay (remember the solution of  $g_A$  quenching puzzle)

*J. Menendez et al. PRL 107 (2011)*

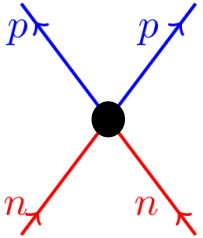


*L. Wang et al. PRC 98 (2018)*

- Neutron can be excited to any empty level, leading to linear divergence

Unknown high-energy  $\nu$  exchange physics prevents complete analysis of 2-body currents

## Short-range contact term



*V. Cirigliano et al. PRL120 (2018)*

- Usual light neutrino exchange must be supplemented, even at leading order in chiral EFT, by short-range operator (representing high-energy  $\nu$  exchange)

- Very recent first estimation of the coupling coefficient

*V. Cirigliano et al. 2012.11602 ; 2102.03371*

New (computational very demanding) studies on two-body currents contributions put in stand-by until a firm handle on the value of the contact term coefficient

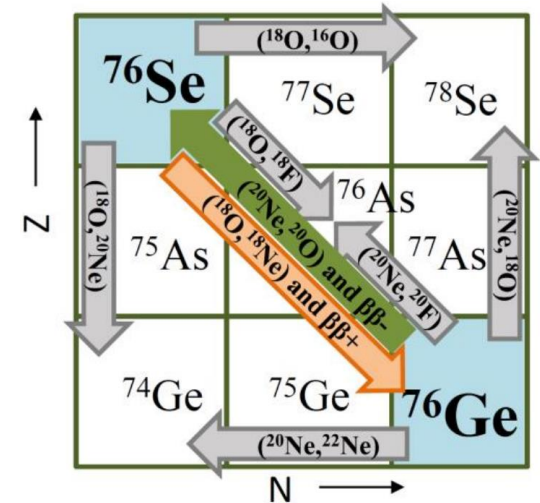
Contrasting effects from physics neglected up to now are expected

Significant work remains to do before any claims to final NME can be made

# $0\nu\beta\beta$ decay and Double charge exchange reactions

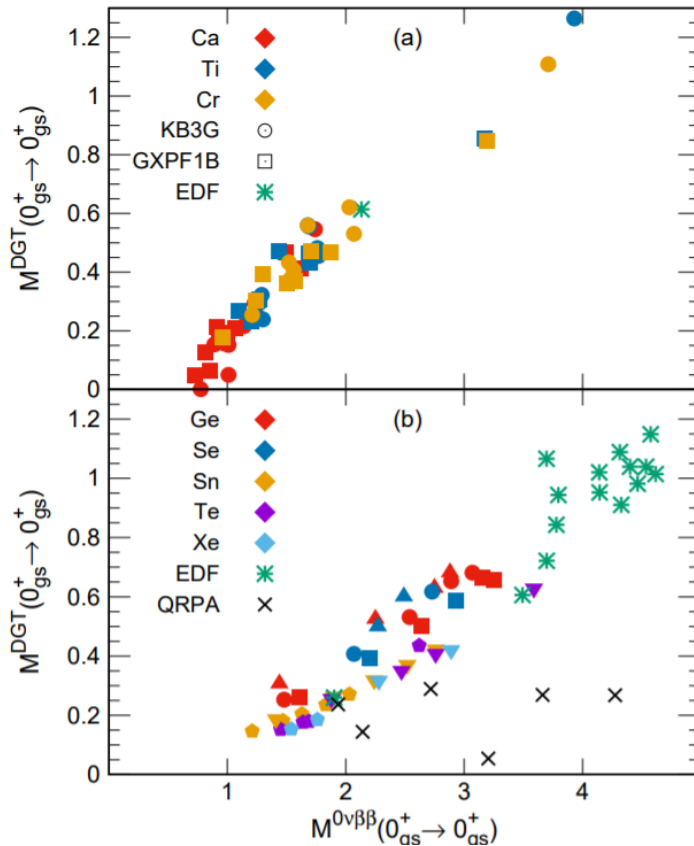
Recent experimental plans at INFN-LNS Catania and RIKEN on double charge exchange reactions, such as  $^{76}\text{Ge}(^{20}\text{Ne}, ^{20}\text{O})^{76}\text{Se}$

The initial and final state nucleus wave functions in strong double charge exchange reactions and  $0\nu\beta\beta$  decay are the same and the transition operators are similar



F. Cappuzzello et al., EPJA 54 (2018)

N. Shimizu, J. Menendez, K. Yako PRL 120 (2018)



Correlations of  $0\nu\beta\beta$  with Double Gamow-Teller matrix elements holds across nuclear chart

Experiments may access Double GT transitions

[Francesco Cappuzzello talk]



# Nuclear models for LBL oscillation experiments and $0\nu\beta\beta$ decay

## Summary

Nuclear correlations and two-body currents play a central role for a precise determination of neutrino cross sections and  $0\nu\beta\beta$  Nuclear Matrix Elements

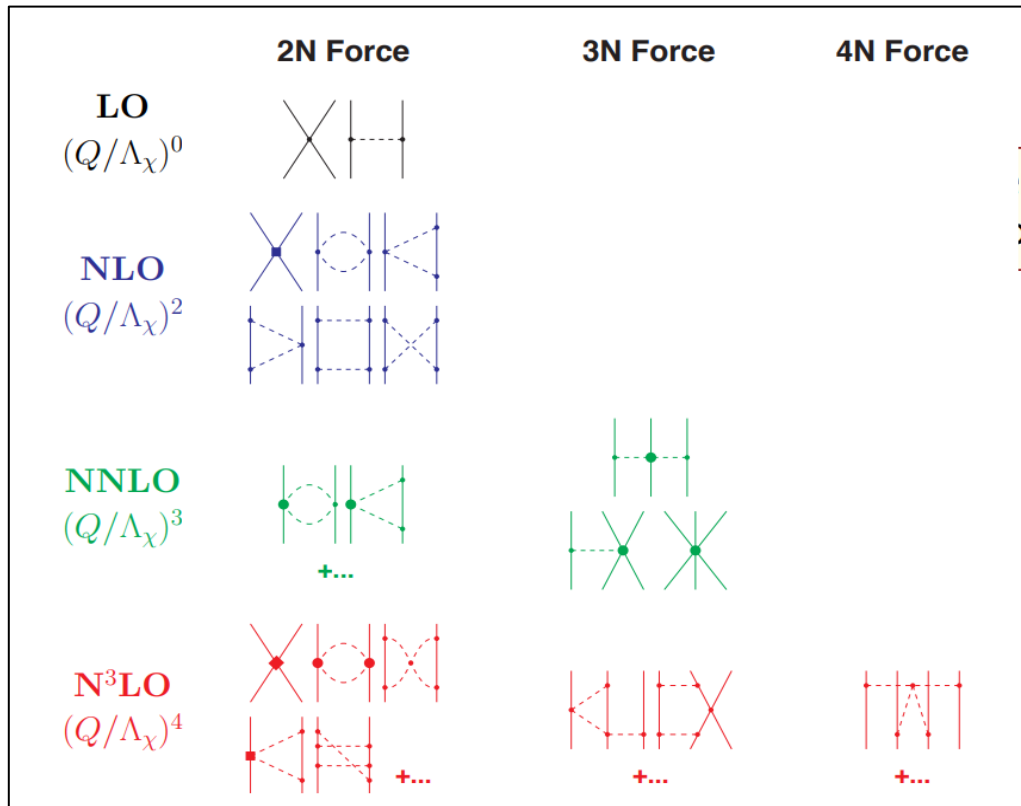
- Important results in these last years by phenomenological and ab initio approaches
- Recent ab initio calculations allowed complete inclusion of nuclear correlations
- Some limitations on two-body currents employed in ab-initio approaches when large energy and/or momentum transfer are considered

Important work remains to do before to reach the few-percent precision in neutrino cross sections required by next generation LBL experiment and before any claims on final values of  $0\nu\beta\beta$  Nuclear Matrix Elements can be made

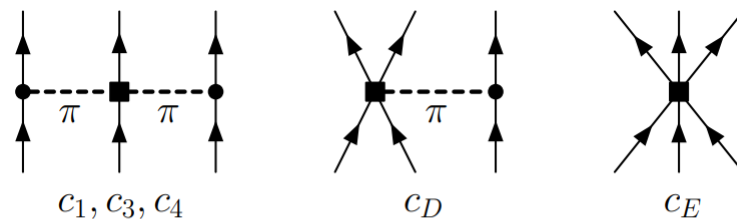
# Spares

# Chiral forces and currents

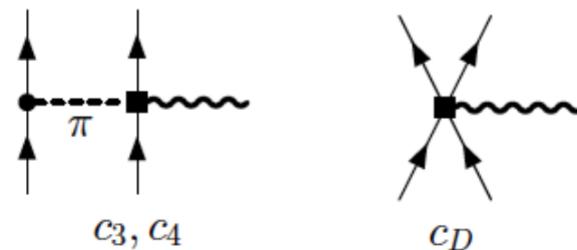
Consistent treatment of nuclear many-body forces and electroweak currents



3-body force at NNLO:



2-body currents:



P.S. Three related diagrammatic representations

