

The Hidden Geometry of Particle Collisions

Jesse Thaler



Virtual Particle Physics in Paris — February 23, 2021

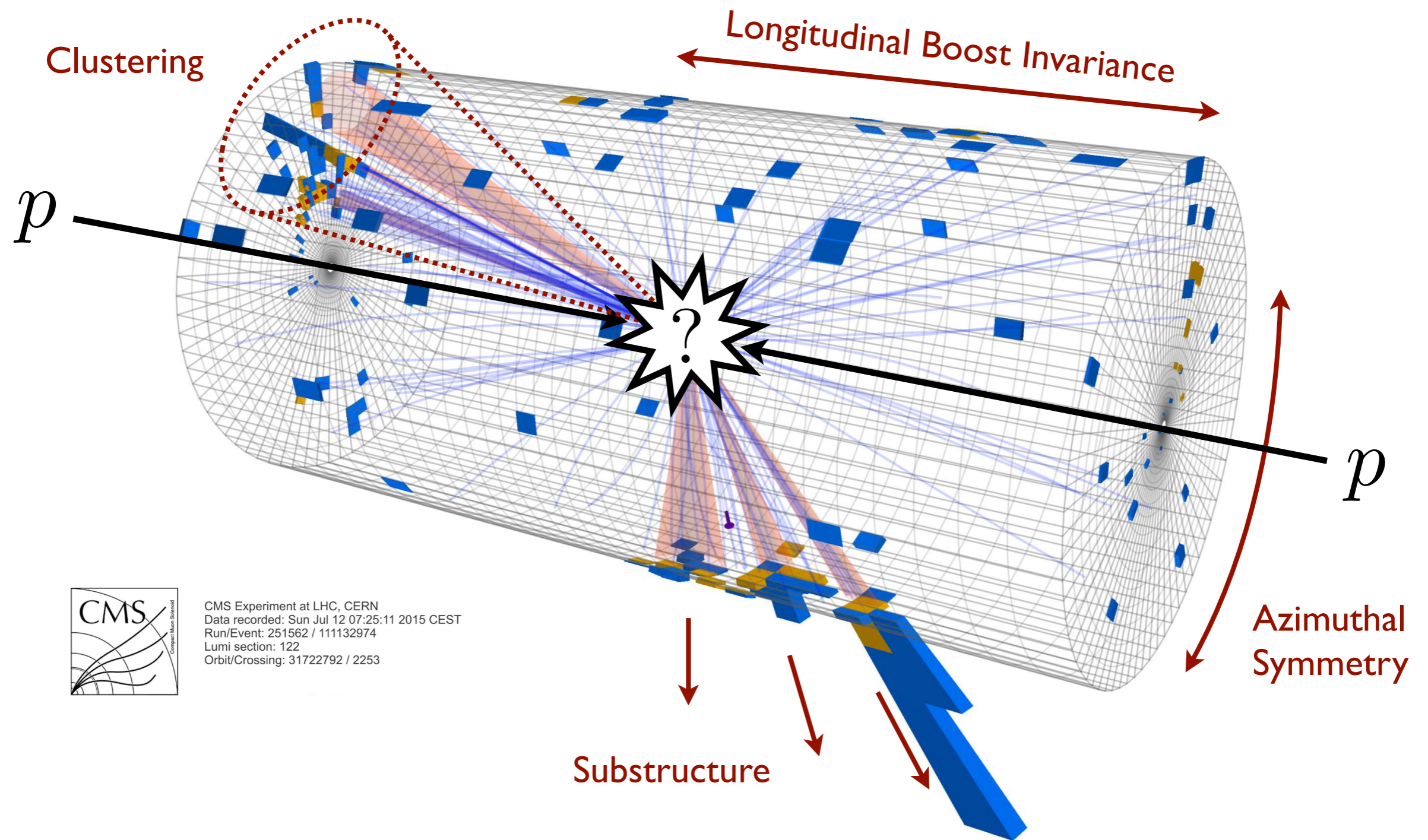
The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI) *“eye-phi”*

Advance physics knowledge — from the smallest building blocks of nature to the largest structures in the universe — and galvanize AI research innovation

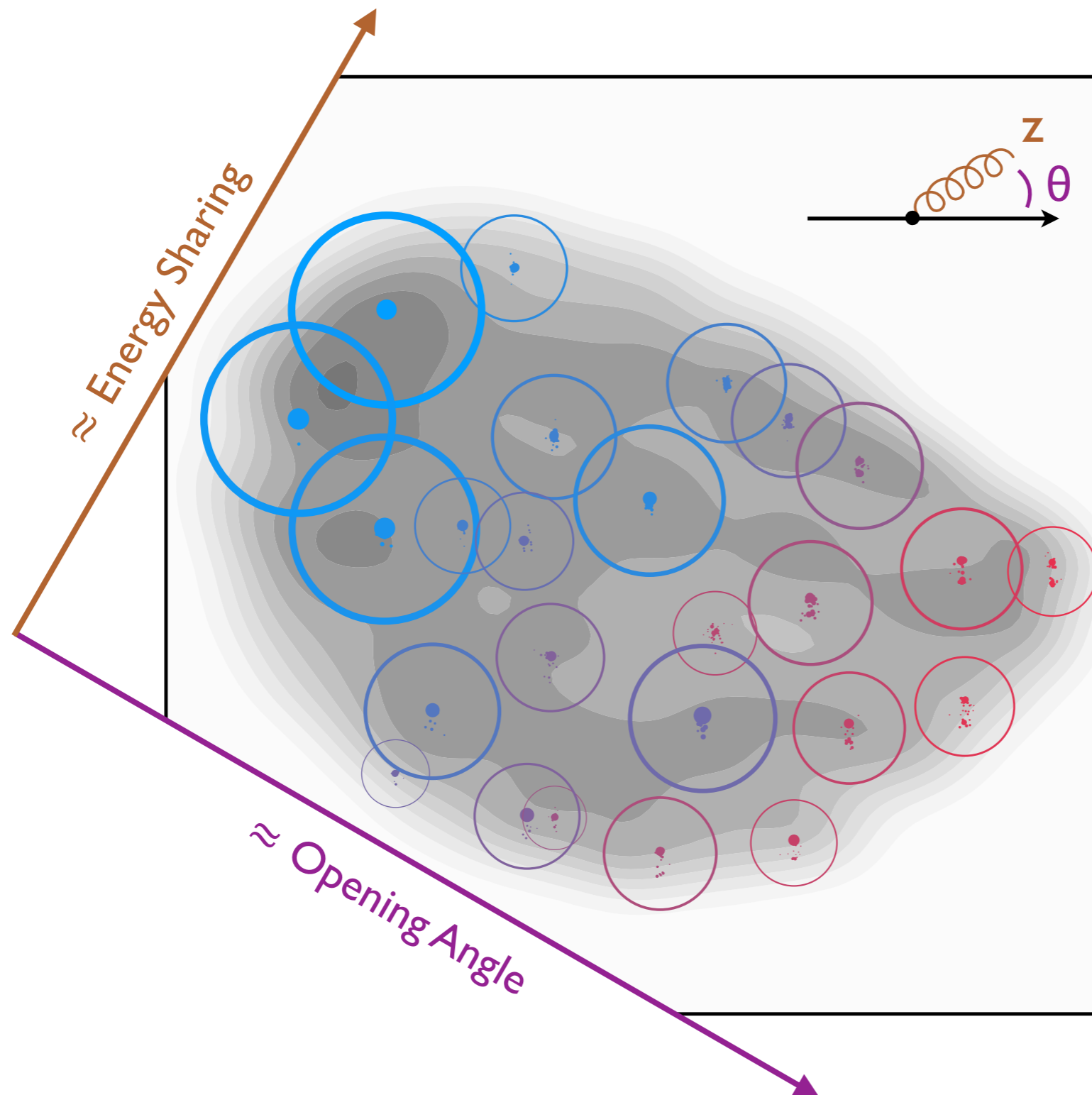


[<http://iaifi.org>, MIT News Announcement]

The Manifest Geometry of One Collision



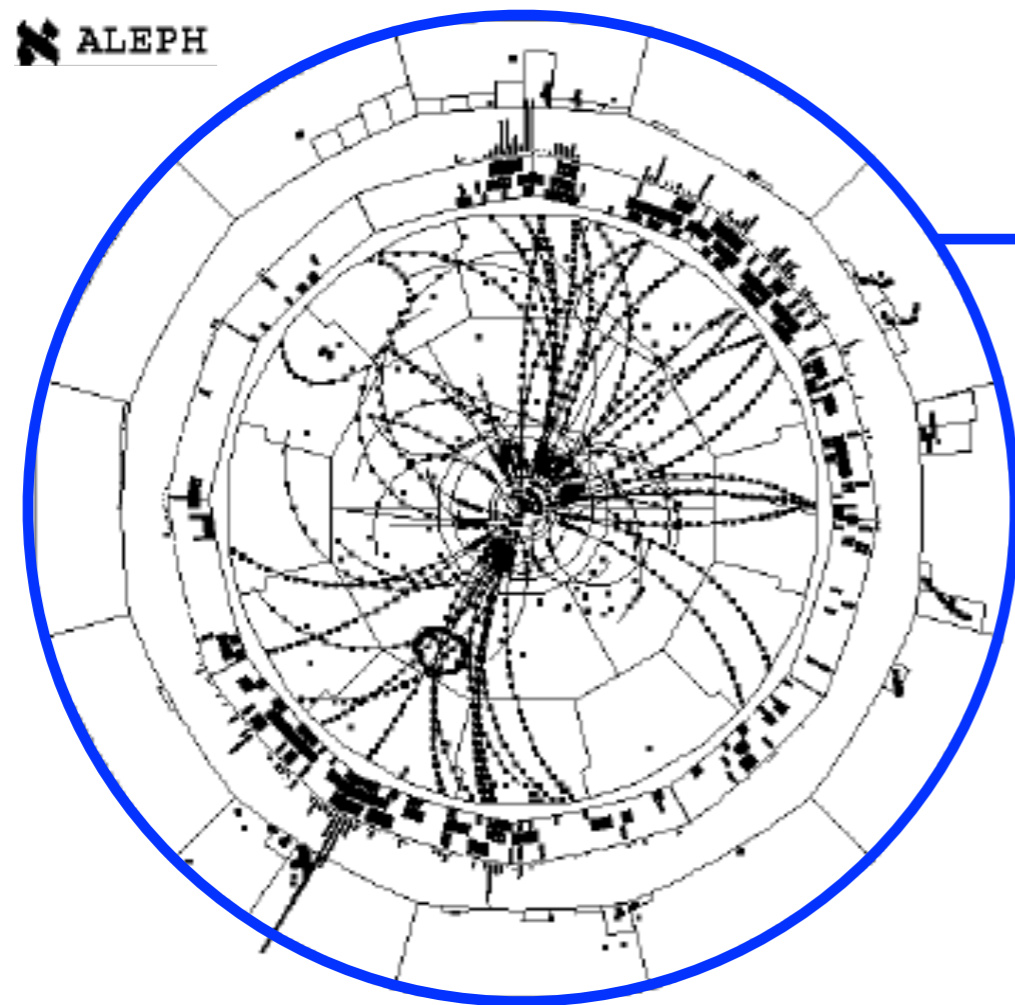
The Emergent Geometry of Many Collisions



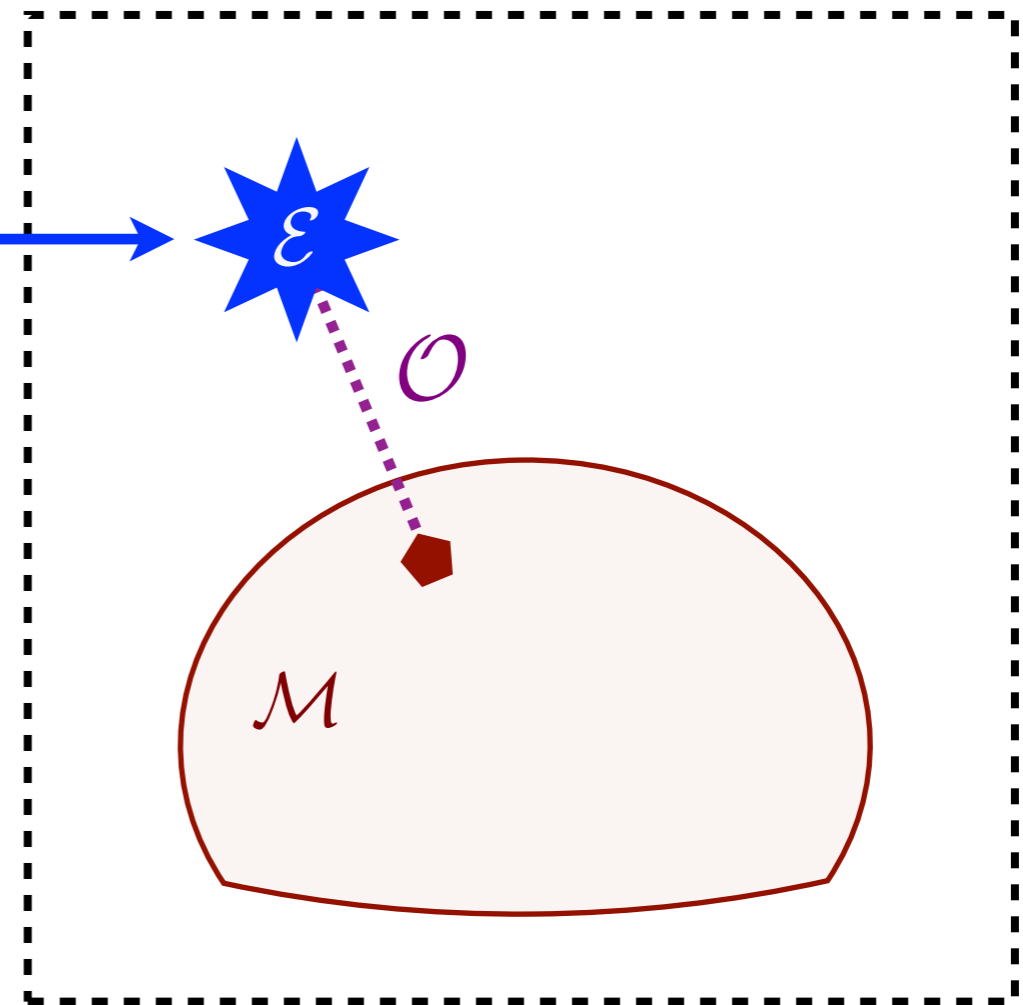
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
based on Komiske, Metodiev, JDT, PRL 2019; using EnergyFlow and CMS Open Data]

The Hidden Geometry of Particle Collisions

E.g. Classic QCD Event Shapes



One Electron-Positron Event



Distance to a Manifold in Event Space

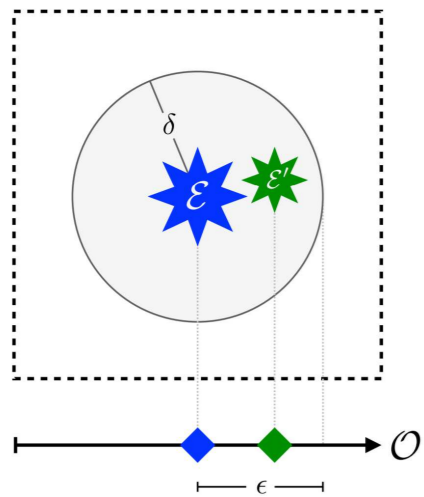
[Komiske, Metodiev, JDT, JHEP 2020]

[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977]



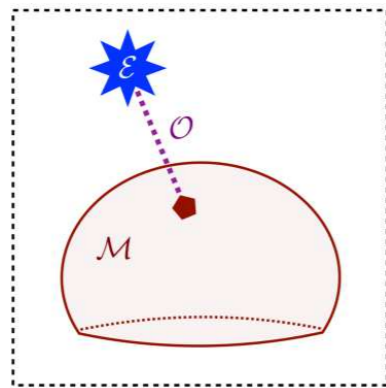
Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events



Taming infinities

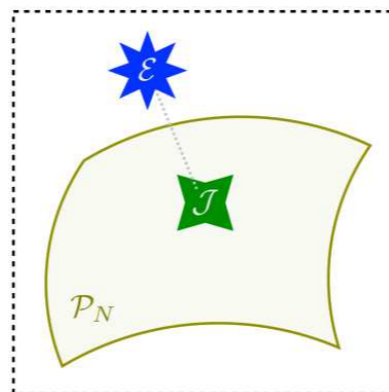
Event shapes are distances from events to manifolds.



$$O(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

Event Shapes

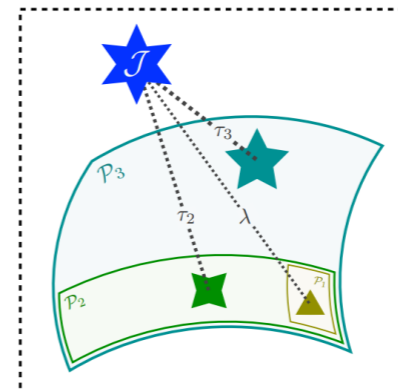
Jets are projections to few-particle manifolds.



$$J = \operatorname{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

Jet Algorithms

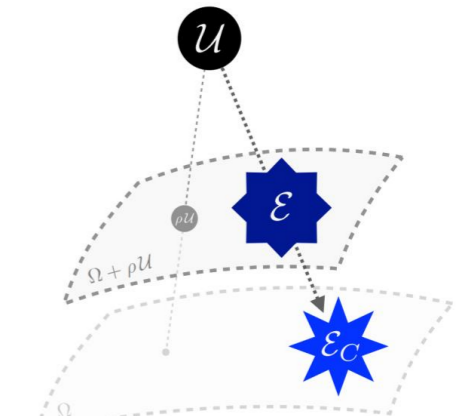
Substructure resolves emissions within the jet.



$$\tau(J) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{E}')$$

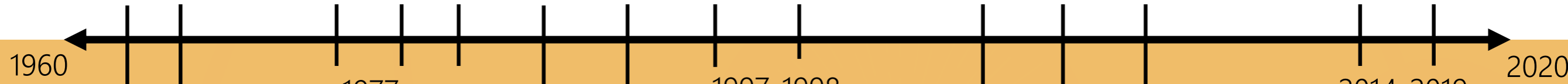
Jet Substructure

Pileup mitigation moves away from uniform radiation.



$$\mathcal{E}_C = \operatorname{argmin}_{\mathcal{E}'} \text{EMD}(\mathcal{E}, \mathcal{E}' + \rho \mathcal{U}).$$

Pileup



1962-1964

Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977

Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993

k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998

C/A jet clustering
[Wobisch, Wengler, 1998]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997]

2010-2015

N-(sub)jettiness, X Cone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

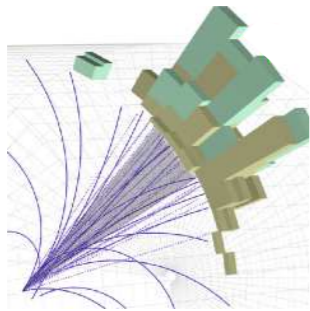
2014-2019

Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

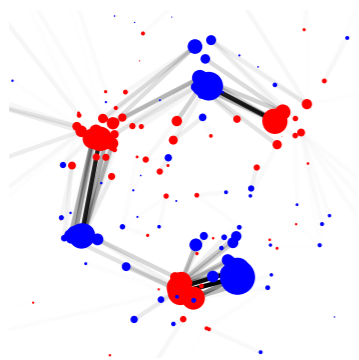
And many more!

[timeline from Eric Metodiev]

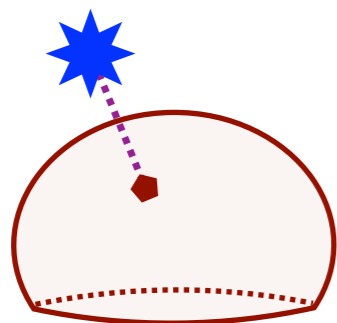
Outline



What is a Collider Event?



When are Events Similar?



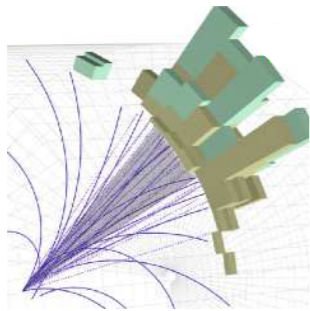
What can be Geometrized?

Pause

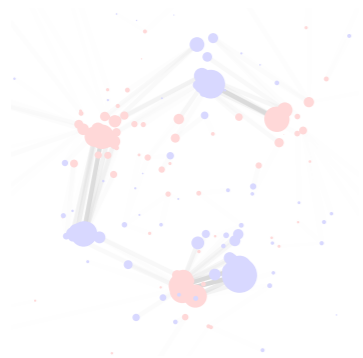
Interrupt me or drop questions in the chat, and I'll try to answer them as I go

Anticipating a coda...

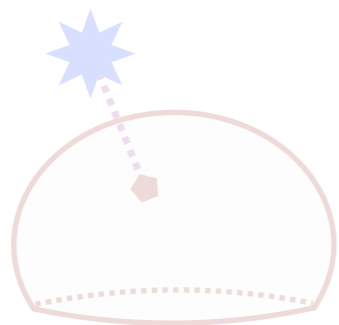




What is a Collider Event?



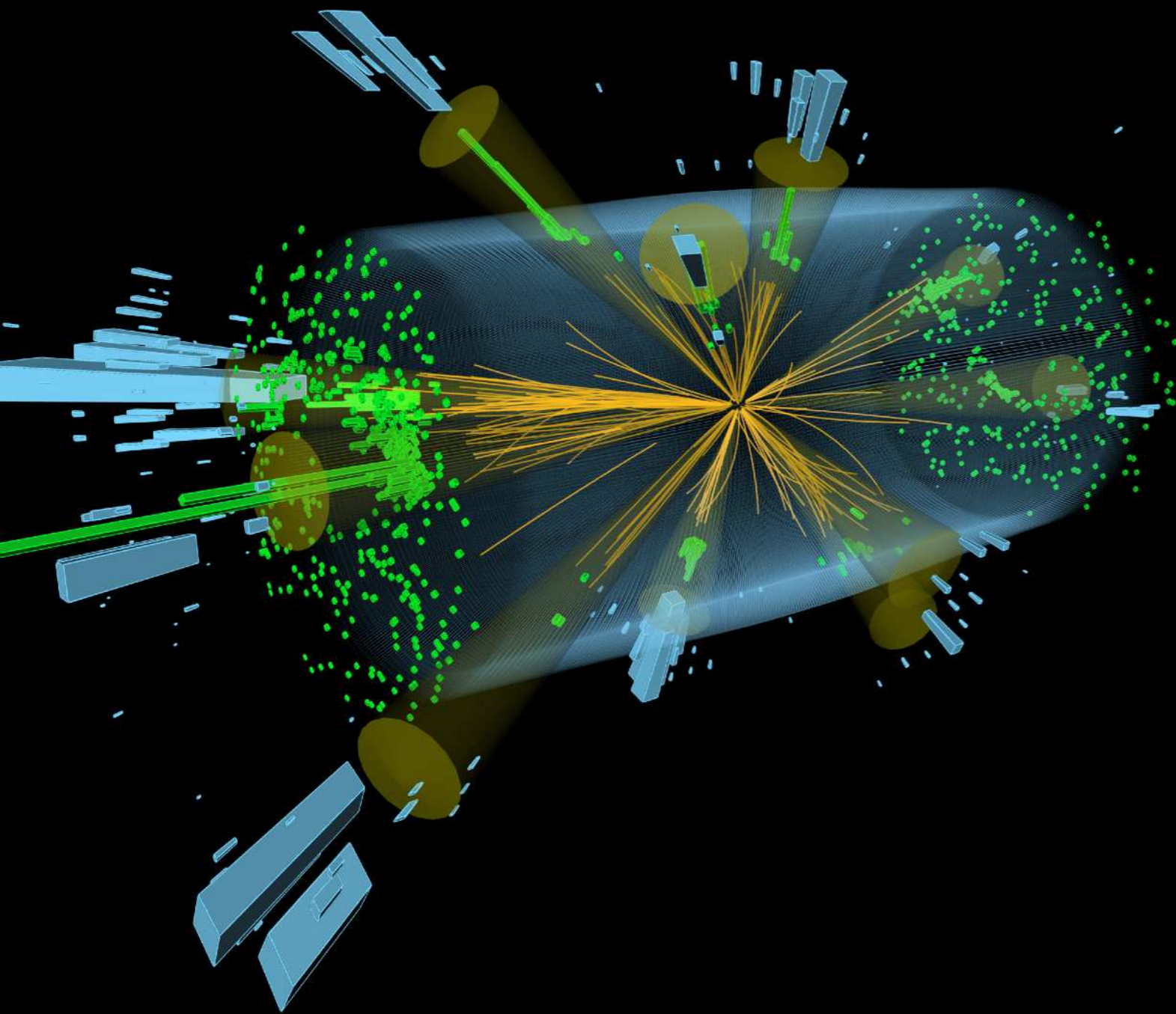
When are Events Similar?



What can be Geometrized?

Collider Event

Collection of points in momentum space



T E H M

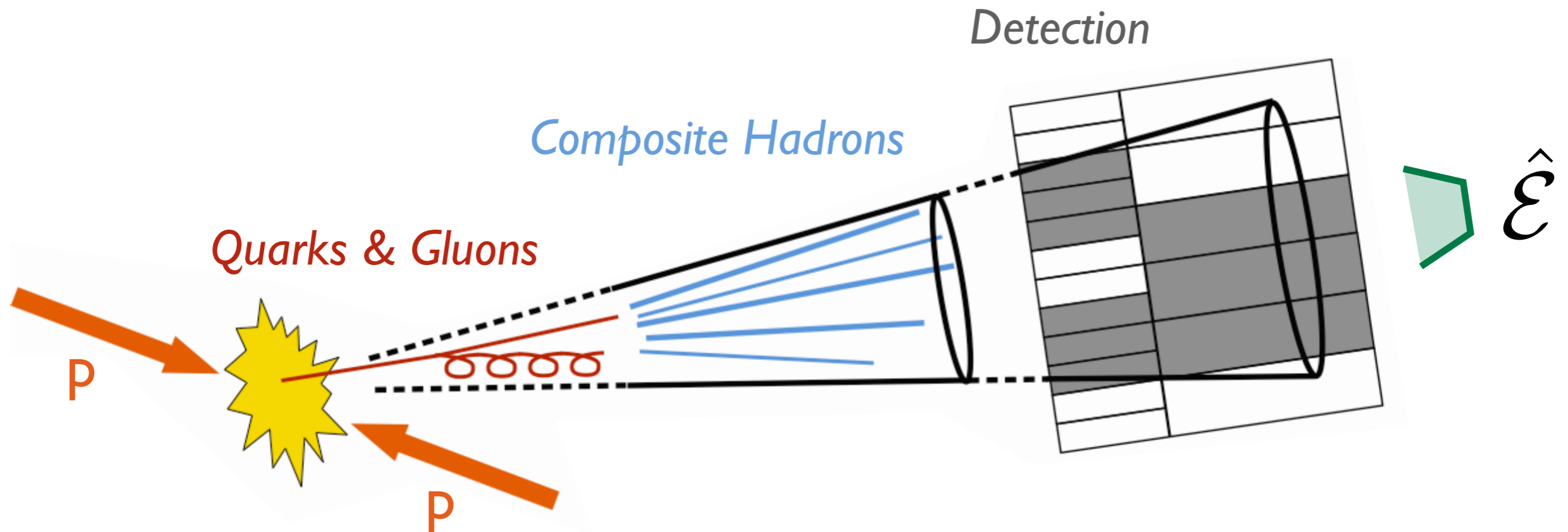
	●	γ	photon	elementary	
●	●	e^{\pm}	electron		
●	●	●	μ^{\pm}		muon
●	●	●	π^{\pm}	pion	composite
●	●	●	K^{\pm}	kaon	
	●	●	K_L^0	K-long	
●	●	●	p/\bar{p}	proton	
	●	●	n/\bar{n}	neutron	

elementary

composite

Jet Formation from QCD

Theory



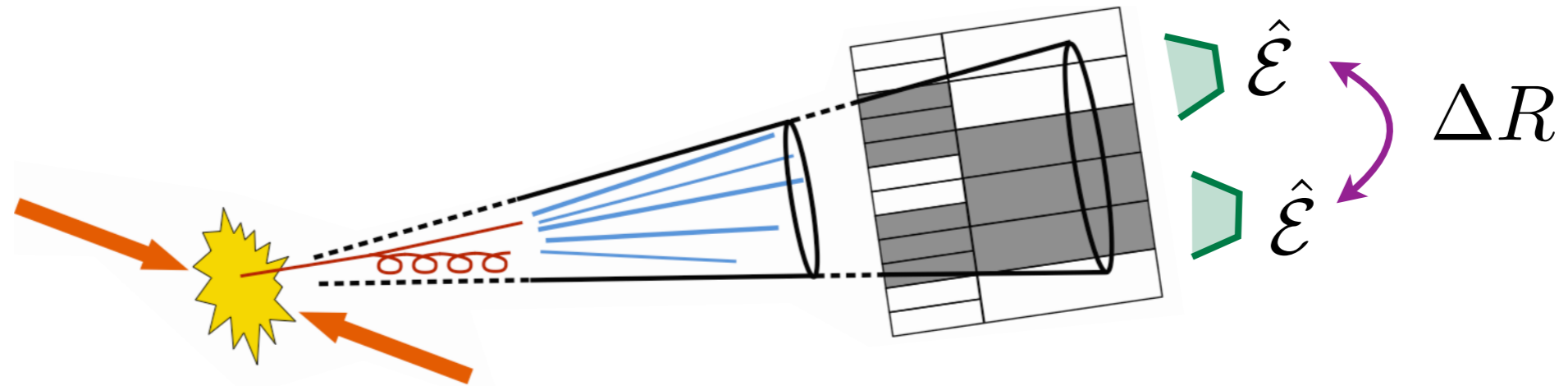
Energy Flow:

Robust to hadronization and detector effects
Well-defined for massless gauge theories

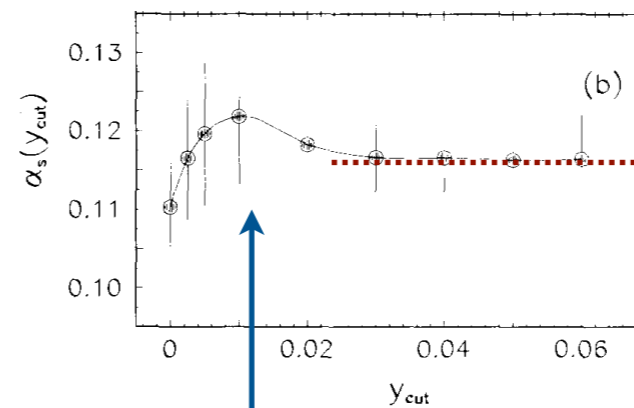
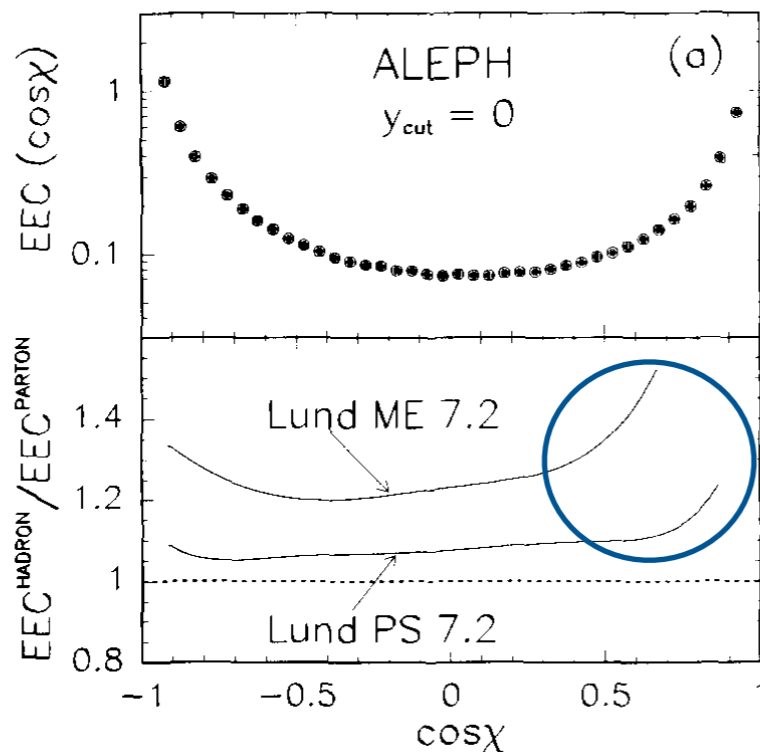
$$\hat{\mathcal{E}} \simeq \lim_{t \rightarrow \infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

[see e.g. Sveshnikov, Tkachov, [PLB 1996](#); Hofman, Maldacena, [JHEP 2008](#); Mateu, Stewart, [JDT, PRD 2013](#); Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, [PRL 2014](#); Chen, Moul, Zhang, Zhu, [PRD 2020](#)]

Energy-Energy Correlators



A long history in probing collinear dynamics of QCD

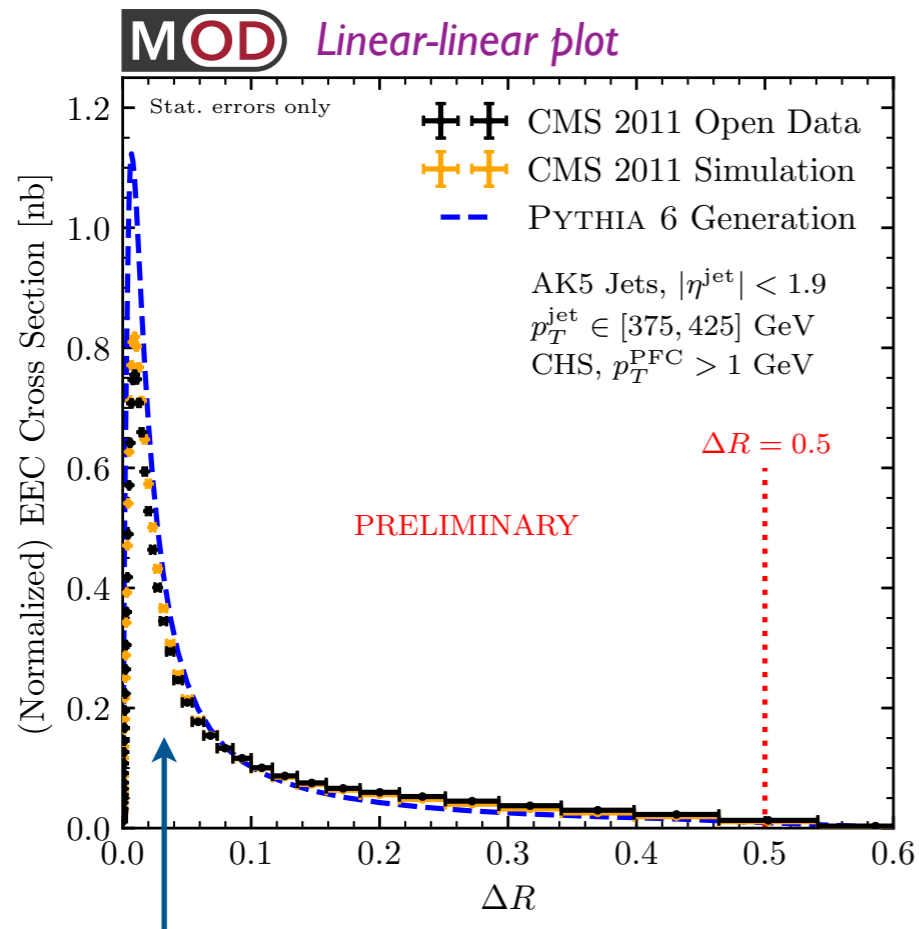


Extracting the strong coupling constant

Theoretical challenges with small angle (collinear) limit

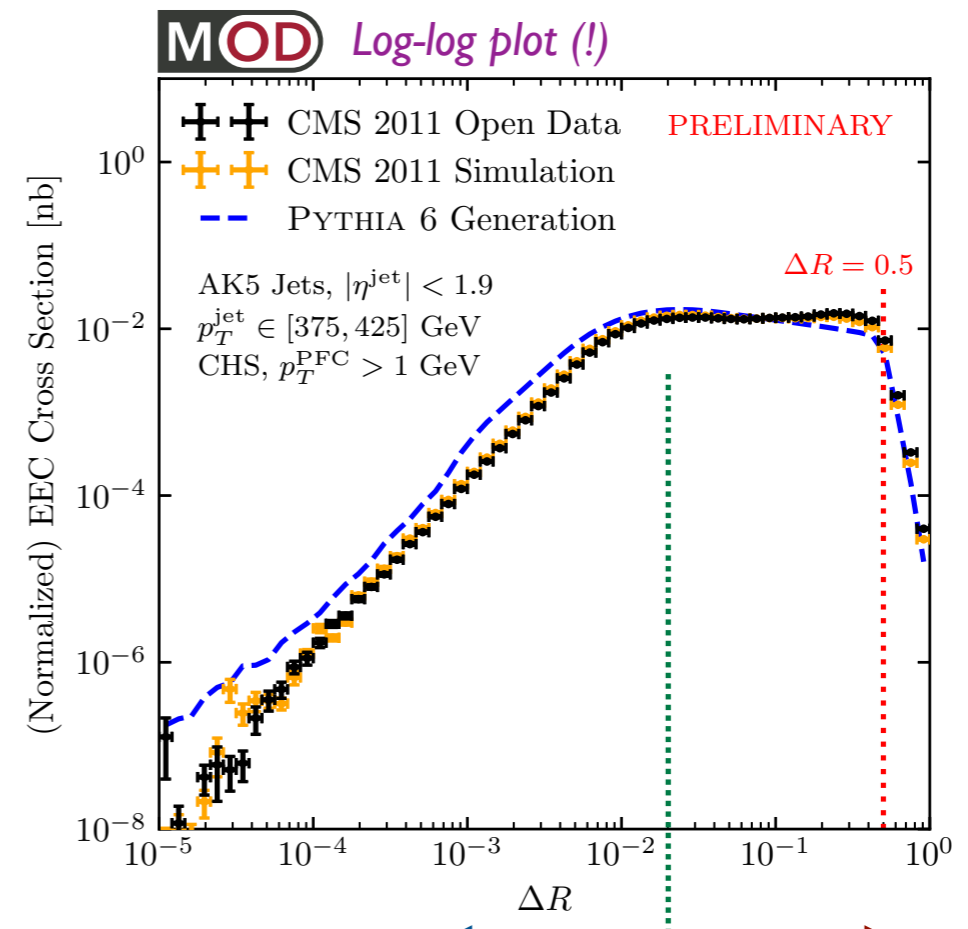
[Basham, Brown, Ellis, Love, *PRL* 1978; ALEPH, *PLB* 1991; see Chen, Mout, Zhang, Zhu, *PRD* 2020]

QCD Phase Transition in Jets?



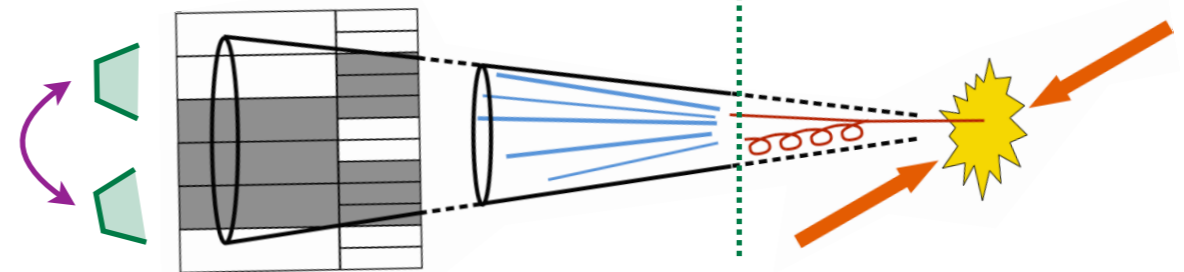
Are we learning something about small angle limit of QCD?

First Jet EEC Plot from the LHC (!)



Hadronic Phase

Partonic Phase



[Komiske, Mout, JDT, Zhu, in progress; see talks by Mout, BOOST 2019, BOOST 2020]

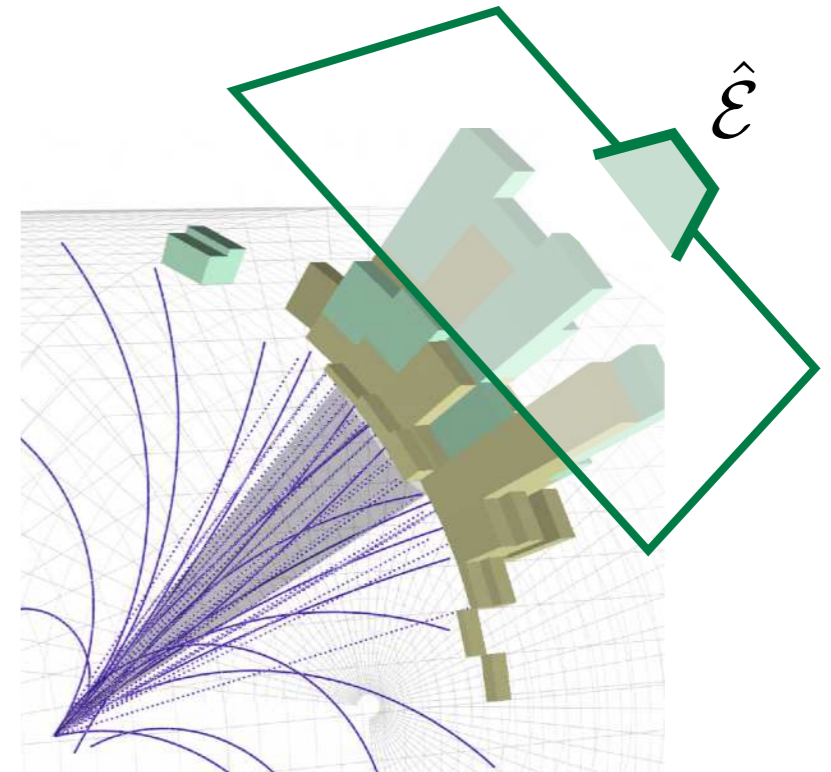


Jets as **Weighted Point Clouds**

- **Energy-Weighted Directions**

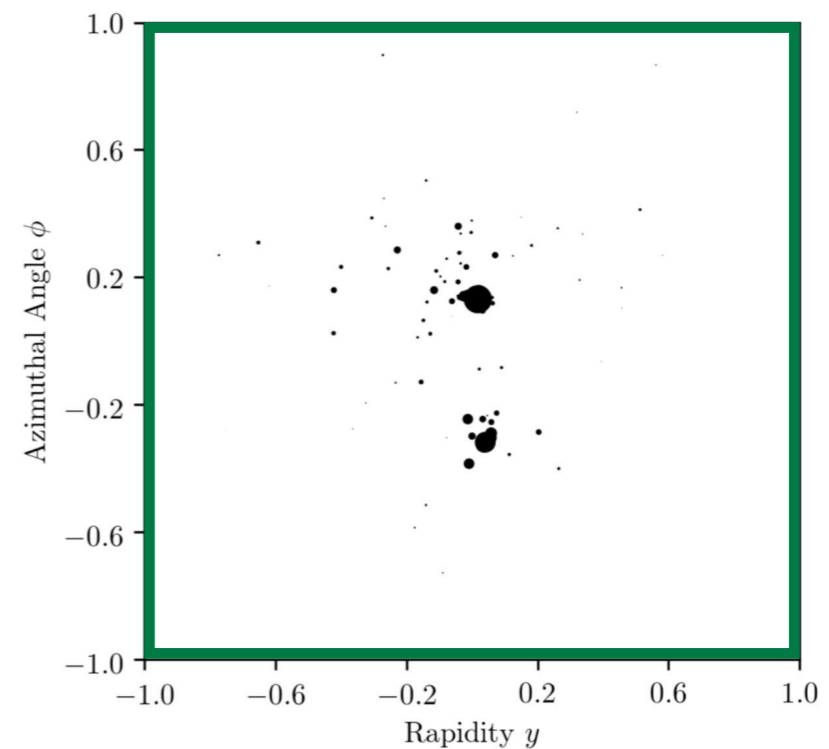
$$\vec{p} = \left\{ \underset{\substack{\uparrow \\ \text{Energy}}}{E}, \underbrace{\hat{n}_x, \hat{n}_y, \hat{n}_z}_{\text{Direction}} \right\}$$

(suppressing “unsafe” charge/flavor information)



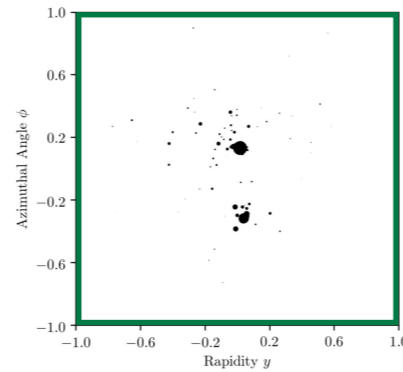
- Equivalently: **Energy Density**

$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} \underset{\substack{\uparrow \\ \text{Energy}}}{E_i} \delta^{(2)}(\hat{n} - \underset{\substack{\uparrow \\ \text{Direction}}}{\hat{n}_i})$$



Pause

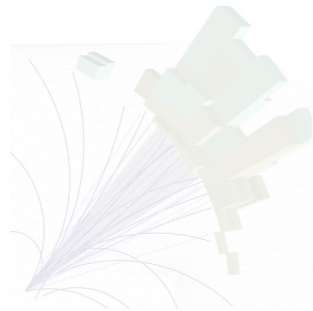
What is a Collider Event?



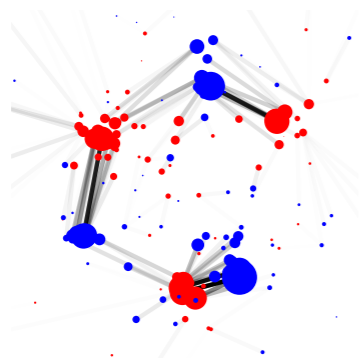
“Calo” Energy Density

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

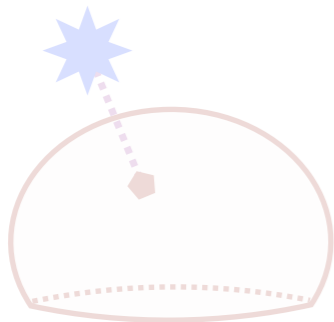
(see backup for relevance to ML)



What is a Collider Event?



When are Events Similar?

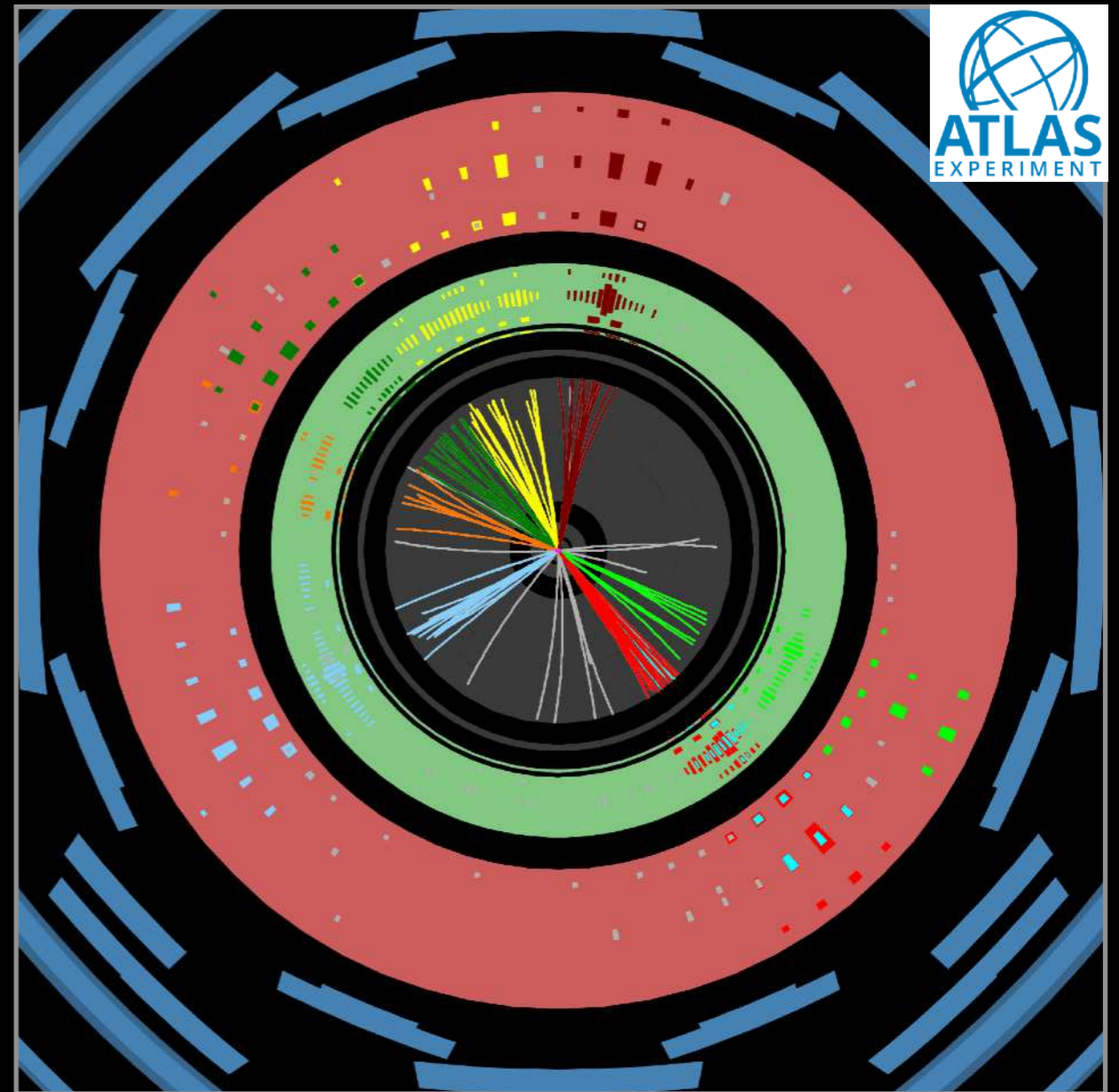
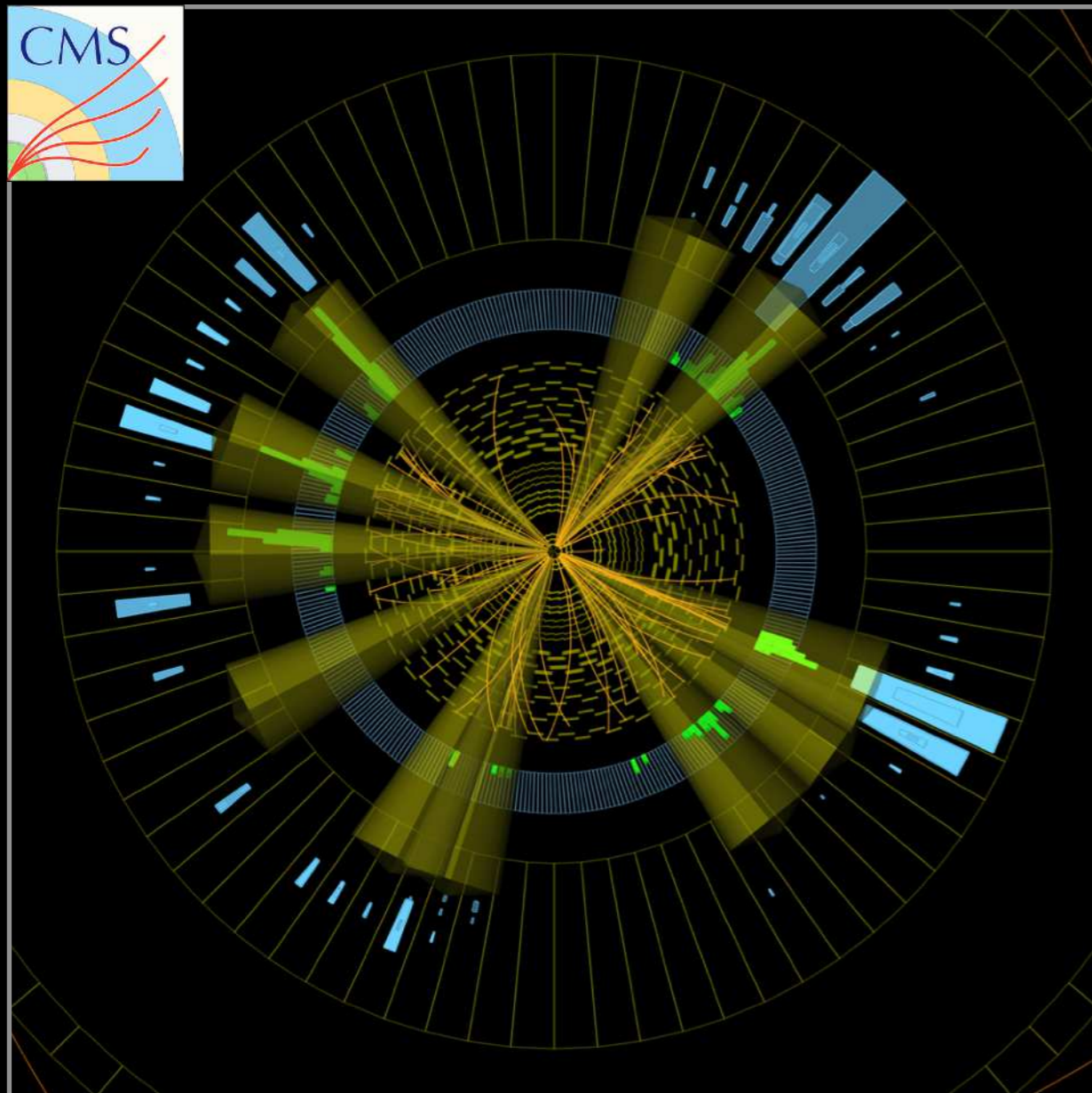
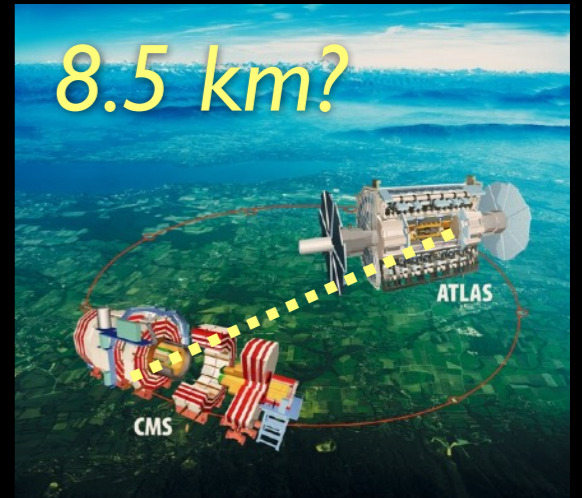


What can be Geometrized?

Two Collider Events

Two collections of points in (momentum) space

How “close” are these?



The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff x distance) to make one distribution look like another distribution



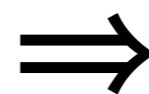
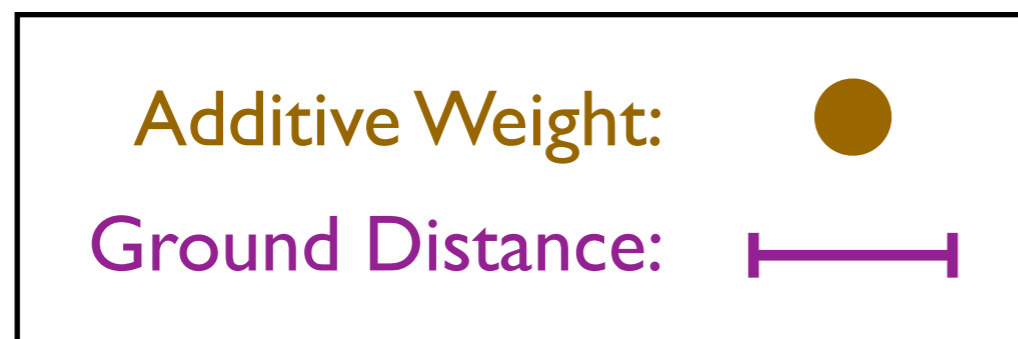
[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Vaserštejn, 1969; [Wikipedia](#)]

The Earth Mover's Distance

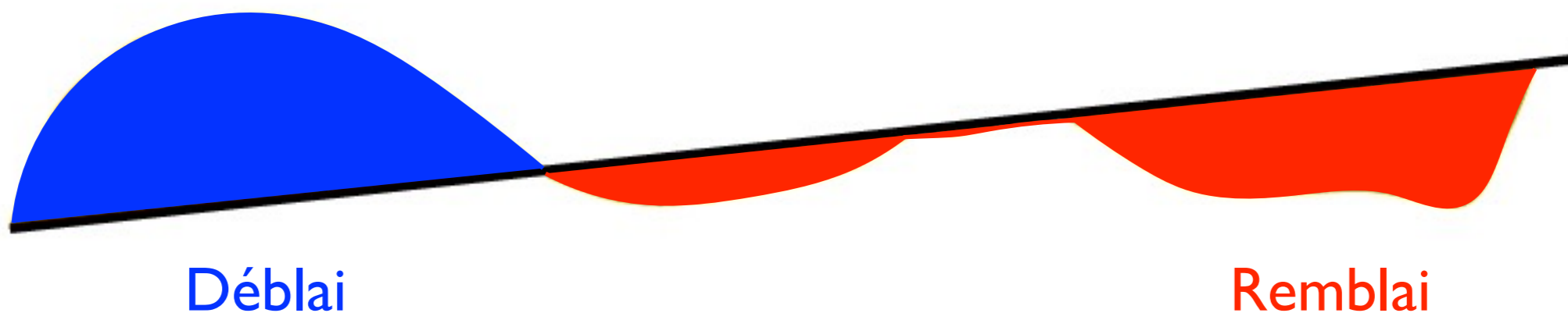
Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff x distance) to make
one distribution look like another distribution

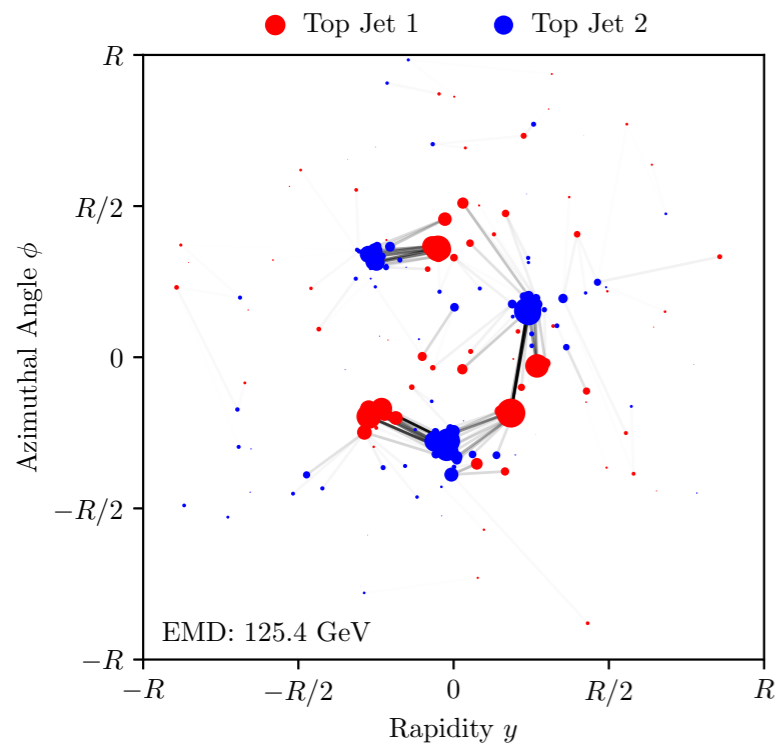
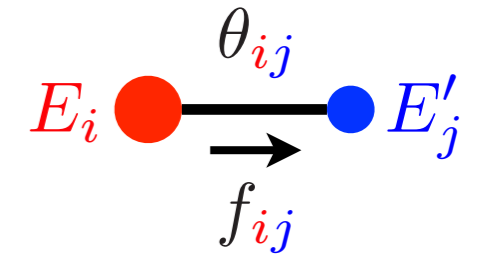


Distance Between
Distributions



[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Vaserštejn, 1969; [Wikipedia](#)]

The Energy Mover's Distance



Optimal transport between energy flows...

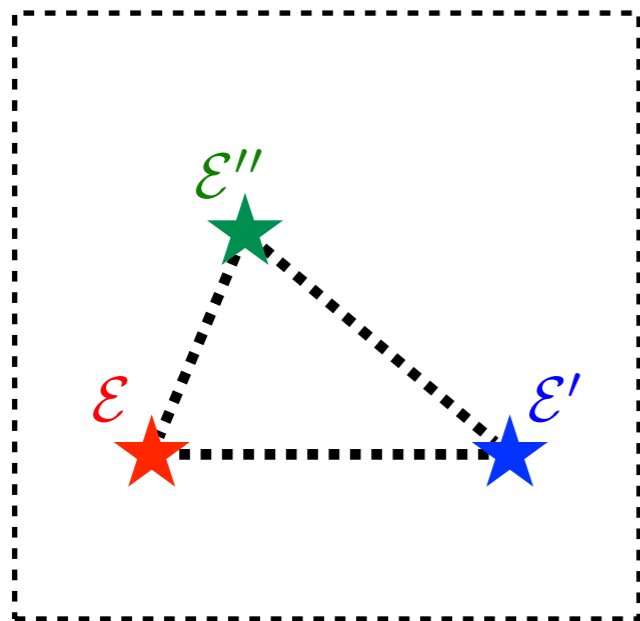
$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \underbrace{\sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R}}_{\text{Cost to move energy}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost to create energy}}$$

↑
in GeV

...defines a metric on the space of events

$$0 \leq \text{EMD}(\mathcal{E}, \mathcal{E}') \leq \text{EMD}(\mathcal{E}, \mathcal{E}'') + \text{EMD}(\mathcal{E}', \mathcal{E}'')$$

(assuming $R \geq \theta_{\max}/2$, i.e. $R \geq$ jet radius for conical jets)

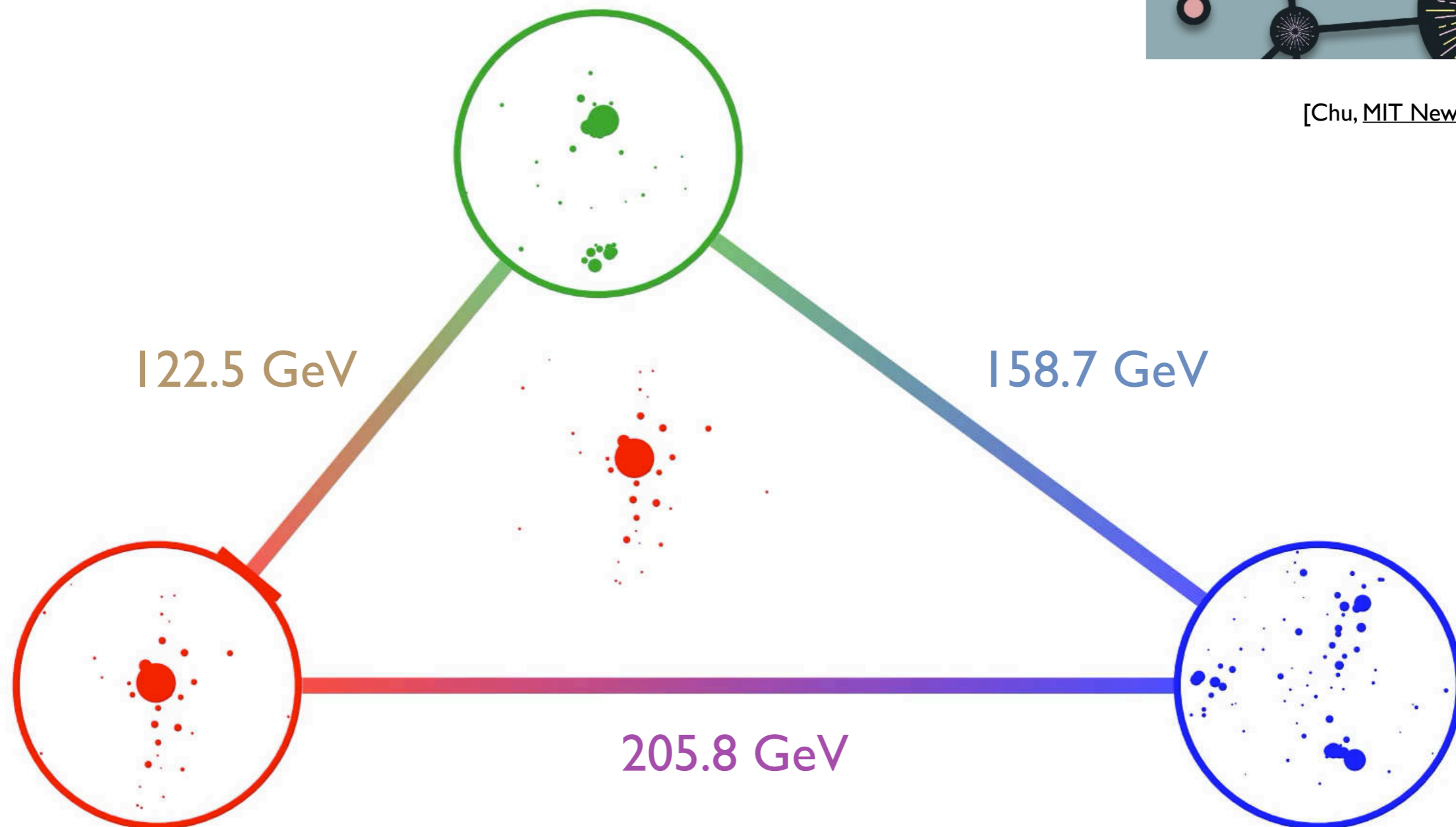


[Komiske, Metodiev, JDT, [PRL 2019](#);
see also Pele, Werman, [ECCV 2008](#); Pele, Taskar, [GSI 2013](#);
[see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, [arXiv 2020](#)]
[see computational speed up in Cai, Cheng, Craig, Craig, [arXiv 2020](#)]

Similarity of Three Energy Flows?



[Chu, [MIT News July 2019](#)]



[Komiske, Metodiev, JDT, [PRL 2019](#); code at Komiske, Metodiev, JDT, [energyflow.network](#);
see alternative graph network approach in Mullin, Pacey, Parker, White, Williams, [arXiv 2019](#)]

Dimensionality of Space of Jets



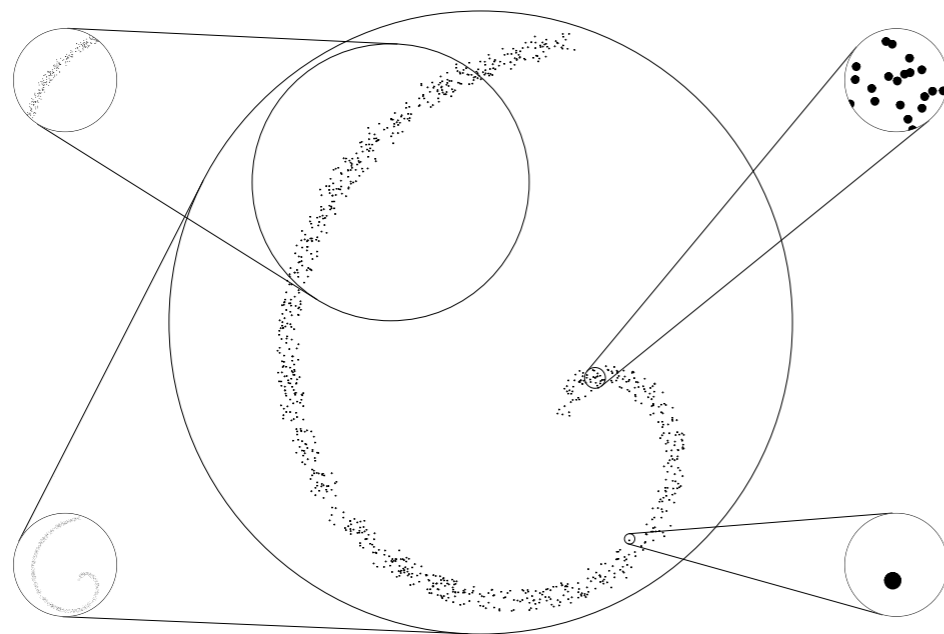
$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

$$\Rightarrow \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

dim ≈ 1

dim ≈ 2

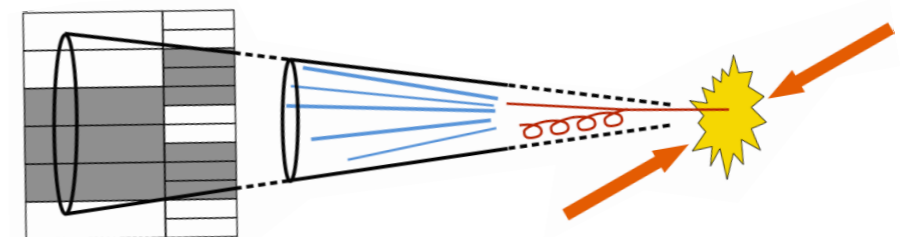
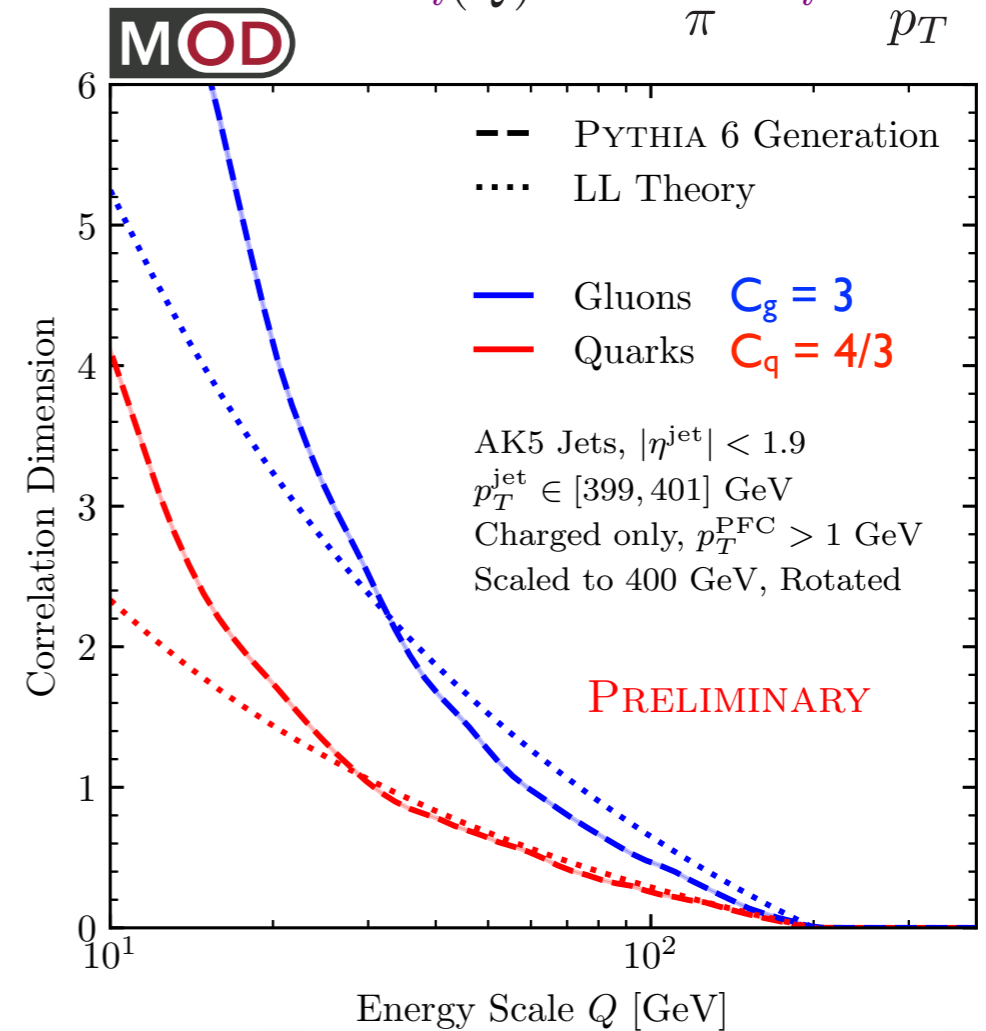


dim ≈ 2

dim ≈ 0

(eventually 0)

$$\text{dim}_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T}$$



Dimensionality of Space of Jets



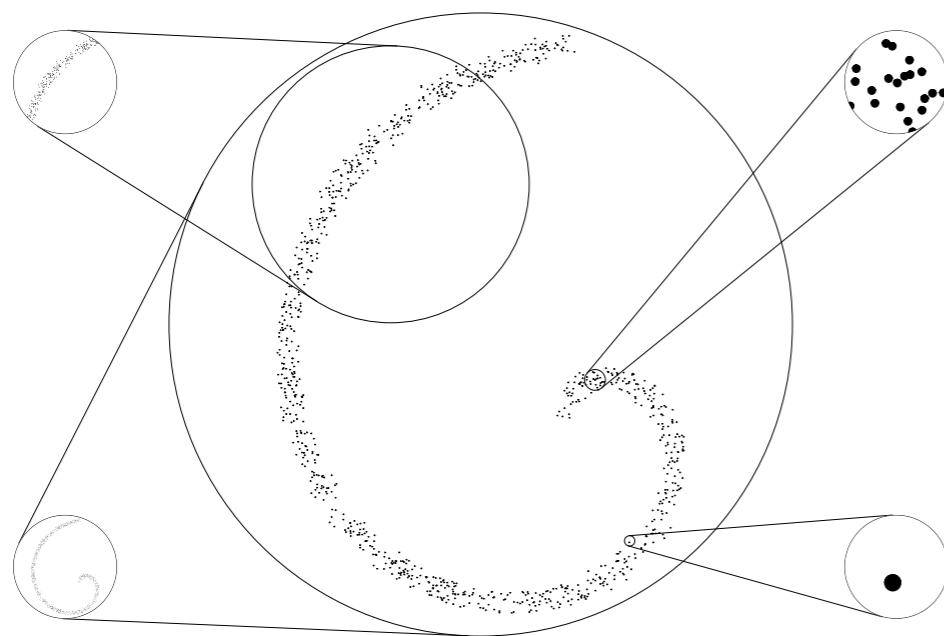
$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

$$\Rightarrow \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

dim ≈ 1

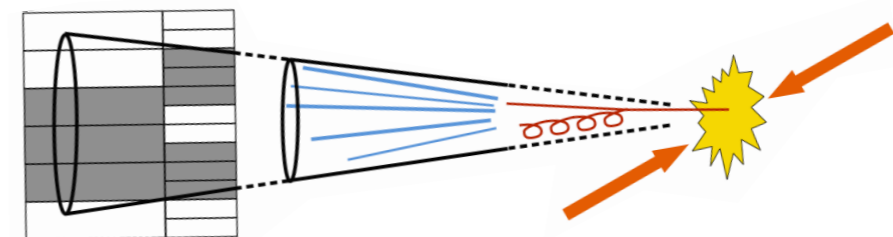
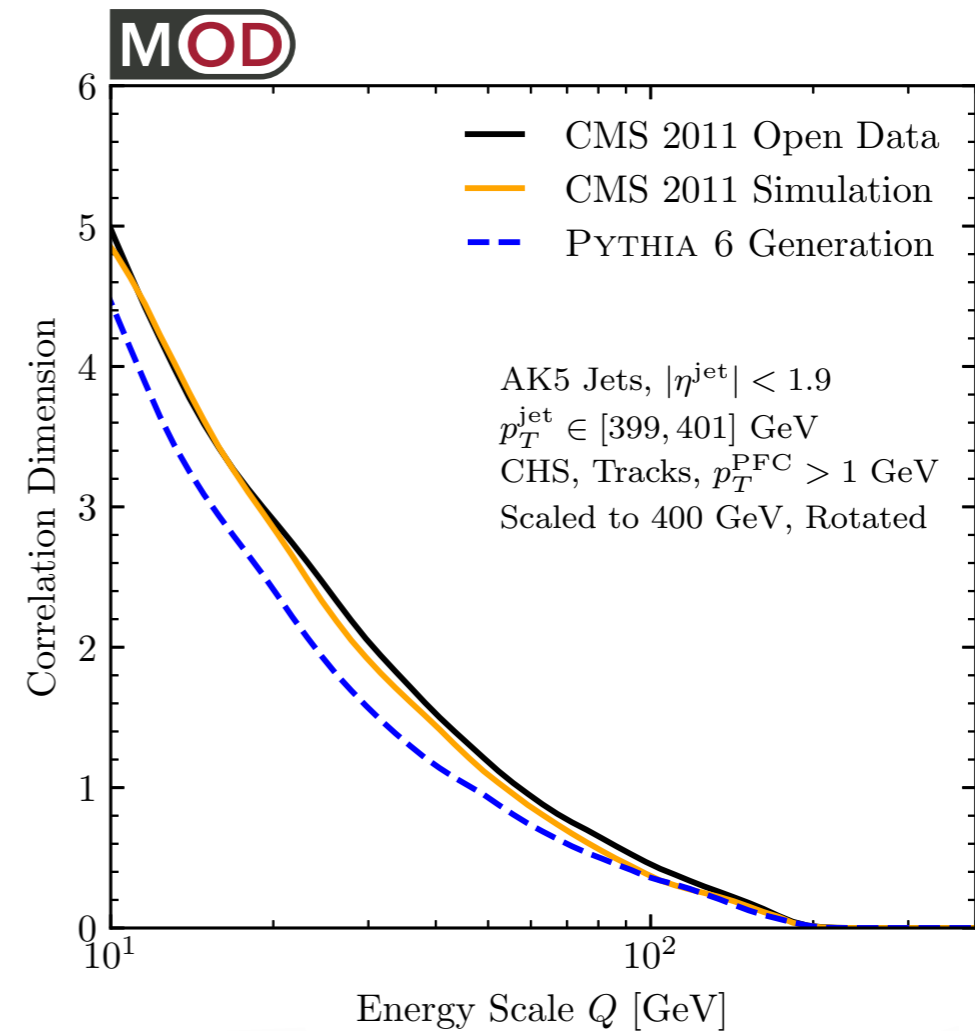
dim ≈ 2



dim ≈ 2

dim ≈ 0

(eventually 0)

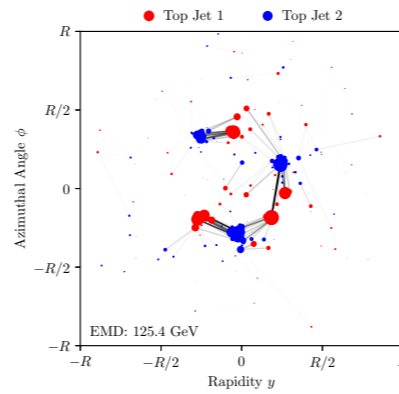


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using CMS Open Data]



Pause

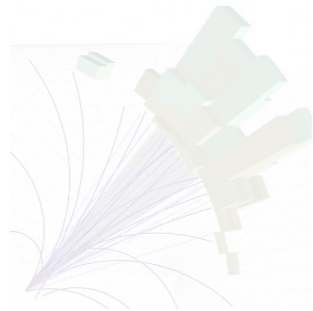
When are Events Similar?



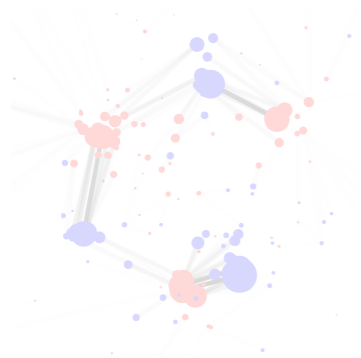
Small Energy Mover's Distance

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_i E_i - \sum_j E'_j \right|$$

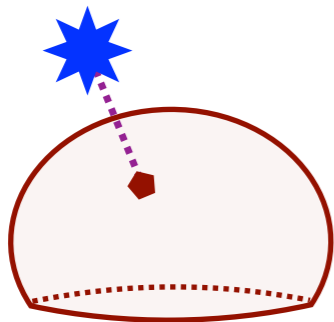
(see backup for more applications)



What is a Collider Event?



When are Events Similar?

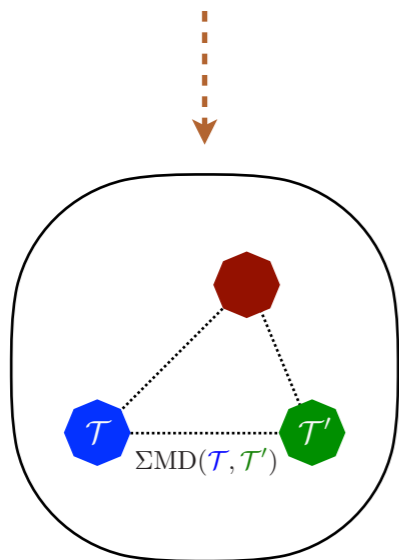


What can be Geometrized?

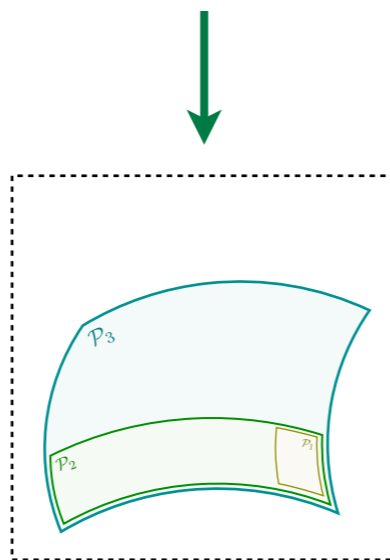
Master Formula for Collider Physics

$$\sigma_{\text{obs}} \simeq \frac{1}{2E_{\text{CM}}^2} \sum_{n=2}^{\infty} \int d\Phi_n |\mathcal{M}_{AB \rightarrow 12\dots n}|^2 f_{\text{obs}}(\Phi_n)$$

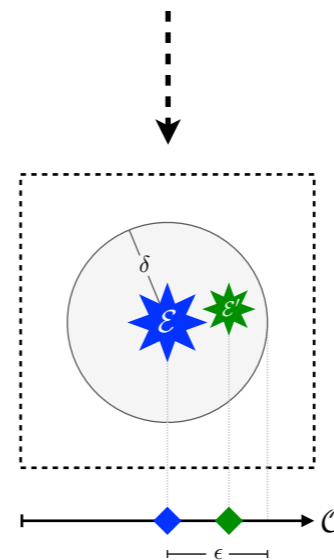
cross section



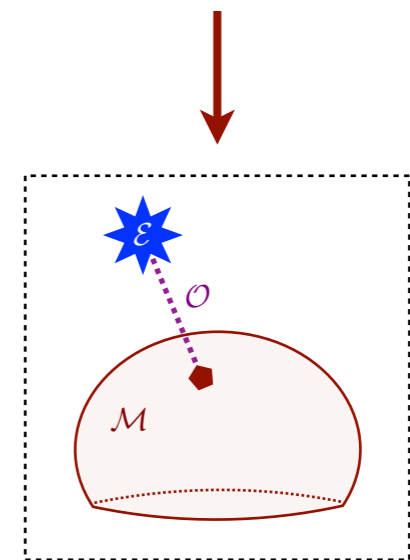
phase space



amplitude



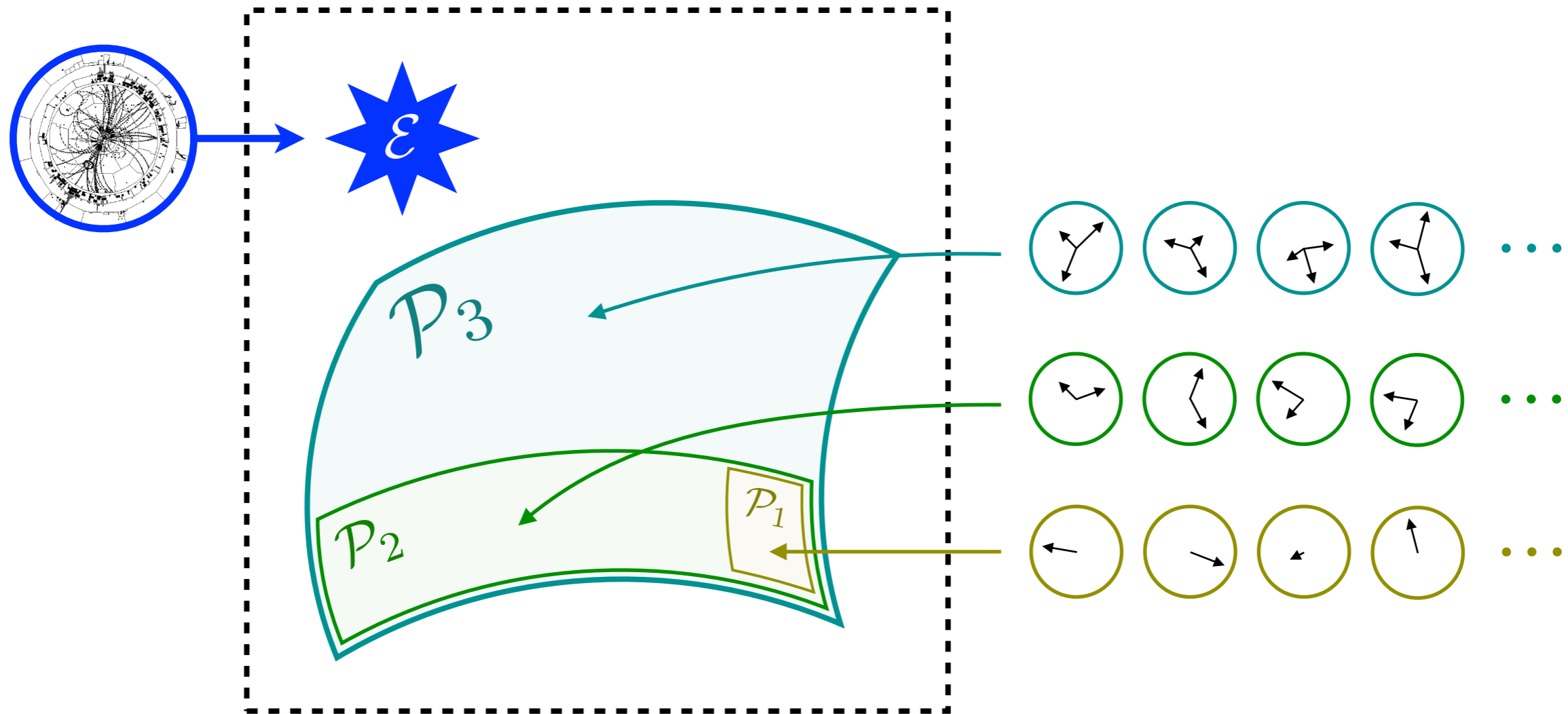
observable



Introducing N-particle Manifolds

$$\sum_{n=2}^{\infty} \int d\Phi_n$$

\mathcal{P}_N = set of all N-particle configurations



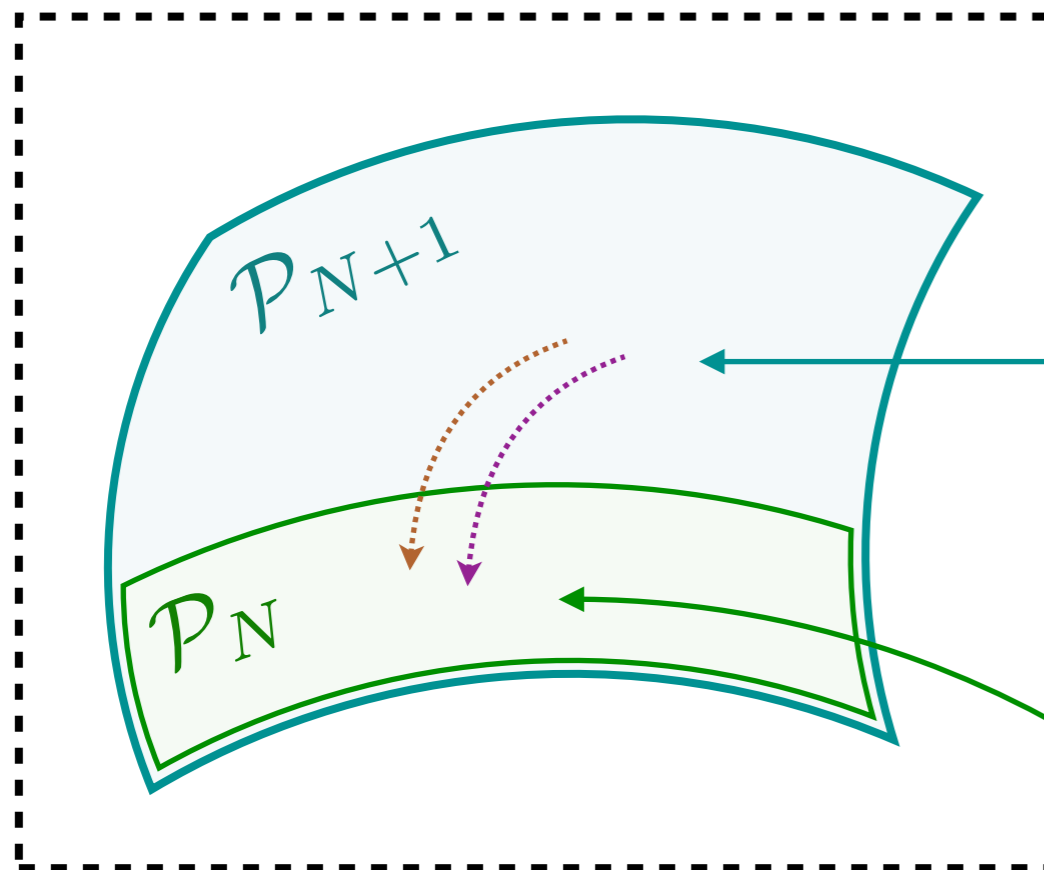
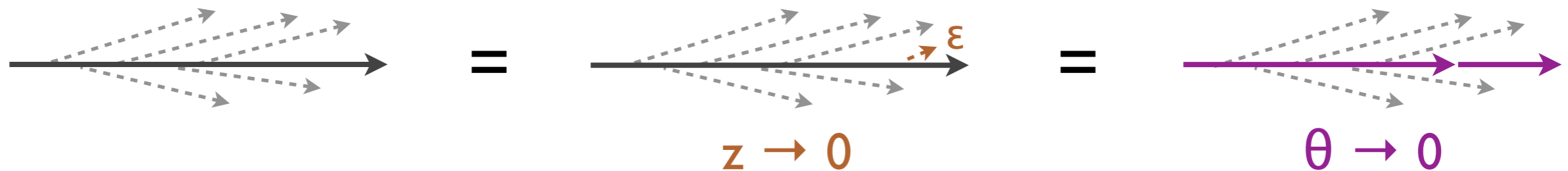
$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \dots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

[see related discussion in Larkoski, Melia, PRD 2020]

When are Two Events **the Same**?

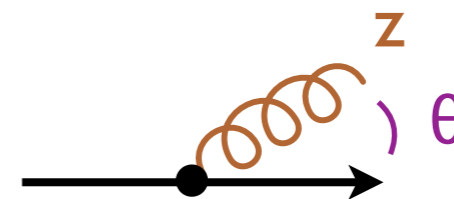
$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

Energy Flow unchanged by infinitesimal **soft/collinear** emissions



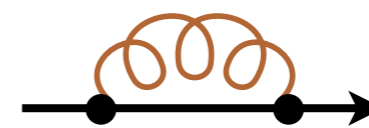
Infrared divergences “live” together!

Real:



$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{dz}{z} \frac{d\theta}{\theta}$$

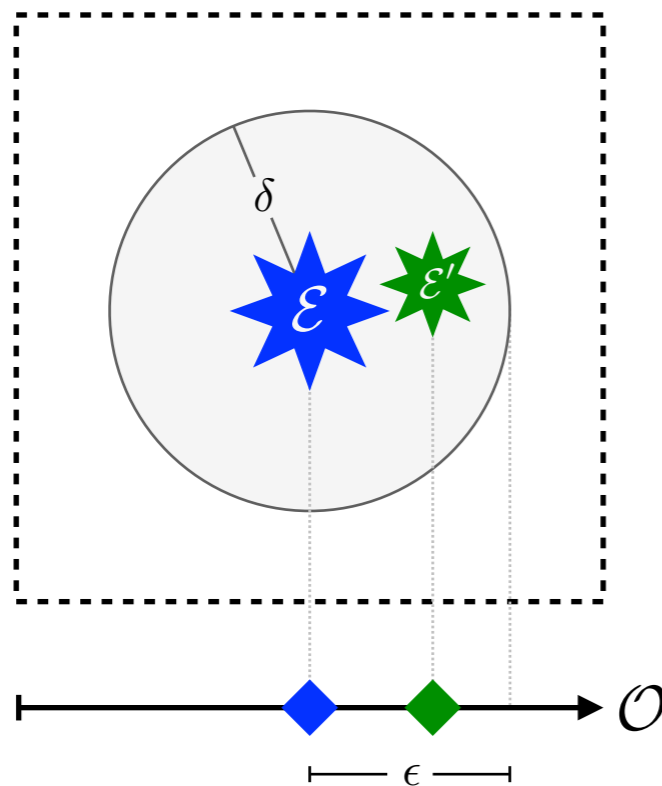
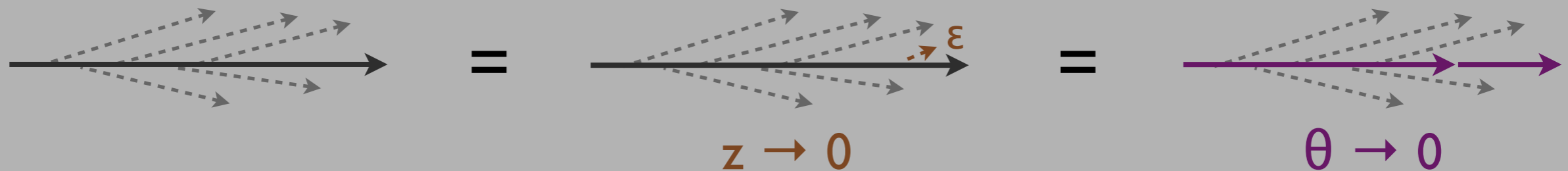
Virtual:



When are Two Events the Same?

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

Energy Flow unchanged by infinitesimal *soft/collinear* emissions



Infrared & Collinear Safety

≈ calculable in perturbative quantum field theory

is^* ← (see backup for subtleties)

Continuity in EMD Space

[Komiske, Metodiev, JDT, JHEP 2020]

[Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]

[see also Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

*EMD seems to define the “natural”
geometry for massless gauge theories*

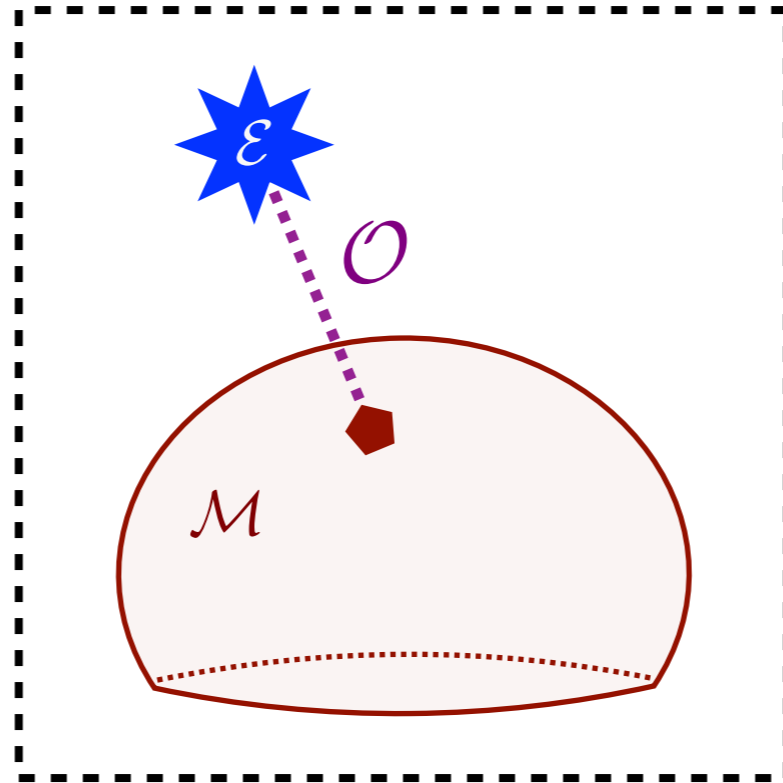
Open question: Can you define $|\mathcal{M}_{AB \rightarrow 12 \dots n}|^2$ directly in this space?

What does it mean to “integrate” in this space?

Manifolds for Observables

$f_{\text{obs}}(\Phi_n)$

One Event



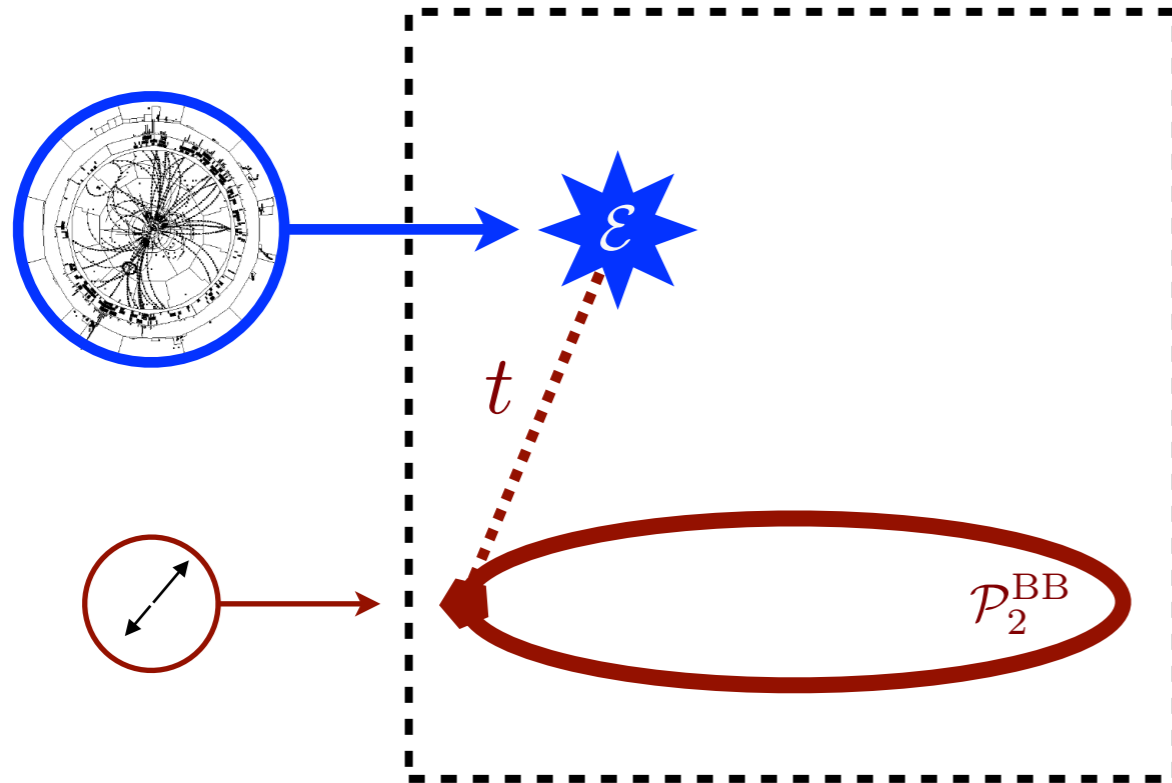
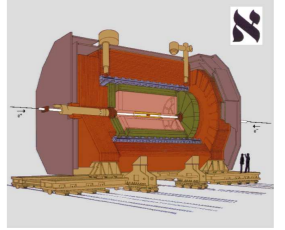
Set of Events

Distance of Closest Approach \Rightarrow Observable

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}(\mathcal{E}, \mathcal{E}')$$

E.g. Thrust

How dijet-like is an event?

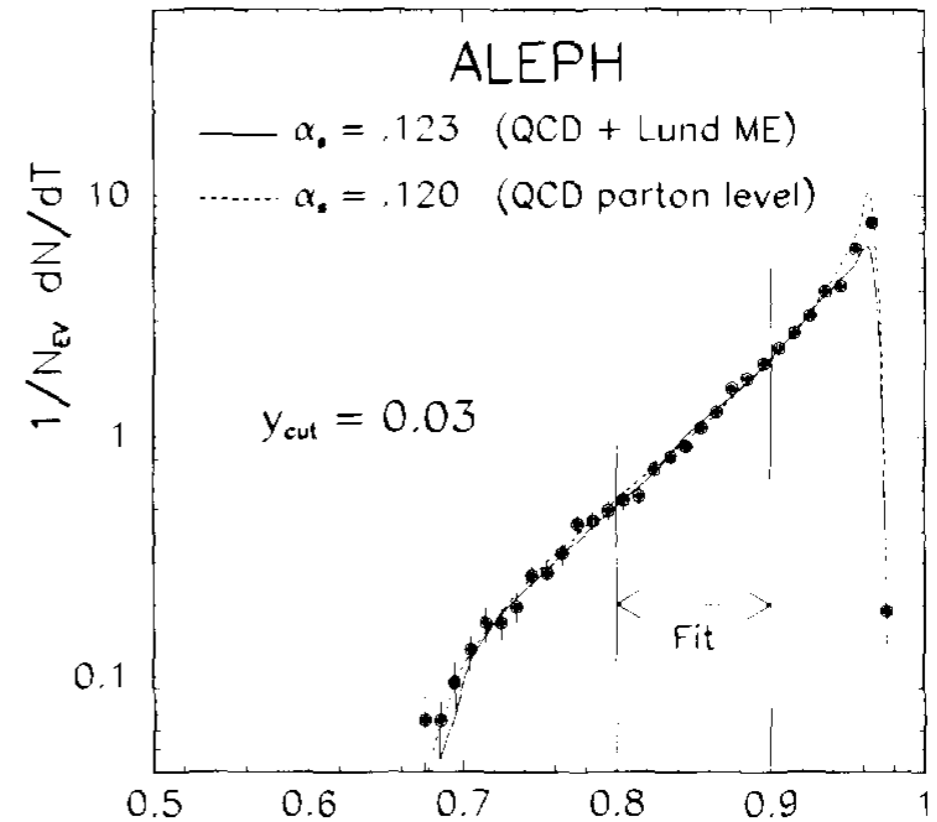


All Back-to-Back Two Particle Configurations

$$\mathcal{P}_2^{\text{BB}} = \left\{ \begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \end{array} \right\}$$

(using $\beta=2$ EMD variant)

$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_2(\mathcal{E}, \mathcal{E}')$$



$$1 - \frac{t}{2E_{\text{CM}}}$$

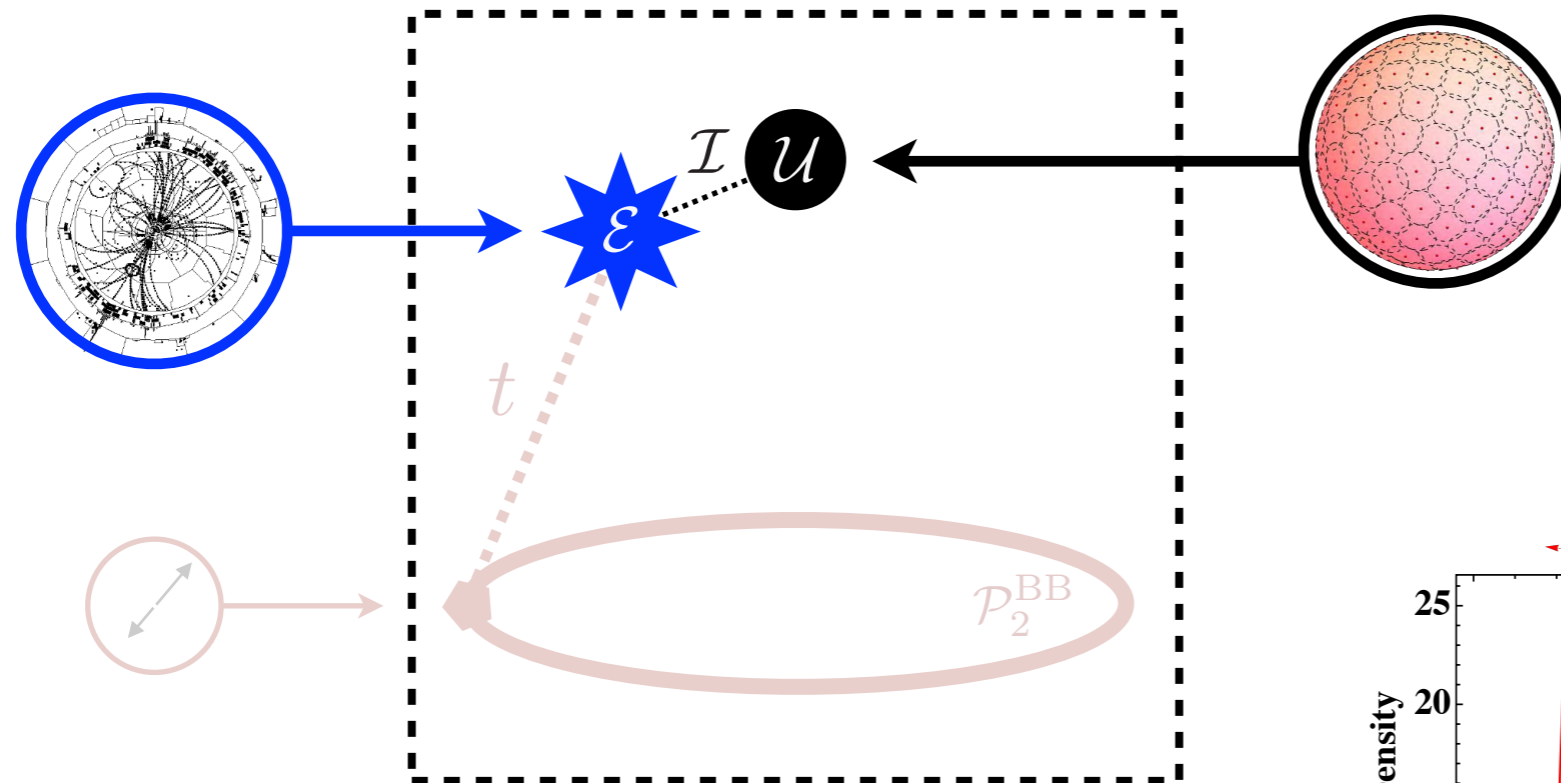
$$\text{cf. } T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$$

[Komiske, Metodiev, JDT, JHEP 2020]

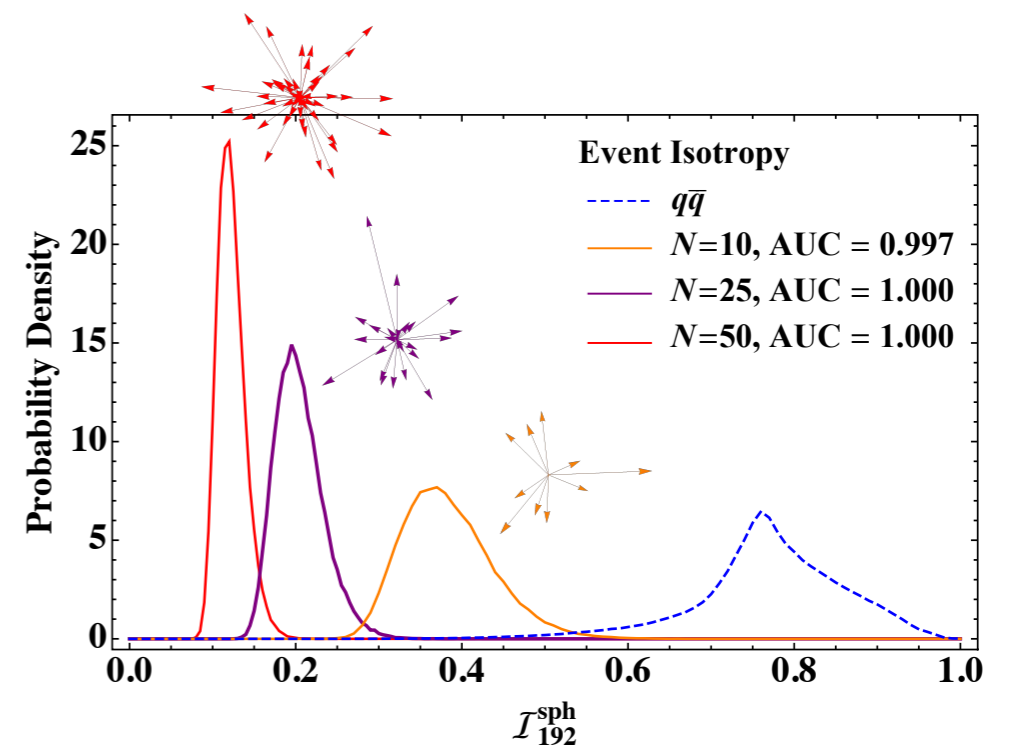
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

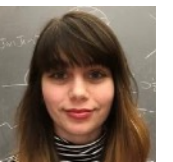
How isotropic is an event?



$$\mathcal{I}(\mathcal{E}) = \text{EMD}(\mathcal{E}, \mathcal{U})$$

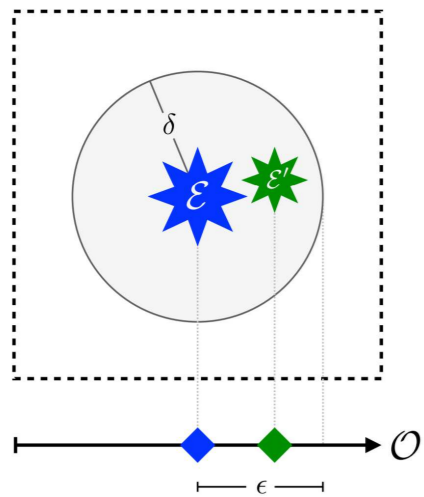


[Cesarotti, JDT, JHEP 2020;
see also Cesarotti, Reece, Strassler, arXiv 2020]



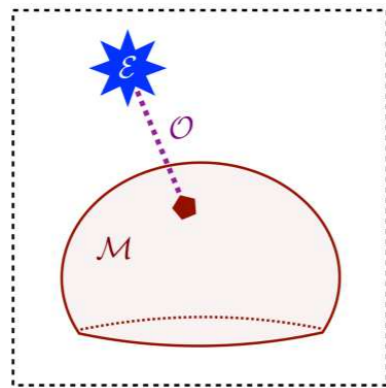
Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events



Taming infinities

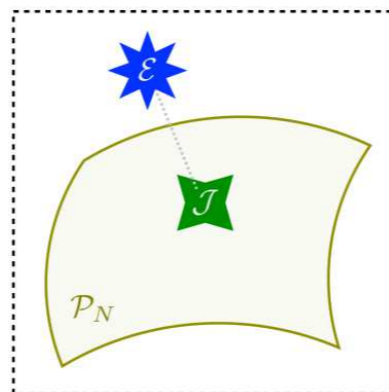
Event shapes are distances from events to manifolds.



$$O(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

Event Shapes

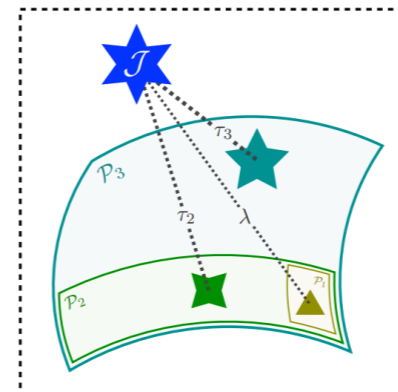
Jets are projections to few-particle manifolds.



$$J = \operatorname{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

Jet Algorithms

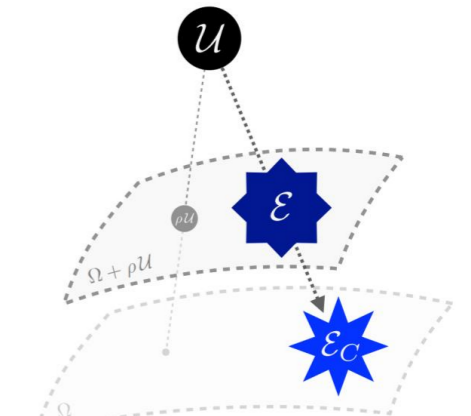
Substructure resolves emissions within the jet.



$$\tau(J) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{E}')$$

Jet Substructure

Pileup mitigation moves away from uniform radiation.



$$\mathcal{E}_C = \operatorname{argmin}_{\mathcal{E}'} \text{EMD}(\mathcal{E}, \mathcal{E}' + \rho \mathcal{U}).$$

Pileup

1960

1962-1964

Infrared Safety

[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977

Thrust, Sphericity

[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993

k_T jet clustering

[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998

C/A jet clustering

[Wobisch, Wengler, 1998]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997]

2010-2015

N-(sub)jettiness, X Cone

[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019

Constituent Subtraction

[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

2020

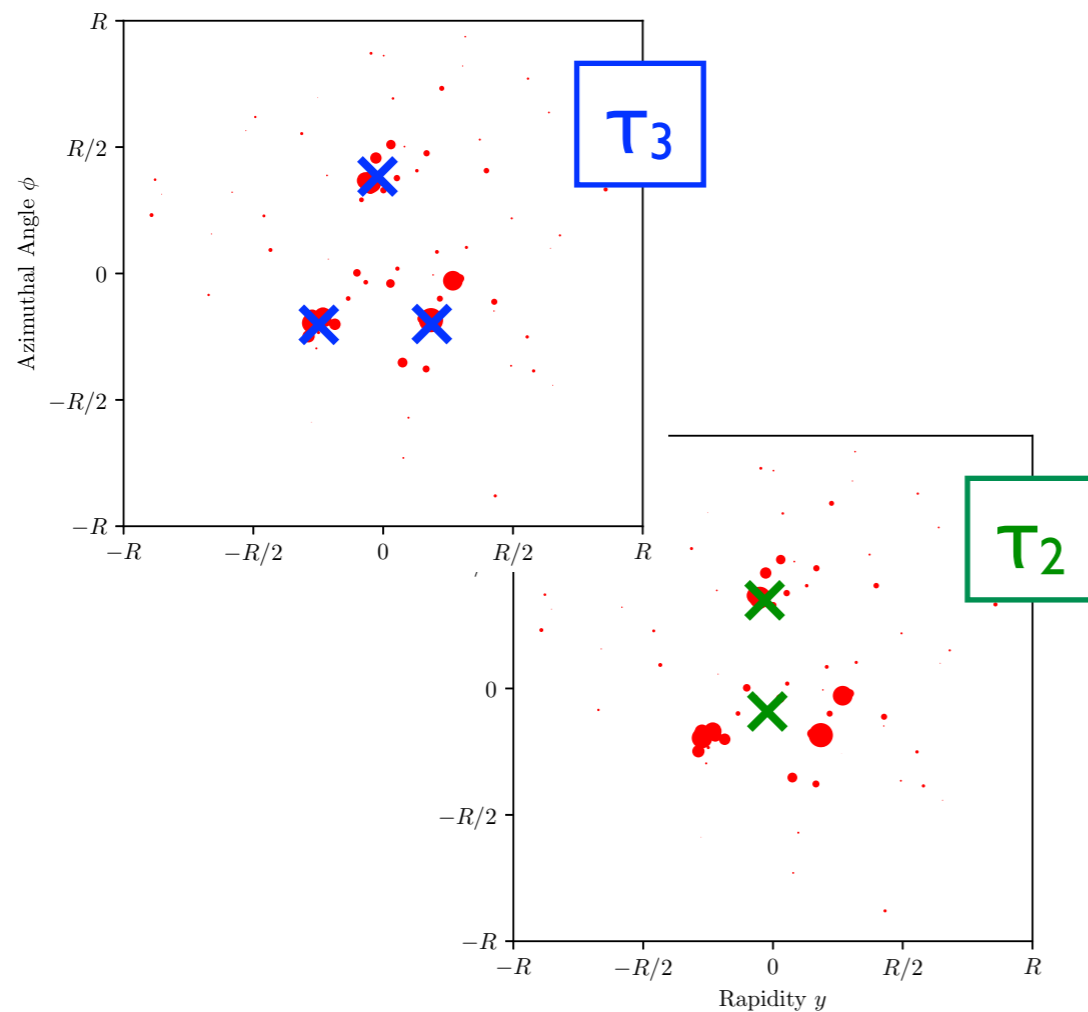
And many more!

[timeline from Eric Metodiev]

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade...

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_i E_i \min \{ \theta_{1,i}, \theta_{2,i}, \dots, \theta_{N,i} \}$$



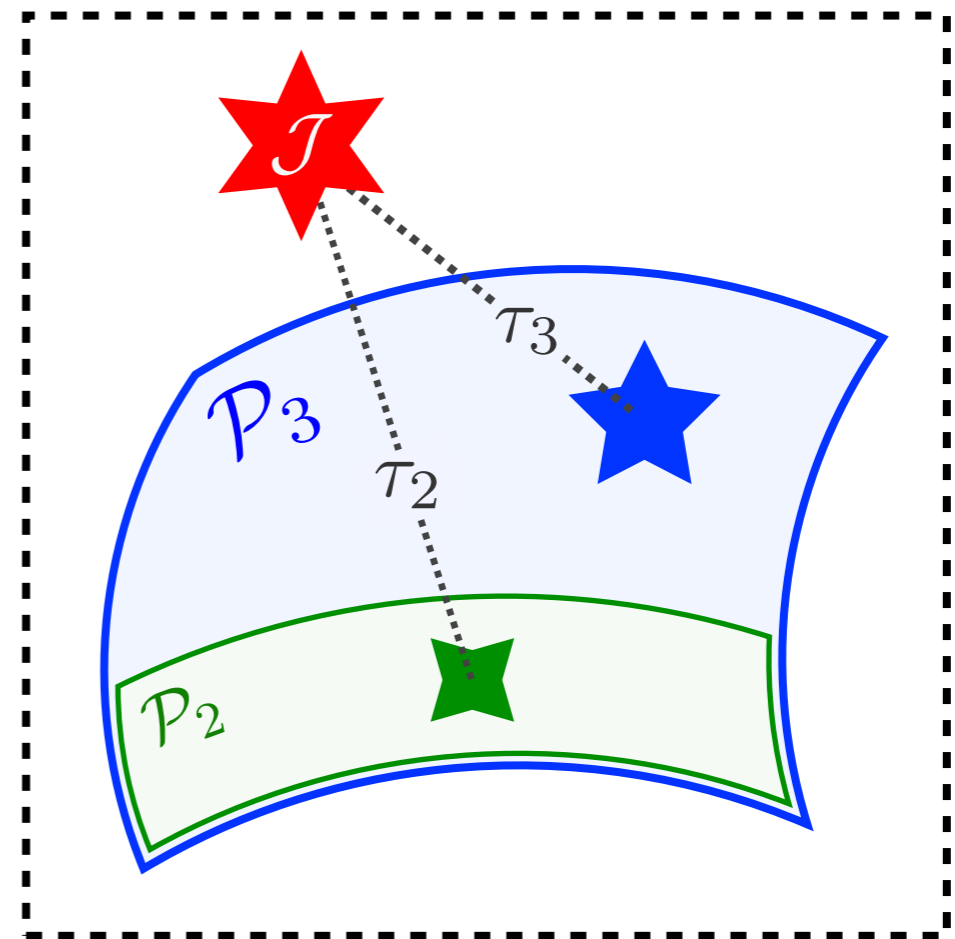
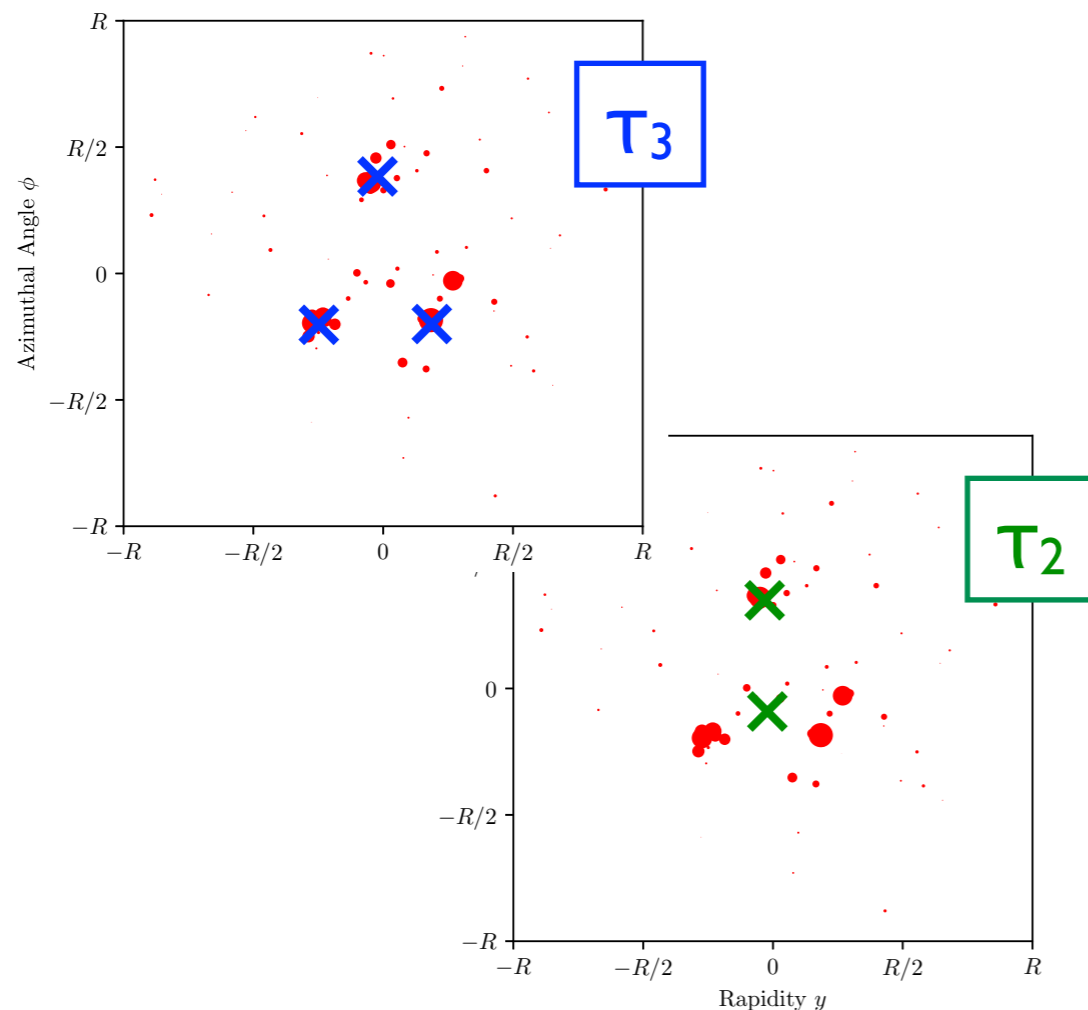
[JDT, Van Tilburg, [JHEP 2011](#), [JHEP 2012](#);
based on Brandt, Dahmen, [ZPC 1979](#); Stewart, Tackmann, Waalewijn, [PRL 2010](#)]



N-subjettiness = Point to Manifold EMD

...is secretly an optimal transport problem

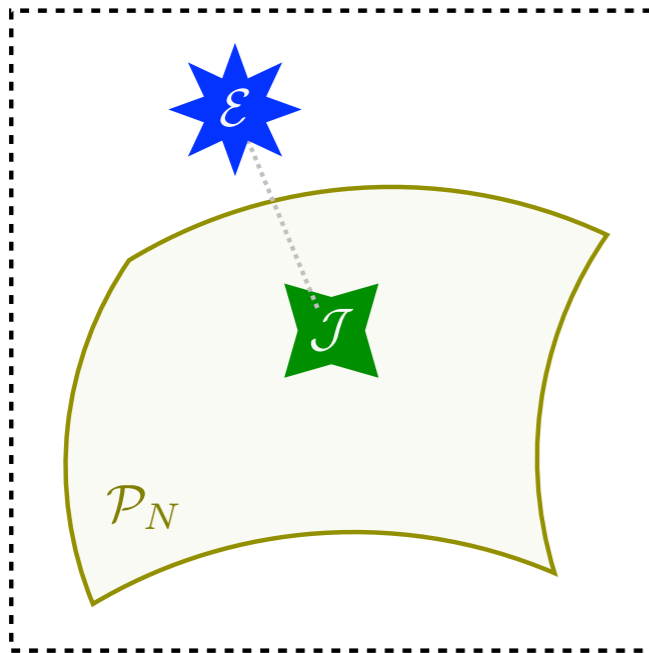
$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \text{EMD}(\mathcal{J}, \mathcal{J}')$$



[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]



More Fun with N-particle Manifolds



N-jettiness

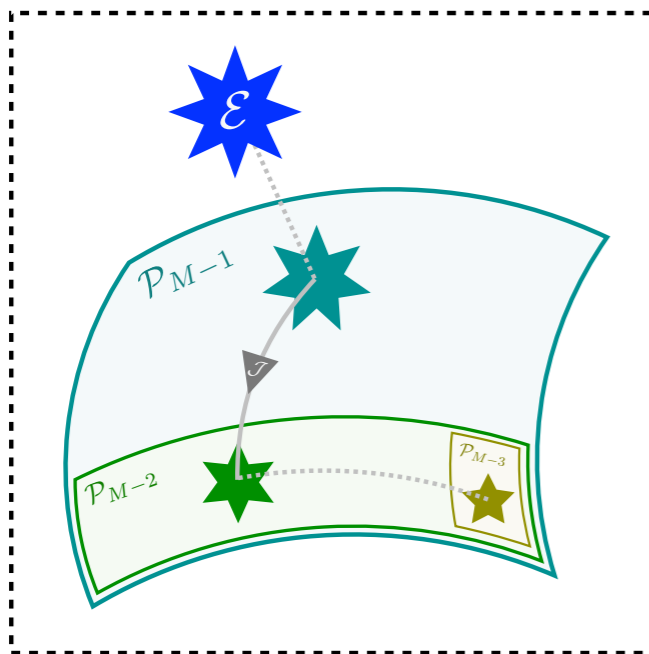
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, JHEP 2015]



Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

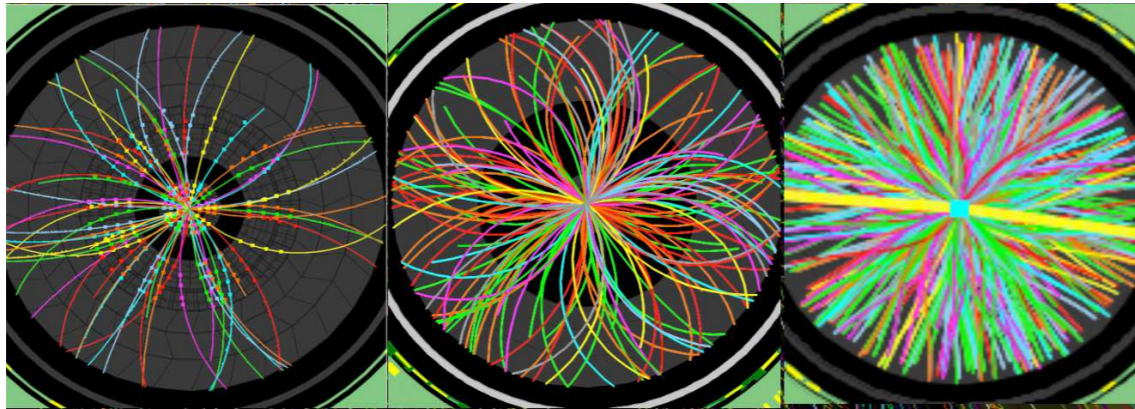
[Catani, Dokshitzer, Seymour, Webber, NPB 1993; Ellis, Soper, PRD 1993]

[Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Wobisch, Wengler, arXiv 1999]

[Butterworth, Couchman, Cox, Waugh, CPC 2003; Larkoski, Neill, JDT, JHEP 2014]

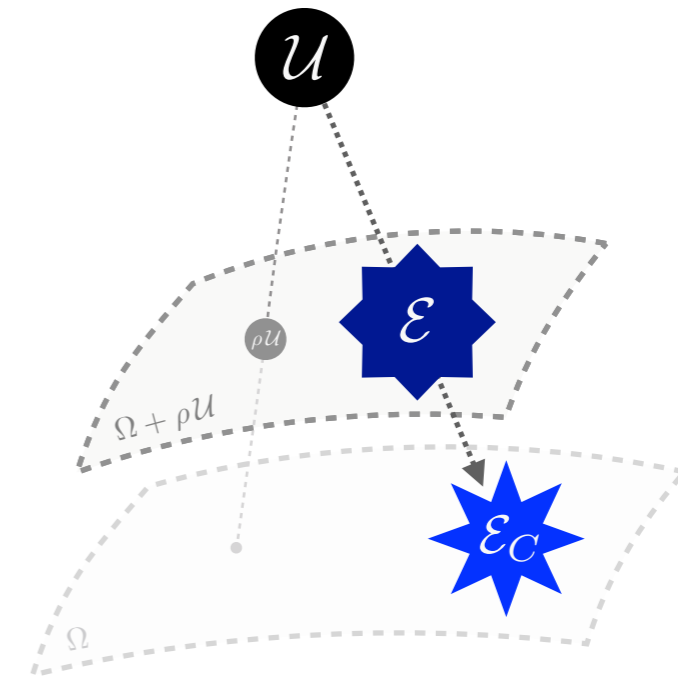
[Komiske, Metodiev, JDT, JHEP 2020]

Pileup Mitigation



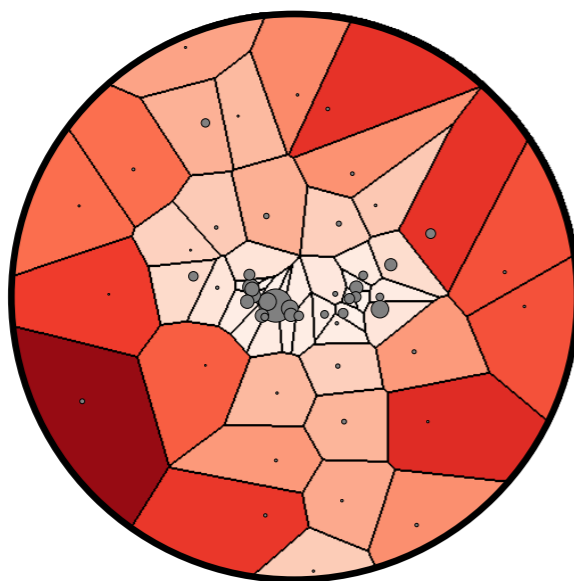
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



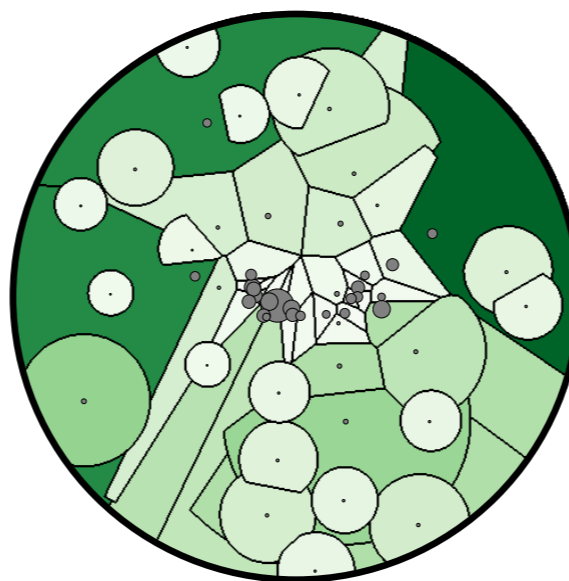
Pileup Mitigation:
“Move away” from uniform event

Voronoi



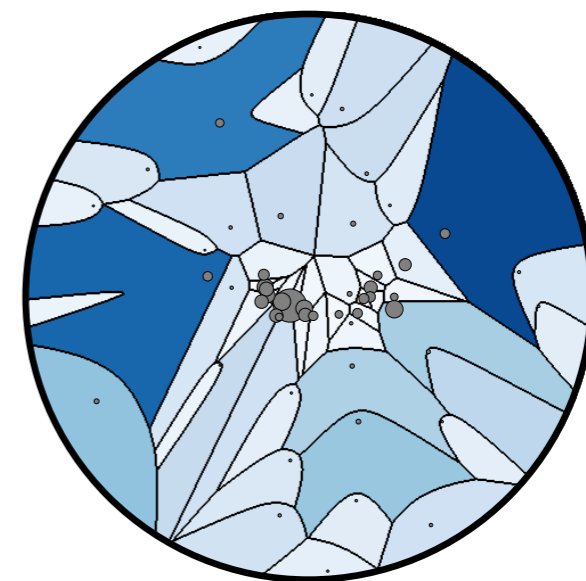
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction



[Berta, Spousta, Miller, Leitner, JHEP 2014]

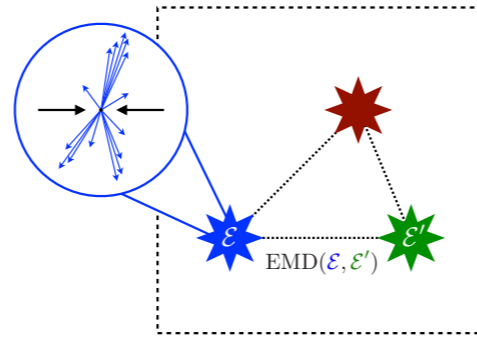
Apollonius



[Komiske, Metodiev, JDT, JHEP 2020]

Pause

What can be Geometrized?

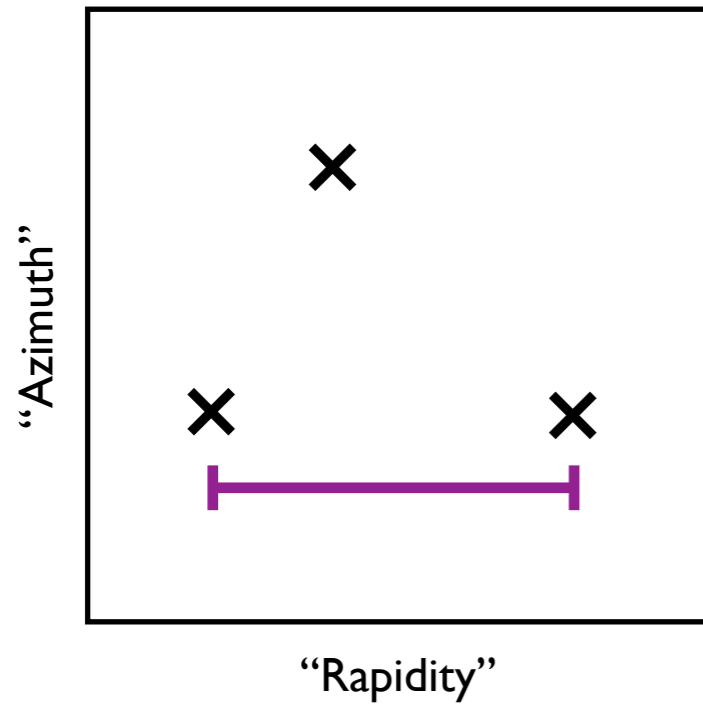


IRC Safety, Observables,
Jet Algorithms, Pileup Mitigation

And now, the grand finale...

How far down does this rabbit hole go?

Direction Space



x = Direction

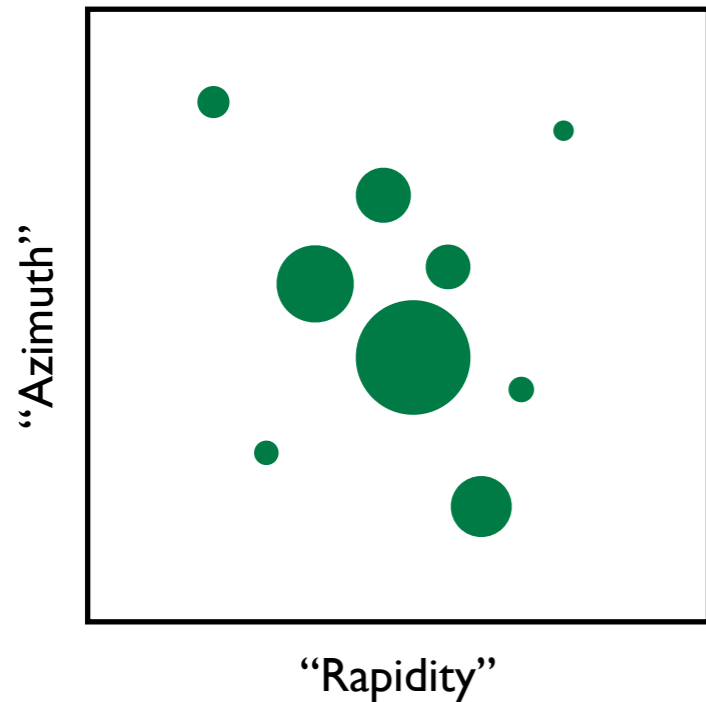
— = Angular Distance

$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$\theta_{ij} = \sqrt{2n_i^\mu n_{j\mu}}$$

(for massless particles)

Direction Space Distribution



● = Weighted Direction

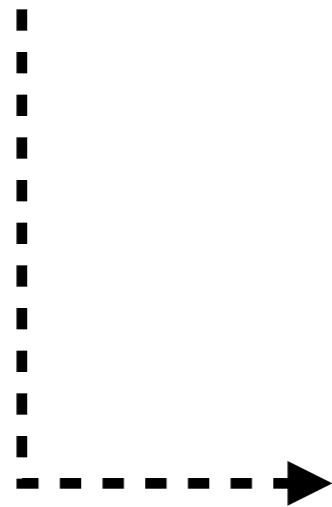
— = Angular Distance

$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$w_i = E_i$$

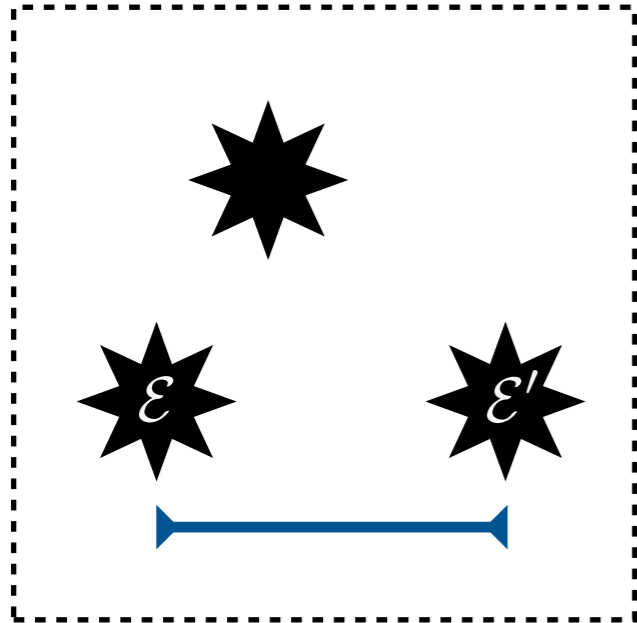
$$\theta_{ij} = \sqrt{2n_i^\mu n_{j\mu}}$$

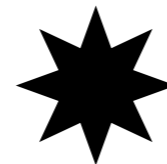
(for massless particles)



★ \mathcal{E} = Event

Event Space



 = Event

 = EMD
Energy
Mover's Distance


$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

Event Space Distribution



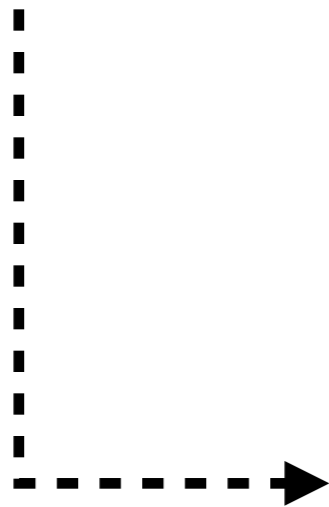
 = **Weighted Event**


$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

$$w_a = \sigma_a$$

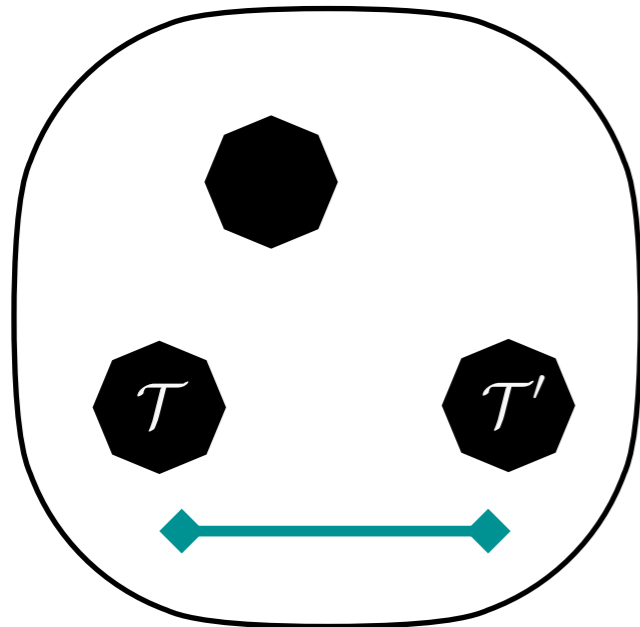
 = **EMD**
 Energy
 Mover's Distance


$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$
 (for equal total energy)



 = **Theory**

Theory Space



 = Theory

 = ΣMD
Cross-Section
Mover's Distance

$$\mathcal{T}(\mathcal{E}) = \sum_a \sigma_a \delta(\mathcal{E} - \mathcal{E}_a)$$

$$\Sigma\text{MD}(\mathcal{T}, \mathcal{T}') = \min_{\{\mathcal{F}\}} \sum_a \sum_b \mathcal{F}_{ab} \text{EMD}(\mathcal{E}_a, \mathcal{E}'_b)$$

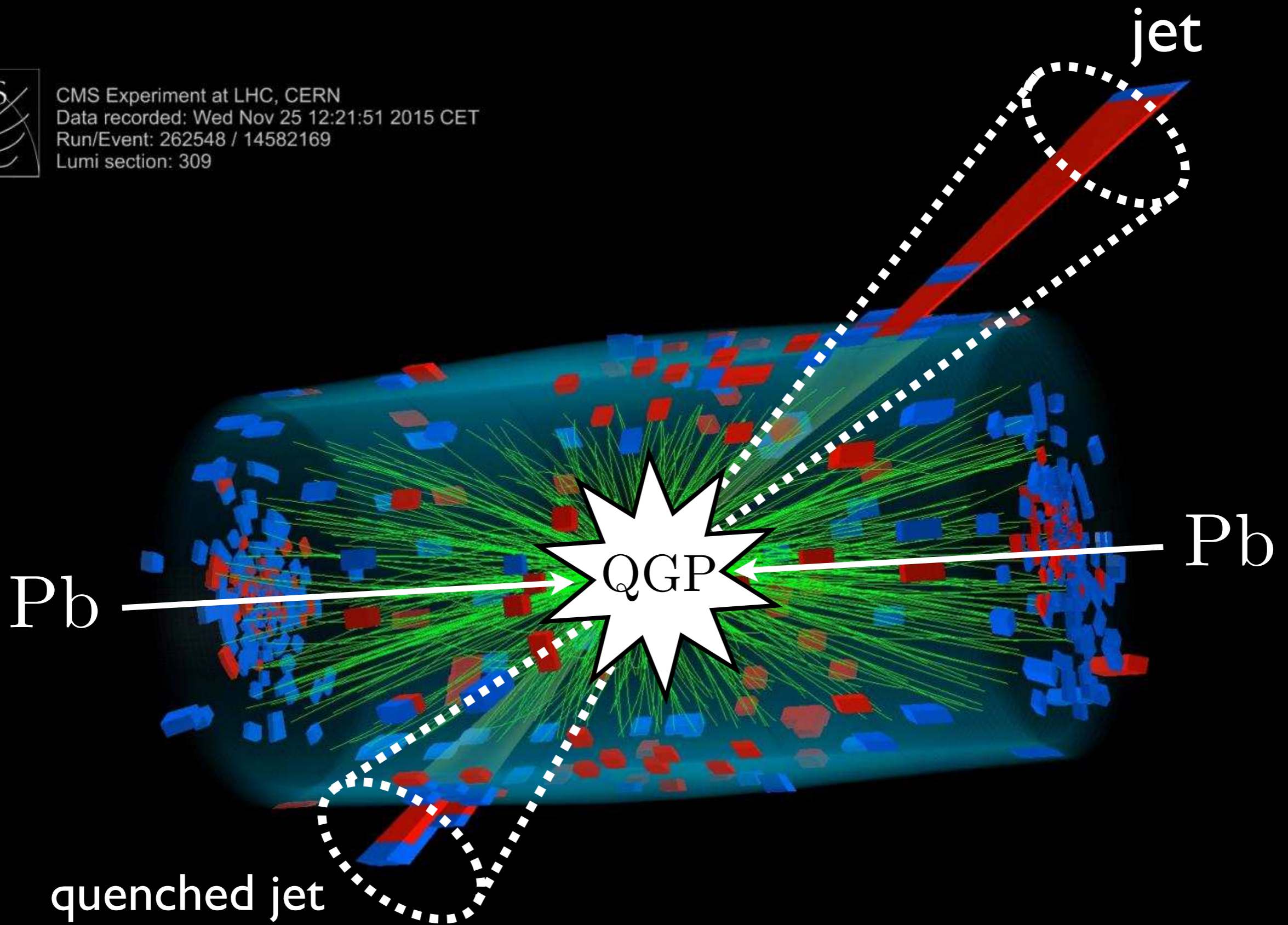
(for equal total xsec)

A distance between theories!

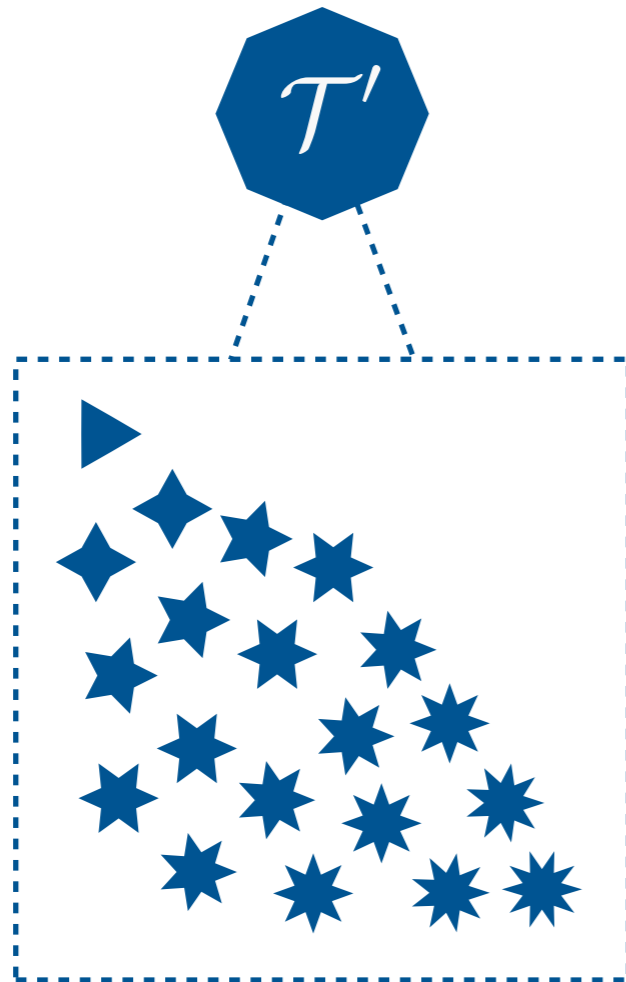
(e.g. EMD : N-jettiness :: ΣMD : k-eventiness)



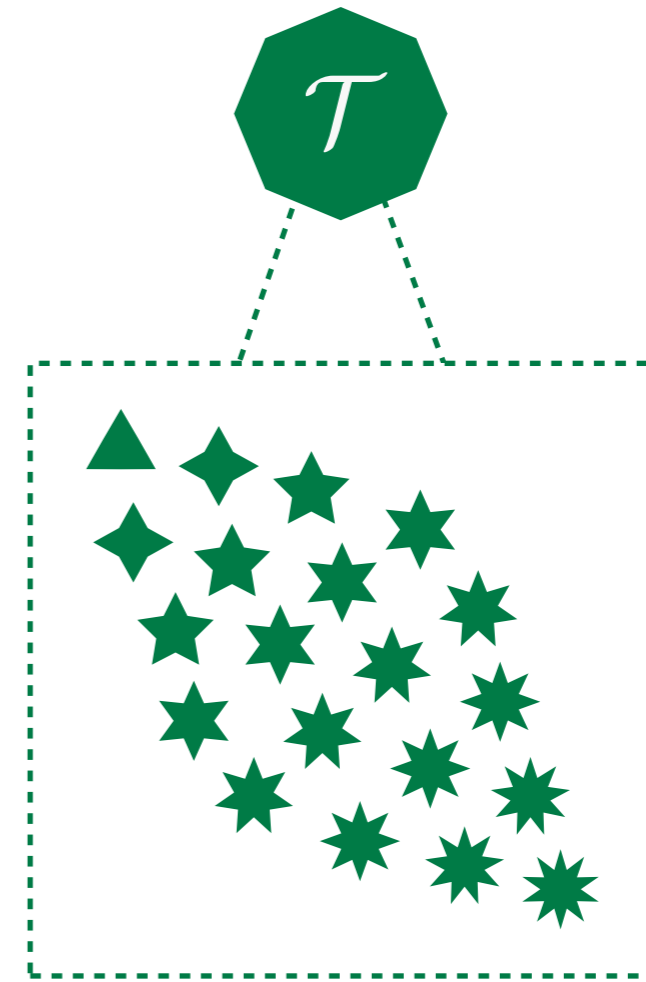
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 25 12:21:51 2015 CET
Run/Event: 262548 / 14582169
Lumi section: 309



Theory Prime: In-Medium QCD



Theory: Vacuum QCD



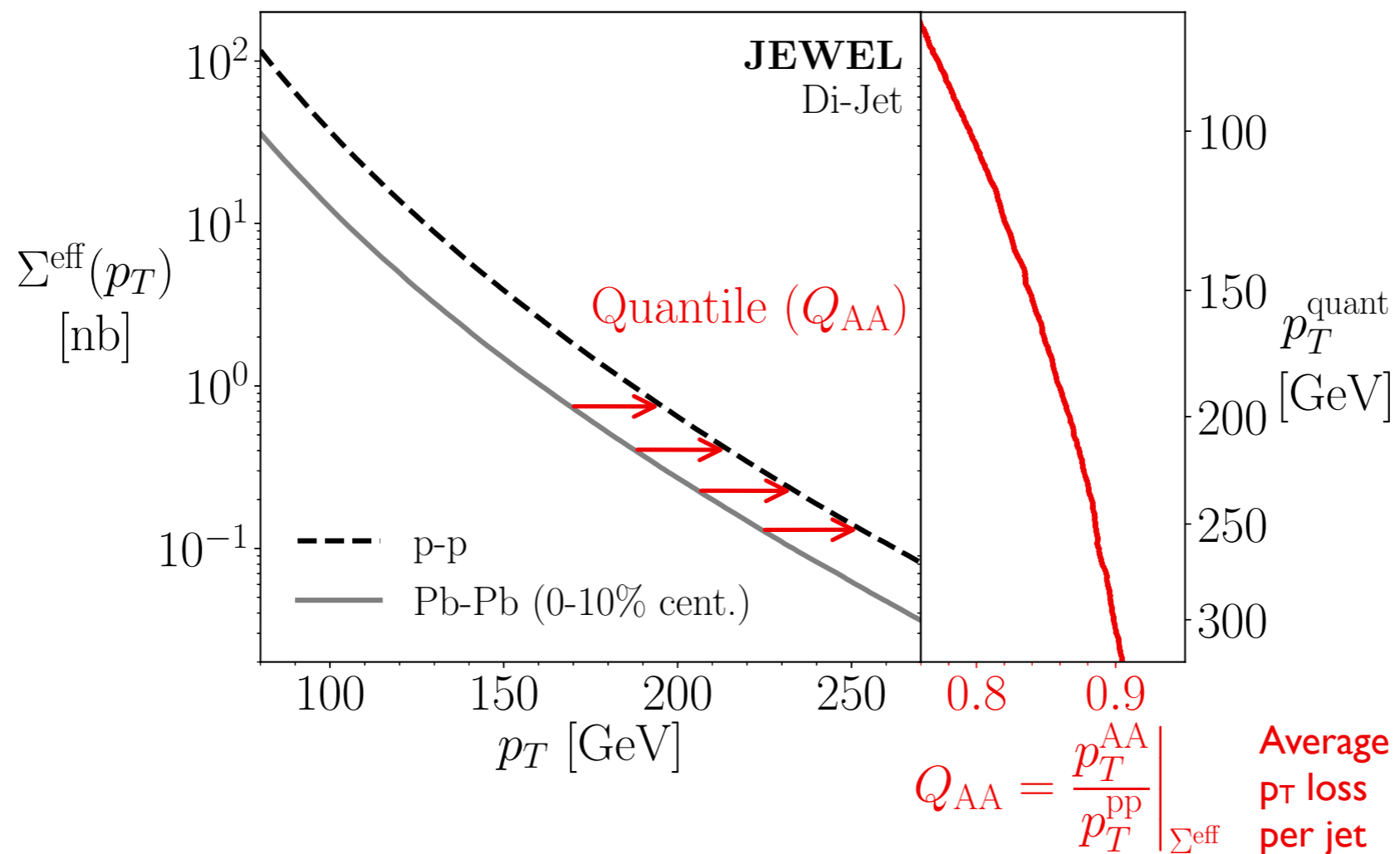
Σ MMD



*Optimal transportation plan defines mapping
between in-medium jets and vacuum jets!*

Jet Quenching via Quantile Matching

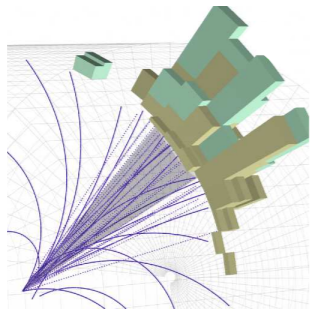
Equivalent to following a geodesic in theory space (!)



[Brewer, Milhano, JDT, PRL 2019]



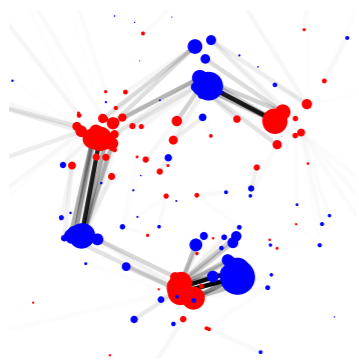
Summary



What is a Collider Event?

An unordered set of particles that describes the energy flow away from the collision point

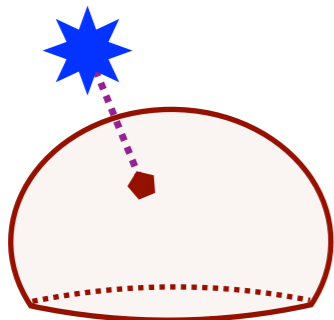
(ask me about ML relevance)



When are Events Similar?

When they are close in the geometric space triangulated by the energy mover's distance

(ask me more about EMD)



What can be Geometrized?

Many concepts/techniques in quantum field theory and collider physics from the last half century

(ask me more about safety)

Fin

Whew!



Backup Slides

Point Cloud

Collection of points in position space



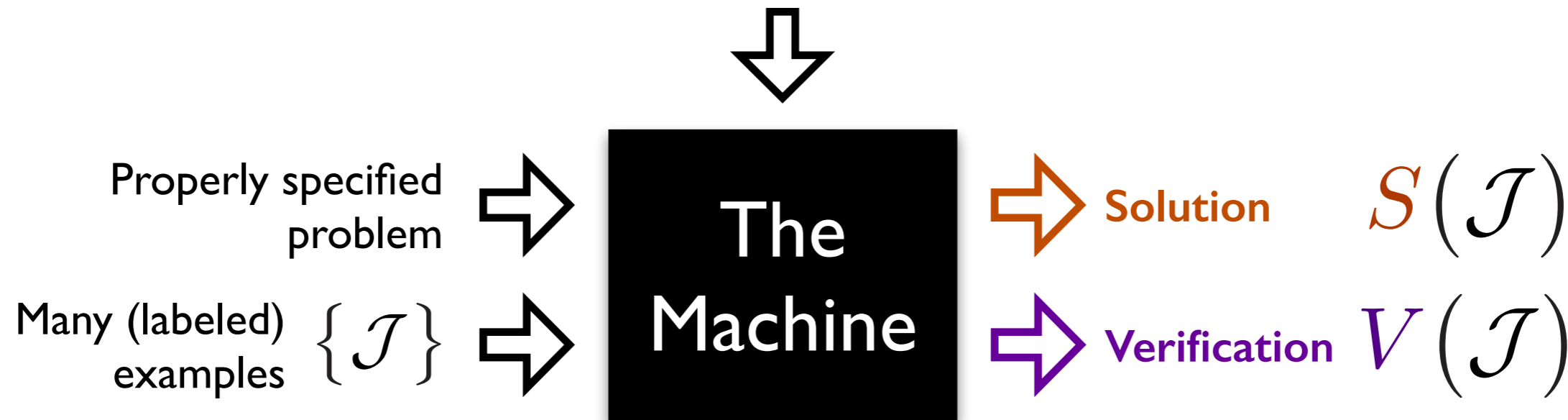
[Popular Science, 2013]

Aside: Machine Learning for Jets

“ML4jets”
NYU, January 2020

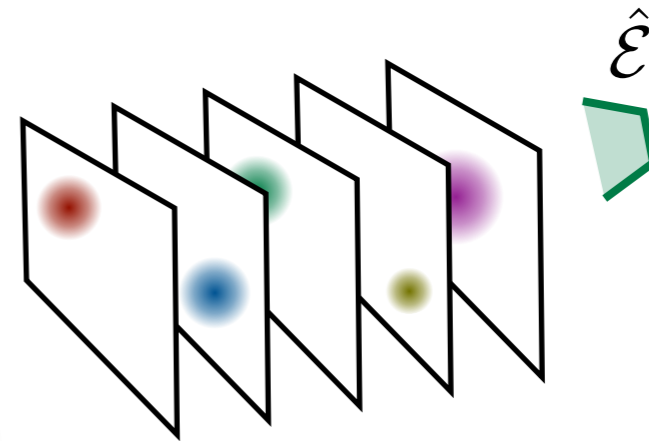
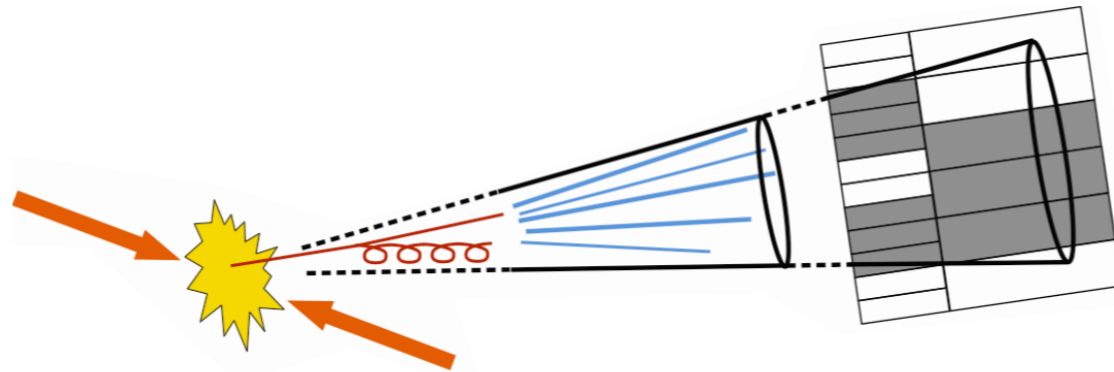
Symmetry: $\mathcal{J} = \{ \vec{p}_1, \vec{p}_2, \vec{p}_3, \dots, \vec{p}_N \}$
Unordered, Variable Length Set (QM!)

Safety: $\vec{p} = \{ E, \hat{n}_x, \hat{n}_y, \hat{n}_z \}$
Energy weighting (QFT!)

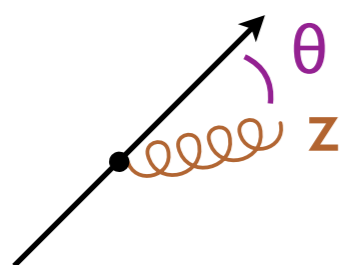
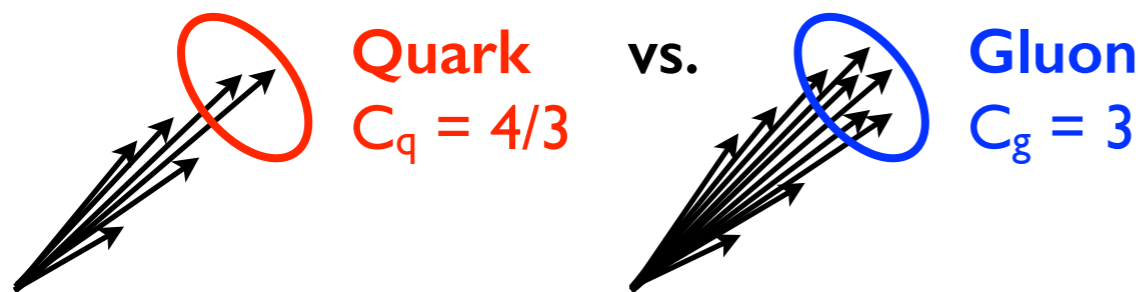


*Check that answer
is physically sensible*

E.g. Energy Flow Networks

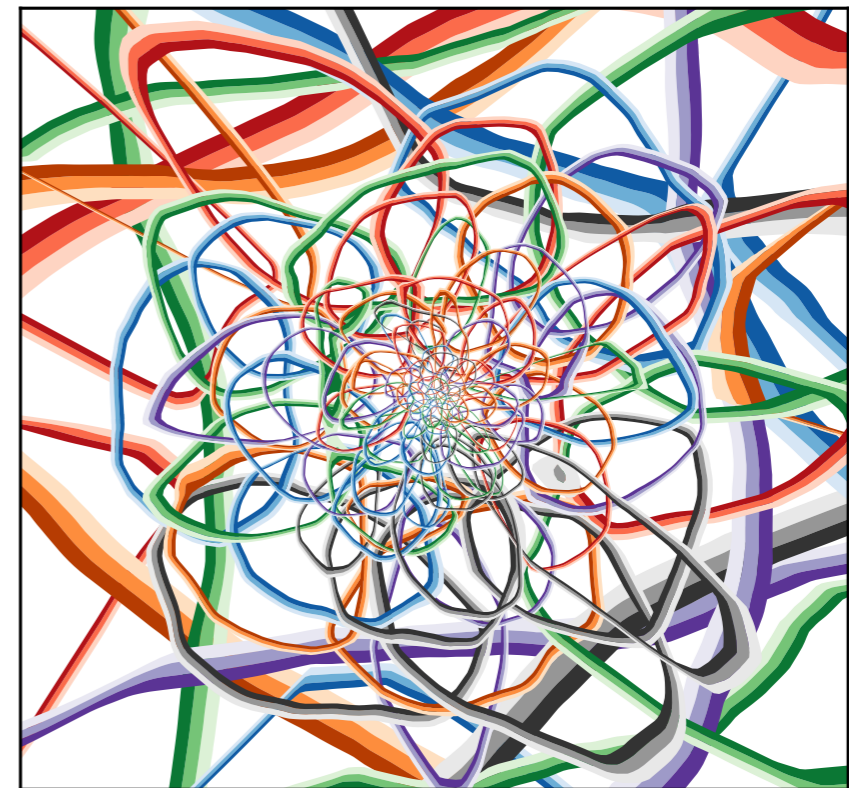


Learning QCD singularities!



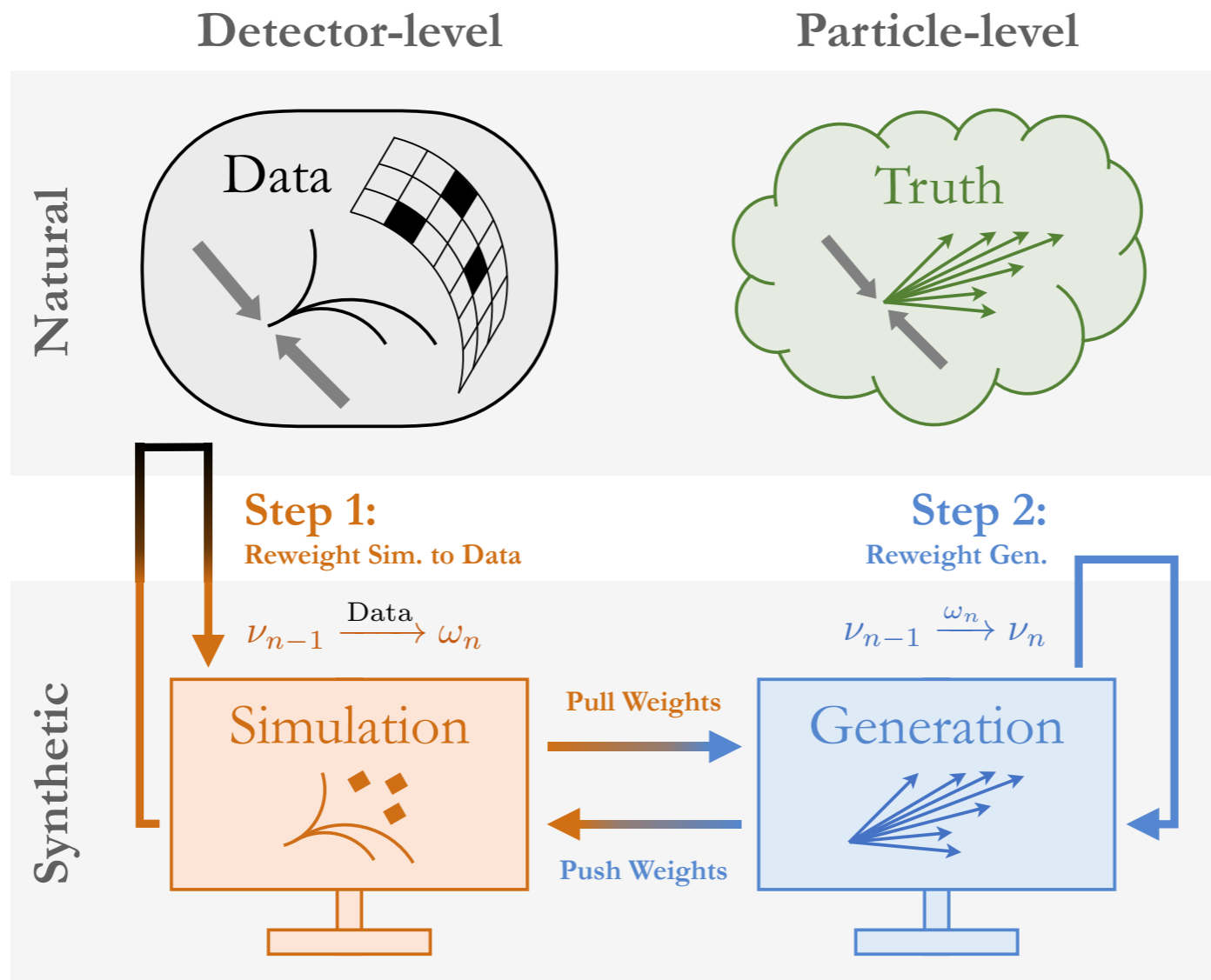
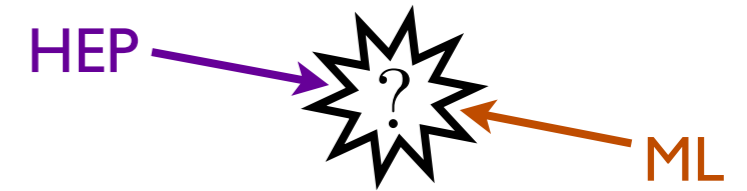
AP splitting probability:

$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$



[Komiske, Metodiev, JDT, JHEP 2019; see also Komiske, Metodiev, JDT, JHEP 2018, PRD 2020; special case of Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, NIPS 2017]

OmniFold



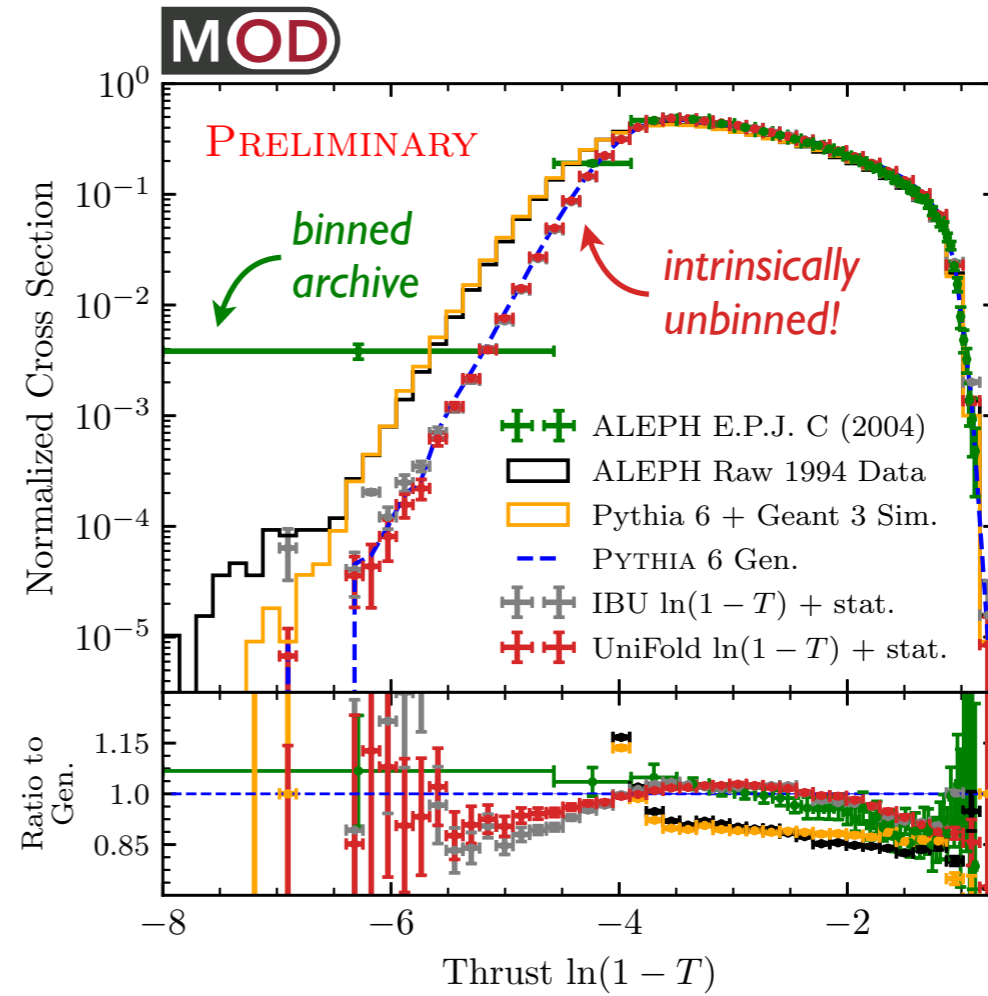
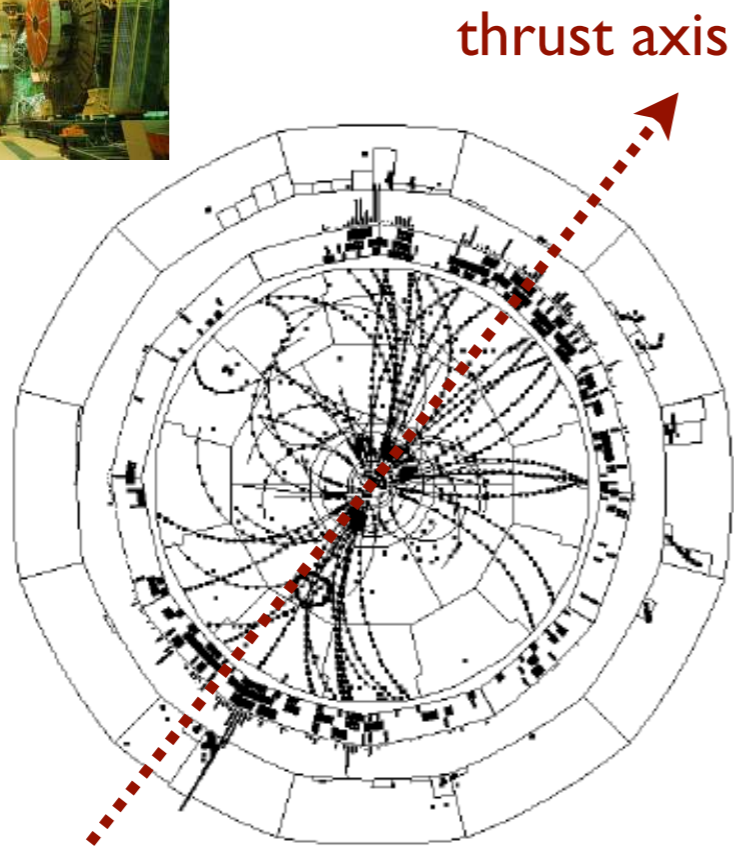
Using, e.g.,
Energy Flow
Networks

Multi-dimensional unbinned detector corrections via iterated binary classification

[Andreassen, Komiske, Metodiev, Nachman, JDT, PRL 2020]

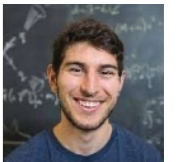


Back to the Future with ALEPH Archival Data

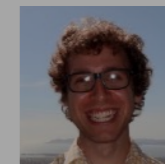


[talk by Badea, [ICHEP 2020](#); cf. ALEPH, [EPJC 2004](#)]

[see also Badea, Baty, Chang, Innocenti, Maggi, McGinn, Peters, Sheng, [JDT, Lee, PRL 2019](#)]

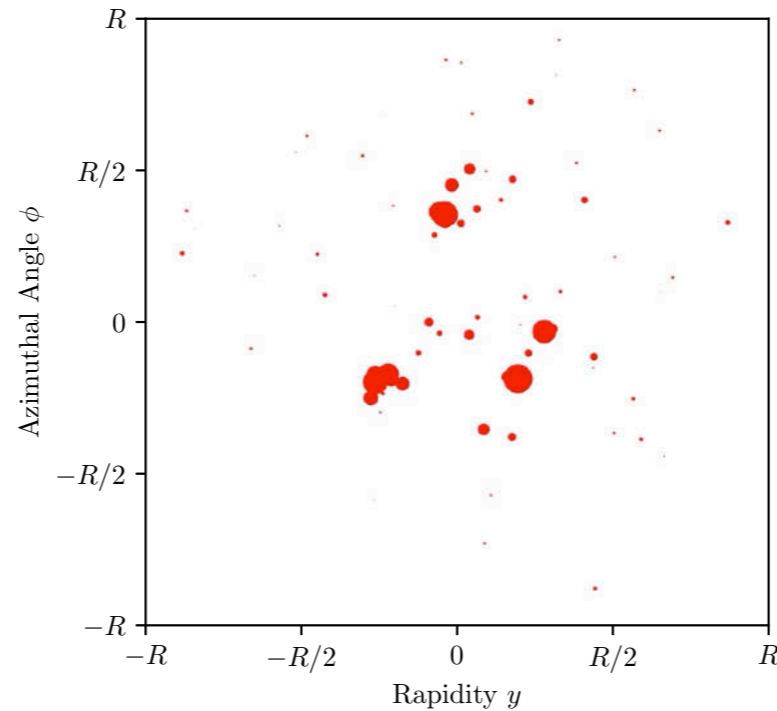


[Andreassen, Komiske, Metodiev, Nachman, [JDT, PRL 2020](#)]



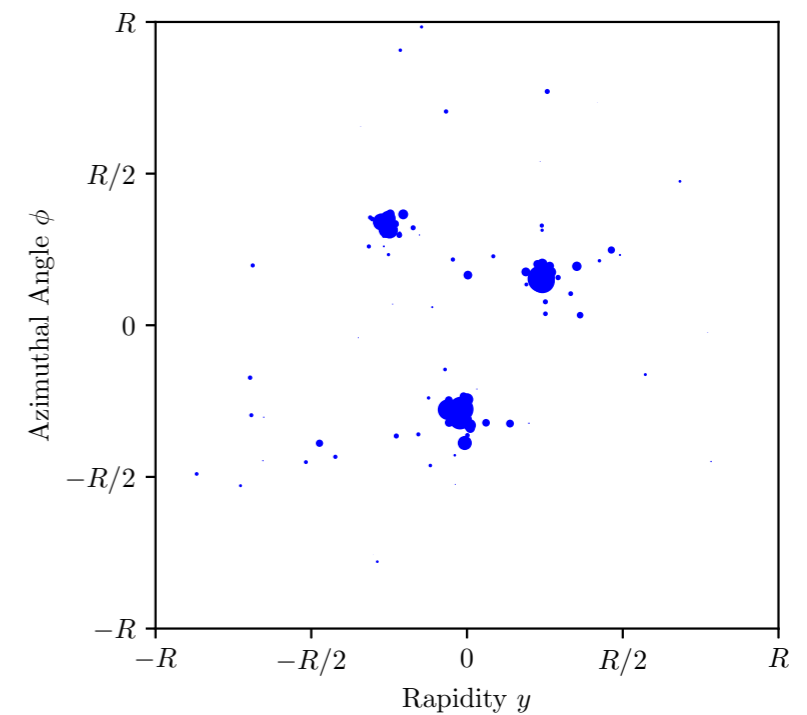
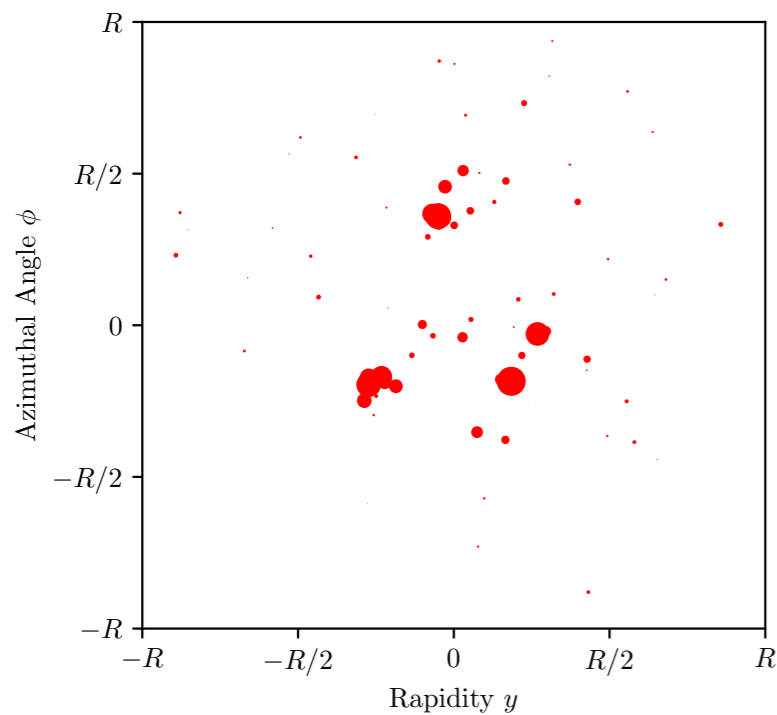
Similarity of Two Energy Flows?

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$



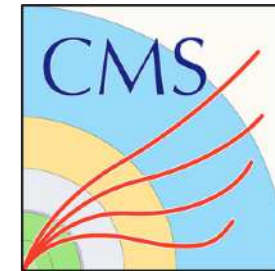
Optimal Transport:

Earth Mover's Distance
a.k.a. *1-Wasserstein metric*

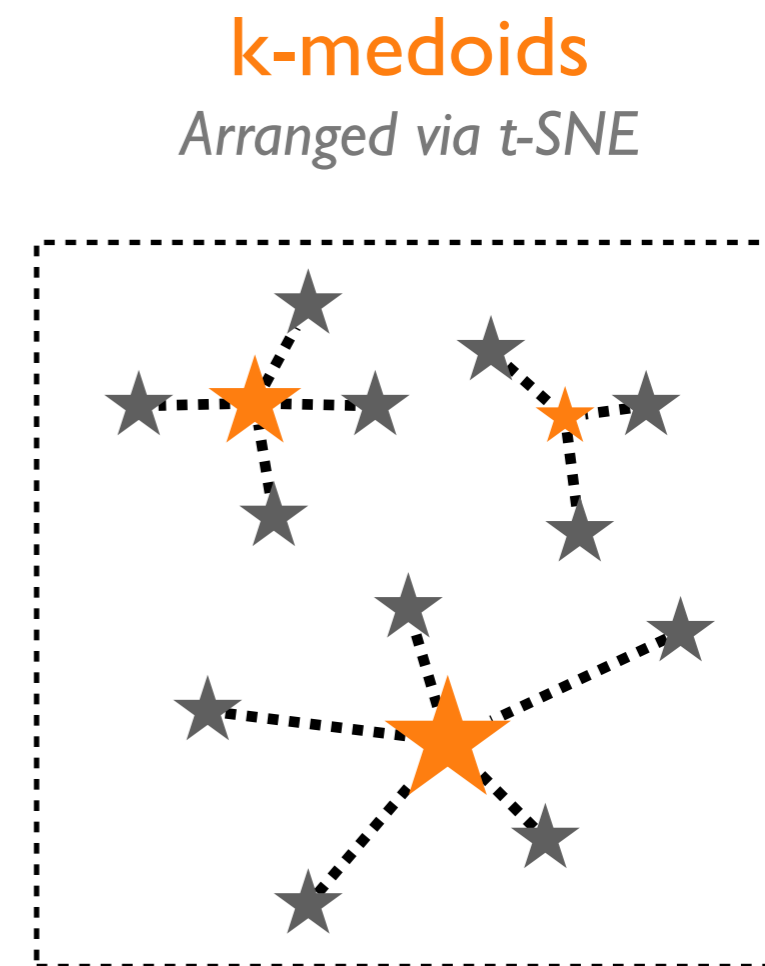
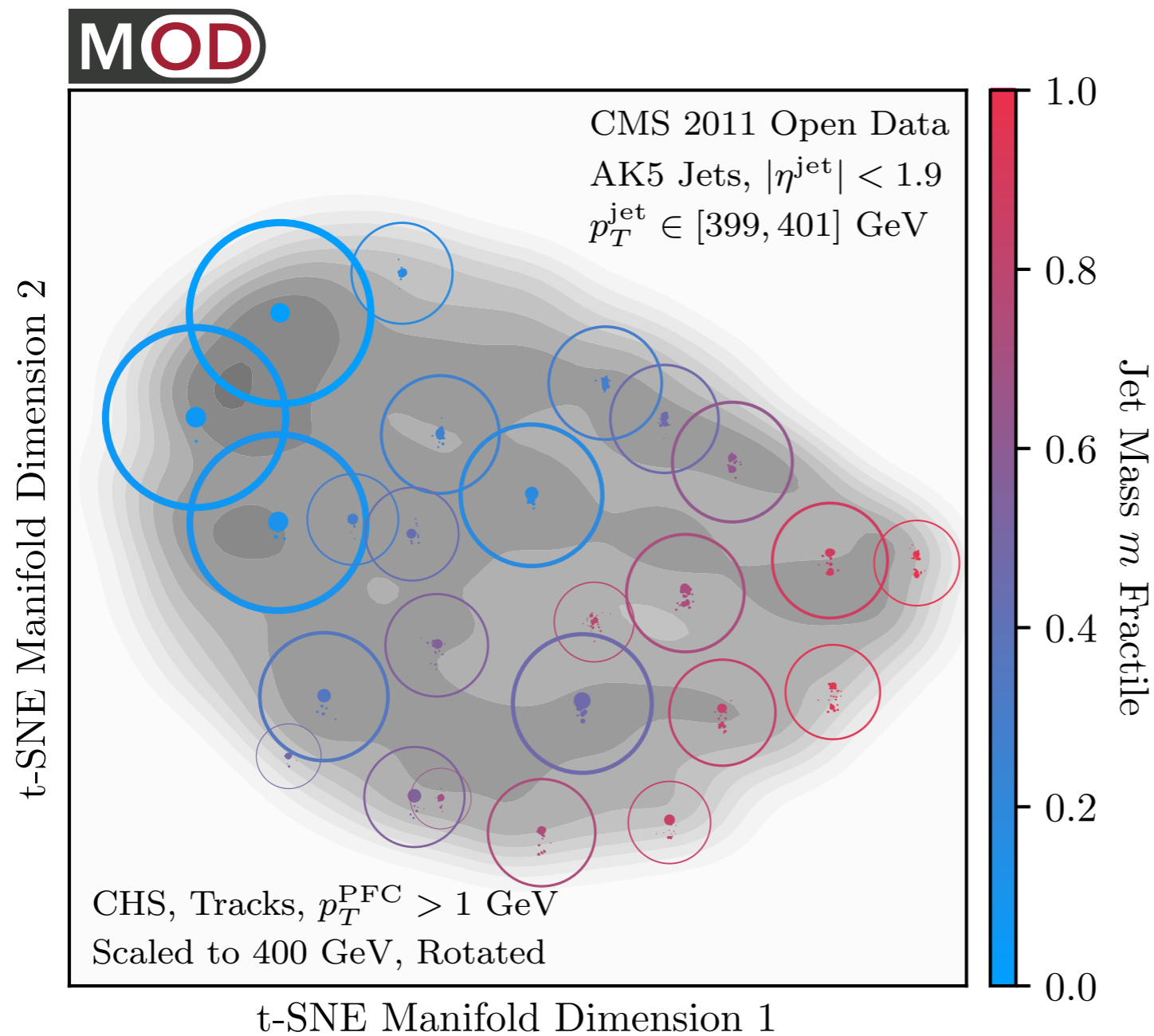


[Komiske, Metodiev, JDT, [PRL 2019](#); code at Komiske, Metodiev, JDT, [energyflow.network](#)]

Most Representative Jets

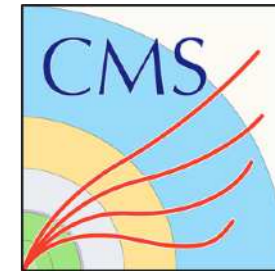


[<http://opendata.cern.ch/>]

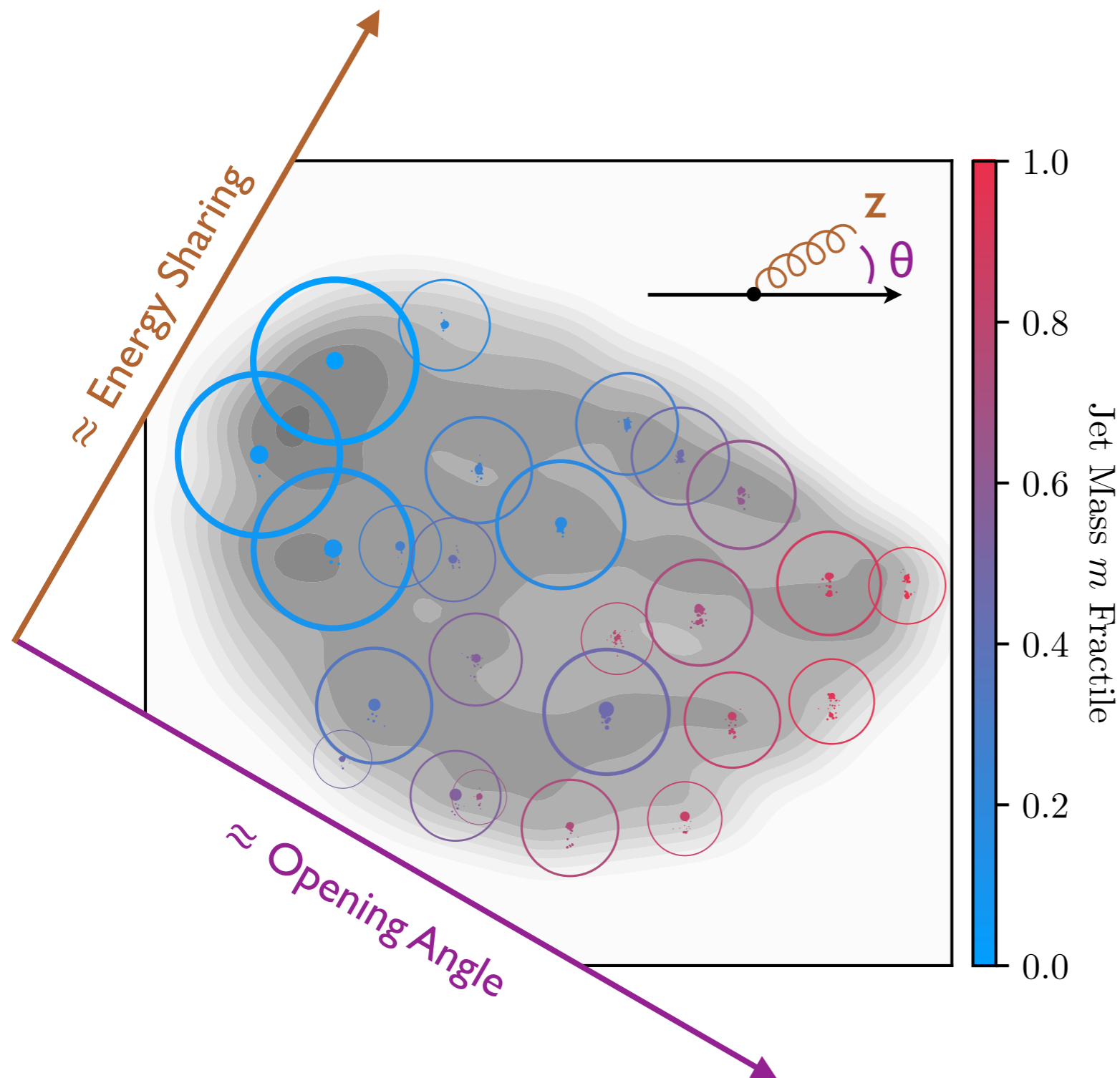


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

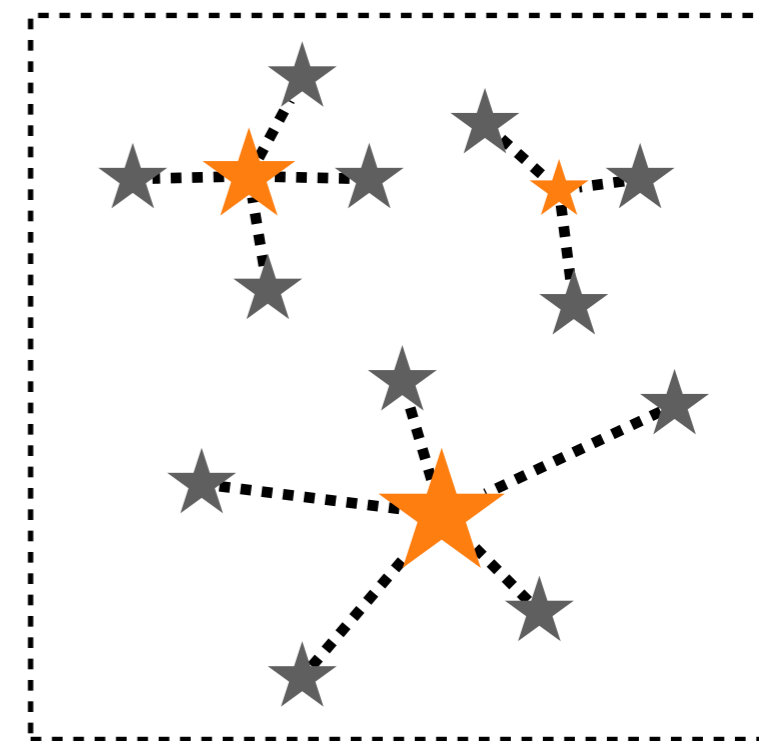
Most Representative Jets



[<http://opendata.cern.ch/>]



k-medoids
Arranged via *t*-SNE

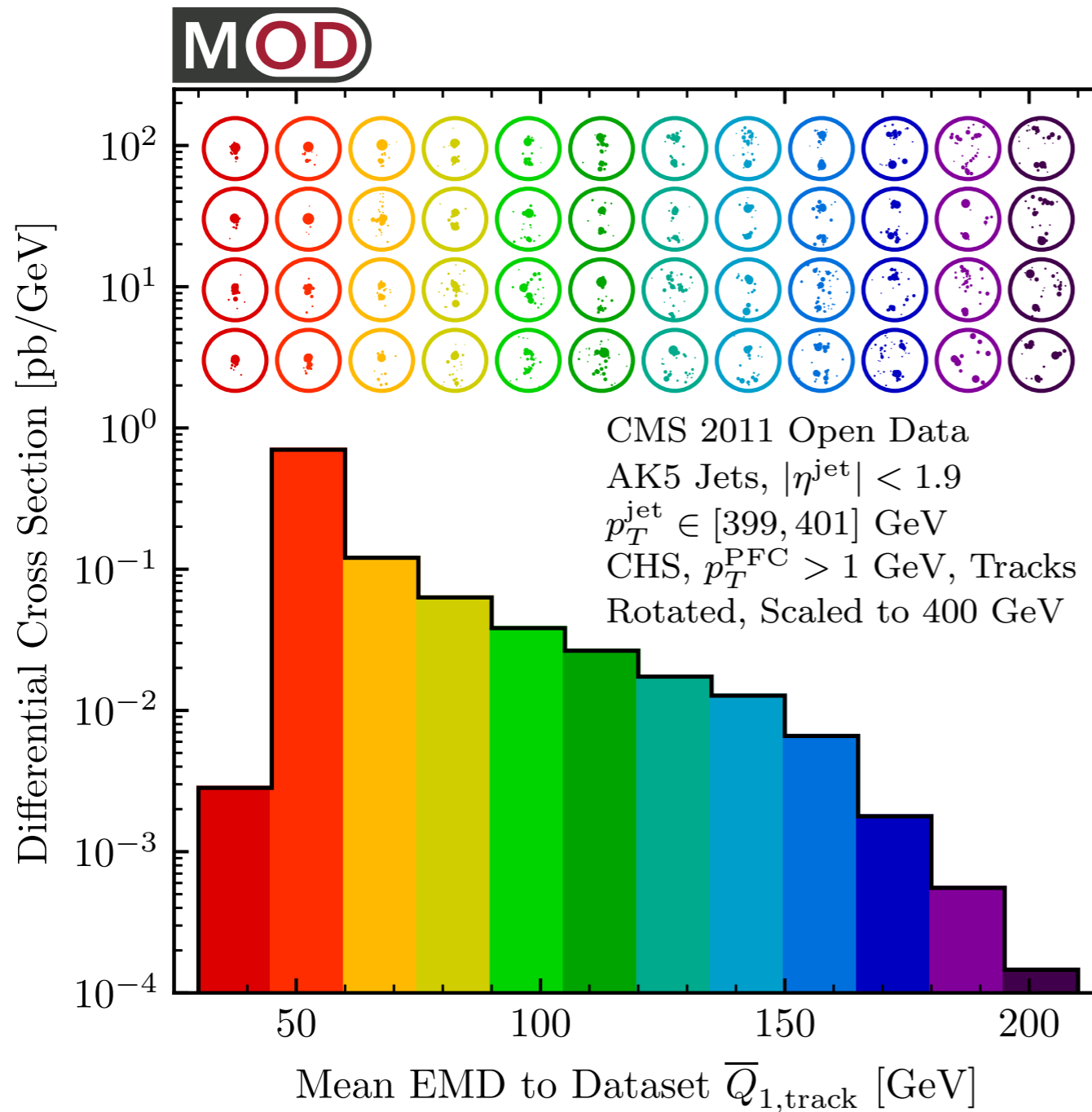


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

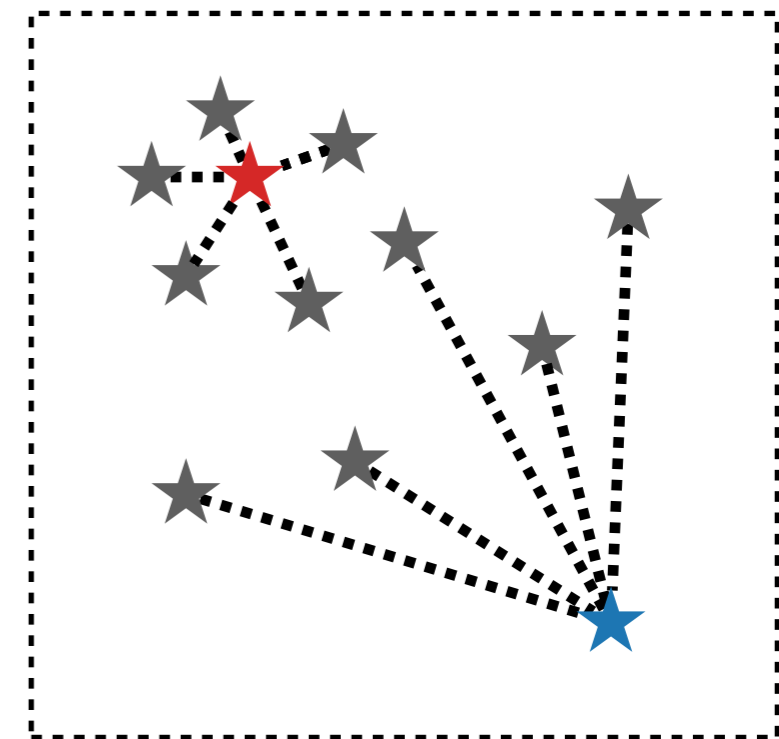
Least Representative Jets



[<http://opendata.cern.ch/>]

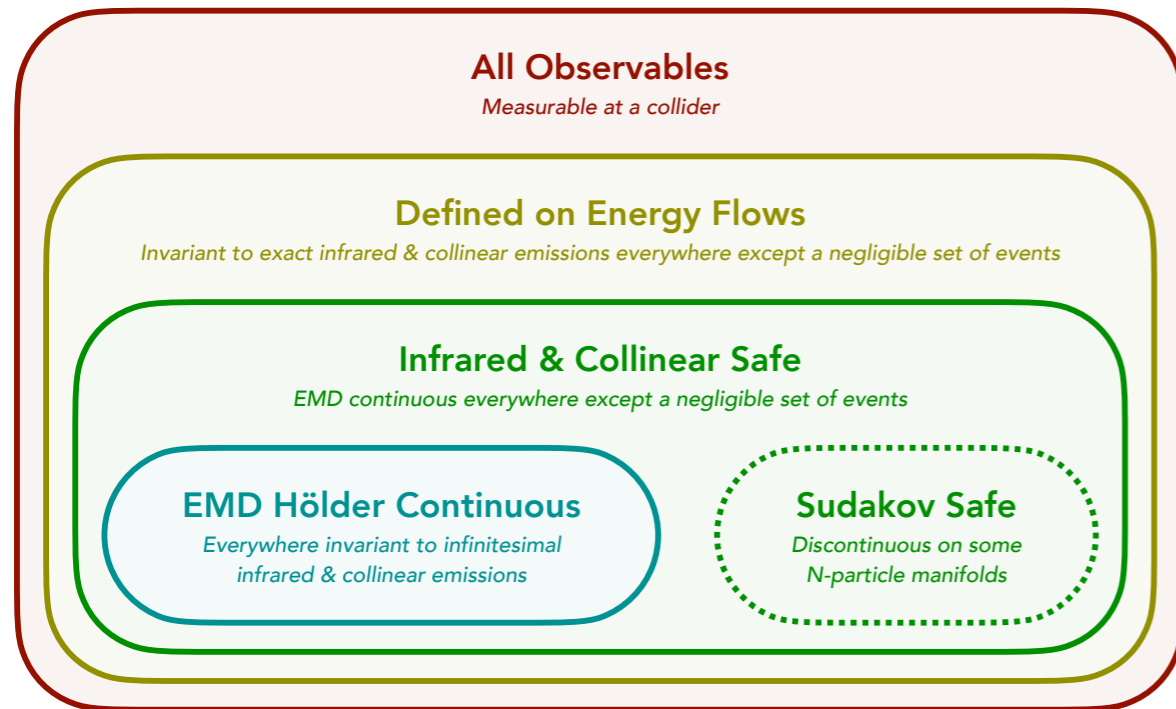


New Physics?
 Or tails of QCD?



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

Observable Taxonomy



All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe

Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions

Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary

Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N -subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N -particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold

EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Sphericity [42]	
Angularities [70]	
N -jettiness [44] (\mathcal{T}_N)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

[Komiske, Metodiev, JDT, JHEP 2020; cf. Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]