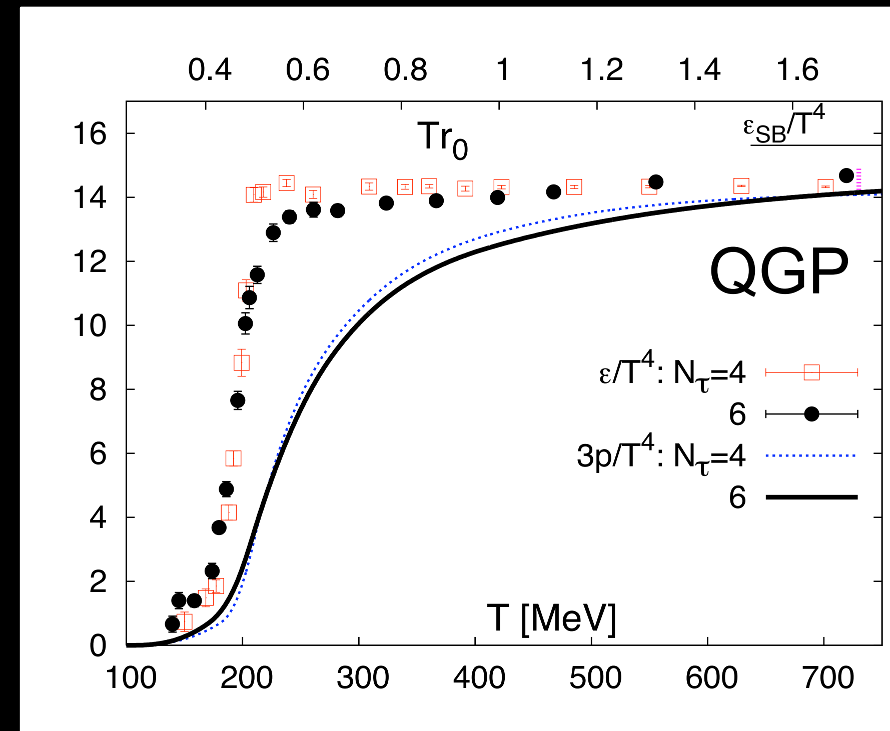
Stochastic String Motion Above and Below the World Sheet Horizon

Jorge Casalderrey-Solana

Work in collaboration with K-Y Kim and D. Teaney

Why Strong Coupling?: Flow



Cheng et al. (08)

Impact parameter

$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T}\frac{1}{2\pi}\left[1 + 2v_2(p_T)\cos(2\phi) + \ldots\right]$$

Collectivity: anisotropy in space is transfered to momentum.

It is described by (almost) ideal hydrodynamics.

The viscosity at RHIC is much smaller than any other known substance.

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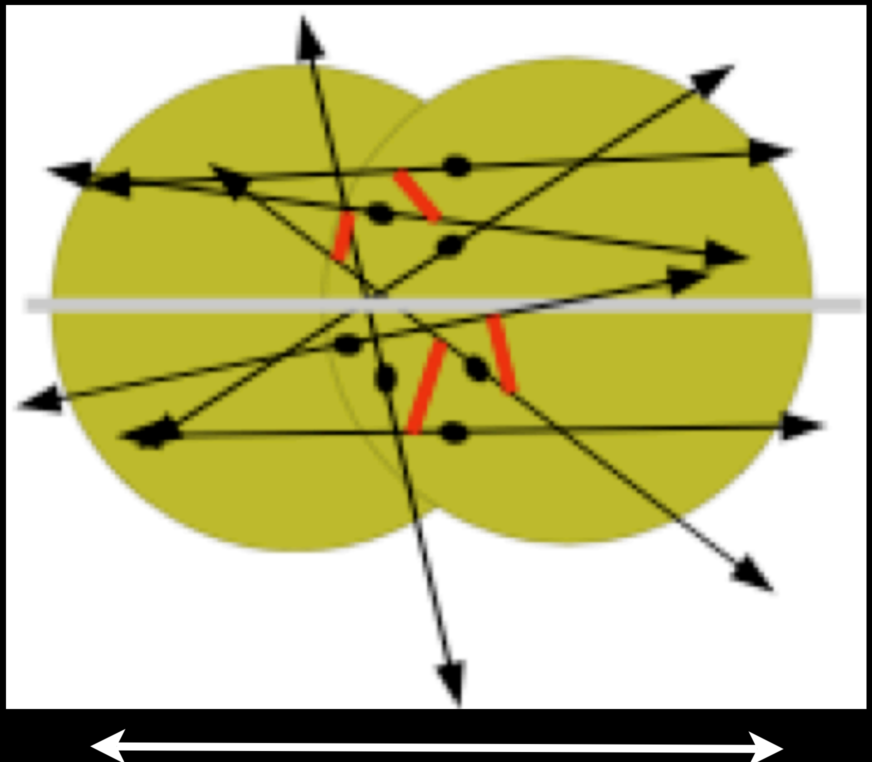

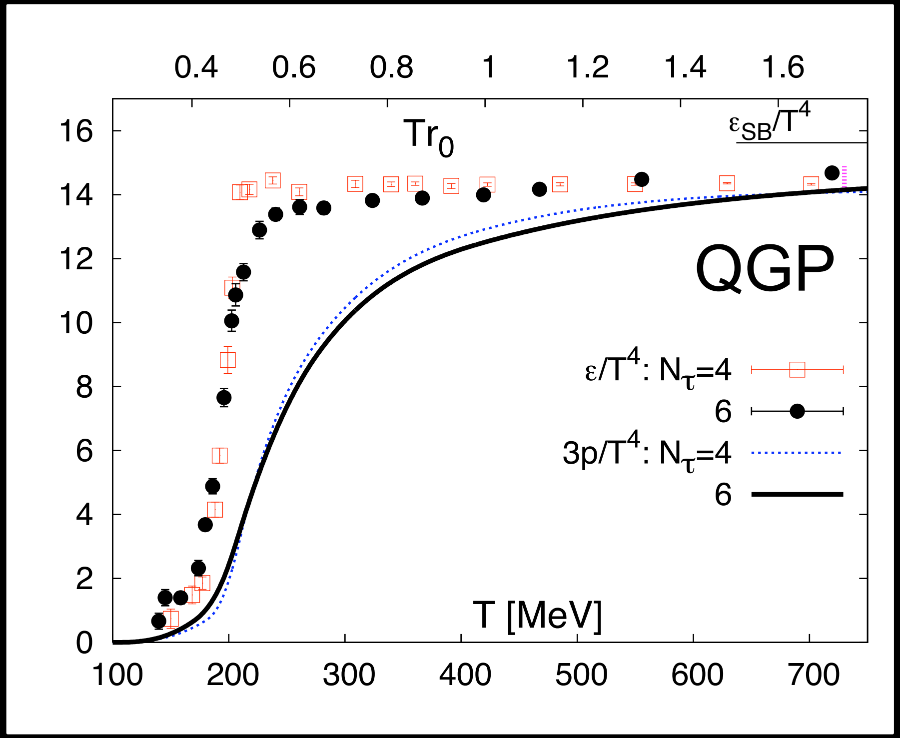

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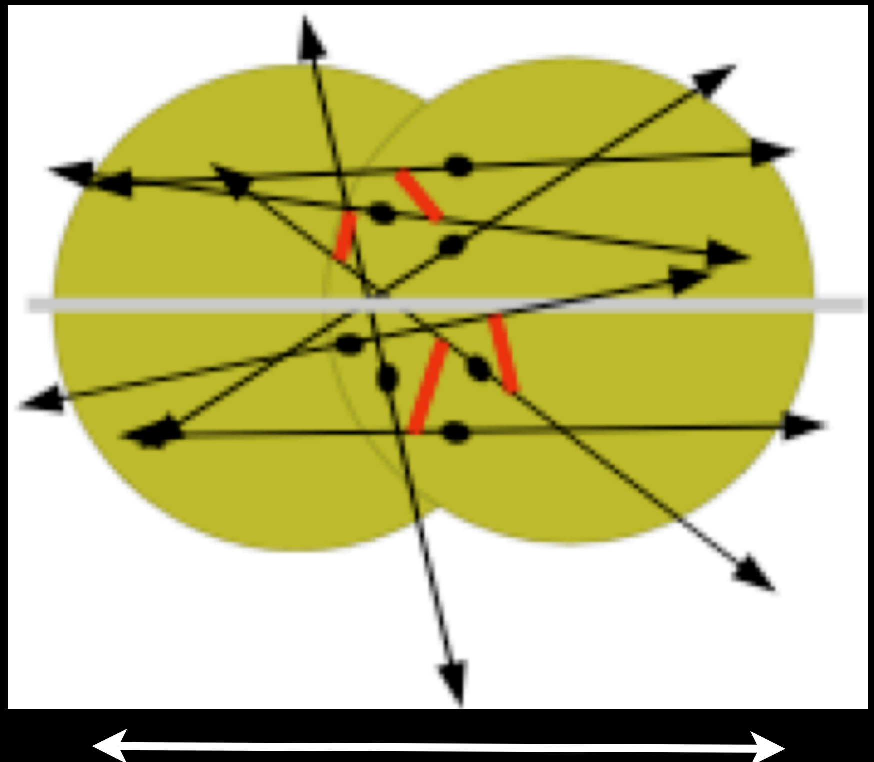

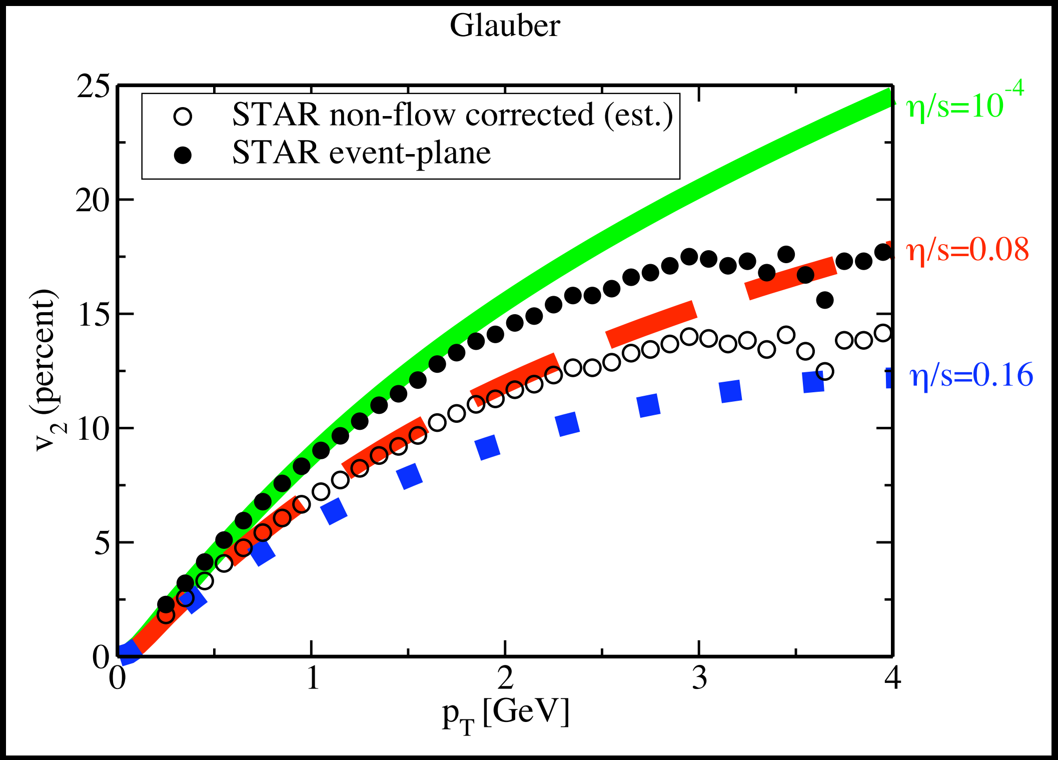

Romatschke 08

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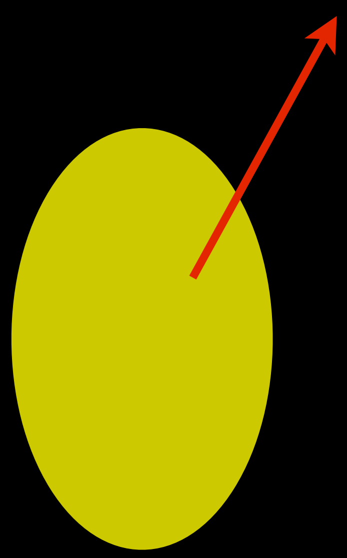
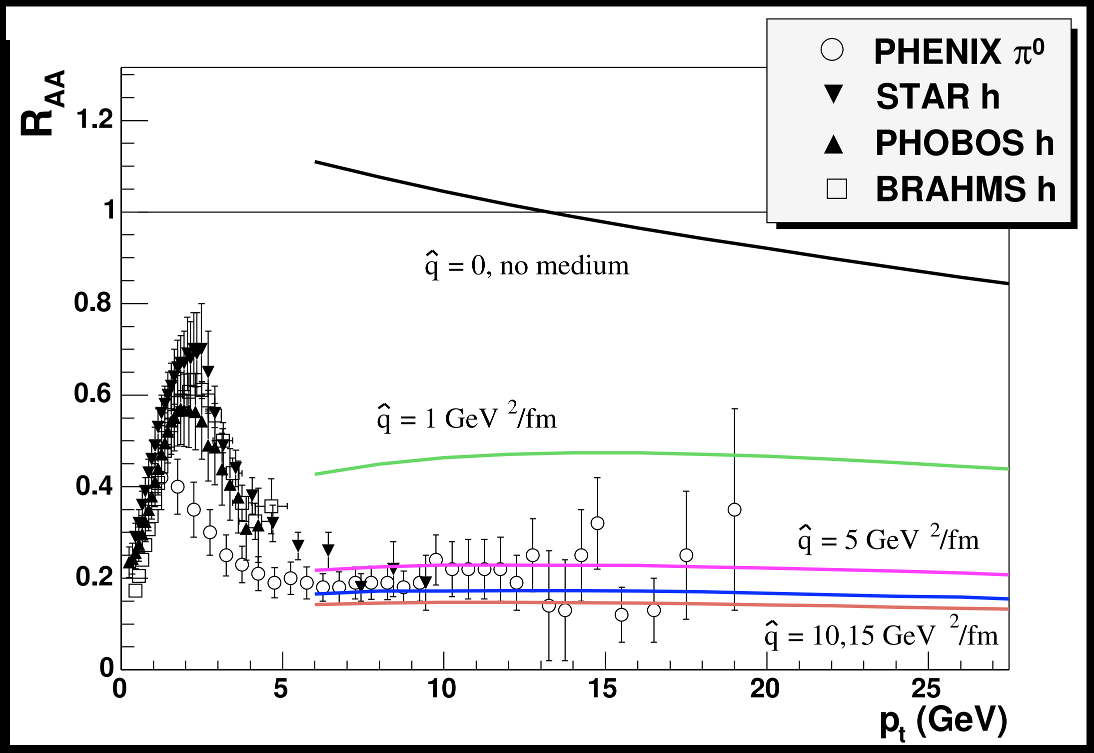
Why Strong Coupling?: Quenching





Eskola et al.

$$R_{AA} = \frac{\text{Number of particles in A} - \text{A}}{\text{Number of collision} \times \text{Number of particles in p} - \text{p}}$$

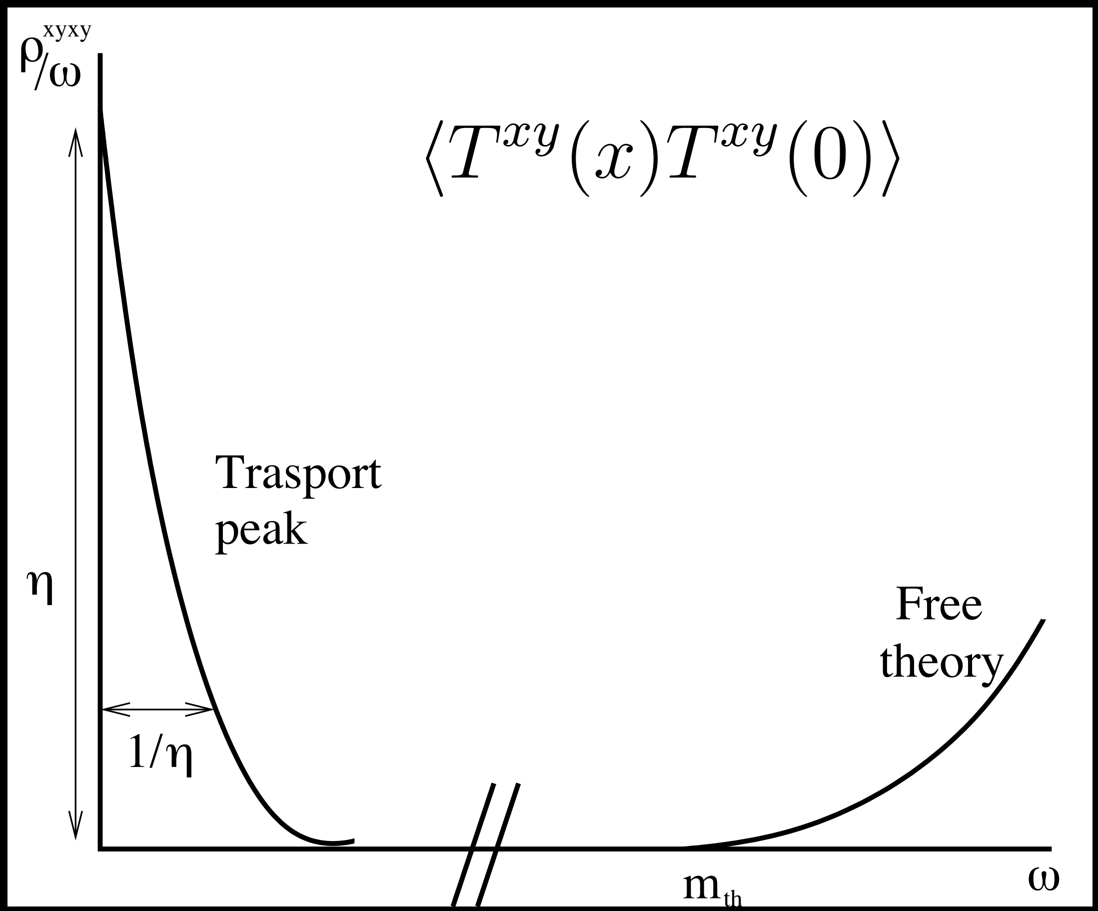
Matter is very opaque to high energy probes.

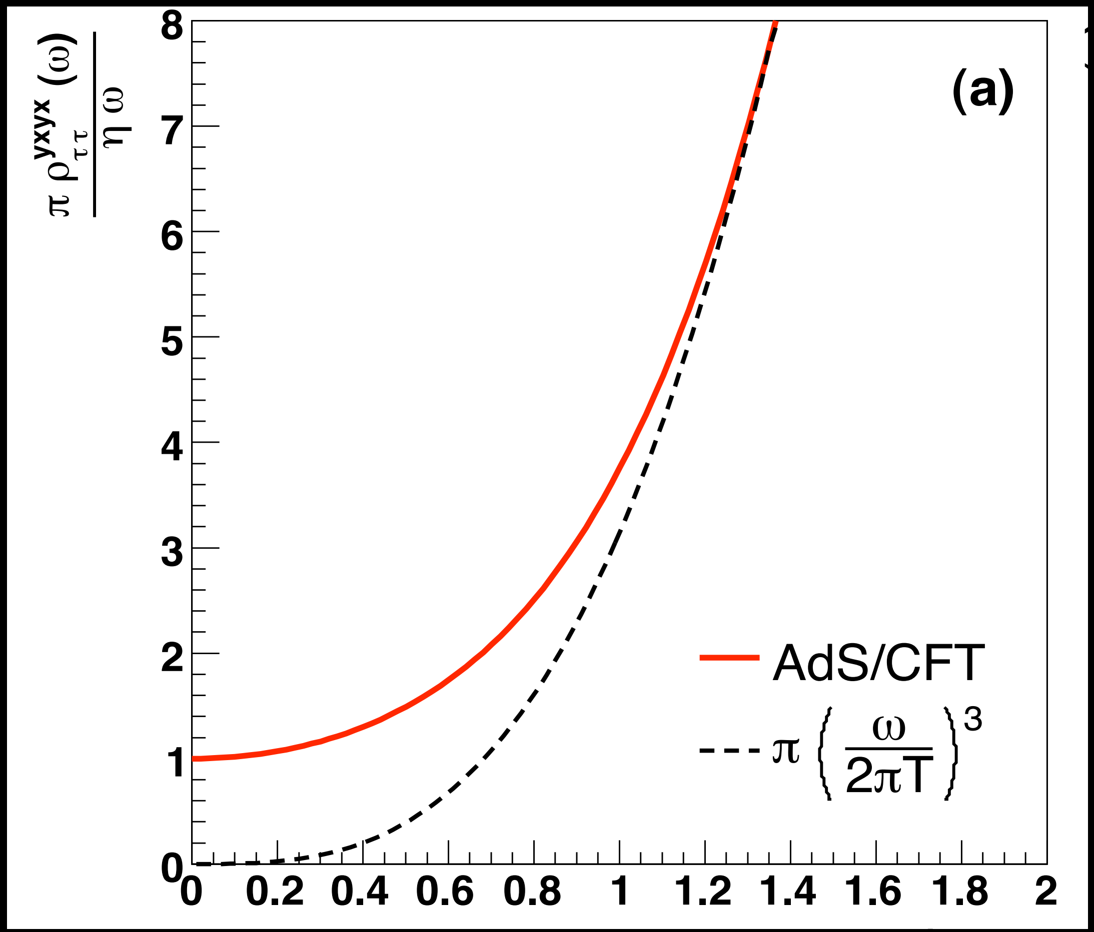
It is hard to accommodate these facts with pertubation theory

Temperatures at RHIC are not much higher than $\Lambda_{\rm QCD}$

The coupling constant seems to be large (or at least not small)

Why AdS/CFT?





AdS/CFT allows to study N=4 SYM in the $N_c \rightarrow \infty$ at infinitely strong coupling $\lambda \rightarrow \infty$

It is not QCD but we can address the theory in a regime that is not tractable in QCD

We study a strongly coupled deconfined gauge theory plasma.

Some features are solely dependent on the strong coupling such as the absence of quasiparticles. In gauge theories the separation of scales that allow the quasiparticle picture is due to $\lambda \ll 1$

Langevin Model for Heavy Quarks

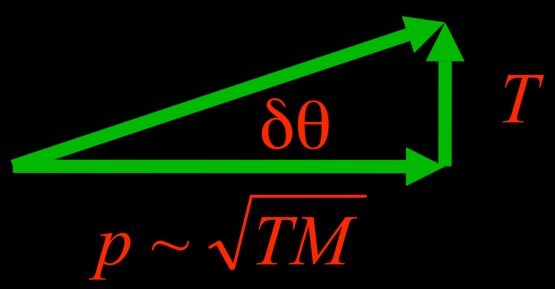
Moore, Teaney 03

Heavy quark with v<<1⇒neglect radiation

$$\lambda = \frac{h}{\sqrt{TM}} \ll \frac{h}{T}$$ ⇒ HQ classical on medium correlations scale

$$\frac{dp}{dt} = -\eta_D p + \xi$$

η_D: drag; ξ: random force



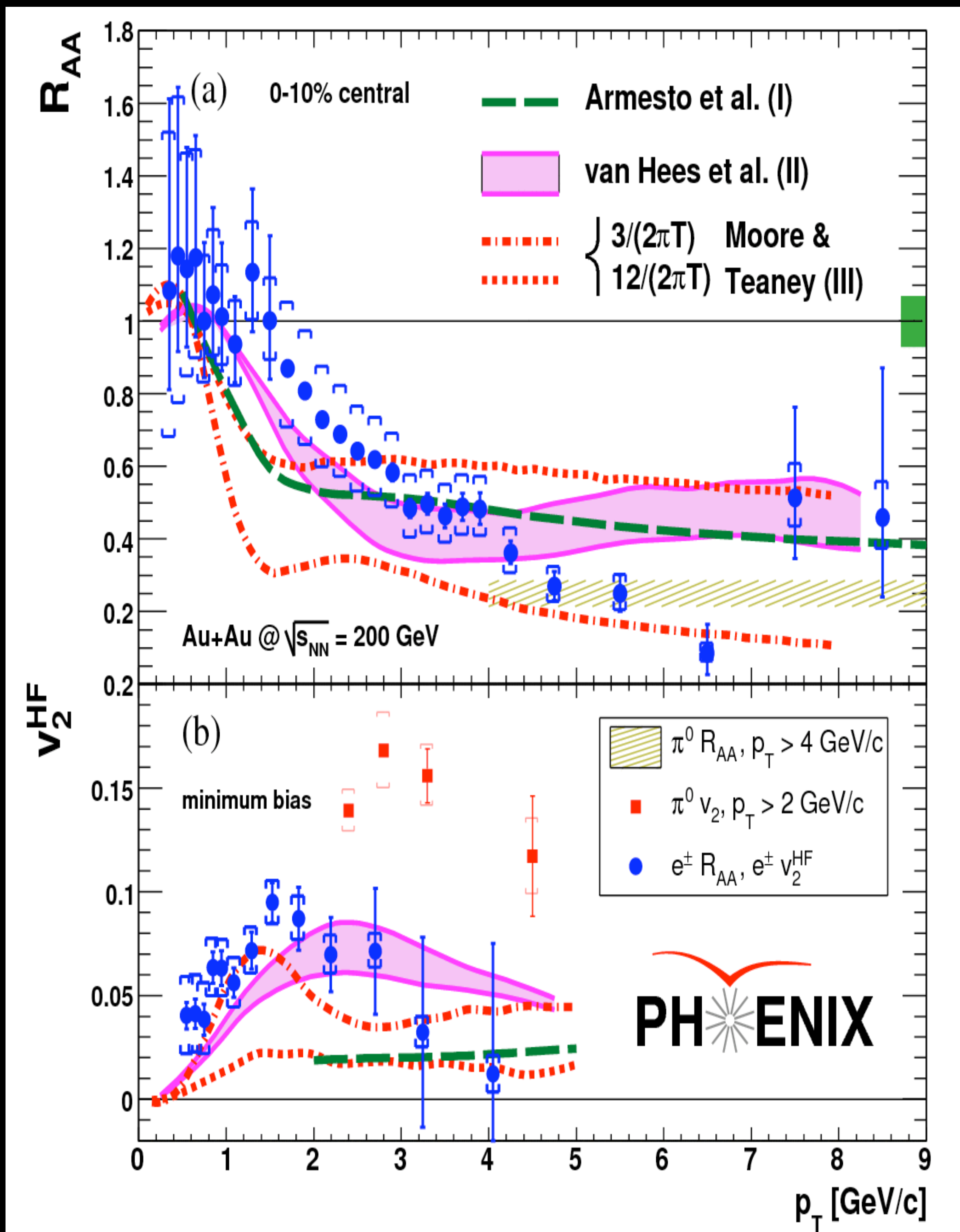
$$\delta\theta \sim \sqrt{\frac{T}{M}} \ll 1$$ ⇒ white noise ⇒ $$\langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

κ: Mean transfer momentum from the medium

Einstein relations: $$\eta_D = \frac{\kappa}{2MT}$$ $$D = \frac{2T^2}{\kappa}$$

Heavy Quarks at RHIC

Heavy Quarks are suppressed and participate on collective motion



Fit to elliptic flow:

$$D = \frac{2T^2}{\kappa} = \frac{3-6}{2\pi T}$$

However: R_{AA} and v_2
cannot be fitted simultaneously

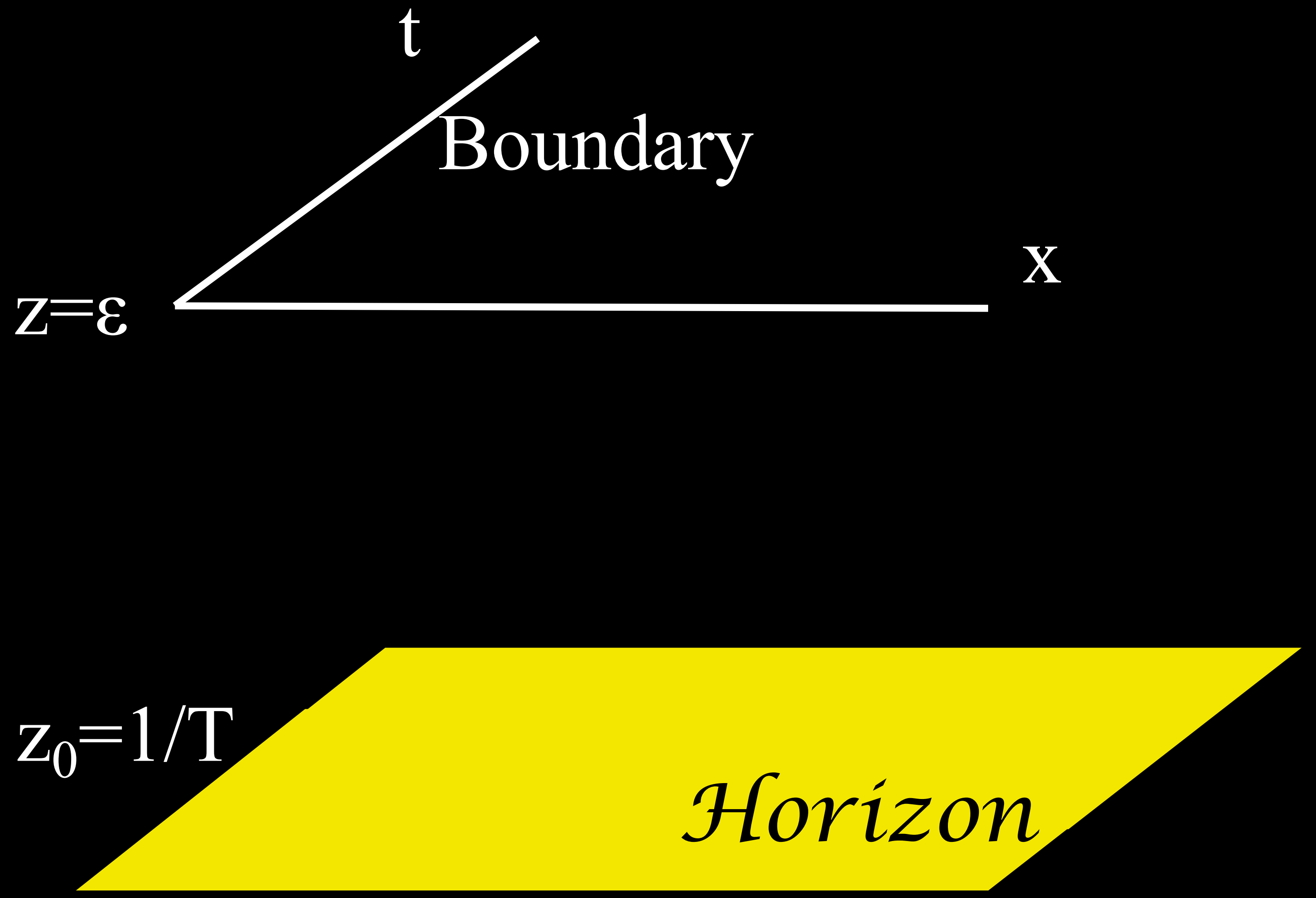
Goal:

Compute D in AdS

Can we go beyond Langevin?

Can we study correlations?

Brief Introduction to AdS/CFT



The gauge theory is dual to classical gravity in 1(+) more dimension

The fifth dimension (z) is interpreted as the scale

A thermal state is obtained by introducing a black brane.

Flavor is introduced by adding a D7 brane⇒ introduces a new scale

A heavy quark is a classical string that "pends" down from the brane

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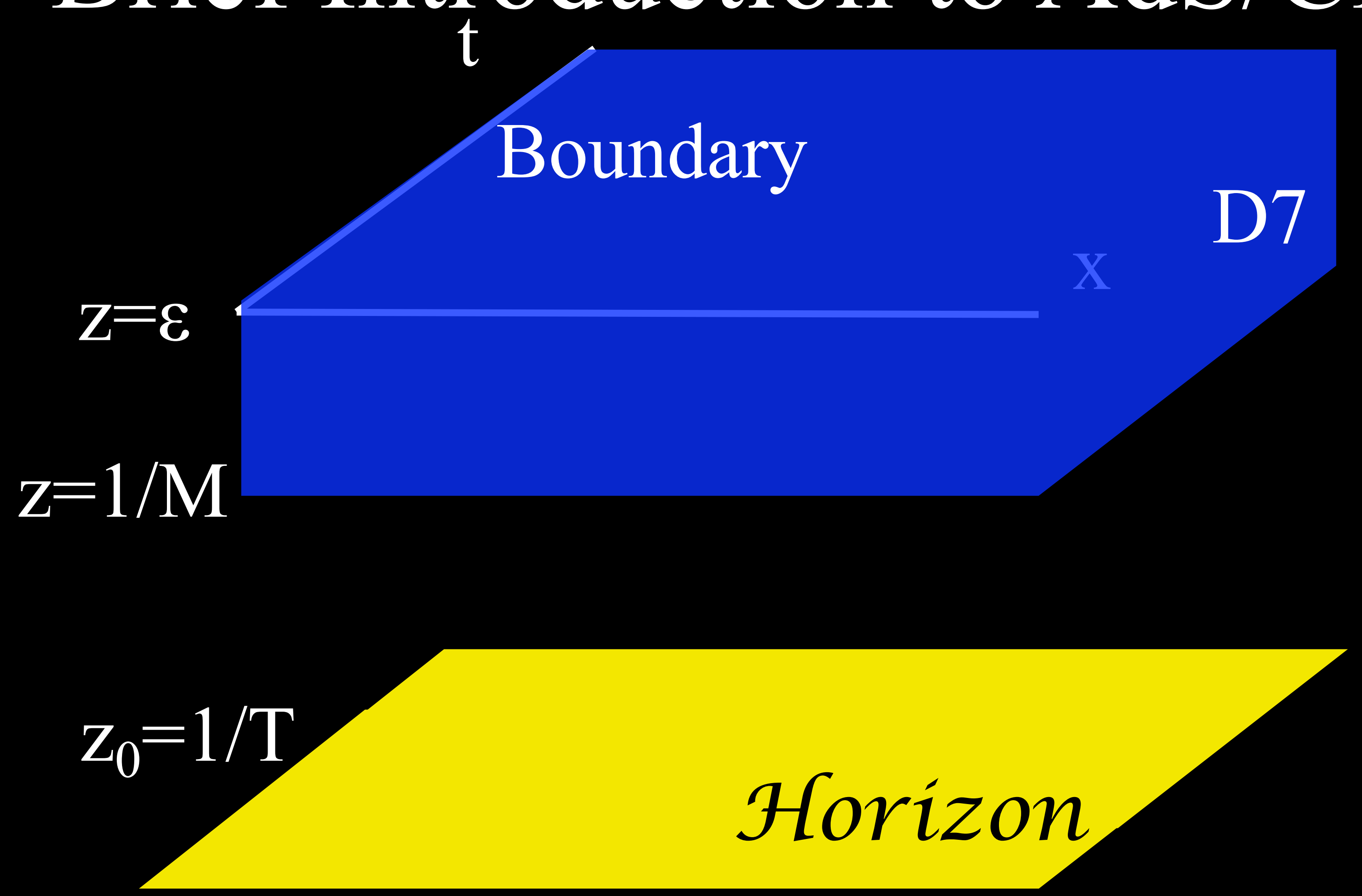

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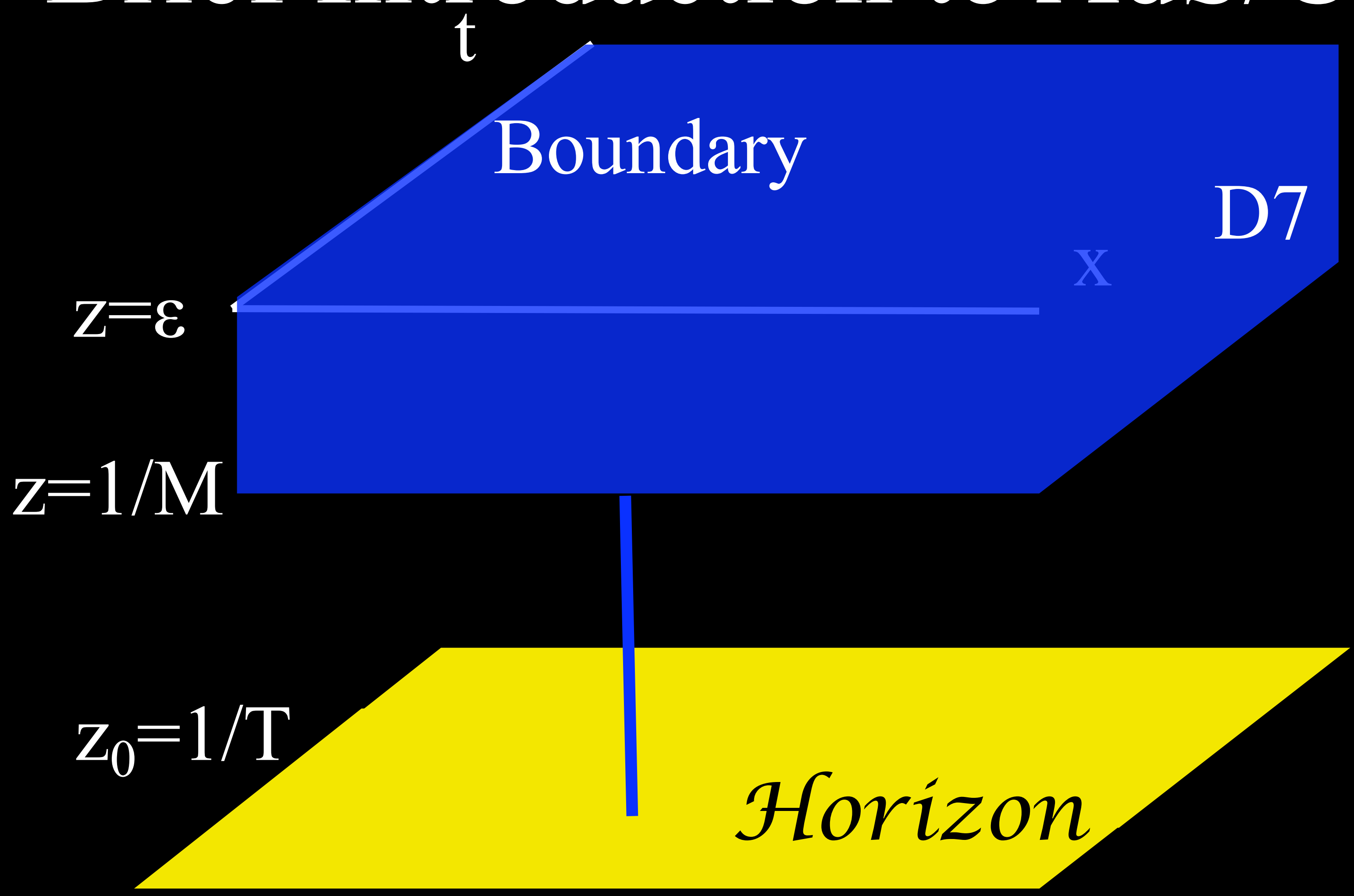

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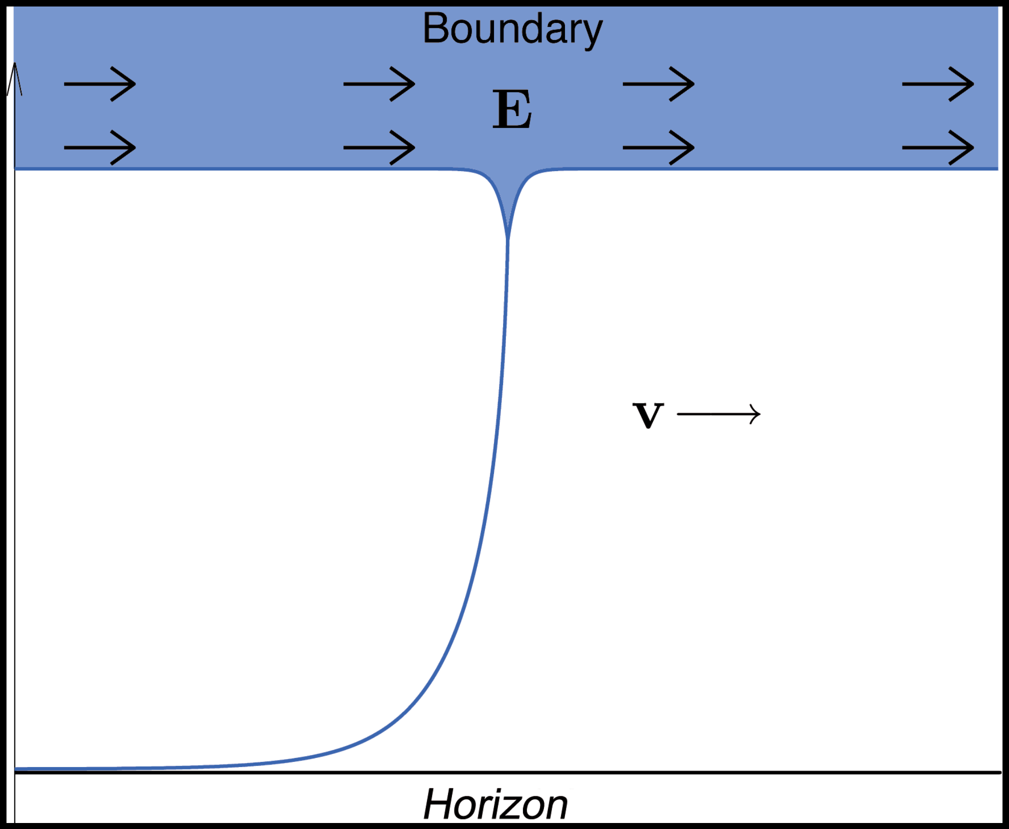
Drag Force on Heavy Quarks

(Herzog, Karch, Kovtun, Kozcaz and Yaffe ; Gubser)



$$\frac{dp}{dt} = -\eta_D p$$

$$\eta_D = \frac{\sqrt{\lambda}\pi T^2}{2M}$$

Even for U-relativistic!

Forced motion of the quark by an external E field (stationary)

The string bends and lags behind the quark endpoint

The work over the string tension leads to energy loss

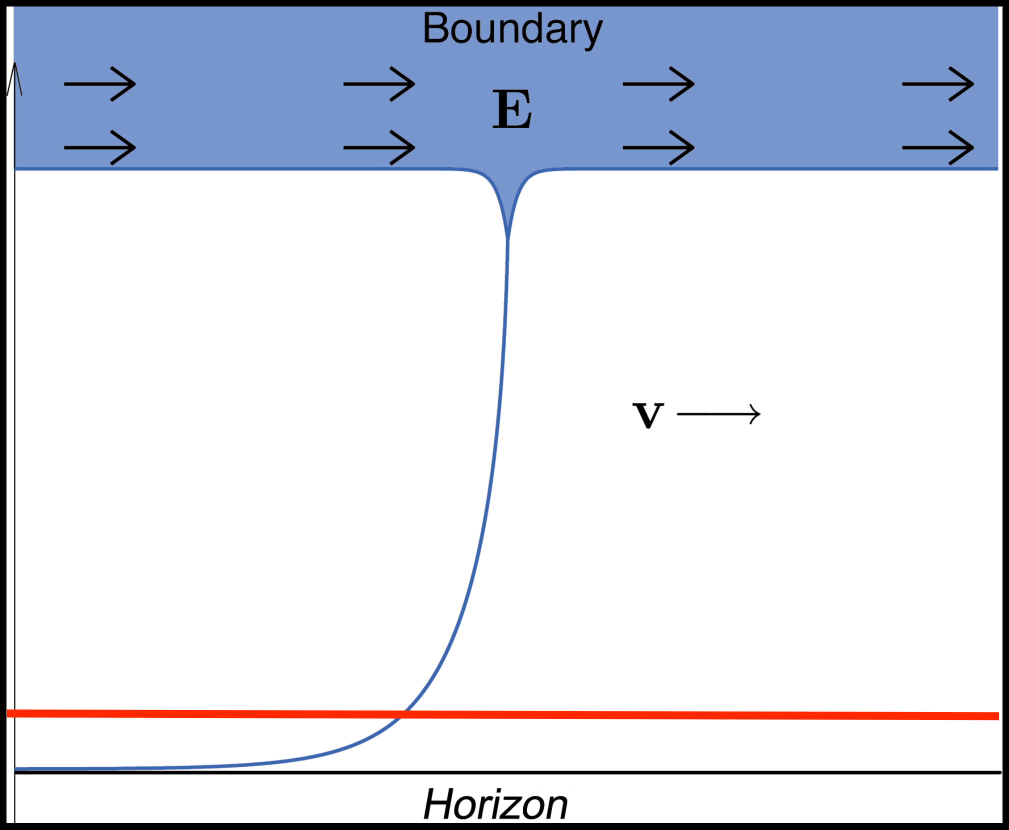
The string develops as new scale $z_{ws} \sim 1/\sqrt{\gamma}T$

Drag Force on Heavy Quarks

(Herzog, Karch, Kovtun, Kozcaz and Yaffe ; Gubser)

Upper and lower ends are causally disconnected

$$c_{z_{ws}} = \sqrt{-\frac{g_{tt}}{g_{xx}}} = v$$



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Discussion on the Drag Result

The drag coefficient is

$$\eta_D = \frac{\kappa}{2MT} = \frac{\sqrt{\lambda}\pi T^2}{2M}$$

Assuming Brownian motions, the Einstein relations

$$\kappa = \sqrt{\lambda}\pi T^3 \qquad\qquad D = \frac{2}{\sqrt{\lambda}\pi T}$$

(direct computation by JCS and Teaney 06)

Putting numbers:

$$D \simeq \frac{1.0}{2\pi T}\left(\frac{1.5}{\alpha_{SYM} N}\right)^{1/2}$$

Comparable to RHIC value!

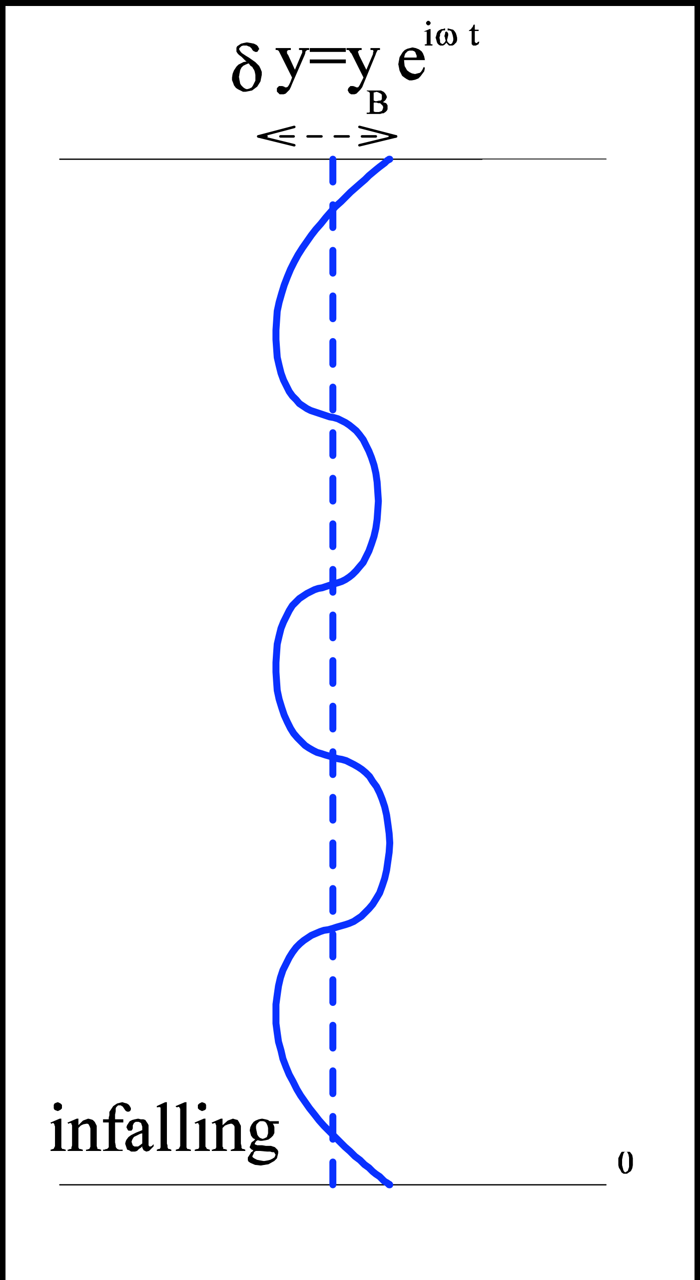
Where is the noise?

(de Boer, Hubeny, Rangamani and Shigemori; Son & Teaney)

What are the memory effects?

String Fluctuations

Small string fluctuations on top of the drag solution
(hat coordinates are a convenient redefinition)



The fluctuations "live" in the world-sheet

$$S_{NG} = -\frac{R^2}{2\pi l_s^2}\int \frac{d\hat{t}d\hat{z}}{\hat{z}^2}\left[1 - \frac{1}{2}\left(\frac{(\dot{\hat{y}})^2}{f(\hat{z})} - f(\hat{z})(\hat{y}')^2\right)\right]$$

S_2^T

Close to the world-sheet horizon the solution behaves as

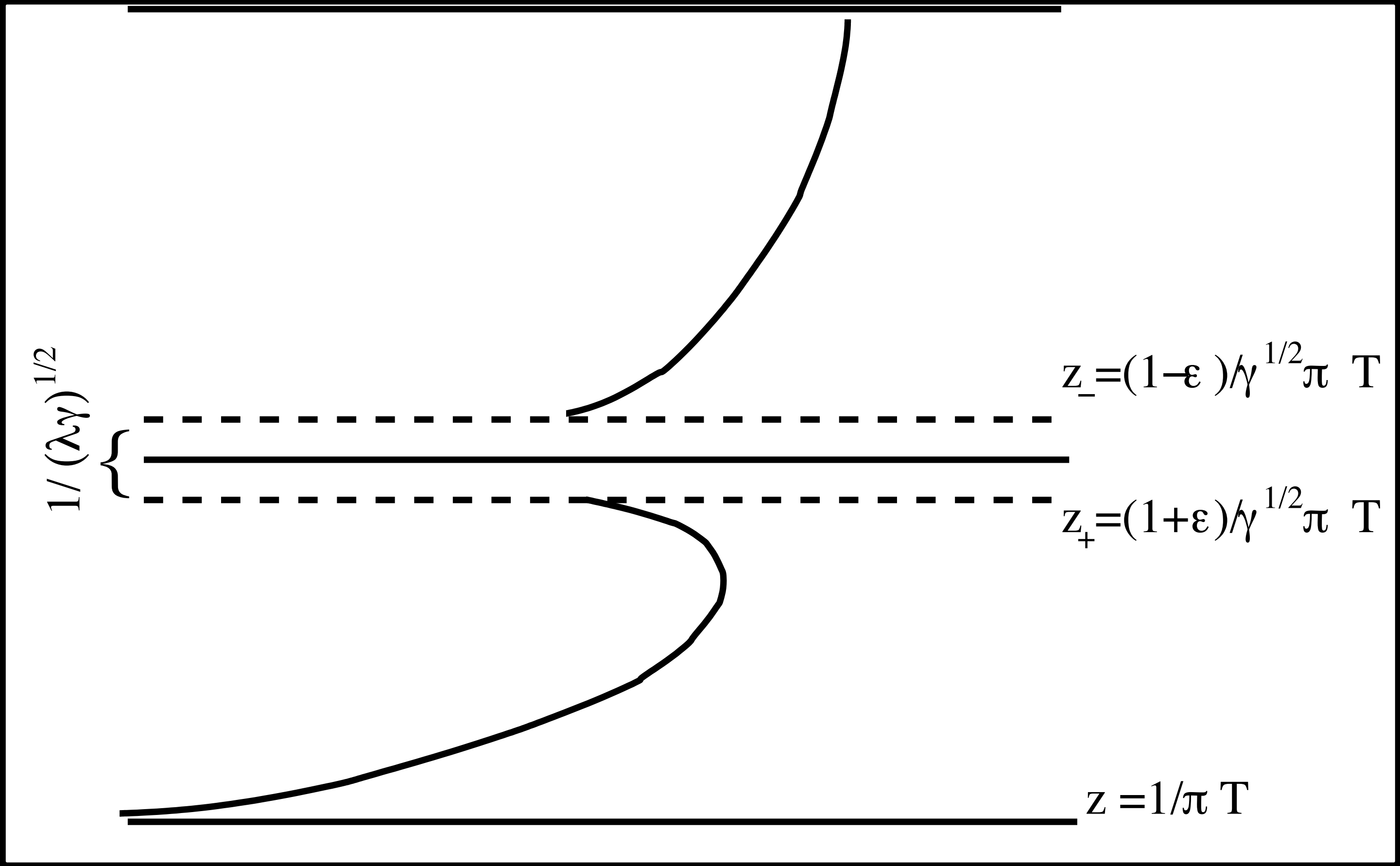
$$F_{\hat{w}}(\hat{z}) = e^{i\hat{w}t}\,(1-\hat{z})^{-i\hat{w}/4}$$ infalling

$$F^*_{\hat{w}}(\hat{z}) = e^{i\hat{w}t}\,(1-\hat{z})^{i\hat{w}/4}$$ outgoing

$$\hat{\omega} = \sqrt{\gamma}\omega \qquad \hat{z} = \sqrt{\gamma}\pi T z$$

The fluctuations are log divergent close to the w-s horizon.

Origin of Fluctuations



- The small fluctuations grow close to the world-sheet horizon
- Strategy: Integrate out a strip above and below the horizon (stretched horizon).
- The strip can be small $\varepsilon > (\lambda\Upsilon)^{-1/2}$.
- The integration of these modes leads to stochasticity.

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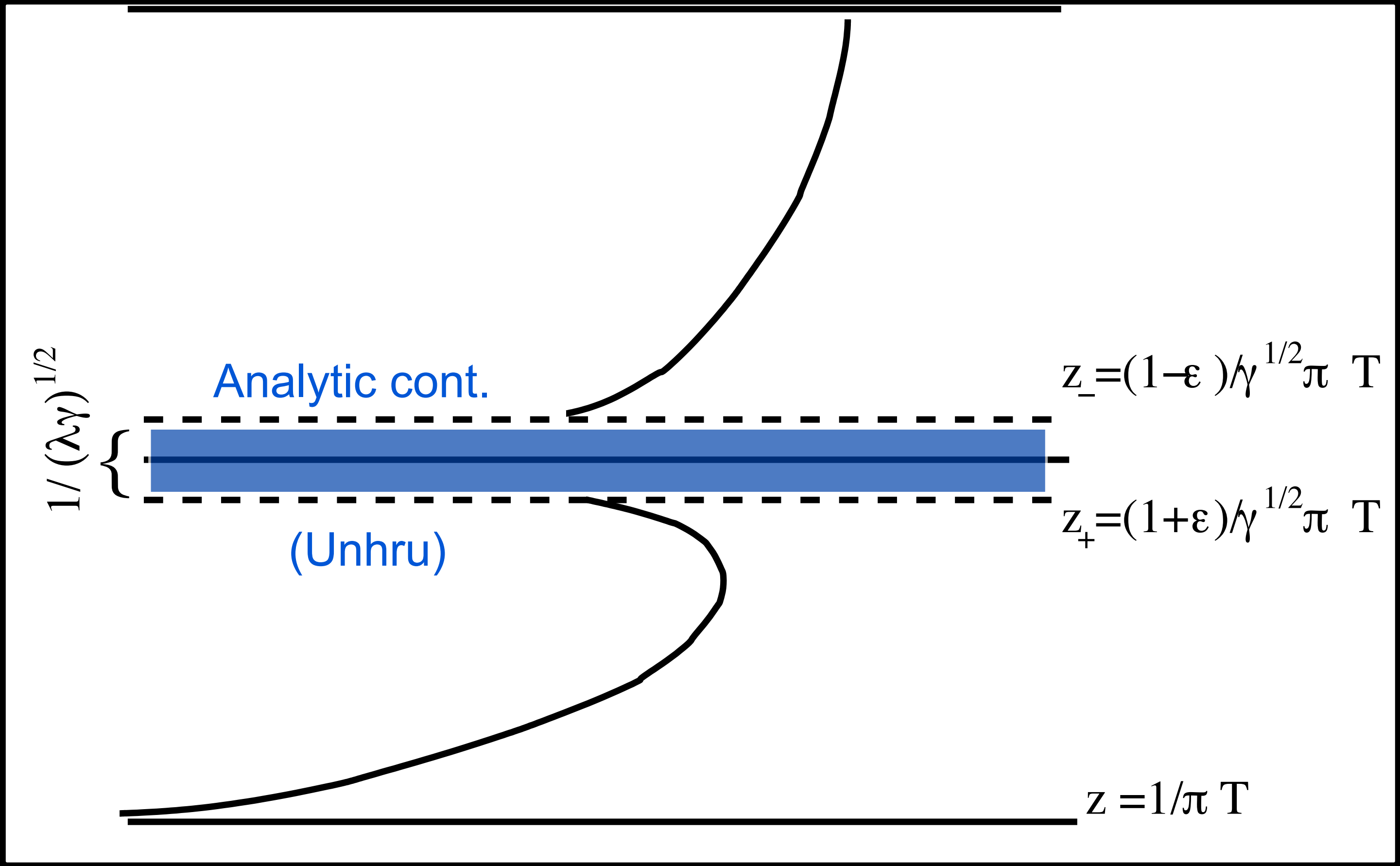

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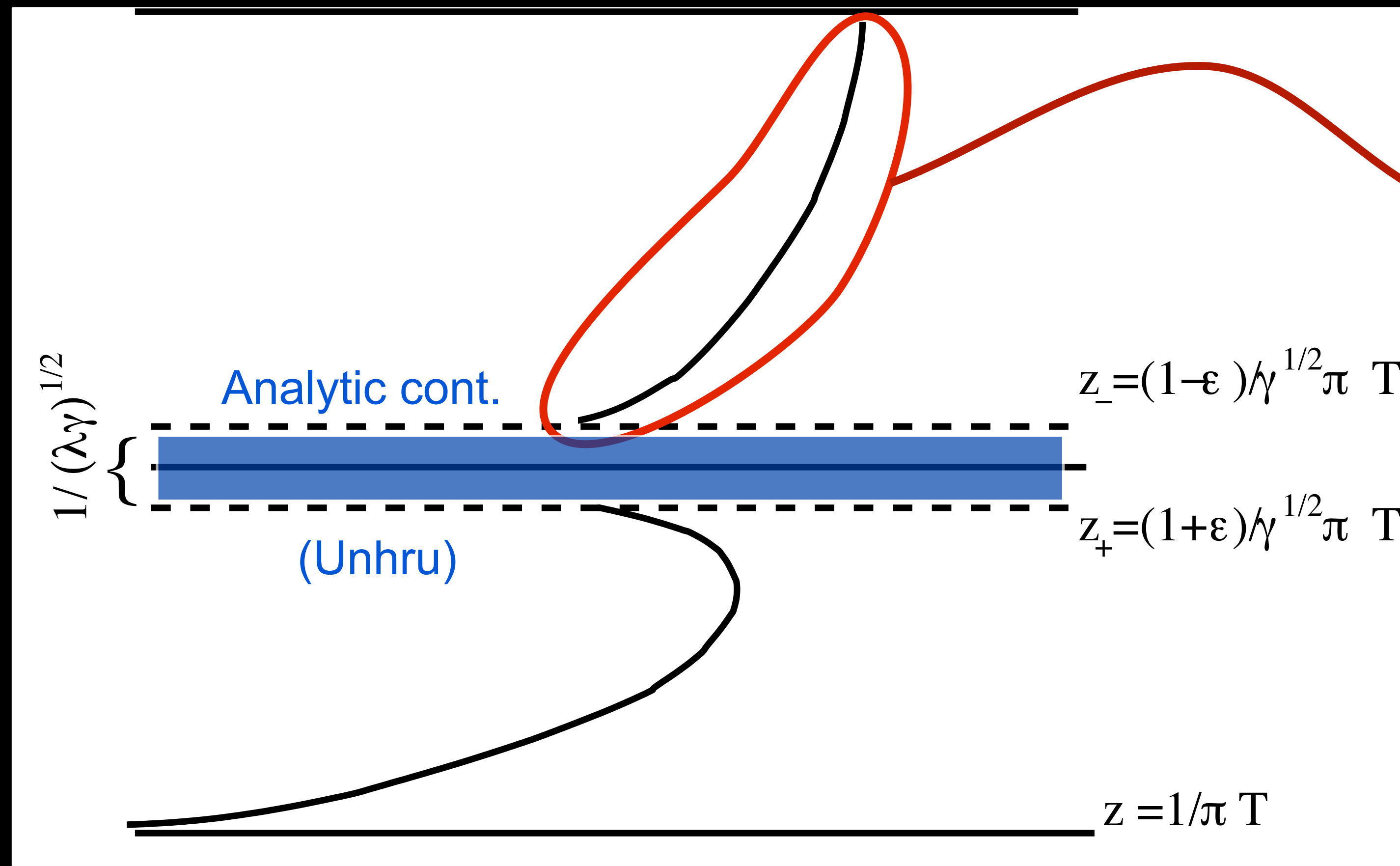

Giecold, Iancu, Mueller 09

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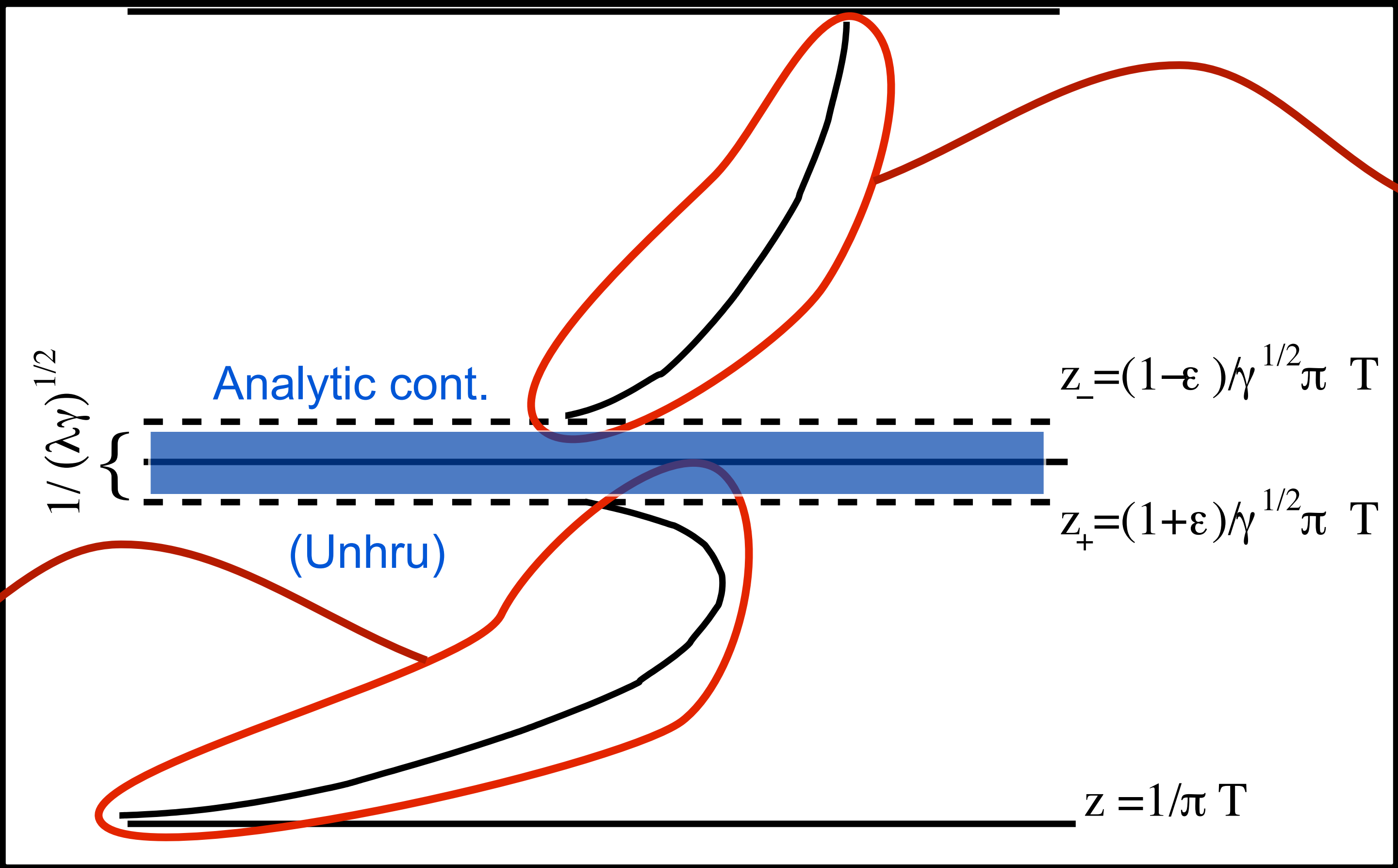

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JCS, K-Y Kim, Teaney (09)

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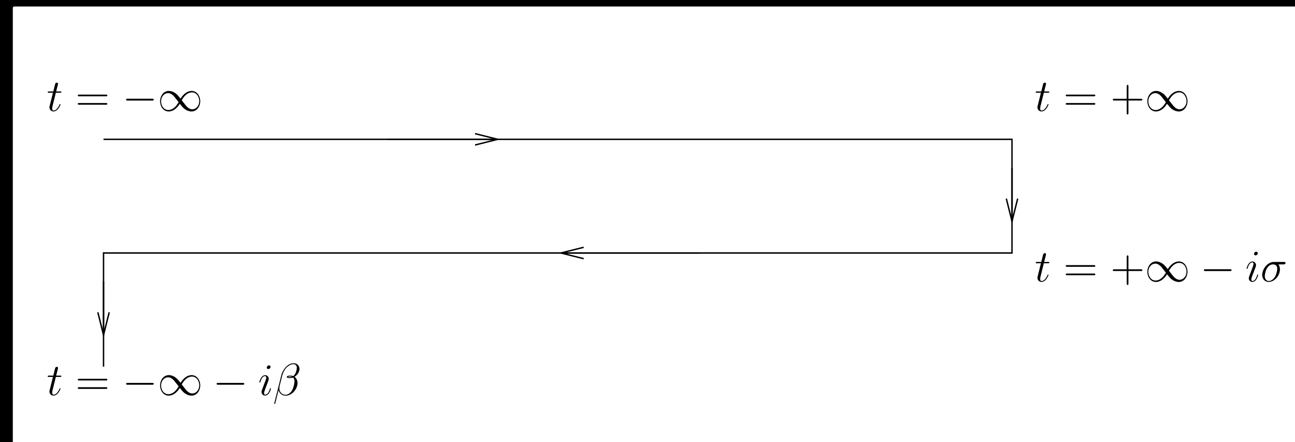
Schwinger-Keldysh: Quantum Particle



A particle state in the plasma is described by a density matrix:

$$\hat{\rho}(t)=e^{-iHt}\hat{\rho}(0)e^{iHt}$$

$$\rho(x_1,x_2;t)=\langle x_1|\,\hat{\rho}(t)\,|x_2\rangle$$

$$\rho(x_2,x_1;t)=\int dy_1dy_2\,\langle x_1|\,e^{-iHt}\,|y_1\rangle\,\rho(y_1,y_2;t)\,\langle y_2|\,e^{iHt}\,|x_2\rangle$$

"Normal" PI "Time reverted" PI

Two types of PI ⇒ Two sets of fields "1" and "2"

The Schwinger-Keldysh contour "connects" both contours into a single one.

Brownian Motion for a Quantum Particle (I)

Motion of a particle under a random force:

$$S[x(t)] = \int dt \left(\frac{1}{2} M \dot{x}(t)^2 + \mathcal{F}(t) x(t) \right) \quad \Rightarrow \quad Z = \left\langle \int \mathcal{D}x_1 \mathcal{D}x_2 e^{iS[x_1(t_1)] - iS[x_2(t_2)]} \right\rangle_{bath}$$

Since the mass is large, the force induces small changes:

$$Z = \int \mathcal{D}x_1 \mathcal{D}x_2 e^{i \int dt_1 \frac{M}{2} \dot{x}_1^2(t_1) - i \int dt_2 \frac{M}{2} \dot{x}_2^2(t_2) dt_2} e^{-\frac{1}{2} \int dt dt' x_s(t) \langle \mathcal{F}_s(t) \mathcal{F}_{s'}(t') \rangle x_{s'}(t')}$$

The partition function depends on 4 correlators. At finite T they are not independent.

$$\begin{aligned} iG_{11}(t,t') &= \left\langle \mathcal{T} \mathcal{F}_1(t) \mathcal{F}_1(t') \right\rangle \\ iG_{12}(t,t') &= \left\langle \mathcal{F}_2(t') \mathcal{F}_1(t) \right\rangle \\ iG_{21}(t,t') &= \left\langle \mathcal{F}_2(t) \mathcal{F}_1(t') \right\rangle \\ iG_{22}(t,t') &= \left\langle \mathcal{T}^* \mathcal{F}_2(t) \mathcal{F}_2(t') \right\rangle \end{aligned}$$

$$\begin{aligned} iG_{11}(\omega) &= +i\mathrm{Re}G_R(\omega) - (1+2n)\,\mathrm{Im}G_R(\omega) \\ iG_{22}(\omega) &= -i\mathrm{Re}G_R(\omega) - (1+2n)\,\mathrm{Im}G_R(\omega) \\ iG_{12}(\omega) &= -2ne^{\omega\sigma}\mathrm{Im}G_R(\omega) \\ iG_{21}(\omega) &= -2\,(1+n)\,e^{-\omega\sigma}\mathrm{Im}G_R(\omega) \end{aligned}$$

Brownian Motion for a Quantum Particle (II)

Introduce the coordinates $x_r = \frac{x_1 + x_2}{2}$ $x_a = x_1 - x_2$

$$Z = \int \mathcal{D}x_a \mathcal{D}x_r e^{-i\int \frac{d\omega}{2\pi} x_a(-\omega)\left(-M\omega^2 + G_R(\omega)\right)x_r(\omega)} e^{-\frac{1}{2}\int \frac{d\omega}{2\pi} x_a(-\omega) G_{sym}(\omega) x_a(\omega)}$$

$$G_{sym}(\omega) = -(1+2n)\mathrm{Im}G_R(\omega)$$

Linearizing the quadratic term

$$Z = \int \mathcal{D}x_a \mathcal{D}x_r \mathcal{D}\xi e^{-i\int \frac{d\omega}{2\pi} \frac{\xi(-\omega)\xi(\omega)}{G_{sym}(\omega)}}$$

$$\times \exp\left(-\int \frac{d\omega}{2\pi} x_a(-\omega)\left(-M\omega^2 x_r(\omega) + G_R(\omega) - \xi(\omega)\right)\right) x_r(\omega)$$

At small frequency we obtain Langevin

$$G_R(\omega) = -i\omega\eta \qquad \langle \xi(t)\xi(t')\rangle = G_{sym}(t-t')$$

Doubling of fields in AdS

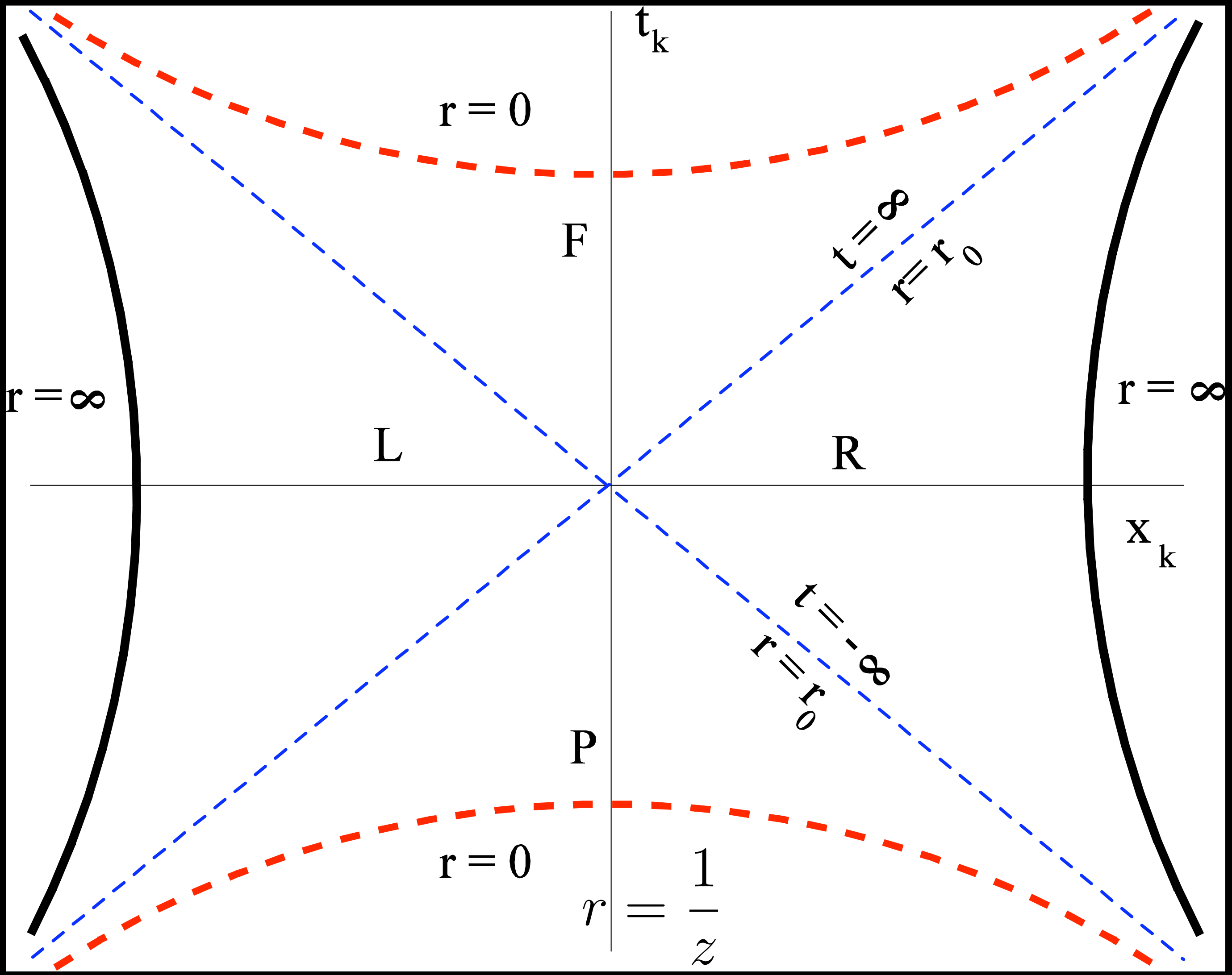
As in black holes, (t, z) –coordinates are only defined for $z<z_0$

Proper definition of coordinate, two copies of (t, z) related by time reversal.

In the presence of black branes the space has two boundaries (L and R)

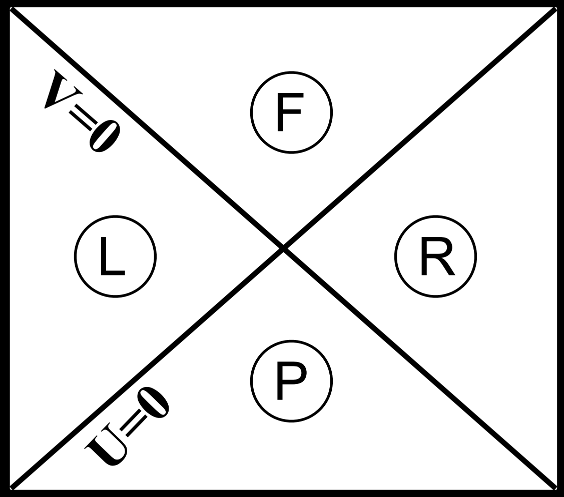
SUGRA fields on L and R boundaries are type 1 and 2 sources

Maldacena
Herzog, Son (02)



The presence of two boundaries leads to properly defined thermal correlators (KMS relations)

Matching Fluctuations



Extension of solutions to the full Kruskal plane

Unhru prescription:

Positive energy modes ⇒ infalling $U^{i\omega}$

Negative energy modes ⇒ outgoing $V^{-i\omega}$

The prescription leads to the correct thermal correlators Herzog, Son (02)

It provides a prescription for going around the $\log{(1-\hat{z})}$

String fluctuations are decomposed in infalling/outgoing basis

$$\hat{X}_i(\hat{\omega},\hat{z}) = a_i(\hat{\omega})e^{\delta_i\theta(\hat{z}-1)\hat{\omega}/2T}F^*_\omega(\hat{z}) + b_i(\hat{\omega})F_\omega(\hat{z})$$

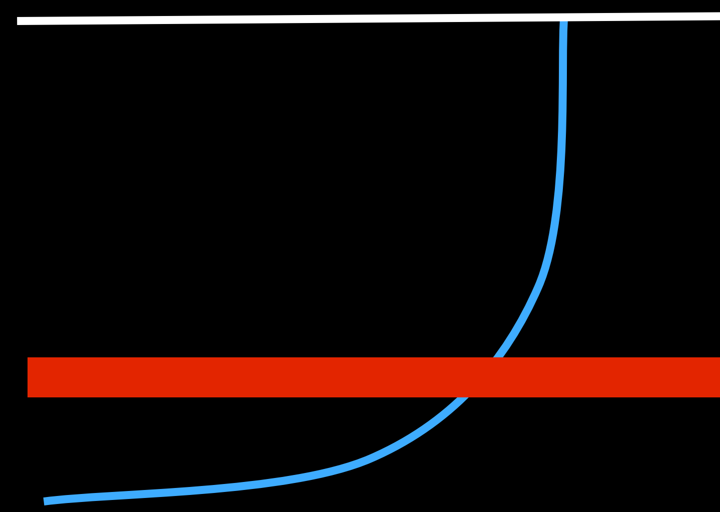
$$a_2(\hat{\omega}) = e^{-\omega\sigma}e^{\hat{\omega}/T}a_1(\hat{\omega})$$

$$b_2(\hat{\omega}) = e^{-\omega\sigma}b_1(\hat{\omega})$$

$$\delta_1 = 1 \qquad \delta_2 = -1$$

Integrating String Fluctuations

Partition function for string fluctuations



$$\mathcal{Z}_\alpha = \int \mathcal{D}_s \hat{X}_1(\hat{t},\hat{z}) \mathcal{D}_s \hat{X}_2(\hat{t},\hat{z}) e^{iS_\alpha[\hat{X}_1] - iS_\alpha[\hat{X}_2]}$$

We use the quadratic action above and below the w-s horizon.

The fluctuations exist in both sides of the Kruskal plane

The connection of the different fluctuations is performed via analytic continuation

Integrating out x_a

$$\begin{aligned}\mathcal{Z} = \int \mathcal{D}\hat{x}^b_r \, \mathcal{D}\hat{x}^b_a \, \mathcal{D}\hat{x}^w_{r-} \mathcal{D}_s \hat{X}_r \Big\langle & e^{i\int \frac{d\hat{\omega}}{2\pi} T_o(z_b)\hat{x}^b_a(-\hat{\omega})\partial_z \hat{X}_r(\hat{\omega},\hat{\epsilon})} \\ & \times\delta\left(\partial_{\hat{z}}\left(T_o(\hat{z})\partial_{\hat{z}}\hat{X}_r(\hat{\omega},\hat{z})\right) + \frac{m\hat{\omega}^2}{\pi T \hat{z}^2 f(\hat{z})}\hat{X}_r(\hat{\omega},\hat{z})\right) \\ & \times\delta\left(T_o(\hat{z}_-)\partial_{\hat{z}}\hat{X}_r(\hat{\omega},\hat{z}_-) - \hat{\xi}^w(\hat{\omega}) - T_o(z_w)\frac{\partial_{\hat{z}}F^w_{\hat{\omega}}}{F^w_{\hat{\omega}}}\left(\hat{x}^w_{r-}(\hat{\omega}) + \Delta(\hat{\omega})\right)\right) \\ & \times\delta\left(T_o(\hat{z}_+)\partial_{\hat{z}}\hat{X}_r(\hat{\omega},\hat{z}_+) - \hat{\xi}^w(\hat{\omega}) - T_o(z_w)\frac{\partial_{\hat{z}}F^w_{\hat{\omega}}}{F^w_{\hat{\omega}}}\hat{x}^w_{r-}(\hat{\omega})\right)\Big\rangle_{\hat{\xi},\Delta},\end{aligned}$$

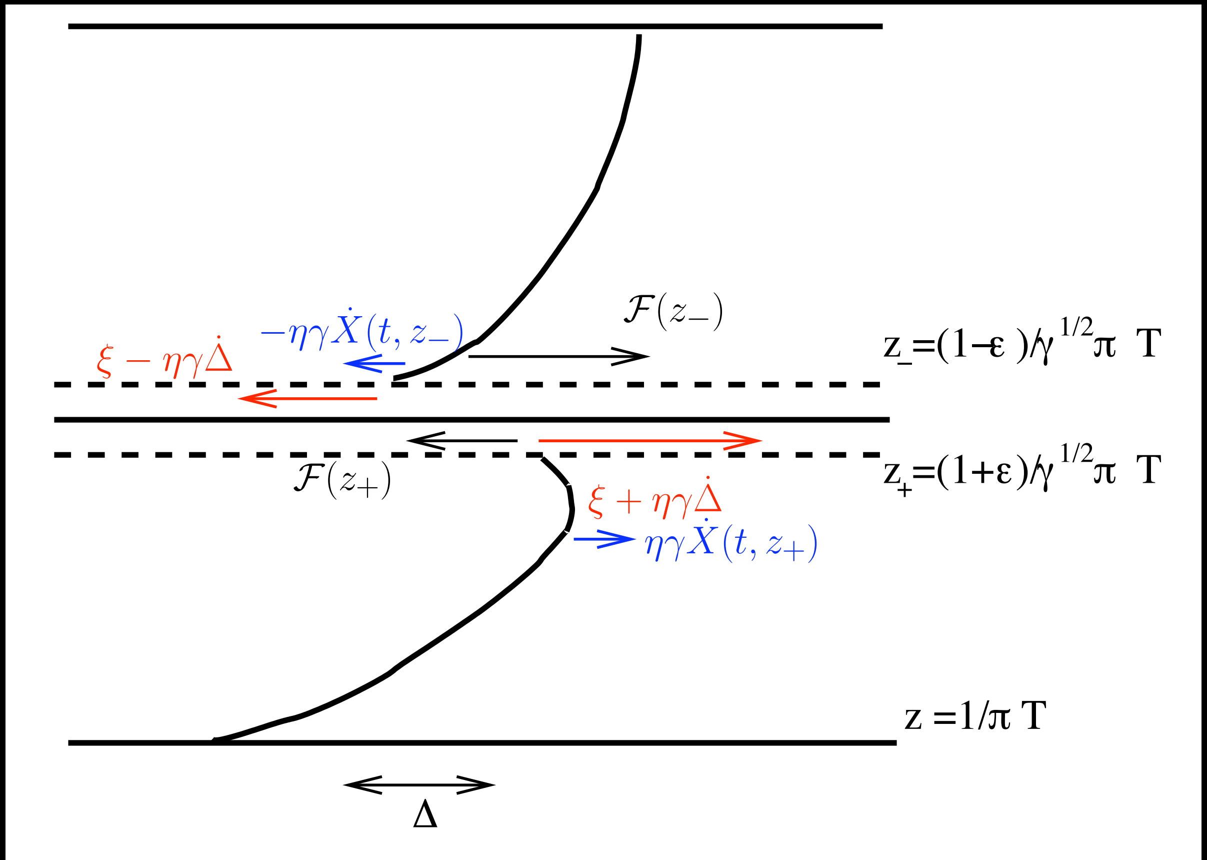
quark partition function

string equation

boundary condition

boundary condition

Balance of Forces



- Effect of the horizon
  - Random force on the string
  - Random discontinuity

$$\langle \xi^{wsh}(-\omega)\xi^{wsh}(\omega)\rangle = \gamma\frac{\sqrt{\lambda}\pi T^2\omega}{4}\frac{e^{\omega\sqrt{\gamma}/2T}+1}{e^{\omega\sqrt{\gamma}/2T}-1}$$

$$\langle \Delta(-\omega)\Delta(\omega)\rangle = \frac{1}{\sqrt{\gamma\lambda}\pi T^2\omega}\frac{e^{\omega\sqrt{\gamma}/2T}-1}{e^{\omega\sqrt{\gamma}/2T}+1}$$

- The force on the boundary is a combination of these two.

The Stochastic String

- String solution under the horizon forze.

Normalizable

$$\hat{X}_{L\,r} = \hat{x}^b_{L\,r} F_{\hat{\omega}}(\hat{z}) + \frac{\mathrm{Im} F_{\hat{\omega}}(\hat{z})}{-\gamma^2 \mathrm{Im} G_R(\hat{\omega})} F^{wsh}_{\hat{\omega}} \left(\xi^{wsh}_L + \gamma^2 T_0{}^{wsh} \frac{\partial_{\hat{z}} F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}} \Delta_L(\hat{\omega}) \right)$$
$$+\theta(\hat{z}-1)\Delta_L(\hat{\omega}) \frac{\partial_{\hat{z}} F^*_{\hat{\omega}}(\hat{z})}{F^{wsh*}},$$

Effective force

Jump

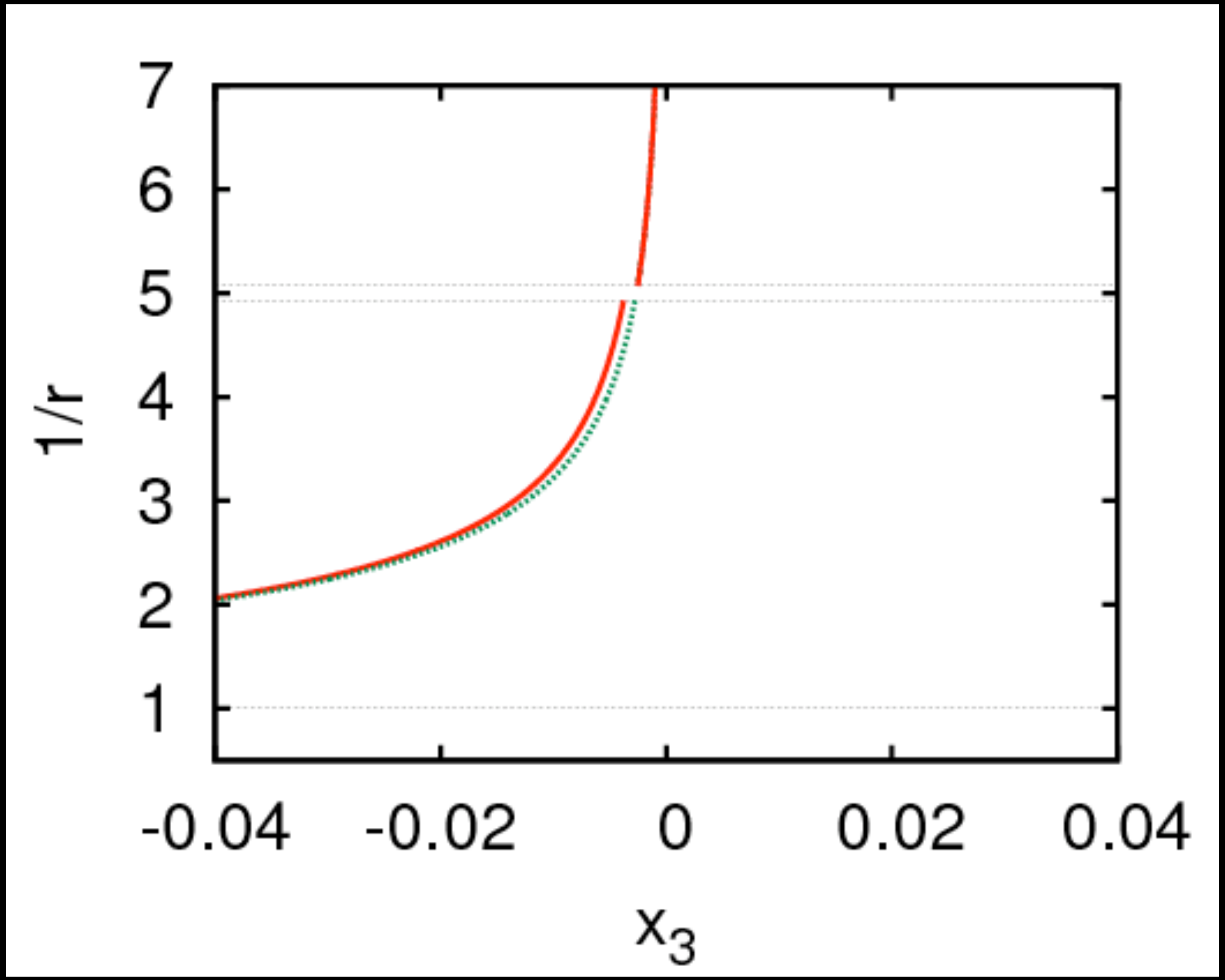
- Correlation function induced by the random force/jump

$$\begin{aligned}
\hat{G}_{T\,sym}(\hat{\omega}, \hat{z}, \hat{z}') &\equiv \left\langle \Delta \hat{X}_r(\hat{\omega}, \hat{z}) \Delta \hat{X}_r(-\hat{\omega}, \hat{z}') \right\rangle \\
&= -\frac{\mathrm{Im} F_{\hat{\omega}}(\hat{z}) \mathrm{Im} F_{\hat{\omega}}(\hat{z}')}{\mathrm{Im} G_R(\hat{\omega})} (1+2\hat{n}) \\
&\quad + \theta(\hat{z}\pi T - 1) \frac{1}{2} \frac{\mathrm{Im} F_{\hat{\omega}}(\hat{z}') i F^*_{\hat{\omega}}(\hat{z})}{\mathrm{Im} G_R(\hat{\omega})} \frac{e^{\hat{\omega}/2T} - 1}{e^{\hat{\omega}/2T} + 1} \\
&\quad + \theta(\hat{z}'\pi T - 1) \frac{1}{2} \frac{\mathrm{Im} F_{\hat{\omega}}(\hat{z}) (-i) F_{\hat{\omega}}(\hat{z}')}{\mathrm{Im} G_R(\hat{\omega})} \frac{e^{\hat{\omega}/2T} - 1}{e^{\hat{\omega}/2T} + 1} \\
&\quad - \theta(\hat{z}'\pi T - 1) \theta(\hat{z}\pi T - 1) \frac{1}{2} \frac{F^*_{\hat{\omega}}(\hat{z}) F_{\hat{\omega}}(\hat{z}')}{\mathrm{Im} G_R(\hat{\omega})} \frac{e^{\hat{\omega}/2T} - 1}{e^{\hat{\omega}/2T} + 1};
\end{aligned}$$

$$\mathrm{Im} G_R(\omega) = \sqrt{\lambda} \left(-i \frac{T^2}{2} \omega + \mathcal{O}(\omega^2) \right)$$

- The correlations vanish at infinite coupling

Fluctuations



- The string fluctuates around the trailing string.
- There is a random momentum flux through the horizon
- The fluctuations above and below the w-s horizon are separated by a random variable

Effect on the Quark Motion

(Giecold, Iancu, Mueller 09)

- The random momentum flux leads to the noise at the boundary.

$$\mathbf{x}_Q = (vt + x_l \, , \mathbf{x}_r)$$

$$\gamma M_0^Q \frac{d^2 x_r^b}{dt^2} + \sqrt{\gamma} \int dt' G_R \left(\frac{t-t'}{\sqrt{\gamma}} \right) x_r^b(t') = \xi(t)$$

$$\langle \xi(t) \xi(t') \rangle = G_{sym}^{v=0} \left(\frac{t-t'}{\sqrt{\gamma}} \right)$$

- The dynamics are very similar at zero and finite velocity.
- The "memory effects" grow with velocity

$$\tau_C \sim \frac{\sqrt{\gamma}}{\pi T}$$

- In the low frequency limit

The effective mass is v dependent!

$$\sqrt{\gamma} G_R(\sqrt{\gamma}\omega) = -i\gamma \frac{\sqrt{\lambda}\pi T^2}{2}\omega + \gamma^{3/2} \frac{\sqrt{\lambda} T}{2}\omega^2 \quad \Longrightarrow \quad M_{kin} = M_Q^0 - \frac{\sqrt{\gamma\lambda} T}{2}$$

- The effective equation of motion at long times is given by

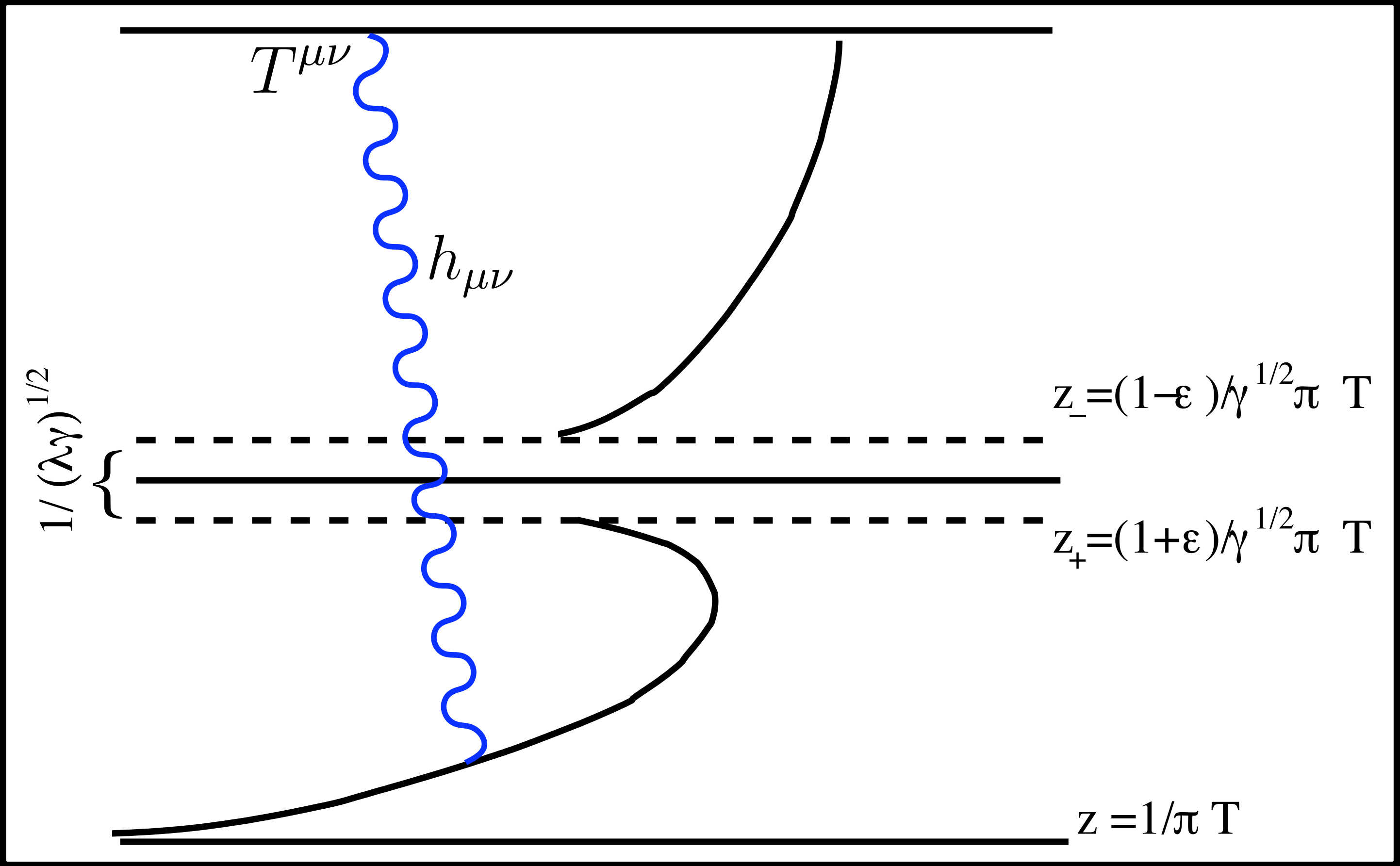
$$\frac{d\mathbf{P}}{dt} = -\mu \mathbf{P} + \mathcal{E}\hat{\mathbf{x}} + \xi \qquad \langle \xi(t) \xi(t') \rangle \; = \; \sqrt{\gamma\lambda} \pi T^3 \delta(t-t')$$

(JCS, Teaney 07)

Limits on Validity

- The calculation is valid for $$\sqrt{\gamma\lambda} \ll \frac{M_0^Q}{T}$$
- Many wrong things happen at this scale
  - Force correlation time is comparable to the relaxation time $$\tau_C \sim \tau_R \sim \frac{M}{\sqrt{\lambda}T^2}$$
  - The thermal correction of to the mass is large $$\frac{\Delta M}{M_Q^0} \sim 1$$
  - The action for the excitation of normalizable modes (states) is large $$\langle S_2[\Delta X]\rangle \sim 1$$
- The system enters in a different regime
  - My guess: radiation dominated, emission of string states.

Outlook



- The effective horizon is only a property of the world sheet.
- The string fluctuation above and below the horizon lead to stress tensor fluctuations on the boundary

$$\left\langle T^{\mu\nu}(x) T^{\mu'\nu'}(x') \right\rangle_{\xi,\Delta}$$

- These correlators are reflected in the particle distribution of particles associated to jets

Conclusions

The momentum broadening κ of the heavy quark is large and depends on the velocity.

Its numerical value is comparable to values extracted from RHIC data on v_2

Quantum fluctuation of the string lead to the appearance of the noise distribution.

We have obtained finite frequency corrections to Langevin dynamics. These may be used in phenomenology.

We have computed the string fluctuations below the (w-s) horizon. These are causally connected to the boundary

The fluctuations might be reflected in the particle correlations.

Change of coordinates

$$\hat{t} = \frac{1}{\sqrt{\gamma}}\left(t + \frac{1}{2}\arctan(z) - \frac{1}{2}\sqrt{\gamma}\arctan(\sqrt{\gamma}z) - \frac{1}{2}\mathrm{arctanh}(\mathrm{z}) + \frac{1}{2}\sqrt{\gamma}\mathrm{artanh}(\sqrt{\gamma}\mathrm{z})\right)$$

$$\hat{z} = \sqrt{\gamma}z$$

The Effective Action

$$
\begin{aligned}
iS^{eff} \;=\; & i\frac{1}{2}T_0{}^{wsh}\int\frac{d\hat{\omega}}{2\pi}\times \qquad (3.9)\\
& \Big[2i\mathrm{Im}\left\{\frac{\partial_{\hat{z}}F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}}\right\}\frac{e^{\hat{\omega}/2T}+1}{e^{\hat{\omega}/2T}-1}\left(\hat{x}^{wsh}_{r-}(-\hat{\omega})-\hat{x}^{wsh}_{r+}(-\hat{\omega})\right)\left(\hat{x}^{wsh}_{r-}(\hat{\omega})-\hat{x}^{wsh}_{r+}(\hat{\omega})\right)\\
& +\frac{i}{2}\mathrm{Im}\left\{\frac{\partial_{\hat{z}}F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}}\right\}\frac{e^{\hat{\omega}/2T}+1}{e^{\hat{\omega}/2T}-1}\left(\hat{x}^{wsh}_{a-}(-\hat{\omega})-\hat{x}^{wsh}_{a+}(-\hat{\omega})\right)\left(\hat{x}^{wsh}_{a-}(\hat{\omega})-\hat{x}^{wsh}_{a+}(\hat{\omega})\right)\\
& +\hat{x}^{wsh}_{a-}(-\hat{\omega})\left(\hat{x}^{wsh}_{r-}(\hat{\omega})\left(\frac{\partial_{\hat{z}}F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}}+\frac{\partial F^{wsh*}_{\hat{\omega}}}{F^{wsh*}_{\hat{\omega}}}\right)+\hat{x}^{wsh}_{r+}(\hat{\omega})\left(\frac{\partial_{\hat{z}}F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}}-\frac{\partial F^{wsh*}_{\hat{\omega}}}{F^{wsh*}_{\hat{\omega}}}\right)\right)\\
& -\hat{x}^{wsh}_{a+}(-\hat{\omega})\left(\hat{x}^{wsh}_{r+}(\hat{\omega})\left(\frac{\partial_{\hat{z}}F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}}+\frac{\partial F^{wsh*}_{\hat{\omega}}}{F^{wsh*}_{\hat{\omega}}}\right)+\hat{x}^{wsh}_{r-}(\hat{\omega})\left(\frac{\partial_{\hat{z}}F^{wsh}_{\hat{\omega}}}{F^{wsh}_{\hat{\omega}}}-\frac{\partial F^{wsh*}_{\hat{\omega}}}{F^{wsh*}_{w}}\right)\right)\Big].
\end{aligned}
$$

Where we have defined (Feynman-Vernon)

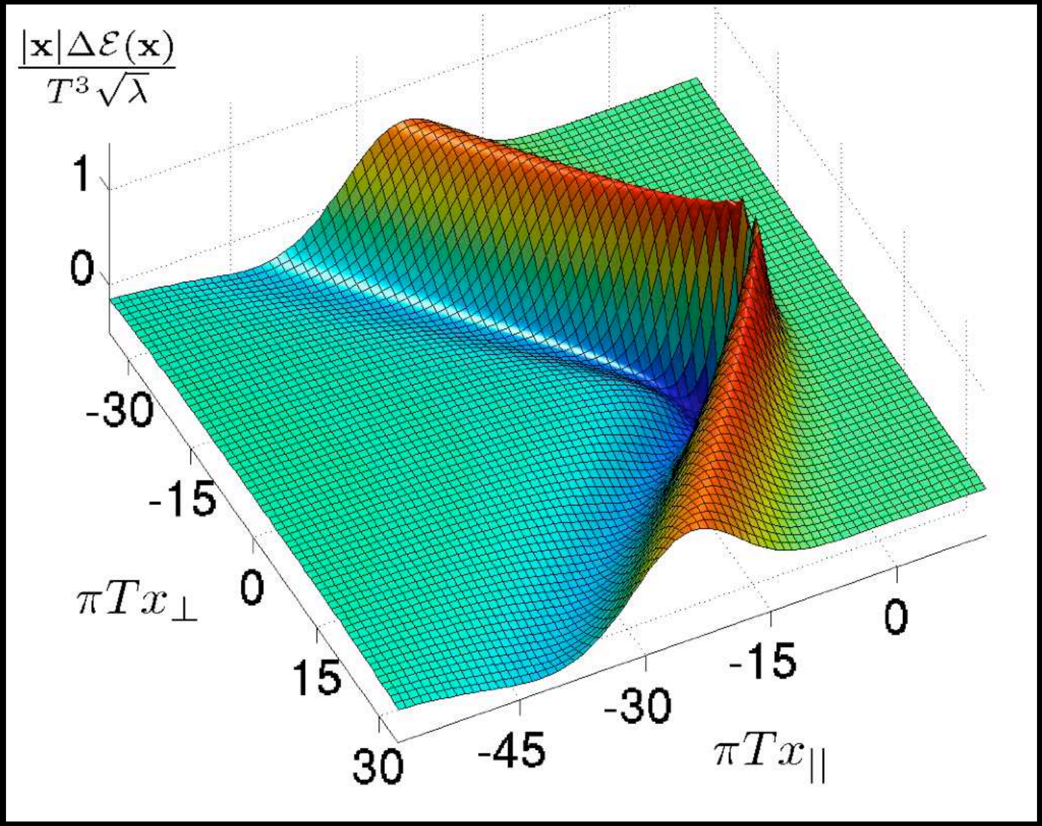
$$x^{wsh}_{r\pm}=\frac{x_{1\pm}+x_{2\pm}}{2}$$

(average position)

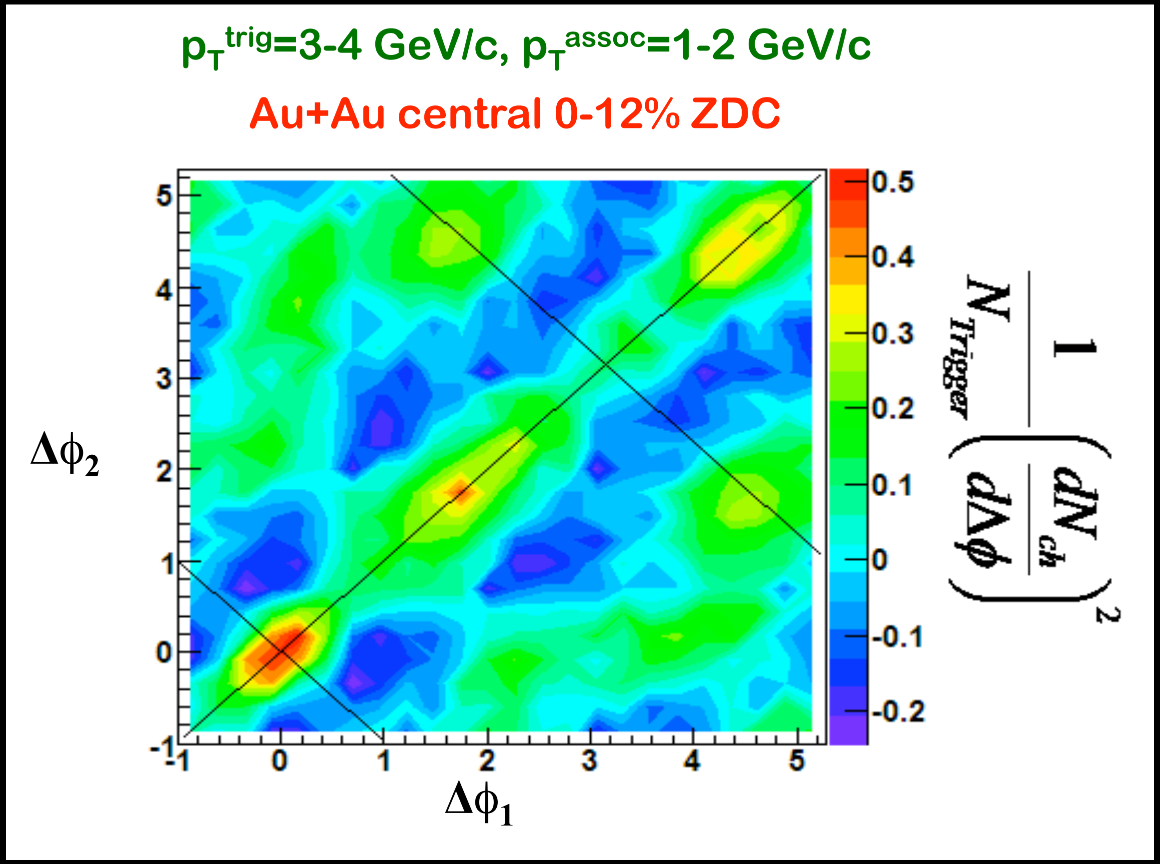
$$x^{wsh}_{a\pm}=x_{1\pm}-x_{2\pm}$$

(Conjugated to momentum)

Chesler & Yaffe; Gubser et al



STAR

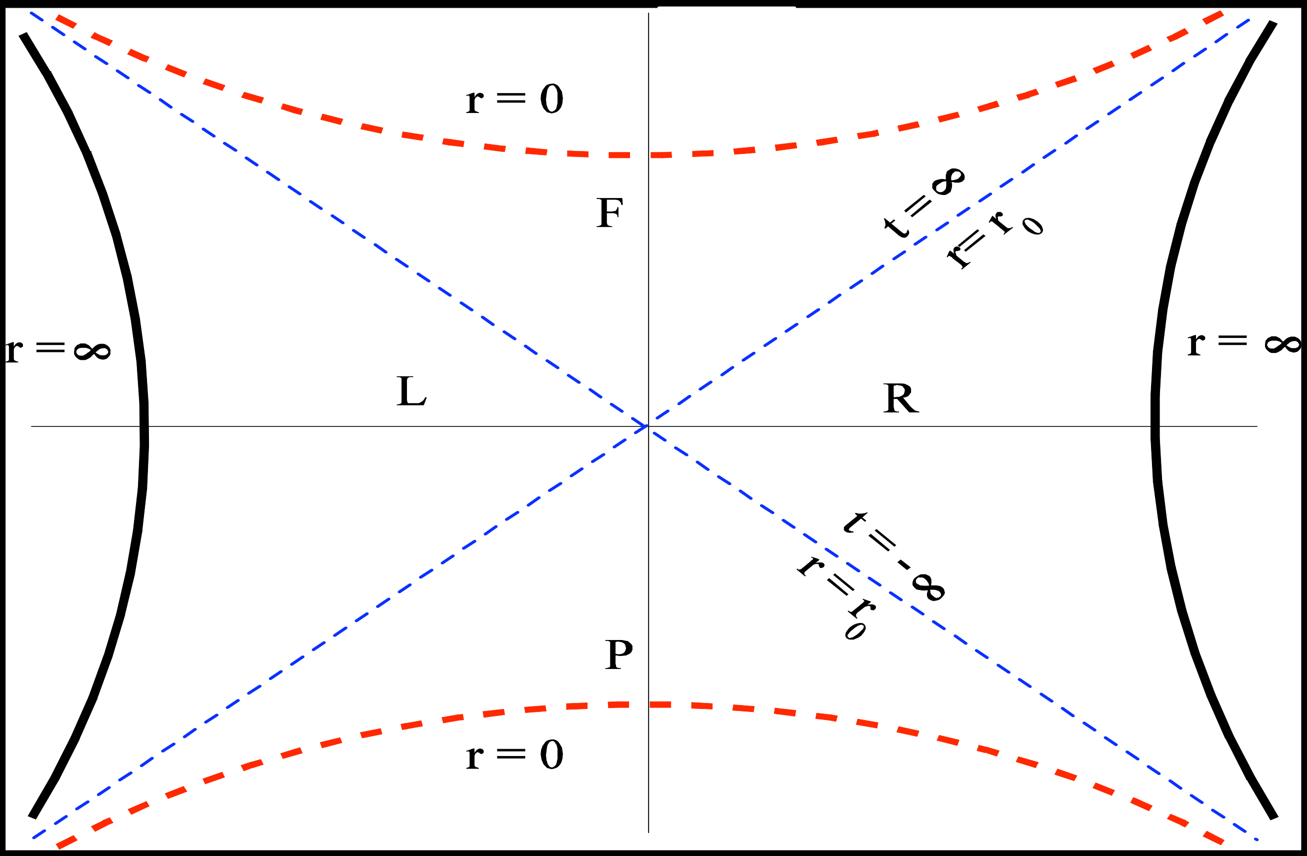


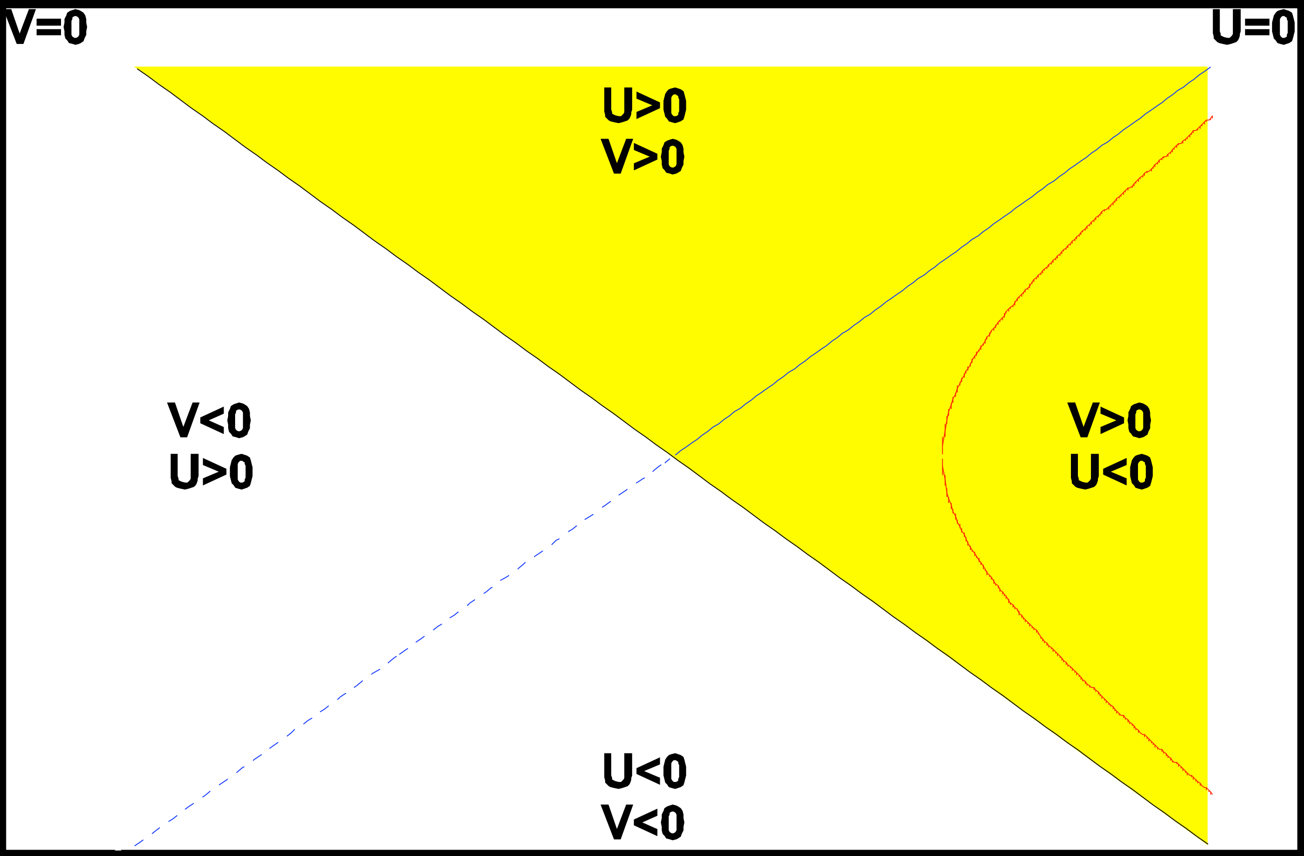
The (average) trailing string leads to Mach Cones

The fluctuations lead to non trivial correlations
They could be reflected in the structure of the 3-particle correlations

String Solution in Global Coordinates

$$x_3 = vt + \xi(z) \quad \longrightarrow \quad x_3 = \frac{v}{2}\log(V) + v\arctan(z)$$





v=0 smooth crossing

v≠0 logarithmic divergence in past horizon. Artifact!

(probes moving from -∞)

String boundaries in L, R universes are type 1, 2 Wilson Lines.

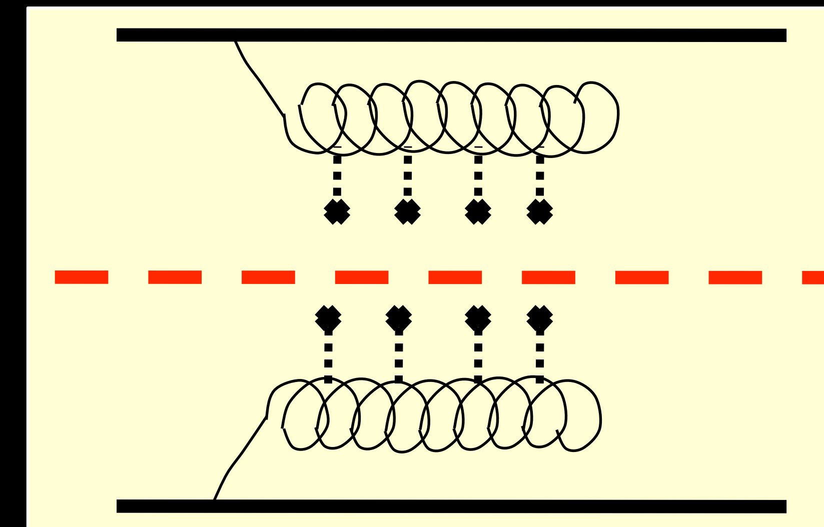
Transverse fluctuations transmit from L ↔ R

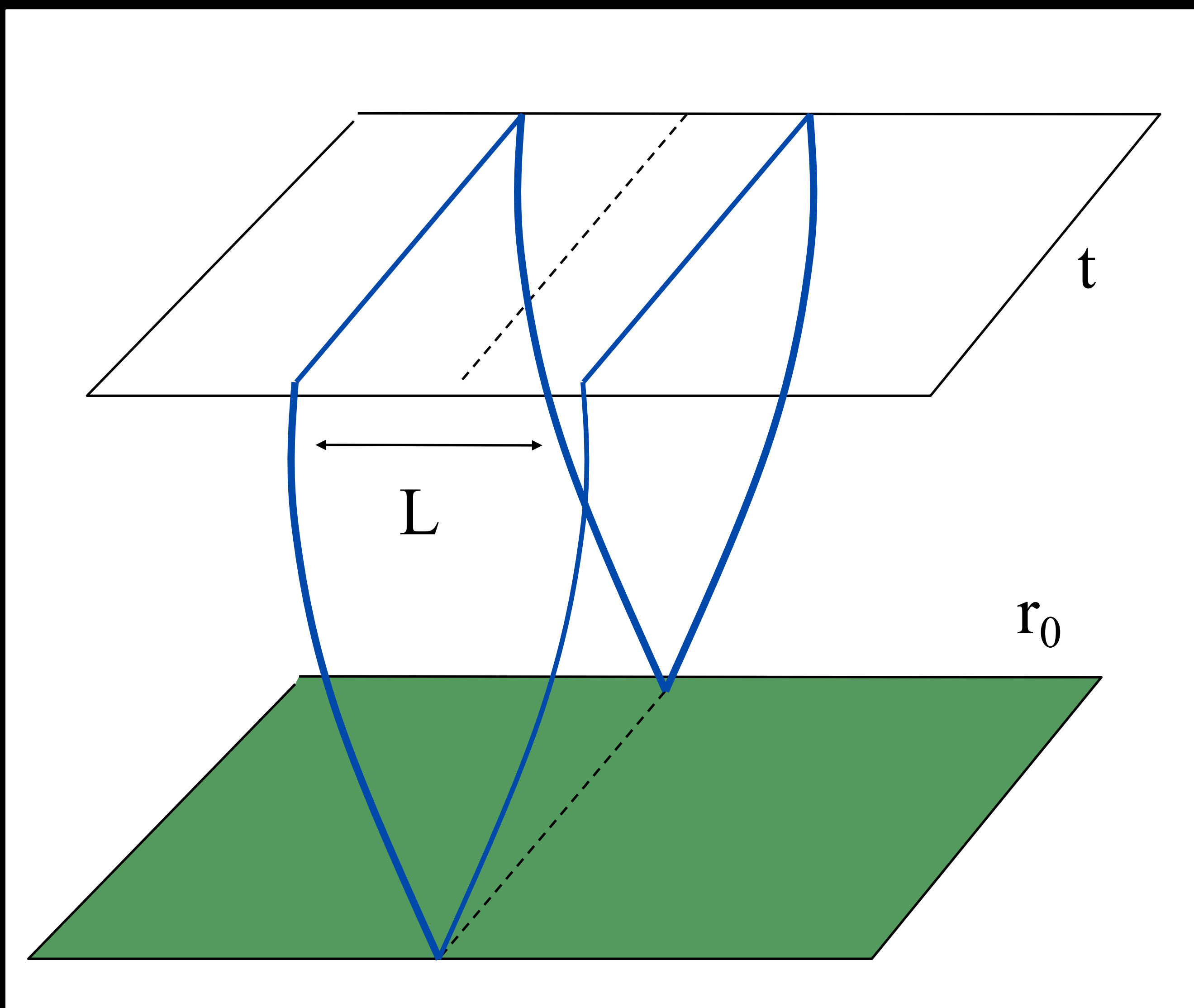
Computation of $\hat{q}$ (Radiative Energy Loss)

(Liu, Rajagopal, Wiedemann)

Dipole amplitude:

two parallel Wilson lines in the light cone:





Order of limits:

1) v ® 1

2) M ® ∞

String action becomes imaginary for

$$\gamma > \left(M/\sqrt{\lambda}T\right)^2$$

For small transverse distance:

$$\langle W \rangle = e^{-S} = e^{-\frac{1}{4}\hat{q}LL^-}$$

entropy scaling

$$\hat{q}_{SYM} = 5.3\sqrt{g^2 N}T^3 \Longrightarrow \hat{q}_{QCD} \approx 6-12 \quad GeV^2/fm$$