# Determination of Dark Matter properties at future $e^+e^-$ colliders

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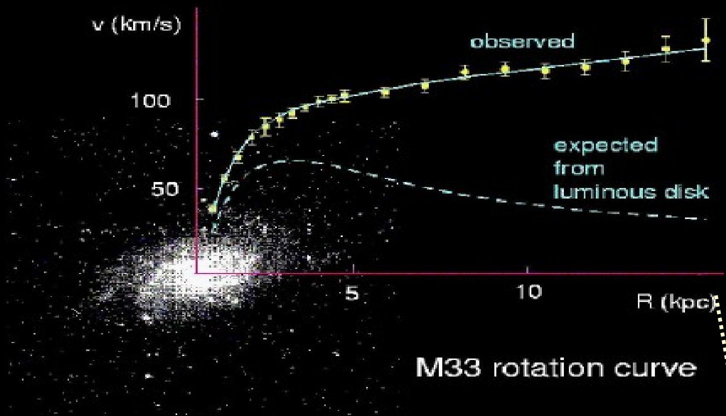
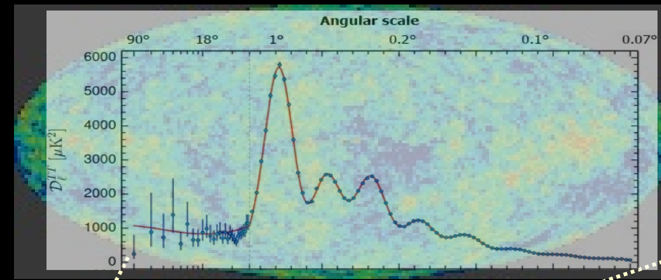
3rd FCC-France / Higgs & ElectroWeak Factory Workshop, Annecy, Nov.30-Dec.2 2021

# The existence of Dark Matter is confirmed by several independent observations at cosmological scale

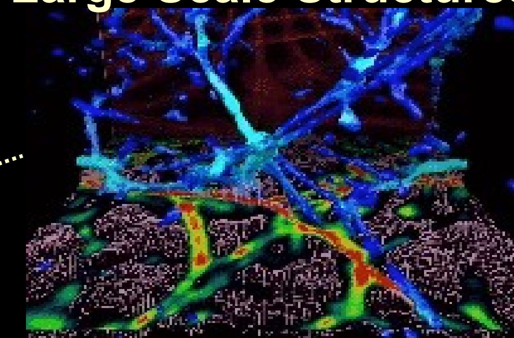
## Galactic rotation curves

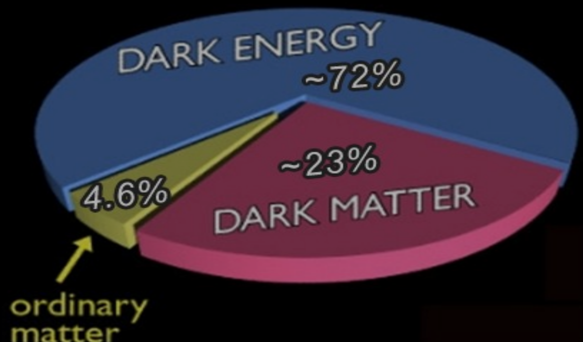



## CMB: WMAP and PLANCK



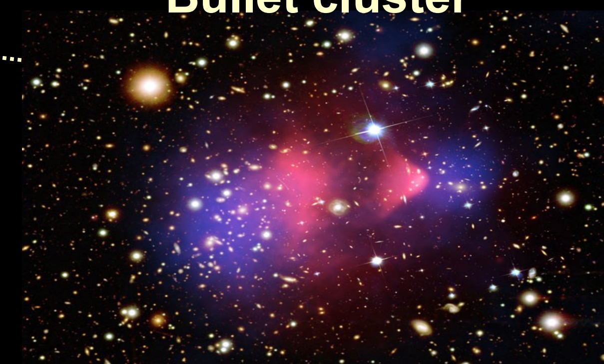

## Large Scale Structures






## Gravitational lensing

## Bullet cluster



# DM is very appealing even though we know almost nothing about it!

*Spin* [?]

*Mass* [?]

*Stable*

Yes [?] No [?]

symmetry behind stability [?]

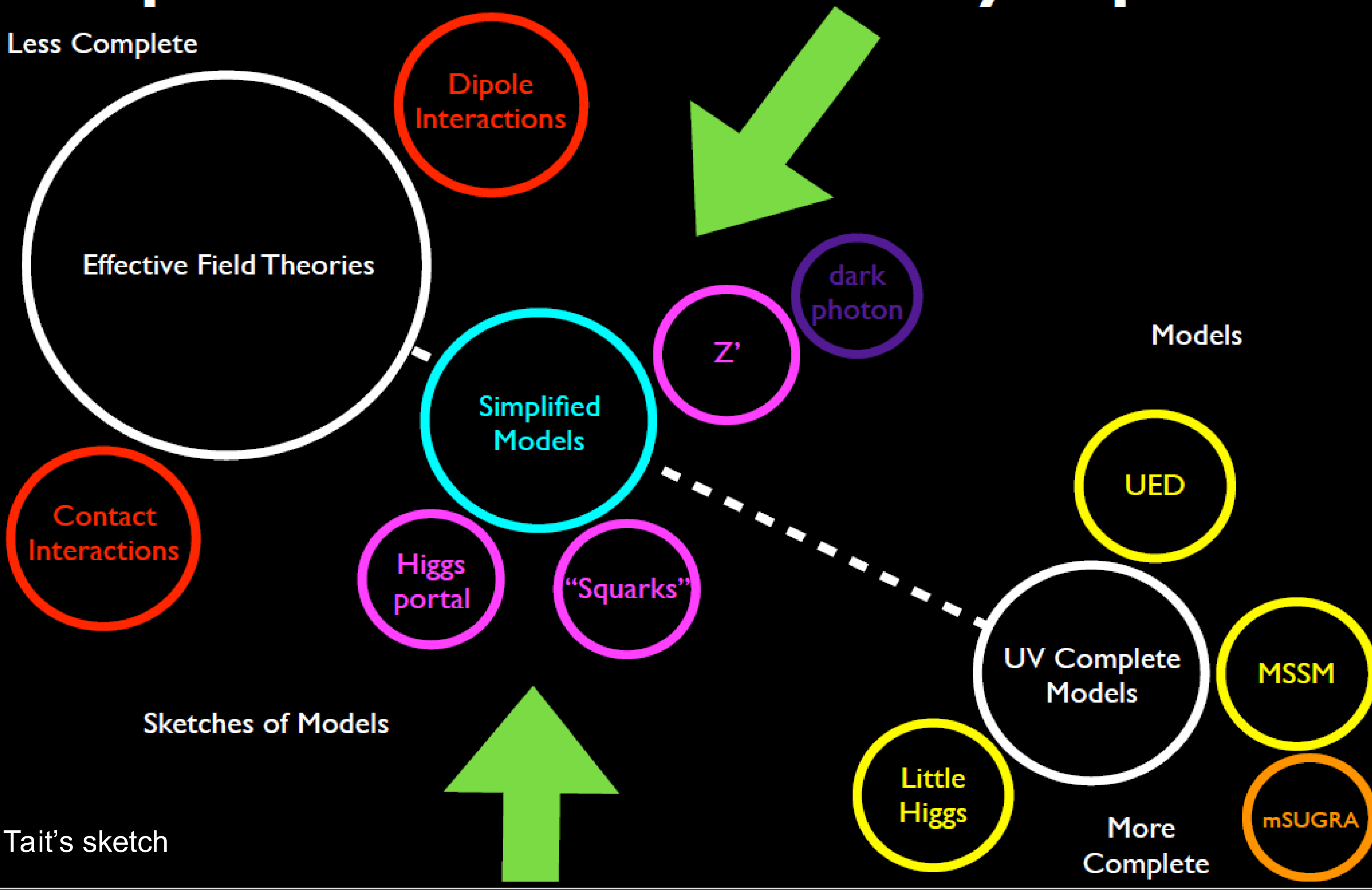
*Couplings*

- gravity [V]
- weak [?]
- higgs [?]
- quarks/gluons [?]
- leptons [?]
- New mediators [?]

*Thermal relic*

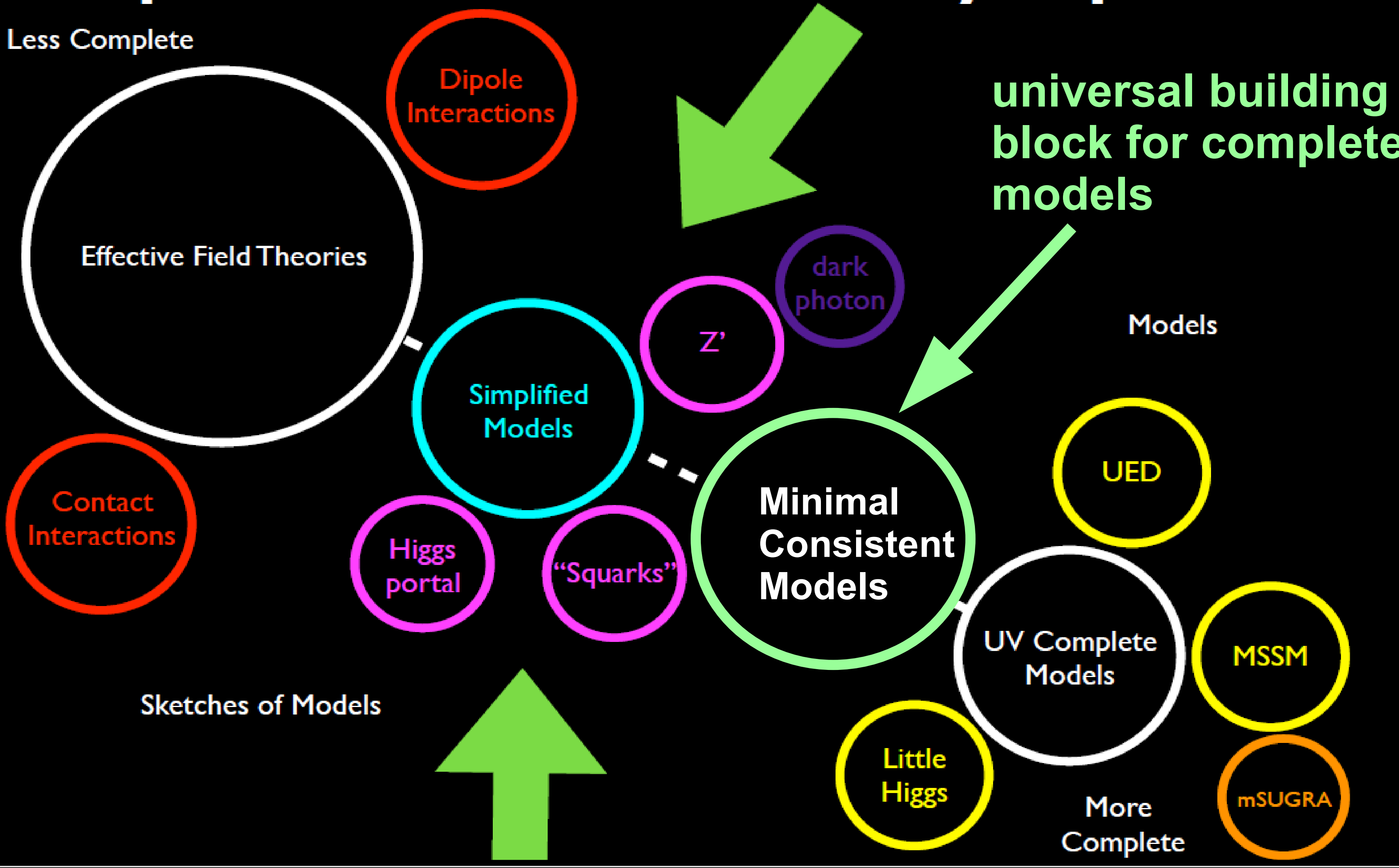
Yes [?] No [?]

# Spectrum of Theory Space




Tim Tait's sketch

# Spectrum of Theory Space

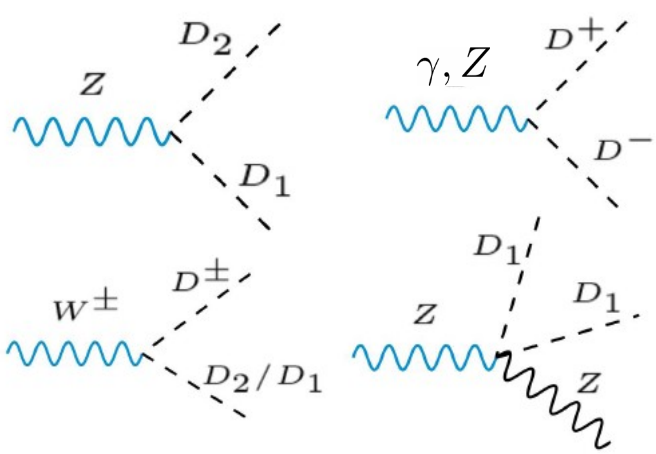


# Inert 2 Higgs Doublet model (i2HDM)

$\tilde{S}^{1/2}_{1/2}$

$$\mathcal{L}_\phi = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - V(\phi_1, \phi_2)$$

$$\phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}, \quad \phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}D^+ \\ D_1 + iD_2 \end{pmatrix}$$




$$[M_{D1}, \Delta M^+ = M_{D+} - M_{D1}, \Delta M^0 = M_{D2} - M_{D+}]$$

# Minimal fermion DM model (MFDM)

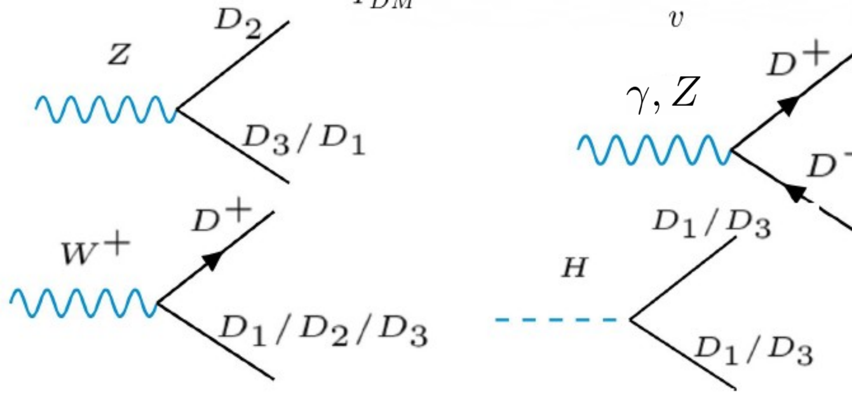
$\widetilde{F}^{1/2}_{1/2}\widetilde{M}^0_0$

$$\mathcal{L}_{FDM} = \mathcal{L}_{SM} + \bar{\psi}(i\not{D} - m_\psi)\psi + \frac{1}{2}\bar{\chi}^0_s(i\not{\partial} - m_s)\chi^0_s - (Y_{DM}(\bar{\psi}\Phi\chi^0_s) + h.c.)$$

$$\psi = \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}}(\chi^0_1 + i\chi^0_2) \end{pmatrix}$$

Majorana singlet $\chi^0_s$

$$Y_{DM} = \frac{\sqrt{(m_{D3} - m_{D+})(m_{D+} - m_{D1})}}{v}$$




$$[M_{D1}, \Delta M^+ = M_{D+} - M_{D1}, \Delta M^0 = M_{D3} - M_{D+}]$$

# Benchmarks and tools

- CalcHEP+PYTHIA8+Delphes3
- ISR+Beamstrahlung (CalcHEP)

  ILC 500 design (from ILC TDR)

```
<
 ISR scale          = 1.00E+00*sqrtS
 Beamstralung              ON
 Bunch x+y sizes (nm)= 500.0
 Bunch lenght (mm)    = 0.300
 Number of particles = 2.0e+10
          *   N_gamma = 1.71
          *   Upsilon = 0.06
 Beamstrahlung F(x) plot
 Beamstrahlung F(x)*(1-x)^(2/3)
```

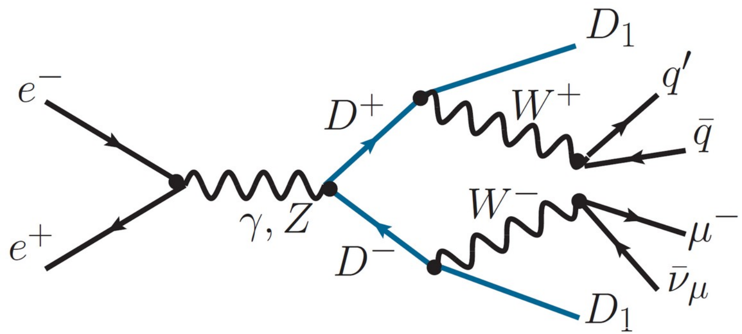
| Parameters \ Benchmarks | | BP1 | BP2 |
|---|---|---|---|
| $M_D$ | | 60 | 60 |
| $M_+$ | | 160 | 120 |
| $M_{D_2}$ | | 160.85 | 120.85 |
| I2HDM parameters | | | |
| $\lambda_{345}$ | | $6.5 \times 10^{-4}$ | $7.0 \times 10^{-4}$ |
| $\lambda_2$ | | 1.0 | 1.0 |
| DM observables | | | |
| $\Omega h^2$ | $SDM$ | 0.111 | 0.112 |
| | $FDM$ | 0.108 | 0.109 |
| $\sigma^p_{SI}$[pb] | $SDM$ | $6.17 \times 10^{-13}$ | $6.17 \times 10^{-13}$ |
| | $FDM$ | $1.67 \times 10^{-11}$ | $1.65 \times 10^{-11}$ |

- MicrOMEGAs
  - relic density
  - DM DD and ID detection
  - Invisible Higgs decay

    (under control – the small value of $M_{D2} - M_+$ split)
- CheckMATE
  - test against LHC current limits

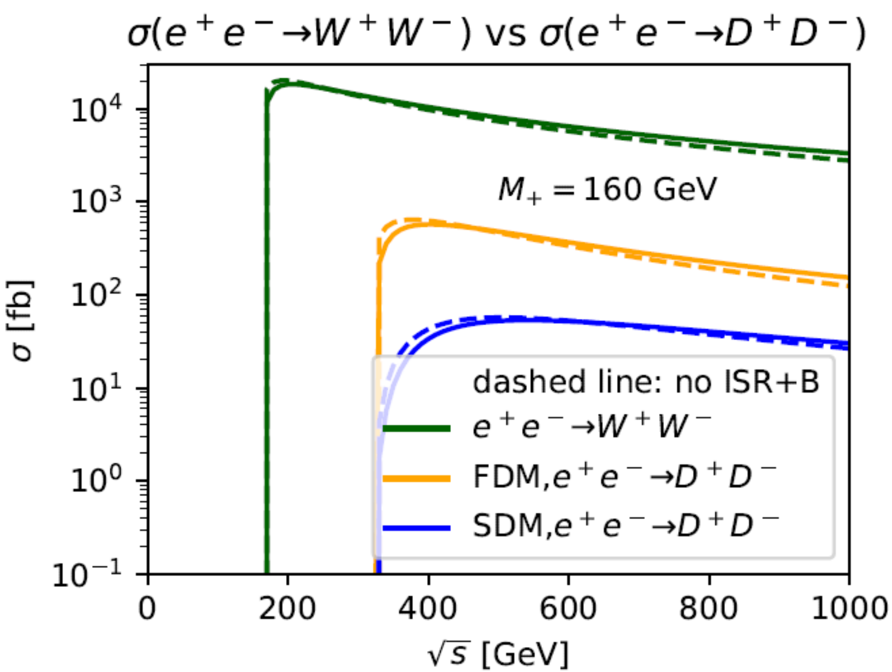
# The process under study

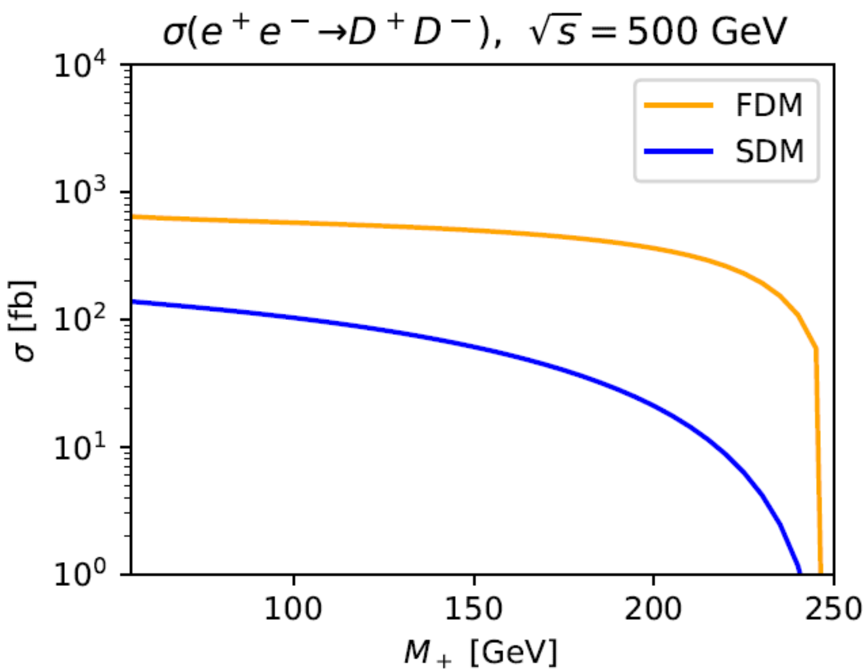
$$e^+e^- \to D^+D^- \to D_1D_1W^+W^- \to D_1D_1q'\bar{q}\mu\bar{\nu}$$




- Di-jet + muon + MET signature

$$\sigma_{\gamma\gamma} = \begin{cases} \sigma_0\beta_+\left[1 + \frac{2M_+^2}{s}\right] & \text{if } s_D = \frac{1}{2} \\ \sigma_0\frac{\beta_+^3}{4} & \text{if } s_D = 0 \end{cases}$$

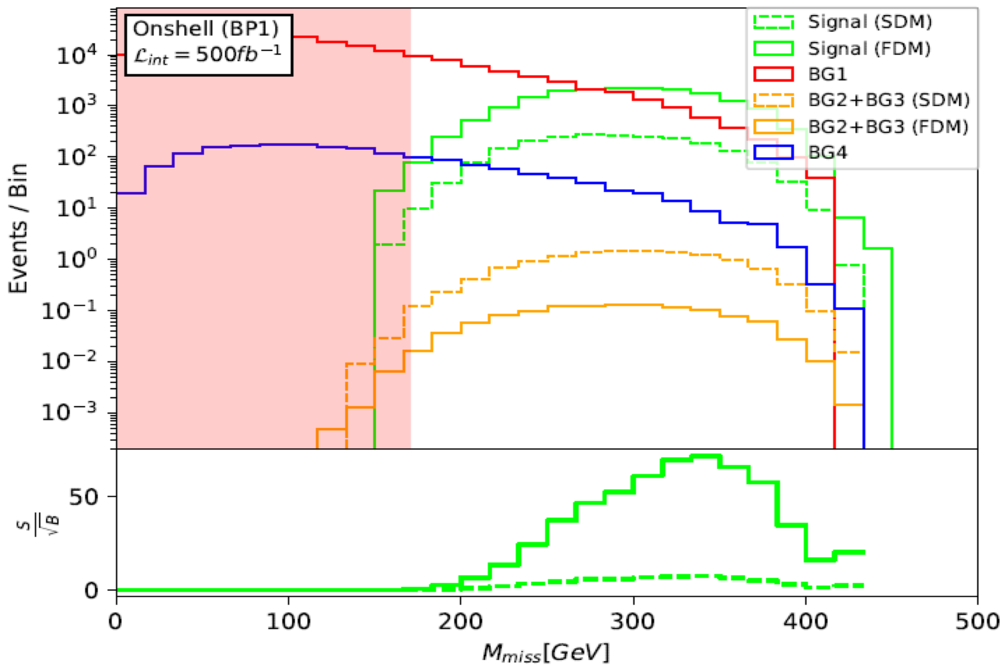


# Observables

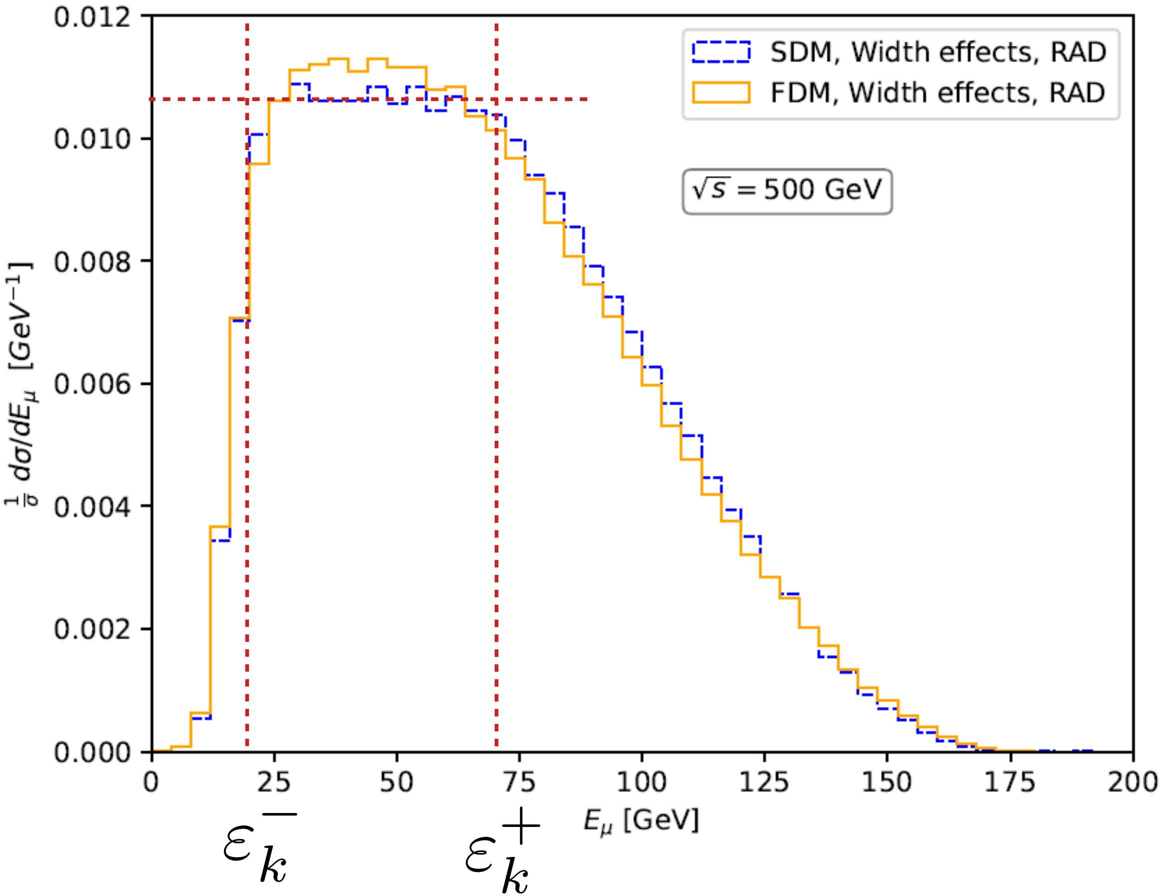
$$e^+e^- \to D^+D^- \to D_1D_1W^+W^- \to D_1D_1q'\bar{q}\mu\bar{\nu}$$

- Di-jet + muon + MET signature
  - $\sqrt{s}$ Is fixed (up to ISR+BRM effects)
  - $M_{miss}$ can be reconstructed: $M_{miss} = \sqrt{\left((\sqrt{s},0,0,0) - \sum_{vis} P_{vis}\right)^2}$
  - Missing transverse momentum, $\not{E}_T$
  - charged lepton energy (muon), $E_\mu$
  - angle of reconstructed W-boson in the LAB system, $\cos\theta_W$
  - the energy of W-boson reconstructed from the di-jet pair, $E_{jj}$
  - The cross section itself, which includes spin factors



# W-boson and charged lepton energy distributions

$$e^+e^- \to D^+D^- \to D_1 D_1 W^+ W^- \to D_1 D_1 q' \bar{q} \mu \bar{\nu}$$




- Edges in W energy distribution (from $D^+$ decay) have edges

$$E_{on}^{\pm} = \frac{\sqrt{s}}{2M_+}\left(E^{rest} \pm \beta_+ |\vec{p}|_W^{rest}\right)$$

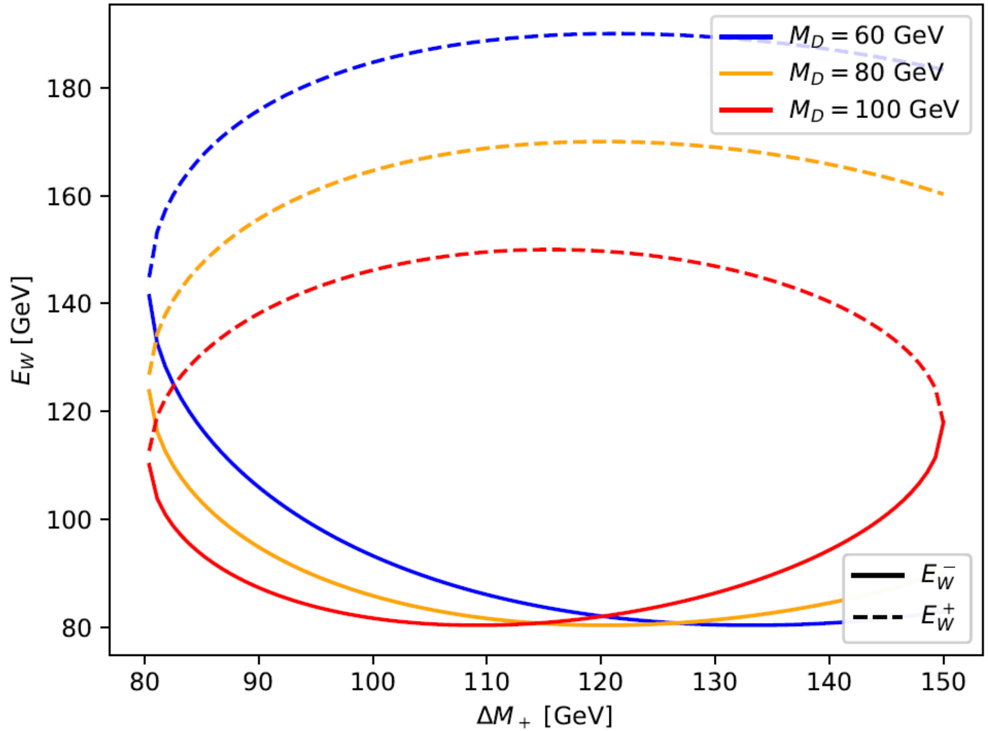
  which lead to kinks in muon energy distributions

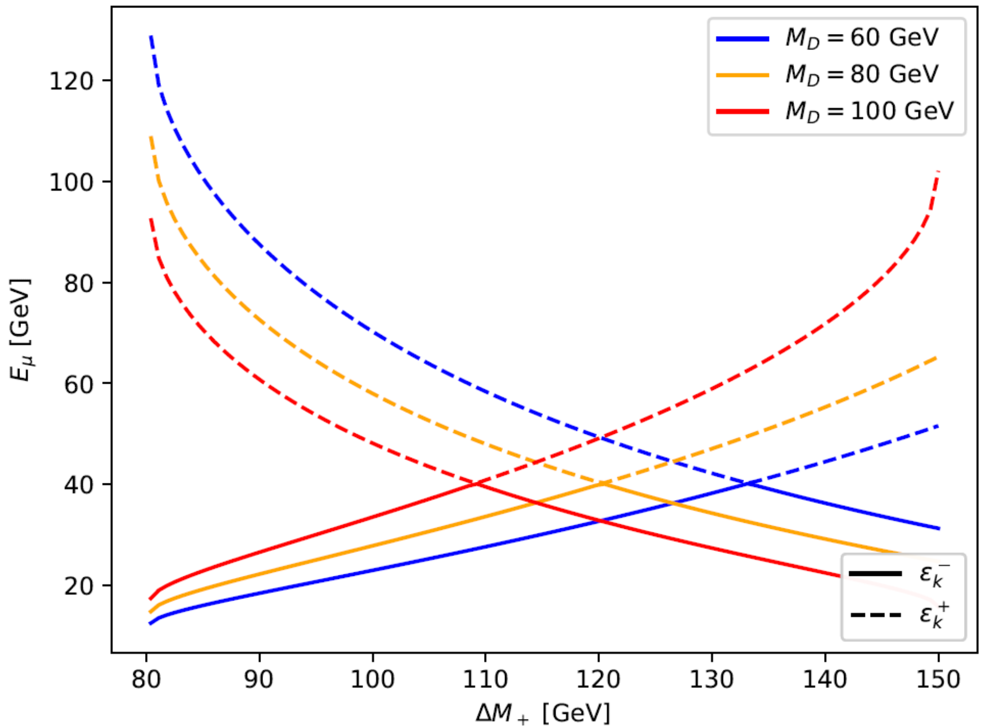
$$\varepsilon_k^{\pm} \equiv E_\mu^{(\pm)} = \frac{E_W^{(-)} \pm \sqrt{(E_W^{(-)})^2 - M_W^2}}{2}$$

- between these kinks distribution is approximately flat

- the positions of the upper edge of the di-jet (W) energy distribution and the lower kink in the muon energy distribution give two equations to determine $M_D$ and M+

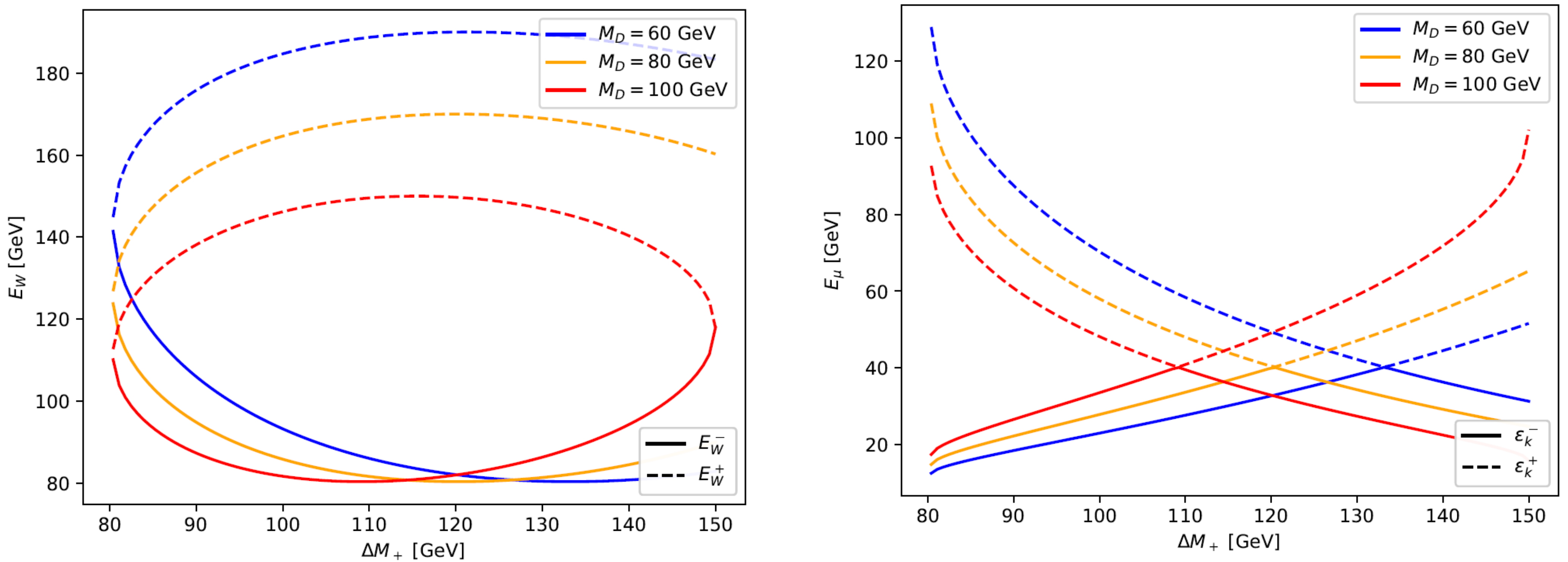
# Kinks and $M_D$ and $M_+$ determination




- Either of two edges in E(W) or in E(muon) distributions can be used to determine $M_D$ and M+
- However, for certain kinematics edges either in E(W) or in E(mu) can overlap

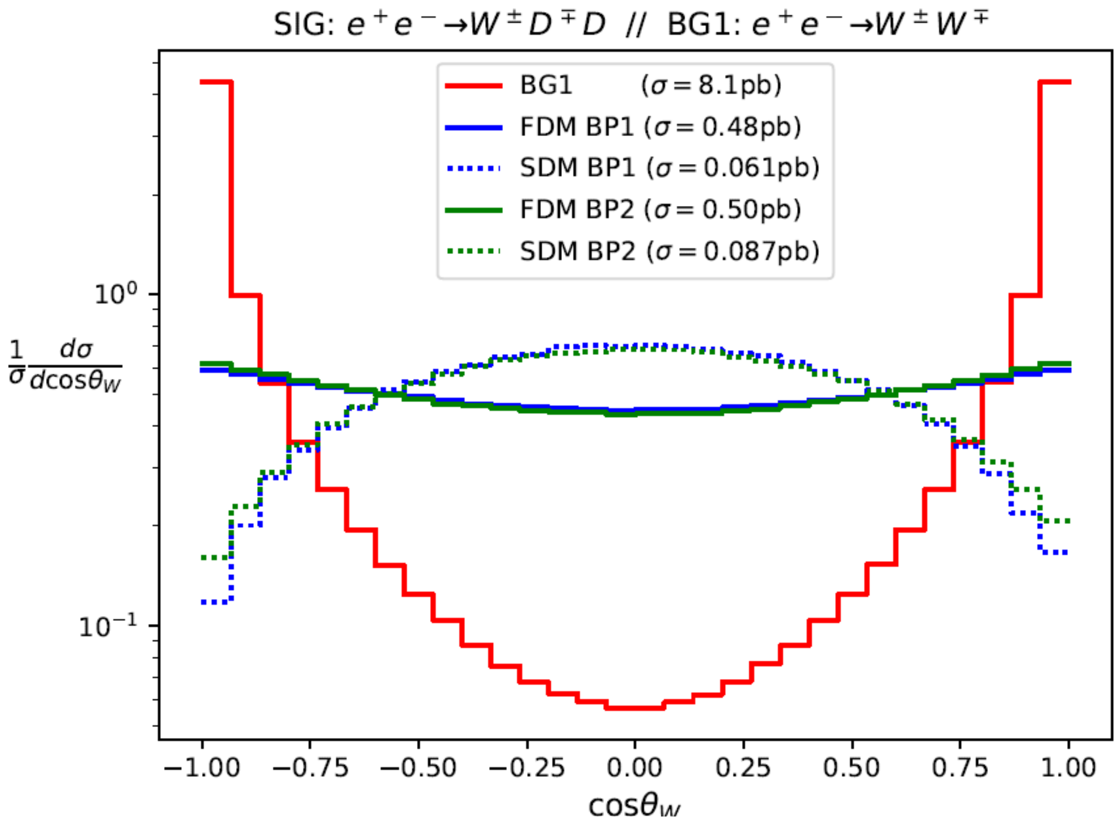
# Kinks and $M_D$ and $M_+$ determination



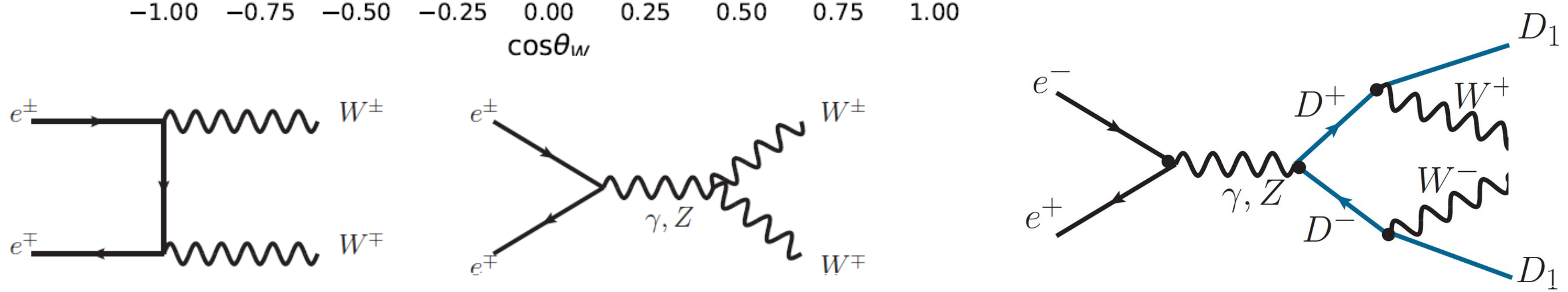

- Either of two edges in E(W) or  in E(muon) distributions can be used to determine $M_D$ and M+
- However, for certain kinematics  edges either in E(W) or in E(mu) can overlap

- Important point:  the edges in E(W) and E(muon) never overlap simultaneously: if distance between edges in E(W) distribution is small, the distance between edges in E(mu) is maximal and vice versa – so the $M_D$ and M+ can always be determined

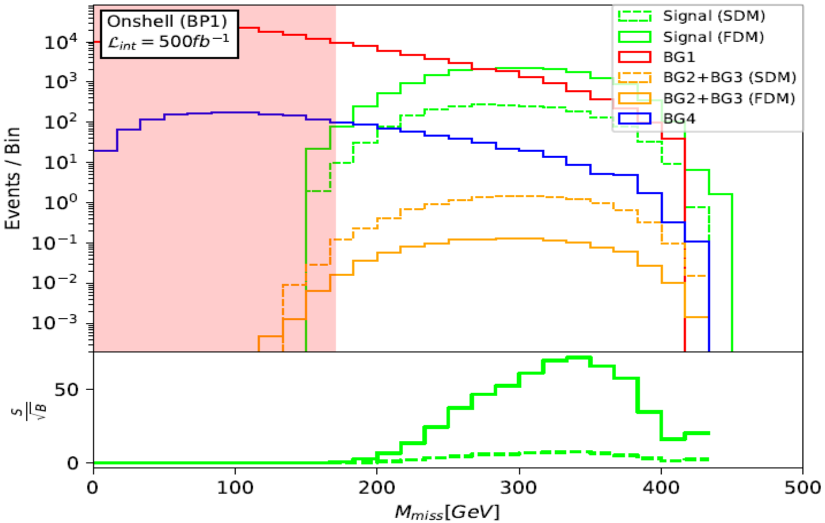
# DM spin determination




- The angular W-boson distribution (either for real or virtual W) is found to be very important descriminator between DM spin as well as the main BG
- The shape of angular W-boson distribution is the same for two benchmarks for DM of the same spins
- The $\cos\theta_W$ distribution for SDM is the most distinct one



# Signal vs BG analysis

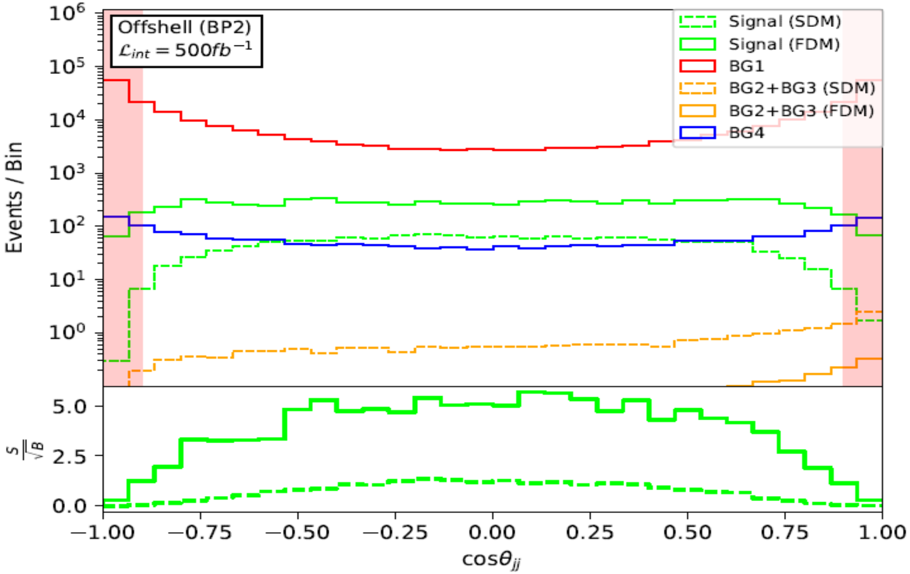


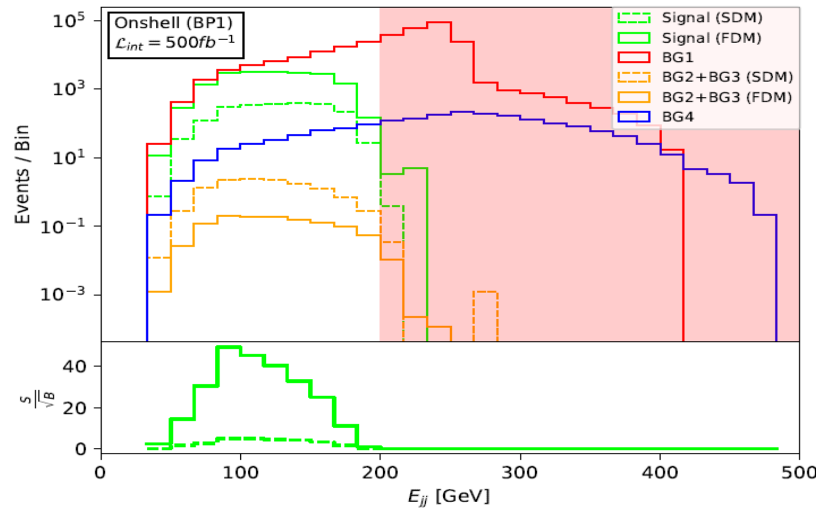

**BP1 cut flow**

| | SDM | | | | FDM | | | |
|---|---|---|---|---|---|---|---|---|
| | S | B | $S/B$ | $\alpha$ | S | B | $S/B$ | $\alpha$ |
| No cuts | 2098 | 332588 | 0.006 | 3.63 | 17962 | 334254 | 0.054 | 30.27 |
| $M_{miss} > 170$ | 2095 | 83143 | 0.025 | 7.18 | 17922 | 84810 | 0.211 | 55.92 |
| $E_{jj} < 200$ | 2094 | 67130 | 0.031 | 7.96 | 17917 | 68796 | 0.260 | 60.84 |
| $\lvert\cos\theta_{jj}\rvert < 0.9$ | 2046 | 29526 | 0.069 | 11.52 | 15993 | 31038 | 0.515 | 73.75 |
| $\lvert\cos\theta_{\mu}\rvert < 0.9$ | 1947 | 24306 | 0.081 | 12.02 | 14893 | 25766 | 0.578 | 73.86 |

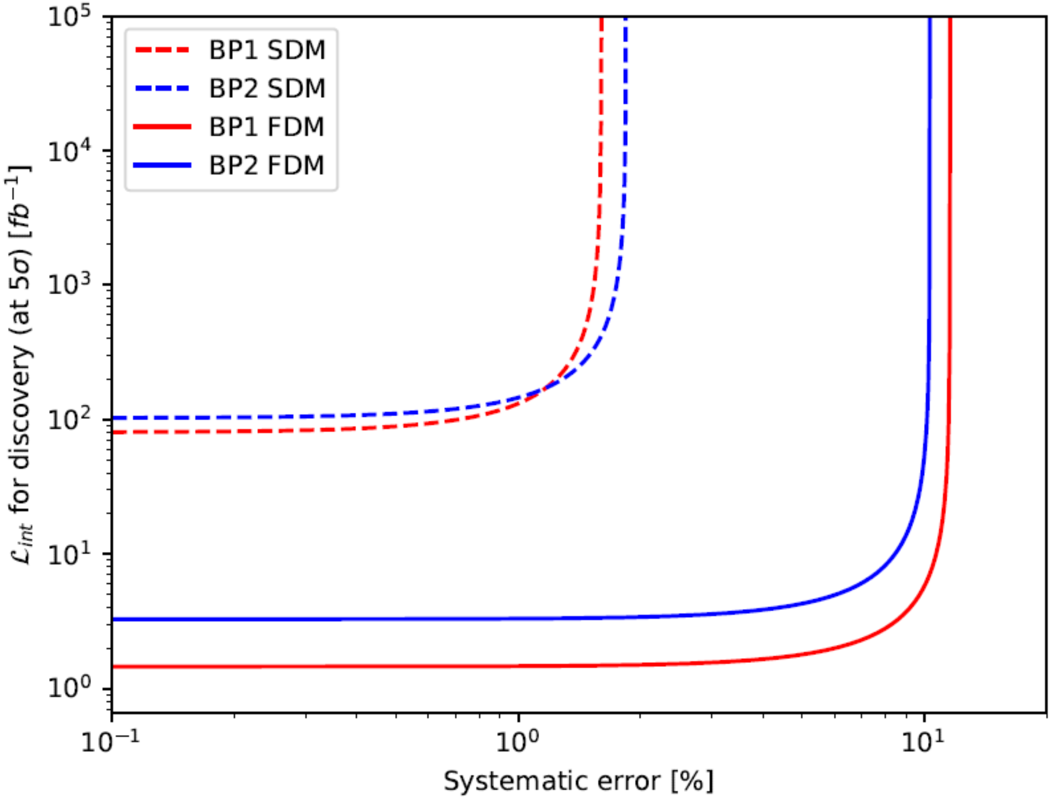
**BP2 cut flow**

| | SDM | | | | FDM | | | |
|---|---|---|---|---|---|---|---|---|
| | S | B | $S/B$ | $\alpha$ | S | B | $S/B$ | $\alpha$ |
| No cuts | 1370 | 284290 | 0.005 | 2.56 | 8138 | 284273 | 0.029 | 15.05 |
| $M_{miss} > 170$ | 1370 | 39323 | 0.0349 | 6.79 | 8136 | 39307 | 0.207 | 37.35 |
| $E_{jj} < 200$ | 1369 | 36177 | 0.0379 | 7.06 | 8123 | 36161 | 0.225 | 38.60 |
| $\lvert\cos\theta_{jj}\rvert < 0.9$ | 1360 | 18647 | 0.0730 | 9.62 | 7815 | 18634 | 0.419 | 48.06 |
| $\lvert\cos\theta_{\mu}\rvert < 0.9$ | 1326 | 14398 | 0.0922 | 10.58 | 7420 | 14386 | 0.516 | 50.25 |



# Signal vs BG analysis

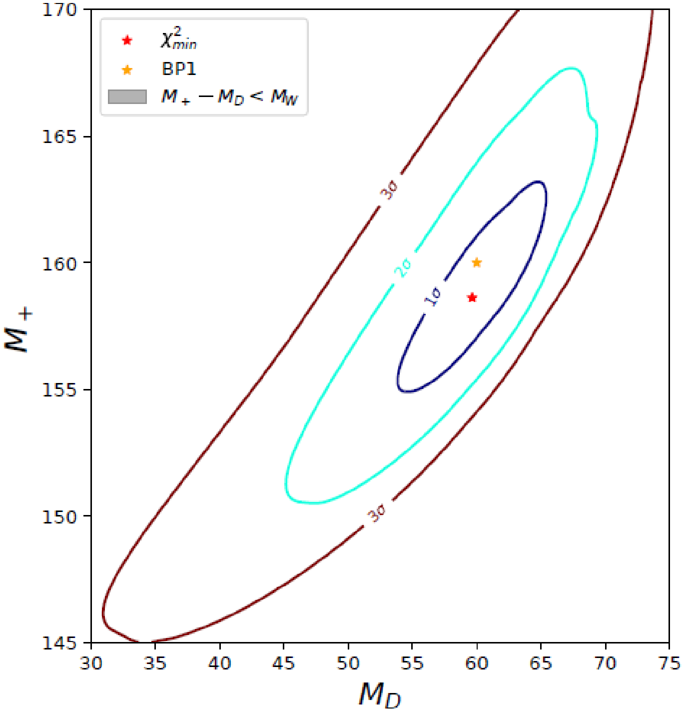


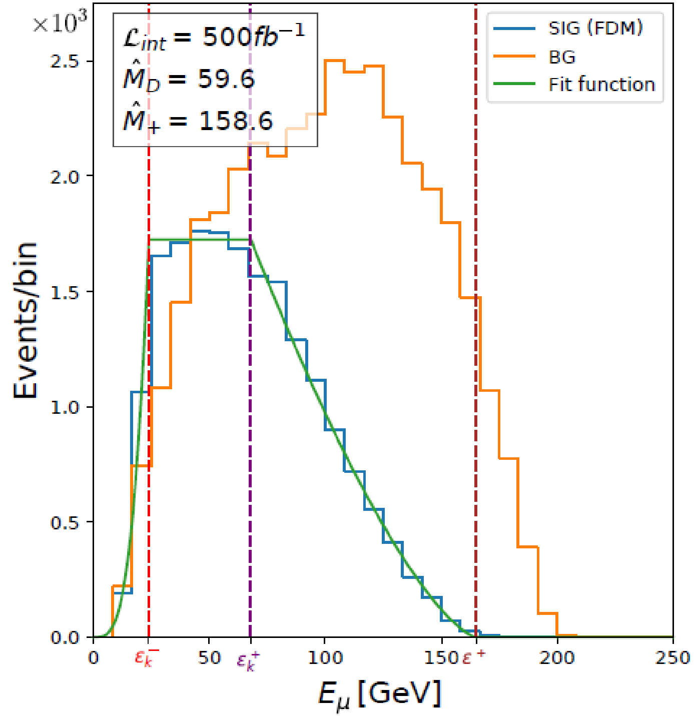

$$\alpha_a = \frac{S}{\sqrt{B + \delta^2_{Sys} B^2}}$$

$$\alpha_b = \frac{S}{\sqrt{S + B + \delta^2_{Sys}(S+B)^2}}$$

| | | Luminosity required for discovery (at $5\sigma$)/$fb^{-1}$ | |
|---|---|---|---|
| | | $\alpha_a$ | $\alpha_b$ |
| SDM | BP1 | 131.31 | 158.70 |
| | BP2 | 145.14 | 172.39 |
| FDM | BP1 | 1.46 | 2.33 |
| | BP2 | 3.30 | 5.06 |

# Mass determination




| | | $500 fb^{-1}$ | $20 ab^{-1}$ |
|---|---|---|---|
| **FDM** | $M_D$ | $58.4^{+5.7}_{-6.0}$ | $57.6^{+1.9}_{-2.2}$ |
| | $M_+$ | $158.1^{+4.0}_{-3.7}$ | $157.4^{+2.7}_{-2.4}$ |
| **SDM** | $M_D$ | $66.0^{+19.2}_{-64.3}$ | $64.3^{+3.2}_{-6.1}$ |
| | $M_+$ | $161.3^{+14.7}_{-52.8}$ | $161.0^{+3.3}_{-3.9}$ |

| | | $500 fb^{-1}$ | $20 ab^{-1}$ |
|---|---|---|---|
| FDM | $M_D$ | $60.0^{+0.7}_{-0.8}$ | $60.0^{+0.1}_{-0.1}$ |
| | $M_+$ | $120.0^{+1.5}_{-1.7}$ | $120.0^{+0.2}_{-0.3}$ |
| SDM | $M_D$ | $60.0^{+24.1}_{-19.7}$ | $60.0^{+4.4}_{-1.3}$ |
| | $M_+$ | $120.0^{+22.3}_{-45.9}$ | $120.0^{+2.3}_{-2.7}$ |

$$f(\epsilon) = \begin{cases} b\left(\frac{\epsilon}{\epsilon_k^-}\right)^a & \text{if } \epsilon \leq \epsilon_k^- \\ b & \text{if } \epsilon_k^- < \epsilon < \epsilon_k^+ \\ b\left(1 - \frac{\epsilon - \epsilon_k^+}{\epsilon^+ - \epsilon_k^+}\right)^c & \text{if } \epsilon_k^+ \leq \epsilon < \epsilon^+ \\ 0 & \text{if } \epsilon^+ \leq \epsilon \end{cases}$$

The profile $\chi^2$ is calculated by minimising over nuisance parameters $a, b, c$.
The minimum of this profiled $\chi^2$ corresponds to the global minimum for the fit, when $M_D, M_+$ are also allowed to vary.

# Spin discrimination

| | $\mathcal{L}_{int}$ to differentiate at 95% CL / $fb^{-1}$ | | | |
|---|---|---|---|---|
| | Shape only | | Shape and cross-section | |
| Assumed nature | SDM | FDM | SDM | FDM |
| BP1 | 974.9 | 30.08 | 1.9 | 3.4 |
| BP2 | 2320.2 | 117.9 | 9.6 | 13.2 |

We assume that the mass of the DM is precisely known: a more complete treatment would involve a simultaneous fit of mass and spin.

Events are generated with the model assigned to "Assumed nature", before statistical comparison with the alternative model is conducted.

We perform the analysis for two cases:
1) using only the shape: signal strength becomes a nuisance parameter $\mu$
2 )using the signal strength predicted by the specific model realisations.

we present the luminosity required to exclude a given hypothesis at the expected 95% CL

# Conclusions and Outlook

- Future $e^+e^-$ colliders has unique power to determine the properties of DM
- Two minimal models with DM spin ½ and 0 as an example of the case study
- Kinks and edges in W and charged lepton distributions:

  are very complementary since the edges in E(W) and E(mu) never overlap simultaneously, so the $M_D$ and M+ can always be determined

- Powerful kinematical variables are explored: E(mu), angular distribution of W, missing mass which allows to
  - discover 100 GeV FDM (SDM) with the few (hundered) inverse fb luminosity
  - determine mass of DM with up to a percent accuracy
  - discriminate DM spin
- Next steps
  - explore bigger theory space (vector DM case) and more generic parameter space

# Thank you!

# Backup slides

It is convenient to use the cross section for SM process

$$\sigma_0 \equiv \sigma(e^+e^- \to \gamma \to \mu^+\mu^-) = 4\pi\alpha^2/3s$$

as a normalizer for the cross sections of the $e^+e^- \to D^+D^-$ processes under study. For $\gamma$-factors and velocities of $D^+$,

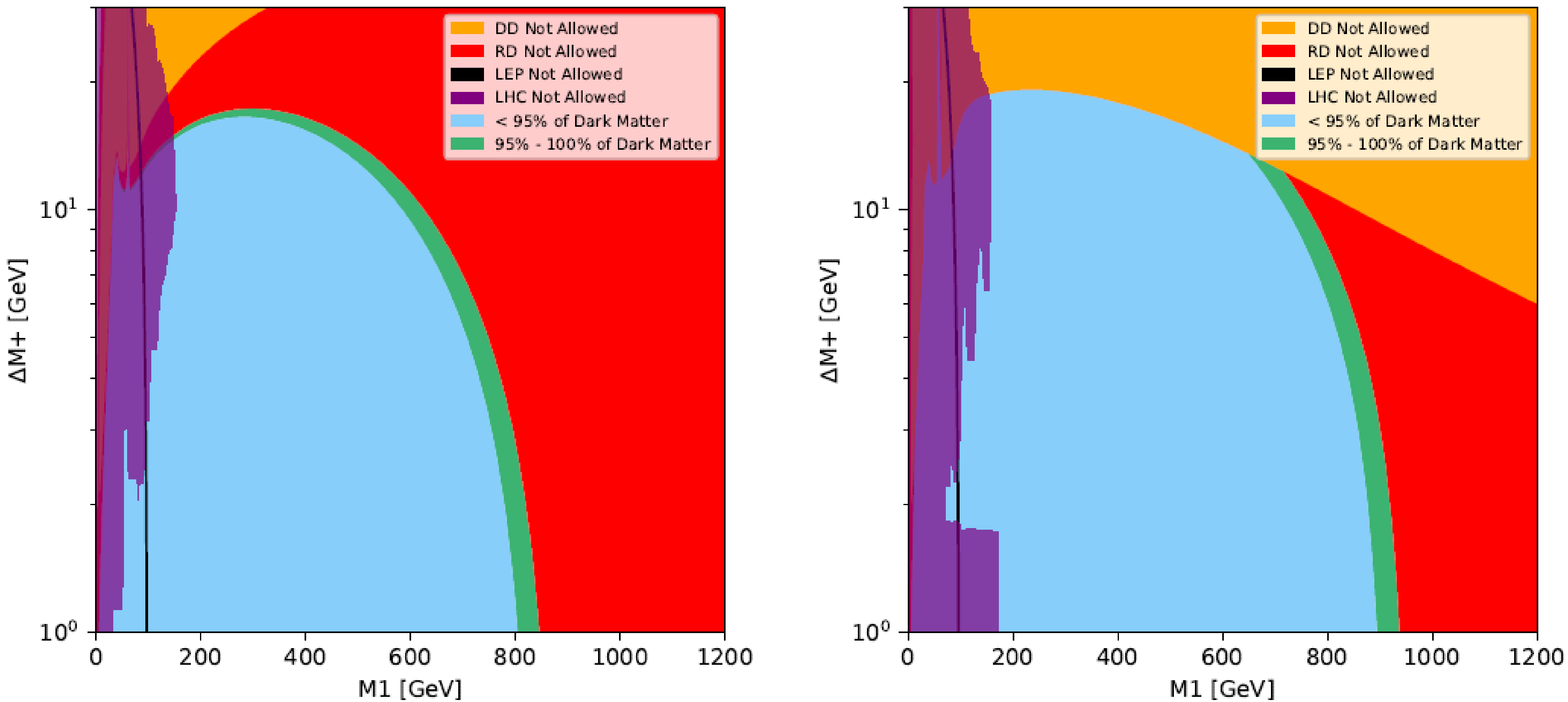
$$\gamma_+ = \frac{\sqrt{s}}{2M_+}, \qquad \beta_+ = \sqrt{1 - 4M_+^2/s}$$

the QED cross section of $e^+e^- \to D^+D^-$ process from the squared amplitude with the photon exchange only is given by

$$\sigma_{\gamma\gamma} = \begin{cases} \sigma_0\beta_+ \left[1 + \frac{2M_+^2}{s}\right] & \text{if } s_D = \frac{1}{2} \\ \sigma_0\frac{\beta_+^3}{4} & \text{if } s_D = 0 \end{cases},$$

while the total cross section is given by

$$\sigma = \sigma_{\gamma\gamma} + \sigma_{\gamma Z} + \sigma_{ZZ} = \sigma_{\gamma\gamma}\left[1 + \frac{\kappa_{\gamma Z}}{1 - \frac{M_Z^2}{s}} + \frac{\kappa_{ZZ}}{\left(1 - \frac{M_Z^2}{s}\right)^2}\right],$$



DD Not Allowed
RD Not Allowed
LEP Not Allowed
LHC Not Allowed
< 95% of Dark Matter
95% - 100% of Dark Matter
ΔM+ [GeV]
M1 [GeV]
$10^1$
$10^0$
0
200
400
600
800
1000
1200
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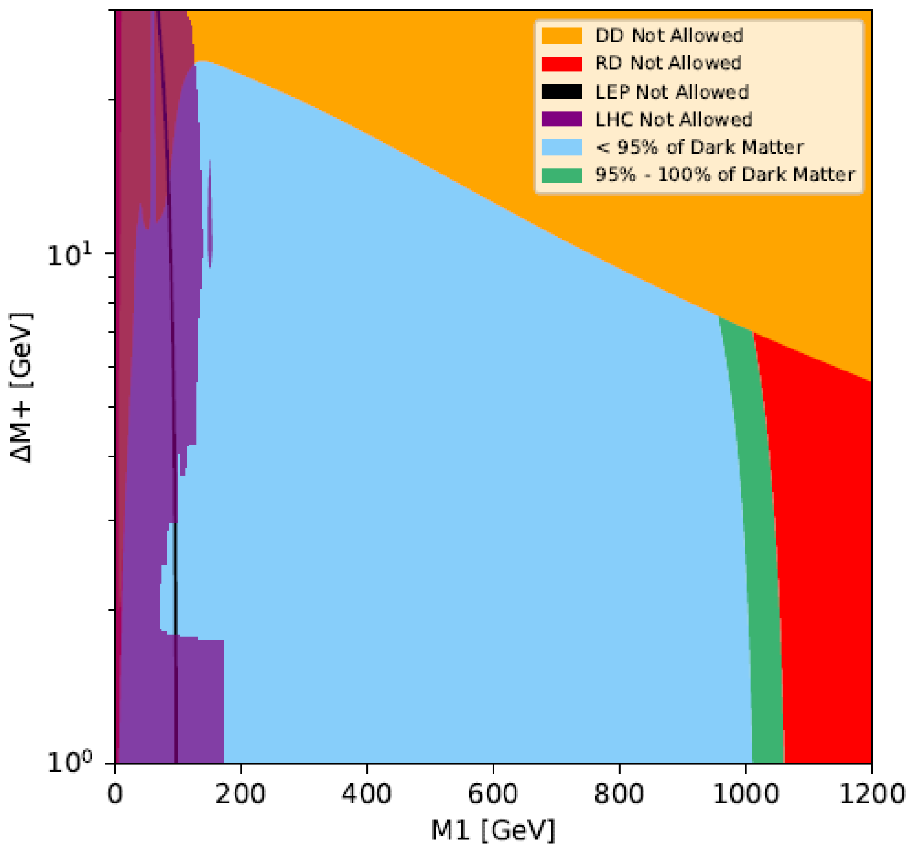
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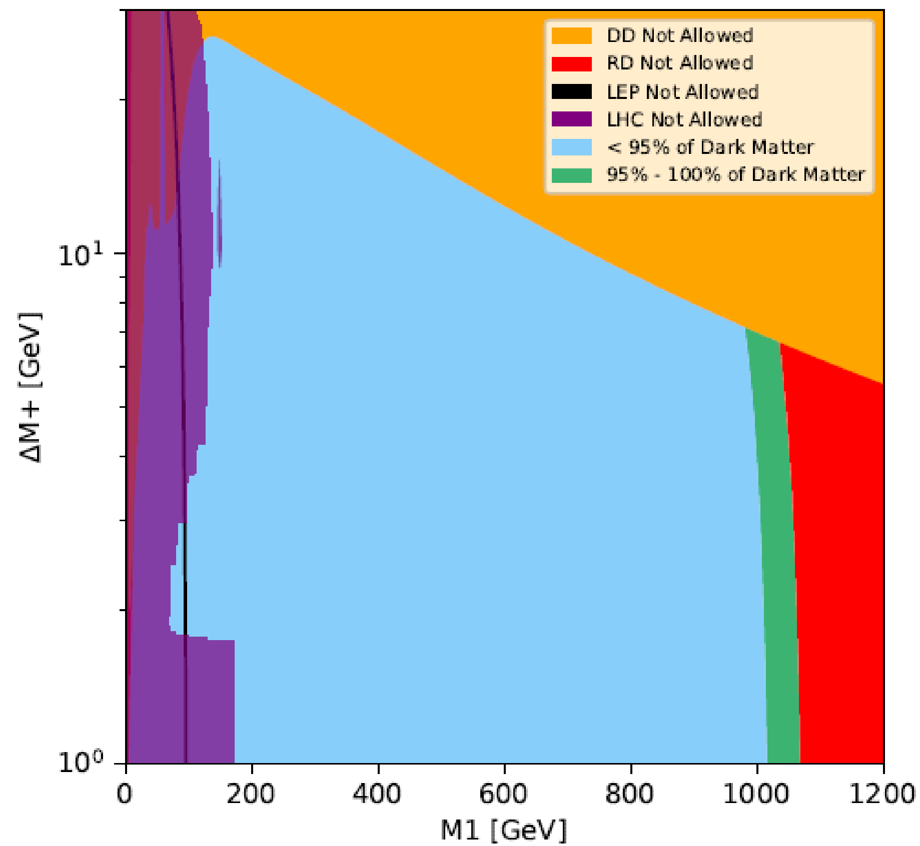
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