Time Delay Interferometry

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including in particular development from INREP group,
in particular N. Quang Dam, Jean-Baptiste Bayle & Olaf Hartwig

Cours LISA France
Remote - 8 janvier 2021

Overview

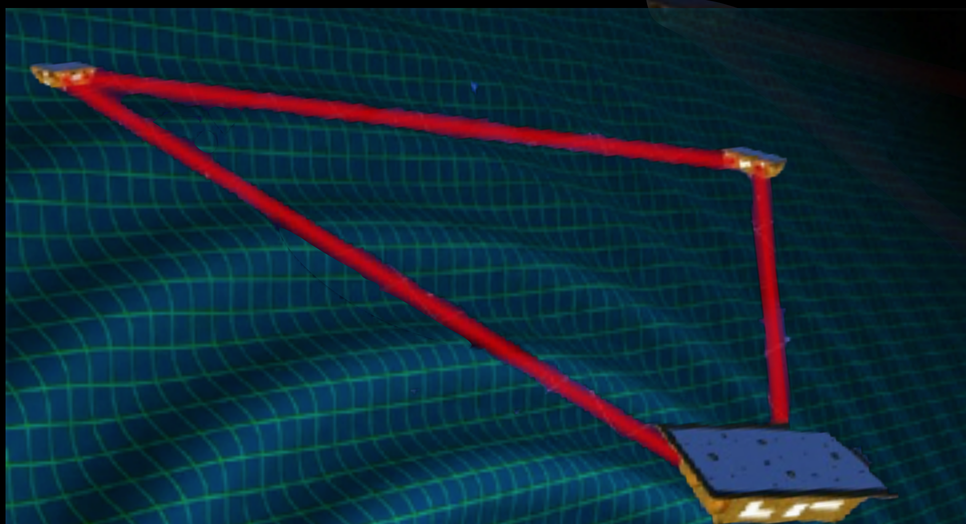
- Introduction on LISA data processing
- Basic principle of TDI
- TDI with the current design
- TDI generators
- Propagation of noises and signal through TDI
- Laser locking
- Key ingredients of TDI
- New convention

LISA data

'Survey' type observatory

Gravitational wave sources emitting between 0.02mHz and 1 Hz

LISA data



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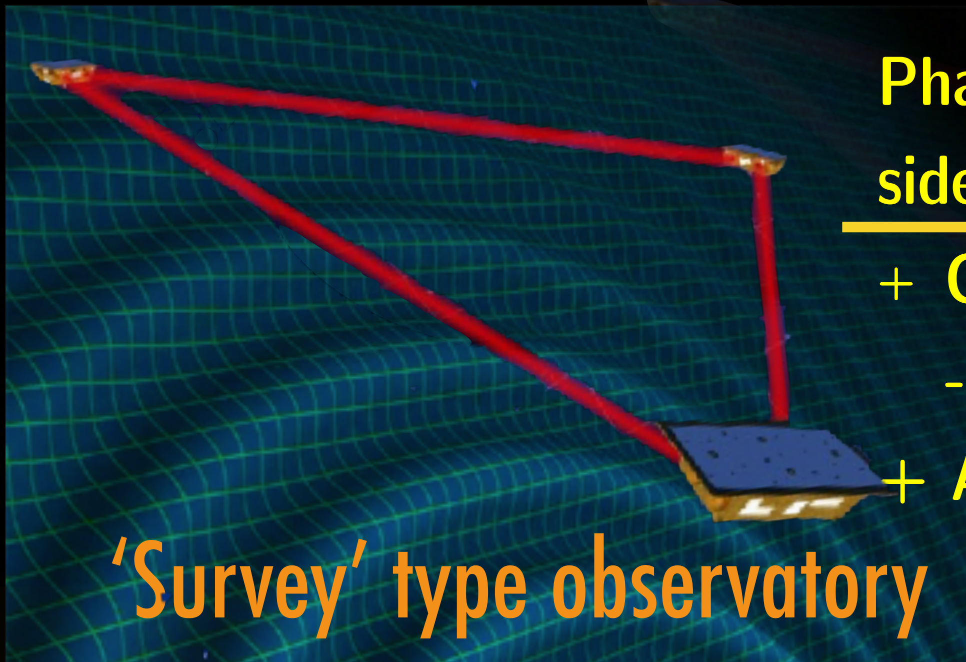

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Phasemeters (carrier, sidebands, armlengths)

+ Gravitational Reference Sensor
+ Auxiliary measurements

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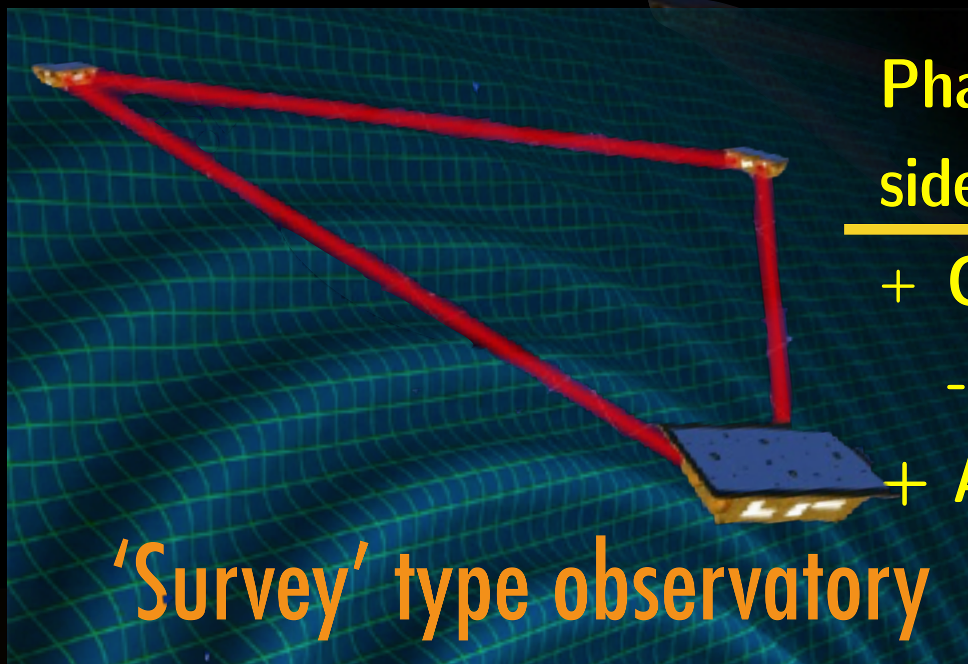

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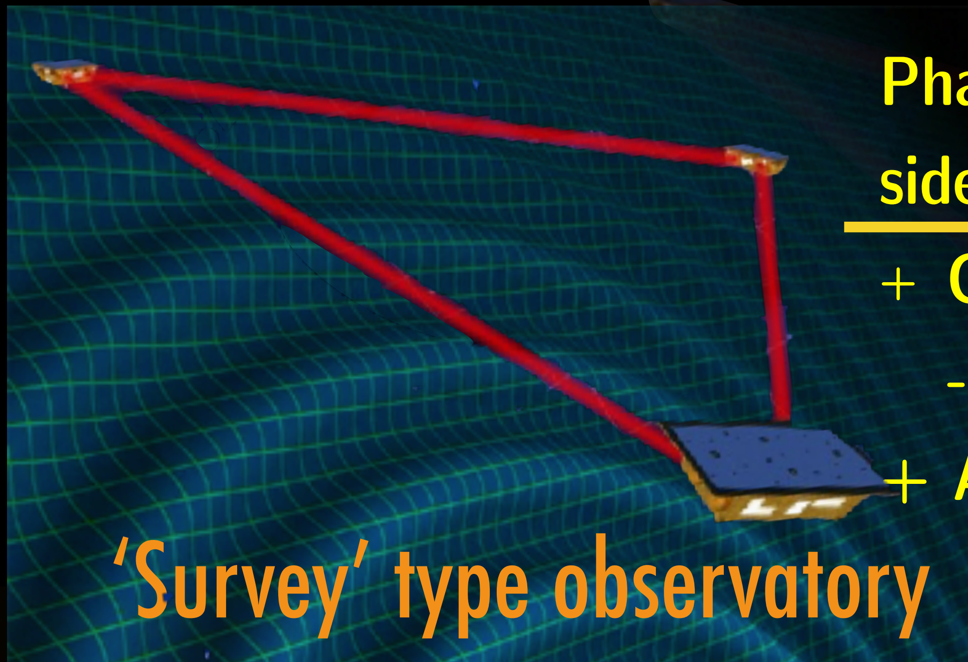

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Calibrations

Resynchronisation (clocks)

Time-Delay Interferometry
reduce laser noise

3 TDI time series

Gravitational wave sources emitting between 0.02mHz and 1 Hz

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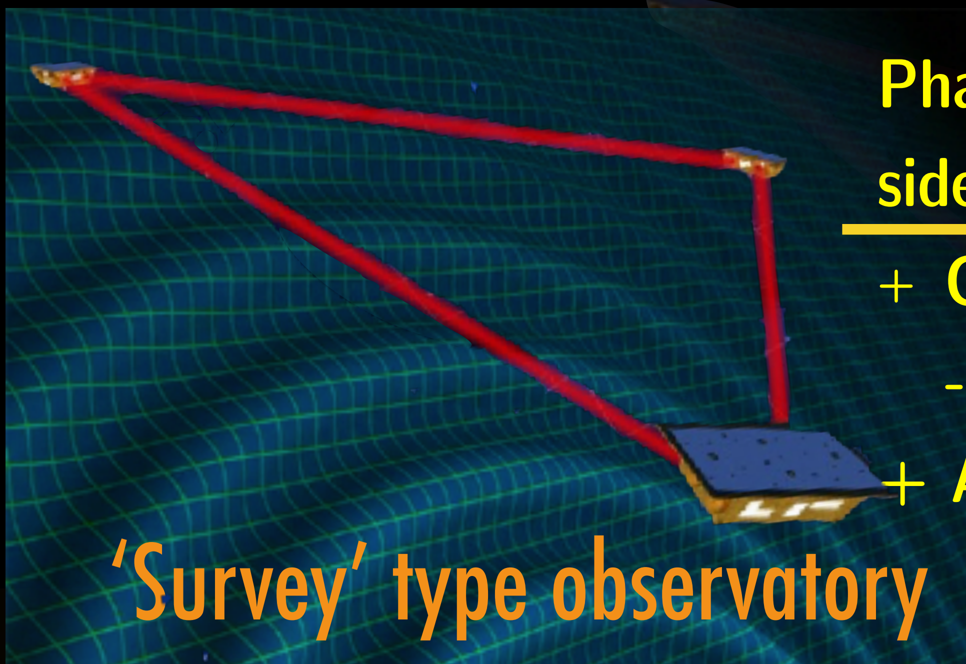

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GWs analysis

Catalogs of GW sources (posteriors & correlations), waveforms, etc

Gravitational wave sources emitting between 0.02mHz and 1 Hz

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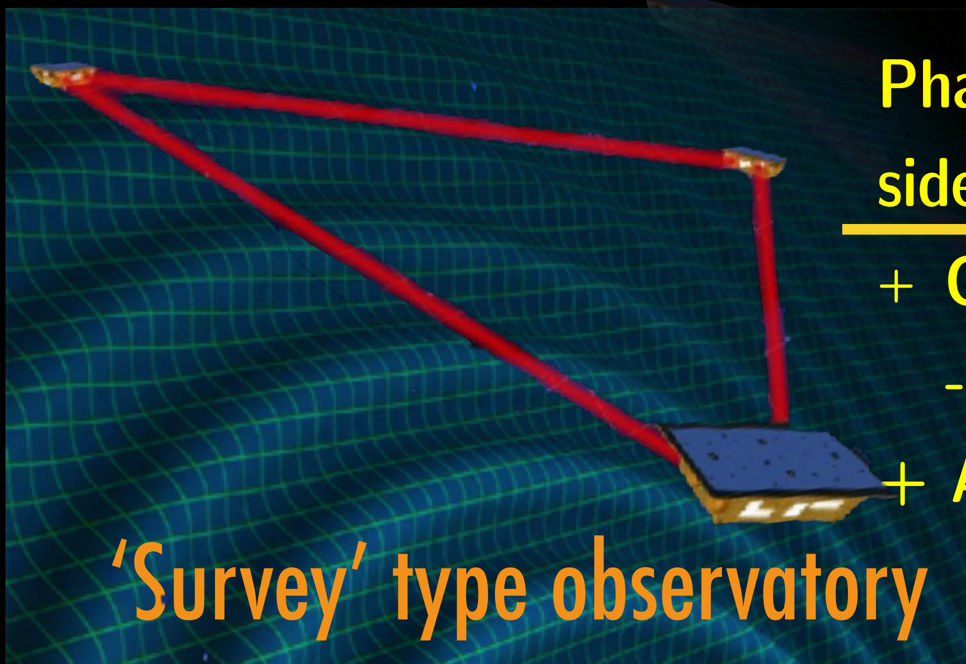

Phasemeters (carrier, sidebands, armlengths)
+ Gravitational Refe- -rence Sensor
+ Auxiliary measurements

L0

Calibrations

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Time-Delay Interferometry
reduce laser noise

L1

3 TDI time series

GWs analysis

L2

L3

Catalogs of GW sources (posteriors & correlations), waveforms, etc

Gravitational wave sources emitting between 0.02mHz and 1 Hz

LISA data

Phasemeters (carrier, sidebands, armlengths)

Mission Operation Centre

al Refe-
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+ Auxiliary measurements

'Survey' type observatory

Science Operation Centre

Gravitational wave sources emitting between 0.02mHz and 1 Hz

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reduce laser noise

L1

3 TDI time series

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Science Operation Centre

Gravitational wave sources emitting between 0.02mHz

Distributed Data Processing Centre

L0 → Calibrations → Resynchronisation (clocks) → Time-Delay Interferometry reduce laser noise

L1: 3 TDI time series

L2: GWs analysis

L3: Catalogs of GW sources (posteriors & correlations), waveforms, etc

LISA data

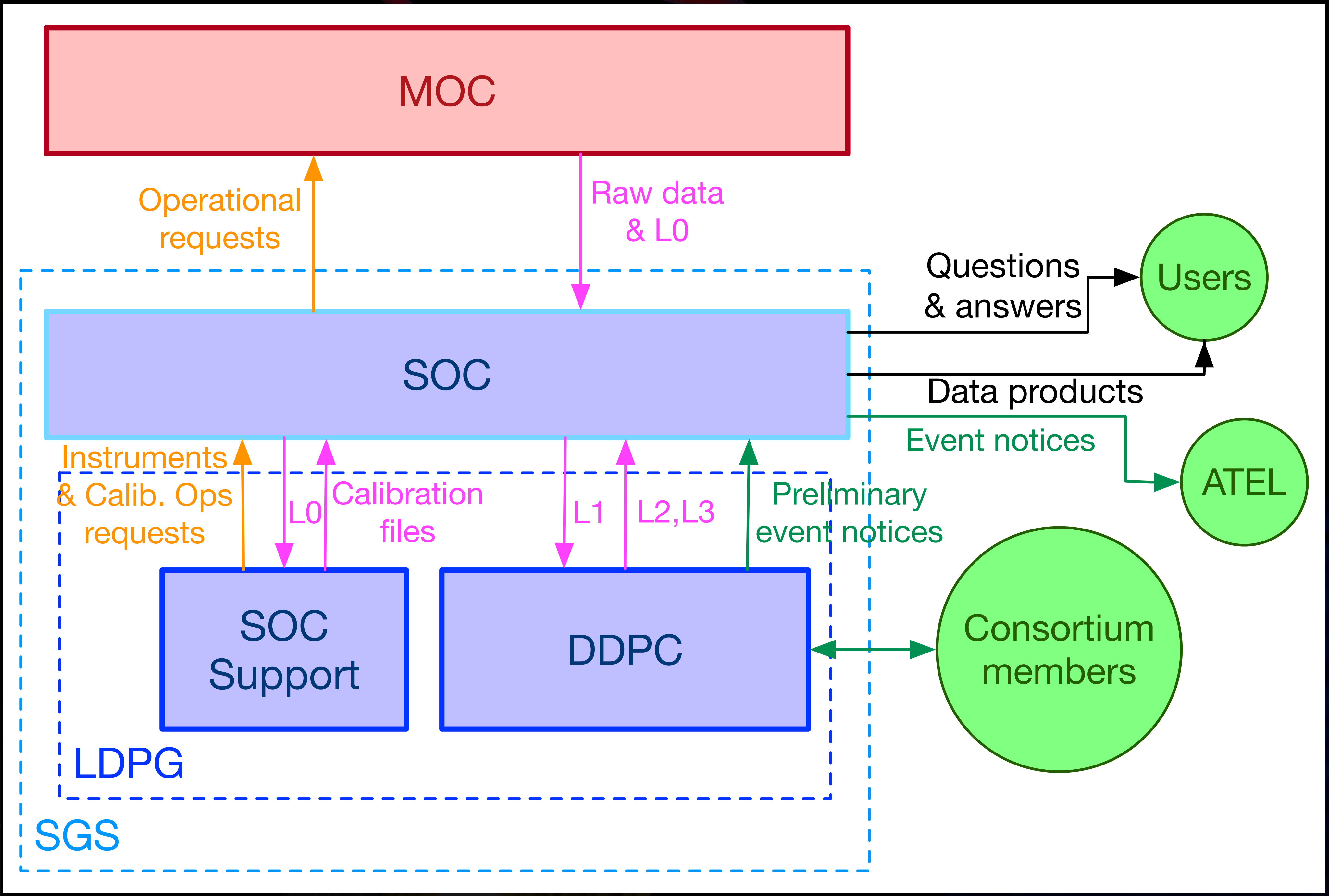
- Generation rates:

System	Downlink rate [bit/s]
Phasemeter	5120,00
LA	64,00
DFACS	1088,00
GRS FEE	-
CGT	-
CMS	544,00
SciDiag	307,09
Housekeeping	4000,00
Total	11123,09

- Generation volumes:

Volume	GB
Telemetered volume generated per day per S/C	0,20
Telemetered volume generated per day for the constellation	0,81
Onboard volume generated per day per S/C	2,79
Onboard Storage Required per S/C	39,12

LISA Ground Segment



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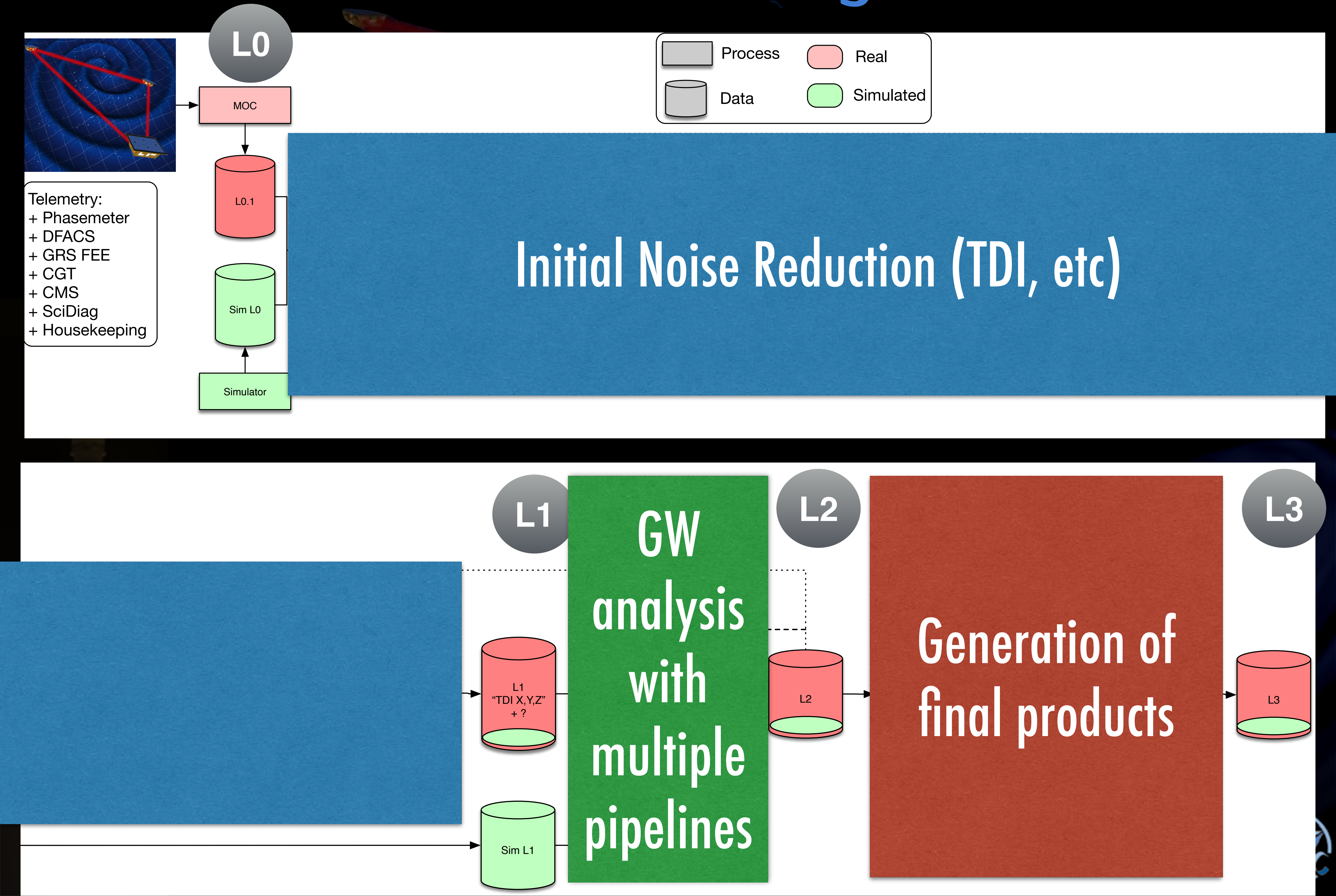

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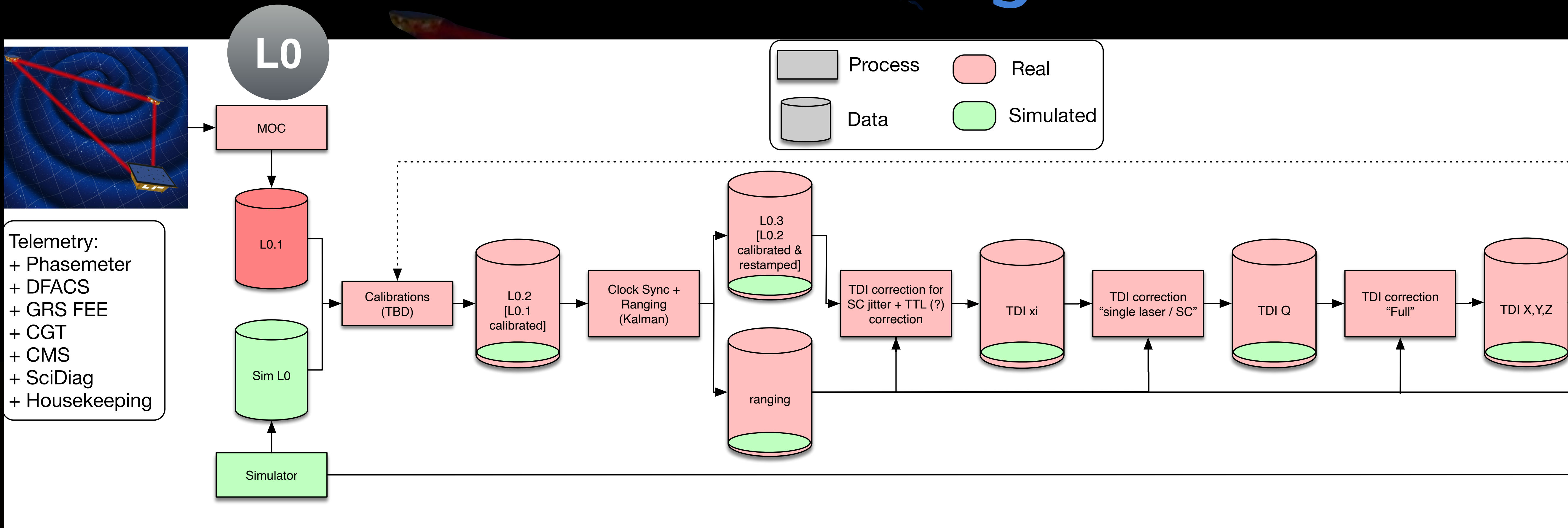
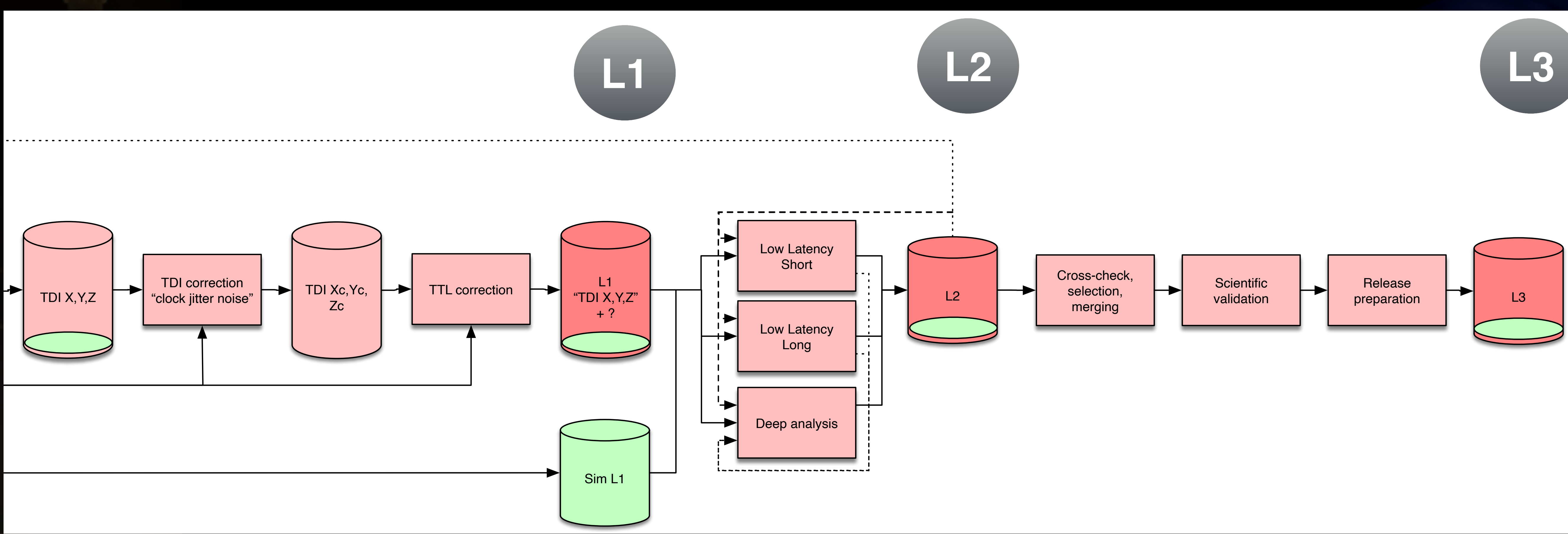
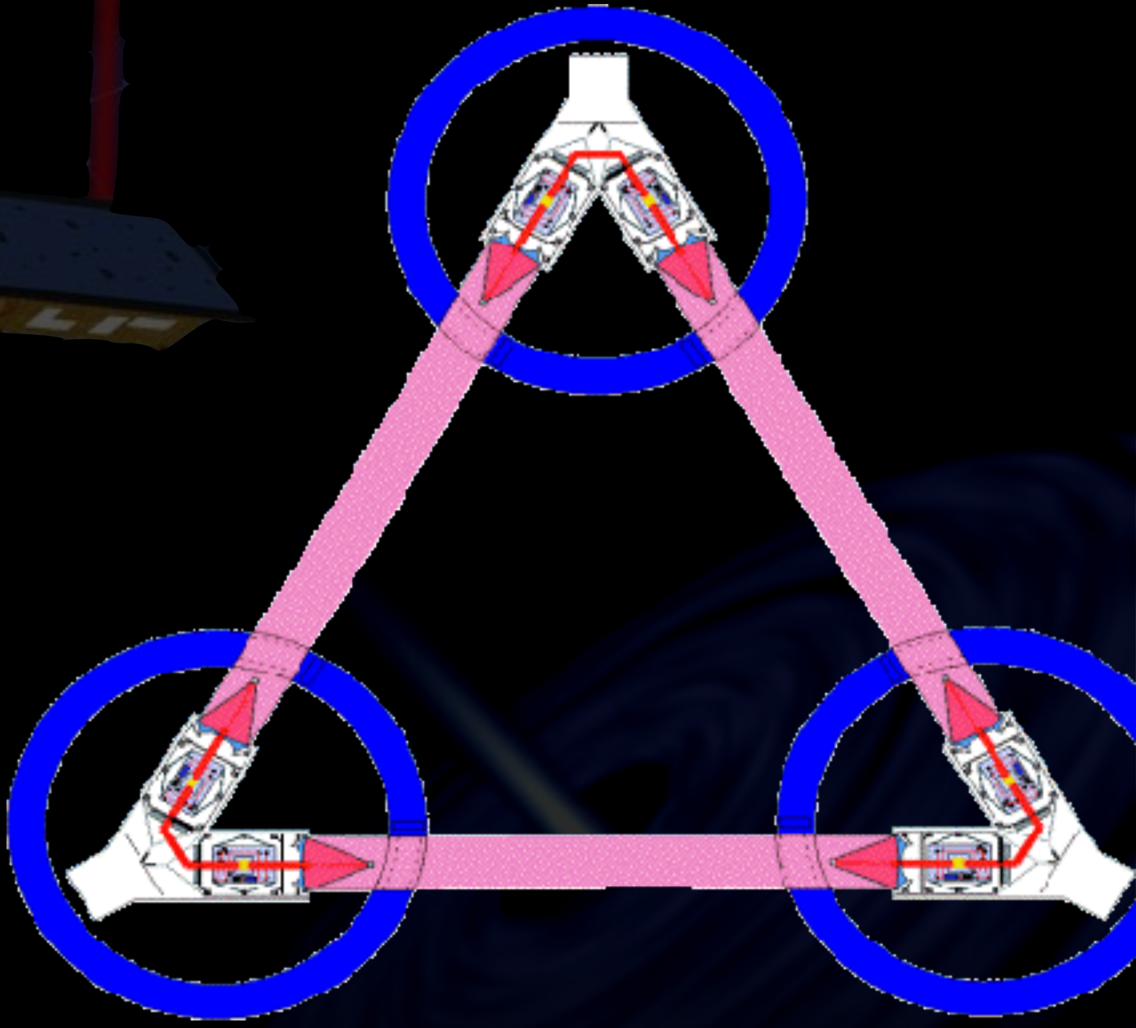




LISA : Measurements

- Problem with 2.5×10^9 m : A laser beam cannot make a round trip because too much intensity is lost.
  - 100pW received for 1 Watt emitted.

- Measurement with one arm and interference between two incoherent lasers in phase :
  - Distant laser
  - Local laser.



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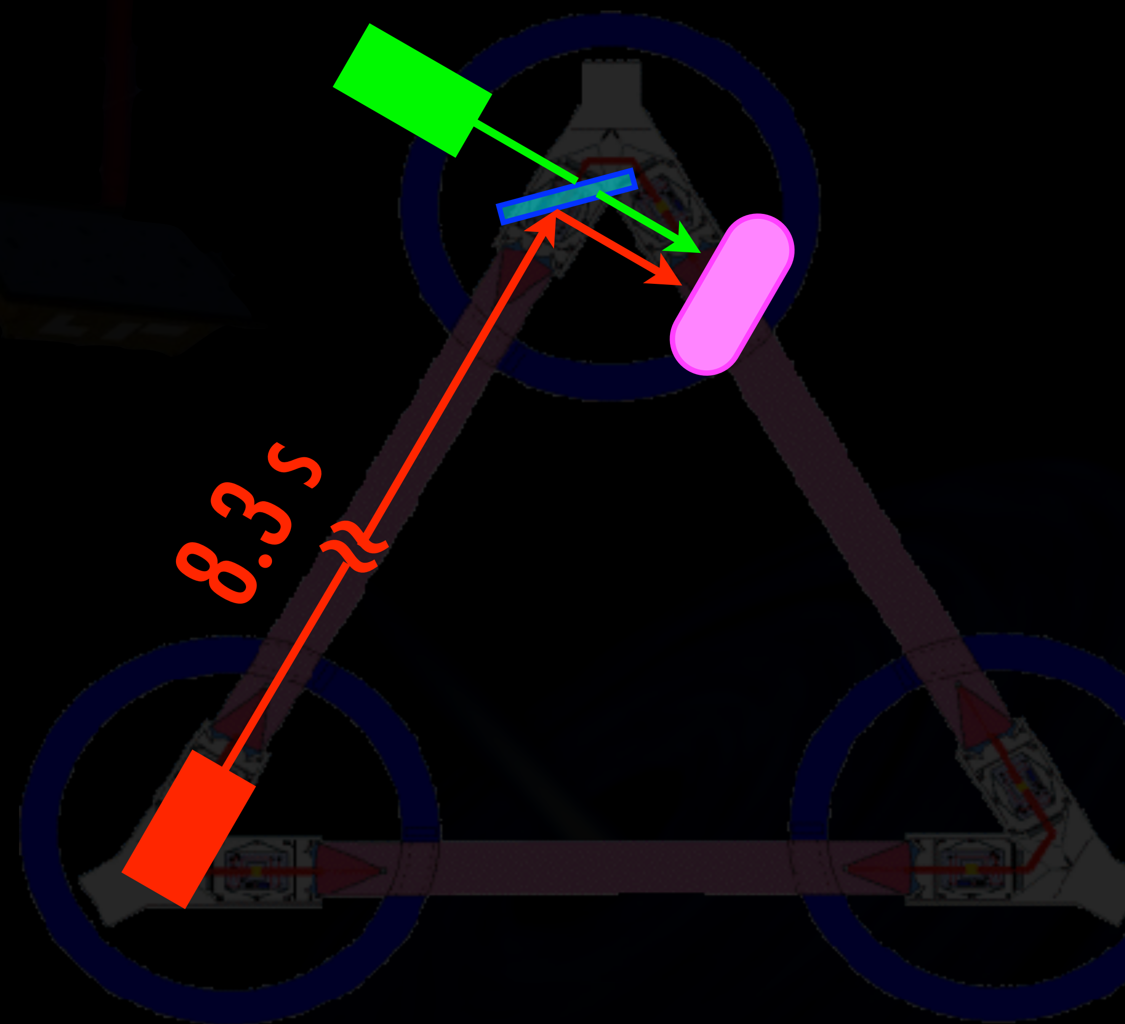

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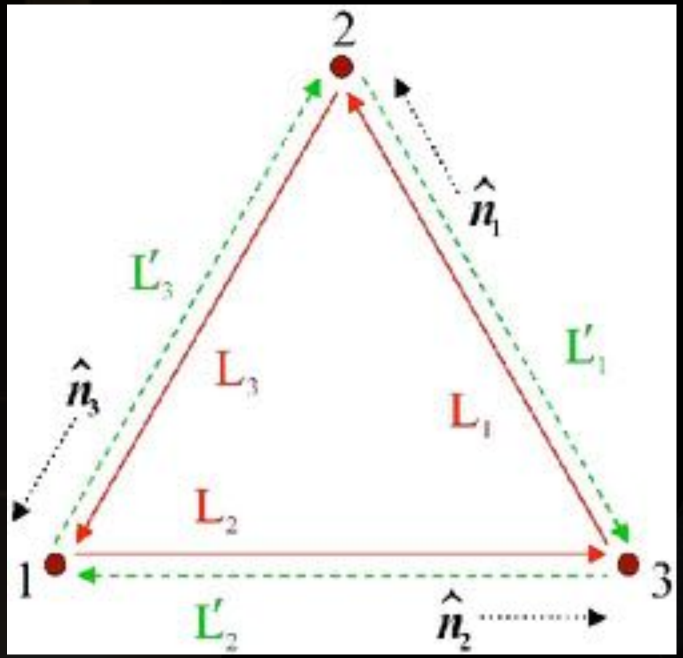
- Phase shift between the two beams measured by phasemeter.
- Beams from an external spacecraft, are delayed :
  - delay operator D_i^{real} :

$$D_i^{real}\ x(t) = x\left(t - \frac{L_i^{real}}{c}\right).$$

- The measurement :

$$s_1 = s_1^{GW} + s_1^{ShotNoise} + D_3^{real}\ p'^{lasernoise}_2 - p_1^{lasernoise} - 2\delta^{Acc.Noise}$$



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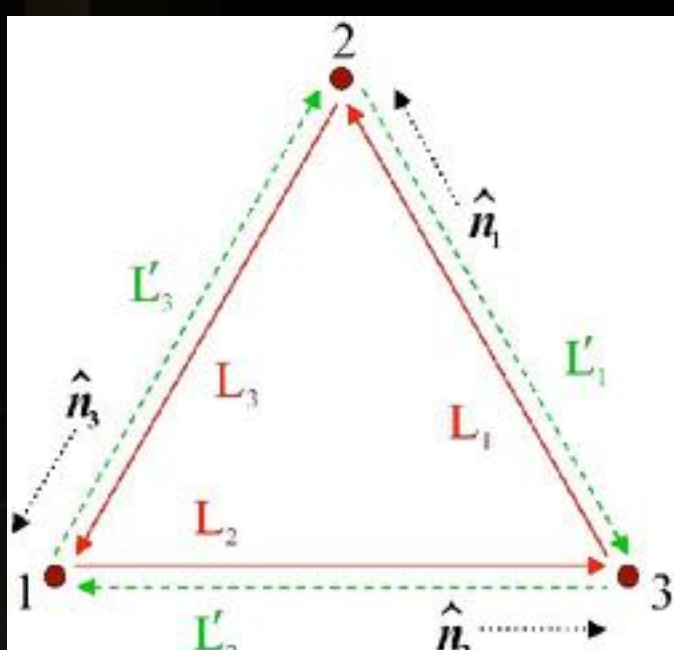

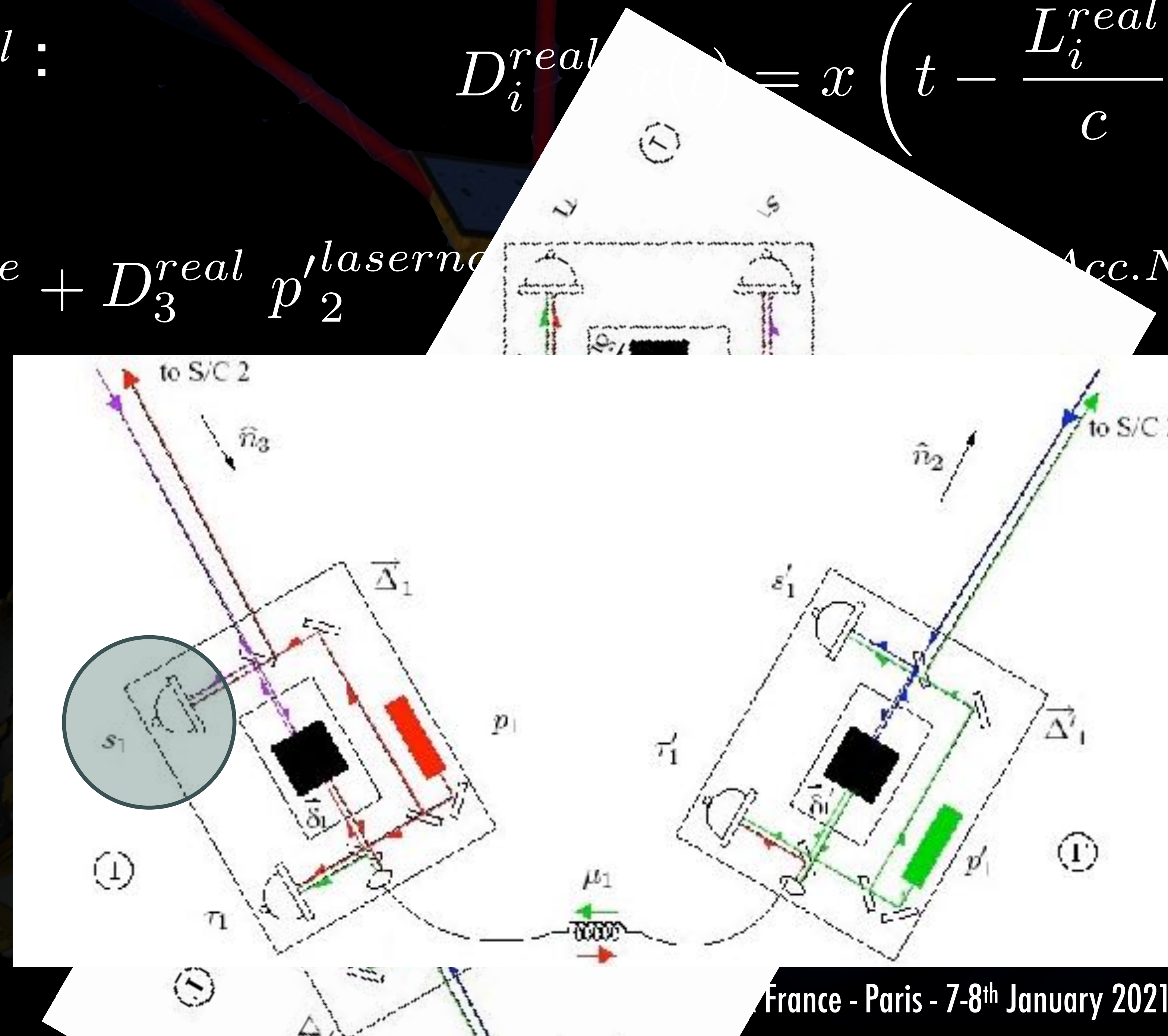

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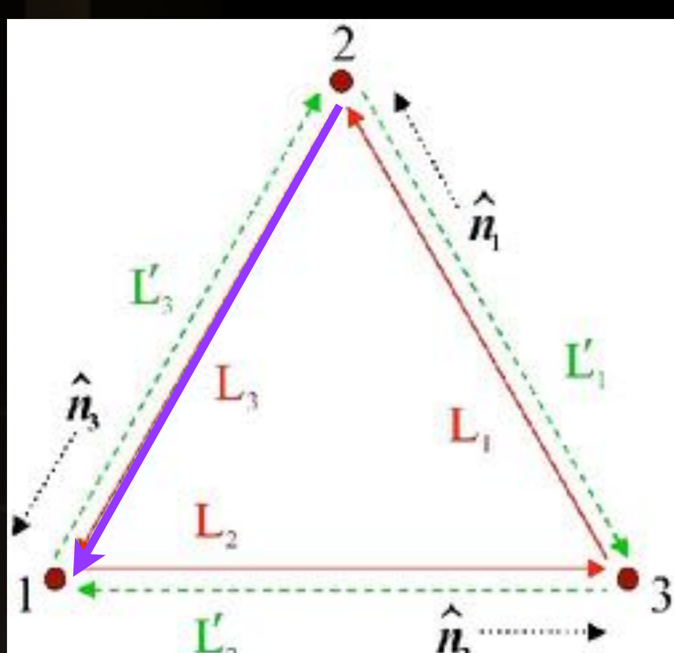

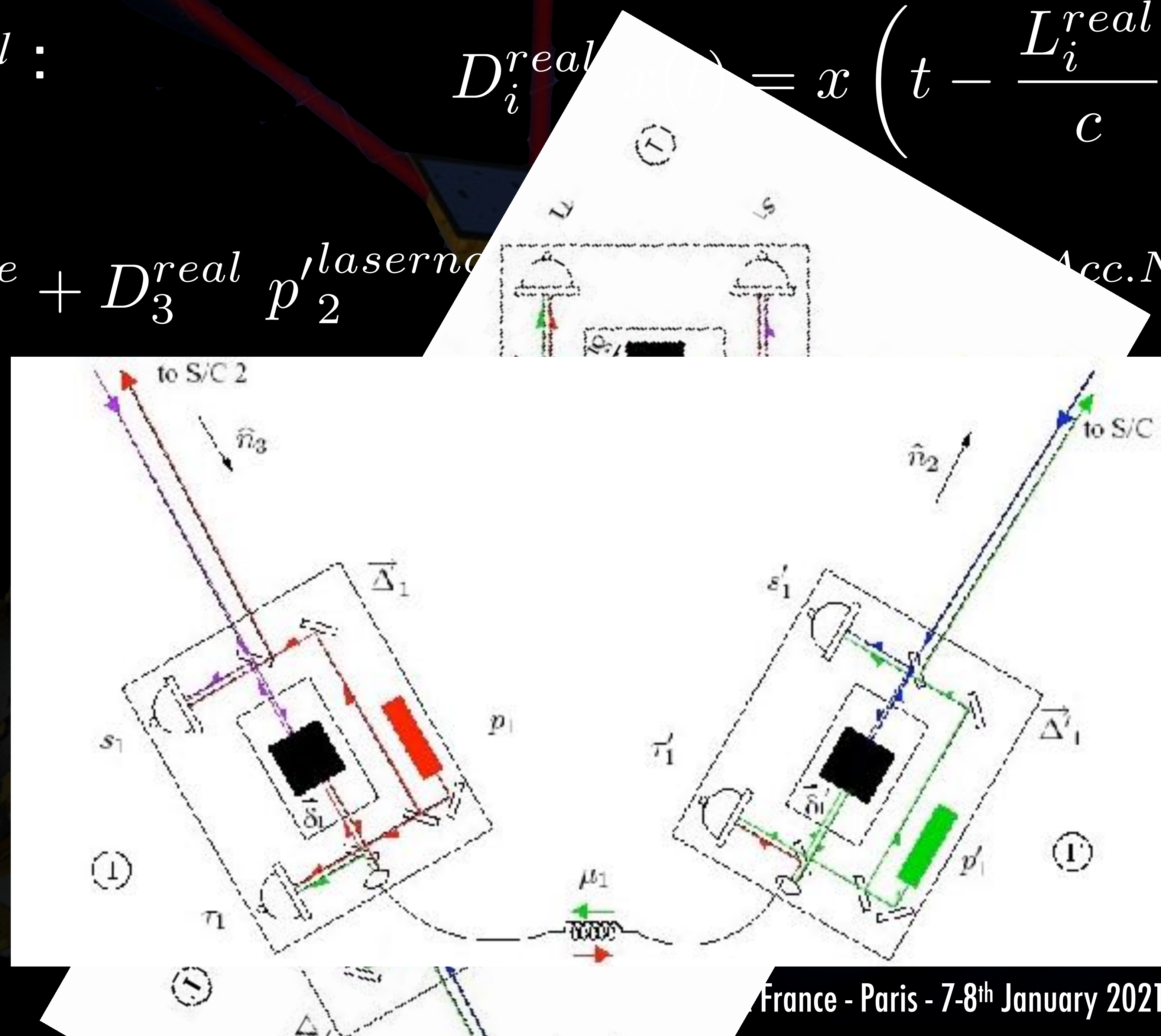

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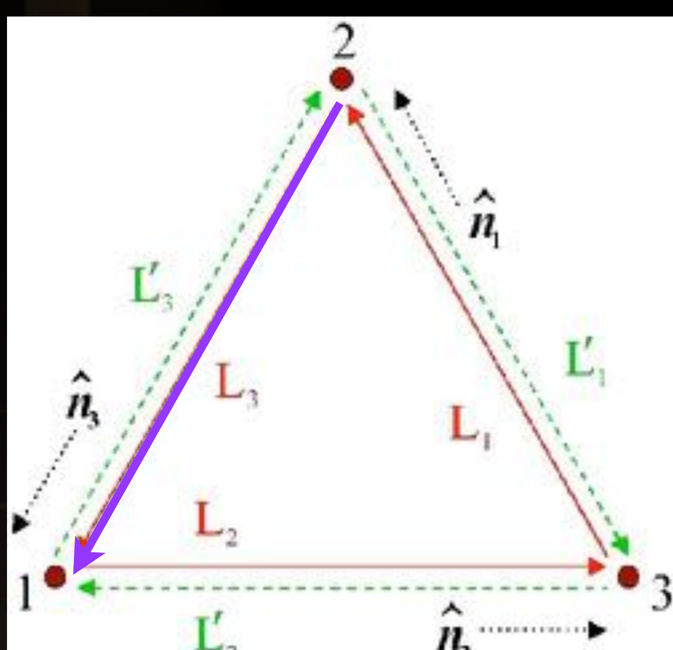

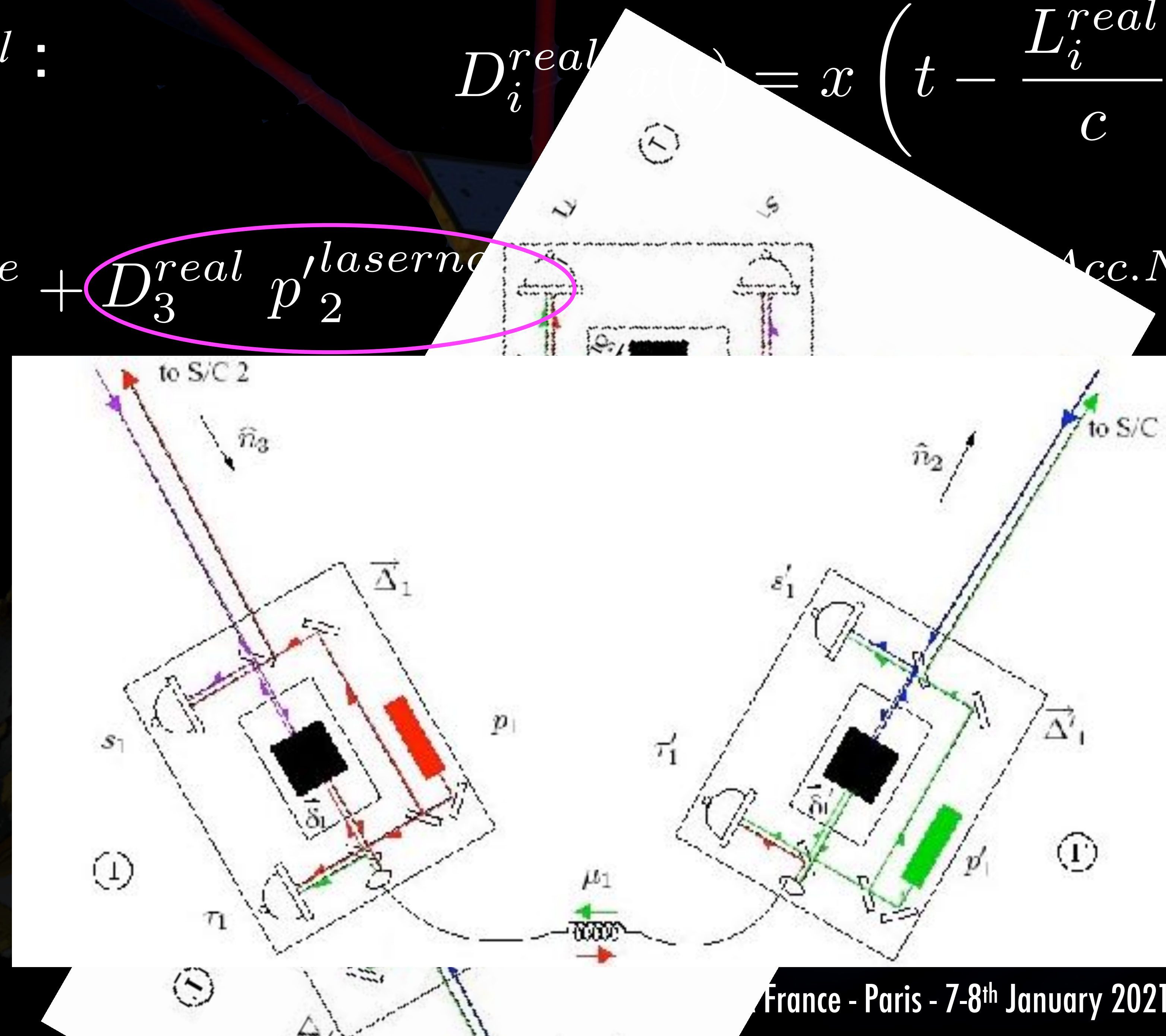

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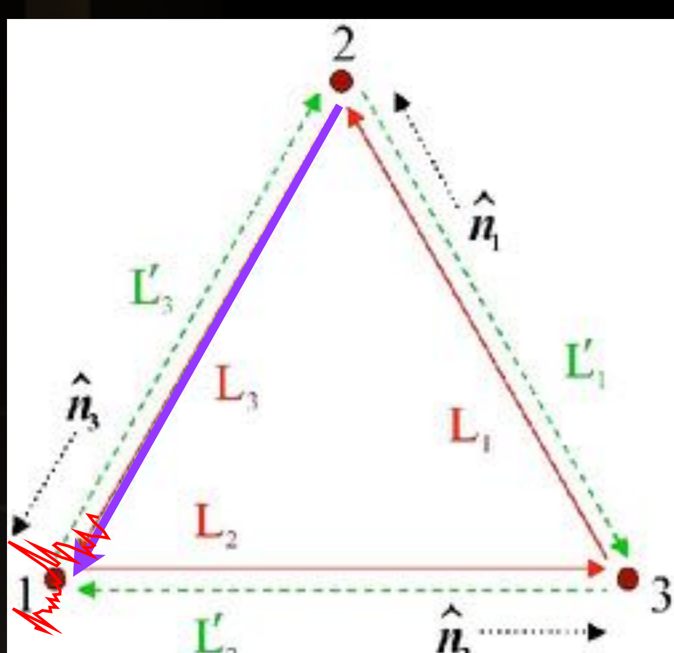

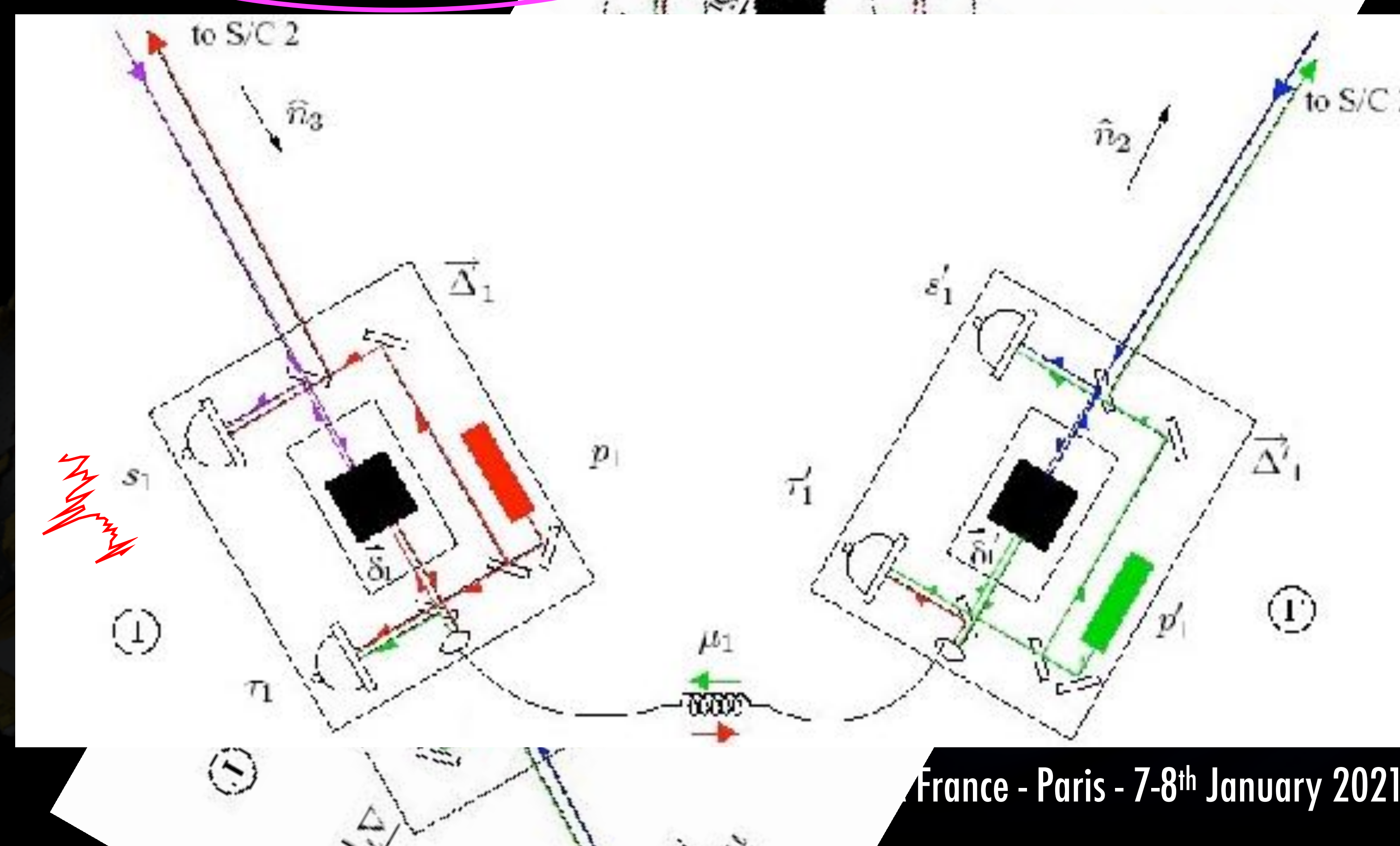

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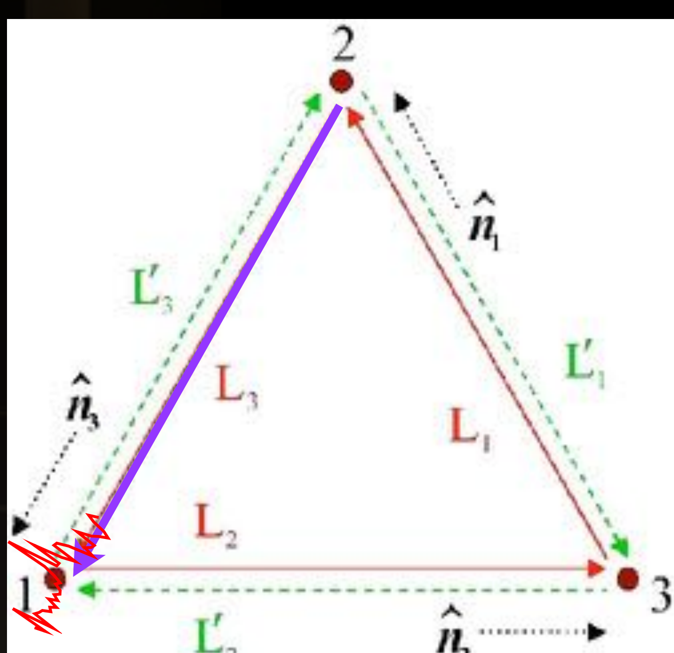

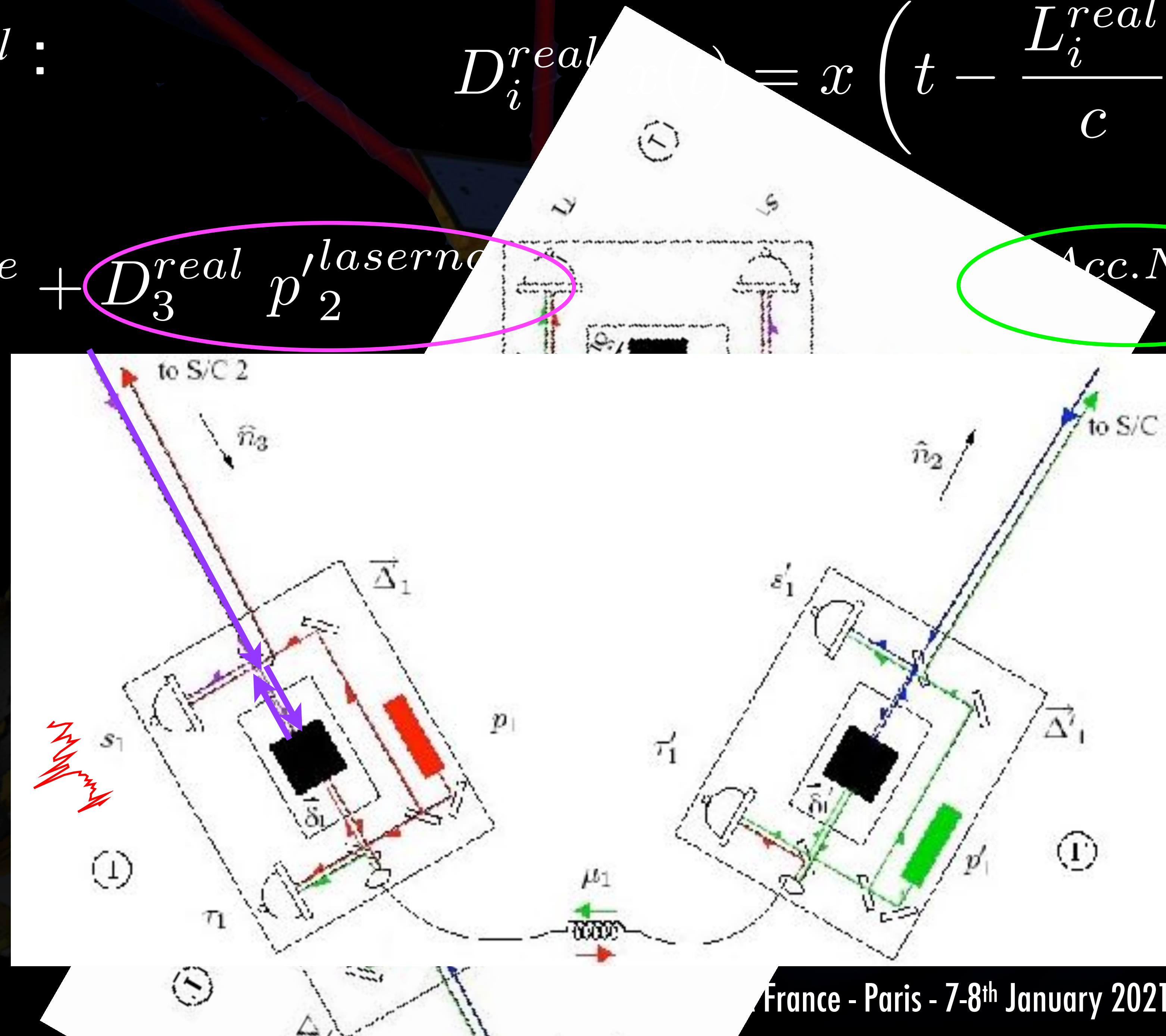

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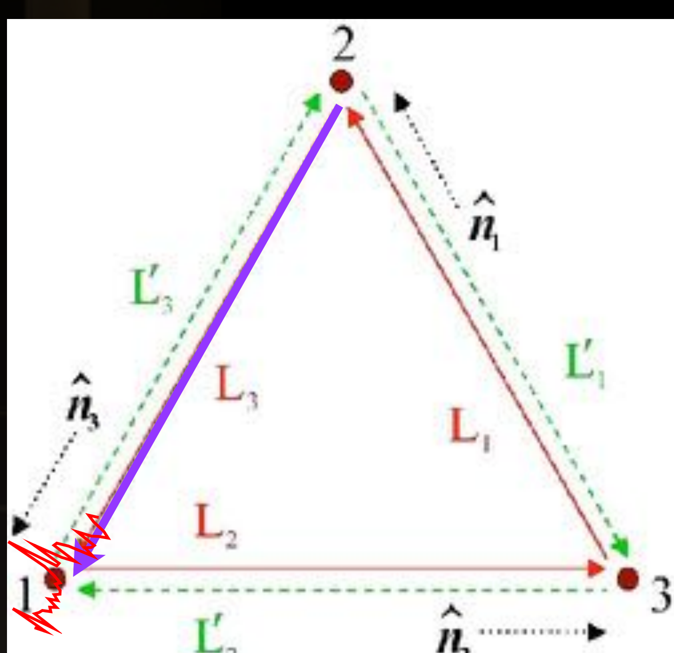

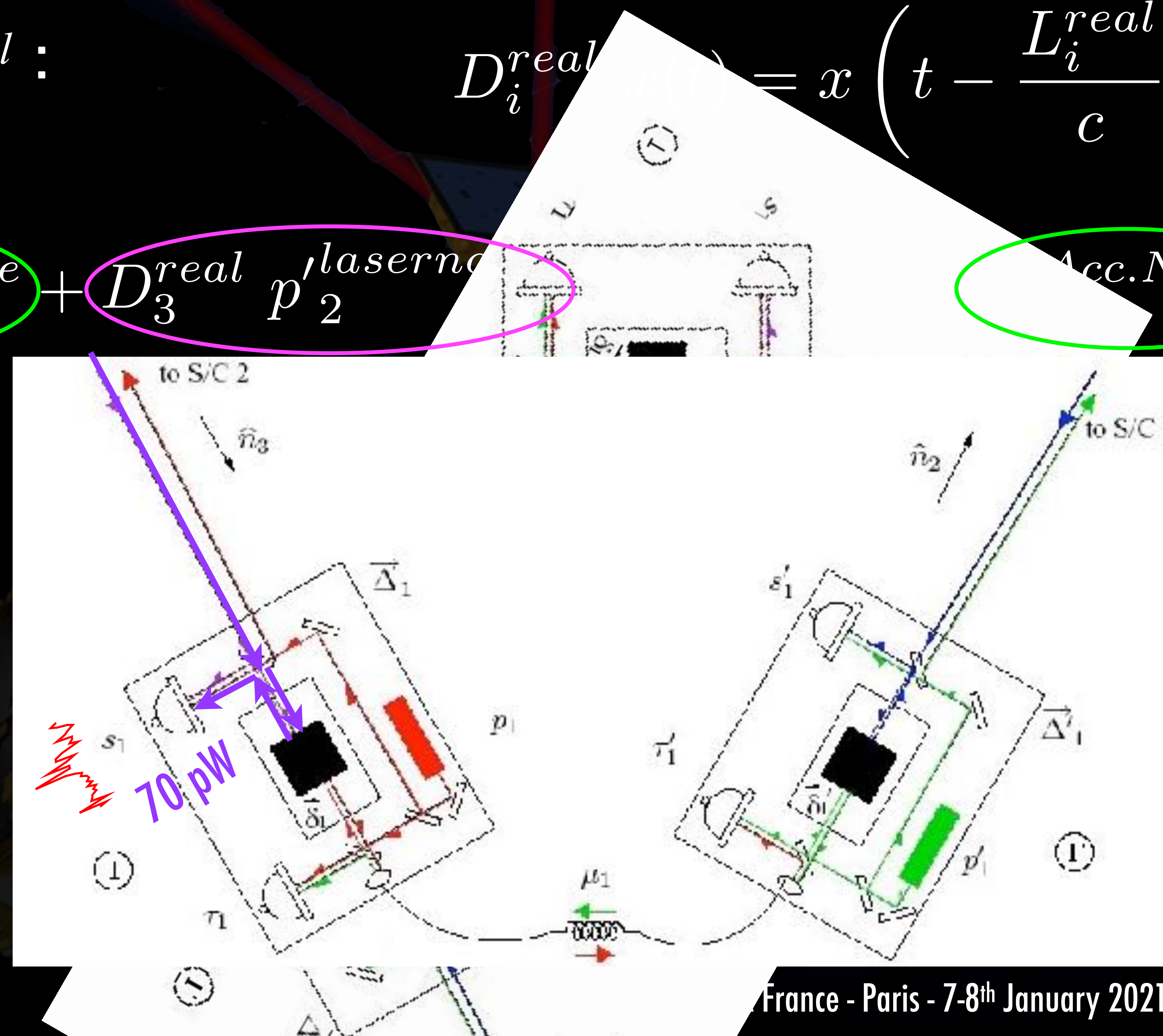

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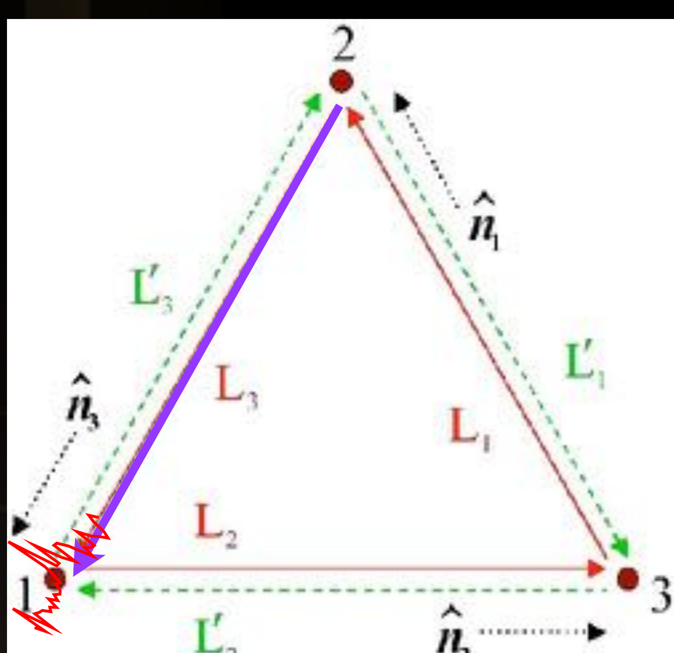

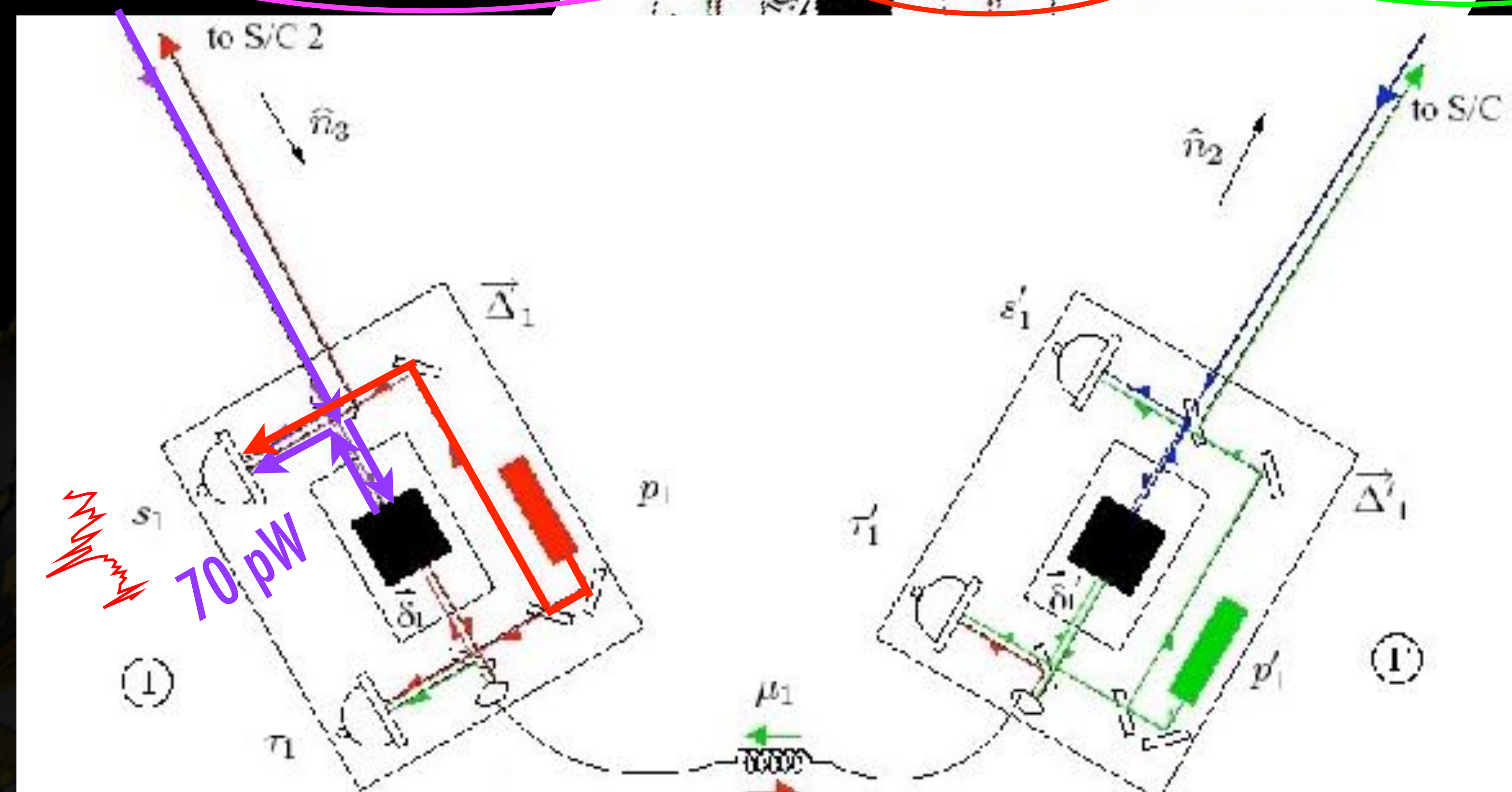

Time Delay Interferometry

- Pre-processing of the science data,

Tinto & Durandhar, Revue *Living Rev. Rel. 8 p 4* (2005)
Durandhar, Nayak & Vinet, *PRD 65 102002* (2002)

- Combinations of delayed measurements to reduce laser noise:

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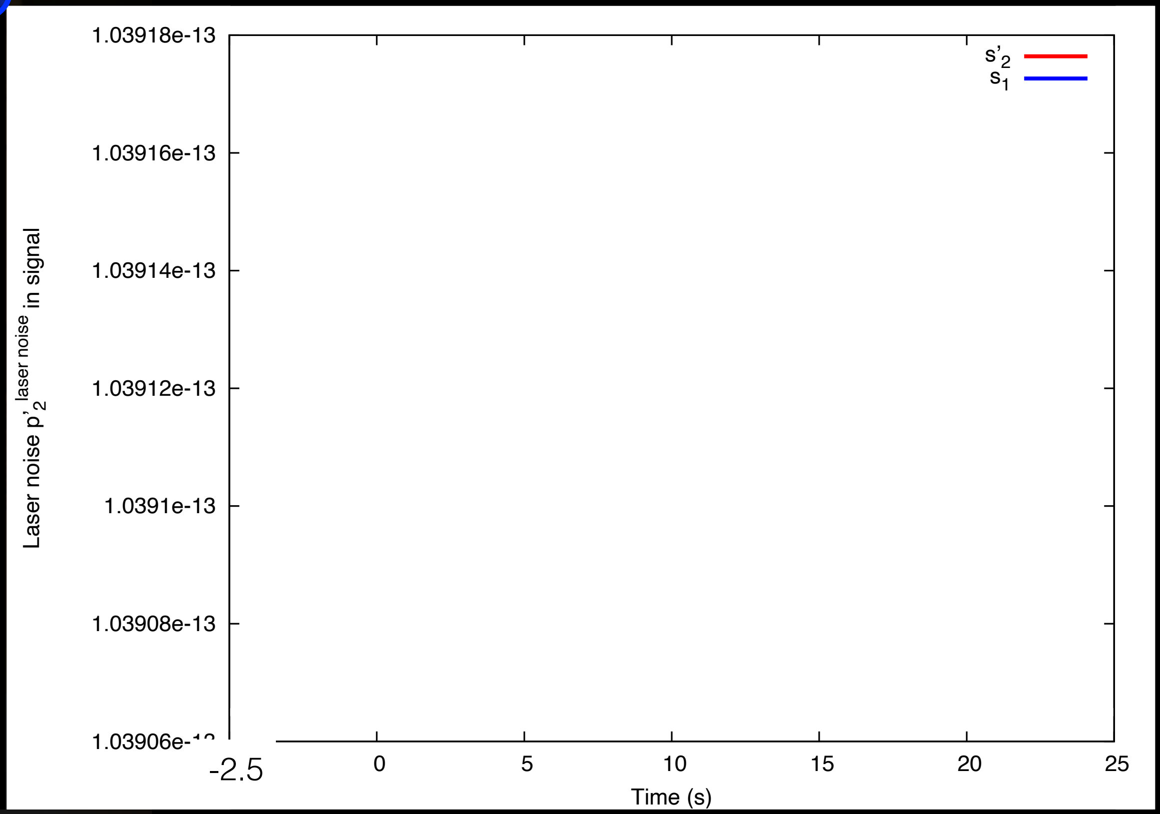

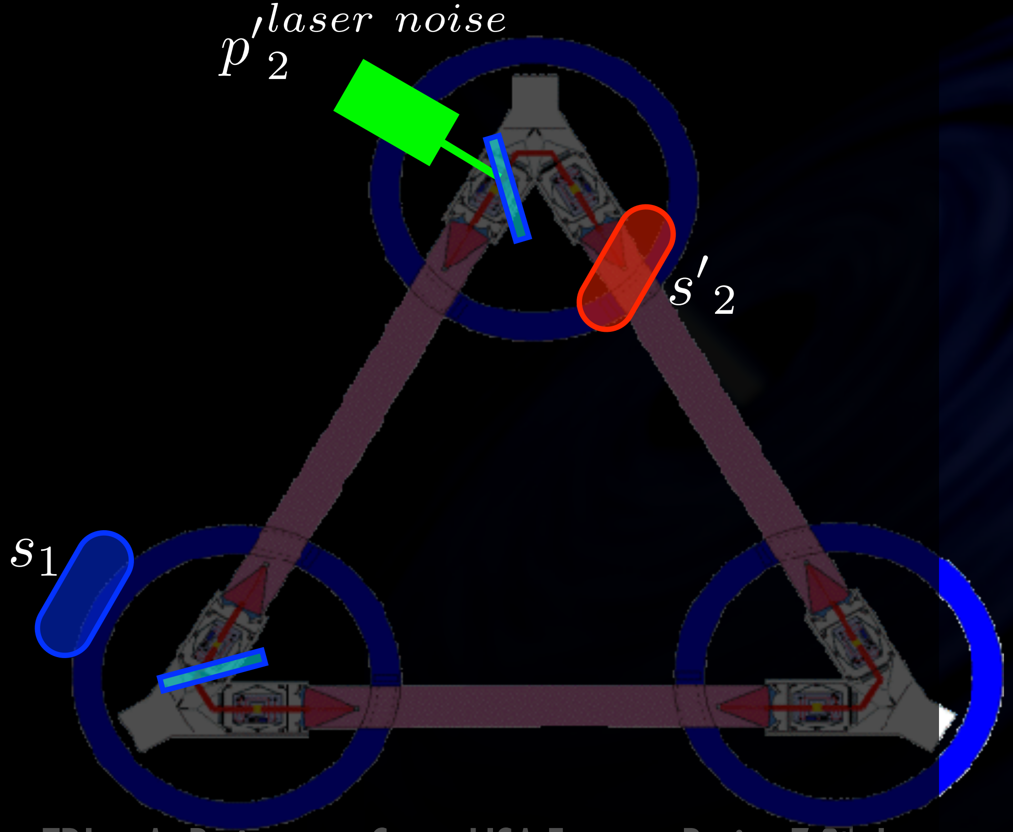

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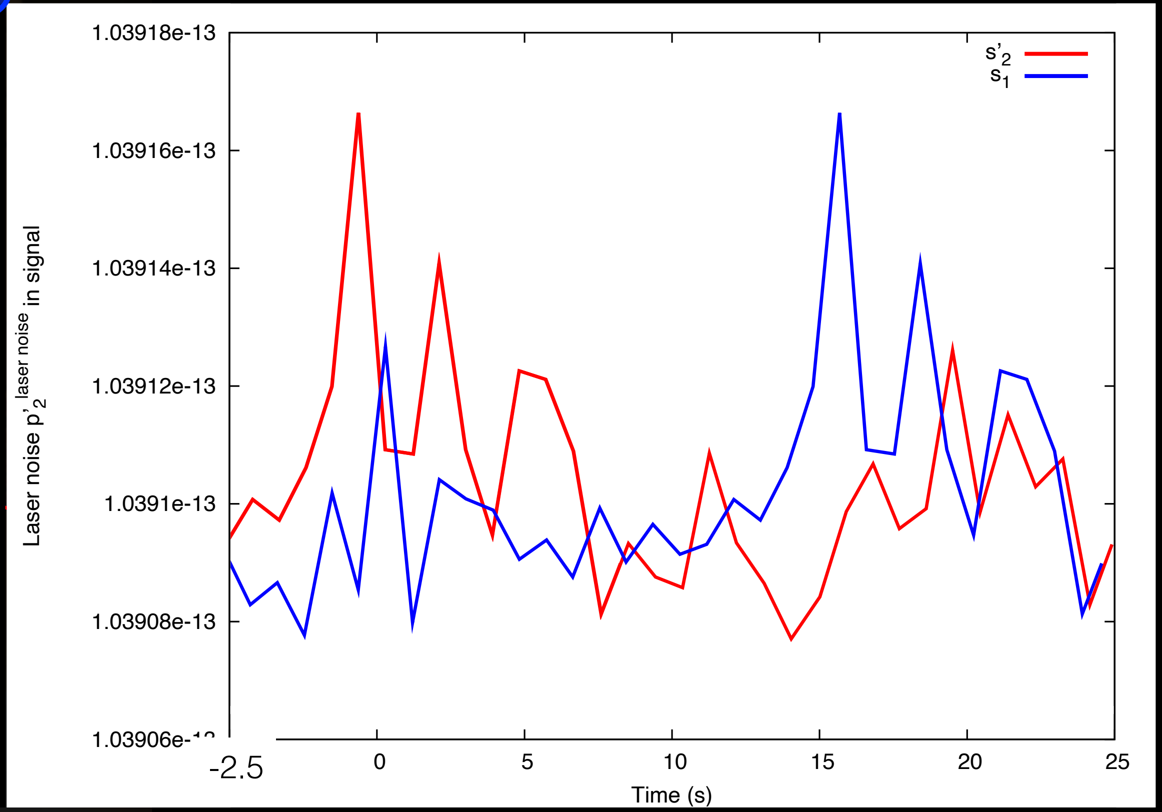

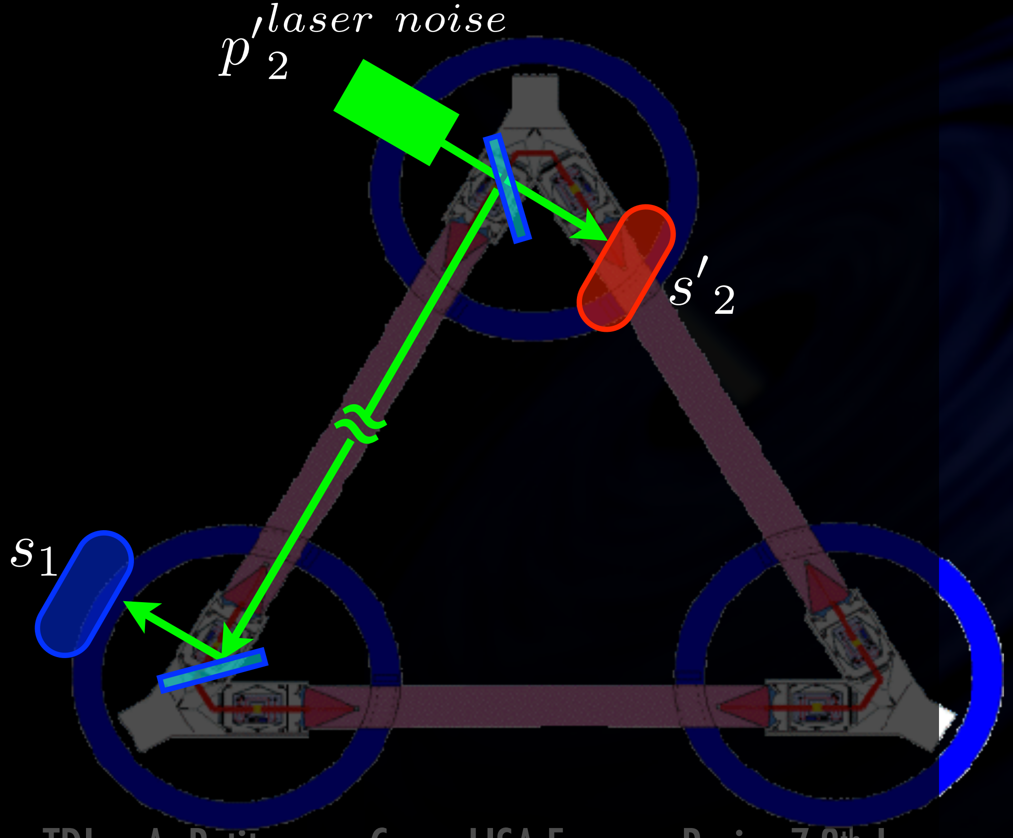

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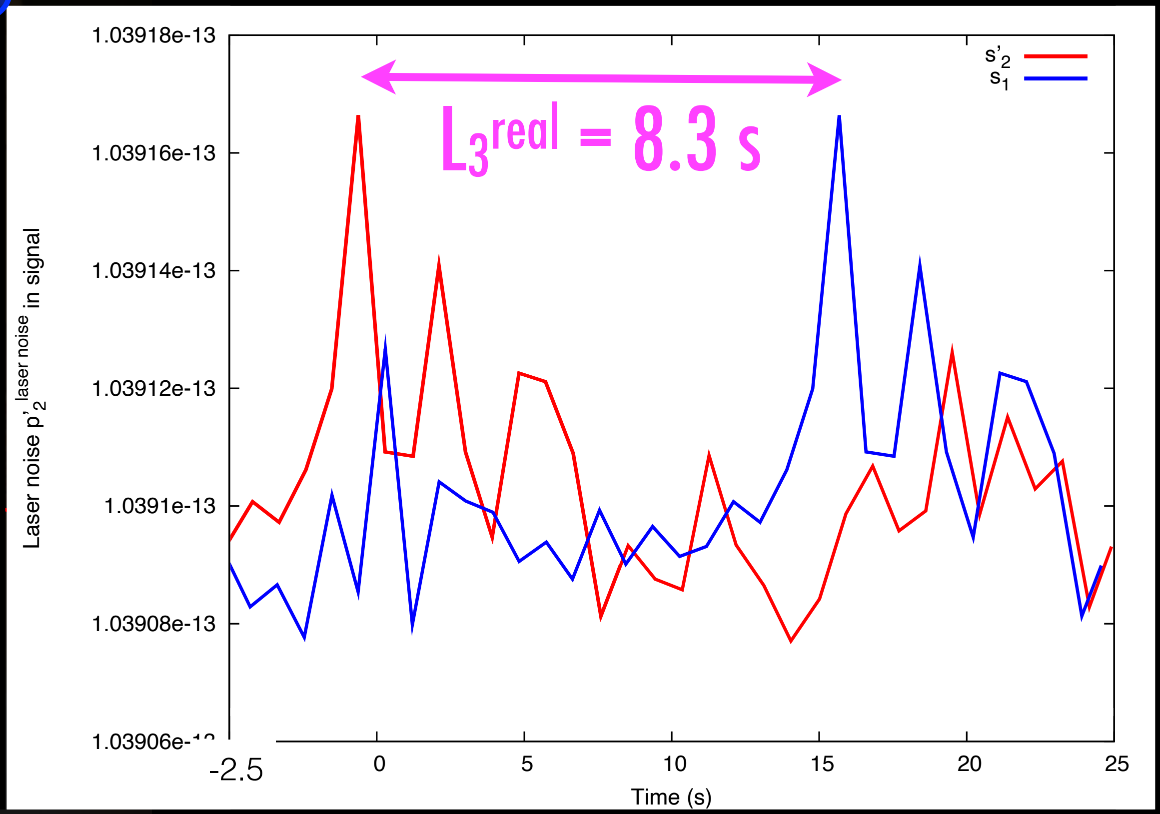

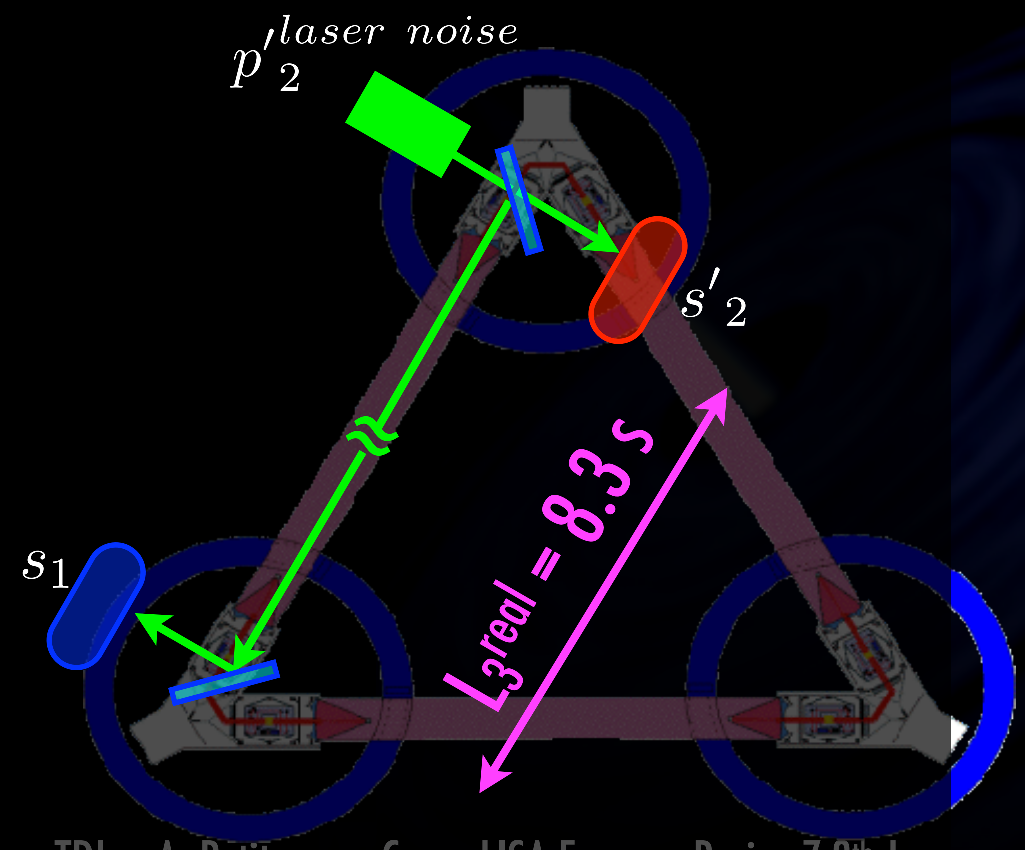

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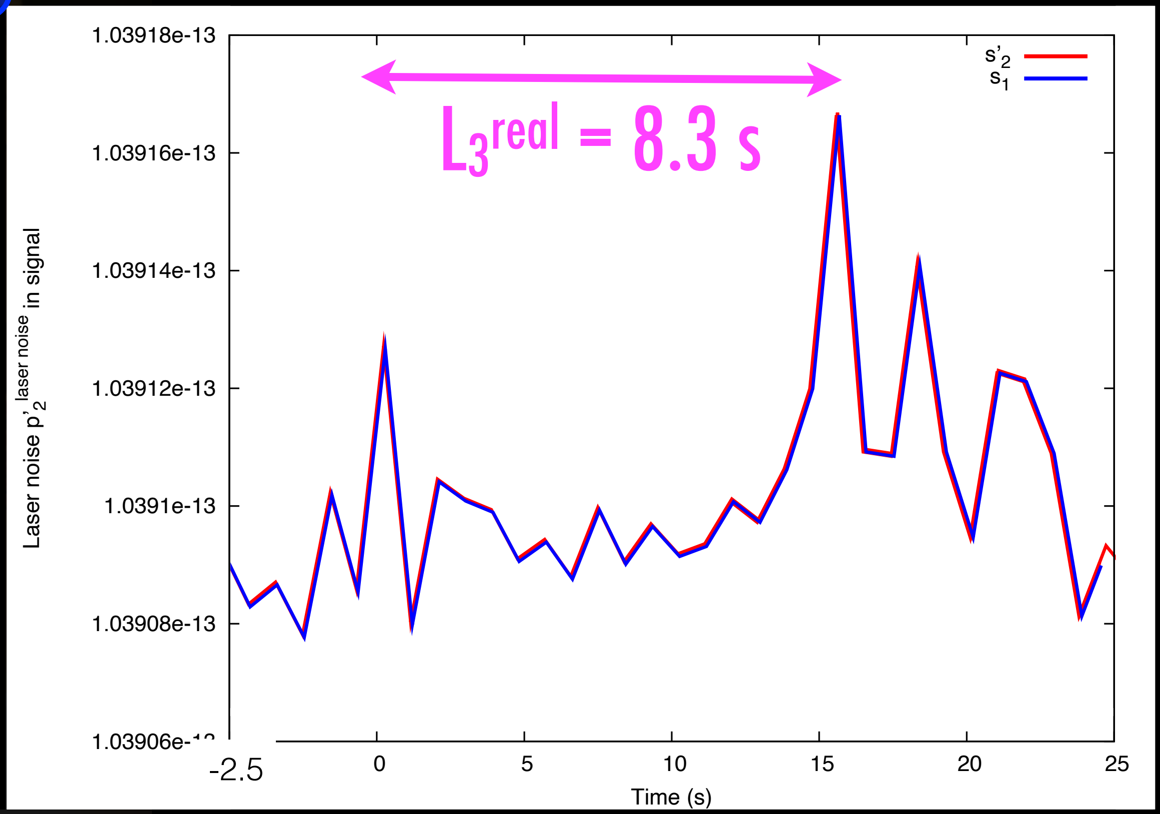

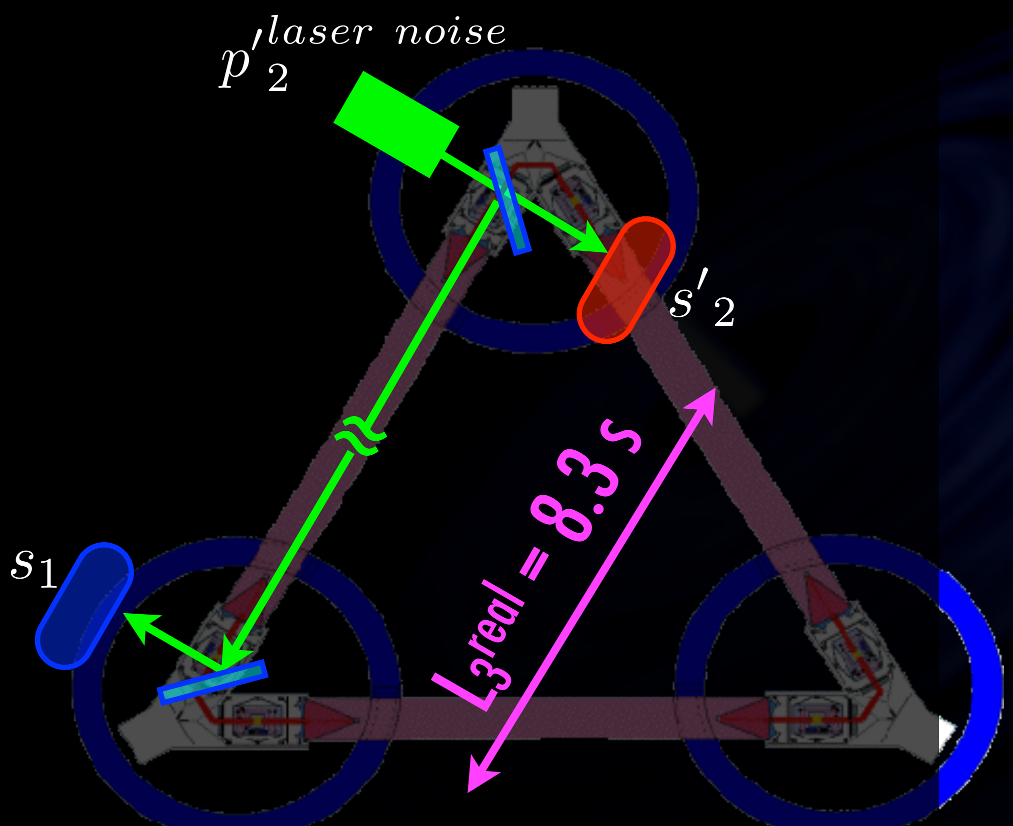

Time Delay Interferometry

- Pr
- Co

... p 4 (2005)

2002)

TDI requires :

- knowledge of delays : $L_i^{TDI} = L_i^{real}$
- interpolation due to the sampling of phasemeter signal

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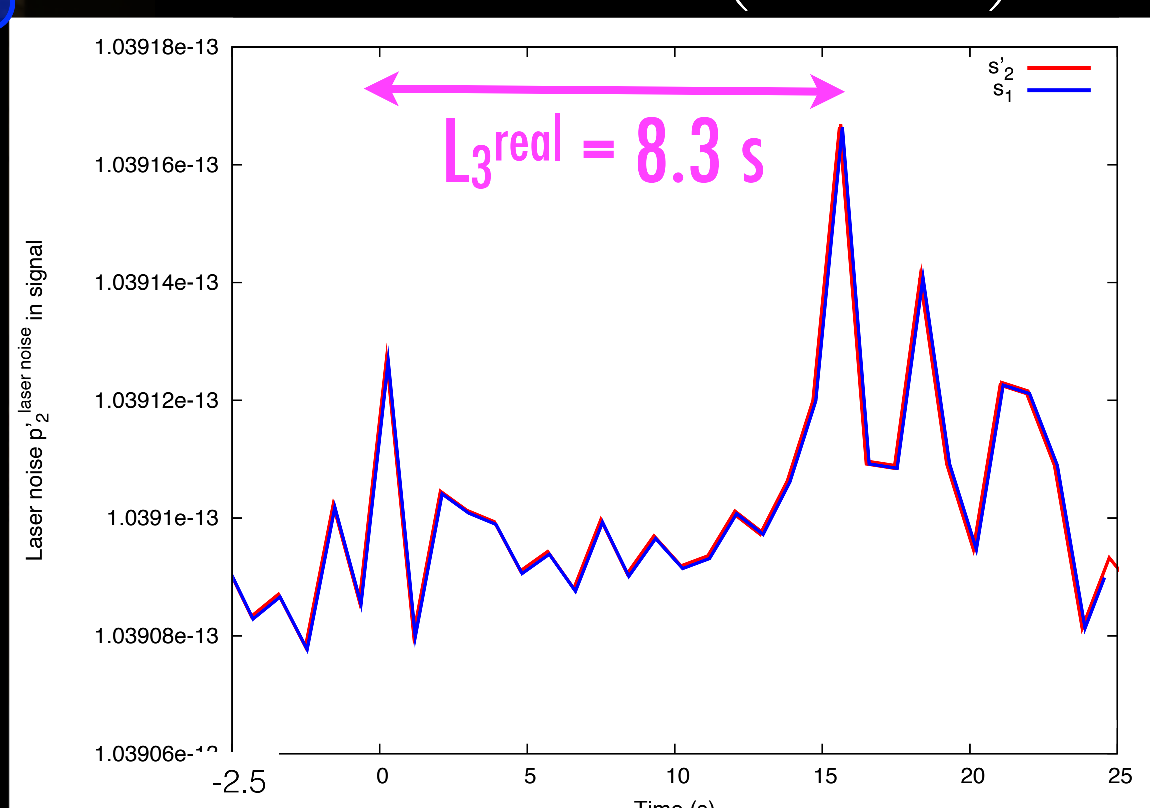
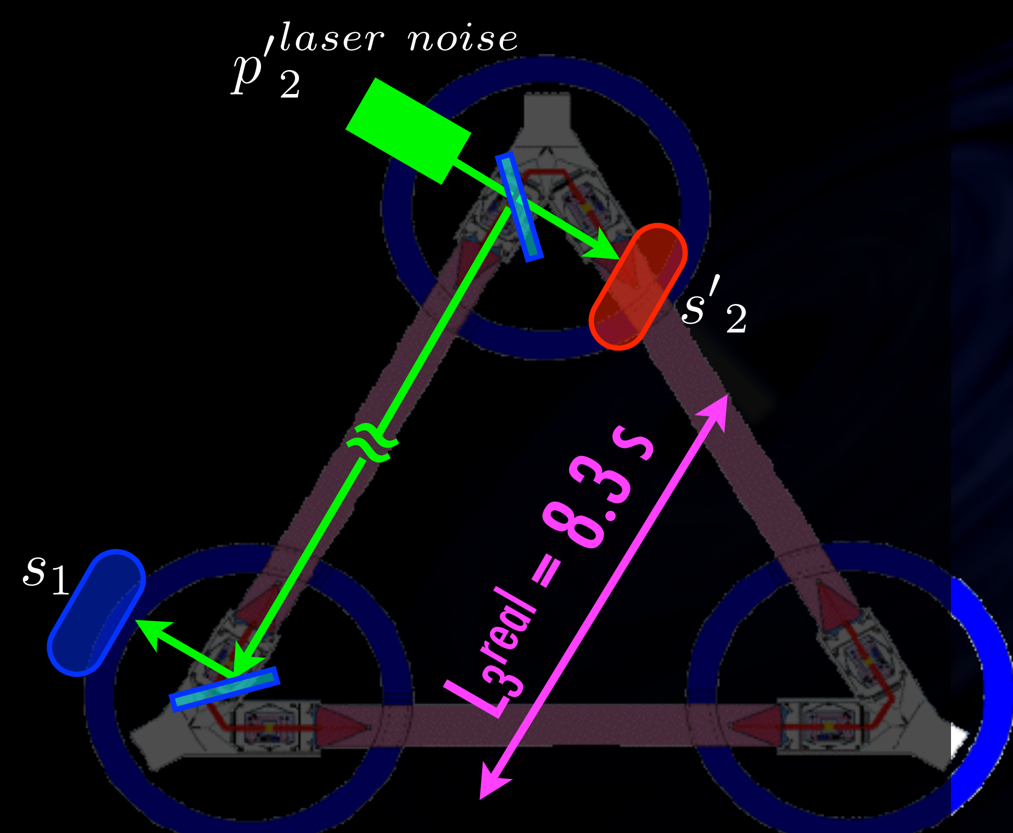
$$s_1(t) = D_3^{real} p'_2(t) = p'_2\left(t - \frac{L_3^{real}}{c}\right)$$

$$s_1(t) + D_3^{TDI}\ s'_2(t) = D_3^{real} p'_2 - D_3^{TDI} p'_2 \simeq 0$$





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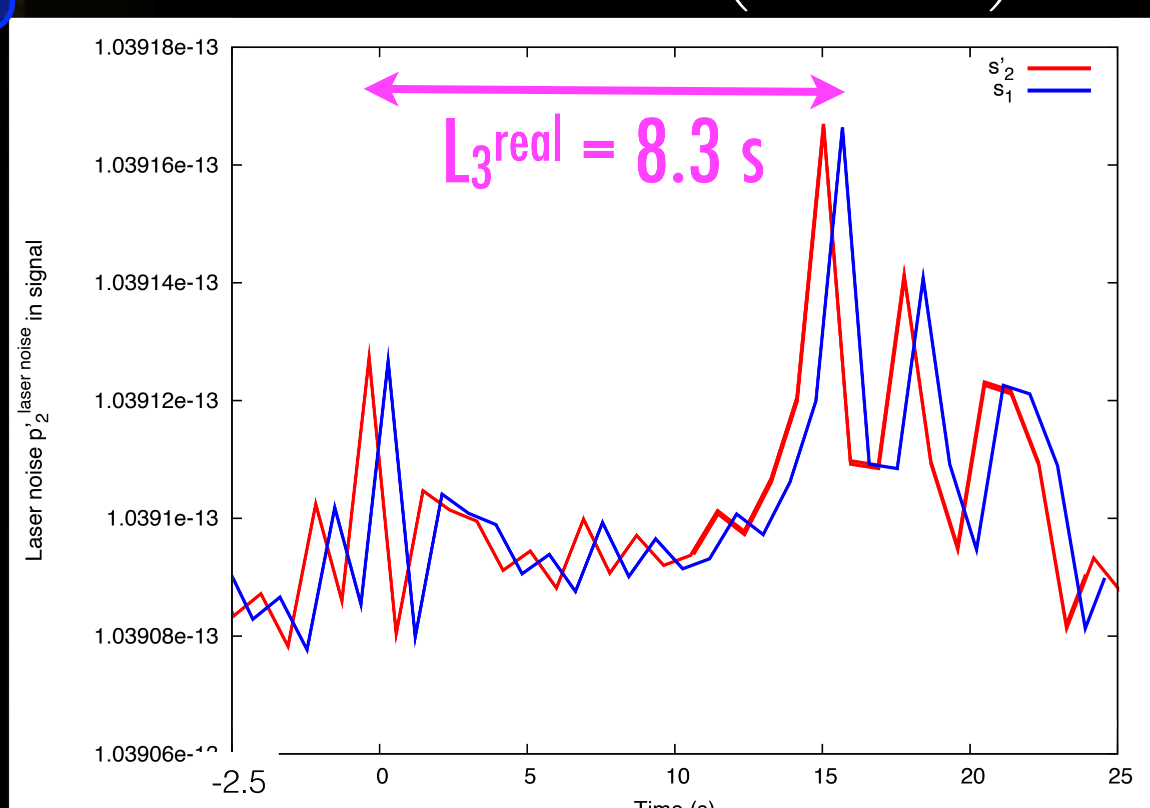
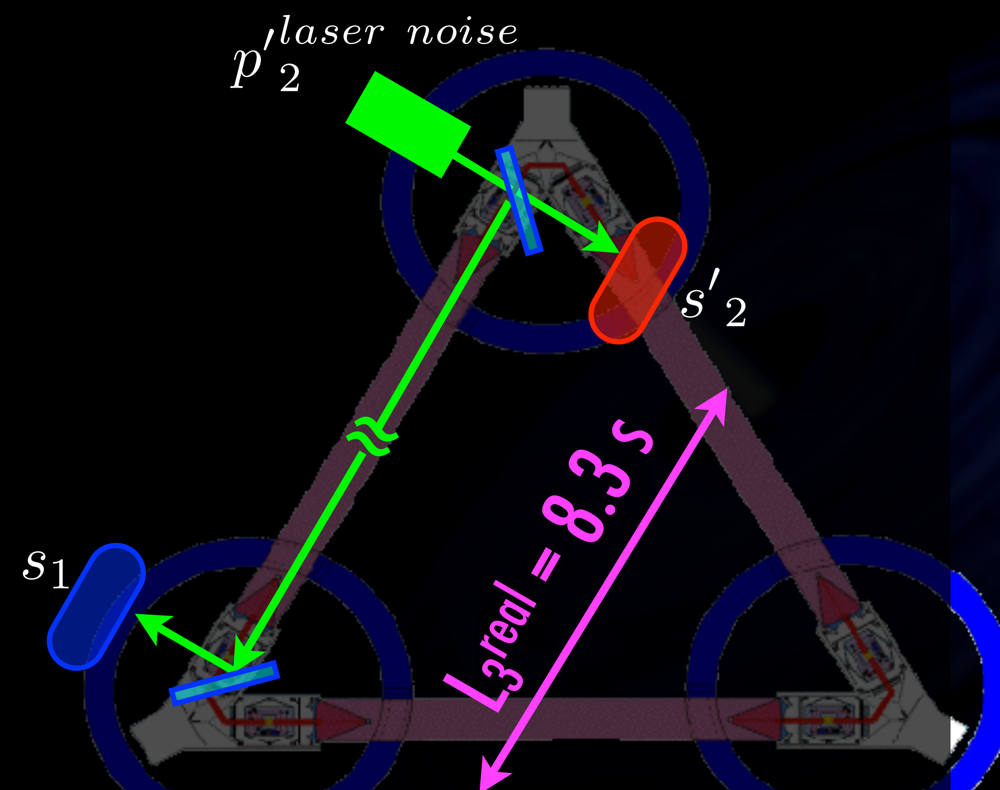
$$\begin{aligned} s_1(t) \ &+ \ D_3^{TDI}\ s'_2(t) \\ &= \ D_3^{real} p'_2 - D_3^{TDI} p'_2 \\ &\simeq \ \text{residual laser noise} \end{aligned}$$





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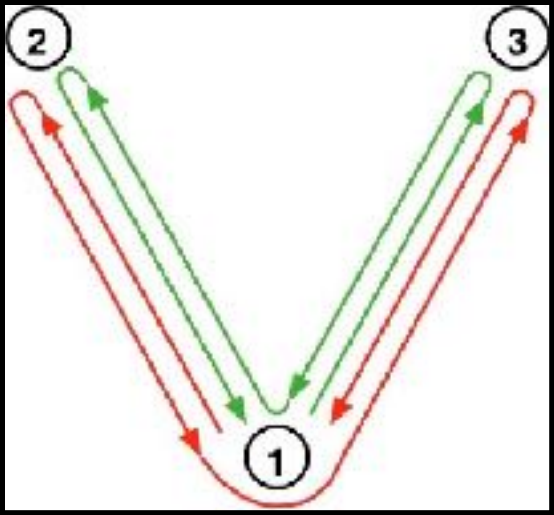
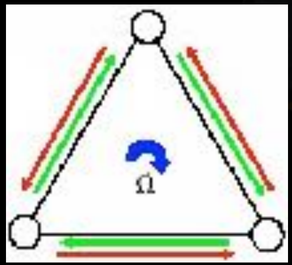
Vallisneri, *gr-qc/0504145* (2005)

- Time Delay Interferometry:
  - Combine delayed measurements to reduce laser noises, optical bench noises, ... ?
  - Algebraic development: many combinations (generators)

$$\begin{aligned} X &= -s_1 - D_3\, s'_2 - D_3 D_{3'}\, s'_1 - D_3 D_{3'} D_{2'}\, s_3 \\ &\quad + s'_1 + D_{2'}\, s_3 - D_{2'} D_2\, s_1 - D_{2'} D_2 D_3\, s_{2'} \\ &\simeq 0 \end{aligned}$$



  - Different precisions level
    - 1st generation: rigid formation of LISA : $D_{i'}\, s = D_i\, s$,
    - generation 1.5: Sagnac effect : $D_{i'}\, s \neq D_i\, s$ but $D_j D_i\, s = D_i D_j\, s$,
    - 2nd generation: flexing and Sagnac effect : $D_j D_i\, s \neq D_i D_j\, s$



Time Delay Interferometry

- TDI generation 1

$$X_{1st} = \left(1 - D_2^2, 0, -D_2 + D_2 D_3^2, -1 + D_3^2, D_3 - D_2^2 D_3, 0\right)$$

- TDI generation 1.5

$$X_{1.5} = (1 - D_2 D'_2, 0, -D'_2 + D'_2 D'_3 D_3, -1 + D'_3 D_3, D_3 - D_2 D'_2\ D_3, 0)$$

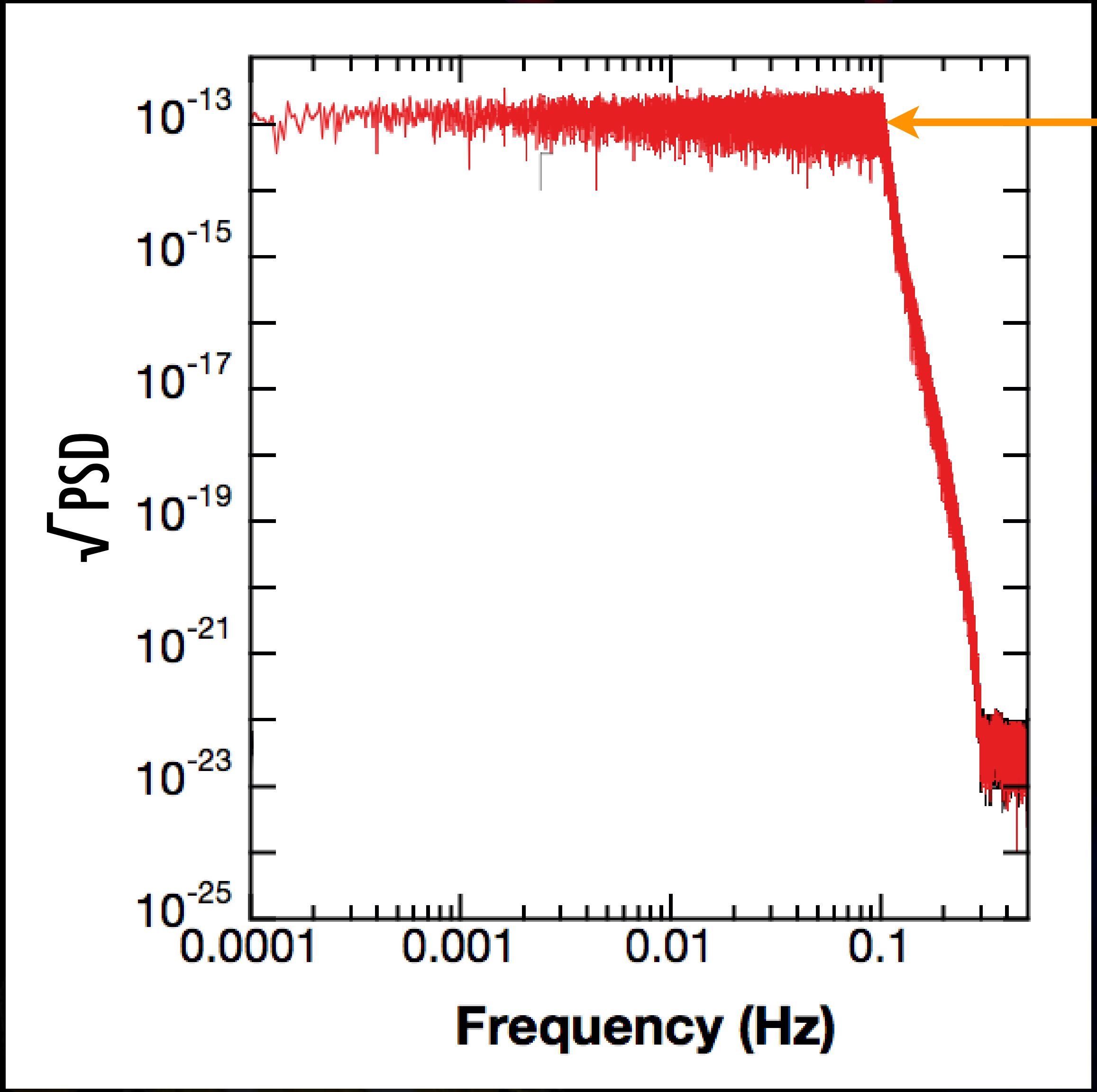
- TDI 2nd generation: until 7 delay operators combined

$$\begin{aligned} X_{2nd} = (&1 + D_3 D'_3 D'_2 D_2 D'_2 D_2 - D'_2 D_2 - D'_2 D_2 D_3 D'_3, \\ &0, \\ &D_3 D'_3 D'_2 + D_3 D'_3 D'_2 D_2 D'_2 - D'_2 - D'_2 D_2 D_3 D'_3 D_3 D'_3 D'_2, \\ &D_3 D'_3 + D_3 D'_3 D'_2 D_2 - 1 - D'_2 D_2 D_3 D'_3 D_3 D'_3, \\ &D_3 + D_3 D'_3 D'_2 D_2 D'_2 D_2 D_3 - D'_2 D_2 D_3 - D'_2 D_2 D_3 D'_3 D_3, \\ &0) \end{aligned}$$

Time Delay Interferometry

- Reduction of laser noises by 8 orders of magnitude !

A GW is hidden here !



Phasemeter
(cut off due to the filter required for digitalization of signal)

Petiteau & al, Phys. Rev. D (2008)

Time Delay Interferometry

- Reduction of laser noises by 8 orders of magnitude !

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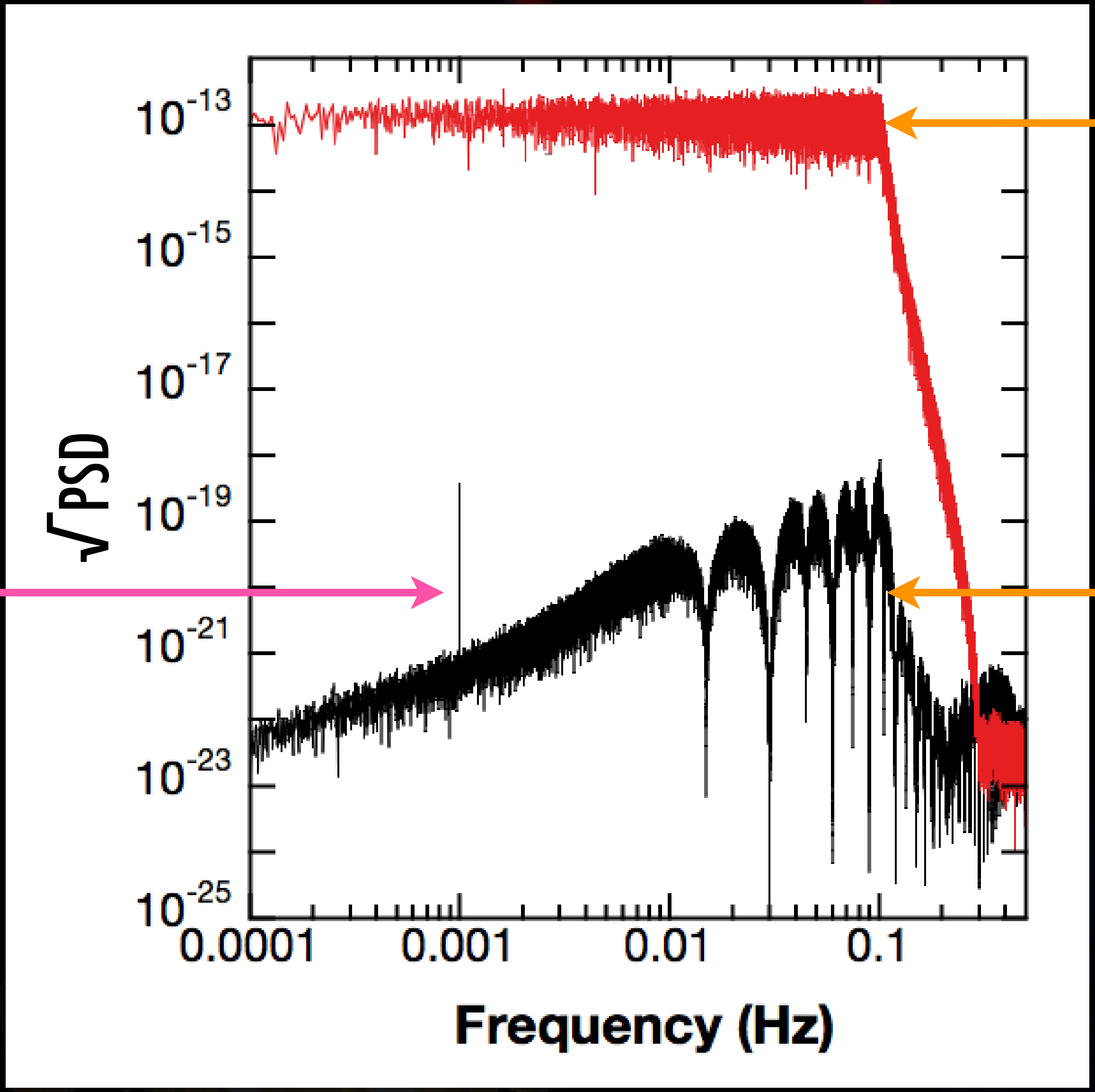
Here it is !



Phasemeter
(cut off due to the filter required for digitalization of signal)

TDI Michelson

Petiteau & al, Phys. Rev. D (2008)

Time Delay Interferometry

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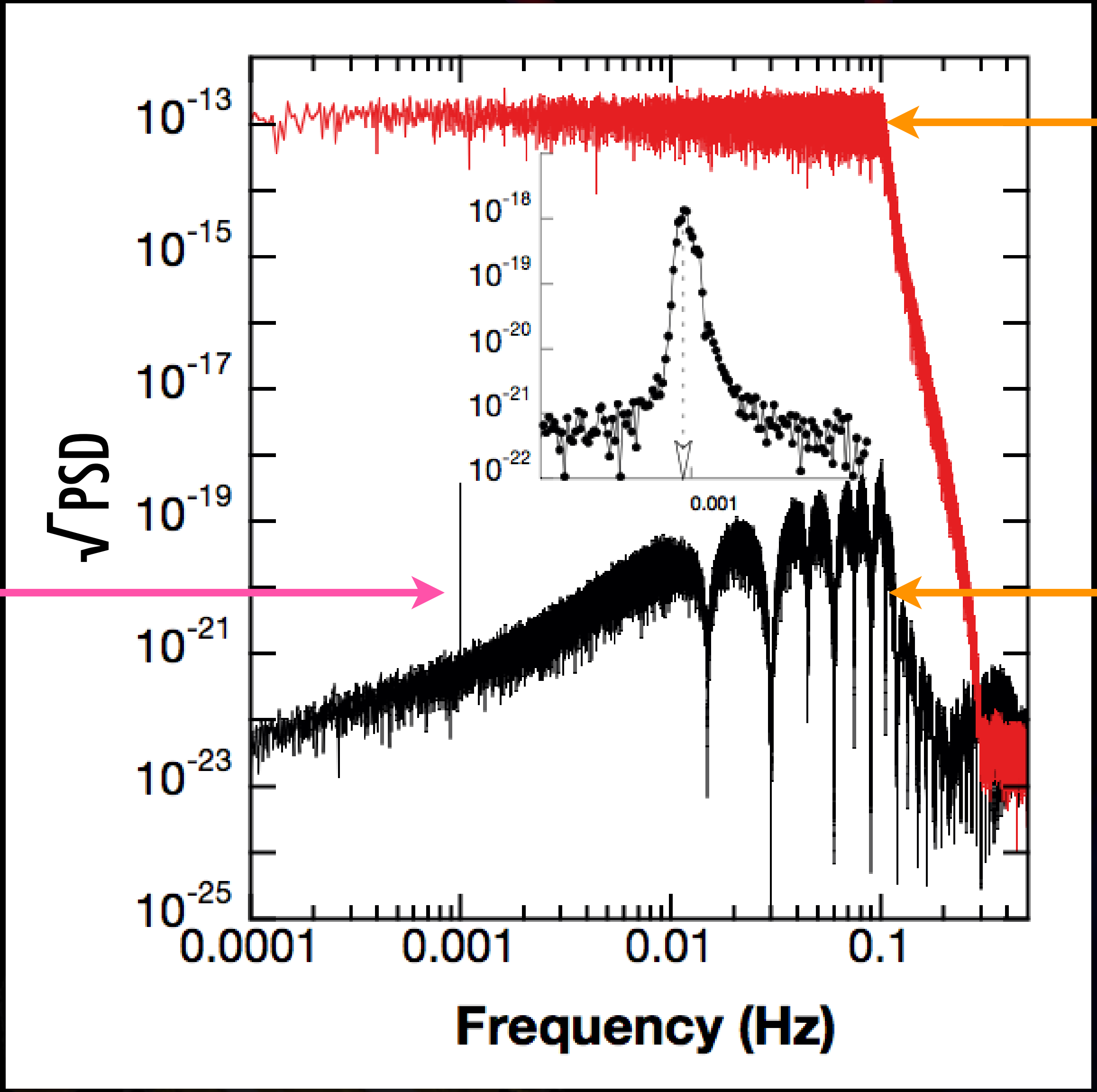


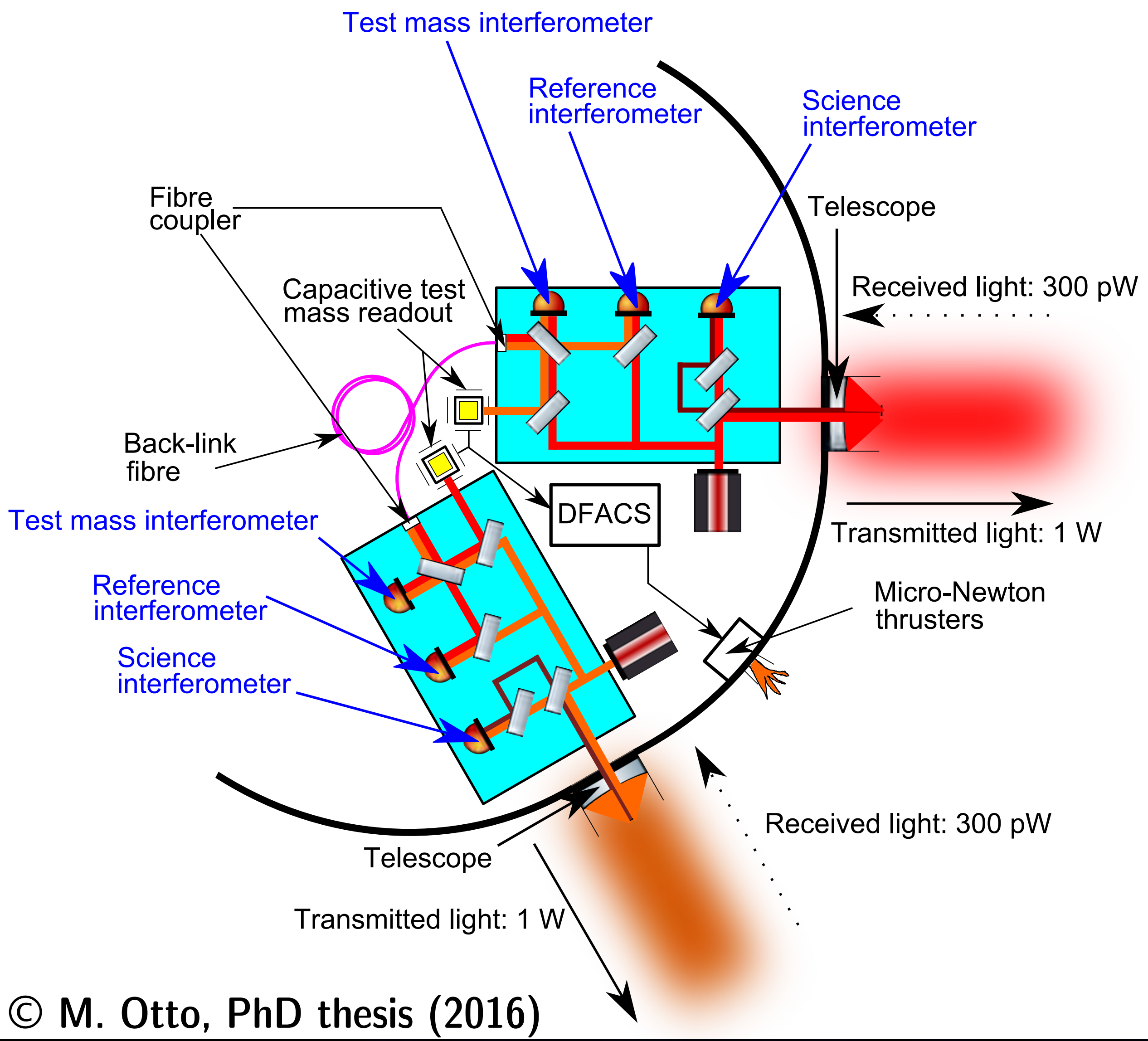
Phasemeter (cut off due to the filter required for digitalization of signal)

TDI Michelson

Petiteau & al, Phys. Rev. D (2008)

LISA measurements

- Exchange of laser beam to form several interferometers
- Phasemeter measurements on each of the 6 Optical Benches:
  - Distant OB vs local OB
  - Test-mass vs OB
  - Reference using adjacent OB
  - Transmission using sidebands
  - Distance between spacecrafts
- Noises sources:
  - Laser noise : 10^{-13} (vs 10^{-21})
  - Clock noise (3 clocks)
  - Acceleration noise (see LPF)
  - Read-out noises
  - Optical path noises



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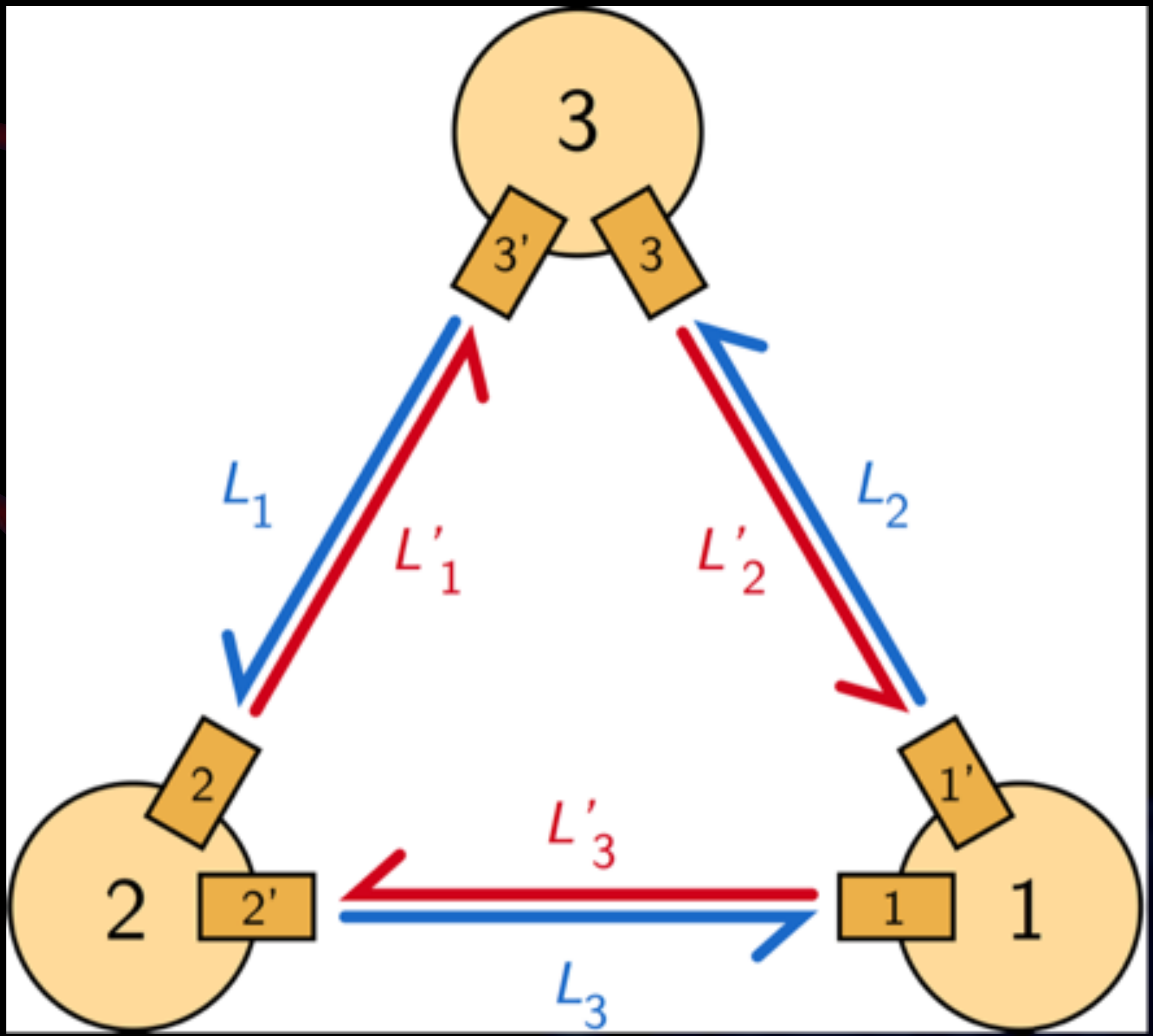
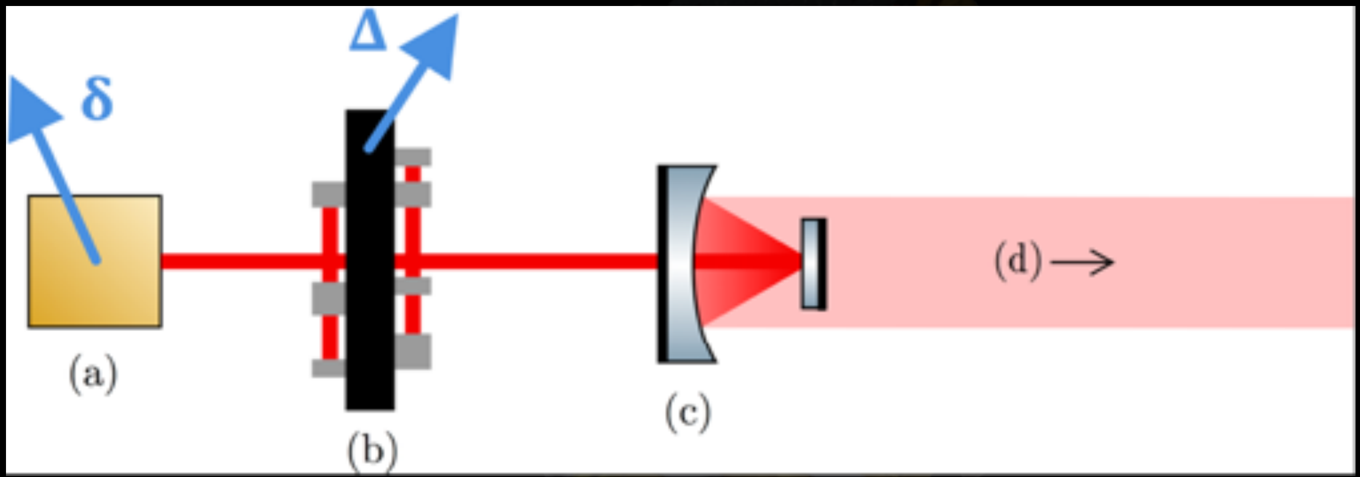
LISA measurements

- Configuration and conventions (from Jean-Baptiste Bayle's PhD)

(WARNING: not following the new LISA Consortium conventions)

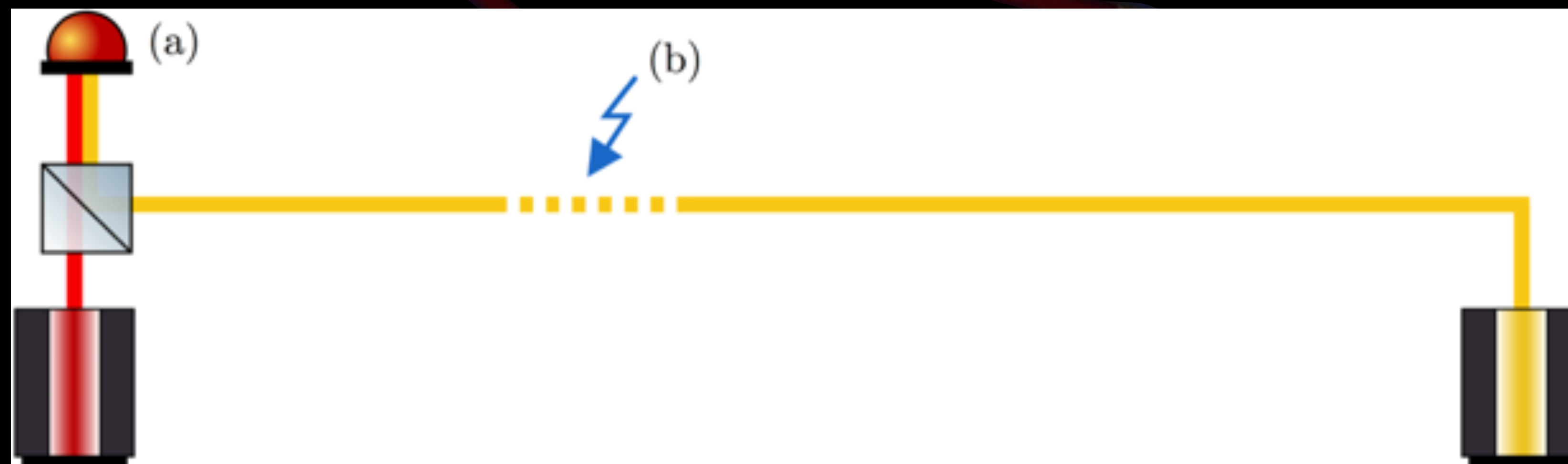




LISA measurements

- Long arm interferometer (IFO)

Local



Distant spacecraft

GW — laser noise — spacecraft jitter — readout noises

$$s_1 = \theta_1^s \left[H_1 + \mathbf{D}_3 p_{2'} - p_1 - \left(\hat{\mathbf{n}}_3 \cdot \mathbf{D}_3 \frac{\mathbf{v}_{\Delta_{2'}}}{c} + \hat{\mathbf{n}}_{3'} \cdot \frac{\mathbf{v}_{\Delta_1}}{c} \right) \right] + N_1^s,$$

$$s_{1'} = \theta_{1'}^s \left[H_{1'} + \mathbf{D}_{2'} p_3 - p_{1'} - \left(\hat{\mathbf{n}}_{2'} \cdot \mathbf{D}_{2'} \frac{\mathbf{v}_{\Delta_3}}{c} + \hat{\mathbf{n}}_2 \cdot \frac{\mathbf{v}_{\Delta_{1'}}}{c} \right) \right] + N_{1'}^s$$

delay operator: $D_i x = D_i x(t) = x\left(t - \frac{L_i}{c}\right)$

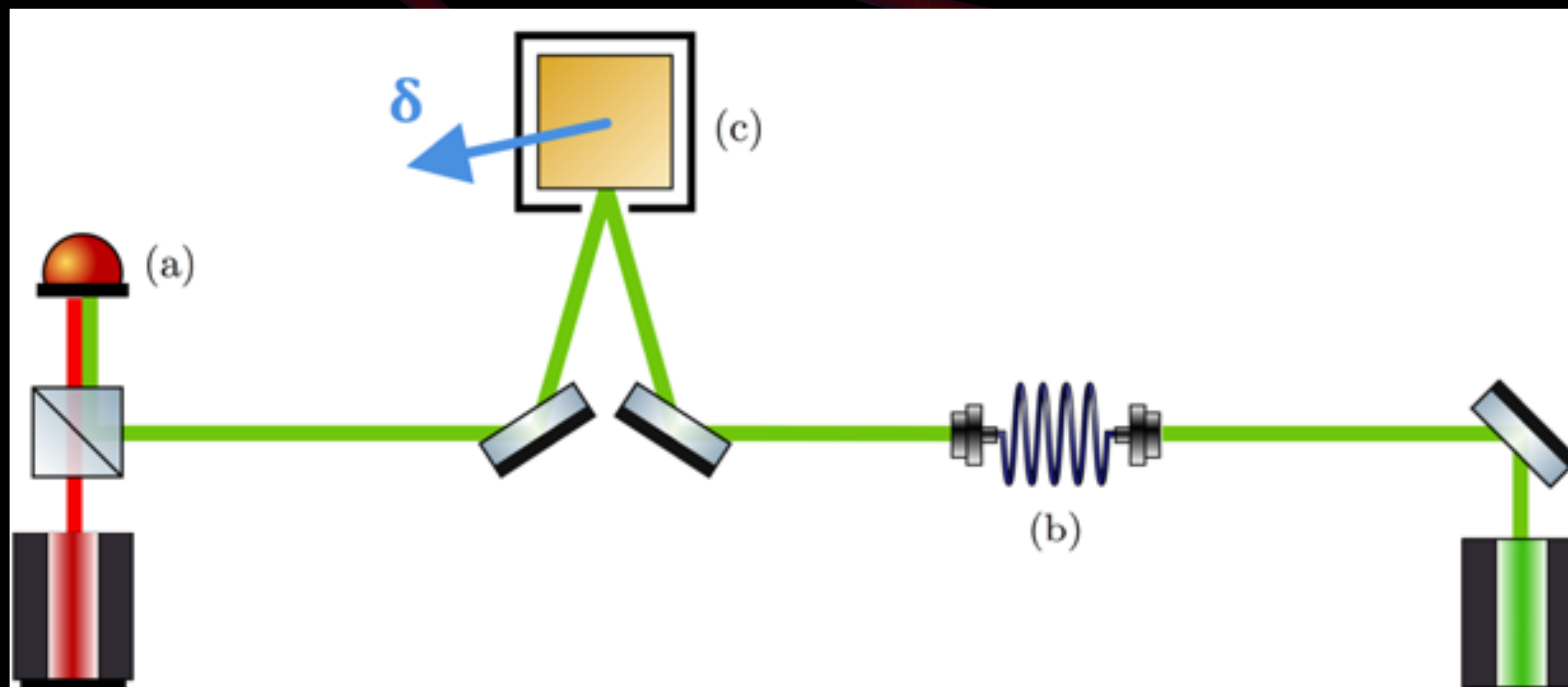
with the sign of the difference between frequencies

$$\theta_1^s = \text{sign}(\nu_{2'} - \nu_1) \quad \text{and} \quad \theta_{1'}^s = \text{sign}(\nu_3 - \nu_{1'})$$

LISA measurements

- Test-Mass Interferometer (IFO)

Local



Other MOSA on the same spacecraft

laser noise

spacecraft jitter

Test-Mass noises = LPF

$$\epsilon_1 = \theta_1^{\tau}\left[p_{1'} - p_1 + 2\hat{\mathbf{n}}_{3'} \cdot \left(\frac{\mathbf{v}_{\Delta_1}}{c} - \frac{\mathbf{v}_{\delta_1}}{c}\right) + \mu_{1'}\right] + N_1^{\epsilon}$$

$$\epsilon_{1'} = \theta_{1'}^{\tau}\left[p_1 - p_{1'} + 2\hat{\mathbf{n}}_2 \cdot \left(\frac{\mathbf{v}_{\Delta_{1'}}}{c} - \frac{\mathbf{v}_{\delta_{1'}}}{c}\right) + \mu_1\right] + N_{1'}^{\epsilon}$$

readout noises

optical fiber noises

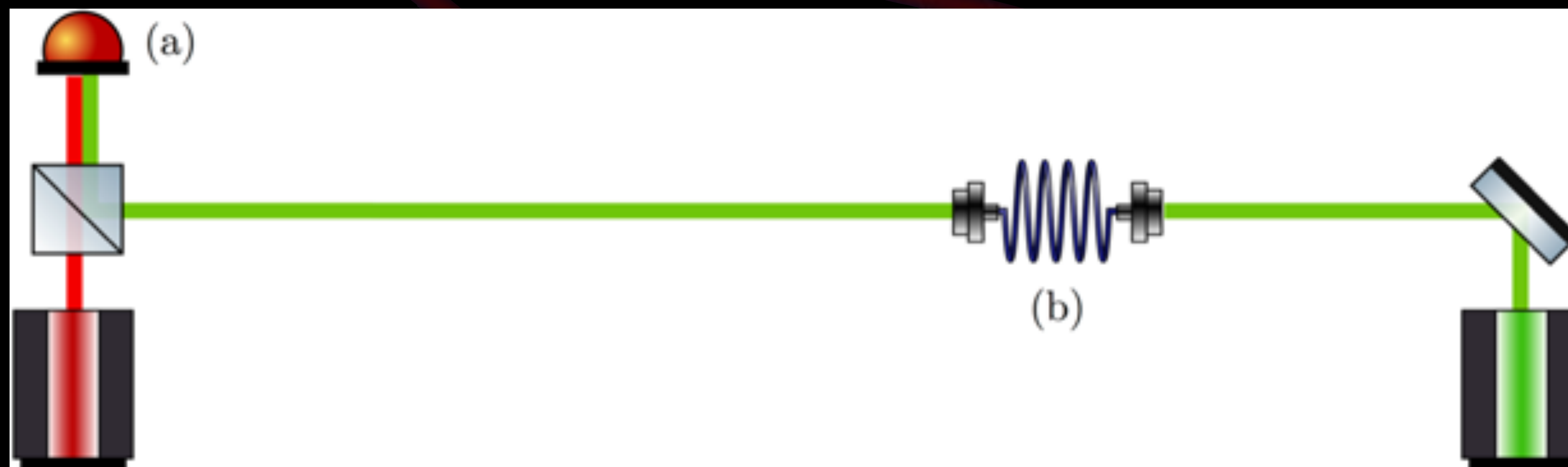
with the sign of the difference between frequencies

$$\theta_1^{\tau} = \mathrm{sign}(\nu_{1'} - \nu_1) \quad \text{and} \quad \theta_{1'}^{\tau} = \mathrm{sign}(\nu_1 - \nu_{1'}) = -\theta_1^{\tau},$$

LISA measurements

- Reference Interferometer (IFO)

Local



Other MOSA on the same spacecraft

laser noise

optical fiber noises

readout noises

$$\tau_1 = \theta_1^{\tau}[p_{1'} - p_1 + \mu_{1'}] + N_1^{\tau},$$
$$\tau_{1'} = \theta_{1'}^{\tau}[p_1 - p_{1'} + \mu_1] + N_{1'}^{\tau}.$$

LISA Measurements

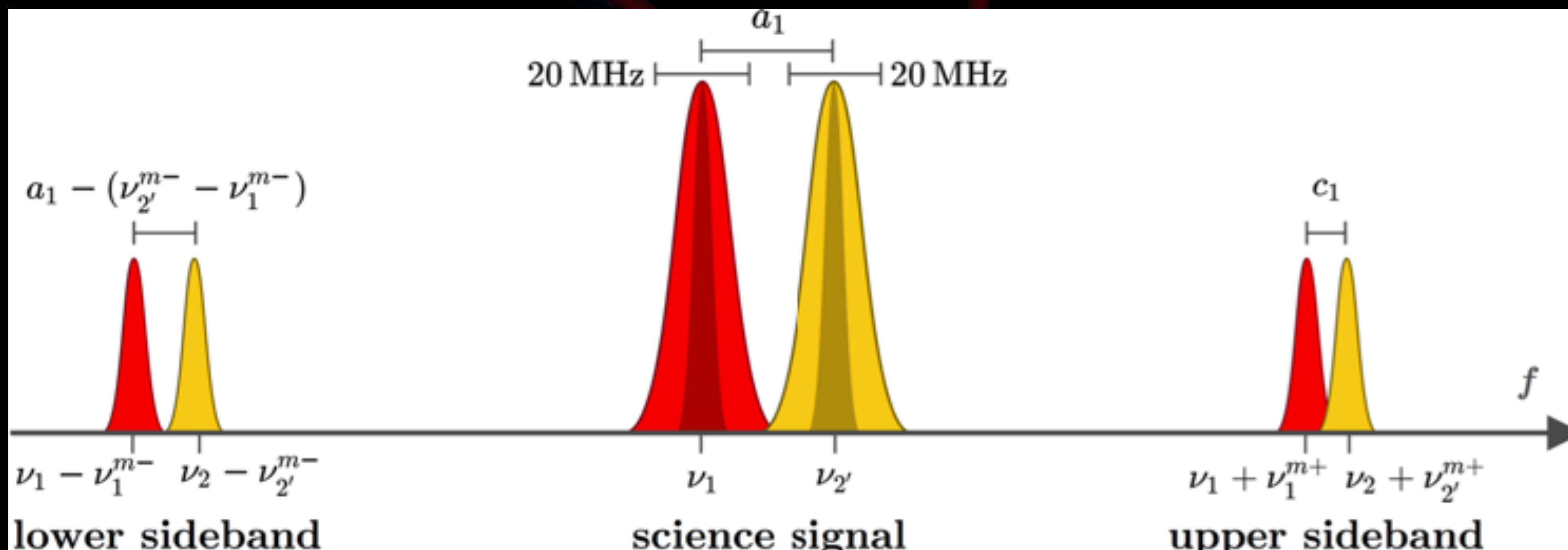
- With clock noises: 3 Ultra-Stable Oscillator (USO)
  - Pilot tone sampled by the ADC and subtracted from measurement to cancel ADC timing jitter
  - USO used to generate pilot tone
  - => USO noises in measurements as scaled by the relevant beat note frequency

$$s_1 = \theta^s_1\left[H_1 + \mathbf{D}_3 p_{2'} - p_1 - \left(\hat{\mathbf{n}}_3 \cdot \mathbf{D}_3 \frac{\mathbf{v}_{\Delta_{2'}}}{c} + \hat{\mathbf{n}}_{3'} \cdot \frac{\mathbf{v}_{\Delta_1}}{c}\right) - a_1 q_1\right] + N^s_1\,,$$
$$s_{1'} = \theta^s_{1'}\left[H_{1'} + \mathbf{D}_{2'} p_3 - p_{1'} - \left(\hat{\mathbf{n}}_{2'} \cdot \mathbf{D}_{2'} \frac{\mathbf{v}_{\Delta_3}}{c} + \hat{\mathbf{n}}_2 \cdot \frac{\mathbf{v}_{\Delta_{1'}}}{c}\right) - a_{1'} q_1\right] + N^s_{1'}$$
$$\epsilon_1 = \theta^\tau_1\left[p_{1'} - p_1 + 2\hat{\mathbf{n}}_{3'} \cdot \left(\frac{\mathbf{v}_{\Delta_1}}{c} - \frac{\mathbf{v}_{\delta_1}}{c}\right) + \mu_{1'} - b_1 q_1\right] + N^\epsilon_1\,,$$
$$\epsilon_{1'} = \theta^\tau_{1'}\left[p_1 - p_{1'} + 2\hat{\mathbf{n}}_2 \cdot \left(\frac{\mathbf{v}_{\Delta_{1'}}}{c} - \frac{\mathbf{v}_{\delta_{1'}}}{c}\right) + \mu_1 - b_{1'} q_1\right] + N^\epsilon_{1'}\,,$$
$$\tau_1 = \theta^\tau_1\left[p_{1'} - p_1 + \mu_{1'} - b_1 q_1\right] + N^\tau_1\,,$$
$$\tau_{1'} = \theta^\tau_{1'}\left[p_1 - p_{1'} + \mu_1 - b_{1'} q_1\right] + N^\tau_{1'}\,.$$

$$a_1 = \nu_{2'} - \nu_1$$
$$a_{1'} = \nu_3 - \nu_{1'}$$
$$b_1 = \nu_{1'} - \nu_1$$
$$b_{1'} = -b_1 = \nu_1 - \nu_{1'}$$

LISA measurements

- Side-band measurements on science interferometers
  - Amplification of clock noises on sidebands by factor m_i



$$s_1^{\mathrm{sb}} = \theta_1^s \Big[H_1 + \mathbf{D}_3 p_{2'} - p_1 + \mathbf{D}_3 m_{2'} q_2 - m_1 q_1 - \Big(\hat{\mathbf{n}}_3 \cdot \mathbf{D}_3 \frac{\mathbf{v}_{\Delta_{2'}}}{c} + \hat{\mathbf{n}}_{3'} \cdot \frac{\mathbf{v}_{\Delta_1}}{c}\Big) - c_1 q_1\Big] + N_1^{\mathrm{sb}},$$

$$s_{1'}^{\mathrm{sb}} = \theta_{1'}^s \Big[H_{1'} + \mathbf{D}_{2'} p_3 - p_{1'} + \mathbf{D}_{2'} m_3 q_3 - m_{1'} q_1 - \Big(\hat{\mathbf{n}}_{2'} \cdot \mathbf{D}_{2'} \frac{\mathbf{v}_{\Delta_3}}{c} + \hat{\mathbf{n}}_2 \cdot \frac{\mathbf{v}_{\Delta_{1'}}}{c}\Big) - c_{1'} q_1\Big] + N_{1'}^{\mathrm{sb}},$$

LISA measurements

- Phasemetre
  - Input data from quadrant photodiode at 80 MHz
  - Heterodyne interferometry
  - Phase Locked Loop
  - Output: phase and/or relative frequency fluctuation at 30 Hz

- Drag Free Attitude Control System
  - Use the output of the phasemeter: TM IFO & beam angles
  - To control the spacecraft motion and Test-Mass motion on all degrees of freedom except the sensitive one

LISA measurements

- On-board computer
  - Anti-aliasing filtering of the phase meter data then downsampling around few Hz for telemetry (2.5, 3.2, 4 or 5 Hz)
  - Strong contraints on the filter; example attenuation > 240 dB
  - Impact on the data: small delays added

$$s_1 = \mathcal{F}\left\{\theta_1^s\left[H_1 + \mathbf{D}_3 p_{2'} - p_1 - \left(\hat{\mathbf{n}}_3 \cdot \mathbf{D}_3 \frac{\mathbf{v}_{\Delta_{2'}}}{c} + \hat{\mathbf{n}}_{3'} \cdot \frac{\mathbf{v}_{\Delta_1}}{c}\right) - a_1 q_1\right] + N_1^s\right\}$$

$$s_{1'} = \mathcal{F}\left\{\theta_{1'}^s\left[H_{1'} + \mathbf{D}_{2'} p_3 - p_{1'} - \left(\hat{\mathbf{n}}_{2'} \cdot \mathbf{D}_{2'} \frac{\mathbf{v}_{\Delta_3}}{c} + \hat{\mathbf{n}}_2 \cdot \frac{\mathbf{v}_{\Delta_{1'}}}{c}\right) - a_{1'} q_1\right] + N_{1'}^s\right\}$$

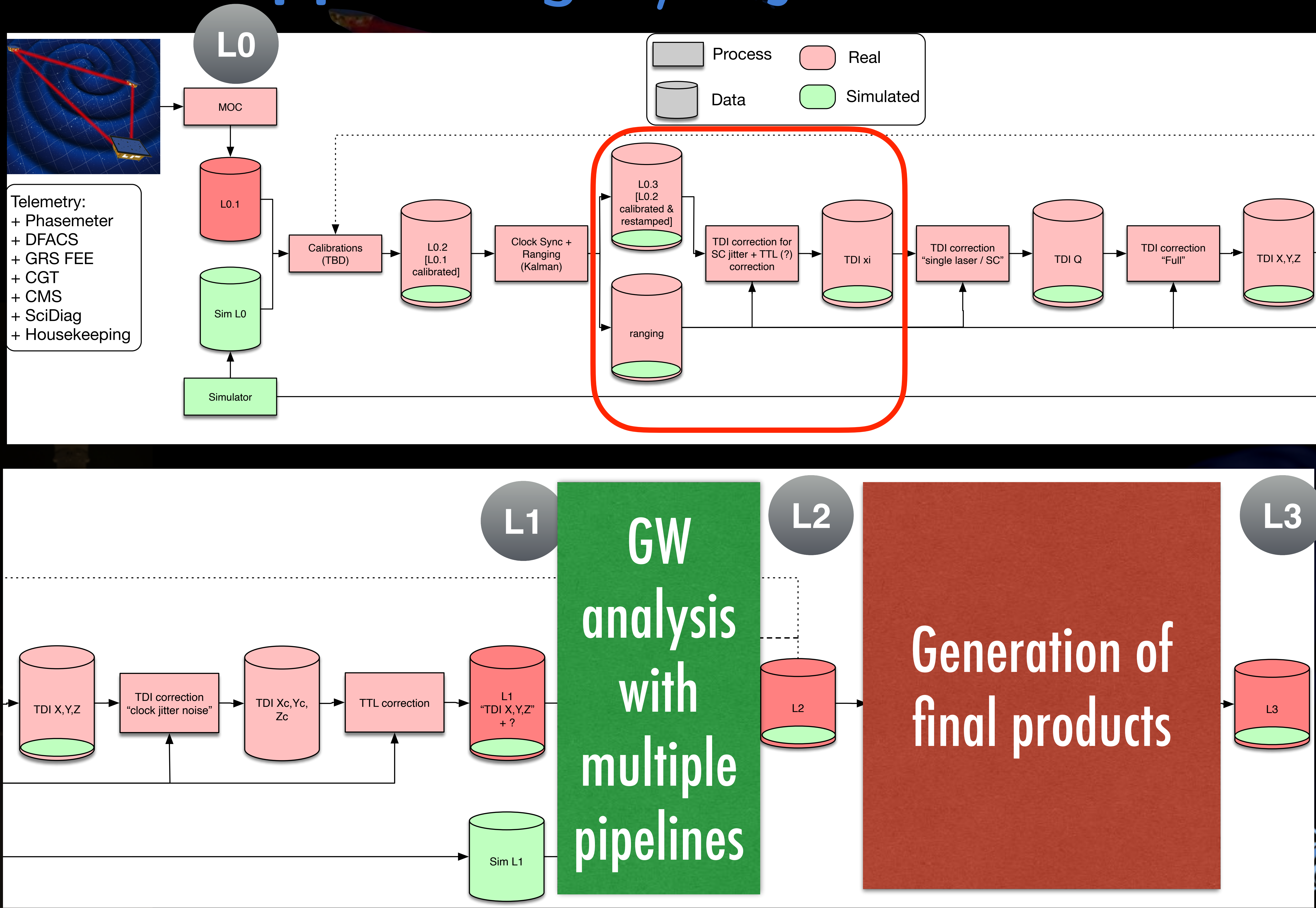
$$\epsilon_1 = \mathcal{F}\left\{\theta_1^\epsilon\left[p_{1'} - p_1 + 2\hat{\mathbf{n}}_{3'} \cdot \left(\frac{\mathbf{v}_{\Delta_1}}{c} - \frac{\mathbf{v}_{\delta_1}}{c}\right) + \mu_{1'} - b_1 q_1\right] + N_1^\epsilon\right\}$$

$$\epsilon_{1'} = \mathcal{F}\left\{\theta_{1'}^\epsilon\left[p_1 - p_{1'} + 2\hat{\mathbf{n}}_2 \cdot \left(\frac{\mathbf{v}_{\Delta_{1'}}}{c} - \frac{\mathbf{v}_{\delta_{1'}}}{c}\right) + \mu_1 - b_{1'} q_1\right] + N_{1'}^\epsilon\right\}$$

$$\tau_1 = \mathcal{F}\{\theta_1^\tau[p_{1'} - p_1 + \mu_{1'} - b_1 q_1] + N_1^\tau\}$$

$$\tau_{1'} = \mathcal{F}\{\theta_{1'}^\tau[p_1 - p_{1'} + \mu_1 - b_{1'} q_1] + N_{1'}^\tau\}$$

Suppressing S/C jitter noise



Suppressing S/C jitter noise

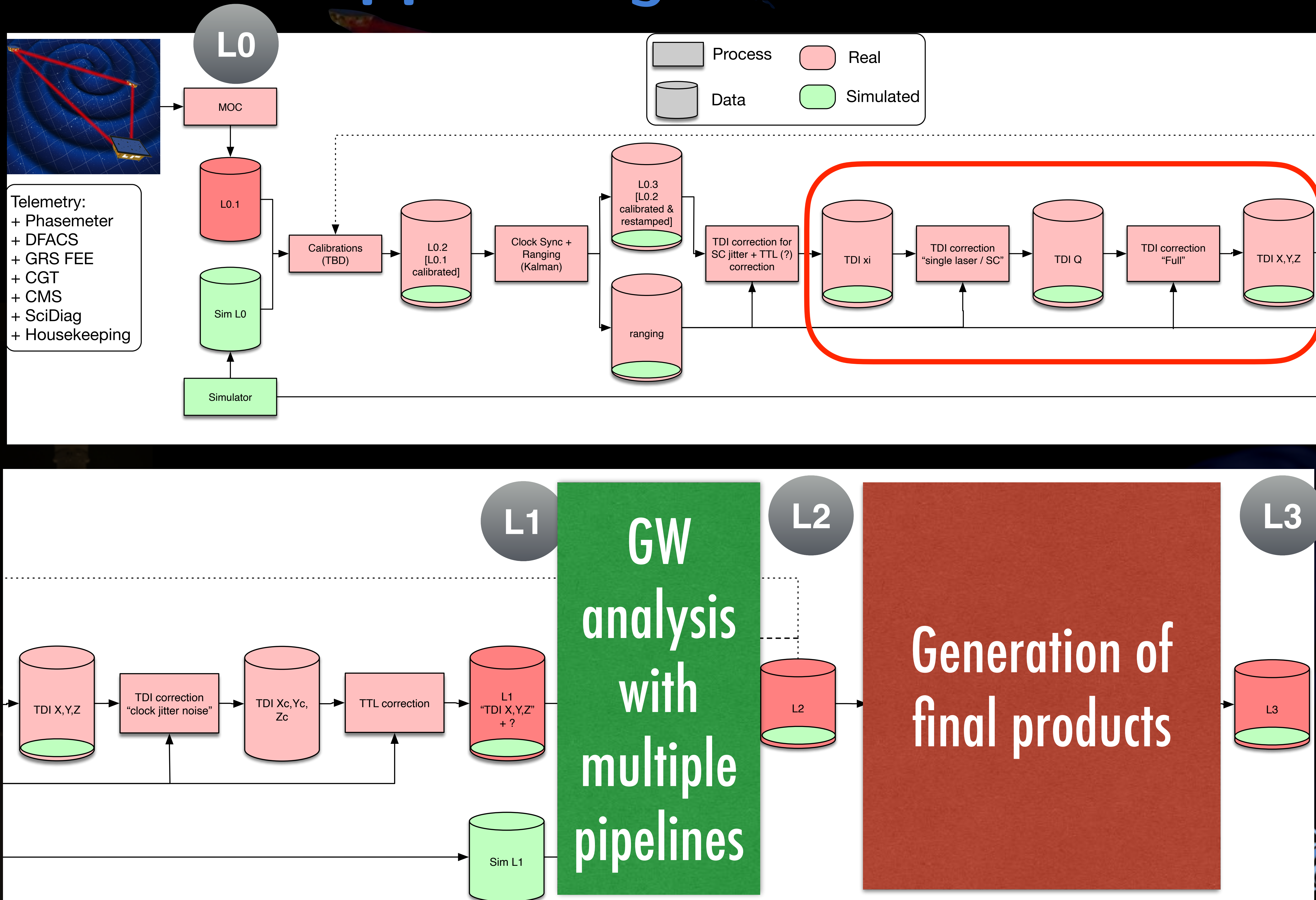
- TDI step 1: removing spacecraft jitter
  - Extract the spacecraft jitter by subtracting reference IFO from Test-Mass IFO
  - Then correcting the science IFO

$$\xi_1 = s_1 + \theta^s_1 \theta^\tau_1 \frac{\epsilon_1 - \tau_1}{2} + \theta^s_1 \theta^\tau_{2'} \frac{\mathcal{D}_3(\epsilon_{2'} - \tau_{2'})}{2},$$
$$\xi_{1'} = s_{1'} + \theta^s_{1'} \theta^\tau_{1'} \frac{\epsilon_{1'} - \tau_{1'}}{2} + \theta^s_{1'} \theta^\tau_3 \frac{\mathcal{D}_{2'}(\epsilon_3 - \tau_3)}{2}$$

  - Residual spacecraft jitter :

$$\xi^\Delta_1 = \theta^s_1 (\mathcal{F}\mathbf{D}_3 - \mathcal{D}_3\mathcal{F})(\hat{\mathbf{n}}_3 \cdot \mathbf{v}_{\Delta_{2'}}),$$
$$\xi^\Delta_{1'} = \theta^s_{1'} (\mathcal{D}_{2'}\mathcal{F} - \mathcal{F}\mathbf{D}_{2'})(\hat{\mathbf{n}}_{2'} \cdot \mathbf{v}_{\Delta_3}).$$

Suppressing laser noise



Suppressing laser noise

- TDI step 2: removing half of the laser noise
  - Combining reference IFOs and combining with the result of the previous TDI step

$$\eta_1 = \theta_1^s \xi_1 + \frac{\theta_{2'}^T \mathcal{D}_3 (\tau_{2'} + \tau_2)}{2},$$
$$\eta_{1'} = \theta_{1'}^s \xi_{1'} + \frac{\theta_1^T (\tau_{1'} + \tau_1)}{2}.$$

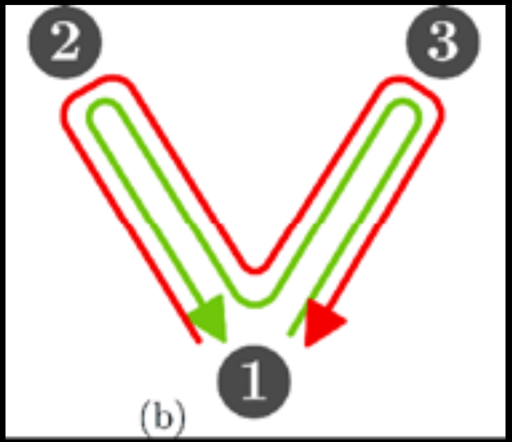
$$\tau_1 = \mathcal{F}\{\theta_1^T [p_{1'} - p_1 + \mu_{1'} - b_1 q_1] + N_1^T\}$$
$$\tau_{1'} = \mathcal{F}\{\theta_{1'}^T [p_1 - p_{1'} + \mu_1 - b_{1'} q_1] + N_{1'}^T\}$$

=> Cancel all $p_{i'}$

=> Situation equivalent of having one laser per spacecraft

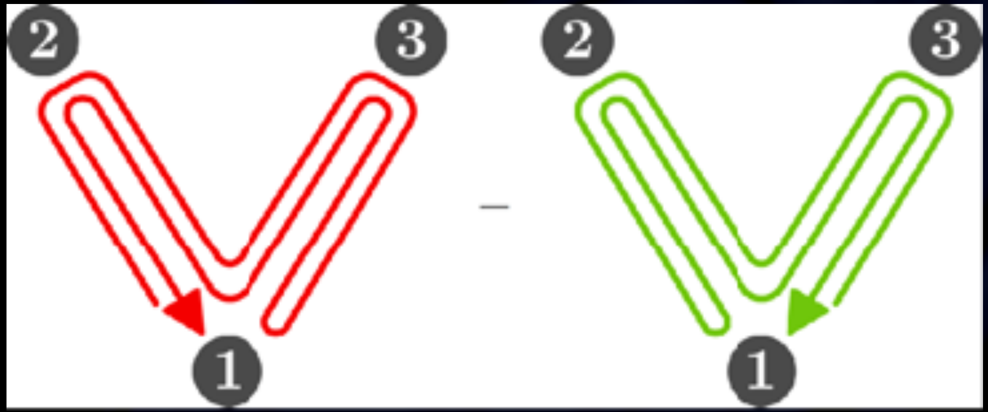
Suppressing laser noise

- Reducing all laser noises
  - Several complex combinations
  - Can be seen as virtual interferometer
  - TDI generation 1.5 takes into account the unequal arms



$$X_1 = \eta_{1'} + \mathcal{D}_{2'}\eta_3 + \mathcal{D}_{2'2}\eta_1 + \mathcal{D}_{2'23}\eta_{2'} - \eta_1 - \mathcal{D}_3\eta_{2'} - \mathcal{D}_{33'}\eta_{1'} - \mathcal{D}_{33'2'}\eta_3$$

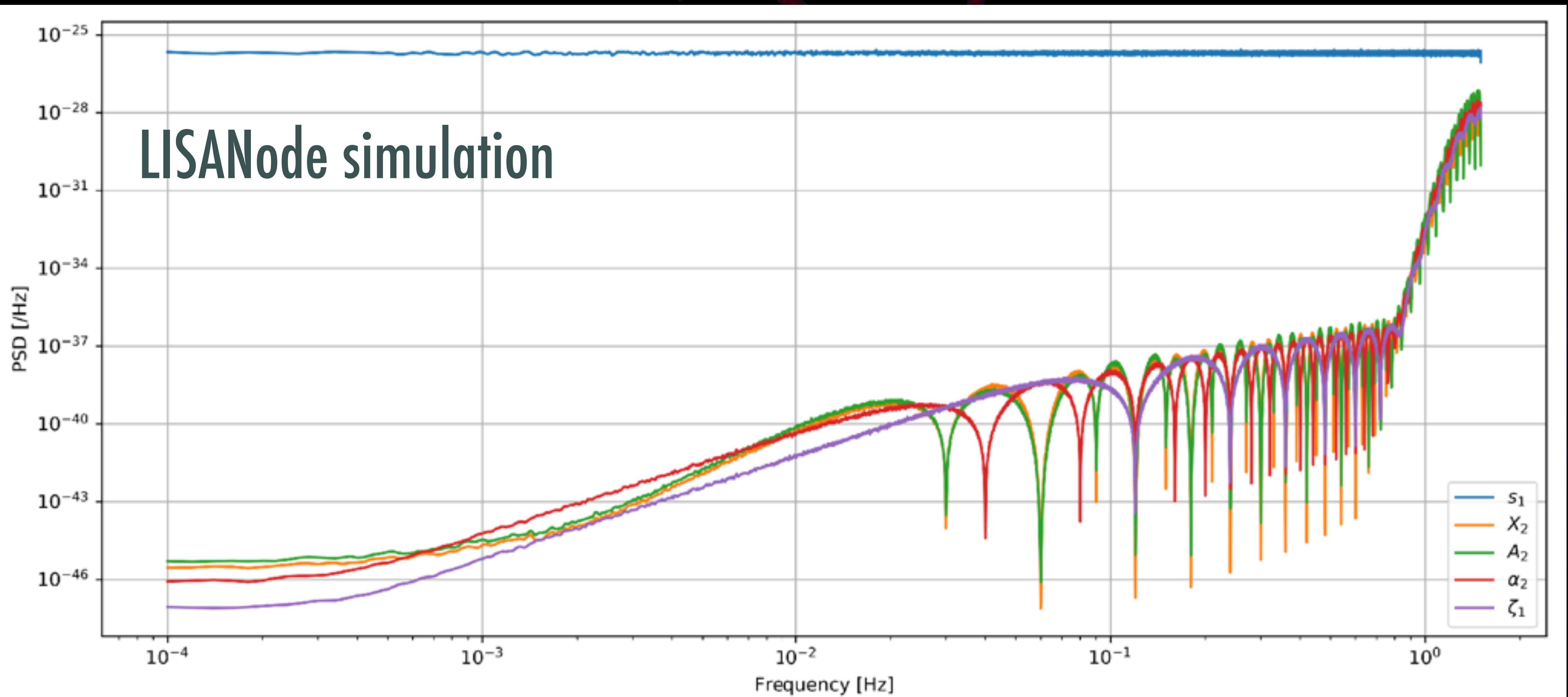
  - TDI generation 2 takes into account first order time evolution of arm length



$$\begin{aligned} X_2 = \eta_{1'} &+ \mathcal{D}_{2'}\eta_3 + \mathcal{D}_{2'2}\eta_1 - \mathcal{D}_{2'23}\eta_{2'} + \mathcal{D}_{2'233'}\eta_1 \\ &+ \mathcal{D}_{2'233'3}\eta_{2'} + \mathcal{D}_{2'233'33'}\eta_{1'} + \mathcal{D}_{2'233'33'2'}\eta_3 \\ &- \eta_1 - \mathcal{D}_3\eta_{2'} - \mathcal{D}_{33'}\eta_{1'} - \mathcal{D}_{33'2'}\eta_3 - \mathcal{D}_{33'2'2}\eta_{1'} \\ &- \mathcal{D}_{33'2'22'}\eta_3 - \mathcal{D}_{33'2'22'2}\eta_1 - \mathcal{D}_{33'2'22'23}\eta_{2'} \,. \end{aligned}$$

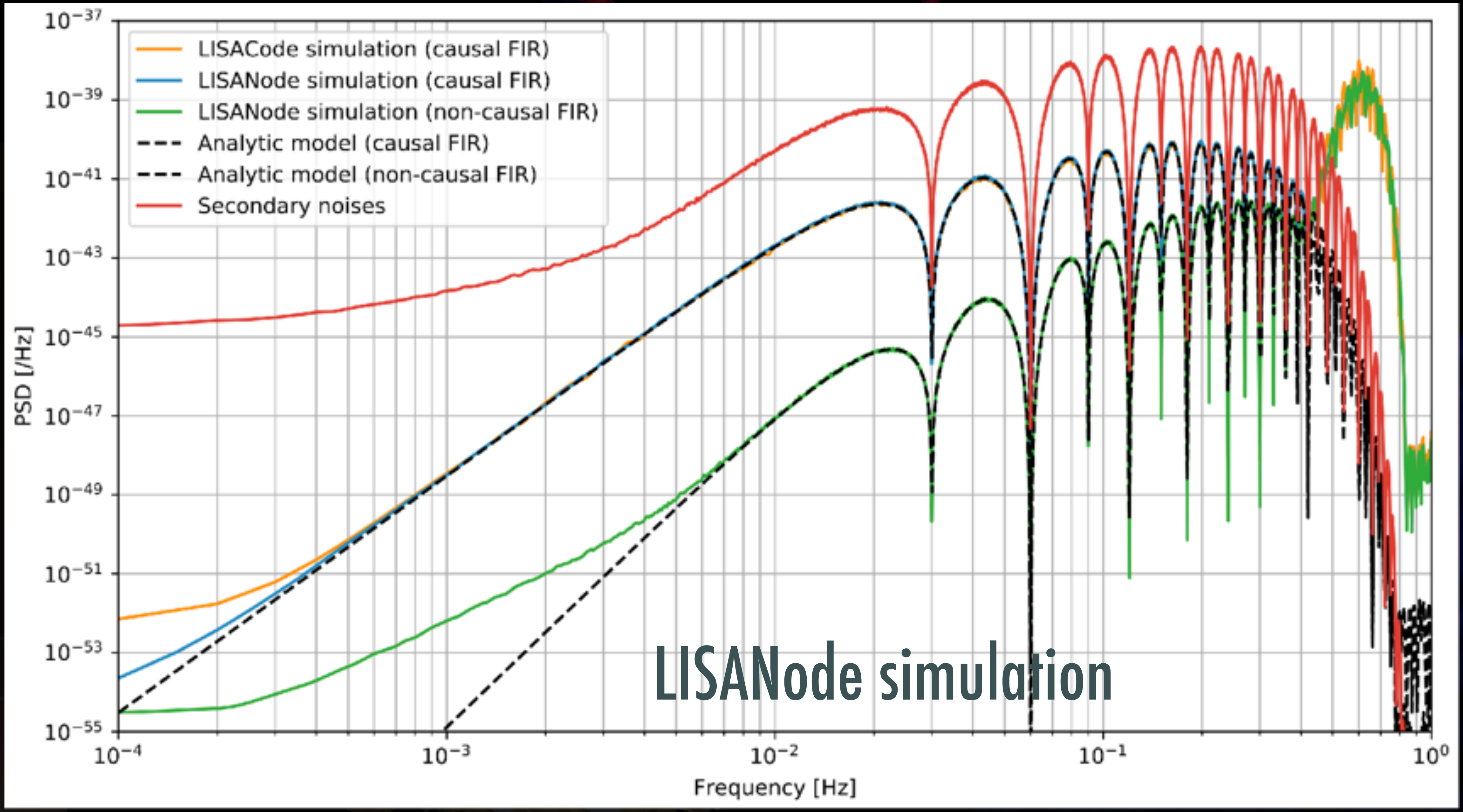
Noises after TDI

- Assuming no clock noise and perfect ranging, laser noise and spacecraft jitter are reduced below other noises, i.e. acceleration and readout (here all IFO noises)



Flexing-filtering effect

- Coupling between filter and delay J.-B. Bayle et al., PRD (2019)
  - => residual laser noise depend on the filter



Flexing-filtering effect

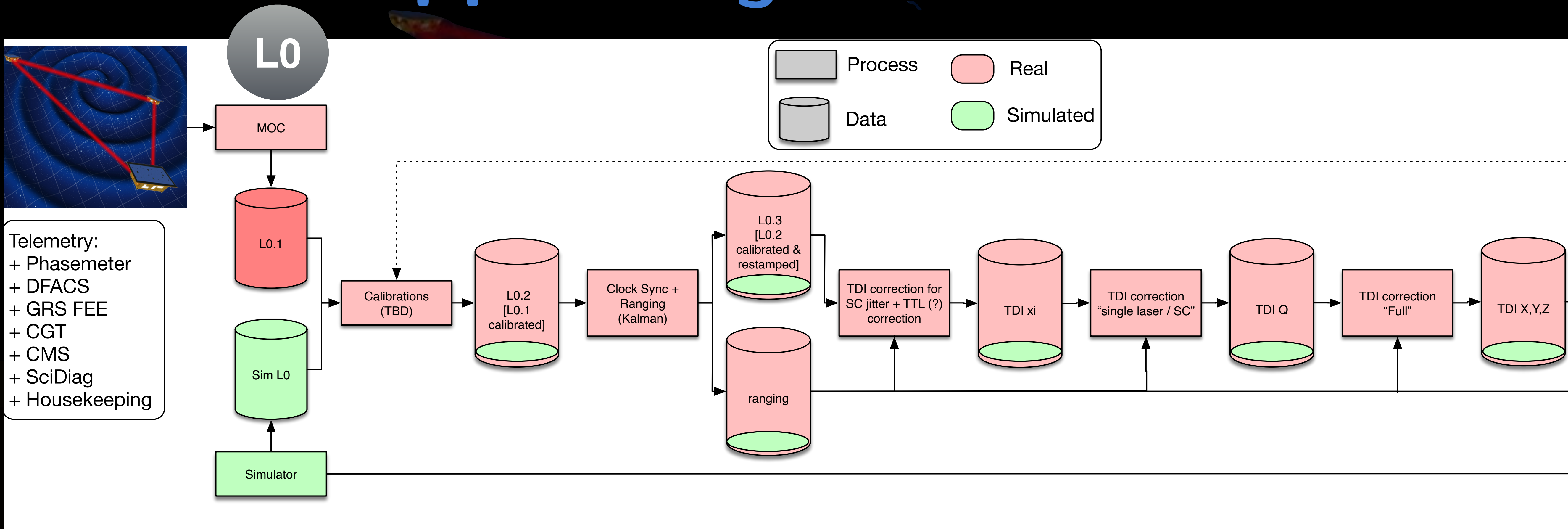
- Analytic approximation (linear delay):

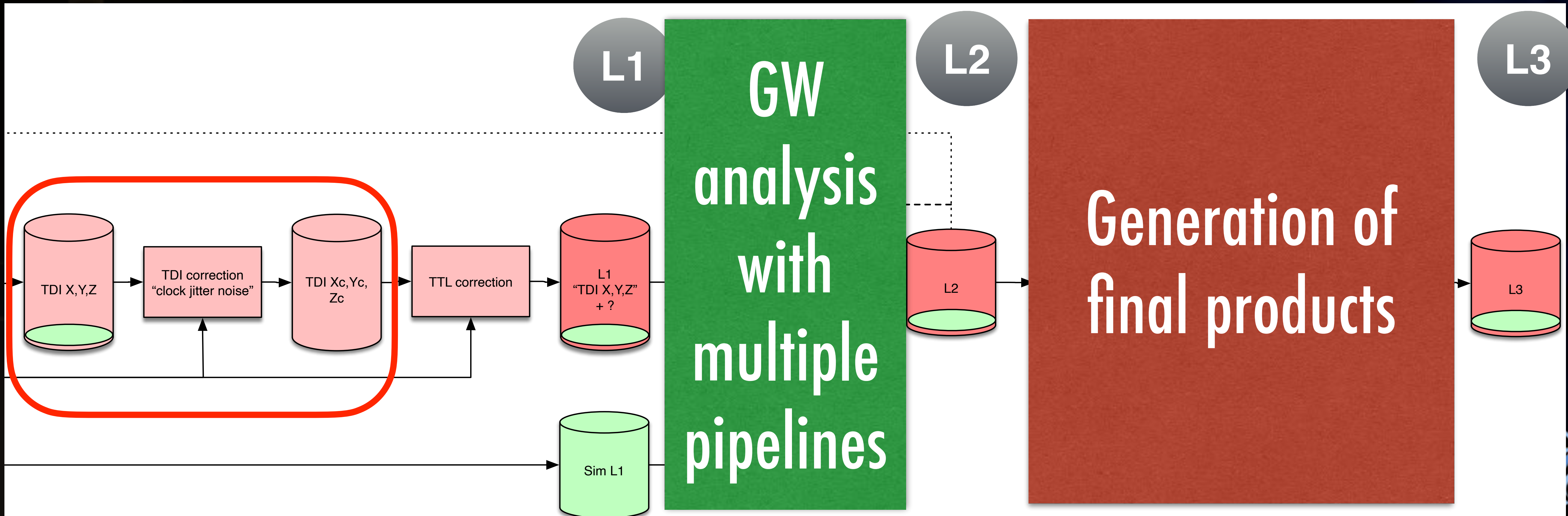
$$S_{X_2}(\omega) \approx 32 S_p \omega^2 \sin^2(\omega L) \sin^2(2\omega L) \left(\dot{L_2}^2 + \dot{L_3}^2\right) K_{\mathcal{F}}(\omega)$$

  - with:
    - L : travel time along arms
    - S_p : pre-stabilised laser noise
    - filter term: $$K_{\mathcal{F}}(\omega) = 4 f_s^{-2} \left| \sum_{k=1}^{N} k \alpha_{N+k} \sin\left(\frac{k\omega}{f_s}\right) \right|^2$$
    - α_k : the 2 N + 1 filter coefficients of the FIR filter
    - Filter output are $$y_n = \sum_{k=0}^{2N} \alpha_k x_{n-k}$$

J.-B. Bayle et al., PRD (2019)

Suppressing clock noise





Suppressing clock noise

Hartwig et Bayle, arXiv:2005.02430

- Use the sideband signal where the clock noise is amplified to build a calibration signal similar to TDI but clock noise only then subtract it from the standard TDI to form the corrected TDI.

$$\mathrm{TDI}^c = \mathrm{TDI} - \left(\sum_{i=1}^{3} - (a_i + b_{(i+1)'}) R_i - (a_{i'} - b_{i'}) R_{i'} \right)$$

with

$$r_1 = \theta_1^s \frac{s_1 - s_1^{\mathrm{sb}}}{\nu_{2'}^m}, \quad r_{1'} = \theta_{1'}^s \frac{s_{1'} - s_{1'}^{\mathrm{sb}}}{\nu_3^m}.$$

R_i and $R_{i'}$ combination of r_i and $r_{i'}$ such that $R_i = P_i q_i$ & $R_{i'} = P_{i'} q_i$ with P_i and $P_{i'}$ polynomial of delays.

- Noise level:

$$S_{X_1^{c,\mathcal{F}}}(\omega) \approx 4\omega^2((A_1^2 + b_{1'}^2)\dot{L}_2^2 + a_1^2 \dot{L}_3^2) \times \sin^2(\omega L) K_f(\omega) S_q(\omega),$$

$$S_{X_2^{c,\mathcal{F}}}(\omega) \approx 4\sin^2(2\omega L) S_{X_1^{c,\mathcal{F}}}(\omega)$$

Suppressing modulation noise

Hartwig et Bayle, arXiv:2005.02430

- M_i are additional deviations present in the sidebands relative to that same pilot tone, e.g., due to noise added by the electro-optical modulators or the fibre amplifiers.

$$
\begin{aligned}
s_1^{\text{sb}} &= \mathcal{F}\big[\theta_1^s\big(\nu_{2'}^m \mathcal{D}_3(q_2 + M_{2'}) \\
&\quad - \nu_1^m(q_1 + M_1) \\
&\quad - (a_1 + \nu_{2'}^m - \nu_1^m)q_1\big) + N_1^{\text{sb}}\big], \\
s_{1'}^{\text{sb}} &= \mathcal{F}\big[\theta_{1'}^s\big(\nu_3^m \mathcal{D}_{2'}(q_3 + M_3) \\
&\quad - \nu_{1'}^m(q_1 + M_{1'}) \\
&\quad - (a_{1'} + \nu_3^m - \nu_{1'}^m)q_1\big) + N_{1'}^{\text{sb}}\big]
\end{aligned}
$$

$$
\begin{aligned}
\tau_1^{\text{sb}} &= \mathcal{F}\big[\theta_1^\tau\big(\nu_{1'}^m(q_1 + M_{1'}) \\
&\quad - \nu_1^m(q_1 + M_1) \\
&\quad - (b_1 + \nu_{1'}^m - \nu_1^m)q_1\big) + N_1^{\text{sb},\tau}\big] \\
\tau_{1'}^{\text{sb}} &= \mathcal{F}\big[\theta_{1'}^\tau\big(\nu_1^m(q_1 + M_1) \\
&\quad - \nu_{1'}^m(q_1 + M_{1'}) \\
&\quad - (b_{1'} + \nu_1^m - \nu_{1'}^m)q_1\big) + N_{1'}^{\text{sb},\tau}\big]
\end{aligned}
$$

- The EOM imprinting the clock noise on side band are working at different frequency on primed and unprimed OB, for example M_i at 2.4 GHz and $M_{i'}$ at 2.401 GHz.

Suppressing modulation noise

Hartwig et Bayle, arXiv:2005.02430

- To correct at the first order the modulation error, we form new quantities:

$$\Delta M_1 = \theta^{\tau}_{1'} \left[\frac{\tau^{\text{sb}}_1 - \tau_1}{2} + \frac{\tau^{\text{sb}}_{1'} - \tau_{1'}}{2} \right]$$

$$r_1 = \theta^s_1 \frac{s_1 - s^{\text{sb}}_1}{\nu^m_{2'}} ,$$
$$r_{1'} = \theta^s_{1'} \frac{s_{1'} - s^{\text{sb}}_{1'}}{\nu^m_3} .$$

$$r^c_1 = r_1 - \frac{\mathcal{D}_3 \Delta M_2}{\nu^m_{2'}}$$
$$r^c_{1'} = r_{1'} + \frac{\Delta M_1}{\nu^m_3} .$$

- Then subtract a polynomial combination to the standard TDI to form the corrected TDI

$$\mathrm{TDI}^c = \mathrm{TDI} - \left(\sum_{i=1}^{3} b_{(i+1)'} P_i r^c_i - (a_i + b_{(i+1)'}) R_i - (a_{i'} - b_{i'}) R_{i'} \right)$$

- Noise level:

$$S_{X^M_1}(\omega) \approx 4 \sin^2(\omega L) \left| \tilde{f}(\omega) \right|^2 S_M(\omega) \times \left[(a_1 - a_{1'})^2 + a^2_{2'} + a^2_3 + 4 b_{1'} (a_1 - a_{1'} + b_{1'}) \sin^2(\omega L) \right]$$

$$S_{X^M_2}(\omega) \approx 4 \sin^2(2\omega L) S_{X^M_1}(\omega) ,$$

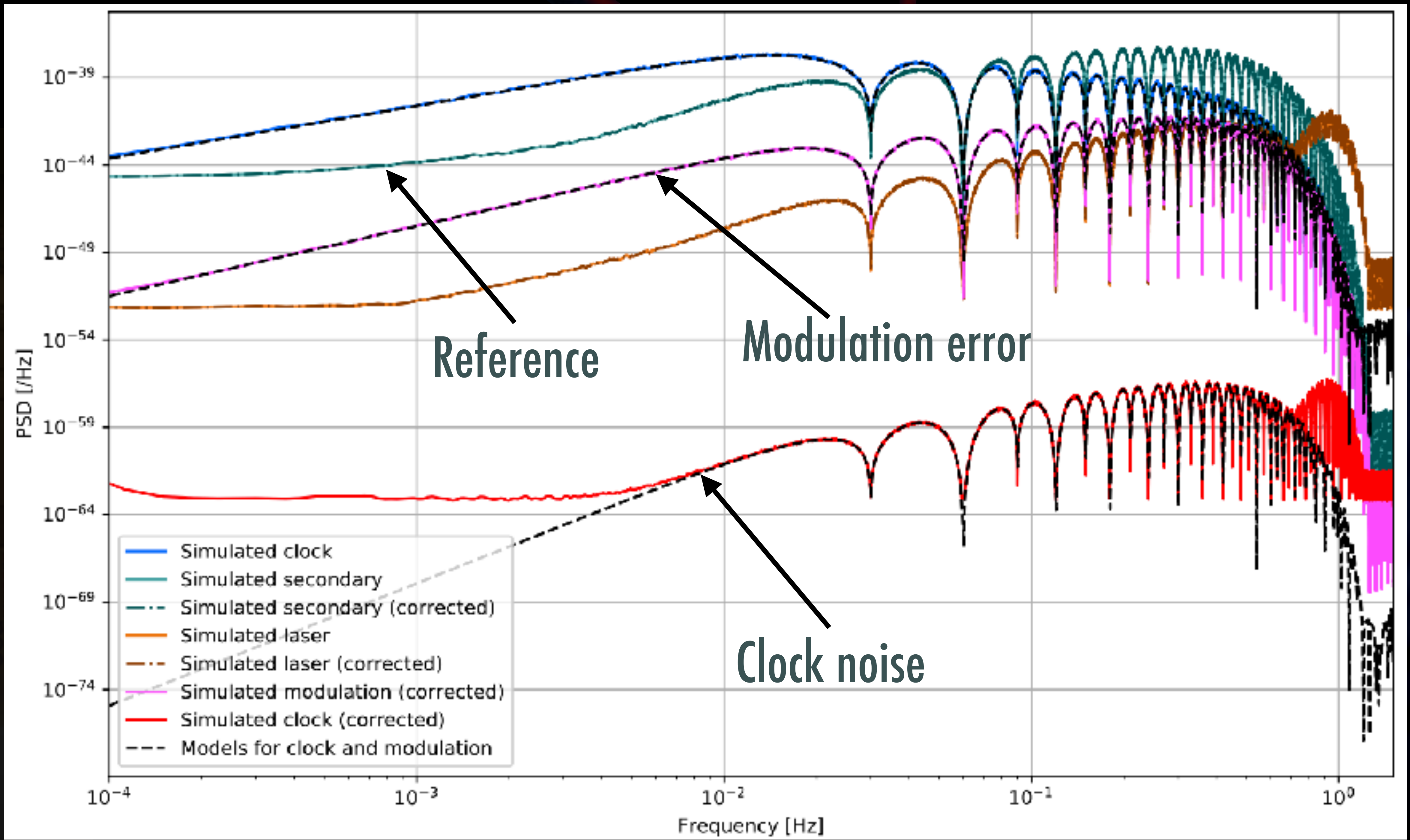
with

$$\left| \tilde{f}(\omega) \right|^2 = \left| \sum_{k=0}^{2N} \alpha_k \exp^{-jk\omega/f_s} \right|^2$$

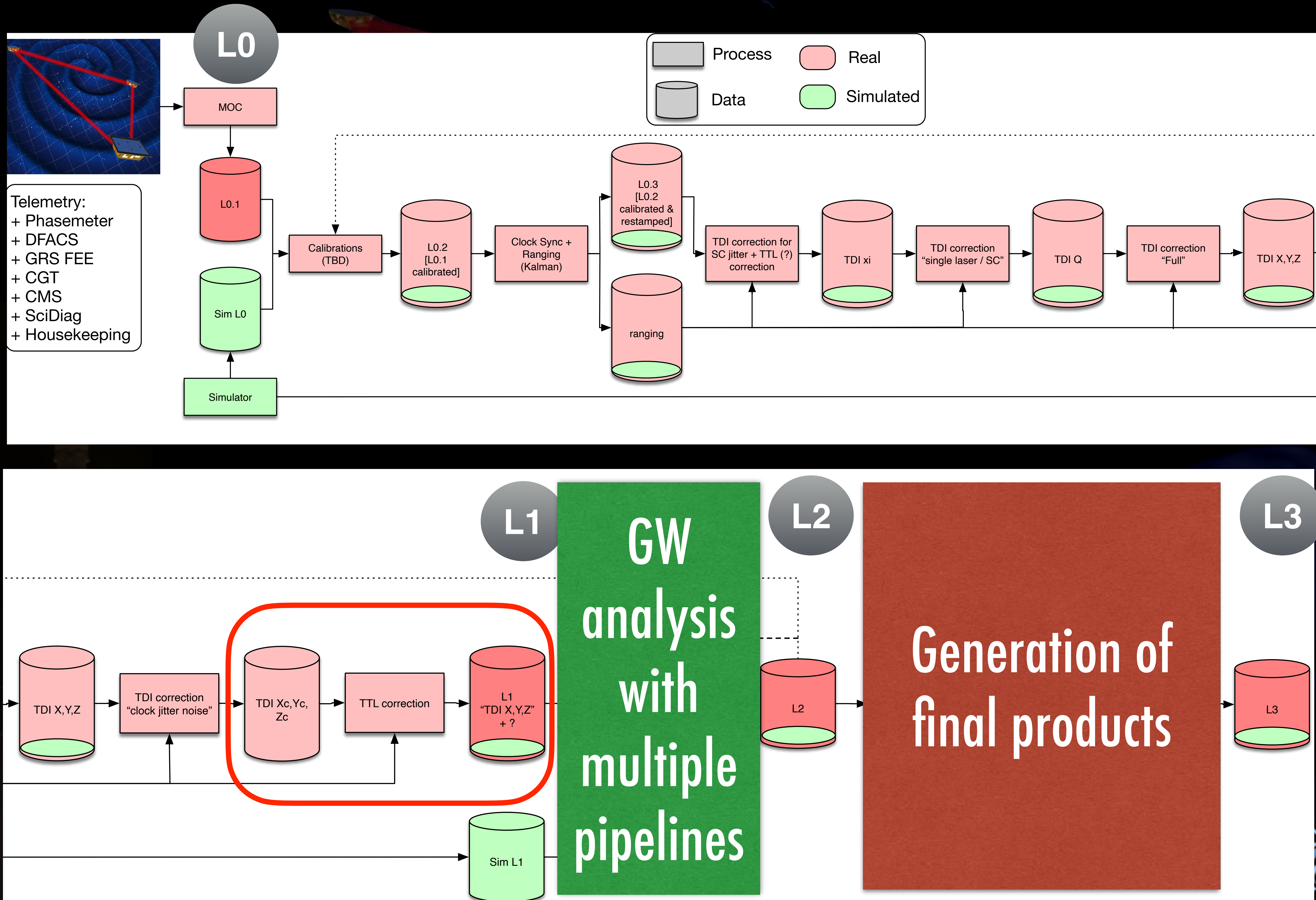
Suppressing clock noise

Hartwig et Bayle, arXiv:2005.02430

- Results: clock noise well suppressed but modulation residual not negligible



TTL correction



TTL correction

- TTL: coupling between rotation of beam with respect to the optical system and longitudinal phase measured by IFO
- See Hubert's slides
- Subtraction of correlation with pointing noise measured with DWS => done on ground after TDI
- Under development
- Preliminary noise budget existing (see dedicated TN by Ewan Fitzimons and performance TN)

TDI generators

- With 6 links, there is a large numbers of possible TDI combinations suppressing laser noise: generators
- X, Y, Z: Michelson equivalent
- α, β, γ : Sagnac

$$\alpha_1 = \eta_1 - \eta_{1'} + \mathcal{D}_3\eta_2 - \mathcal{D}_{2'1'}\eta_{2'} + \mathcal{D}_{31}\eta_3 - \mathcal{D}_{2'}\eta_{3'}$$

$$\begin{aligned}\alpha_2 = &(1 - \mathcal{D}_{2'1'3'})\eta_1 - (1 - \mathcal{D}_{312})\eta_{1'} + (1 - \mathcal{D}_{2'1'3'})\mathcal{D}_3\eta_2 \\ &- (1 - \mathcal{D}_{312})\mathcal{D}_{2'1'}\eta_{2'} + (1 - \mathcal{D}_{2'1'3'})\mathcal{D}_{31}\eta_3 - (1 - \mathcal{D}_{312})\mathcal{D}_{2'}\eta_{3'}\end{aligned}$$

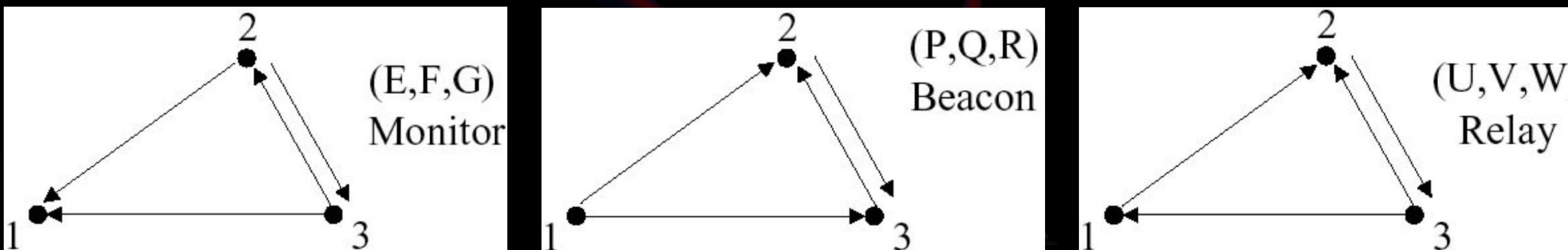
- ζ : fully symmetric Sagnac: senses the constellation rotation, & combines all interferometric signals with exactly one delay

$$\zeta_1 = \mathcal{D}_1\eta_1 + \mathcal{D}_2\eta_2 + \mathcal{D}_3\eta_3 - \mathcal{D}_{1'}\eta_{1'} - \mathcal{D}_{2'}\eta_{2'} - \mathcal{D}_{3'}\eta_{3'}$$

$$\begin{aligned}\zeta_2 = &(\mathcal{D}_{11'} - \mathcal{D}_{2'3'1'})\eta_1 + (\mathcal{D}_{1'2'} - \mathcal{D}_{322'})\eta_2 \\ &+ (\mathcal{D}_{13} - \mathcal{D}_{2'3'3})\eta_3 - (\mathcal{D}_{1'1} - \mathcal{D}_{321})\eta_{1'} \\ &- (\mathcal{D}_{1'2'} - \mathcal{D}_{322'})\eta_{2'} - (\mathcal{D}_{13} - \mathcal{D}_{2'3'3})\eta_{3'}\end{aligned}$$

TDI generators

- Specific generators when 2 links are missing



- TDI 1.5:

$$
\begin{aligned}
P_1 &= -(\mathcal{D}_2 - \mathcal{D}_{3'1'})\eta_2 + (\mathcal{D}_2 - \mathcal{D}_{11'2})\eta_{2'} - (\mathcal{D}_{3'} - \mathcal{D}_{11'3'})\eta_3 + (\mathcal{D}_{3'} - \mathcal{D}_{21})\eta_{3'} \,, \\
E_1 &= -(1 - \mathcal{D}_{11'})\eta_1 + (1 - \mathcal{D}_{11'})\eta_{1'} - (\mathcal{D}_3 - \mathcal{D}_{2'1'})\eta_2 + (\mathcal{D}_{2'} - \mathcal{D}_{31})\eta_{3'} \,, \\
U_1 &= -(\mathcal{D}_{3'} - \mathcal{D}_{11'3'})\eta_{1'} + (1 - \mathcal{D}_{3'2'1'})\eta_2 - (1 - \mathcal{D}_{11'})\eta_{2'} + (\mathcal{D}_1 - \mathcal{D}_{3'2'})\eta_{3'} \,.
\end{aligned}
$$

- TDI 2.0 in Hartwig et Bayle, arXiv:2005.02430

TDI generators

- Noise uncorrelated generators:
  - Form a basis of TDI generators by diagonalising the noise correlation matrix

$$\begin{pmatrix} S_{XX} & S_{XY} & S_{XZ} \\ S_{XY} & S_{YY} & S_{YZ} \\ S_{XZ} & S_{YZ} & S_{ZZ} \end{pmatrix}$$

  - If all the noise of the same type are at the same level, identical PSD, $S_{XX} = S_{YY} = S_{ZZ}$, and CSD, $S_{XY} = S_{XZ} = S_{YZ}$

$$\begin{pmatrix} S_{XX} & S_{XY} & S_{XY} \\ S_{XY} & S_{XX} & S_{XY} \\ S_{XY} & S_{XY} & S_{XX} \end{pmatrix}$$

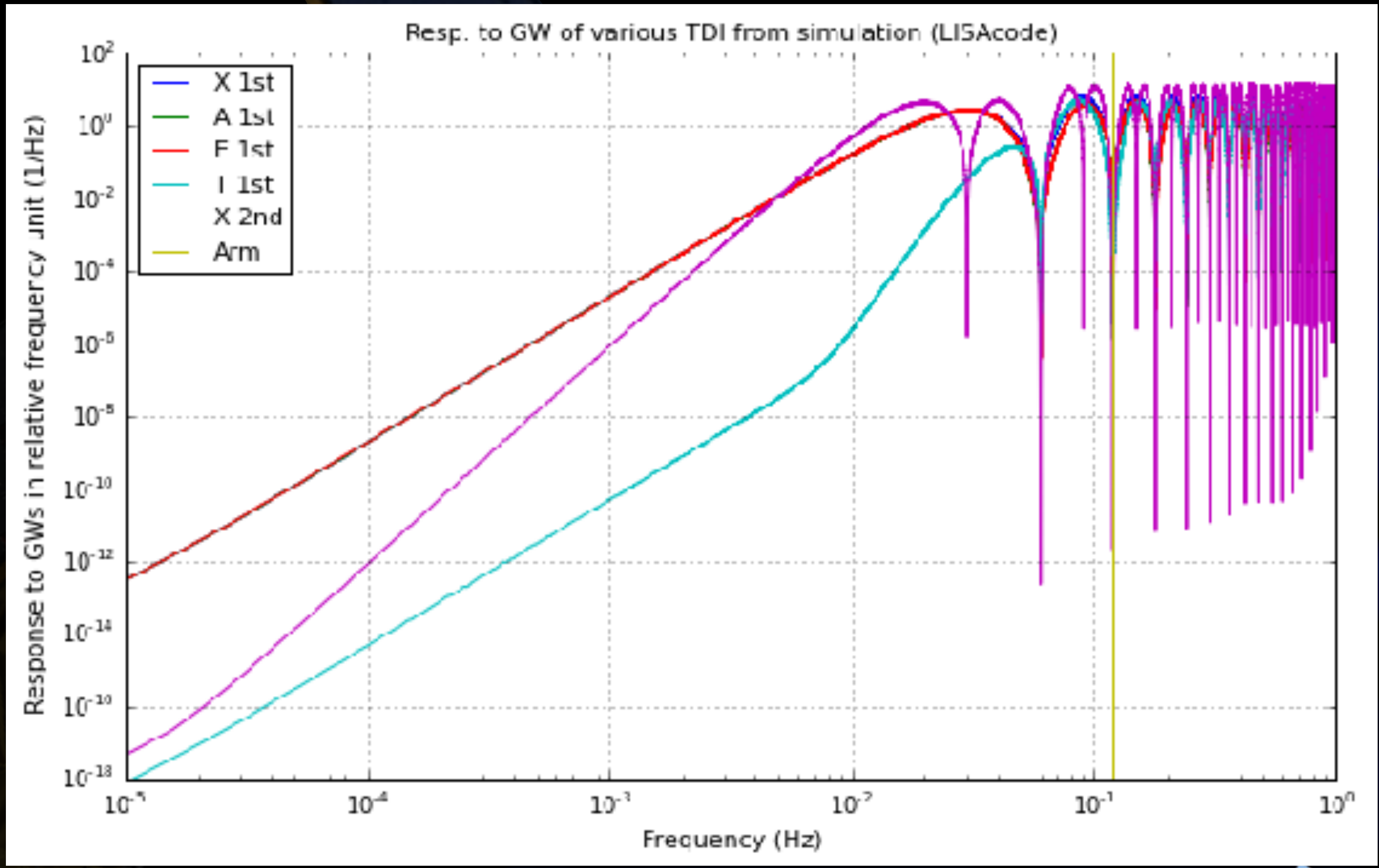
  - Diagonalisation => generator A, E and T.

TDI generators

- Several possibilities for A, E, T
- Standard one, convention in the Consortium, used in LDC

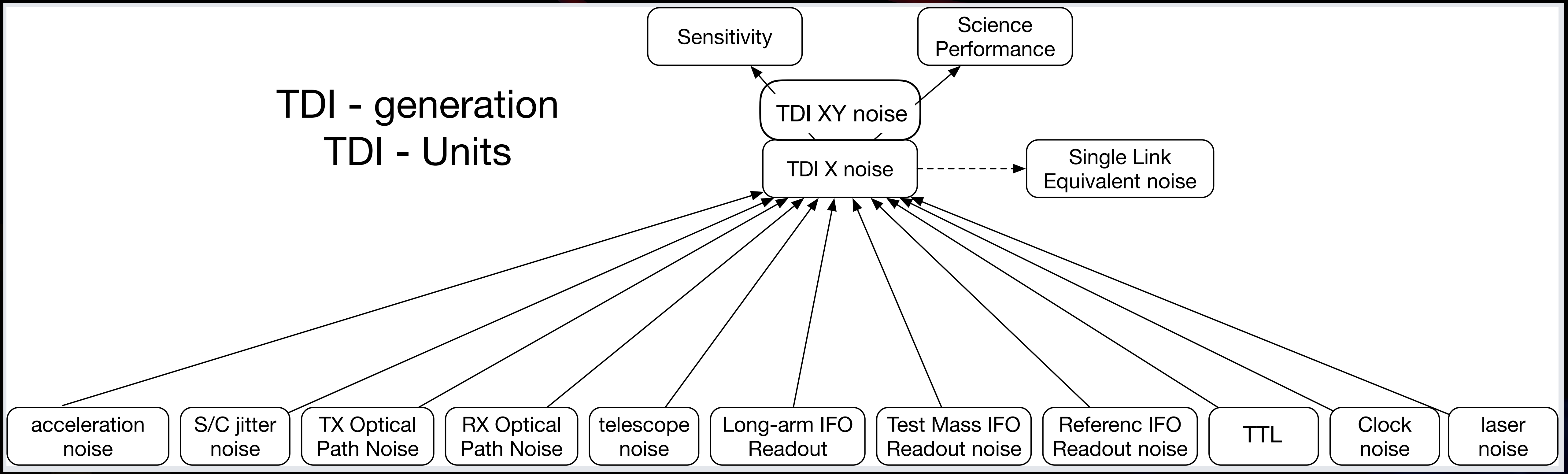
$$E = \frac{X - 2Y + Z}{\sqrt{6}}, \quad A = \frac{Z - X}{\sqrt{2}}, \quad T = \frac{X + Y + Z}{\sqrt{3}}$$

- Noise level:
  - $S_A = S_E = S_{XX} - S_{XY}$
  - $S_T = S_{XX} + 2\, S_{XY}$
- Response to GW:
  - T very weak
  - A, E => main generators used for science



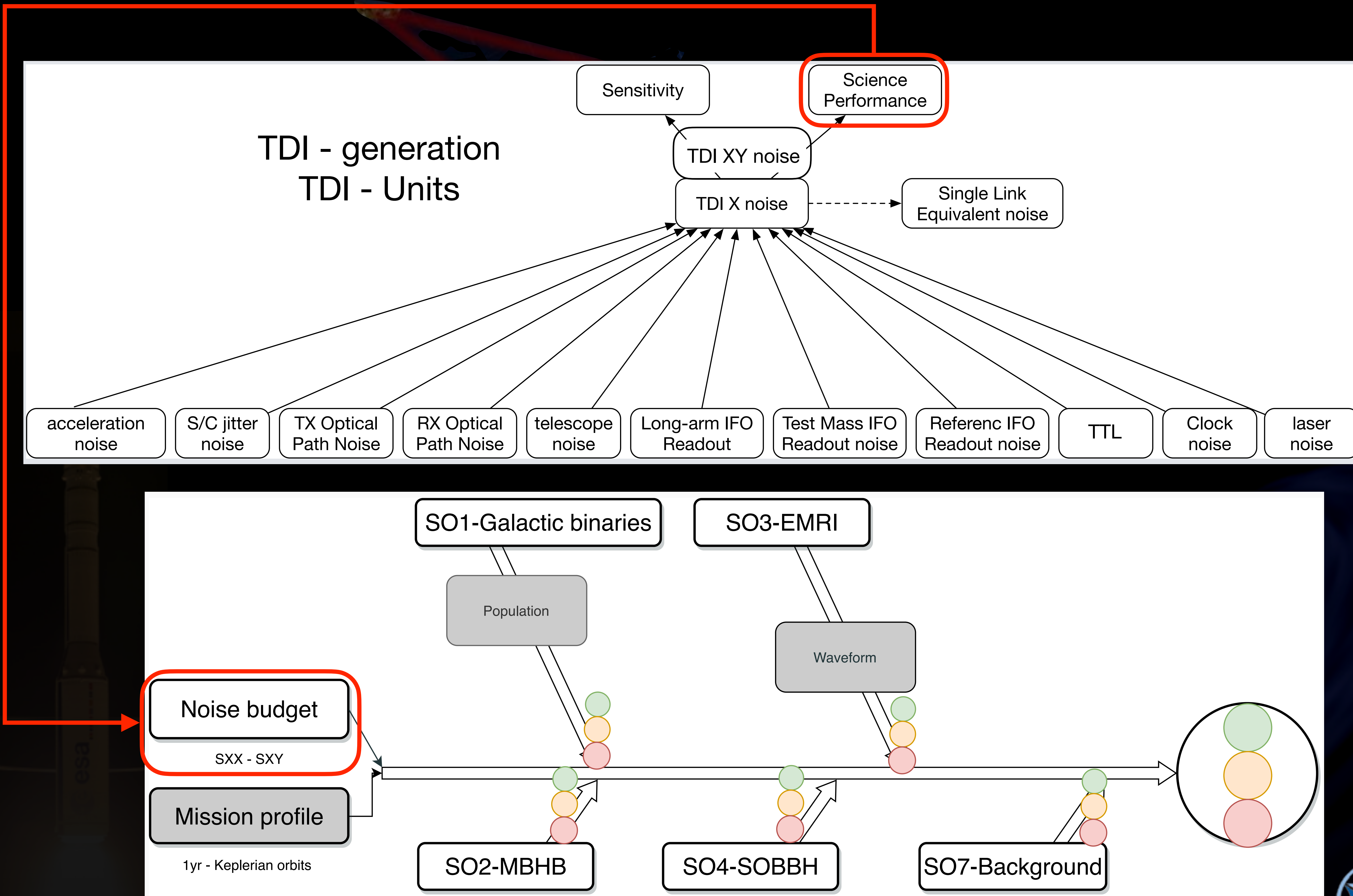
Noises & TDI

- Noise budget to be done at the TDI level



- Input of science analysis :
  - Data analysis
  - Figures of Merits
  - …

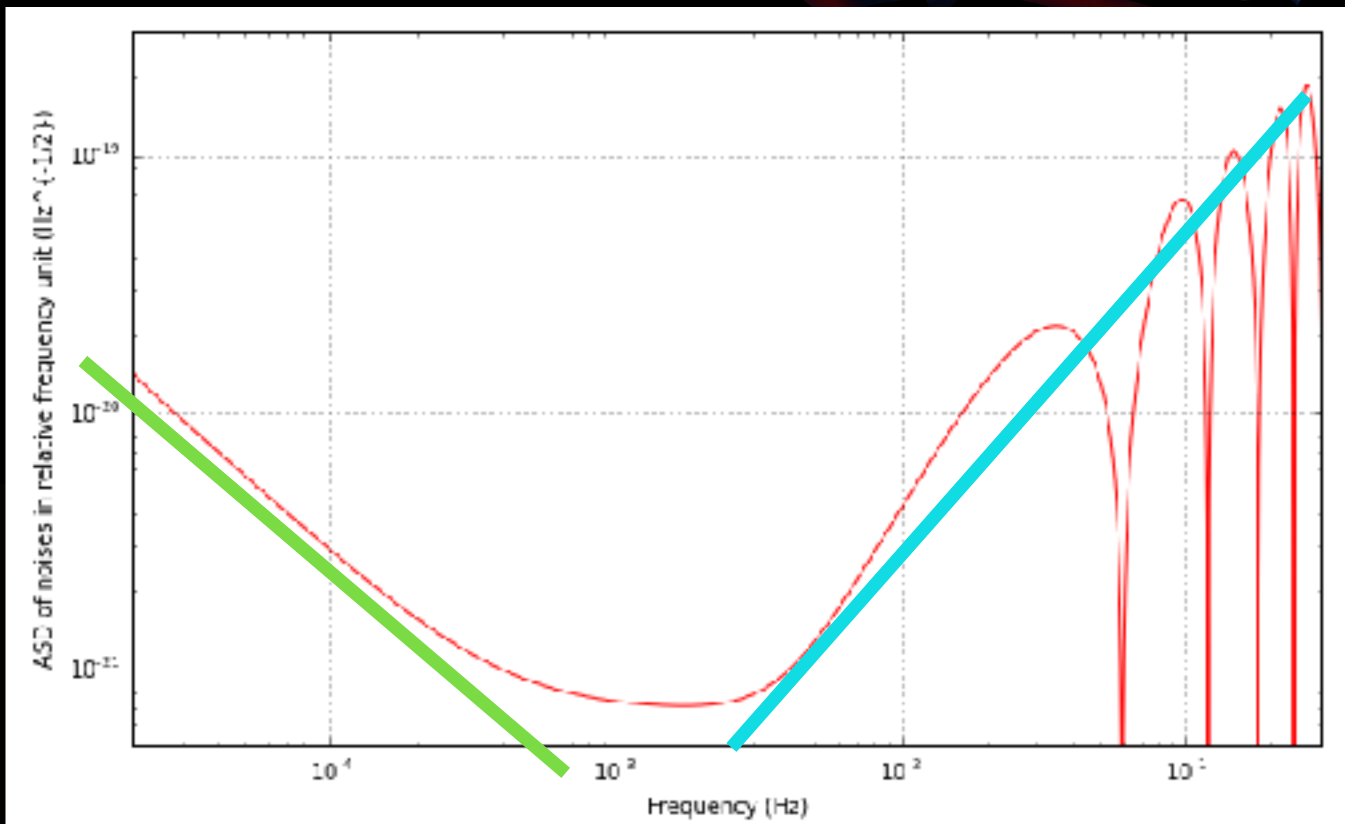
Noises & TDI



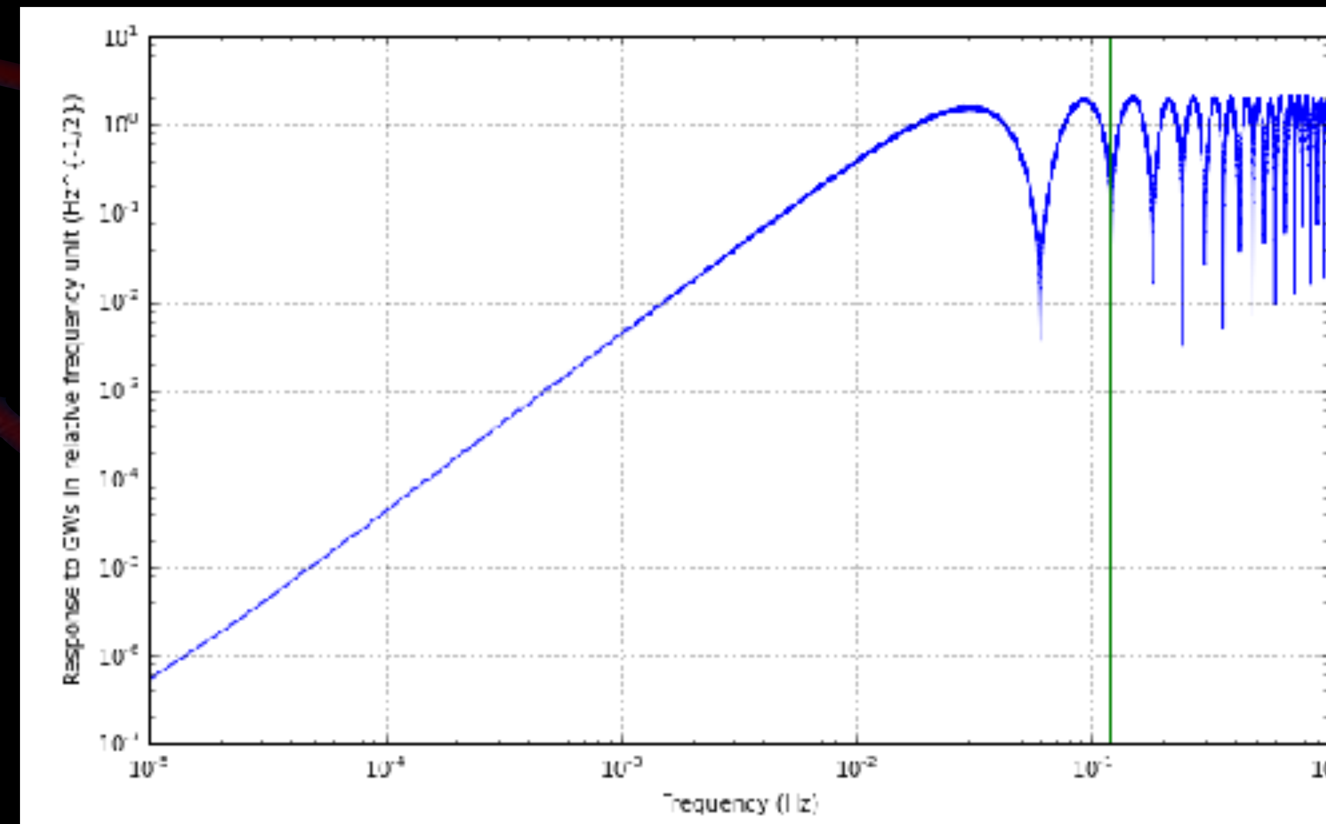
Sensitivity
Science Performance
TDI - generation
TDI - Units
TDI XY noise
TDI X noise
Single Link Equivalent noise
acceleration noise
S/C jitter noise
TX Optical Path Noise
RX Optical Path Noise
telescope noise
Long-arm IFO Readout
Test Mass IFO Readout noise
Referenc IFO Readout noise
TTL
Clock noise
laser noise
SO1-Galactic binaries
SO3-EMRI
Population
Waveform
Noise budget
SXX - SXY
Mission profile
1yr - Keplerian orbits
SO2-MBHB
SO4-SOBBH
SO7-Background

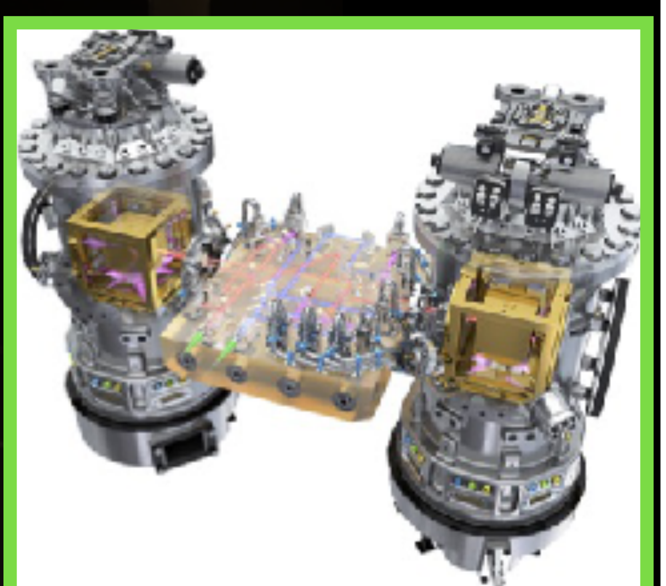
Sensitivity

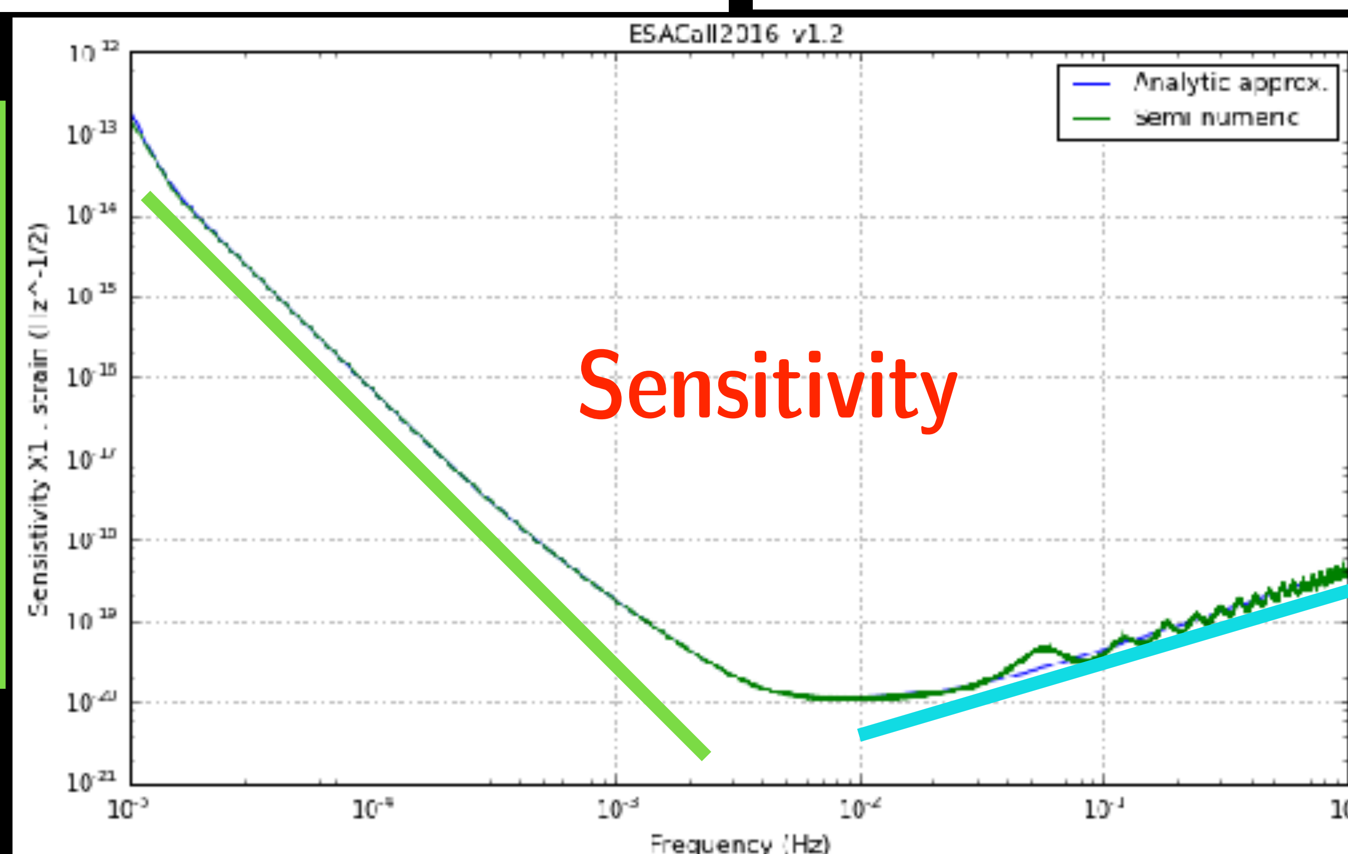
Noises

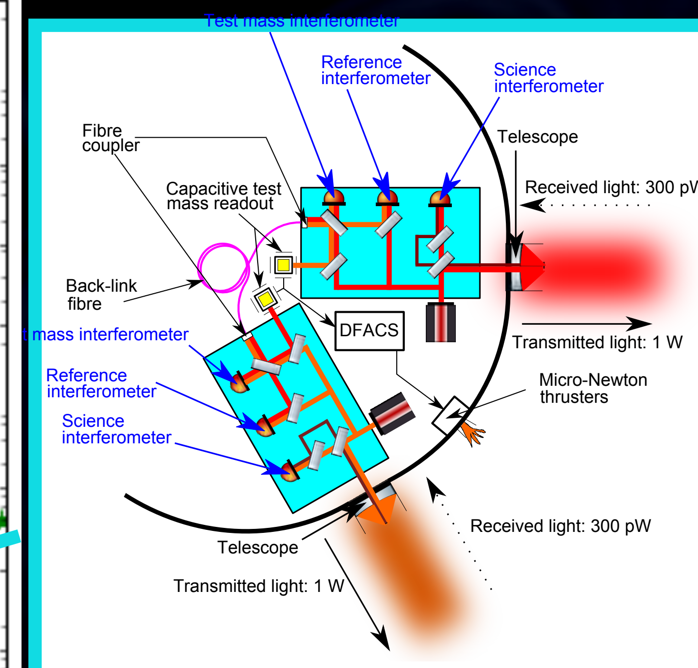


Response of the detector to GWs









TDI transfert function for noises

- Study the propagation of acceleration and noises through TDI
- Others noises neglected or assumed to be cancelled by TDI
- 4 interferometric measurements in spacecraft 1 (simplified version)

$$\left\{\begin{array}{lcl} s_1^c & = & \theta_1^{2'} N_1 \\ \tau_1 & = & 0 \\ \epsilon_1 & = & 2\theta_1^{1'} \delta_1 \\ s_1^{sb} & = & \theta_1^{2'} N_1 \end{array}\right. \qquad \left\{\begin{array}{lcl} s_{1'}^c & = & \theta_{1'}^3 N_{1'} \\ \tau_{1'} & = & 0 \\ \epsilon_{1'} & = & -2\theta_{1'}^1 \delta_{1'} \\ s_{1'}^{sb} & = & \theta_{1'}^3 N_{1'} \end{array}\right.$$

TDI transfert function for noises

- Suppressing spacecraft jitter noise

$$
\begin{aligned}
\xi_1 &= s_1^c - \theta_1^{2'}\theta_1^{1'}\frac{\epsilon_1 - \tau_1}{2} - \theta_1^{2'}\theta_{2'}^{2}\frac{D_3\epsilon_{2'}(t) - D_3\tau_{2'}}{2} \\
&= \theta_1^{2'}\left(N_1 - \delta_1 + D_3\delta_{2'}\right) \\
\xi_{1'} &= s_{1'}^c - \theta_{1'}^{3}\theta_{1'}^{1}\frac{\epsilon_{1'}(t) - \tau_{1'}(t)}{2} - \theta_{1'}^{3}\theta_3^{3'}\frac{D_{2'}\epsilon_3 - D_{2'}\tau_3}{2} \\
&= \theta_{1'}^{3}\left(N_{1'} + \delta_{1'} - D_{2'}\delta_3\right)
\end{aligned}
$$

- Suppressing half of the laser noise:

$$
\begin{aligned}
\eta_1 &= \theta_1^{2'}\xi_1 + \frac{\theta_{2'}^{2}D_3\tau_{2'} - \theta_2^{2'}D_3\tau_2}{2} \\
&= N_1 - \delta_1 + D_3\delta_{2'} \\
\eta_{1'} &= \theta_{1'}^{3}\xi_{1'} - \frac{\theta_{1'}^{1}\tau_{1'} - \theta_1^{1'}\tau_1}{2} \\
&= N_{1'} + \delta_{1'} - D_{2'}\delta_3
\end{aligned}
$$

TDI transfert function for noises

- TDI X 1.5 (time domain)

$$
\begin{aligned}
X_{1.5} &= \eta_{1'} + D_{2'}\eta_3 + D_{2'}D_2\eta_1 + D_{2'}D_2D_3\eta_{2'} - \eta_1 - D_3\eta_{2'} - D_3D_{3'}\eta_{1'} - D_3D_{3'}D_{2'}\eta_3 \\
&= N_{1'} + \delta_{1'} - D_{2'}\delta_3 \\
&\quad + D_{2'}N_3 - D_{2'}\delta_3 + D_{2'}D_2\delta_{1'} \\
&\quad + D_{2'}D_2N_1 - D_{2'}D_2\delta_1 + D_{2'}D_2D_3\delta_{2'} \\
&\quad + D_{2'}D_2D_3N_{2'} + D_{2'}D_2D_3\delta_{2'} - D_{2'}D_2D_3D_{3'}\delta_1 \\
&\quad - N_1 + \delta_1 - D_3\delta_{2'} \\
&\quad - D_3N_{2'} - D_3\delta_{2'} + D_3D_{3'}\delta_1 \\
&\quad - D_3D_{3'}N_{1'} - D_3D_{3'}\delta_{1'} + D_3D_{3'}D_{2'}\delta_3 \\
&\quad - D_3D_{3'}D_{2'}N_3 + D_3D_{3'}D_{2'}\delta_3 - D_3D_{3'}D_{2'}D_2\delta_{1'} \\
X_{1.5} &= (1 - D_3D_{3'})N_{1'} - (1 - D_{2'}D_2)N_1 + (D_{2'} - D_3D_{3'}D_{2'})N_3 - (D_3 - D_{2'}D_2D_3)N_{2'} \\
&\quad + (1 + D_3D_{3'} - D_{2'}D_2 - D_{2'}D_2D_3D_{3'})\delta_1 + (1 - D_3D_{3'} + D_{2'}D_2 - D_3D_{3'}D_{2'}D_2)\delta_{1'} \\
&\quad - 2(D_3 - D_{2'}D_2D_3)\delta_{2'} - 2(D_{2'} - D_3D_{3'}D_{2'})\delta_3
\end{aligned}
$$

TDI transfert function for noises

- Computation of the PSD:
  - OMS terms

$$
\begin{aligned}
PSD\left[(1 - D_{2'}D_2)N_1\right] &= \left\langle \left(1 - e^{-i\omega(L_{2'}+L_2)}\right)\left(1 - e^{i\omega(L_{2'}+L_2)}\right)\tilde{N}_1\tilde{N}_1^* \right\rangle \\
&= \left\langle \left(e^{-i\omega\frac{L_{2'}+L_2}{2}} - e^{i\omega\frac{L_{2'}+L_2}{2}}\right)\tilde{N}_1\tilde{N}_1^* \right\rangle \\
&= 4\sin^2\left(\omega\frac{L_{2'}+L_2}{2}\right) S_{OMS_i}
\end{aligned}
$$

  - Acceleration terms

$$
\begin{aligned}
PSD\left[(1 + D_3D_{3'} - D_{2'}D_2 - D_{2'}D_2D_3D_{3'})\delta_1\right] &= \left\langle \left(1 + e^{-i\omega(L_3+L_{3'})} - e^{-i\omega(L_{2'}+L_2)} - e^{-i\omega(L_{2'}+L_2+L_3+L_{3'})}\right)(\ldots)^*\,\tilde{\delta}_1\tilde{\delta}_1^* \right\rangle \\
&= \left\langle \left(\left(1 + e^{-i\omega(L_3+L_{3'})}\right)\left(1 - e^{-i\omega(L_{2'}+L_2)}\right)\right)(\ldots)^*\,\tilde{\delta}_1\tilde{\delta}_1^* \right\rangle \\
&= \left\langle \left(e^{-i\omega\frac{L_3+L_{3'}}{2}} + e^{i\omega\frac{L_3+L_{3'}}{2}}\right)^2 \left(-e^{-i\omega\frac{L_{2'}+L_2}{2}} + e^{i\omega\frac{L_{2'}+L_2}{2}}\right)^2 \delta_1\delta_1^* \right\rangle \\
&= 16\cos^2\left(\omega\frac{L_3+L_{3'}}{2}\right)\sin^2\left(\omega\frac{L_{2'}+L_2}{2}\right) S_{acc_i} \qquad (97)
\end{aligned}
$$

TDI transfert function for noises

- Computation of PSD:

$$
\begin{aligned}
PSD_{X_{1.5}} = & \; 4\sin^2\left(\omega\frac{L_2+L_{2'}}{2}\right)\left(S_{OMS_1}+S_{OMS_{2'}}+4S_{acc_{2'}}+4\cos^2\left(\omega\frac{L_3+L_{3'}}{2}\right)S_{acc_1}\right) \\
& +4\sin^2\left(\omega\frac{L_{3'}+L_3}{2}\right)\left(S_{OMS_{1'}}+S_{OMS_3}+4S_{acc_3}+4\cos^2\left(\omega\frac{L_2+L_{2'}}{2}\right)S_{acc_{1'}}\right)
\end{aligned}
$$

- Approx. : All armlength equal

$$PSD_{X_{1.5}} = 4\sin^2(\omega L)\left(S_{OMS_1}+S_{OMS_{2'}}+S_{OMS_{1'}}+S_{OMS_3}+4\left(S_{acc_{2'}}+S_{acc_3}+\cos^2(\omega L)\left(S_{acc_1}+S_{acc_{1'}}\right)\right)\right)$$

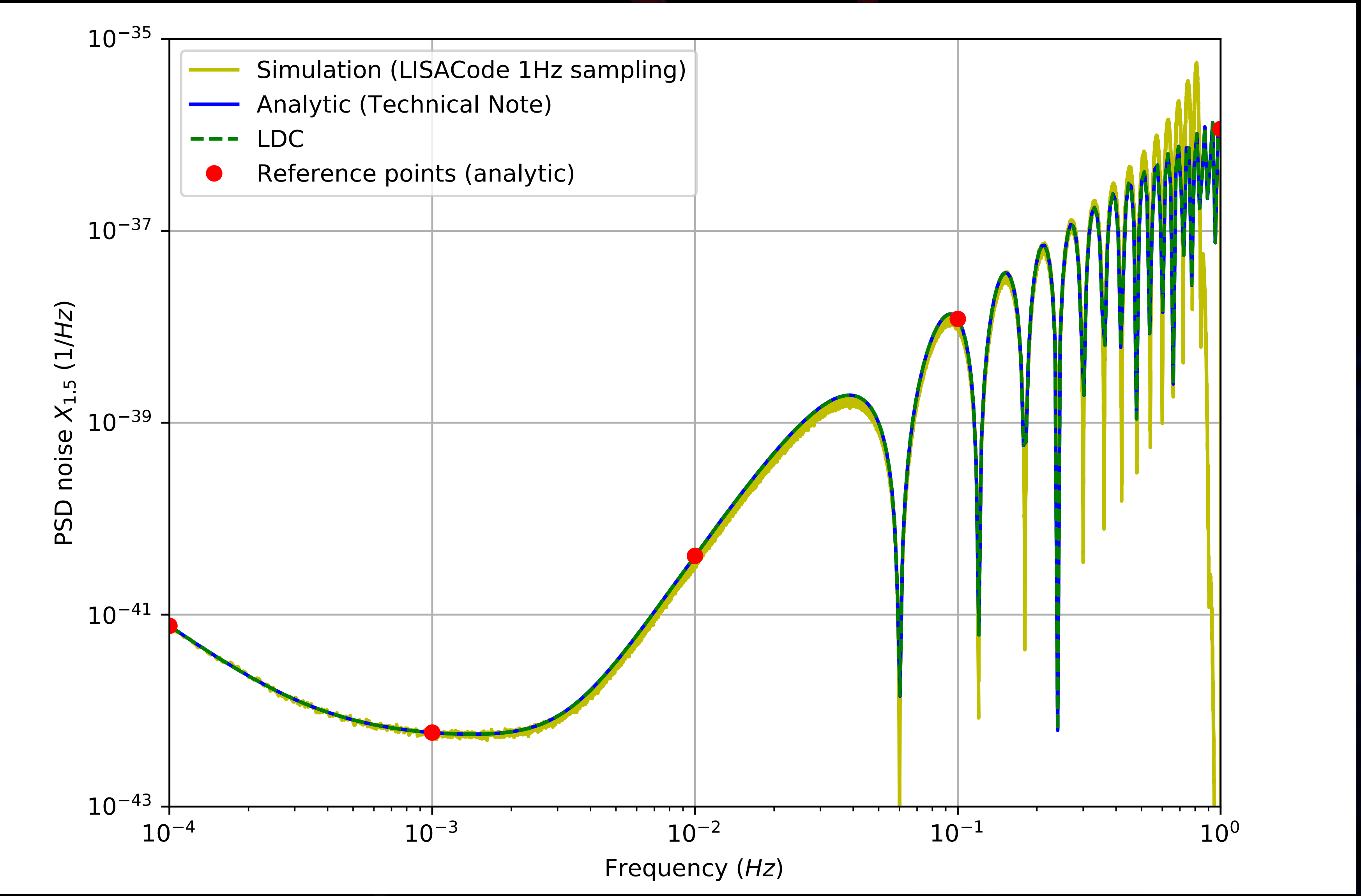
- Approx. : All noises of the same type have the same PSD

$$PSD_{X_{1.5}} = 16\sin^2(\omega L)\left(S_{OMS}+(3+\cos(2\omega L))S_{acc}\right)$$

TDI transfert function for noises

- TDI X 1.5:

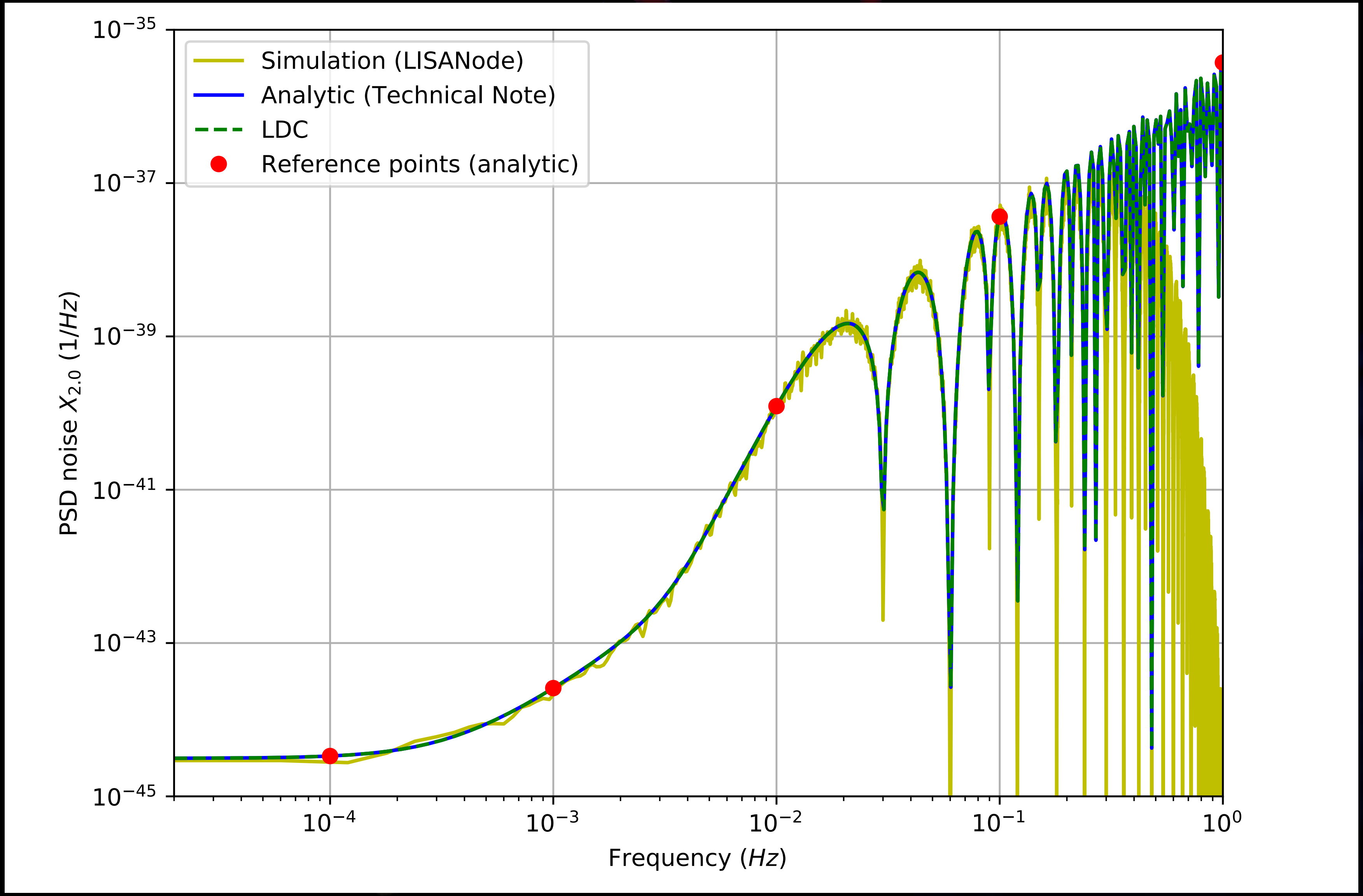
$$PSD_{X_{1.5}} = 16\sin^2(\omega L)\left(S_{OMS} + (3+\cos(2\omega L))\,S_{acc}\right)$$



TDI transfert function for noises

- **TDI X 2.0:**

$$PSD_{X_{2.0}} = 64 \sin^2(\omega L) \sin^2(2\omega L) \left(S_{OMS} + (3 + \cos(2\omega L)) S_{acc} \right)$$



TDI transfer function in practice

- Model:
  - multiple beams. Example of MOSA 1:

$$
\begin{aligned}
b_{sc,2'\to 1} &= \mathbf{D}_3 \left[p_{2'} + N^{\mathrm{op}}_{TX,2'} - k_{2'} \hat{\mathbf{n}}_3 . \frac{[K\ \vec{\delta}_{2'} + \vec{\mathbf{\Delta}}_{2'}]}{1+K} + \frac{K}{1+K} N^{sens}_{2'} \right] \\
&\quad + ds_3 + gw_3 - k_{2'} \hat{\mathbf{n}}_{3'} . \frac{[K\ \vec{\delta}_1 + \vec{\mathbf{\Delta}}_1]}{1+K} + \frac{K}{1+K} N^{sens}_1 + N^{\mathrm{op}}_{RX,1} \\
b_{TM,1'\to 1} &= p_{1'} + \mu_{1'\to 1} + N^{\mathrm{op}}_{TM,1} \\
b_{ref,1'\to 1} &= p_{1'} + \mu_{1'\to 1} + N^{\mathrm{op}}_{ref,1} \\
b_{sc,1\to 1} &= p_1 + N^{\mathrm{op}}_{loc\to sc,1} \\
b_{TM,1\to 1} &= p_1 + 2k_1 \hat{\mathbf{n}}_{3'} . \frac{(\vec{\mathbf{\Delta}}_1 - \vec{\delta}_1)}{1+K} + N^{\mathrm{op}}_{loc\to TM,1} \\
b_{ref,1\to 1} &= p_1 + N^{\mathrm{op}}_{loc\to ref,1}
\end{aligned}
$$

TDI transfer function in practice

- Model:
  - Measurements. Example of MOSA 1:

$$
\begin{aligned}
s_1^c &= \mathcal{F}\left[\theta_1^{2'}\left(b_{sc,2'\to 1} - b_{sc,1\to 1}\right) + N^{ro}_{sc,1}\right] \\
\epsilon_1 &= \mathcal{F}\left[\theta_1^{1'}\left(b_{TM,1'\to 1} - b_{TM,1\to 1}\right) + N^{ro}_{\epsilon,1}\right] \\
\tau_1 &= \mathcal{F}\left[\theta_1^{1'}\left(b_{ref,1'\to 1} - b_{ref,1\to 1}\right) + N^{ro}_{\tau,1}\right] \\
s_1^{sb} &= \mathcal{F}\left[\theta_1^{2'}\left(b_{2'\to 1} + \mathbf{D}_3[m_3 q_3] - b_{1\to 1} - m_1 q_1\right) + N^{ro}_{s^{sb},1}\right]
\end{aligned}
$$

TDI transfer function in practice

- Doing the same procedure as the one described before, we get transfer function TDI X2.0 for all noises sources
- To be used in performance model (see Joseph's talk)
- Approximations:
  - There is no coupling between S/C and DFACS (two independent bodies): K = 0
  - All armlengths of the constellation are equal.
  - All noises of the same type have the same PSD, i.e

$$PSD\{p_{12}\} = PSD\{p_{21}\} = PSD\{p_{13}\} = PSD\{p_{31}\} = PSD\{p_{32}\} = PSD\{p_{23}\} = S_p$$

  - The laser nominal frequencies are almost constant and equal, so are the laser wavelengths.

TDI transfer function in practice

- for unsuppressed noise

Noise category	Noise symbol	Transfer function in PSD
Test-mass acceleration noise ✓	δ	$4C(\omega)\left(3+\cos\left(2\omega\frac{L}{c}\right)\right)$
Backlink fiber noise ✓	μ	$2C(\omega)\sin^2(\omega\frac{L}{c})$
Science and reference IFO noise ✓	N^{ro}_{sci}, N^{op}_{RX}, N^{op}_{TX}, N^{op}_{ref}, $N^{op}_{loc\to sci}$, $N^{op}_{loc\to ref}$	$4C(\omega)$
Test-mass IFO noise ✓	N^{op}_{tm}, $N^{op}_{loc\to tm}$, N^{ro}_{tm}	$C(\omega)\left(3+\cos\left(2\omega\frac{L}{c}\right)\right)$
Telescope noise	N^{op}_{tel}	$4C(\omega)\left(3+\cos\left(2\omega\frac{L}{c}\right)\right)$

with
$$C(\omega) = 16\sin^2\left(\omega\frac{L}{c}\right)\sin^2\left(2\omega\frac{L}{c}\right)$$

TDI transfer function in practice

- for suspressed noise

Noise category	Noise symbol	Transfer function in PSD
Residual laser frequency noise ✓	p	$4C(\omega)\omega^2\left(\left(\frac{\dot{L}}{c}\right)^2 K_{\mathcal{F}}(\omega) + \left(\frac{\bar{\mu}}{c}\right)^2\right)$
Residual S/C translational jitter noise	Δ	$4C(\omega)\ \omega^2 \left(\frac{\dot{L}}{c}\right)^2 K_{\mathcal{F}}(\omega)$
Residual clock jitter noise	q	$2C(\omega)\ \omega^2 \left(\frac{\dot{L}}{c}\right)^2 K_{\mathcal{F}}(\omega)$ $+C(\omega)\ \delta\nu_{BN}^2 \left(6 + 12\sin^2\left(\omega\frac{L}{c}\right)\right)$
Residual modulation errors	m	$C(\omega)\left\lvert\tilde{f}(\omega)\right\rvert^2 \left(6 + 12\sin^2\left(\omega\frac{L}{c}\right)\right)$
TTL		$4C(\omega)$

- $\dot{L}_i$, the time derivative of the armlength L_i (in m.s^{-1});
- $\omega = 2\pi f$;
- $\bar{\mu}$, the average ranging bias (in m);
- ν_{BN}, the beatnote frequency (Hz);
- $\delta\nu_{BN}$, the fractional error in measured value of beatnote frequency
- $K_{\mathcal{F}}$ the delay-filter factor (in s^2),

$$K_{\mathcal{F}}(\omega) = 4f_s^{-2}\left\lvert\sum_{k=1}^{N} k\alpha_{N+k}\sin\left(\frac{k\omega}{f_s}\right)\right\rvert^2$$

- $\left\lvert\tilde{f}(\omega)\right\rvert^2$, the filter response (dimensionless):

$$\left\lvert\tilde{f}(\omega)\right\rvert^2 = \left\lvert\sum_{k=0}^{2N}\alpha_k \exp^{-jk\omega/f_s}\right\rvert^2 \quad (24)$$

- f_s, the sampling frequency (in Hz),
- α_k, the coefficients of the Finite Impulse Response filter (dimensionless),
- $2N+1$ the number of coefficients of the filter (dimensionless),

We can write the filter output y_n as a function of the past input samples x_{n+k} and $2N+1$ coefficients α_k

$$y_n = \sum_{k=0}^{2N}\alpha_k x_{n-k} \quad (25)$$

TDI transfert function for noises

- Correlations:
  - If a noise source appears in several terms when doing combining measurements at TDI level correlation appears while it was not present in the measurement.
  - Examples:
    - thermal noise presents on acceleration noise and OMS noise
    - telescope noise for emitted beam and received beam:

$$N^{op}_{TX,i}(t) = \alpha N^{op}_{RX,i}(t)$$

$$PSD\left[XX^{\text{corr tels op}}_{1.5}\right] = 16\left[1+\alpha^2+2\alpha\cos^2(\omega L)\right]\sin^2(\omega L) S_{N^{op}_{RX}}$$

$$PSD\left[YY^{\text{corr tels op}}_{1.5}\right] = PSD\left[ZZ^{\text{corr tels op}}_{1.5}\right] = PSD\left[XX^{\text{corr tels op}}_{1.5}\right]$$

    - ...

GW signal

- The single link response (the laser light emitted by "s"ender to the "r"eceiver) to GW is given as:

$$y_{rs}^{GW} = \frac{\Phi_{rs}(t - \hat{k}.\vec{R}_s - L_{rs}) - \Phi_{rs}(t - \hat{k}.\vec{R}_r)}{2(1 - \hat{k}.\hat{n}_{rs})}$$

$$\Phi_{rs} = \hat{n}_{rs} h_{ij} \hat{n}_{rs}$$

- where:

$$h_{ij}^{SSB} = (h_+ \cos 2\psi - h_\times \sin 2\psi)\epsilon_{ij}^+ + (h_+ \sin 2\psi + h_\times \cos 2\psi)\epsilon_{ij}^+$$

  - k is direction of GW propagation,
  - $R_{r,s}$ vector position of a sender/receiver
  - n_{rs} unit vector of the link
  - Source:
    - ecliptic latitude β and longitude λ
    - polarisation ψ

$$\epsilon_{ij}^+ = (\hat{u} \otimes \hat{u} - \hat{v} \otimes \hat{v})_{ij}$$
$$\epsilon_{ij}^\times = (\hat{u} \otimes \hat{v} + \hat{v} \otimes \hat{u})_{ij}$$

$$\hat{u} = -\hat{e}_\phi \sim \frac{\partial \hat{k}}{\partial \lambda}$$
$$\hat{v} = -\hat{e}_\theta \sim \frac{\partial \hat{k}}{\partial \beta}$$

$$\hat{k} = -\hat{e}_r = -\{\cos\beta\cos\lambda, \cos\beta\sin\lambda, \sin\beta\}$$

- Apply TDI on the y_{rs} directly

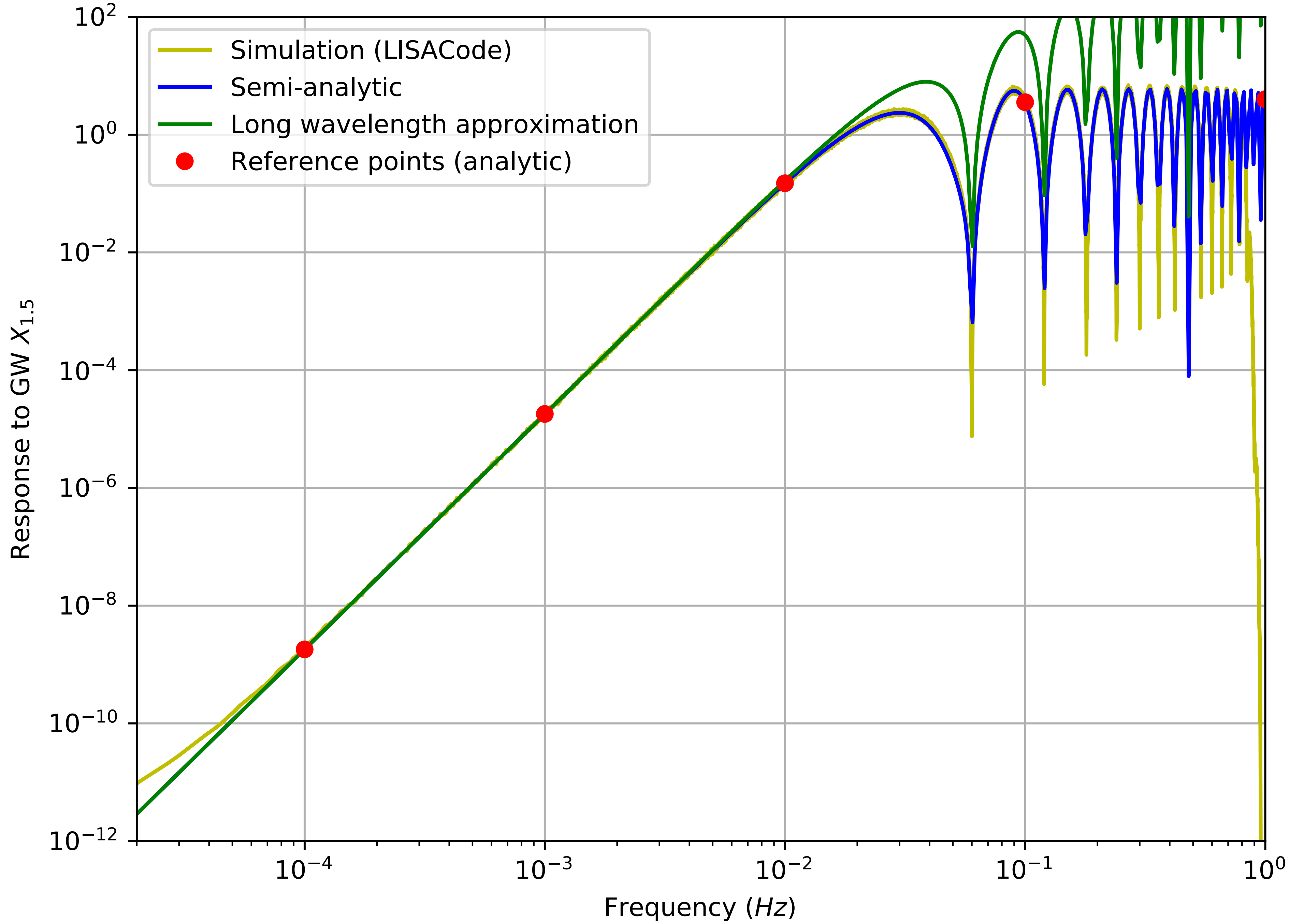
GW signal & TDI

- TDI on the y_{rs} : signal for a particular source (h_{ij}, β, λ, ψ)
- General TDI transfer function to GW: average over polarisation and sky. Complex to compute:
  - Semi-analytical computation $$< R_L(f) >= \left(4 \sin\left(\frac{2\pi f L}{c}\right)\right)^2 \left(\frac{L}{c}\right)^2 (2\pi f)^2 R_\Sigma(f, L)$$

    with R_Σ : average response in phase for a standard Michelson. Semi-analytical computation with numerical integral (see Larson et al. 2000, PRD 62(6):062001)
  - Numerical simulation using white noise stochastic background emitted from many sources regularly distributed over the sky
  - Analytical computation in long wavelength approximation

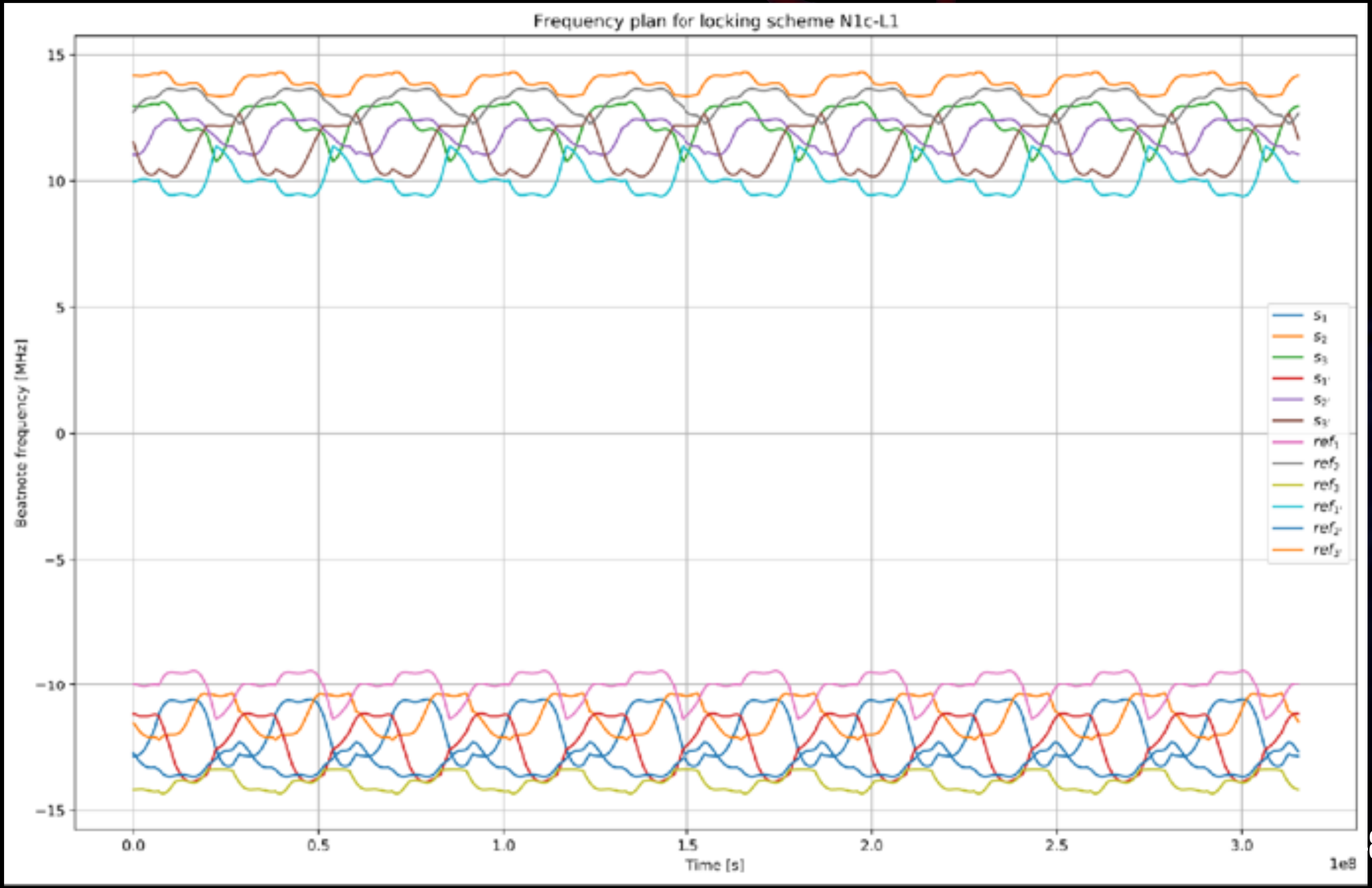
    $$R_{LW} = \frac{3}{20}(\omega L)^2 \sin^2(\omega L)$$

GW signal & TDI



Laser locking / frequency planning

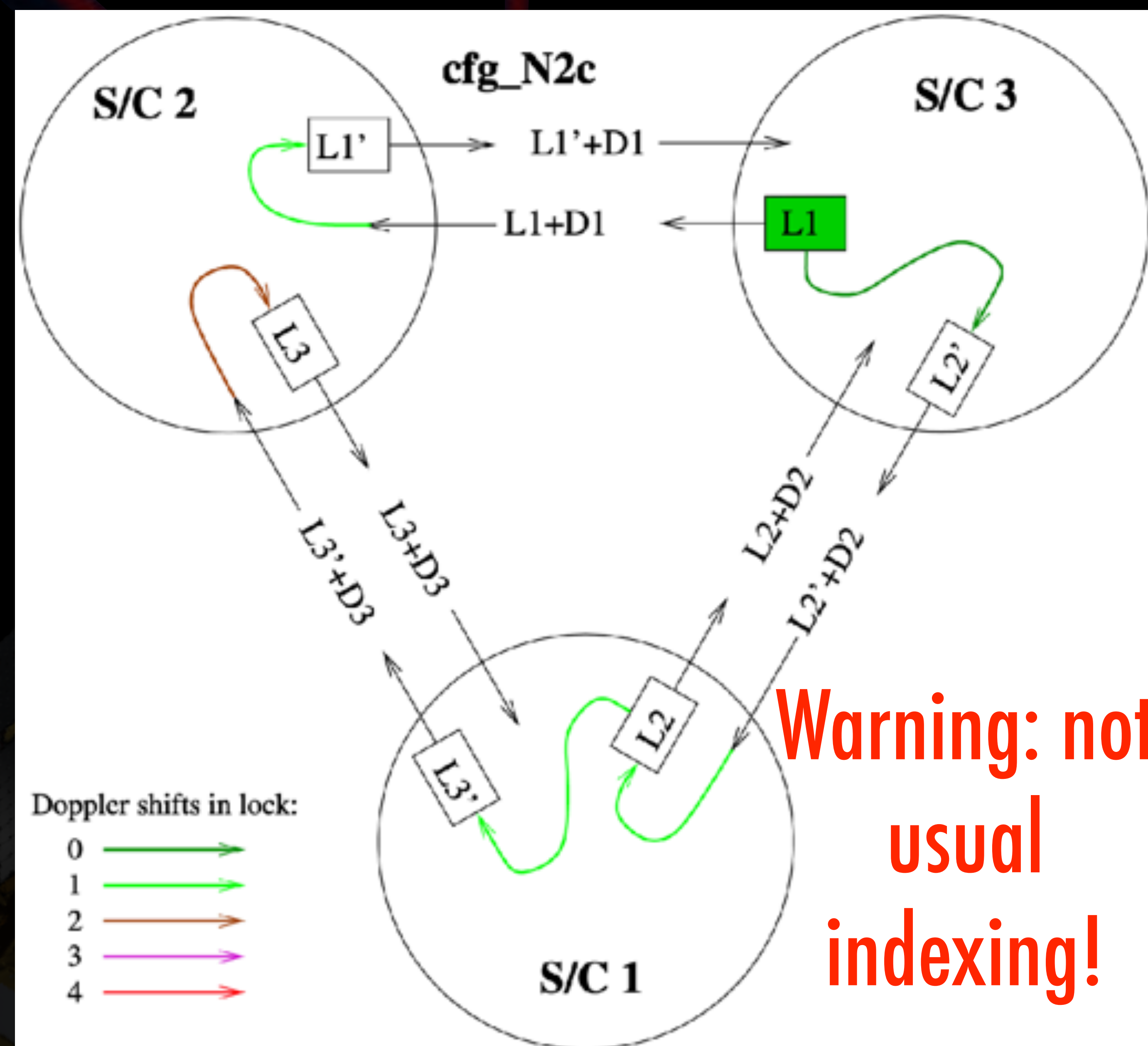
- In reality the laser are not 6 free running lasers but there is one main laser on which one the other are locked with offsets
- The time evolution of offset: frequency planning



Laser locking / frequency planning

- Different locking schemes (TN from Gerhard Heinzel)
  - Example

$$s_2 = os_{3'\to 2}$$
$$\tau_3 = os_{3'\to 3}$$
$$s_{1'} = os_{3\to 1'}$$
$$\tau_1 = os_{1'\to 1}$$
$$s_{2'} = os_{1\to 2'}$$



Laser locking / frequency planning

$$\begin{aligned}
\cancel{s_2} &= os_{3'\rightarrow 2} \\
\cancel{\tau_3} &= os_{3'\rightarrow 3} \\
\cancel{s_{1'}} &= os_{3\rightarrow 1'} \\
\cancel{\tau_1} &= os_{1'\rightarrow 1} \\
\cancel{s_{2'}} &= os_{1\rightarrow 2'}
\end{aligned}$$

- Provide the following formulation for the 5 locked lasers:

$$\begin{aligned}
p_2 &= \theta_2^{3'}(N_{s,2}^{ro} - os_{3'\rightarrow 2}) + b_{sc,3'\rightarrow 2} - N_{loc\rightarrow sc,2}^{op} \\
p_3 &= \theta_3^{3'}(N_{\tau,3}^{ro} - os_{3'\rightarrow 3}) + b_{ref,3'\rightarrow 3} - N_{loc\rightarrow ref,3}^{op} \\
p_{1'} &= \theta_{1'}^{3}(N_{s,1'}^{ro} - os_{3\rightarrow 1'}) + b_{sc,3\rightarrow 1'} - N_{loc\rightarrow sc,1'}^{op} \\
p_1 &= \theta_1^{1'}(N_{\tau,1}^{ro} - os_{1'\rightarrow 1}) + b_{ref,1'\rightarrow 1} - N_{loc\rightarrow ref,1}^{op} \\
p_{2'} &= \theta_{2'}^{1}(N_{s,2'}^{ro} - os_{1\rightarrow 2'}) + b_{sc,1\rightarrow 2'} - N_{loc\rightarrow sc,2'}^{op}.
\end{aligned}$$

Laser locking / frequency planning

- For unsuppressed noises (readout noise, optical path noise, acceleration noise, ...) => same PSD as for free running lasers
- For suppressed noise:
  - depend on the locking configuration
  - not the same for X, Y & Z
  - Work in progress via analytical formulation and simulation (locking implemented in last version of LISANode)

INREP: Analytical formulations

- TDI transfer functions without & with phase locking, PSD & CSD, TDI1.5 & TDI2.0:
  - Unsuppressed noises => done
  - Suppressed noises:
    - laser frequency noise => partially (flexing-filtering, armlength uncertainties bias & stochastic, interpolation)
    - Translational spacecraft jitter noise => done
    - Clock jitter noise => done
    - Clock correction residuals => partially (modulation, sideband readout noise)
    - TTL => partially
  - ...

INREP: Analytical formulations

- TDI transfer functions without & with phase locking, PSD & CSD, TDI1.5 & TDI2.0:
  - …
  - Correlated noises:
    - Telescope => done
    - Backlink => done
    - correlation need to be identified first => on demand
- Cross-checking with LISANode:
  - 70% done for PSD
  - To do for CSD

Key ingredients of TDI

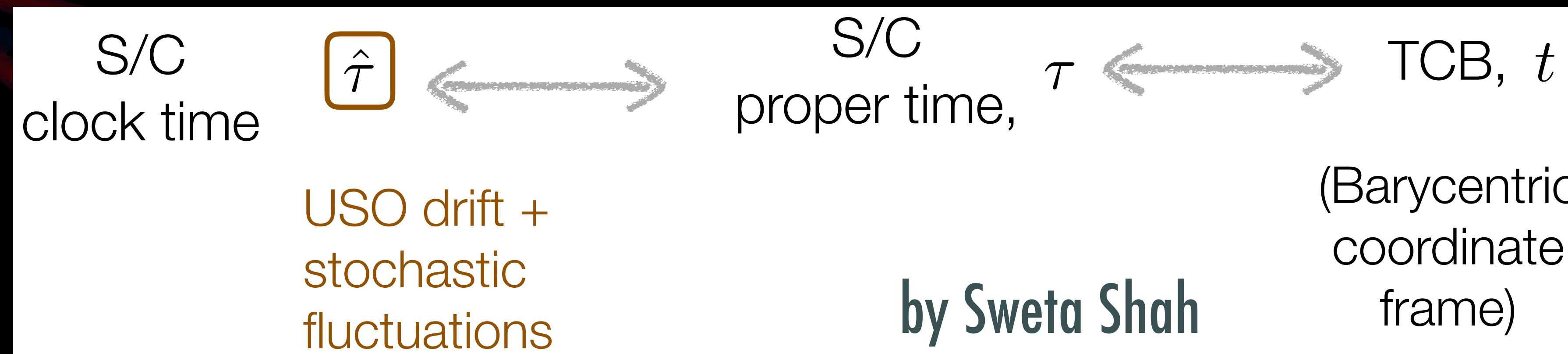
- Measurements
- Arm length knowledge
- Synchronisation of data
- No aliasing => onboard filtering
- Interpolation
- Mitigate the impact of artefacts, i.e. gaps and glitches

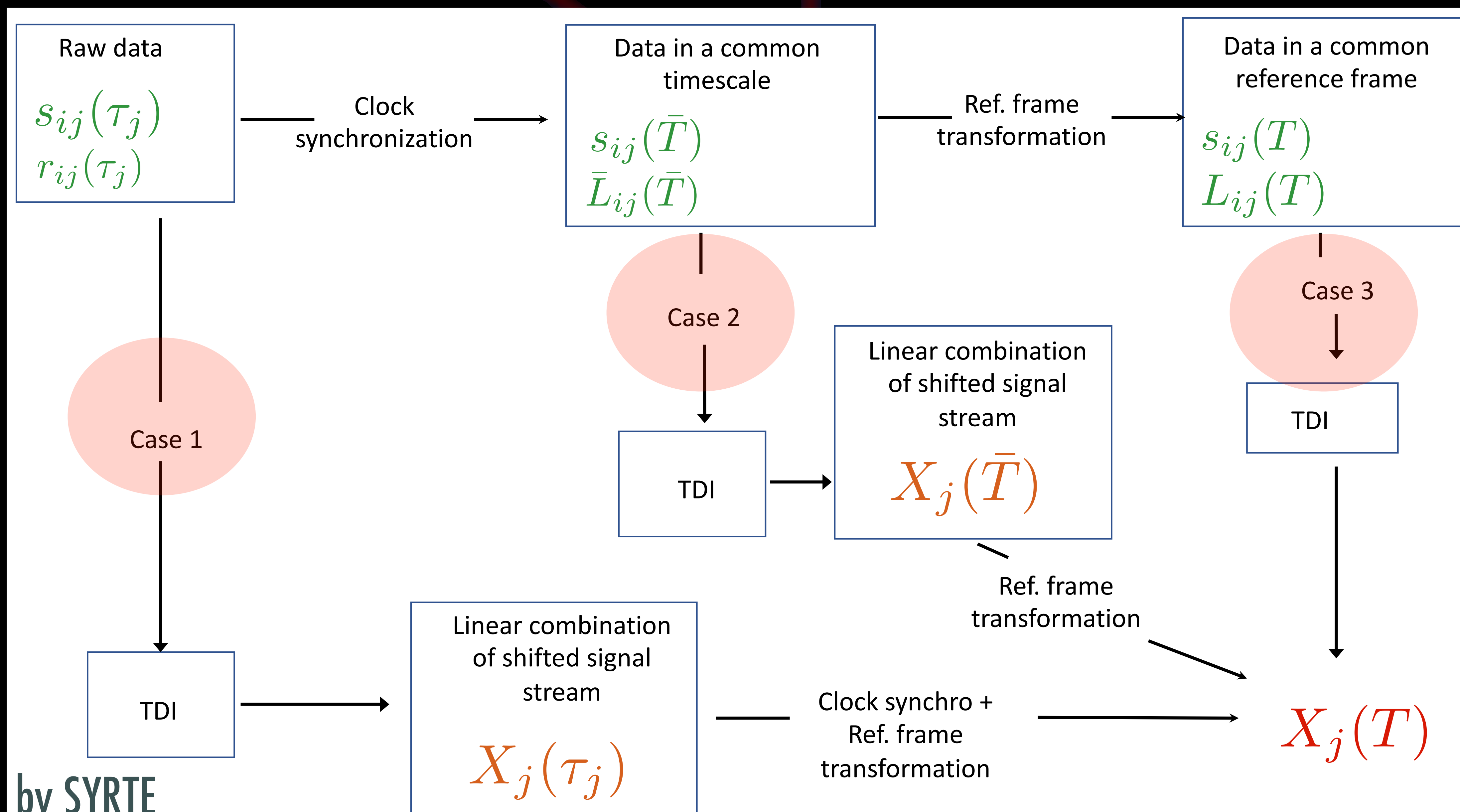
Armlength

- Knowledge of the absolute armlength is crucial
- Several combined technics:
  - Direct measurement: pseudo-random code imprinted on laser beam => precision at the meter level (TBC)
  - Kalman filter:
    - combine a dynamic model of the whole LISA constellation (orbits, PSD of clock jitters) and phasemeter raw measurements
    - => precision at cm level
  - TDI-Ranging: minimisation of noises in post-processing (data analysis) with armlength treated as parameters

INREP: Clock synchronisation

- Study TDI vs clock synchronisation



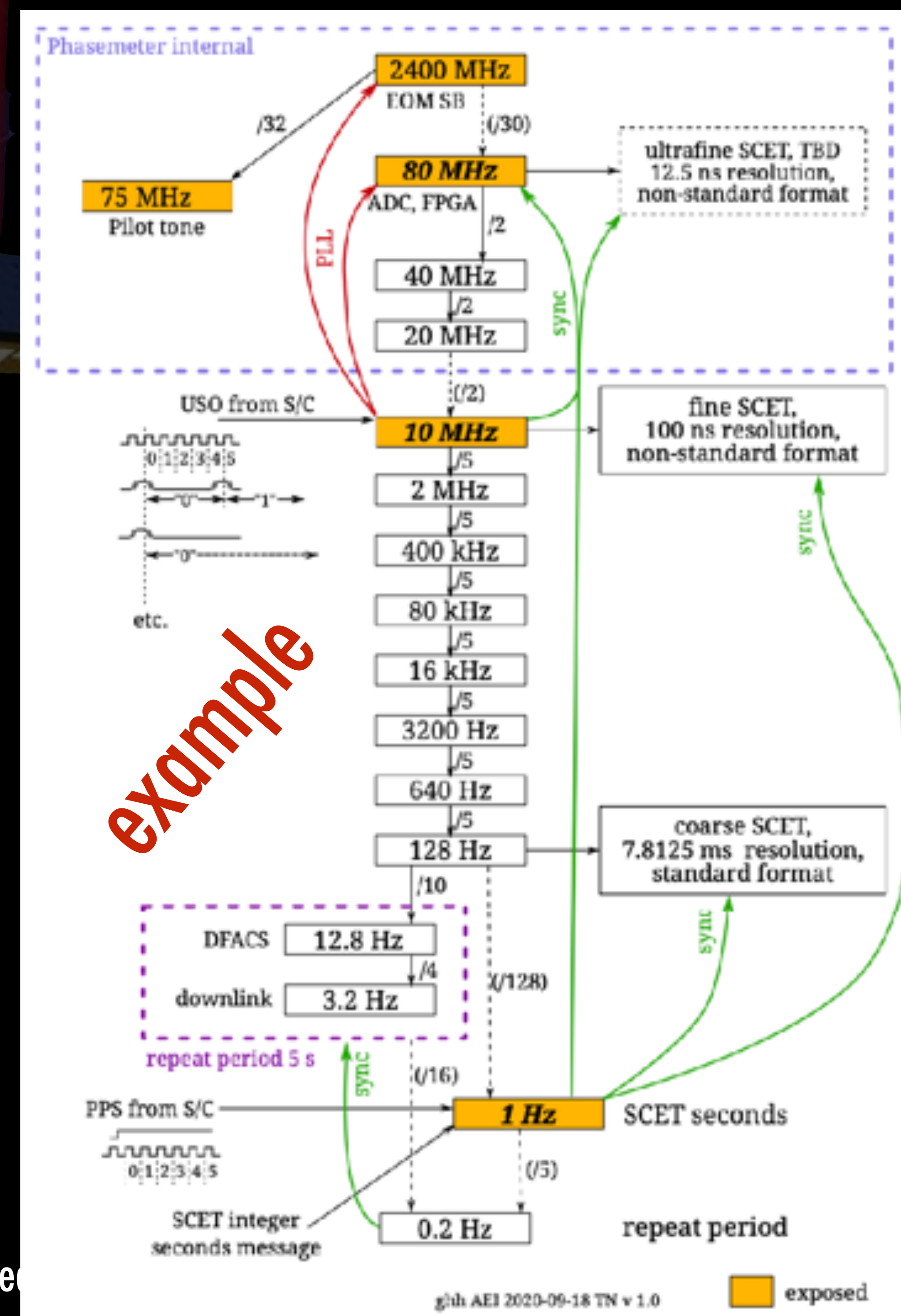


Anti-aliasing filtering

- On-board sampling rate at few tens Hz (10Hz, 12.8Hz, 16Hz ?)
- Telemetry sampling rate at few Hz (2.5Hz, 3.2Hz, 4Hz, 5Hz ?)

  => on-board filtering
- To avoid any aliasing residual at TDI level, strong reduction requested

  => typically > 240 dB
- Linear response => FIR
- Coupling with delay used in TDI (flexing -filtering)
- Computing cost

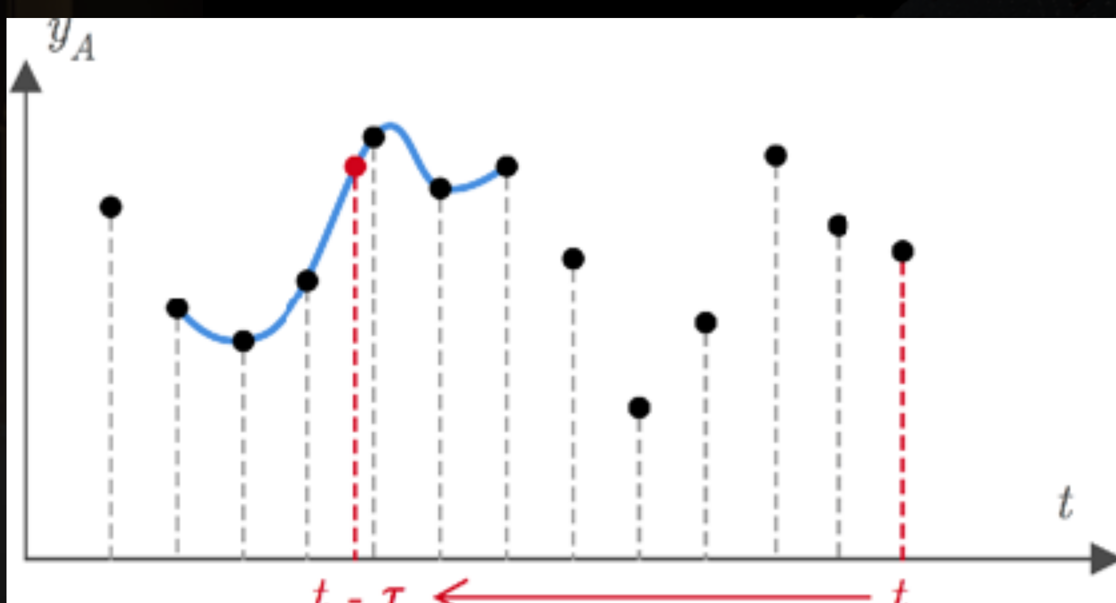


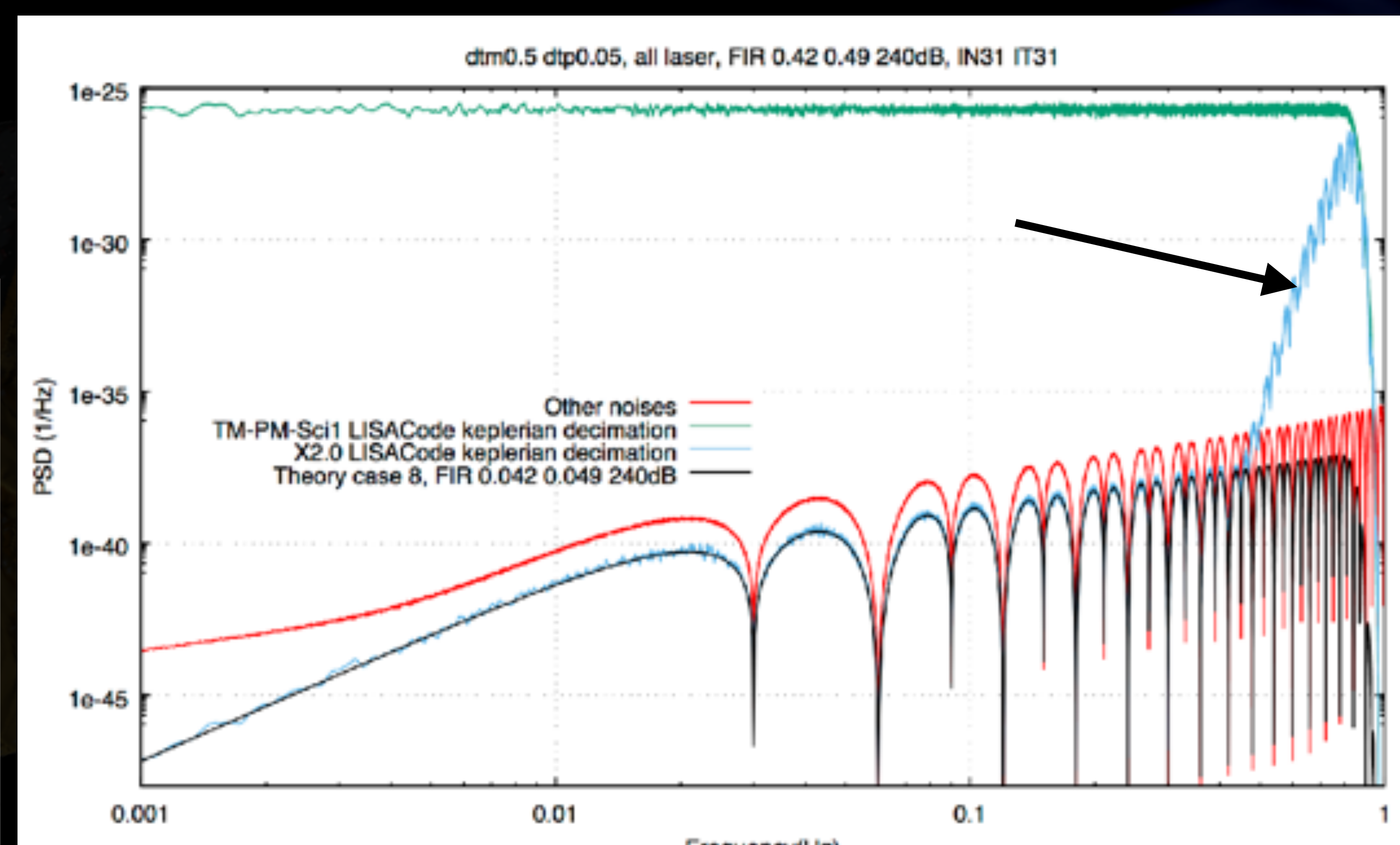
Interpolation

- Discrete input timeseries=> we need to have a very precise interpolation in order to get measurement between samples
- Which method ?

$$D_i x = D_i x(t) = x\left(t - \frac{L_i}{c}\right)$$

  - we are using Lagrange interpolation with 32 points but other methods could be tested
- Importance of the sampling rate





Artefacts

- We are expecting artefacts in LISA data:
  - Gaps
  - Glitches
  - ...

=> Interaction of TDI with these artefacts have to be studied

- For example it could increase the gap size

Data analysis & TDI

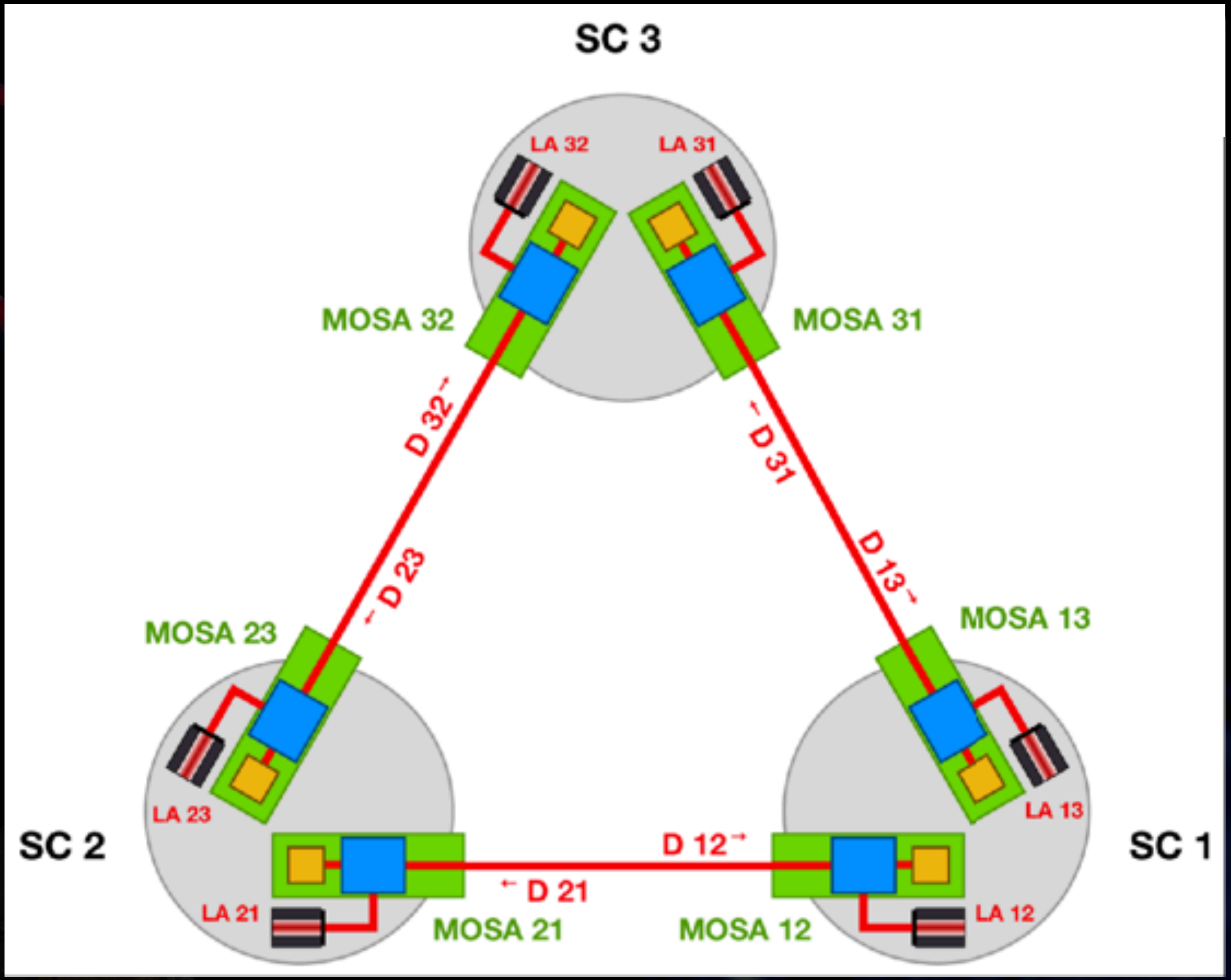
- The input of GW search pipelines are TDI data:
  - X, Y, Z
  - A, E, T : assuming all noise level are the same, noise matrix is diagonal => likelihood computation is just a sum
- Most of the methods are using matched filtering => TDI has to be included in the template:
  - Computation of template have to be very fast
    - approximation in the TDI implementation
    - very fast TDI ?

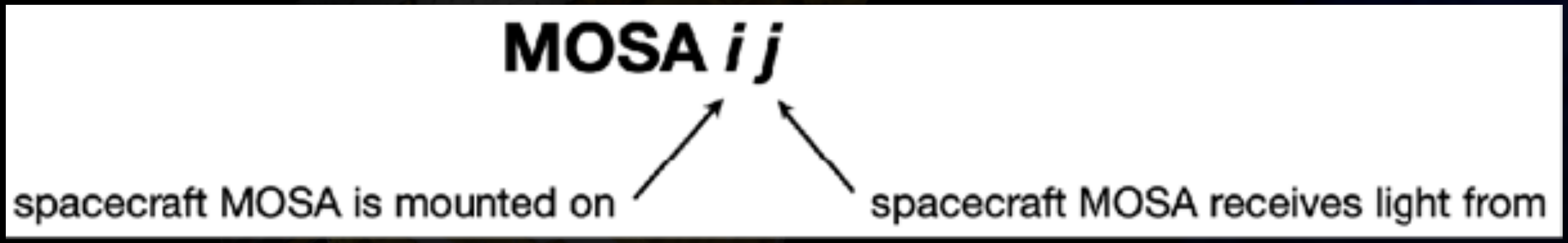
TDI infinity

- Main idea: likelihood directly in terms of the basic phase measurements with marginalization over the laser phase noises in the limit of infinite laser-noise variance
- 2 studies:
  - Vallisneri et al. 2020 arXiv:2008.12343
  - Baghi et al. 2020 arXiv:2010.07224

New conventions

- Ongoing uniformisation of the convention in the LISA Consortium
- Spacecraft, indexed from 1 to 3 clockwise looking down the z-axis, onto the solar panels
- MOSAs, indexed with two numbers ij





New conventions

- Measurements

$$\begin{cases} s^c_{12} &= \theta^{21}_{12}\, N_{12} \\ \tau_{12} &= 0 \\ \epsilon_{12} &= 2\theta^{13}_{12}\, \delta_{12} \\ s^{sb}_{12} &= \theta^{21}_{12}\, N_{12} \end{cases} \qquad \begin{cases} s^c_{13} &= \theta^{31}_{13}\, N_{13} \\ \tau_{13} &= 0 \\ \epsilon_{13} &= -2\theta^{12}_{13}\, \delta_{13} \\ s^{sb}_{13} &= \theta^{31}_{13}\, N_{13} \end{cases}$$

- TDI
  - Advantage: natural chaining of indices when concatenating delays

$$X_{1.5} = \eta_{13} + D_{13}\eta_{31} + D_{13}D_{31}\eta_{12} + D_{13}D_{31}D_{12}\eta_{21} - \eta_{12} - D_{12}\eta_{21} - D_{12}D_{21}\eta_{13} - D_{12}D_{21}D_{13}\eta_{31}$$

$$X_{2.0} = \eta_{13} + D_{13}\eta_{31} + D_{13}D_{31}\eta_{12} + D_{13}D_{31}D_{12}\eta_{21} + D_{13}D_{31}D_{12}D_{21}\eta_{12} + D_{13}D_{31}D_{12}D_{21}D_{12}\eta_{21} + D_{13}D_{31}D_{12}D_{21}D_{12}D_{21}\eta_{13} + D_{13}D_{31}D_{12}D_{21}D_{12}D_{21}D_{13}\eta_{31}$$