Electroweak corrections to tri-boson production at the ILC

Sun Hao

Work in collaboration with Fawzi Boudjema, Duc Ninh Le, M. Marcus Weber
Laboratoire d'Annecy-le-Vieux de Physique Théorique

January 26, 2010

Outline

- **Motivation**

Outline

- **Motivation**
- **Calculations:** $e^+e^- \to ZZZ\,,WWZ$

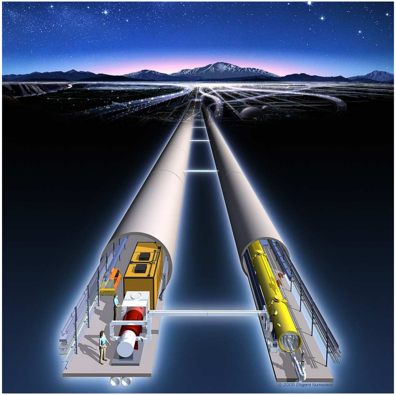
Outline

- **Motivation**
- **Calculations:** $e^+e^- \to ZZZ\,,WWZ$
- **Results**

Outline

- **Motivation**
- **Calculations:** $e^+e^- \to ZZZ\,, WWZ$
- **Results**
- **Conclusions**

The LCs



The International Linear Collider:

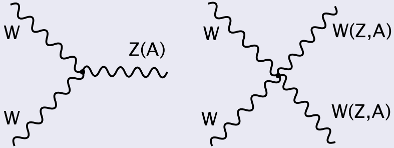
- CM energy: $500 \div 1000\text{GeV}$.
- Luminosity: of the order $10^{34}\text{cm}^{-2}\text{s}^{-1}$.

The Compact LInear Collider:

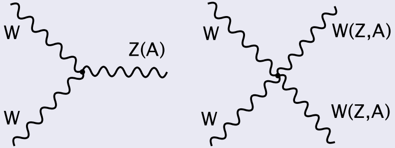
- CM energy: $0.5 \div 3\text{TeV}$.
- Luminosity: also of the order $10^{34}\text{cm}^{-2}\text{s}^{-1}$.

e^+e^- colliders are high precision machines.

SM trilinear and quartic gauge couplings

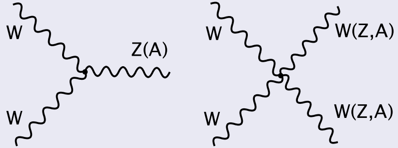


SM trilinear and quartic gauge couplings



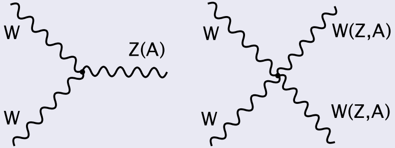
- **Trilinear couplings:** **checking the non-abelian gauge structure.**

SM trilinear and quartic gauge couplings

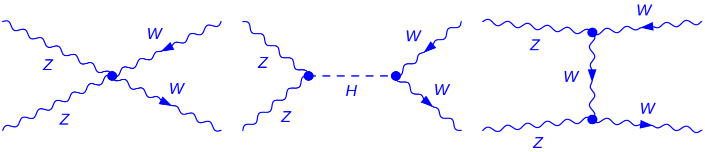


- **Trilinear couplings:** **checking the non-abelian gauge structure.**
- **Quartic couplings:** **also give a window on the spontaneous symmetry breaking (SSB) mechanism.**

SM trilinear and quartic gauge couplings

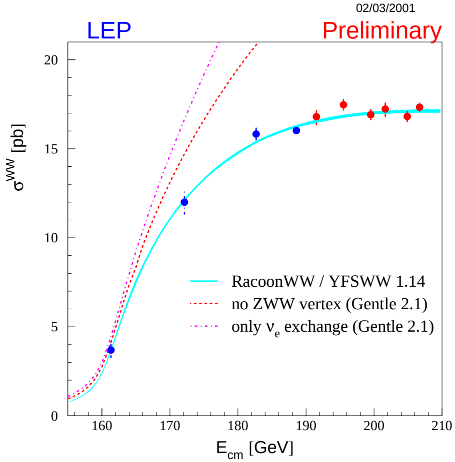


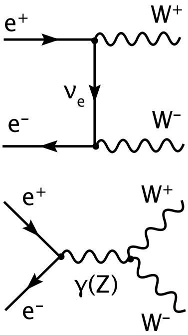
- **Trilinear couplings:** checking the non-abelian gauge structure.
- **Quartic couplings:** also give a window on the spontaneous symmetry breaking (SSB) mechanism.
- SM: unitarity constraints and the perturbative condition (*and* experimental facts) indicate the existence of a small-mass Higgs or new physics at TeV scale.



$\Longrightarrow$ this suggests some connection between the Higgs, new physics and quartic gauge couplings.

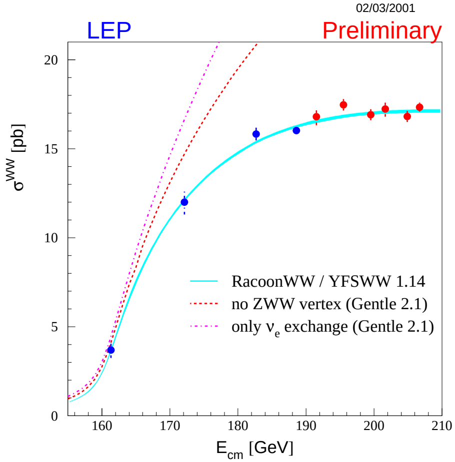
WW production at LEP

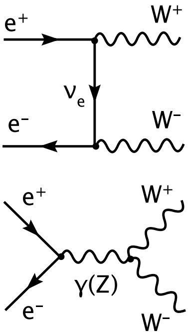




- SM trilinear couplings: well tested at LEP.

WW production at LEP





- SM trilinear couplings: well tested at LEP.
- What about the quartic gauge couplings? Not well tested.

Motivation to calculate tri-boson production

- if the SM Higgs is heavier($> 2\ M_{boson}$), tri-boson productions are where the higgs signal should be looked for.

Motivation to calculate tri-boson production

- if the SM Higgs is heavier($> 2\ M_{boson}$), tri-boson productions are where the higgs signal should be looked for.
- if production of the Higgs boson is not allowed kinematically, precise studies of the gauge boson self-couplings will provide useful information to reveal the underlying mechanism.

Motivation to calculate tri-boson production

- if the SM Higgs is heavier($> 2\ M_{boson}$), tri-boson productions are where the higgs signal should be looked for.
- if production of the Higgs boson is not allowed kinematically, precise studies of the gauge boson self-couplings will provide useful information to reveal the underlying mechanism.
- Obvious from the LEP's results, at the LCs, the radiative electroweak loop corrections should and must be taken into account.

1-loop multi-leg calculation: theoretical challenges

- 1-loop multi-leg calculation: electroweak and QCD corrections.

1-loop multi-leg calculation: theoretical challenges

- 1-loop multi-leg calculation: electroweak and QCD corrections.
- In some respects, electroweak radiative corrections difficult than QCD corrections (many different mass scales $\longrightarrow$ different mass configurations).

1-loop multi-leg calculation: theoretical challenges

- 1-loop multi-leg calculation: electroweak and QCD corrections.
- In some respects, electroweak radiative corrections difficult than QCD corrections (many different mass scales $\longrightarrow$ different mass configurations).
- At present, the frontier of one-loop multi-leg calculation are $2 \longrightarrow 3$ processes (only a few $2 \longrightarrow 4$ processes are available).

1-loop multi-leg calculation: theoretical challenges

- 1-loop multi-leg calculation: electroweak and QCD corrections.
- In some respects, electroweak radiative corrections difficult than QCD corrections (many different mass scales $\longrightarrow$ different mass configurations).
- At present, the frontier of one-loop multi-leg calculation are $2 \longrightarrow 3$ processes (only a few $2 \longrightarrow 4$ processes are available).
- Gauge boson production is an important topic and attracts a lot of interests.

1-loop multi-leg calculation: theoretical challenges

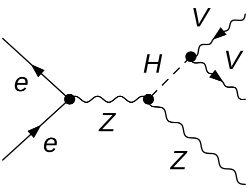
- 1-loop multi-leg calculation: electroweak and QCD corrections.
- In some respects, electroweak radiative corrections difficult than QCD corrections (many different mass scales $\rightarrow$ different mass configurations).
- At present, the frontier of one-loop multi-leg calculation are $2 \rightarrow 3$ processes (only a few $2 \rightarrow 4$ processes are available).
- Gauge boson production is an important topic and attracts a lot of interests.
- 1-loop QCD correction to $pp \rightarrow VVV$ at the LHC: Binoth, Ossola, Papadopoulos and Pittau (2008); Hankele and Zeppenfeld (2008); Lazopoulos, Melnikov and Petriello (2007) ...

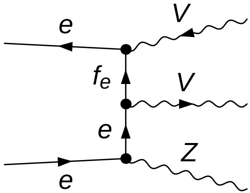
1-loop multi-leg calculation: theoretical challenges

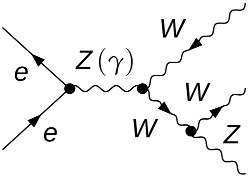
- 1-loop multi-leg calculation: electroweak and QCD corrections.
- In some respects, electroweak radiative corrections difficult than QCD corrections (many different mass scales $\rightarrow$ different mass configurations).
- At present, the frontier of one-loop multi-leg calculation are $2 \rightarrow 3$ processes (only a few $2 \rightarrow 4$ processes are available).
- Gauge boson production is an important topic and attracts a lot of interests.
- 1-loop QCD correction to $pp \rightarrow VVV$ at the LHC: Binoth, Ossola, Papadopoulos and Pittau (2008); Hankele and Zeppenfeld (2008); Lazopoulos, Melnikov and Petriello (2007) ...
- 1-loop EW correction to $e^+e^- \rightarrow VVZ$ at LCs: Ma's Group (2009) ; Our Group(2009)

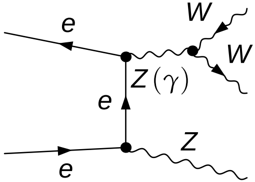
$e^+e^- \to VVZ$: tree diagrams

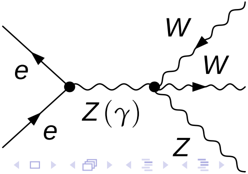
- ZZZ: 9 diagrams, no trilinear and quartic couplings in SM
- WWZ: 20 diagrams, trilinear and quartic couplings contribute in SM





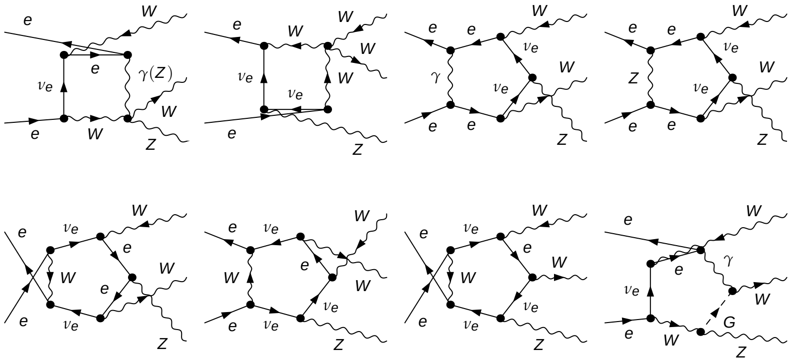






$e^+e^- \rightarrow VVZ$: one-loop diagrams

neglecting $< eeS >$ couplings:



Topology	$ZZZ\,(1767)$	$WWZ\,(2736)$
Loop Amp. (`FormCalc-6.0`)	6.4MB	6.9MB
4-point	384	396
5-point	64	109

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

- UV-divergence is regularised by using on-shell renormalisation.

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

- UV-divergence is regularised by using on-shell renormalisation.
- IR-divergences cancel out.

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

- UV-divergence is regularised by using on-shell renormalisation.
- IR-divergences cancel out.
- `FeynArts-3.4` , `FormCalc-6.0` (`Mathematica` + `FORM`) to generate Feynman diagrams and to get the helicity amplitude expressions.

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

- UV-divergence is regularised by using on-shell renormalisation.
- IR-divergences cancel out.
- `FeynArts-3.4` , `FormCalc-6.0` (`Mathematica` + `FORM`) to generate Feynman diagrams and to get the helicity amplitude expressions.
- `SloopS` (Baro, Boudjema and Semenov) to make sure that the amplitudes are correct by checking gauge invariance (using NLG Feynman rules). More later.

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

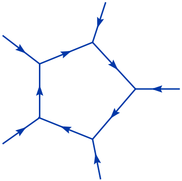
- UV-divergence is regularised by using on-shell renormalisation.
- IR-divergences cancel out.
- `FeynArts-3.4` , `FormCalc-6.0` (`Mathematica` + `FORM`) to generate Feynman diagrams and to get the helicity amplitude expressions.
- `SloopS` (Baro, Boudjema and Semenov) to make sure that the amplitudes are correct by checking gauge invariance (using NLG Feynman rules). More later.
- Improved `LoopTools` (FF + Tensor reduction + ...) to calculate all one-loop integrals. [Hahn, van Oldenborgh and Vermaseren]

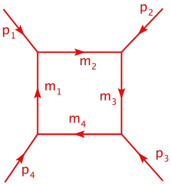
calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} \quad = \quad d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

- UV-divergence is regularised by using on-shell renormalisation.
- IR-divergences cancel out.
- `FeynArts-3.4` , `FormCalc-6.0` (`Mathematica` + `FORM`) to generate Feynman diagrams and to get the helicity amplitude expressions.
- `SloopS` (Baro, Boudjema and Semenov) to make sure that the amplitudes are correct by checking gauge invariance (using NLG Feynman rules). More later.
- Improved `LoopTools` (FF + Tensor reduction + ...) to calculate all one-loop integrals. [Hahn, van Oldenborgh and Vermaseren]
- BASES (Kawabata) to do phase space integration and to get distributions.

LoopTools integrals



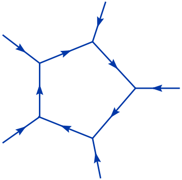


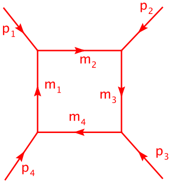
One loop tensor N-point integrals have the general form:

$$T^{N,\mu_1\cdots\mu_P}\left(k_1,\ldots,k_{N-1},m_0,\ldots,m_{N-1}\right)=\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q^{\mu_1}\ldots q^{\mu_P}}{N_0N_1\ldots N_{N-1}}$$

$k_i=\sum_{j=1}^{i-1}p_j,\ i=1,2,3,\ldots,\qquad N_k=(q+k_k)^2-m_k^2$

LoopTools integrals





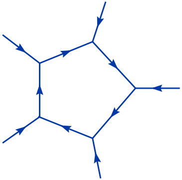
One loop tensor N-point integrals have the general form:

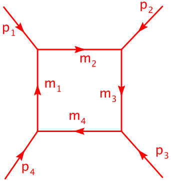
$$T^{N,\mu_1\cdots\mu_P}\left(k_1,\ldots,k_{N-1},m_0,\ldots,m_{N-1}\right)=\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q^{\mu_1}\ldots q^{\mu_P}}{N_0N_1\ldots N_{N-1}}$$

$$k_i=\sum_{j=1}^{i-1}p_j,\ i=1,2,3,\ldots,\qquad N_k=(q+k_k)^2-m_k^2$$

- 1-point and 2-point integrals, explicit numerically stable results are used.

LoopTools integrals





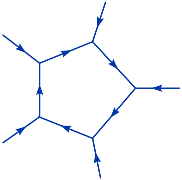
One loop tensor N-point integrals have the general form:

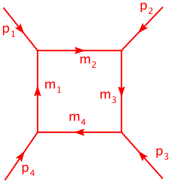
$$T^{N,\mu_1\cdots\mu_P}(k_1,\ldots,k_{N-1},m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q^{\mu_1}\ldots q^{\mu_P}}{N_0N_1\ldots N_{N-1}}$$

$$k_i = \sum_{j=1}^{i-1} p_j,\ i = 1,2,3,\ldots, \qquad N_k = (q+k_k)^2 - m_k^2$$

- 1-point and 2-point integrals, explicit numerically stable results are used.
- 3-point and 4-point scalar integrals, expecially the scalar 4-point integrals: tricky (different mass scales, in practise the log- and Spence- arguments can be very close to 0, leading to numerical problems (observed in WWZ).

LoopTools integrals





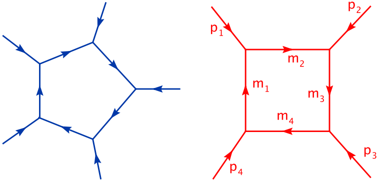
One loop tensor N-point integrals have the general form:

$$T^{N,\mu_1\cdots\mu_P}(k_1,\ldots,k_{N-1},m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q^{\mu_1}\ldots q^{\mu_P}}{N_0N_1\ldots N_{N-1}}$$

$$k_i = \sum_{j=1}^{i-1} p_j,\ i = 1,2,3,\ldots, \qquad N_k = (q+k_k)^2 - m_k^2$$

- 1-point and 2-point integrals, explicit numerically stable results are used.
- 3-point and 4-point scalar integrals, expecially the scalar 4-point integrals: tricky (different mass scales, in practise the log- and Spence- arguments can be very close to 0, leading to numerical problems (observed in WWZ).
- 3-point and 4-point tensor integrals are reduced to scalar integrals with the PV reduction (in most points in phase space, small Gram problem).

LoopTools integrals



One loop tensor N-point integrals have the general form:

$$T^{N,\mu_1\cdots\mu_P}(k_1,\ldots,k_{N-1},m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q^{\mu_1}\ldots q^{\mu_P}}{N_0N_1\ldots N_{N-1}}$$

$$k_i = \sum_{j=1}^{i-1} p_j,\ i = 1,2,3,\ldots, \qquad N_k = (q+k_k)^2 - m_k^2$$

- 1-point and 2-point integrals, explicit numerically stable results are used.
- 3-point and 4-point scalar integrals, expecially the scalar 4-point integrals: tricky (different mass scales, in practise the log- and Spence- arguments can be very close to 0, leading to numerical problems (observed in WWZ).
- 3-point and 4-point tensor integrals are reduced to scalar integrals with the PV reduction (in most points in phase space, small Gram problem).
- 5-point integrals are reduced in terms of five 4pt functions.

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
    - singularities can be introduced as $\ln(m_\gamma^2)$

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package
- Regulate the divergence in dimensional regularization.

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package
- Regulate the divergence in dimensional regularization.
  - singularity pole can be introduced interms of $\frac{1}{\epsilon^2}a_2 + \frac{1}{\epsilon}a_1 + a_0$

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package
- Regulate the divergence in dimensional regularization.
  - singularity pole can be introduced interms of $\frac{1}{\epsilon^2}a_2 + \frac{1}{\epsilon}a_1 + a_0$
  - method of choice in QCD loop processes (light fermion masses taken massless in high energy limits)

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package
- Regulate the divergence in dimensional regularization.
  - singularity pole can be introduced interms of $\frac{1}{\epsilon^2}a_2 + \frac{1}{\epsilon}a_1 + a_0$
  - method of choice in QCD loop processes (light fermion masses taken massless in high energy limits)
  - available interface: QCDLoop library and OneLOop library

LoopTools integrals: scalar integrals

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package
- Regulate the divergence in dimensional regularization.
  - singularity pole can be introduced interms of $\frac{1}{\epsilon^2}a_2 + \frac{1}{\epsilon}a_1 + a_0$
  - method of choice in QCD loop processes (light fermion masses taken massless in high energy limits)
  - available interface: QCDLoop library and OneLOop library

small mass parameter scheme $\Longleftrightarrow$ dimensional regularization scheme

$$\ln(m_\gamma^2) \Longleftrightarrow \frac{1}{\epsilon}$$

Loop integrals and numerical instabilities: small Gram Problem(I)

Gram Matrix

Gram Matrix:

$$G = \begin{pmatrix} 2k_1k_1 & \dots & 2k_1k_N \\ \dots & \dots & \dots \\ 2k_Nk_1 & \dots & 2k_Nk_N \end{pmatrix}$$

determinant of Gram Matrix:

$$detG = \begin{vmatrix} 2k_1k_1 & \dots & 2k_1k_N \\ \dots & \dots & \dots \\ 2k_Nk_1 & \dots & 2k_Nk_N \end{vmatrix}$$

D_{ijkl} tensor integral

- $\boxed{D_{ijkl} = f(p_i, m_i)/detG}$

Loop integrals and numerical instabilities: small Gram Problem(I)

Gram Matrix

Gram Matrix:

$$G = \begin{pmatrix} 2k_1k_1 & \dots & 2k_1k_N \\ \dots & \dots & \dots \\ 2k_Nk_1 & \dots & 2k_Nk_N \end{pmatrix}$$

determinant of Gram Matrix:

$$detG = \begin{vmatrix} 2k_1k_1 & \dots & 2k_1k_N \\ \dots & \dots & \dots \\ 2k_Nk_1 & \dots & 2k_Nk_N \end{vmatrix}$$

D_{ijkl} tensor integral

$$\boxed{D_{ijkl} \;=\; f(p_i, m_i)/detG}$$

- $\Longrightarrow$ numerical instabilities occur when *detG* become small

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)
- quadruple precision (32 digits kept while double precision 16 digits kept, slow, more CPU time, computer dependent)

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)
- quadruple precision (32 digits kept while double precision 16 digits kept, slow, more CPU time, computer dependent)
- QD library (higher precision used when numerical instability occurs)

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)
- quadruple precision (32 digits kept while double precision 16 digits kept, slow, more CPU time, computer dependent)
- QD library (higher precision used when numerical instability occurs)
- DD approach [A. Denner, S. Dittmaier, Nucl.Phys.B734:62-115,2006] (split the PS into different regions, use different expansion, $e^+e^- \to 4f$ electroweak correction, not easy to implement, slow)

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)
- quadruple precision (32 digits kept while double precision 16 digits kept, slow, more CPU time, computer dependent)
- QD library (higher precision used when numerical instability occurs)
- DD approach [A. Denner, S. Dittmaier, Nucl.Phys.B734:62-115,2006] (split the PS into different regions, use different expansion, $e^+e^- \to 4f$ electroweak correction, not easy to implement, slow)
- segmentation [F. Boudjema, A.Semenov, D.Temes, Phys.Rev.D72 (2005) 055024] (easy to implement, quicker)

$$\frac{1}{D_0D_1D_2D_3} \to \frac{a_0}{D_1D_2D_3} + \frac{a_1}{D_0D_2D_3} + \frac{a_2}{D_0D_1D_3} + \frac{a_3}{D_0D_1D_2}$$
$$\hookrightarrow \frac{b_0}{D_1D_2} + \frac{b_1}{D_0D_2} + \frac{b_2}{D_0D_1}$$

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)
- quadruple precision (32 digits kept while double precision 16 digits kept, slow, more CPU time, computer dependent)
- QD library (higher precision used when numerical instability occurs)
- DD approach [A. Denner, S. Dittmaier, Nucl.Phys.B734:62-115,2006] (split the PS into different regions, use different expansion, $e^+e^- \to 4f$ electroweak correction, not easy to implement, slow)
- segmentation [F. Boudjema, A.Semenov, D.Temes, Phys.Rev.D72 (2005) 055024] (easy to implement, quicker)

$$\frac{1}{D_0D_1D_2D_3} \to \frac{a_0}{D_1D_2D_3} + \frac{a_1}{D_0D_2D_3} + \frac{a_2}{D_0D_1D_3} + \frac{a_3}{D_0D_1D_2}$$
$$\hookrightarrow \frac{b_0}{D_1D_2} + \frac{b_1}{D_0D_2} + \frac{b_2}{D_0D_1}$$

- We have Finished implementing and trying all these methods. To tri-boson production, we use QD library and segmentation method. Within integration error, we can get perfect agreements.

Real correction (I): Two Cuts

1) Two cutoff phase space slicing approach: easy to implement

$$\begin{aligned} d\sigma_{real}^{e^+e^-\to W^+W^-Z\gamma} &= d\sigma_{soft}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) + d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s), \\ d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) &= d\sigma_{coll}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c) + d\sigma_{fin}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c) \end{aligned}$$

Soft part: $E_\gamma < \delta_s\sqrt{s}/2 = \Delta E$,

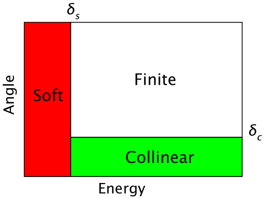
$$d\sigma_{soft} = -d\sigma_{Born}\frac{\alpha}{2\pi^2}\sum_{i,j=1}^{4}\int_{|\mathbf{k}|<\Delta E}\frac{d^3k}{2\omega_k}\frac{\pm p_i p_j Q_i Q_j}{(p_i.k)(p_j.k)}.$$

Collinear part: $\{E_\gamma \geq \Delta E, \cos\theta_{\gamma f} > 1-\delta_c\}$, $\hat{s} = xs$,

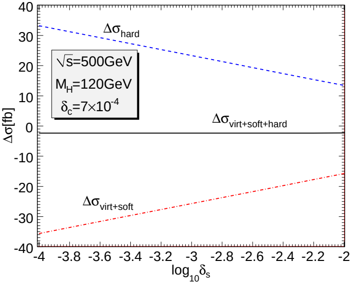
$$d\sigma_{coll} = \sum_{i=1}^{2}\frac{\alpha}{2\pi}Q_i^2\int_0^{1-\delta_s}dx\,d\sigma_{Born}(\hat{s})\left[\frac{1+x^2}{1-x}\ln\frac{\hat{s}\delta_c}{2m_i^2 x} - \frac{2x}{1-x}\right]$$

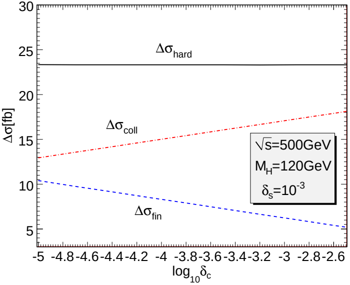
Finite part: $\{E_\gamma \geq \Delta E, \cos\theta_{\gamma f} \leq 1-\delta_c\}$, numerical integration using Monte Carlo BASES.

Real correction (II): Two Cuts



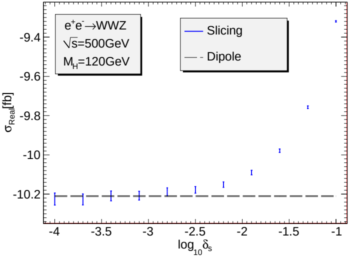
- Real correction is cutoff-independent.
- Factorization condition:δ_S and δ_C are sufficiently small. And $\delta_C \gg 2m_e^2/s$ to use the collinear integration formula.





Real correction (III): Dipole

2) Dipole subtraction approach: to cross check



Non-Linear gauge check

General Non-Linear gauge (NLG) fixing Lagrangian (Boudjema, Chopin 1995):

$$\begin{aligned} L_{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} + \xi_W\frac{g}{2}(v + \tilde{\delta}H + i\tilde{\kappa}\chi_3)\chi^+|^2 \\ &\quad -\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}(v + \tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2 \,. \end{aligned}$$

- Choose $\xi_W = \xi_Z = \xi_A = 1$: propagators are the same as in the linear 't Hooft-Feynman gauge, loop tensor structure is simplest.

$$\frac{1}{k^2 - M_W^2}\left[g_{\mu\nu} - (1 - \xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right]$$

Non-Linear gauge check

General Non-Linear gauge (NLG) fixing Lagrangian (Boudjema, Chopin 1995):

$$
\begin{aligned}
L_{GF} \quad = \quad & -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} + \xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+|^2 \\
& -\frac{1}{2\xi_Z}(\partial.Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2 .
\end{aligned}
$$

- Choose $\xi_W = \xi_Z = \xi_A = 1$: propagators are the same as in the linear 't Hooft-Feynman gauge, loop tensor structure is simplest.

$$\frac{1}{k^2 - M_W^2}\left[g_{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right]$$

- New one-loop Feynman rules: trilinear and quartic gauge-Higgs couplings depend on 5 non-linear gauge parameters $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon})$.

Non-Linear gauge check

General Non-Linear gauge (NLG) fixing Lagrangian (Boudjema, Chopin 1995):

$$
\begin{aligned}
L_{GF} \quad = \quad & -\frac{1}{\xi_W}|(\partial_\mu \; - \; ie\tilde{\alpha}A_\mu \; - \; igc_W\tilde{\beta}Z_\mu)W^{\mu+} + \xi_W\frac{g}{2}(v+\tilde{\delta}H + i\tilde{\kappa}\chi_3)\chi^+|^2 \\
& -\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 \; - \; \frac{1}{2\xi_A}(\partial.A)^2 \; .
\end{aligned}
$$

- Choose $\xi_W = \xi_Z = \xi_A = 1$: propagators are the same as in the linear 't Hooft-Feynman gauge, loop tensor structure is simplest.

$$
\frac{1}{k^2 - M_W^2}\left[g_{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right]
$$

- New one-loop Feynman rules: trilinear and quartic gauge-Higgs couplings depend on 5 non-linear gauge parameters $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon})$.
- Squared amplitude: independent of those parameters (gauge invariant).

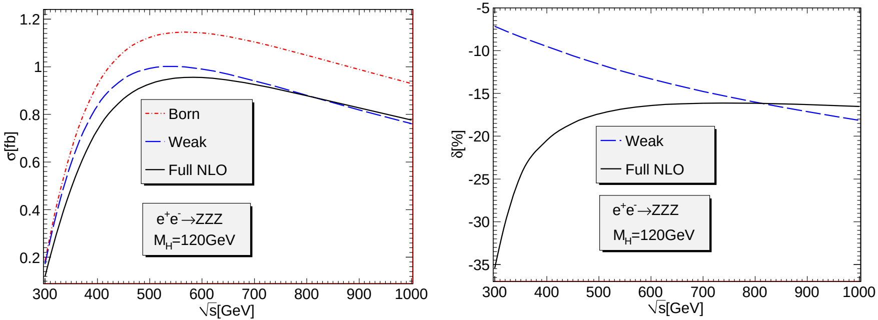
Checks on the results

- gauge invariance check: tree and one-loop squared amplitude level.
- UV and IR finiteness: one-loop squared amplitude level and for the virtual + soft corrections.

$(\tilde{\alpha}, \tilde{\beta})$	ZZZ	WWZ(1)	WWZ(2)
(0,0)	-7.8077709362570481E-4	-6.3768793214220439E-2	5.588092511112647047819820306727217E
(1,0)	-7.8077709362570731E-4	-6.3767676883630841E-2	5.588092511111034991142696308013526E
(0,1)	-7.8077709361534624E-4	-6.3772289648961160E-2	5.588092511114608451016661052972381E

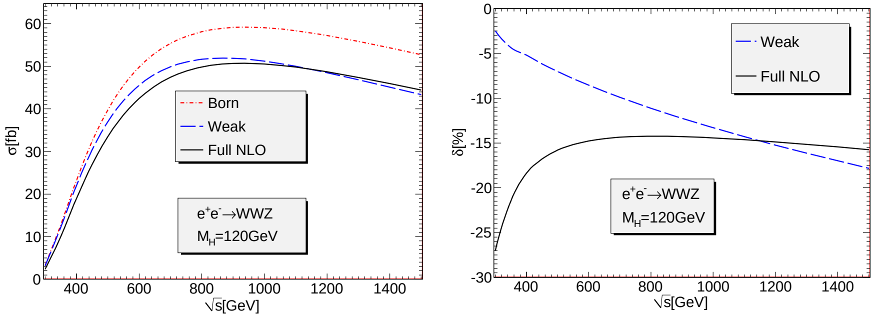
  - *ZZZ*: at least 10 digit agreement at a random point with double precision.
  - *WWZ*: for a DP random point, got only 4 digit agreement. By using QP, got 12 digits. Gauge invariance check is much worse for *WWZ*. This is an indication of numerical instability.
- Two independent calculations: different loop integral libraries/ different photon emission method

$e^+e^- \to ZZZ$: Total Xsection



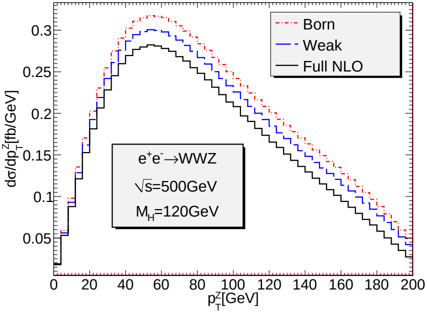
- Total Xsection peak about 1fb is at $\sqrt{s} \approx 550$GeV.
- The weak correction goes from -3.5% to -10% when $\sqrt{s}$ increases from 500GeV to 1TeV.
- The total electroweak corrections can be larger then -15%.

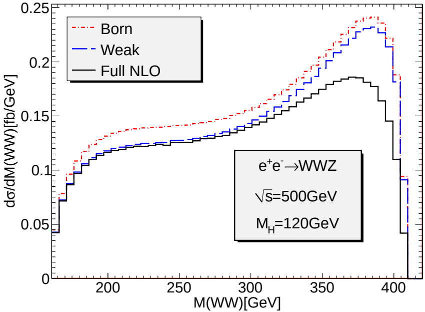
$e^+e^- \to W^+W^-Z$: Total Xsection



- Total Xsection peak about 50fb (50 times larger than σ_{ZZZ}) is at $\sqrt{s} \approx$ 900GeV.
- The weak correction goes from 1.6% to -8.9% when $\sqrt{s}$ increases from 500GeV to 1.5TeV.
- The total electroweak correction larger than -15%, significant and should be taken into account.

$e^+e^- \to W^+W^-Z$: Distributions





- Quite small corrections (less than -5%) at small GeV. At large GeV, large corrections (-30%) due to the hard photon effect [dominant contribution comes from the low-energy photon region (see the δ_S-plot) which corresponds to large p_T^Z and large M_{WW}.]

Conclusions

- Tri-boson production (ZZZ and WWZ) at the ILC is a very important process to test the quartic gauge couplings and the Higss mechanism.

Conclusions

- Tri-boson production (ZZZ and WWZ) at the ILC is a very important process to test the quartic gauge couplings and the Higss mechanism.
- The results of our calculation indicate that the EW corrections are significant and have to be taken into account when doing analysis.

Conclusions

- Tri-boson production (ZZZ and WWZ) at the ILC is a very important process to test the quartic gauge couplings and the Higss mechanism.
- The results of our calculation indicate that the EW corrections are significant and have to be taken into account when doing analysis.
- Our calculation of tri-boson production is also a successful step for our later multi-leg calculations, for example, some $2 \longrightarrow 4$ projects.

Conclusions

- Tri-boson production (ZZZ and WWZ) at the ILC is a very important process to test the quartic gauge couplings and the Higss mechanism.
- The results of our calculation indicate that the EW corrections are significant and have to be taken into account when doing analysis.
- Our calculation of tri-boson production is also a successful step for our later multi-leg calculations, for example, some $2 \to 4$ projects.
- Paper published, and more details can be found arXiv:0912.4234.

Merci!