

# Neutrino mass hierarchies in the double seesaw model

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- 1 Introduction: fermion mass hierarchies
- 2 Neutrino mass hierarchies in the type I seesaw
- 3 Neutrino mass hierarchies in the type I *double* seesaw
- 4 Summary and conclusions

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# Fermion mass hierarchies

## Large hierarchies for charged fermion masses

$$m_t = 171.2 \pm 2.1 \text{ GeV}$$

$$m_c = 1.27_{-0.11}^{+0.07} \text{ GeV}$$

$$m_u = 1.5 - 3.3 \text{ MeV}$$

$$m_t/m_c \sim 125 - 150$$

$$m_c/m_u \sim 350 - 900$$

$$m_b = 4.20_{-0.07}^{+0.17} \text{ GeV}$$

$$m_s = 104_{-34}^{+26} \text{ MeV}$$

$$m_d = 3.5 - 6.0 \text{ MeV}$$

$$m_b/m_s \sim 30 - 60$$

$$m_s/m_d \sim 11 - 37$$

$$m_\tau = 1.78 \text{ GeV}$$

$$m_\mu = 105 \text{ MeV}$$

$$m_e = 0.51 \text{ MeV}$$

$$m_\tau/m_\mu \sim 17$$

$$m_\mu/m_e \sim 205$$

## What about neutrinos?

$$\Delta m_{12}^2 = 7.67 \times 10^{-5} \text{ eV}^2$$

$$|\Delta m_{23}^2| = 2.46 \times 10^{-3} \text{ eV}^2$$

$$1 < h = \frac{m_2}{m_1} < 6$$

$$h' = \frac{m_3}{m_1} > 1$$

( $h = 1$  for degenerate or inverted)

Mild hierarchy for  $\nu$  masses!

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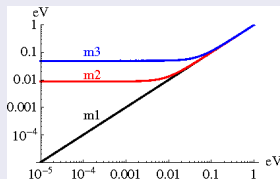
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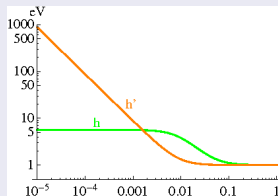
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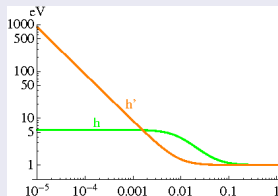
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## Dirac vs Majorana masses?

$$\mathcal{L}^{lepton} = -\bar{\ell}_L H Y_e e_R - \bar{\ell}_L \tilde{H} Y_\nu^\dagger N_R + \text{h.c.}$$

- We do not know the mechanism which generates Yukawa couplings, but it seems it produces them hierarchical (see  $Y_u, Y_d, Y_e$ )
- Hierarchical  $Y_\nu \Rightarrow h \gg 6$   
 $\Rightarrow$  Dirac masses disfavoured from the point of view of hierarchies

## The seesaw mechanism

$$\mathcal{L}^{lepton} = -\bar{\ell}_L H Y_e e_R - \bar{\ell}_L \tilde{H} Y_\nu^\dagger N_R + \frac{1}{2} \bar{N}_R M_N N_R^c + \text{h.c.}$$

- If  $M \gg M_{EW} \Rightarrow m_\nu = \frac{v^2}{2} Y^T M^{-1} Y$   
 $\Rightarrow$  naturally small neutrino masses. What about hierarchies?

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# Hierarches from the seesaw mechanism

## The “naive” seesaw

$$m_1 \sim \frac{v^2}{2} \frac{y_1^2}{M_1} \quad m_2 \sim \frac{v^2}{2} \frac{y_2^2}{M_2} \quad m_3 \sim \frac{v^2}{2} \frac{y_3^2}{M_3} \quad \Rightarrow \quad h = \frac{m_3}{m_2} \sim \frac{y_3^2}{y_2^2} \frac{M_2}{M_3}$$

The seesaw mechanism tends to generate large hierarchies

- All known Yukawa couplings are hierarchical  $\Rightarrow$  also  $Y_{\nu i}$
- Majorana masses: we do not know
  - $M_i$  degenerate  $\rightarrow h \sim (y_3/y_2)^2$  hierarchy is reinforced
  - $M_i$  hierarchical (as  $y_i$ )  $\rightarrow h \sim (y_3/y_2)$   $h$  still hierarchical
  - $M_i$  very hierarchical (as  $y_i^2$ )  $\rightarrow h \sim 1$

## Including mixing effects in the right-handed sector

- Starting in a basis where  $M_N = D_M$

$$m_\nu \propto Y^T D_M^{-1} Y = V_L^* D_Y V_R^T D_M^{-1} V_R D_Y V_L^\dagger$$

- the eigenvalues depend on  $D_M$ ,  $D_Y$  and  $V_R$

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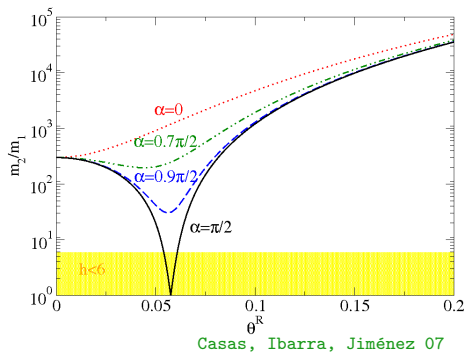
- Varying the  $V_R$  parameters  $\theta_R$  and  $\alpha$ , the hierarchy can vary

$$V_R = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{pmatrix}$$

$$y_1 : y_2 = 1 : 300$$

$$M_1 : M_2 = 1 : 300$$

$$\cos \theta_R = \frac{M_2 y_2^2 - M_1 y_1^2}{(M_1 + M_2)(y_2^2 - y_1^2)} \quad \alpha = \frac{\pi}{2}$$



- Mild hierarchy can be reproduced with fine-tuning of high energy parameters



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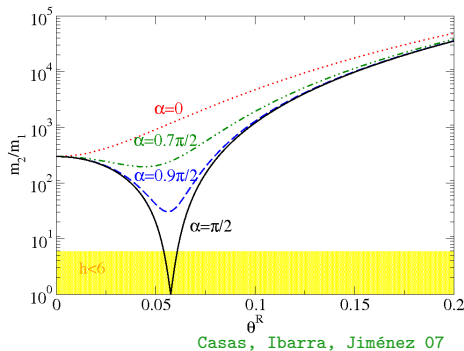
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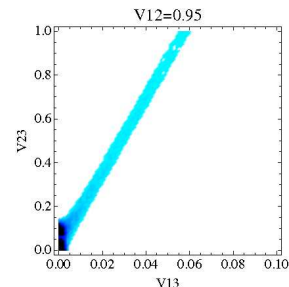
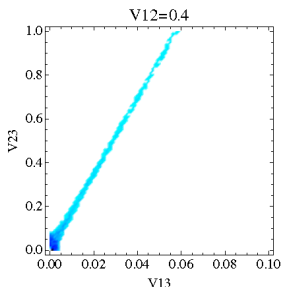
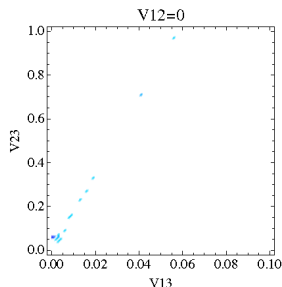


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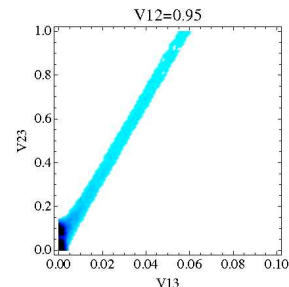
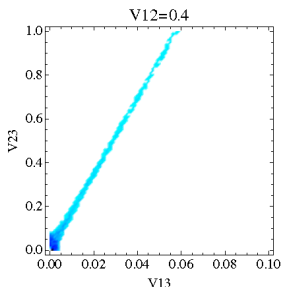
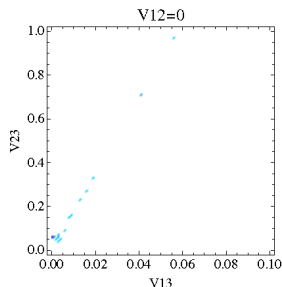
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  - no need of cancellations in the 3 generation case
  - $V_R$  is constrained by the requirement of  $h < 6$  (“CKM-like”)
- Degenerate Majorana masses: soft hierarchy only with cancellations among  $V_R$  elements

Is there any mechanism where mild hierarchies arise more naturally?

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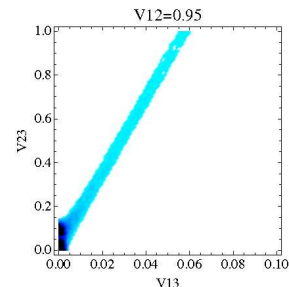
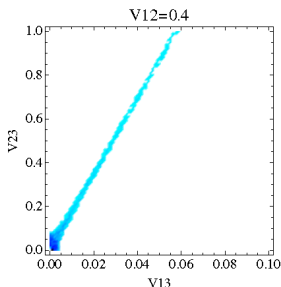
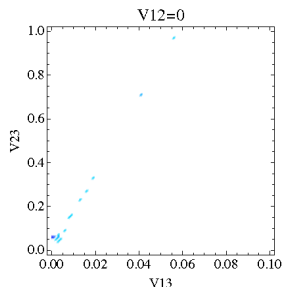
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# The type I *double* seesaw

## The double seesaw

- Add to the SM 3 singlets  $N_R$  and 3 singlets  $S$ :

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- $y_i$  and  $M_i$  are both Dirac parameters  $\Rightarrow$  it's natural they are hierarchical
- $\mu_i$  are Majorana parameters  $\Rightarrow$  can be hierarchical or degenerate
- Ex.:  $\frac{\mu_i}{\mu_j} \sim 1 \quad \frac{y_i}{y_j} \sim \frac{M_i}{M_j} \quad \Rightarrow \quad h \sim 1$

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- Ex.:  $\frac{\mu_i}{\mu_j} \sim 1 \quad \frac{y_i}{y_j} \sim \frac{M_i}{M_j} \quad \Rightarrow \quad h \sim 1$

**The double seesaw naturally gives mild hierarchies!**



$$m_\nu \propto Y^T D_M^{-1} \mu D_M^{-1} Y = V_L^* D_Y V_R^T D_M^{-1} \Omega^\dagger D_\mu \Omega^* V_R D_Y V_L^\dagger$$

## Hierarchical $M$ and degenerate $\mu$

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$$\mu_1 = \mu_2$$

$$h < 6 \text{ for any } \Omega \text{ and } \theta_R \ll \frac{y_1}{y_2}$$

no need of fine-tuning

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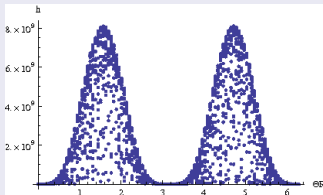
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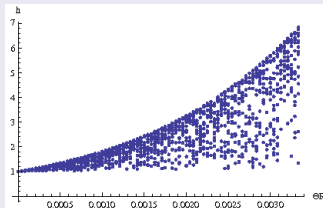
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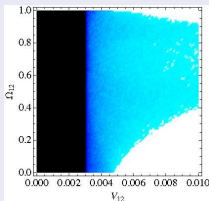
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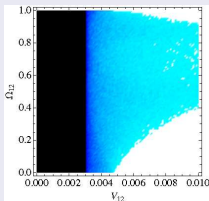
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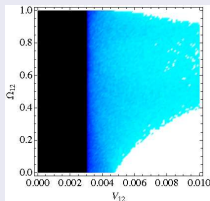
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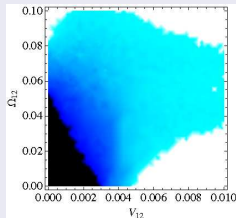
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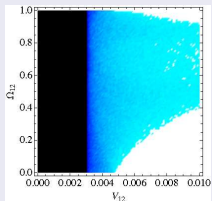
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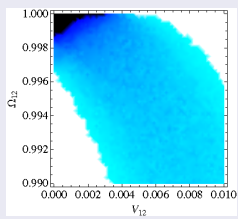
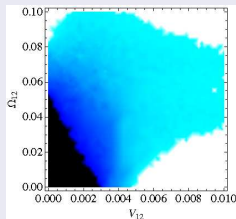
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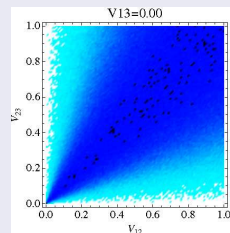


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- constraints on both  $\Omega$  and  $V_R$

EXAMPLE  $\rightarrow$

$$\begin{aligned} \text{hier}(\mu_i) &\sim \text{hier}(y_i) \\ \text{hier}(M_i) &\sim \text{hier}(\sqrt{y_i}) \\ V_{13} &\simeq 0 \\ \Omega &\sim \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

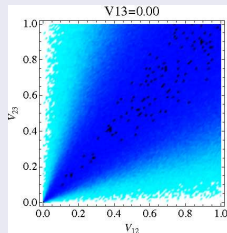


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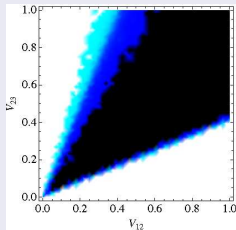
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- 2 Neutrino mass hierarchies in the type I seesaw
- 3 Neutrino mass hierarchies in the type I *double* seesaw
- 4 Summary and conclusions

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