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Introduction du degré de liberté de spin dans les jets de quark

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Why consider spin in quark fragmentation ?

Semi-inclusive deep-inelastic scattering

$$lepton + N\uparrow \;\rightarrow\; q\uparrow \rightarrow\; h$$

can measure $\Delta q(x)$ (helicity) ***and*** $\delta q(x)$ (transversity), provided we have a ***quark polarimeter***

- $q\uparrow \rightarrow \Lambda\uparrow$ +X OK for helicity & transversity, but low statistics.

- $q\uparrow \rightarrow \pi$ (ou 2 π) + X, with *Collins effect*
  = asymmetry in $\phi(\pi) - \phi_S$ or $\phi(\pi-\pi) - \phi_S$
  OK for transversity

- $q\uparrow \rightarrow 2\,\pi$ (ou $3\,\pi$) + X, with *jet handedness*
  = asymmetry in $\phi(\pi_1) - \phi(\pi_2)$
  OK for helicity

$\rightarrow$ we need a jet model for **polarised** quark

following

- Quark fragmentation models *without* spin
- What are Collins effect and jet handedness
- Semi-classical spin mechanism in the string model (Lund $^3P_0$) - application to Collins
- Multiperipheral model with quark exchanges
- How to include vector mesons

Recursive fragmentation model *without* spin

Feynman & Field, Peterson

$q_0 \rightarrow h_1 + q_1$,
$q_1 \rightarrow h_2 + q_2$, etc.

momentum conservation: $\boldsymbol{k}_n = \boldsymbol{p}_{n+1} + \boldsymbol{k}_{n+1}$

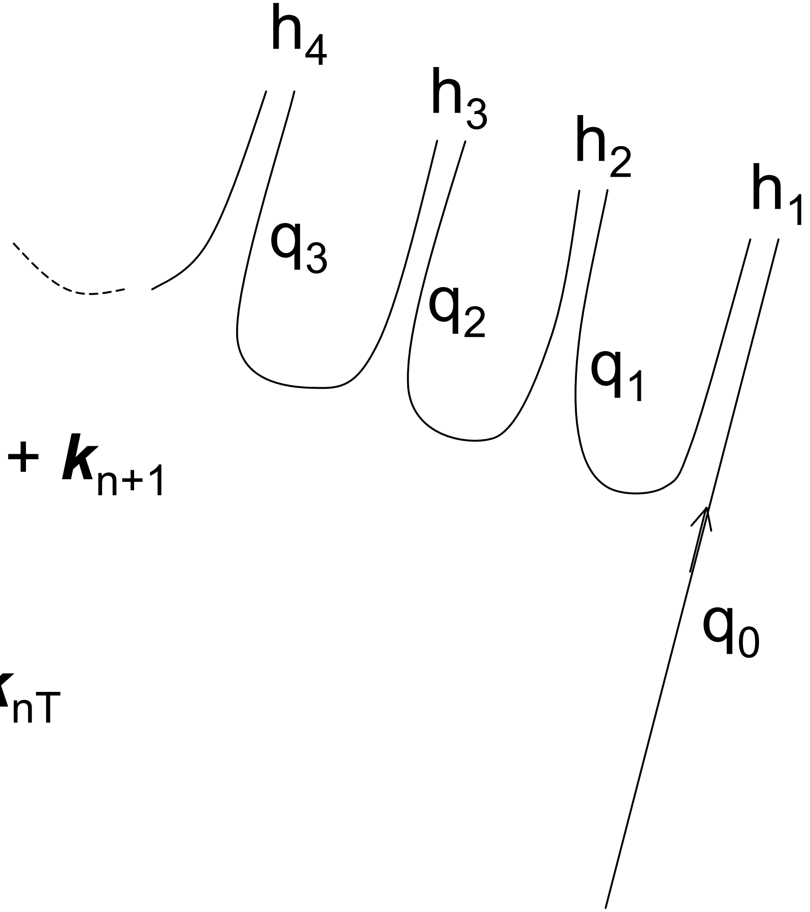
quark *splitting* function :

$$dW = f_n(\zeta_n, \boldsymbol{k}_{n-1,T}, \boldsymbol{k}_{nT})\, d\zeta_n\, d^2\boldsymbol{k}_{nT}$$

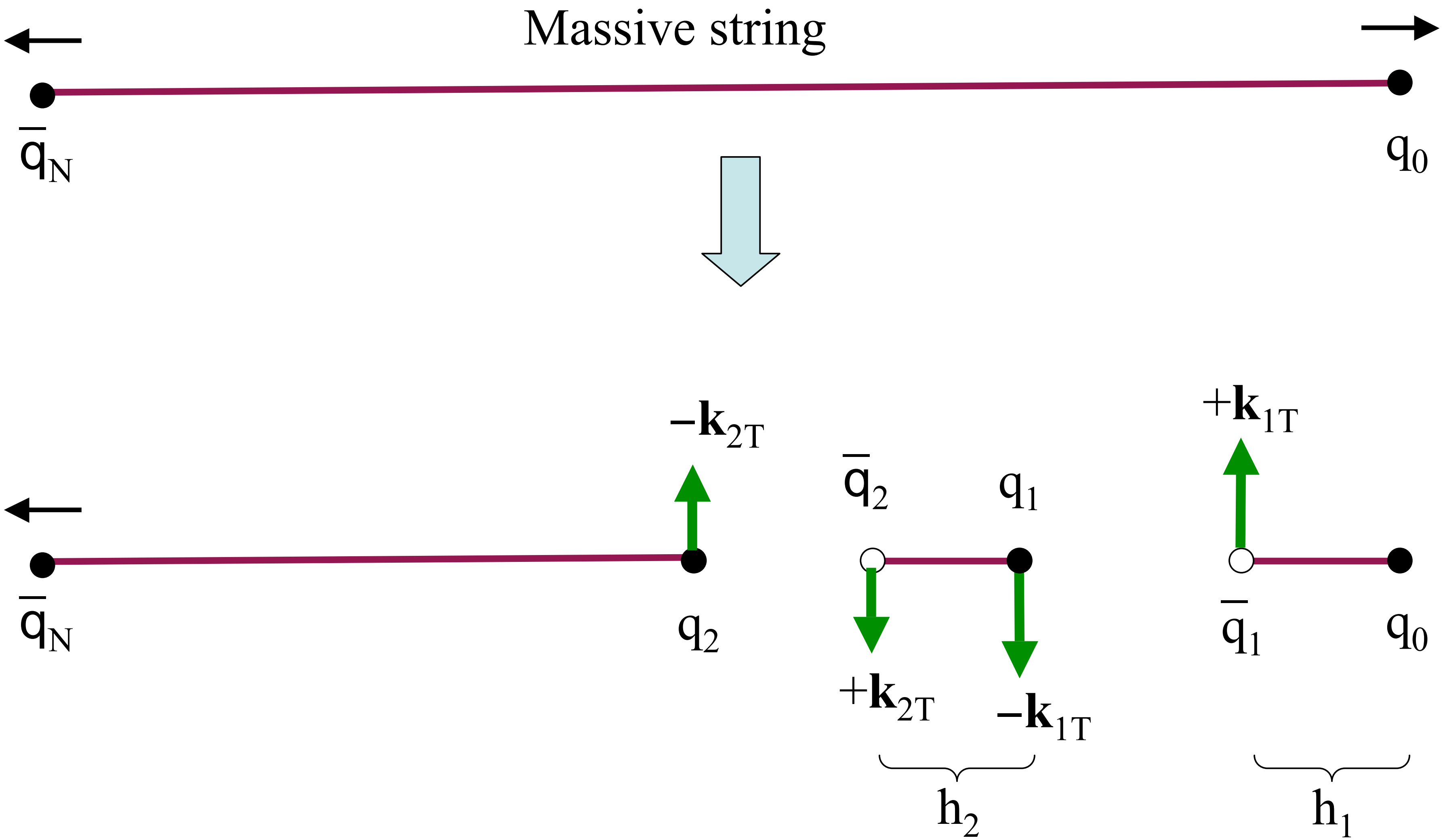
with $\zeta_n = p_{n,z} / k_{n-1,z}$

$\boldsymbol{k}_{nT}$ = transverse momentum of the « left-over » quark q_n.

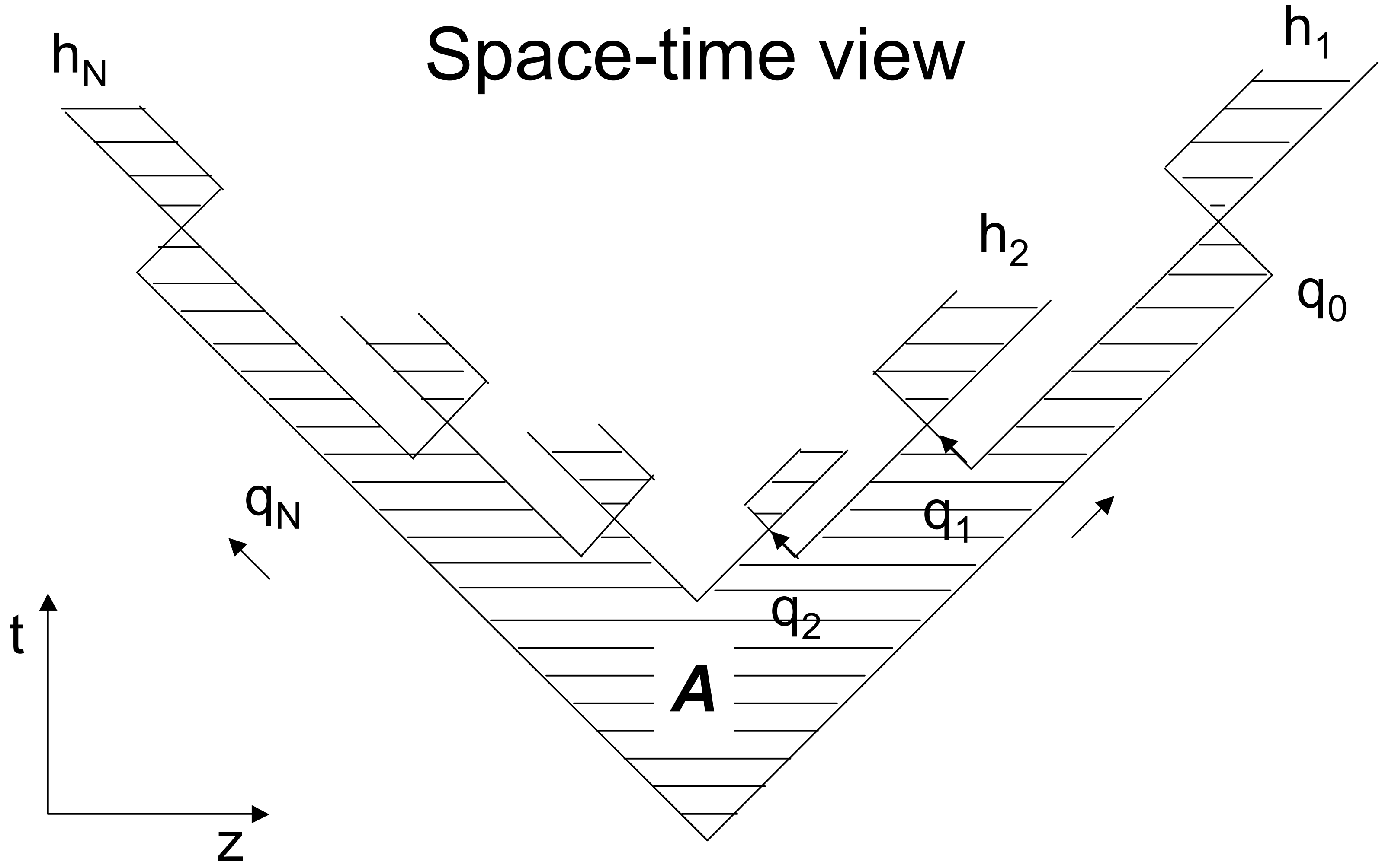
Do not confuse **splitting** function with **fragmentation** function $F(\zeta, \boldsymbol{p}_T)$



Recursive fragmentation in the string model



Space-time view

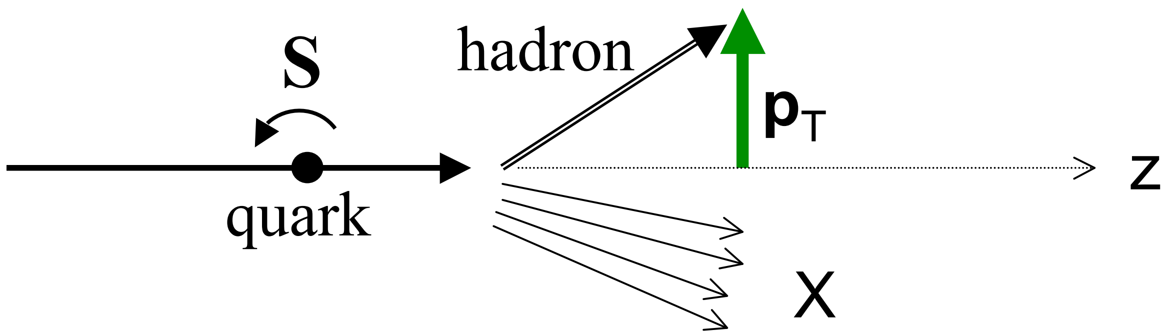


amplitude = exp(-i κ***A***) κ= string tension

probability = exp(- b***A***) b= –2 Im κ

But, up to now, **no quark spin dependence…**

The Collins effect (for transversity)



Transversely polarised fragmentation function:

$$F(z,\mathbf{p}_T;\mathbf{S}_T) = F_0(z,\mathbf{p}_T)\ [\,1+ A_T\ \underbrace{(\mathbf{p}_T\times\mathbf{S}_T)_z\ /p_T}_{\sin(\phi_S - \phi_h)}\,]$$

A_T = analysing power $\quad -1 < A_T < +1$

Jet handedness (for helicity)

(Nachtman ; Efremov, Mankiewicz and Tornqvist)

Longitudinally polarised fragmentation function:

$F(z, \mathbf{p}_T ; z', \mathbf{p}'_T ; \mathbf{S}_L) =$

$F_0(z, \mathbf{p}_T ; z', \mathbf{p}'_T) \times [1+ A_L \mathbf{S}.(\mathbf{p}_T \times \mathbf{p}'_T) / | \mathbf{p}_T \times \mathbf{p}'_T |]$

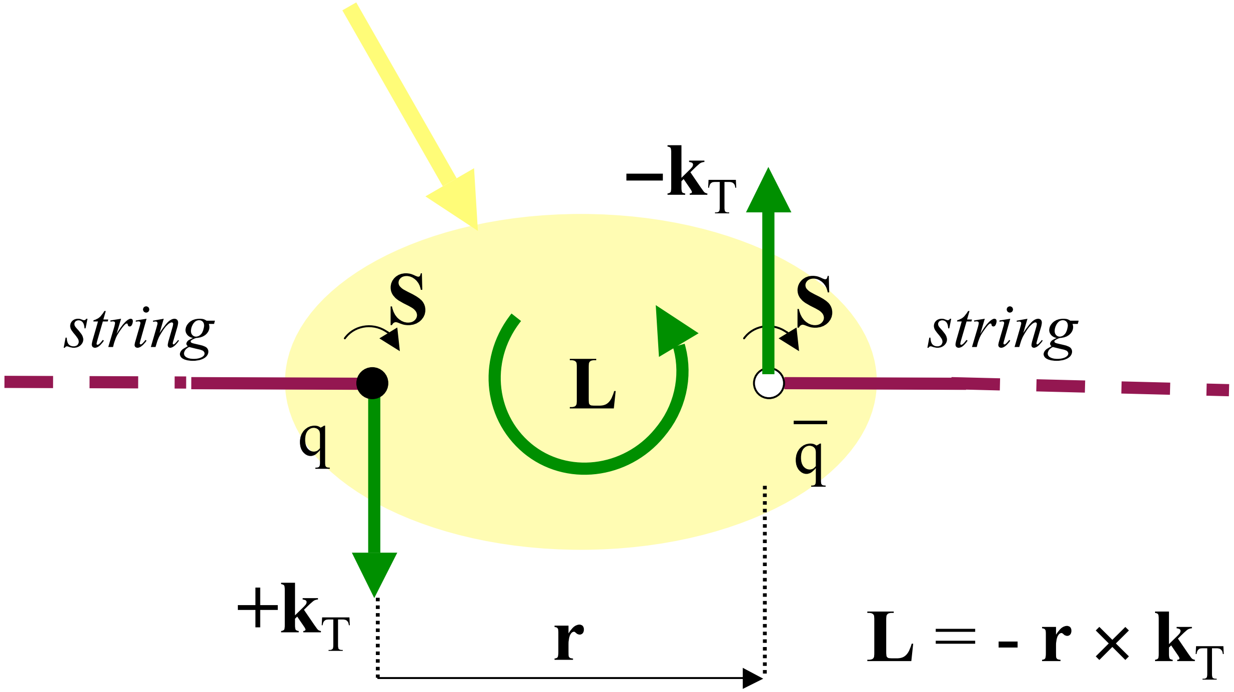
$\mathbf{S}.(\mathbf{p}_T \times \mathbf{p}'_T) / | \mathbf{p}_T \times \mathbf{p}'_T | \rightarrow \sin(\phi'_h - \phi_h)$

with $-1 < A_L < +1$

Collins and jet handedness effects are interesting ***per se***

The Lund $^3P_0$ mechanism in string breaking

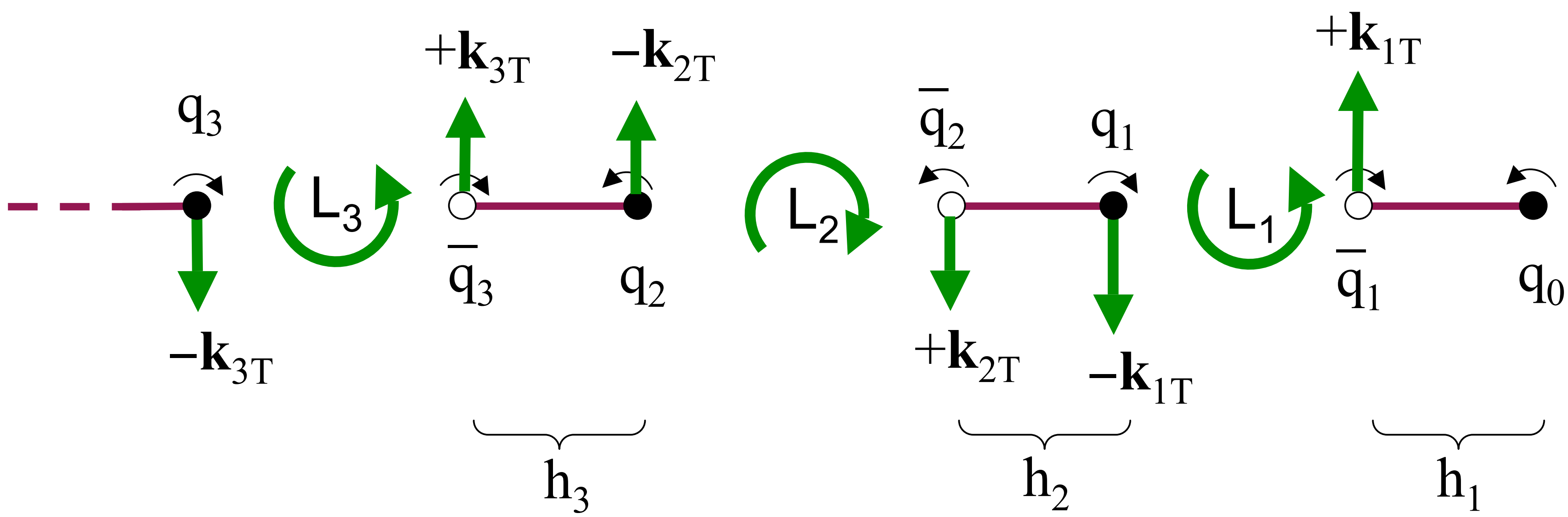
$q\bar{q}$ pair in the $^3P_0$ state (vacuum quantum numbers)



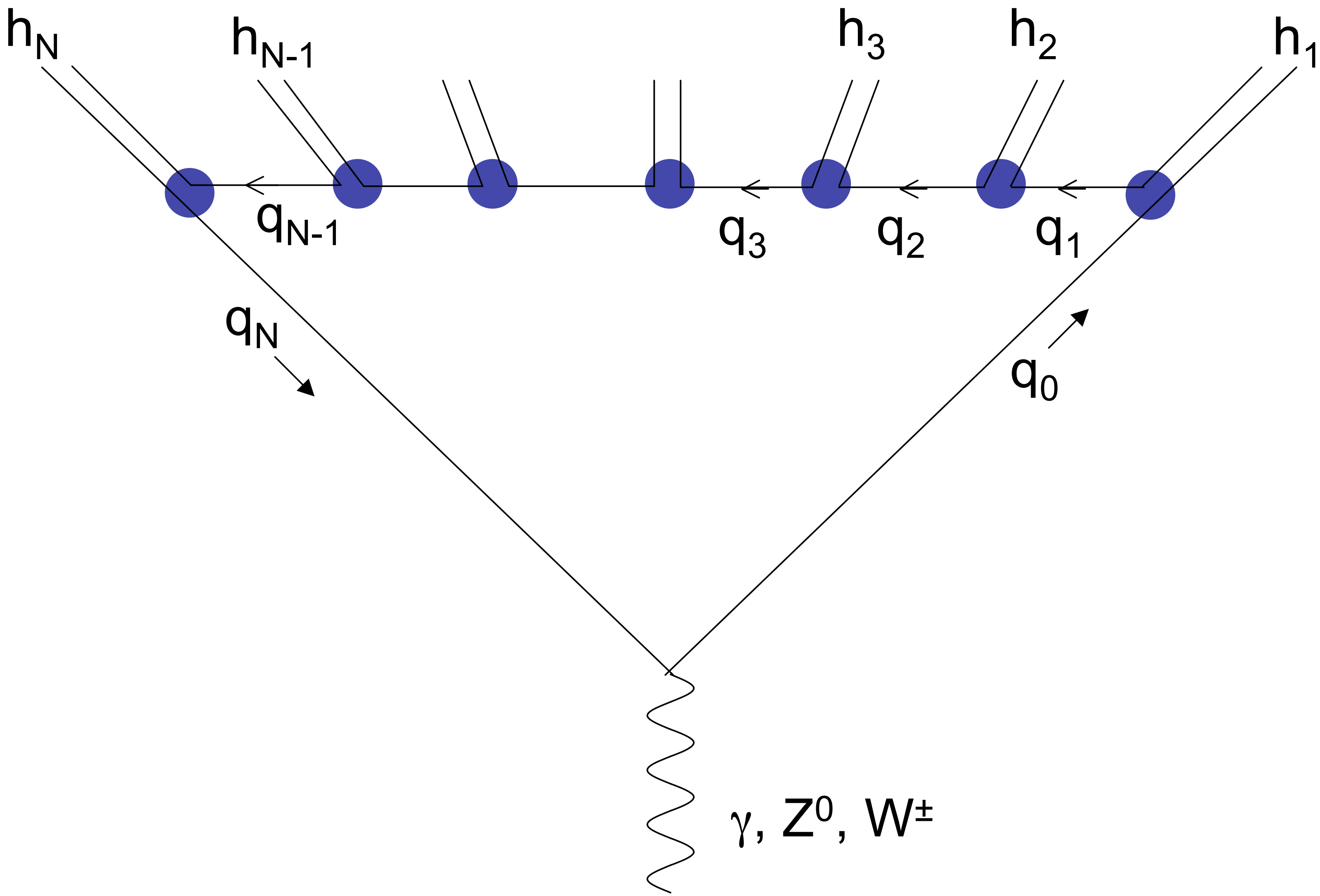
→ correlation between $\mathbf{k}_T$ and $\mathbf{S}_{quark}$

Application to the Collins effect

String decay into **pseudoscalar** mesons :

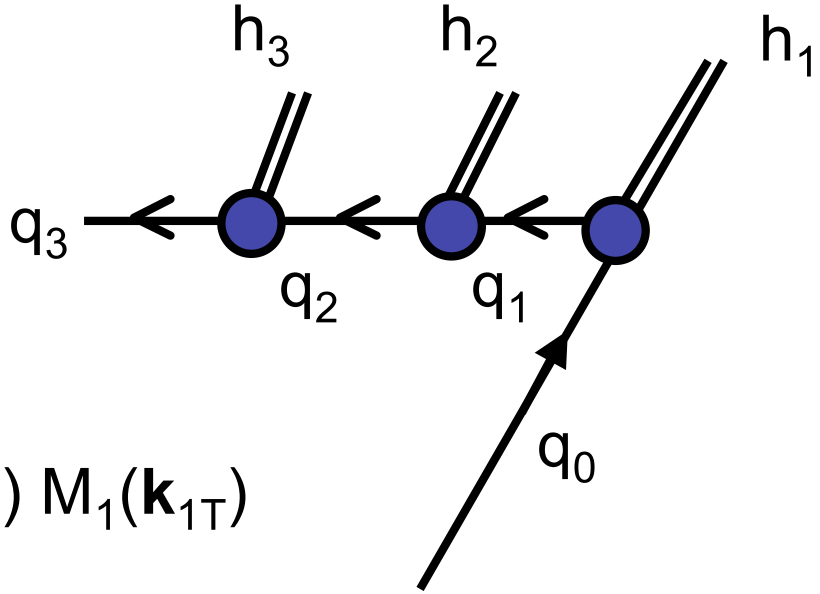


Quantum treatment of quark spin with the multiperipheral model



« Interrupted » multiperipheral amplitude

Simplifications :
- Pauli spinors
- disentangle k_L and $\mathbf{k}_T$
- Gaussian propagators



$$M_{123}\,(\mathbf{k}_{1T}, \mathbf{k}_{2T}, \mathbf{k}_{3T}) = M_3(\mathbf{k}_{3T})\, M_2(\mathbf{k}_{2T})\, M_1(\mathbf{k}_{1T})$$

analogue of $m+\gamma.k$

$$M_n(\mathbf{k}_{nT}) = \exp\{-\,(\mathbf{k}_{nT})^2\}\,(\mu + \sigma\,.\,\mathbf{k}_{nT})\;\sigma_z$$

propagator

pseudoscalar vertex

(analogue of γ_5)

(μ complex)

Joint p_T - distributions of the first n particles

$$J_{12\ldots n}\,(\mathbf{p}_{1T},\,\mathbf{p}_{2T}\,,\ldots\,\mathbf{p}_{nT,}\,)$$

$$= \mathrm{trace}\{\ \underbrace{M(\mathbf{k}_{nT})\ \ldots\ M(\mathbf{k}_{1T})}\ \ \rho_0\ \ M^{\dagger}(\mathbf{k}_{1T})\ \ldots\ M^{\dagger}(\mathbf{k}_{nT})\ \}$$

interrupted amplitude

density matrix of the initial quark = $(1+\sigma.\mathbf{S})/2$

p_T spectrum for the 1rst rank particle

$$J_1(\mathbf{p}_{1T}) = \exp\{-\mathbf{p}_{1T}^2\} \times$$

$$\{ |\mu|^2 + \mathbf{p}_{1T}^2 + \underbrace{2\,\mathrm{Im}(\mu)\,\mathbf{S}.(\mathbf{z}\times\mathbf{p}_{1T})}_{\text{Collins effect}} \}$$

($\mathbf{z}$ = unit vector along the z-axis)

Analysing power :

$$A_T = 2\,\mathrm{Im}(\mu)\,|\mathbf{z} \times \mathbf{p}_{1T}| \;/\; (|\mu|^2 + \mathbf{p}_{1T}^2)$$

The Collins effect is predicted on the same side as in the string model if $\mathrm{Im}(\mu) > 0$

Joint $\mathbf{p}_T$ spectrum for the 1rst and 2nd ranks

$$J_{12}(\mathbf{p}_{1T},\mathbf{p}_{2T}) = \exp\{-\mathbf{p}_{1T}^2 - \mathbf{p}_{2T}^2\} \times \{$$

$$(|\mu|^2 + \mathbf{k}_{1T}^2)(|\mu|^2 + \mathbf{k}_{2T}^2) - 4\,\mathbf{k}_{1T}\,.\,\mathbf{k}_{2T}\,\mathrm{Im}^2(\mu)$$

$$+ 2\,\mathrm{Im}(\mu)\,\mathbf{S}\,.\,(\mathbf{z} \times \mathbf{k}_{1T})\,(2\,\mathbf{k}_{1T}\,.\,\mathbf{k}_{2T} - |\mu|^2 - \mathbf{k}_{2T}^2) \quad \leftarrow \textbf{Collins}$$

$$+ 2\,\mathrm{Im}(\mu)\,\mathbf{S}\,.\,(\mathbf{z} \times \mathbf{k}_{2T})\,(|\mu|^2 - \mathbf{k}_{1T}^2) \quad \leftarrow \textbf{Collins}$$

$$- 2\,\mathrm{Im}(\mu^2)\,\mathbf{S}\,.\,(\mathbf{k}_{1T} \times \mathbf{k}_{2T})\ \} \quad \leftarrow \textbf{handedness}$$

The Collins effect in $\mathbf{k}_{1T}$ and $\mathbf{k}_{2T}$ can be re-grouped in
a *global* C.E. in $\mathbf{k}_{1T} + \mathbf{k}_{2T}$
and a *relative* C.E. in $\mathbf{r}_T = (z_2\,\mathbf{p}_{1T} - z_1\,\mathbf{p}_{2T}) / (z_1+z_2)$

Recursion formula for the cascading quark polarisation (for in Monte-Carlo codes)
1) component normal to the emission plane

Take x-axis along $\mathbf{k}_{nT}$

$$S^y_n = \frac{2|\mathbf{k}_{nT}|\,\mathrm{Im}(\mu) - (|\mu|^2 + \mathbf{k}_{nT}^2)\,S^y_{n-1}}{|\mu|^2 + \mathbf{k}_{nT}^2 - 2\,|\mathbf{k}_{nT}|\,\mathrm{Im}(\mu)\,S^y_{n-1}}$$

2) components in the emission plane

$$\begin{pmatrix} S^x_n \\ S^z_n \end{pmatrix} = D^{-1} \begin{pmatrix} \mathbf{k}_{nT}^2 - |\mu|^2 & -2|\mathbf{k}_{nT}|\,\mathrm{Re}(\mu) \\ -2|\mathbf{k}_{nT}|\,\mathrm{Re}(\mu) & |\mu|^2 - \mathbf{k}_{nT}^2 \end{pmatrix} \begin{pmatrix} S^x_{n-1} \\ S^z_{n-1} \end{pmatrix}$$

helicity is partly converted into transversity and vice-versa (like a rotation in the emission plane).

Jet handedness appears, as the result of two effects :

- partial conversion of $S^z(q_0)$ into $\mathbf{S}^T(q_1)$ along $\mathbf{k}_{1T}$,
- Collins effect in $q_1\uparrow \rightarrow h_2 + q_2$,

Inclusion of spin-1 mesons

Vector meson can be introduced, replacing the pion vertex σ_z by

$$\Gamma = G_L V_z + G_T\, \sigma.\mathbf{V}_T\, \sigma_z$$

V = vector of the meson spin state
(real for linear polarisation)

G_L , G_T = longitudinal and transverse coupling constants
(complex numbers).

Spectrum and linear polarisation of a 1rst-rank ρ meson

$J_1(\mathbf{p}_T,\mathbf{V}) = |G_T|^2 \exp\{-(\mathbf{p}_T)^2\} \times \{$

$(|\alpha|^2 V_z^2 + \mathbf{V}_T^2)\,(|\mu|^2+\mathbf{k}_T^2)$

$-\,4\,\mathrm{Im}(\mu)\,\mathrm{Im}(\alpha)\,V_z\,\mathbf{V}_T.\mathbf{k}_T$

$+\,2\,\mathrm{Im}(\alpha)\,(|\mu|^2+\mathbf{k}_T^2)\,V_z\,\mathbf{V}_T.(\mathbf{z}\times\mathbf{S}_T)$ **(a)**

$-\,4\,\mathrm{Im}(\mu)\,\mathbf{V}_T.\mathbf{k}_T\;\;\mathbf{V}_T.(\mathbf{z}\times\mathbf{S}_T)$ **(b)**

$+\,2\,\mathrm{Im}(\mu)\,(|\alpha|^2 V_z^2 - \mathbf{V}_T^2)\;\;\mathbf{S}_T.(\mathbf{z}\times\mathbf{k}_T)$ **(c)**

$-\;\;4\,\mathrm{Re}(\alpha)\,\mathrm{Im}(\mu)\,V_z\,\mathbf{S}.(\mathbf{V}_T\times\mathbf{k}_T)\;\;\}$ **(d)**

$(\alpha = G_L/G_T)$

The ρ decay products have Collins and jet handedness effects

(a) oblique polarisation in the plane perpendicular to S_T
$\rightarrow$ relative π - π Collins effect.

(b) oblique pol. in transverse plane ; depends on S_T
(a new effect)

(c) global $\pi + \pi$ Collins effect

(d) oblique polarisation in the plane perpendicular to p_T
$\rightarrow \pi \times \pi$ handedness

We obtain qualitatively the same effects as for two successive direct pions **(duality ?)**

Main results

- a very simple model, inspired from the old multiperipheral model of Amati, Fubini and Stanghelini, can implement the spin degree of freedom in quark jet simulation.
- It makes a quantum realisation of the spin effects predicted by the semi-classical string mechanism.
- Jet-handedness and Collins effects are obtained for pions either emitted directly or coming from vector meson decay.
- In the latter case, Collins and handedness effects are associated to oblique linear polarisations of the vector meson.
- Quark spin plays a role even in unpolarized experiments.

Thank you !

Main results (continued)

- In the resonant case, Collins and handedness effects are associated to oblique linear polarisations of the vector meson.
- successive emissions gradually erase the memory of the initial transverse and longitudinal polarisations, but at different rates D_{TT} and D_{LL}.

Discussion

- The model is somewhat over-simplified. The most criticable approximation was to ignore the mass-shell contraints.
- A justification for using Pauli spin instead of Dirac spin would be welcome
- Our method of including the quark degree of freedom could be implemented in Monte-Carlo jet generators like Pythia.

THANK YOU !