Gravitational waves from phase transitions

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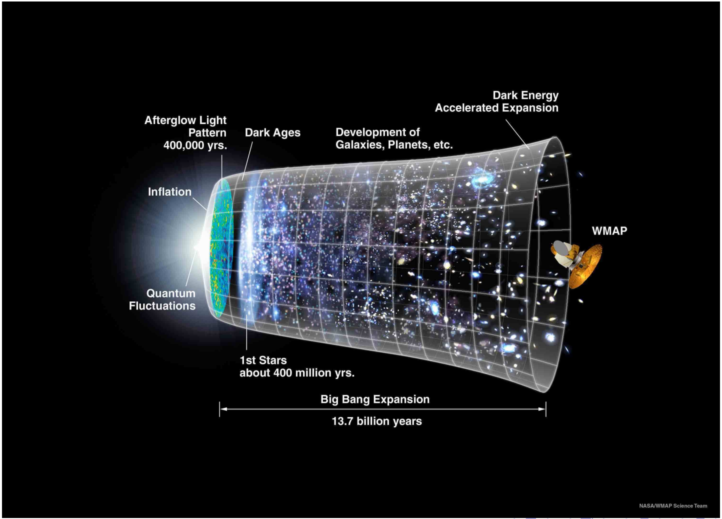
Work in collaboration with: Chiara Caprini, Elisa Fenu, Tina Kahniashvili, Thomas Konstandin, Geraldine Servant
astro-ph/0106244, astro-ph/0304556, astro-ph/0305059, astro-ph/0603476, arXiv:0711.2593, arXiv:0901.1661,arXiv:0906.4772,arXiv:0909.0622

Lyon, janvier 26, 2010

Outline

The Universe has expanded and cooled down from a very hot initial state to (presently) 2.7^oK. It seems likely that it underwent several phase transitions during its evolution of adiabatic expansion.



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In this talk I present a semi-analytical evaluation of the GW signal from a first order phase transition in terms of free parameters.

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- The behaviour of the spectrum on small scales, $k \gg k_*$ depends on the details of the source.

Sources of gravitational waves

Gravitational waves are sourced by fluctuations in the energy momentum tensor which have a non-vanishing spin-2 contribution.

$$ds^2 = a^2 \left(d\eta^2 + (\gamma_{ij} + 2h_{ij}) dx^i dx^j \right)$$

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Einstein's eqn. to first order in h_{ij} give

$$\left(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2\right) h_{ij} = 8\pi G a^2 \Pi_{ij}$$

Here $\Pi_{ij}(\mathbf{k})$ is the Fourier component of the tensors type (spin-2) anisotropic stress and $\mathcal{H} = \frac{a'}{a}$.

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Duration: $\Delta\eta_* < \mathcal{H}^{-1}$, bubble size: $R = v_b \Delta\eta_*$, $v_b =$ bubble velocity.
$k_* = (\Delta\eta_*)^{-1}$ or R^{-1}.

Typical frequencies and the spectrum

- Because of causality, the correlator $\langle \Pi_{ij}(\eta_1, \mathbf{x})\Pi_{lm}(\eta_2, \mathbf{y})\rangle = \mathcal{M}_{ijlm}(\eta_1, \eta_2, \mathbf{x}-\mathbf{y})$ is a function of compact support. For distances $|\mathbf{x}-\mathbf{y}| > \max(\eta_1, \eta_2)$, $\mathcal{M} \equiv 0$.

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- We decompose Π_{ij} into two helicity modes which we assume to be uncorrelated (parity),

$$\Pi_{ij}(\eta, \mathbf{k}) = e^{+}_{ij}\Pi_{+}(\eta, k) + e^{-}_{ij}\Pi_{-}(\eta, k)$$

$$\langle \Pi_{+}(\eta, k)\Pi^{*}_{+}(\eta', k')\rangle = \langle \Pi_{-}(\eta, k)\Pi^{*}_{-}(\eta', k')\rangle = (2\pi)^3\delta^3(\mathbf{k} - \mathbf{k}')\rho_X^2 P(\eta, \eta', k)$$

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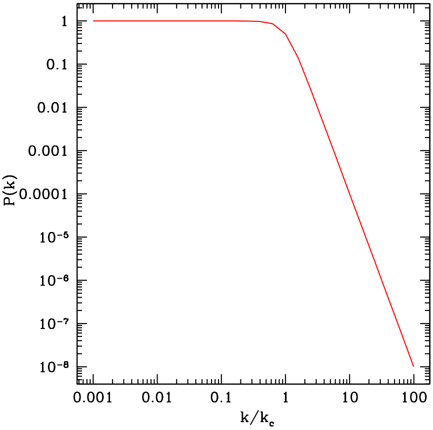
- Causality implies that the function $P(\eta, \eta', k)$ is analytic in k. We therefore expect it to start out as white noise and to decay beyond a certain correlation scale $k_c(\eta, \eta') > \min(1/\eta, 1/\eta')$.


1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
P(k)
0.001
0.01
0.1
1
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100
k/k_c

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- If the Universe is radiation dominated during the phase when the gravitational waves are generated, this gives on large scales, $k < k_c$

$$\frac{d\Omega_{gw}}{d\ln(k)}(\eta_0) = \frac{12\Omega_{\rm rad}(\eta_0)}{\pi^2}\left(\frac{\Omega_X(\eta_*)}{\Omega_{\rm rad}(\eta_*)}\right)^2 \mathcal{H}_*^2 k^3 \mathrm{Re}[P(k,k,k)]\,.$$

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- The behavior or the spectrum close to the peak and its decay rate on smaller scales depends on the source characteristics, on its temporal behavior and its power spectrum.

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- For a source with finite coherence time, $P(\eta,\eta',k) = \sqrt{P(\eta',\eta',k)P(\eta,\eta,k)}\Theta(x_c - |\eta-\eta'|k), \quad x_c \sim 1$ the GW spectrum is again determined by the peak of the spatial Fourier transform of the source.

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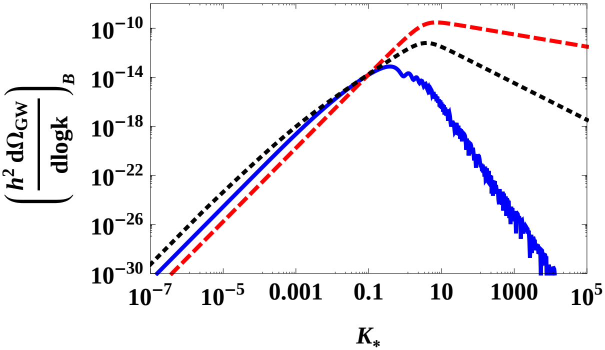

The GW energy density spectrum in the incoherent (red, dashed), tophat (black, dotted) and coherent (blue solid). The parameters are: $T_* = 100$ GeV, $\Delta\eta_*\mathcal{H}_* = 0.01$, $\Omega_X/\Omega_{\rm rad} = 2/9 \quad (\langle v^2\rangle = 1/3)$

. Caprini, RD and Servant, 2009, arXiv:0909.0622

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  The spectrum is supposed to peak at the correlation scale $k_c = \beta \simeq 100/\eta_* \sim 10^{-3}$Hz, which is close to the frequency of the peak sensitivity for the space born gravitational wave antenna LISA, proposed for launch in 2018, a ESA cosmic vision project.

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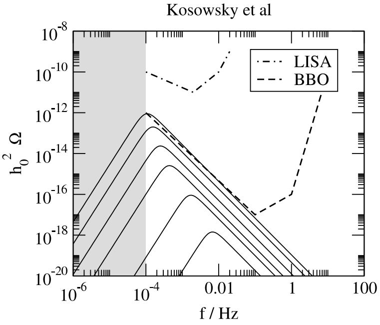

Huber & Konstandin 2008

Ω_{GW} from colliding bubbles, numerical results, $\Omega_X/\Omega_{\rm rad} = 0.03$.

The electroweak phase transition: GW's from turbulence and magnetic fields

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- Because both, the vorticity and the magnetic field are divergence free, causality requires that both, $P_v(k)$ and $P_B(k) \propto k^2$ for small k.
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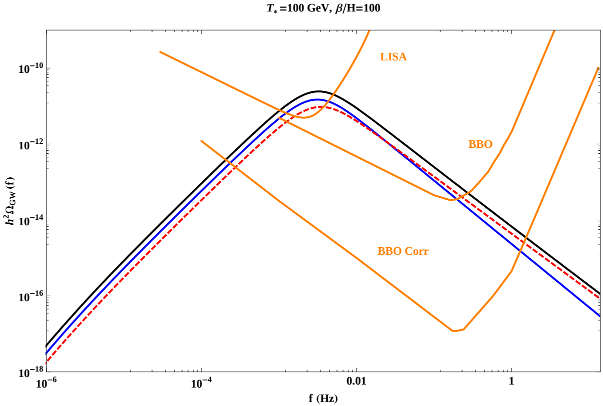
- For the induced GW spectrum this yields

$$\frac{d\Omega_{GW\bullet}(k,\eta_0)}{d\ln(k)} \simeq \Omega_{\rm rad}(\eta_0)\left(\frac{\Omega_\bullet(\eta_*)}{\Omega_{\rm rad}(\eta_*)}\right)^2 \times \begin{cases} (k/k_c)^3 & \text{for } k < k_c \\ (k/k_c)^{-\alpha} & \text{for } k > k_c \end{cases}$$

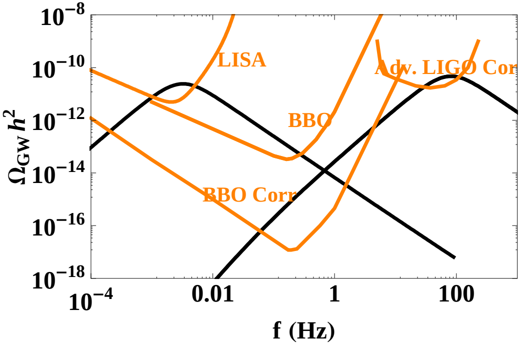
For $\bullet = v$ we have $\alpha = 11/3 - 1 = 8/3$ and for $\bullet = B$ we have $\alpha = 7/2 - 1 = 5/2$. (See Caprini & RD, 2006)

The electroweak phase transition: GW's from turbulence and magnetic fields



Caprini, RD, Servant, arXiv:0909.0622

Ω_{GW} from magnetic fields (red) and turbulence (blue), total (black). Modelling the time-decorrelation of the source (Kraichnan decorrelation) by a 'top-hat' in Fourier space. Sensitivity curves from A. Buonanno 2003.



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We also consider a phase transition at $T = 5 \times 10^6$ GeV with $\Delta\eta_*\mathcal{H}_* = 0.02$.

The electroweak phase transition: GW's from turbulence and magnetic fields

It is difficult to estimate $\Omega_B(\eta_*)$ or $\Omega_v(\eta_*)$, but since causality requires the spectra to be so blue, $\frac{d\Omega_B(k,\eta_*)}{d\ln(k)} \propto k^5$, the limit on gravitational waves (which comes from small scales $k \simeq k_c$ yields very strong limits on primordial magnetic fields on large scales.

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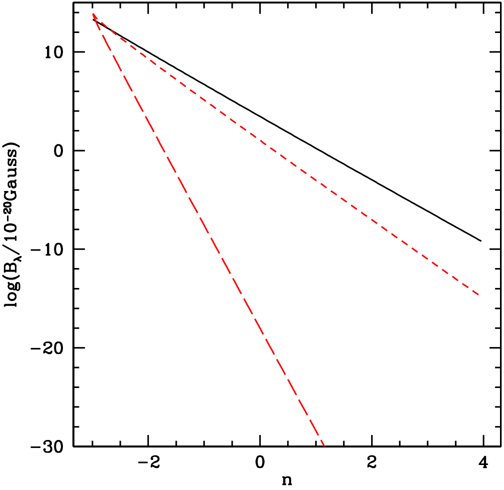
This is purely a consequence of the redness of the magnetic field spectrum. The simple requirement that $\rho_B(t_*) < \rho_c(t_*)$ yields

$$k^{3/2}B(k) < 10^{-29}\mathrm{Gauss}\left(\frac{k\cdot 0.1\mathrm{Mpc}}{k_*\cdot 10^3\mathrm{sec}}\right)^{5/2} .$$

$(10^3\text{sec} = 10^{-11}\text{Mpc})$

The electroweak phase transition: GW's from turbulence and magnetic fields



Caprini & RD., 2001, astro-ph/0106244

- The most optimistic dynamo models for the generation of the magnetic fields of about 10^{-6}Gauss observed in galaxies and clusters, need seed fields of the order of at least 10^{-21}Gauss, which are excluded by this limit.

The electroweak phase transition: helical magnetic fields and parity violation

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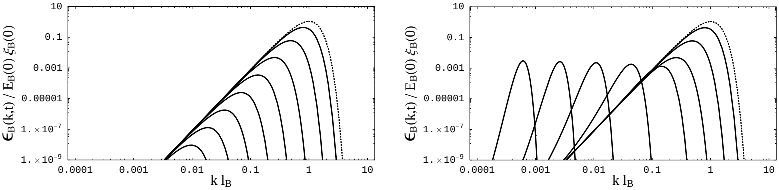
- This relates the baryon number the the magnetic helicity.
- Such helical magnetic fields lead to T-B and E-B correlations in the CMB, and they also generate gravitational waves with non-vanishing helicity (Caprini, Kahniashvili, RD. 2004, astro-ph/0304556).

- Contrary to a non-helical magnetic field, helicity conservation for a helical field does lead to an inverse cascade in the evolution of the magnetic field:



Campanelli, 2007

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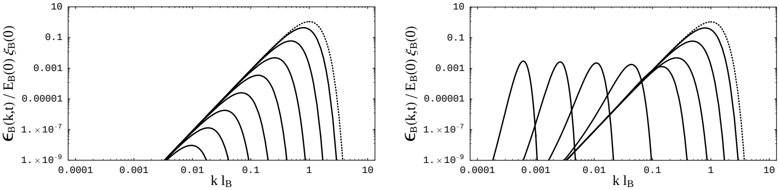

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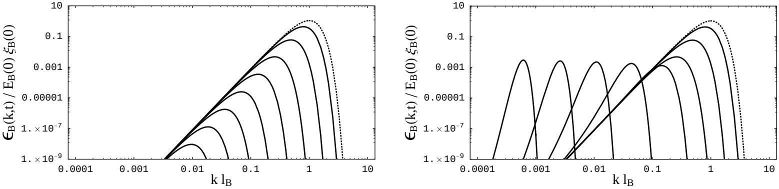

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- In this case, the GW background would not be parity symmetric. There would be more GW's of one helicity than of the other.

Conclusions

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- The spectrum grows like $\frac{d\Omega_{GW}(k,t_0)}{d\ln(k)} \propto k^3$ on large scales and decays on scales smaller than the correlations scale $k_c \sim 1/\eta_*$. The decay law depends of the physics of the source.

Conclusions

- If the SM holds, the electroweak phase transition is not of first order and does (probably) not generate an appreciable gravitational wave background. However, simple deviations from the SM can make it first order (like adding a Higgs singlet (Ashoorioon & Konstandin 2009).

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- In this case we also expect a parity violating gravitational wave background, $|h_+(k)|^2 \neq |h_-(k)|^2$.