Associated production of charged Higgs and top at LHC

the role of the complete electroweak supersymmetric contribution

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Outline

1. Framework: the MSSM
2. Motivations
3. The process $PP \longrightarrow tH^{-} + X$ at EW NLO

Supersymmetry

Symmetry between bosons and fermions: $|\mathbf{B}\rangle \leftrightarrow |\mathbf{F}\rangle$

Minimal Supersymmetric Standard Model

Simplest, phenomenologically acceptable supersymmetric extension of the SM

Superpotential: $W = \mu\epsilon_{ij}H_u^i H_d^j + \epsilon_{ij}Y_l^{IJ}H_d^i L_j^I E^J + \epsilon_{ij}Y_d^{IJ}H_d^i Q_j^I D^J + \epsilon_{ij}Y_u^{IJ}H_u^i Q_j^I U^J$

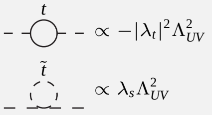
The spectrum is **doubled** →

SM	gauge bosons	fermions	Higgs bosons*
MSSM	gauginos	sfermions	higgsinos

*2 Higgs doublets because the superpotential is an analytic function

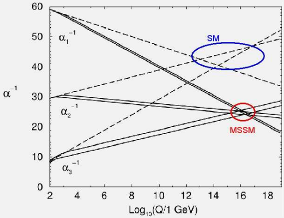
Same supermultiplet → **same gauge quantum numbers**; **same mass**

Hierarchy problem



$\Delta m_H^2 \propto (\lambda_s - |\lambda_t|^2)\Lambda_{UV}^2$

Gauge coupling unification



Dark matter candidate

The Neutralino

χ_0

Provided it is the lightest SUSY particle and it is stable

Supersymmetry Breaking

No experimental evidence of SUSY particles up to now

(where is the superpartner of the electron?)

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Explicit soft breaking terms

Soft = positive mass dimensions (not to spoil the solution to the hierarchy problem)

- **mass terms for scalar fields:**

$$\begin{aligned} \text{Higgs}: &\quad -m_{H_d}^2 (H_d^*)_i (H_d)_i - m_{H_u}^2 (H_u^*)_i (H_u)_i \\ \text{Sleptons}: &\quad -(m_L^2)^{IJ} L_i^{I*} L_i^J - (m_E^2)^{IJ} E^{I*} E^J \\ \text{Squarks}: &\quad -(m_Q^2)^{IJ} Q_i^{I*} Q_i^J - (m_D^2)^{IJ} D^{I*} D^J - (m_U^2)^{IJ} U^{I*} U^J \end{aligned}$$

- **mass terms for gauginos:**

$$-\frac{1}{2} M_1 \lambda_B \lambda_B - \frac{1}{2} M_2 \lambda_A^i \lambda_A^i - \frac{1}{2} M_3 \lambda_G^a \lambda_G^a + h.c.$$

- **bilinear and trilinear couplings of scalar fields:**

$$m_{12}^2 \epsilon_{ij} (H_d)_i (H_u)_j + \epsilon_{ij} A_l^{IJ} (H_d)_i L_j^I E^J + \epsilon_{ij} A_d^{IJ} (H_d)_i Q_j^I D^J + \epsilon_{ij} A_u^{IJ} (H_u)_i Q_j^I U^J$$

The mSUGRA scenario

The MSSM is not so safe...

- 105 new parameters $\longrightarrow$ loss of predictivity
- Dangerous terms in the MSSM soft Lagrangian $\longrightarrow$ CP violation, FCNC

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Possible solution - Universality of Soft parameters at some high energy scale Q:

$$(m^2_Q) = m^2_Q\hat{1} \quad (m^2_U) = m^2_U\hat{1} \quad (m^2_D) = m^2_D\hat{1} \quad (m^2_L) = m^2_L\hat{1} \quad (m^2_E) = m^2_E\hat{1}$$

$$A_u \propto -Y_u \quad A_d \propto -Y_d \quad A_l \propto -Y_l$$

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mSUGRA

SUSY breaking in a "hidden sector" which communicates to the "visible sector" through gravity

Gauge coupling unification at GUT scale M_X ⟶ universality of soft parameters at M_X

- **Universal gaugino mass:** $M_1(M_X) = M_2(M_X) = M_3(M_X) \equiv \mathbf{m_{1/2}}$
- **Universal scalar mass:** $m^2_{\tilde{Q}}(M_X) = m^2_{\tilde{U}}(M_X) = m^2_{\tilde{D}}(M_X) = m^2_{\tilde{L}}(M_X) = m^2_{\tilde{E}}(M_X) = m^2_{H_u}(M_X) = m^2_{H_d}(M_X) \equiv \mathbf{m_0^2}$
- **Universal trilinear coupling:** $A_u(M_X) = A_d(M_X) = A_l(M_X) \equiv \mathbf{A_0}$
- **Bilinear coupling of the Higgs:** $m^2_{1/2}(M_X) = \mathbf{B_0}\mu$

+ E.W. breaking conditions $\Longrightarrow$ $\boxed{\tan\beta (\equiv v_u/v_d) \quad m_{1/2} \quad m_0 \quad A_0 \quad \mathrm{sign}(\mu)}$

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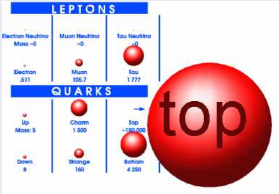
The Top Quark

Identity Card

Discovery	Fermilab – 1995
Decay channel	t $\longrightarrow$ Wb $\sim$ 100%
Lifetime	$\tau \sim$ 0.5 $\times$ 10^{-24} sec $\Lambda_{QCD}^{-1} \gg \tau \Rightarrow$ no top hadrons
Mass	171.3 GeV **The heaviest known particle**
Spin	1/2
Isospin	+1/2
Charge	+2/3



Special Features

- Its **mass** is close to the **electroweak scale**
- **Sizable Yukawa coupling** with the Higgs boson

$\Big\}\longrightarrow$ relevant role in EWSB?

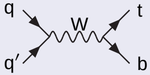
- Top properties are very sensitive to **New Physics corrections**
  $\longrightarrow$ associated production with H^-

Single Top at the LHC

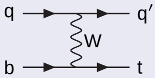
Standard Model Processes

	Tevatron Run II $p\bar{p}$, 1.96 TeV	LHC pp, 14 TeV
A S-Channel	$\sigma \sim$ **0.9 pb**	$\sigma \sim$ **10 pb**
B T-Channel	$\sigma \sim$ **2.0 pb**	$\sigma \sim$ **240 pb**
C tW Production	$\sigma \sim$ **0.12 pb**	$\sigma \sim$ **60 pb**

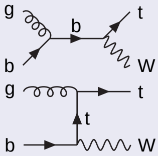
A S-Channel



B T-Channel



C tW Production



At the LHC

$\sigma^{\text{tot}}_{\text{single top}} \sim 310$ pb

$\Downarrow$

$\sim$ 3 millions events (10 fb^{-1}/year)

The LHC will be a Top factory and will allow precision measurements of its properties

The Charged Higgs

Testing the Higgs Sector at the LHC

The discovery of a neutral Higgs would not represent conclusive evidence of BSM physics

Two Higgs Doublets Models (2HDM)
MSSM } → **Extended Higgs sector** → **Charged Higgs**

The discovery of a charged Higgs would signal the existence of new physics!

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$$\left.\begin{array}{c}\text{Two Higgs Doublets Models (2HDM)}\\ \text{MSSM}\end{array}\right\} \longrightarrow \text{Extended Higgs sector} \longrightarrow \text{Charged Higgs}$$

The discovery of a charged Higgs would signal the existence of new physics!

Producing and detecting a charged Higgs at the LHC

$m_{H^-} < m_t - m_b$	$\longrightarrow$	**Top decay**	$\mathbf{t \longrightarrow bH^+}$
$m_{H^-} > m_t - m_b$	$\longrightarrow$	**associated production with heavy quarks**	$\mathbf{bg \longrightarrow tH^-}$

Relevant role of tanβ in the Yukawa coupling with quarks

Decay channels: $\left.\begin{array}{l} H^+ \longrightarrow \tau^+\nu_\tau \\ H^+ \longrightarrow t\bar{b} \end{array}\right\}$ BR's are model-dependent

In the process $bg \longrightarrow tH^-$

2HDM structure can be tested already at tree level

SUSY corrections can be tested only at loop level

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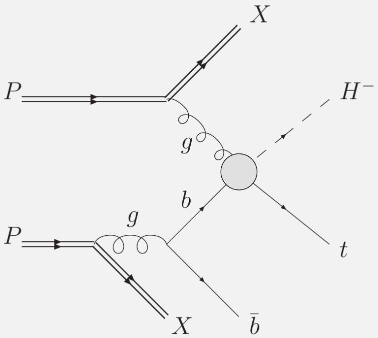
The initial state

4-flavour scheme

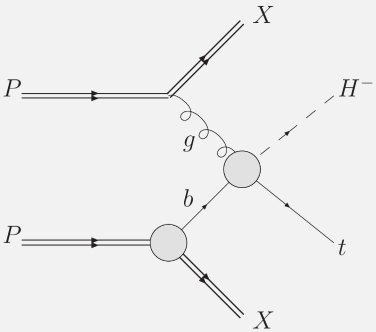
gluon splitting in bottom-antibottom



- three-particle final state
- large logarithms due to collinearities

5-flavour scheme

bottom parton distribution function



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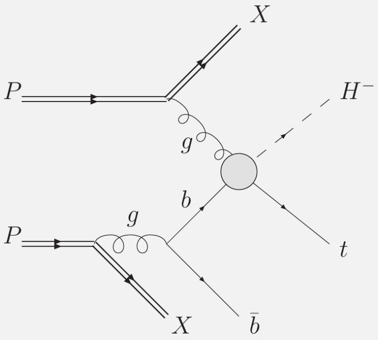

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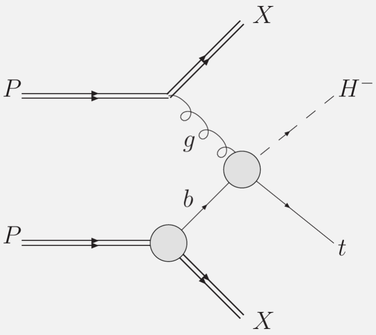

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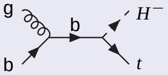
The Born terms

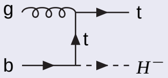
Amplitude

A S-Channel bottom exchange



$$A_s^{\text{Born}} = -\bar{u}(t)\left[c^L_{b\to tH^-}P_L + c^R_{b\to tH^-}P_R\right]\frac{(\not{q}+m_b)}{s-m_b^2}\, g_s\left(\frac{\lambda^l}{2}\right)\not{\epsilon}\; u(b)$$

B Top exchange in the u-channel



$$A_u^{\text{Born}} = -\bar{u}(t)\not{\epsilon}\;\frac{(\not{q}\,'+m_t)}{u-m_t^2}\left[c^L_{b\to tH^-}P_L + c^R_{b\to tH^-}P_R\right] g_s\left(\frac{\lambda^l}{2}\right)u(b)$$

Born level helicity amplitudes

(derived by expanding the t and b spinors and the gluon polarization vector)

$$F_{\lambda_b\lambda_g\lambda_t} = \sum_{\substack{\eta=L,R\\ k=1,2}} N_k^\eta \mathcal{H}^\eta_{k,\ \lambda_b\lambda_g\lambda_t}$$

where $N_{1\ \eta}^{\text{Born s}} = -g_s\left(\frac{\lambda^l}{2}\right)\frac{c^\eta_{b\to tH^-}}{s-m_b^2}$, $N_{1\ \eta}^{\text{Born u}} = -g_s\left(\frac{\lambda^l}{2}\right)\frac{c^\eta_{b\to tH^-}}{u-m_t^2}$ and $N_{2\ \eta}^{\text{Born u}} = -2g_s\left(\frac{\lambda^l}{2}\right)\frac{c^\eta_{b\to tH^-}}{u-m_t^2}$

Partonic differential cross-section

$$\frac{d\sigma_{bg\to tH^-}}{d\cos\theta} = \frac{p_f}{768\pi\ s\ p_i}\sum_{\lambda_b,\lambda_g,\lambda_t}\left|F_{\lambda_b,\lambda_g,\lambda_t}\right|^2$$

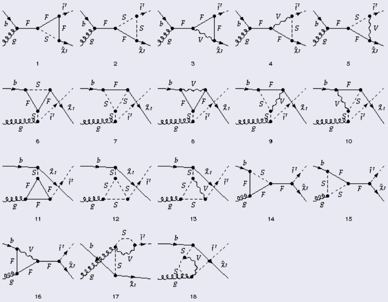
One-loop description

Loop Diagrams

- counter terms for external lines, coupling constants and mixing elements
- self-energy corrections
- s-channel left and right triangles
- u-channels up and down triangles
- direct, crossed and twisted boxes

Checks:

- UV divergences
- IR divergences (next slide)
- Logarithmic behaviour: Sudakov expansion



Differential cross section

$$\frac{d\sigma(PP \overset{bg}{\to} tH^{-} + X)}{dM_{tH-}} = \int dx_1 dx_2 d\cos\theta[\ b(x_1,\mu)g(x_2,\mu) + g(x_1,\mu)b(x_2,\mu)\ ] \times \frac{d\sigma_{bg\to tH-}}{d\cos\theta}\delta(\sqrt{x_1 x_2 S} - M_{tH-})$$

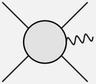
PdF's: CTEQ6L with $\mu = m_t + m_{H-}$

QED radiation

Virtual contributions: IR-divergent → regularized through a small photon mass m_γ

Real contributions:

- **Soft contribution** → $m_\gamma < E_\gamma \leq \Delta E^{\max}_{\gamma_{\rm soft}}$
- **Hard contribution** → $\Delta E^{\max}_{\gamma_{\rm soft}} < E_\gamma$



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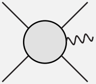

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IR divergences

Virtual + Soft (δ_s): $\left(\mathcal{A}^{\text{Born}}\right)^2 \left(1 + \frac{\alpha}{2\pi}\delta_s\right) + 2\mathcal{A}^{\text{Born}}\mathcal{A}^{1-\text{loop}} =$ IR finite

The cancellation of log terms containing m_γ has been numerically checked for $m_\gamma \to 0$

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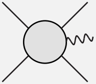

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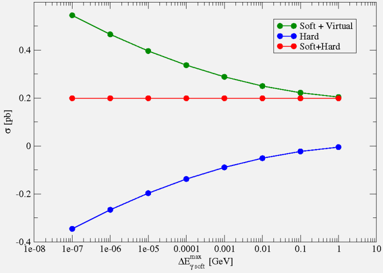
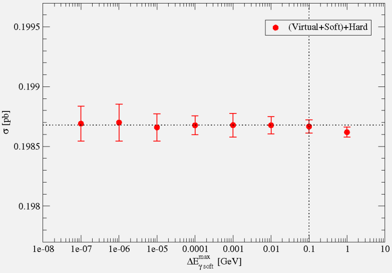
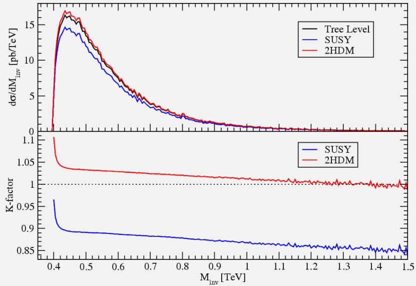
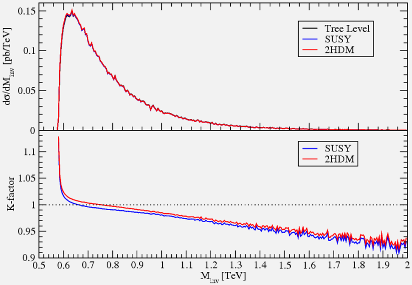
The final cross-section must be independent of $\Delta E^{\max}_{\gamma\text{soft}}$:





One-loop results





LS2 $(\tan\beta = 50 \quad m_{H^-} = 229.6\ GeV)$

SUSY $\neq$ 2HDM

$$\sigma^{\rm Born}_{\rm Tot} = 3.747\ pb$$

$\sigma^{\rm SUSY}_{\rm Tot} = 3.331\ pb$ **K-factor: 0.889**

$\sigma^{\rm 2HDM}_{\rm Tot} = 3.863\ pb$ **K-factor: 1.031**

SPS1 $(\tan\beta = 10 \quad m_{H^-} = 412.1\ GeV)$

SUSY $\approx$ 2HDM

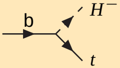
$$\sigma^{\rm Born}_{\rm Tot} = 0.03793\ pb$$

$\sigma^{\rm SUSY}_{\rm Tot} = 0.03738\ pb$ **K-factor: 0.985**

$\sigma^{\rm 2HDM}_{\rm Tot} = 0.03762\ pb$ **K-factor: 0.992**

tanβ renormalization

Yukawa Coupling



$$= \quad i(c^{L}_{b\to t\ H^-} P_L + c^{R}_{b\to t\ H^-} P_R)$$

$$c^{L}_{b\to t\ H^-} = \frac{e\ m_t\ \cot\beta}{\sqrt{2} s_W M_W} \qquad c^{R}_{b\to t\ H^-} = \frac{e\ m_b\ \tan\beta}{\sqrt{2} s_W M_W}$$

Enhanced for large tanβ

$\Downarrow$

Could spoil the perturbative nature of the process

tanβ renormalization

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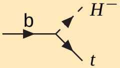

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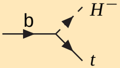
Desirable Properties of tanβ renormalization scheme

(Freitas and Stöckinger hep-ph/0205281)

Gauge Independence — Numerical Stability — Process Independence

$\tan\beta$ renormalization

Yukawa Coupling



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Desirable Properties of tanβ renormalization scheme

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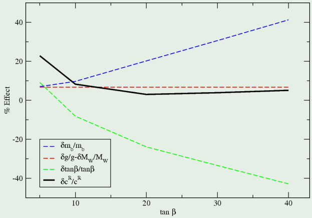
Gauge Independence | Numerical Stability | Process Independence

Our Choice

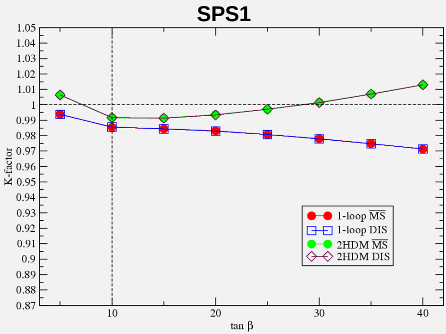
Gauge independent schemes lead to numerical instabilities

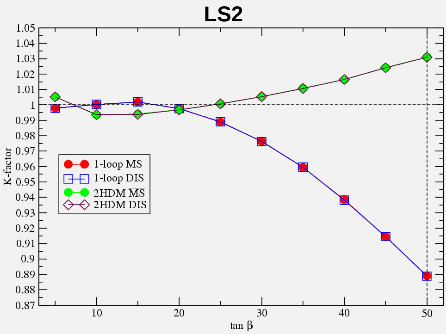
$$\frac{\delta\tan\beta}{\tan\beta} = \frac{Re\Sigma_{H^+W^+}(m^2_{H^+})}{M_W \sin 2\beta} \longrightarrow \begin{cases} \text{Gauge dependent} \\ \textbf{Numerically Stable} \\ \text{Process Independent} \end{cases}$$

Yukawa Counterterm: $\dfrac{\delta c^R}{c^R} = \dfrac{\delta g}{g} - \dfrac{\delta M_W}{M_W} + \dfrac{\delta m_b}{m_b} + \dfrac{\delta\tan\beta}{\tan\beta}$



tanβ dependence

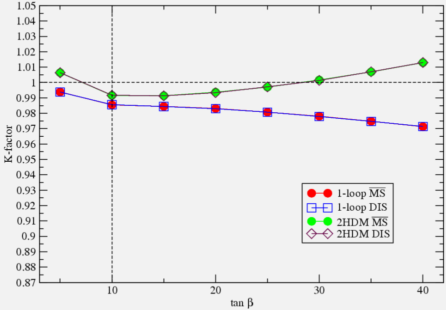




- **Stronger dependence in LS2**
  but under control within a reasonable tanβ range
- **Opposite behaviours**
  - $\longrightarrow$ Decreasing for high tanβ in SUSY
  - $\longrightarrow$ Increasing for high tanβ in 2HDM

$\tan\beta$ dependence

SPS1



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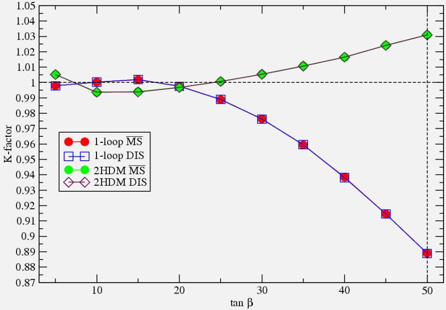

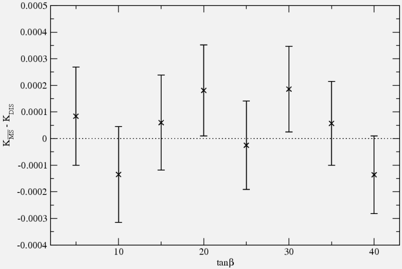

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  - $\longrightarrow$ Decreasing for high $\tan\beta$ in SUSY
  - $\longrightarrow$ Increasing for high $\tan\beta$ in 2HDM
- **Mild dependence on the PDF factorization scheme**
  - $\longrightarrow$ Differences of the order of 0.01%

Conclusions

Motivations

- The **Top** quark will be produced with high statistic at the LHC and is extremely sensitive to new physics corrections
- The **Charged Higgs**, if discovered, would represent clear evidence of new physics beyond the Standard Model

$\Downarrow$

Accurate analysis of the process of associated production of top and charged Higgs and of its dependence on **SUSY parameters**

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Accurate analysis of the process of associated production of top and charged Higgs and of its dependence on **SUSY parameters**

Our results at EW NLO

- Small but hopefully "visible" cross sections ($\sim$3 pb) for light H^- scenarios
- **Sizable SUSY corrections** (up to K=0.89) and **mild 2HDM corrections** (few percents)
- The dependence on tanβ can be significant in some scenarios, but it is numerically under control in the chosen renormalization scheme

NLO EW SUSY Yukawa results for bgth

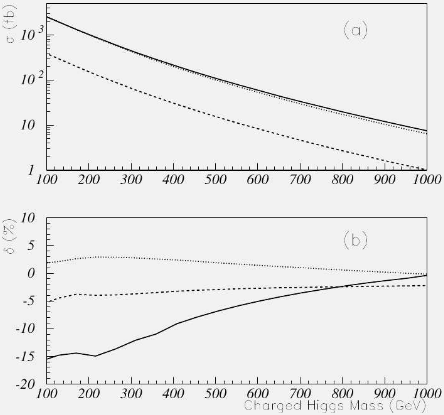


Figure 4: The tree-level total cross sections (a) and relative one-loop corrections (b) versus $mH^{\pm}$ at the LHC with s = 14 TeV. The solid, dashed and dotted lines correspond to tanβ = 2, 10 and 30, respectively.

L. G. Jin, C. S. Li, R. J. Oakes and S. H. Zhu, Phys. Rev. D **62** (2000) 053008

NLO QCD results for bgth

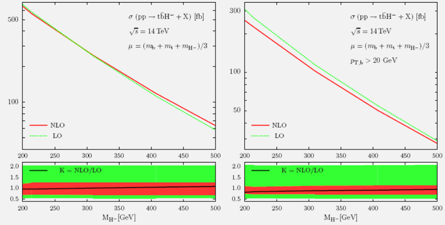


Figure 3: Total LO and NLO cross sections for pp $\to t\bar{b}H^- + X$ at the LHC as a function of the Higgs-boson mass, without (l.h.s.) and with (r.h.s.) a cut of $p_{T,b} > 20$ GeV on the b-quark transverse momentum. The lower plots show the K-factor, $K = \sigma_{NLO}/\sigma_{LO}$, and the scale dependence of the LO and NLO cross section predictions for $\mu_0/3 < \mu < 3\mu_0$.

$M_{H^\pm}$ [GeV]	$\sigma_{NLO} = \sigma_0 \times (1+\delta_{SUSY}^{\tan\beta-\text{resum.}}) \times (1+\delta_{QCD}+\delta_{SUSY}^{\text{remainder}})$				$\sigma_{NLO}^{\text{fixed-order}}$ [fb]
	σ_0 [fb]	δ_{QCD}	$\delta_{SUSY}^{\tan\beta-\text{resum.}}$	$\delta_{SUSY}^{\text{remainder}}$	
214.28	512	0.55	−0.30	−0.0008	562(2)
309.70	224	0.61	−0.30	−0.0012	258(1)
407.33	106	0.61	−0.30	−0.0009	125(1)
505.88	53.3	0.62	−0.30	−0.0002	64.1(2)

Table 2: LO total cross section σ_0 and NLO corrections δ relative to σ_0 for pp $\to t\bar{b}H^- + X$ at the LHC. The error from the Monte Carlo integration on the last digit is given in parenthesis if significant. The MRST pdfs [60] are adopted and the renormalization and factorization scales have been set to $\mu = (m_t + m_b + M_{H^-})/3$. "QCD" denotes the NLO QCD corrections only, "SUSY/$\tan\beta$–resum." the $\tan\beta$-enhanced SUSY corrections, "SUSY/remainder" the remaining one-loop SUSY corrections and "NLO/fixed-order" the complete NLO calculation without summation of the $\tan\beta$-enhanced terms.

S. Dittmaier, M. Kramer, M. Spira and M. Walser, arXiv:0906.2648 [hep-ph]

Minimal Supersymmetric Standard Model

Matter

Fermions	Sfermions
$\Psi^I_Q = \begin{pmatrix} u^I \\ d^I \end{pmatrix}_L$	$Q^I = \begin{pmatrix} \tilde{u}^I \\ \tilde{d}^I \end{pmatrix}_L$
$\Psi^I_U = (u^I_L)^c$	$U^I = \tilde{u}^{I*}_R$
$\Psi^I_D = (d^I_L)^c$	$D^I = \tilde{d}^{I*}_R$
$\Psi^I_L = \begin{pmatrix} \nu^I \\ e^{-I} \end{pmatrix}_L$	$L^I = \begin{pmatrix} \tilde{\nu}^I \\ \tilde{e}^{-I} \end{pmatrix}_L$
$\Psi^I_E = (e^{-I}_L)^c$	$E^I = \tilde{e}^{+I}_R$

Gauge

Gauge Bosons	Gauginos
B_μ	λ_B
A^i_μ	λ^i_A
G^a_μ	λ^a_G

Higgs Sector

Higgs	Higgsinos
$H_d = \begin{pmatrix} H^0_d \\ H^-_d \end{pmatrix}$	$\Psi_{H_d} = \begin{pmatrix} \Psi^0_{H_d} \\ \Psi^-_{H_d} \end{pmatrix}$
$H_u = \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}$	$\Psi_{H_u} = \begin{pmatrix} \Psi^+_{H_u} \\ \Psi^0_{H_u} \end{pmatrix}$

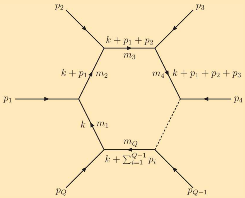
2 Higgs doublets are needed because the superpotential must be analytic

Properties of superpartners

Same supermultiplet $\longrightarrow$ $\begin{cases} \textbf{same gauge quantum numbers} \\ \textbf{same mass} \end{cases}$

Passarino-Veltman Reduction

One-loop tensor integral



$$T_Q^{\mu_1\cdots\mu_p}(p_1,\ldots,p_{Q-1},m_1,\ldots,m_Q) = \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2}\int d^D k \frac{k^{\mu_1}\cdots k^{\mu_p}}{N_1\cdots N_Q}$$

$Q =$ number of propagators in the loop

$$j^{\text{th}}\text{denominator}: N_j = (k+\cdots+p_{j-1})^2 - m_j^2 + i\epsilon$$

Passarino-Veltman Reduction

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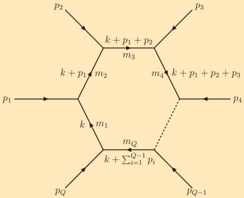

$$T_Q^{\mu_1\cdots\mu_p}(p_1,\ldots,p_{Q-1},m_1,\ldots,m_Q) = \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2}\int d^Dk\,\frac{k^{\mu_1}\cdots k^{\mu_p}}{N_1\cdots N_Q}$$

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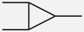
Tadpole: $T_1 \equiv A$

Bubble: $T_2 \equiv B$



Triangle: $T_3 \equiv C$

Box: $T_4 \equiv D$

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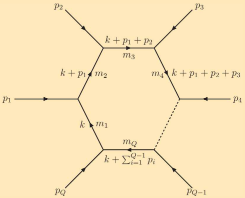
One-loop tensor integral



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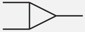

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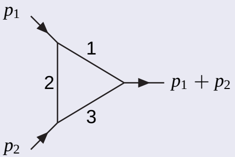
Reduction to scalar functions

Scalar one-point function: $A_0(m_1) = m_1^2\left(\Delta - \log\frac{m_1^2}{\mu^2} + 1\right) + O(\epsilon)$



Triangles:

$$C^\mu(123) = p_1^\mu C_{11}(123) + p_2^\mu C_{12}(123)$$

$$C^{\mu\nu}(123) = p_1^\mu p_1^\nu C_{21}(123) + p_2^\mu p_2^\nu C_{22}(123) + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu)C_{23}(123) + g^{\mu\nu}C_{24}(123)$$

$$C^{\mu\nu\rho}(123) = \ldots$$

Tests based on PV properties

The amplitude must be divergence-free:

- **UV Divergences** ($\Delta = \frac{1}{\epsilon} - \gamma_E + \log 4\pi$)

$$A_0(m_1^2) \sim m_1^2 \Delta \qquad B_{22}(q^2, m_1^2, m_2^2) \sim \frac{\Delta}{4}(m_1^2 + m_2^2 - \frac{q^2}{3})$$

$$B_0(q^2, m_1^2, m_2^2) \sim \Delta \qquad C_{24}(q^2, m_1^2, m_2^2, m_3^2) \sim \frac{\Delta}{4}$$

$$B_1(q^2, m_1^2, m_2^2) \sim -\frac{\Delta}{2} \qquad C_{001}(q^2, m_1^2, m_2^2, m_3^2) \sim -\frac{\Delta}{6}$$

$$B_{21}(q^2, m_1^2, m_2^2) \sim \frac{\Delta}{3} \qquad C_{002}(q^2, m_1^2, m_2^2, m_3^2) \sim -\frac{\Delta}{12}$$

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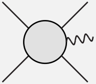
- **IR Divergences** (virtual photons)

Soft photon contribution: $X \rightarrow Y + \gamma_{\text{soft}}$



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\begin{array}{ll}
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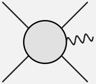

Asymptotic behaviour at high energies:

Sudakov expansion of the cross section:

$$
\sigma_{\text{tot}} \simeq \sigma_{\text{tot}}^{\text{Born}} \left(1 + \sum_i a_i \log^2 \frac{q^2}{M_{ai}^2} + \sum_i b_i \log \frac{q^2}{M_{bi}^2} + \text{constant terms} \right)
$$

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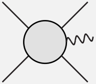

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The coefficients of logarithms can be obtained through:

- Asymptotic expansion of PV functions at the leading logarithmic level
- Analytical methods (splitting functions)