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The AdS/CFT duality and the scalar sector of QCD

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AdS/CFT correspondence provides a new way to address Physics at strong coupling

- **AdS/CFT** correspondence (Maldacena , Witten, Gubser, Klebanov, & Polyakov 1998)

weakly coupled **A**nti **d**e **S**itter Supergravity / strongly coupled (super)**C**onformal **F**ield **T**heory

- Holographic Models of QCD or AdS/QCD correspondence

(Witten 1998, Polchinski & Strassler 2002, Brodsky et al. , Pomarol et al. , Erlich et al. 2005)

QCD
- ~~SUSY~~ : no gluino neither squark
- ~~Conformal~~ : - Dimensionful parameters (m_q , Λ_{QCD})
  - Renormalization (mass scale μ)

⇒ **What is the gravity theory dual to QCD ?** (supposed to exist)

- Hadronic spectrum
  - 0^{++} scalar (1^{--} vector) glueballs
  - 0^{++} scalar mesons $a_0(980)$, $f_0(980)$, $a_0(1450)$,…

- Consistency of AdS/QCD models
  - large-N behaviour
  - chiral symmetry breaking mechanism

Towards a weakly-coupled gravity dual description of the non-perturbative physics of strong interactions

PLB: hep-ph/0703316, PRD: hep-ph/0807.1054, hep-ph/0902.3864, hep-ph/0907.5048, CPC: hep-ph/0912.2659

Maldacena's conjecture (1998) or AdS/CFT correspondence

IIB (oriented closed) superstring theory in $\underbrace{AdS_5}_{\text{Anti de Sitter space}} \times \underbrace{S^5}_{\text{compact manifold}}$ ⟺ $\mathcal{N}$ = 4 supersymmetric YM theory SU(N) in the boundary space ∂AdS_5 (at z → 0)

Anti de Sitter space × compact manifold

Holographic spacetime (*the bulk*)

AdS radius R

holographic coordinate

(<u>no physical</u> extra dim. : dual to an <u>energy scale</u>)

$$\begin{cases} x^M = (x^\mu, z) \\ \eta_{\mu\nu} = \text{diag}(-1,+1,+1,+1) \end{cases}$$

$$\boxed{ds^2 = g_{MN}dx^M dx^N = \underbrace{\frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S^5}}$$

- AdS_5 : solution of empty space Einstein equation $\mathcal{R}_{MN} - \frac{1}{2}g_{MN}\mathcal{R} = \frac{1}{2}g_{MN}\Lambda$

scalar curvature $\mathcal{R} = -\frac{5}{3}\Lambda = -\frac{20}{R^2}$ ⟹ cosmological constant : $\Lambda = \frac{12}{R^2}$ > 0

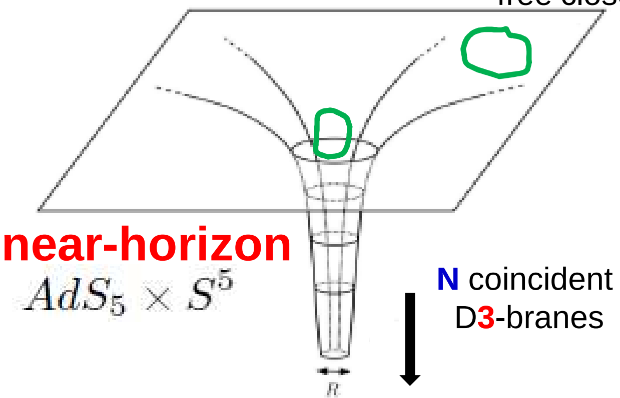
(de Sitter Λ < 0)

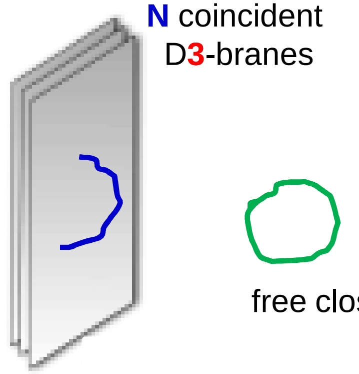
- AdS_5 Isometry group : **SO(2,4)** ⟺ conformal structure of flat boundary space : **SO(2,4)**

  (preserves ds^2) $\mathcal{N}$ = 4 SUSY : conformal theory (β = 0 at 3-loop level)

- S^5 isometry group **SO(6)** ⟺ $\mathcal{N}$ = 4 SUSY : global **SU(4)** R-symmetry

  (not considered for QCD)





SU(N) gauge theory on the (3+1)-dim. worldvolume

Supergravity limit of type - IIB **weakly-coupled superstring** theory in 10d **wrapped** spacetime ⟺ **'t Hooft** limit of $\mathcal{N}$ = 4 **superconformal strongly-coupled Yang-Mills** SU(N) in 4d Minkowski spacetime

Parameter correspondence

(closed) string coupling constant → (g_s, α') ← Regge slope α' (string length ℓ_s)

$\sqrt{\alpha'} \equiv \ell_s$

$$\boxed{\begin{aligned} &(1) \quad 2\pi\, g_s = g_{YM}^2 \\ &(2) \quad \frac{R^4}{\ell_s^4} = 2\, g_{YM}^2 N \end{aligned}}$$

Gauge group of (rank+1) = N → (g_{YM}, N) ← YM coupling

('t Hooft coupling $\lambda \equiv g_{YM}^2 N$)

- 't Hooft limit (**large N** with λ fixed) :

$$g_{YM}^2 = \frac{\lambda}{N} \quad << 1$$

(1) ⟹ **Tree-level** perturbative string theory :

$$g_s \ll 1$$

- **Strong** coupling constant λ >> 1

(2) ⟹ **Small** scalar curvature : $R \gg \ell_s$

Supergravity (string ⟶ • point-like particle)

Operator/field correspondence (Witten, Gubser, Klebanov, Polyakov 1998)

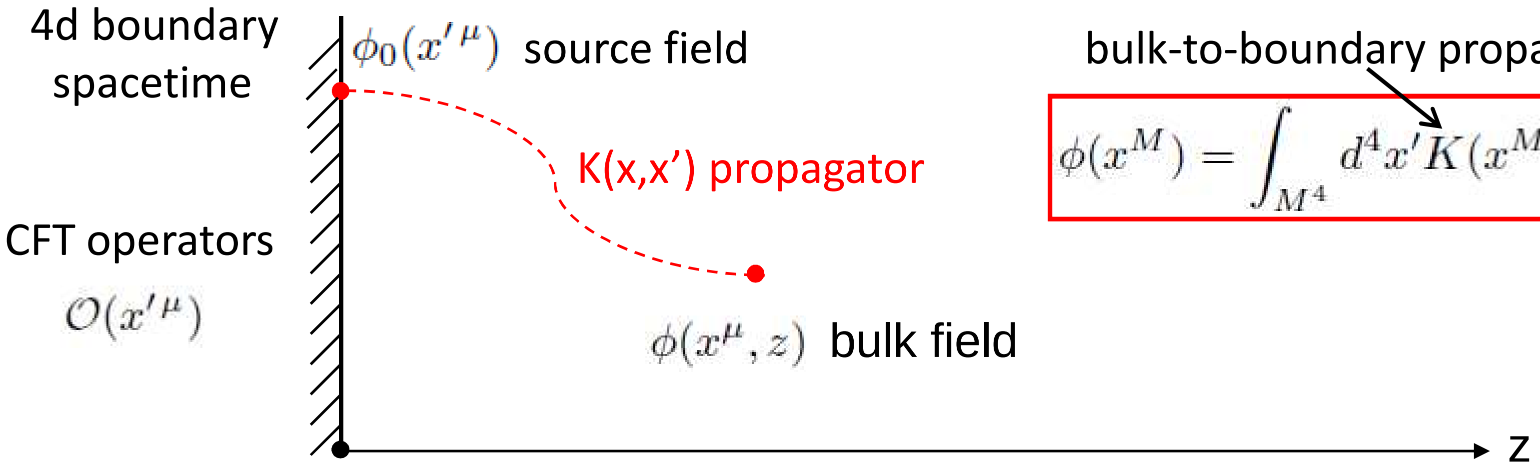
4d boundary operator $\mathcal{O}(x^\mu)$ $\Longleftrightarrow$ 5d bulk field $\phi(x^\mu, z)$ massive, *p*-form

local, gauge invariant, scaling dim. Δ

$\phi(x,z) \underset{z\to 0}{\longrightarrow} z^{4-\Delta}\phi_0(x) + z^{\Delta}\langle\mathcal{O}(x)\rangle$ (if p=0)

$$\boxed{\langle e^{i\int_{\partial AdS_5} d^4x\,\phi_0(x)\mathcal{O}(x)}\rangle_{CFT} = e^{iS_{5d}[\phi(x,z)]}\Big|_{\phi(x,z)\underset{z\to 0}{\longrightarrow}\phi_0(x)}}$$

AdS/CFT provides <u>2 languages for deriving correlation functions</u> (2-,3-,4-points)



bulk-to-boundary propagator K(x,x')

$$\boxed{\phi(x^M) = \int_{M^4} d^4x'\,K(x^M, x'^\mu)\phi_0(x'^\mu)}$$

UV $\boxed{z \to 0}$ **holographic coordinate z** **IR** $\boxed{z \to \infty}$

$$\boxed{R^2 m^2_{AdS} = (\Delta - p)(\Delta + p - 4)}$$

Scale invariance breaking and AdS/QCD

dilatation invariance

$$ds^2_{AdS_5} = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \left\{ \begin{array}{l} x^\mu \to e^{-t}x^\mu \\ z \to e^{-t}z \end{array} \right.$$ (as a spacetime coordinate)

canonical dim.

dilatation charge : $[D, \mathcal{O}(x)] = -i\left(\Delta + x^\mu \partial_\mu\right)\mathcal{O}(x)$ scaling dim. : $\Delta(g) = \Delta_0 + \gamma(g)$

anomalous dim. (AdS/QCD : γ = 0)

⇒ different values of z : different **scales** at which the hadrons are observed

- UV regime : **boundary** space ∂AdS_5 (**z ~ Energy^{-1} → 0**)

- IR regime : max. separation of quarks inside hadrons → **max. value of z**

- ***Hard wall approx.*** (Polchinski & Strassler 2002) : $\boxed{0 < z \leq z_m \sim 1/\Lambda_{QCD}}$

↳ **Kaluza-Klein** mass spectrum (~ QM well potential) : $m_n^2 \propto n^2$

- ***Soft wall approx.*** (Karch et al. 2006) : background dilaton field $\boxed{\Phi(z) = c^2 z^2}$

(Gherghetta et al. 2008 : dynamical justification)

↳ **Linear** Regge trajectories : $m_n^2 \propto n$

$\boxed{(c, z_m) \text{ break conformal inv. of CFT : introduction of QCD scale } \Lambda_{QCD}}$

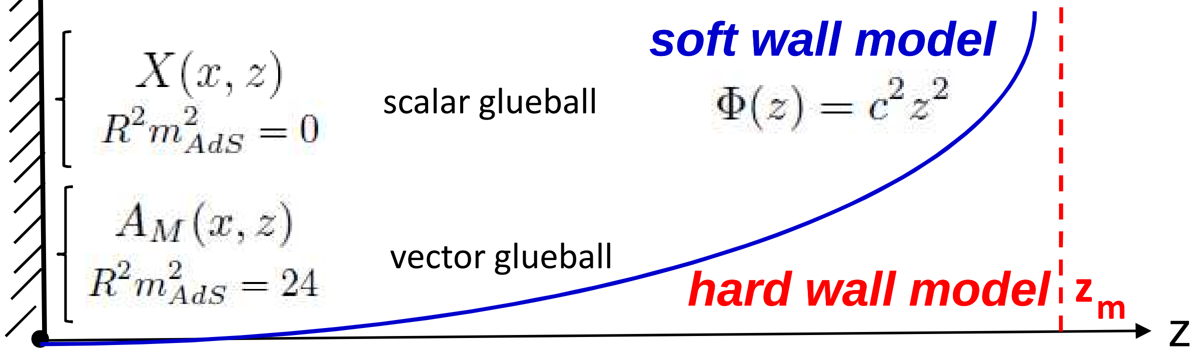
Caveat : strong λ >> 1 at any length scales (no asymptotic freedom of QCD ?)

- AdS/CFT : String-like theories $\longrightarrow$ QCD-like gauge theories (***top-down*** approach)
- AdS/QCD : QCD properties $\longrightarrow$ 5d <u>weakly-coupled</u> dual theory (***bottom-up*** approach)

Holographic models of the scalar sector of QCD

- chiral dynamics of QCD (a few operators)
- Scalar mesons: a_0(980, 1450), f_0(980, 1370, 1505)…
- Scalar (& vector) glueballs : bound-states of gluons (well defined in the large N limit)

M^4 <u>QCD operators</u>	<u>Gravity dual theory in the 5d bulk</u>
• left- and right-handed currents : $j^a_{L\,\mu}$ & $j^a_{R\,\mu}$ (Δ=3,p=1)	$A^a_{L\,\mu}(x,z)$ & $A^a_{R\,\mu}(x,z)$, $R^2 m^2_{AdS} = 0$ ⇒ $V^a_\mu(x,z)$ & $A^a_\mu(x,z)$ vector ρ, axial a1, pseudoscalar modes
• chiral order parameter : $\overline{q}_R q_L$ (Δ=3,p=0)	$X(x,z) = v(z)/2\, e^{2i\pi(x,z)}$, $R^2 m^2_{AdS} = -3$ chiral symmetry breaking function v(z), chiral pion π
• scalar meson operator : $\mathcal{O}^A_S = \overline{q} T^A q$ (Δ=3,p=0)	$X(x,z) = (v(z)/2 + S(x,z))\, e^{2i\pi(x,z)}$ scalar a0
• scalar glueball operator : $\mathcal{O}_S = Tr\,(G^2)$ (Δ=4,p=0)	$X(x,z)$, $R^2 m^2_{AdS} = 0$ scalar glueball
• vector glueball operator : $\mathcal{O}_V = Tr\,(G(DG)G)$ (Δ=7,p=1)	$A_M(x,z)$, $R^2 m^2_{AdS} = 24$ vector glueball



Soft Wall Model of QCD

$$\boxed{S_{5d} = -\frac{1}{k}\int d^5x\sqrt{-g}e^{-\Phi(z)}Tr\left\{|DX|^2 + m^2_{AdS}|X|^2 + \frac{1}{2g_5^2}\left(G_V^2 + G_A^2\right)\right\}}$$

linear eqs. of motion :

- axial-vector : $\tilde{A}^a_\mu(q,z) = \tilde{A}^a_{\mu\perp}(q,z) + iq_\mu\tilde{\phi}^a(q,z)$ $\left\{\begin{array}{l}\text{longitudinal } \tilde{\phi} \text{ : pseudoscalar modes}\\ \text{transverse } A_\perp \text{ : a1 mesons}\end{array}\right.$

$$\left[\partial_z\left(\frac{e^{-\Phi(z)}}{z}\partial_z\tilde{A}^a_\mu\right) - q^2\frac{e^{-\Phi(z)}}{z}\tilde{A}^a_\mu - g_5^2R^2v(z)^2\frac{e^{-\Phi(z)}}{z^3}\tilde{A}^a_\mu\right]_\perp = 0$$

- vector : $\partial_z\left(\frac{e^{-\Phi(z)}}{z}\partial_z\tilde{V}^a_\mu(q,z)\right) - q^2\frac{e^{-\Phi(z)}}{z}\tilde{V}^a_\mu(q,z) = 0$ $\left\{\begin{array}{l}\boxed{q^2 = -m^2_{\rho_n} = -4c^2(n+1)}\\ c = \dfrac{m_\rho}{2} \simeq 385\ \mathrm{MeV}\end{array}\right.$

- chiral symmetry breaking function : $\partial_z\left(\frac{e^{-\Phi(z)}}{z^3}\partial_z v(z)\right) + 3\frac{e^{-\Phi(z)}}{z^5}v(z) = 0$

- pseudoscalar : $\left\{\begin{array}{l}\partial_z\left(\dfrac{e^{-\Phi(z)}}{z}\partial_z\tilde{\phi}^a\right) + g_5^2R^2v(z)^2\dfrac{e^{-\Phi(z)}}{z^3}\left(\tilde{\pi}^a - \tilde{\phi}^a\right) = 0\\ q^2\partial_z\tilde{\phi}^a + g_5^2R^2v(z)^2\dfrac{1}{z^2}\partial_z\tilde{\pi}^a = 0\end{array}\right.$

- scalar : $\partial_z\left(\frac{e^{-\Phi(z)}}{z^3}\partial_z\tilde{S}^A\right) + 3\frac{e^{-\Phi(z)}}{z^5}\tilde{S}^A - q^2\frac{e^{-\Phi(z)}}{z^3}\tilde{S}^A = 0$

Soft Wall Model for scalar mesons

χSB function

a_0 bulk field

scalar 5d bulk field :
$$\boxed{X(x,z) = \Big(\frac{v(z)}{2} + S(x,z)\Big)e^{2i\pi(x,z)} \sim S + S\pi\pi}$$

quadratic eff. action : **<u>spectroscopy</u>**

<u>SPP couplings</u>

n-point correlation functions in terms of bulk-to-boundary propagators

• <u>2-point correlation function</u> :

- QCD : $$\Pi_S^{(QCD)\,AB}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[\mathcal{O}_S^A(x)\mathcal{O}_S^B(0)]|0\rangle$$

- AdS : $$\Pi_S^{(AdS)\,AB}(q^2) = \delta^{AB}\frac{R^3}{k}K\left(\frac{q^2}{c^2},c^2z^2\right)\frac{e^{-\Phi(z)}}{z^3}\partial_z K\left(\frac{q^2}{c^2},c^2z^2\right)\Big|_{z=\epsilon}$$

$$\Pi_S^{(AdS)\,AB}(q^2) = \delta^{AB}\frac{4c^2R}{k}\Big[\frac{1}{4c^2z^2} + \Big(\frac{q^2}{4c^2}+\frac{1}{2}\Big)\ln(c^2z^2) + \gamma_E - \frac{1}{2} + \frac{q^2}{4c^2}\Big(2\gamma_E - \frac{1}{2}\Big)$$
$$+\Big(\frac{q^2}{4c^2}+\frac{1}{2}\Big)\underline{\psi(\frac{q^2}{4c^2}+\frac{3}{2})}\Big]\Big|_{z=\epsilon} \ .$$

- Masses (simple poles of the ψ digamma function) : $\boxed{-q^2 = m^2_{S_n} = c^2(4n+6)}$
  - Ratio (1.612±0.004) : $R_{a0} \equiv \frac{m^2_{a0}}{m^2_{\rho^0}} = \frac{3}{2}$
  - First radial excitation state (1.01±0.04) : $R_{a'_0} = \frac{5}{4}$
- Decay constants (residues) : $\boxed{F_n^2 = \frac{N}{\pi^2} c^4 (n+1)}$
  - current-vacuum matrix elt. (0.21±0.05 GeV^2) : $F_{a_0} \simeq 0.08 \text{ GeV}^2$
  - First radial excitation state : $F_{a'_0} \simeq 0.12 \text{ GeV}^2$
  - $\frac{F^2_{S_n}}{m^2_{S_n}}$ becomes constant as *n* increases

- <u>Large q^2 limit of the 2-point correlation function</u> : pert. contr. + power corrections (condensates)
  - 4-dim. gluon condensate (0,012 GeV^4) : $\boxed{\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{2}{\pi^2} c^4 \simeq 0.004 \text{ GeV}^4}$
  - 6-dim. condensates (QCD $\propto$ **-** $\langle q\bar{q}\rangle^2$) : 6-dim. **positive** condensates

- 3-point correlation functions :

➢ 5d interaction action :

scalar bulk field chiral bulk field

$$iS_{5d}^{(S\pi\pi)} = -i\frac{4}{k}\int d^5x\sqrt{-g}\,e^{-\Phi(z)}g^{MN}\,v(z)Tr\left\{S(\partial_M\pi-\partial_M\phi)(\partial_N\pi-\partial_N\phi)\right\}$$

χSB function

longitudinal component of the axial-vector bulk field

➢ 3-point correlator ⟹ scalar form factor ⟹ SPP couplings :

$$\Pi_{\alpha\beta}^{(QCD)\,abc}(p_1,p_2) = -\frac{p_{1\,\alpha}p_{2\,\beta}}{p_1^2p_2^2}f_\pi^2F_\pi^{abc}(q^2) \quad \& \quad F_\pi^{abc}(q^2) = -d^{abc}\sum_{n=0}^{\infty}\frac{F_ng_{S_n\pi\pi}}{q^2+m_{S_n}^2}$$

$$g_{S_n\pi\pi} = \frac{1}{k}\frac{2}{\mathrm{f}_\pi{}^2}\int_0^\infty dz\frac{R^3}{z^3}e^{-\Phi(z)}v(z)\frac{1}{Rc}\sqrt{\frac{8}{N}}\pi S_n(c^2z^2)\left[\left(\partial_zA(0,c^2z^2)\right)^2+\frac{m_{S_n}^2}{2}A(0,c^2z^2)^2\right]$$

massless pion decay constant scalar holo. wave function axial-vector b-to-b prop. at $q^2 = 0$

$$\boxed{g_{S\pi\pi}^{(0)} = \frac{\sqrt{N_c}}{4\pi}\frac{m_{S_0}^2}{f_\pi^2}Rc^2\int_0^\infty dz\,e^{-c^2z^2}v(z)}$$

- $f_\pi^2 \propto N$: $g_{S_n\pi\pi}^{(0)}$ vanishes in the large N limit
- $\sim$ chiral symmetry breaking function

$$v(z) = \frac{m_q}{R}z\Gamma(\frac{3}{2})U(\frac{1}{2};0;c^2z^2)\underset{z\to 0}{\longrightarrow}\frac{m_q}{R}z+\frac{\sigma}{R}z^3$$

Soft Wall boundary conditions (finite action when $z\to\infty$)

⟹ $\boxed{\text{quark condensate } \sigma \propto m_q \text{ light quark mass}}$

(Gherghetta et al. hep-ph/0908.0725)

Soft Wall Model for the scalar & vector glueballs

$$S_{5d}^{(scalar)} = -\frac{1}{2\kappa_S}\int d^5x\sqrt{-g}e^{-\Phi(z)}g^{MN}\left(\partial_M X\right)\left(\partial_N X\right)$$

$$S_{5d}^{(vector)} = -\frac{1}{2\kappa_V}\int d^5x\sqrt{-g}e^{-\Phi(z)}g^{MN}\left(\frac{1}{2}g^{MN}g^{ST}F_{MS}F_{NT} + m_{AdS}^2 g^{ST}A_S A_T\right)$$

Spectroscopy :

- scalar glueball : $\boxed{m_{G_{0\,n}}^2 = c^2(4n+8)}$ $\boxed{f_{G_{0\,n}}^2 \equiv |\langle 0|\mathcal{O}_S(0)|G_{0\,n}\rangle|^2 = \frac{R^3}{\kappa_S}8(n+1)(n+2)c^3}$
- vector glueball : $\boxed{m_{G_{1\,n}}^2 = c^2(4n+12)}$ $\Longrightarrow$ $m_{G_{1\,n}}^2 - m_{G_{0\,n}}^2 = m_\rho^2 = 4c^2$

AdS/QCD	QCDSR			Lattice QCD	
	Dominguez, Paver ('86)	**Narison** (hep-ph/9612457)	**Hang, Zhang** (hep-ph/9801214)	**Morningstar** (hep-lat/9901004)	**Meyer** (hep-lat/0508002)
m_{G_0} **1.089 GeV**	**< 1**	**1.5 (0.2)**	**1.580(150)**	**1.730(50)(80)**	**1.475(30)(65)**
				Morningstar (hep-lat/9901004)	**Meyer** (hep-lat/0508002)
m_{G_1} **1.334 GeV**				**3.850(50)(190)**	**3.240(330)(150)**

Modification of the background : (λ : perturbative parameter)

- dilaton $\Phi(z)$
- metric function $g_{MN}(z) = e^{2A(z)}\eta_{MN}$

• **UV conformal** behaviour : $ds^2_{bulk} \underset{z\to 0}{\longrightarrow} ds^2_{AdS_5}$

• **IR** regime : **linear Regge** behaviour of the mass spectrum

modification of the dilaton

$$\Phi(z) = c^2 z^2 + \underline{\lambda c z}$$

$$A(z) = -\ln\left(\frac{z}{R}\right)$$

IR subleading

modification of the geometry

$$\Phi(z) = c^2 z^2$$

$$A(z) = -\ln\left(\frac{z}{R}\right) - \underline{\lambda c z}$$

UV subleading

z^α $(0 \leq \alpha < 2)$

Mass splitting

$$m_n^2 = m_{n,(0)}^2 + \lambda m_{n,(1)}^2$$

- **dilaton :** $\boxed{m_{G_1}^2 - m_{G_0}^2 = c^2\left(4 - \frac{3\sqrt{\pi}}{128}\lambda\right)}$
- **geometry :** $\boxed{m_{G_1}^2 - m_{G_0}^2 = c^2\left(4 - \frac{1899\sqrt{\pi}}{128}\lambda\right)}$

$\lambda < 0$

- **Increasing** mass splitting
- Maximun effect : **warped geometry**

$\Rightarrow$ types of constraints on the background

The large N behaviour of the Hard Wall Model
The holographic mechanism of the SχSB

Large-N behaviour :

- ρ meson normalizable mod es : $v_n(z) = \sqrt{2}\frac{z}{z_m}\frac{J_1(m_{\rho_n} z)}{J_1(m_{\rho_n} z_m)}$ ~ $O(N^0)$ $\quad \left\{ \begin{array}{l} v_n(0) = 0 \\ \partial_z v_n(z_m) = 0 \end{array} \right.$

- ρ meson mass spectrum : $\boxed{m_{\rho_n} = \frac{\gamma_{0,n}}{z_m}}$ ~ $O(N^0)$ $\Longrightarrow$ $z_m \simeq 1/323 \text{ MeV}^{-1}$

- decay constants :

$$\left.\begin{array}{l} F^2_{\rho_n} = \frac{R}{kg_5^2}\left(\frac{1}{z}\partial_z v_n(z)\right)^2\Big|_{z=\epsilon} \\ F^2_{a_n} = \frac{R}{kg_5^2}\left(\frac{1}{z}\partial_z a_n(z)\right)^2\Big|_{z=\epsilon} \\ f_\pi^2 = -\frac{R}{kg_5^2}\underline{\frac{1}{z}\partial_z A_\perp(0,z)}\Big|_{z=\epsilon} \end{array}\right\} \sim O(N)$$

- b-to-b propagator :

$$\left.\begin{array}{ll} \text{- timelike} & V(q^2,z) = \sqrt{\frac{kg_5^2}{R}}\sum_{n=1}^{\infty}\frac{F_{\rho_n} v_n(z)}{q^2 - m_{\rho_n} + i\epsilon} \\ \text{- spacelike} & V(Q,z) = Qz\left(K_1(Qz) + \frac{K_0(Qz_m)}{I_0(Qz_m)}I_1(Qz)\right) \end{array}\right\} \sim O(N^0)$$

- form factors : $F_\pi(Q^2)$, $A_\pi(Q^2) \propto \frac{R}{kg_5^2}\frac{1}{f_\pi^2} \times O(N^0)$ ~ $O(N^0)$

- VPP coupling constant : $g_{\rho_n\pi\pi} \propto \sqrt{\frac{R}{kg_5^2}\frac{1}{f_\pi^2}} \times O(N^0)$ ~ $O\left(\sqrt{1/N}\right)$ $\Longrightarrow$ vanishes in the large N limit

The holographic mechanism of the SχSB in the Hard Wall Model :

- χSB function : $v(z) = \frac{\overline{m}_q}{R} z + \frac{\overline{\sigma}}{R} z^3$ $\quad \begin{cases} \overline{m}_q \propto m_q \sim O(N^0) \\ \overline{\sigma} \propto \sigma \equiv -\langle \overline{q} q \rangle \sim O(N) \end{cases}$

pseudoscalar mode eq. of motion : $q^2 \partial_z \phi - g_5^2 R^2 v(z)^2 \frac{1}{z^2} \partial_z \pi = 0$

Gell-Mann-Oakes-Renner relation : $m_\pi^2 f_\pi^2 = 2 m_q \sigma$

$$m_\pi^2 f_\pi^2 = \frac{R}{k} 2 \overline{m}_q \overline{\sigma}$$

Soft & Hard Wall models **:** similar conformal behaviour of the correlation functions

UV pert. contribution of scalar correlator in the Soft Wall model **:** $\frac{R}{k} = \frac{N}{16\pi^2}$

$$\begin{cases} \overline{m}_q = m_q \\ \overline{\sigma} = \frac{k}{R}\sigma = \frac{16\pi^2}{N}\sigma \end{cases} \Longrightarrow \boxed{v(z) = \frac{z}{R}\left(m_q + \frac{16\pi^2}{N} \sigma z^2 \right) \sim O(N^0)}$$

AdS estimate **:** $\sigma \simeq (171 \text{ MeV})^3$

Some open issues

- Holographic description of the **flavour**

- Holographic description of the **UV regime of QCD**

Wilson loop v.e.v. (Maldacena 1998) **:** $W[\mathcal{C}] = Z_{string}[\mathcal{C}]$ (F.J. hep-ph/0812.4903)

AdS/CFT : $V^{(R)}_{Q\overline{Q}}(r) \propto -\frac{\sqrt{\lambda}}{r}$
- **coulomb-like conformal** behaviour $1/r$ at <u>all length scales</u>
- **non-perturbative :** non-polynomial $\sqrt{\lambda}$

AdS/QCD:
- linear confinement at large distances $V^{(R)}(r, z_0^*) = \sigma(z_0^*) r$ when $r(z_0^*)$ explodes
- at short-distances, we want $V_{Q\overline{Q}}(r) \sim -\frac{1}{r \ln(r)}$ *i.e.* QCD running coupling ?

- **Supergravity corrections** $O(\alpha')$ **:** finite $O(1/\sqrt{\lambda})$ corrections

- **Finite temperature QCD :** $\langle \overline{q} q \rangle (T)$ chiral condensate vs. T

Conclusion

AdS/CFT provides a new way to address Physics at strong coupling

$\Longrightarrow$ AdS/QCD **: identify** the main properties of the dual theory of QCD

- **scalar** glueball and meson phenomenology (masses, decay constants, condensates)
  - ➢ surprisingly close pheno. results regarding the relative simplicity of the holographic models
  - ➢ scalar/vector glueball mass splitting **:** modification of the geometry

- **consistency** of the Hard Wall & Soft Wall Models
  - ➢ large-N behaviour (vanishing coupling constants)
  - ➢ SχSB description (χSB function $v(z)$)

- too **drastic** modifications of AdS/CFT to gain AdS/QCD ?

Higher-dimensional gravity theory dual to QCD $\Longrightarrow$ low energy predictions !

Backup Slides

Holographic principle and AdS/CFT, AdS/QCD applications

- **Spectroscopy and Form Factors :**

  Csáki et al. (hep-th/9806021) ; Boschi-Filho et al. (hep-th/0207071) ; Brodsky et al. (hep-ph/0501022)

  Katz et al. (hep-ph/0510388) ; Kwee et al. (hep-ph/0708.4054) ; Grigoryan et al. (hep-ph/0703069)

- **Chiral symmetry breaking mechanism & light mesons :**

  Evans et al. (hep-th/0306018) ; Erlich et al. (hep-ph/0501128) ; Da Rold & Pomarol (hep-ph/0510268)

- **Wilson loop and Heavy quarkonium $Q\bar{Q}$ potential :**

  Maldacena (hep-th/9803002) ; Rey & Yee (hep-th/9803001) ; Sonnenschein et al. (hep-th/ 9803137)

  Andreev & Zakharov (hep-ph/0604204) ; **F. Jugeau (hep-ph/0812.4903)**

- **Heavy-light mesons :**

  Erdmenger et al. (hep-th/0605241) : Herzog et al. (hep-th/0802.2956)

- **Baryons :**

  Hong et al. (hep-ph/0609270) ; Sakai & Sugimoto (hep-th/0701280); Pomarol & Wulzer (hep-ph/0904.2272)

- **Quark-gluon plasma :**

  Son et al. (hep-th/0405231) ; Kiritsis et al. (hep-th/0812.0792)

- **Deep Inelastic Scattering :**

  Braga et al. (hep-th /0807.1917)

- **Condensed matter systems (quantum Hall effect, superconductor, superfluidity) :**

  Herzog, Kovtun & Son (hep-th/0809.4870) ; Hartnoll, Herzog & Horowitz(hep-th/0810.1563)

- **Warped extra dimension Electroweak Physics models**

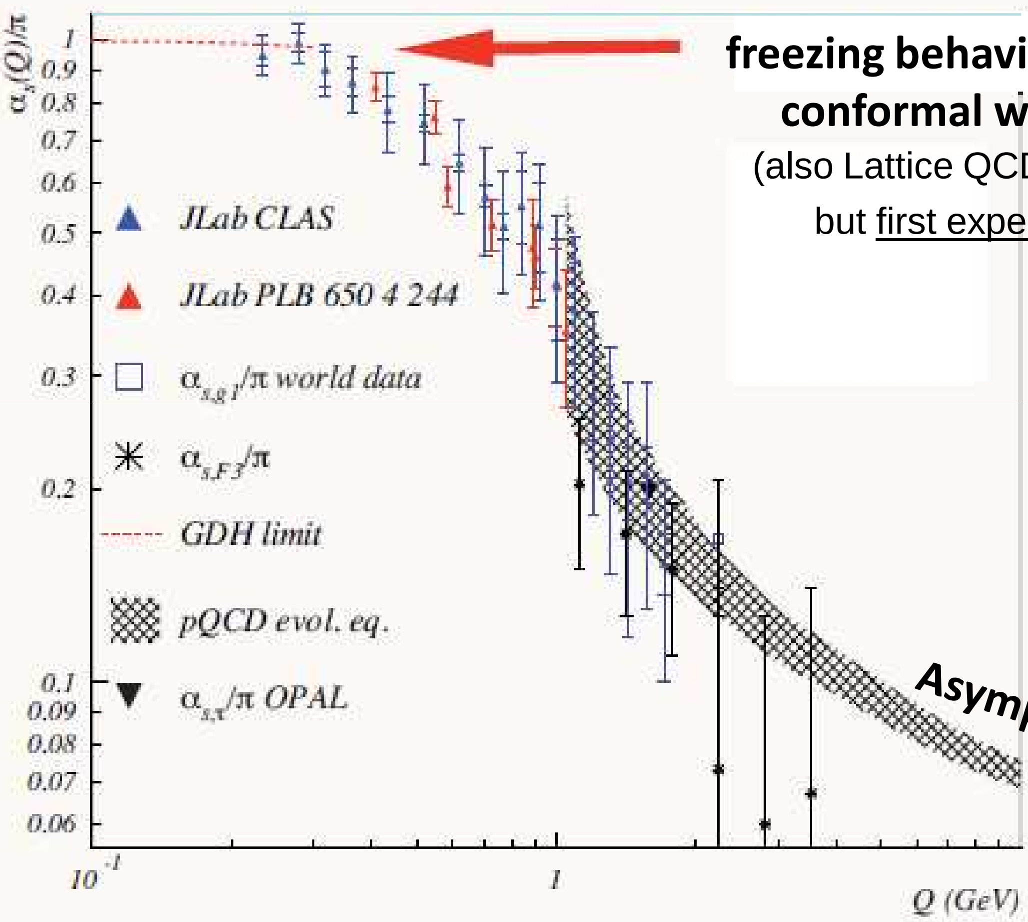
  Gherghetta et al. (hep-ph/0808.3977)

- **Astrophysics : Holographic Dark Matter Model**

  Li (hep-th/0403127)

Freezing behaviour of QCD effective charges at low Q^2

(Deur, Burkert, Chen & Korsch, Phys. Lett. B665:349-351, 2008)



freezing behaviour of the coupling : conformal window for QCD

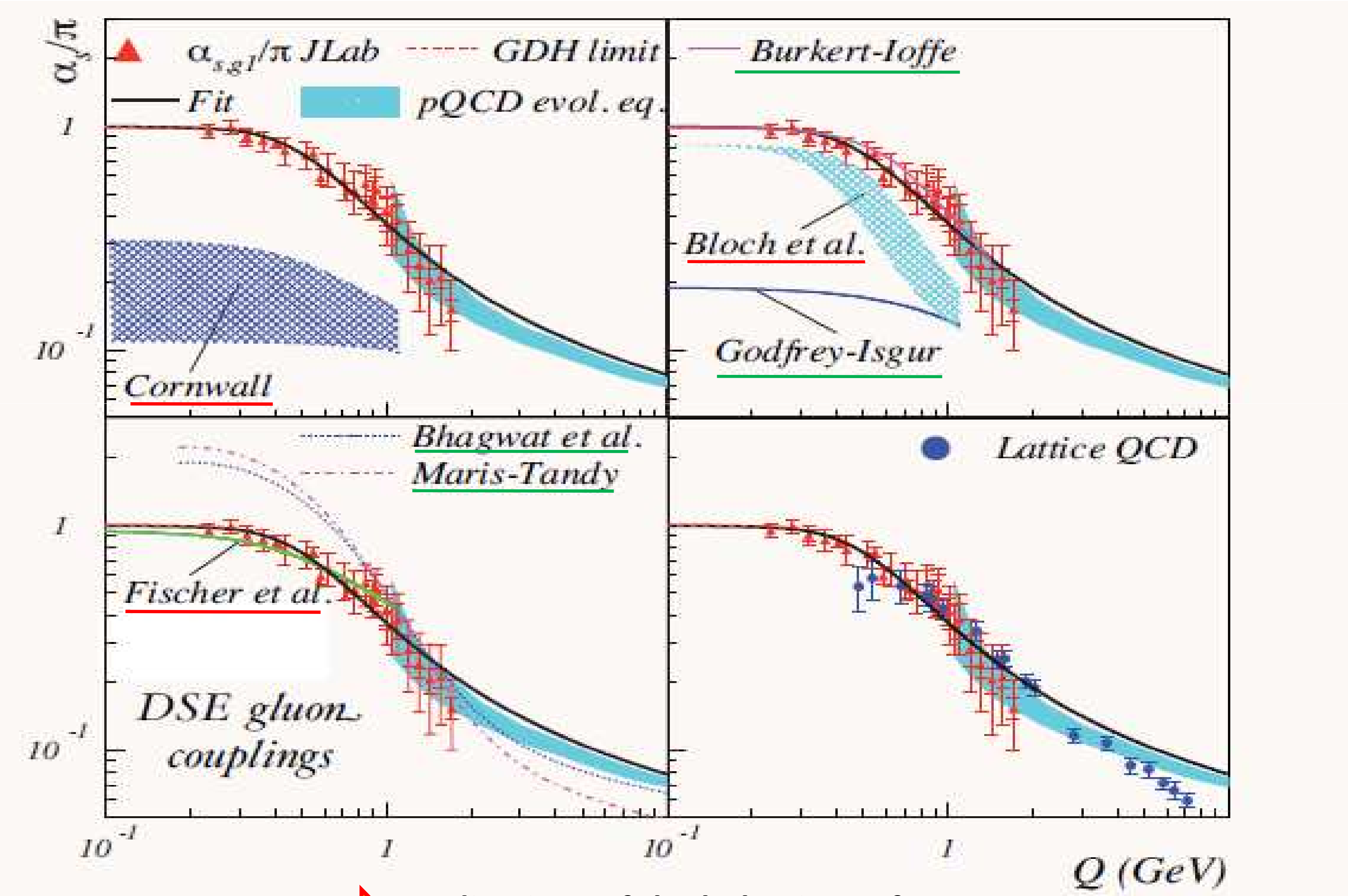
(also Lattice QCD, Schwinger-Dyson eqs. but <u>first experimental evidence</u>)

CFT applicable to study the properties of hadrons (AdS/QCD)

Asymptotic freedom of QCD

hadrons ⟵ **broken conformal behaviour transition region** ⟶ **partons**

Lattice QCD, theoretical calculations and phenomenological models



Indications of the behaviour of α_S

• Large q² limit of the 2-point correlation function : pert. contr. + power corrections (condensates)

$$\ldots\frac{2}{2}, \hat{z}^2) = A\,\widetilde{K}_1(\frac{q^2}{c^2}, \hat{z}^2) + B\,\widetilde{K}_2(\frac{q^2}{c^2}, \hat{z}^2) \cdot \frac{1}{8}\left[-\ln(\frac{q^2}{\nu^2}) + 2 - 2\gamma_E + \ln 4\right]$$

$$+q^2\left[-\frac{c^2}{2}\ln(\frac{q^2}{\nu^2}) + \frac{c^2}{4}(1 - 4\gamma_E + 2\ln 4)\right]$$

$$+\frac{c^4}{6}(12\eta_0 - 5) + \frac{2c^6}{3}\frac{1}{q^2} - \frac{4c^8}{15}\frac{1}{q^4} + O\left(\frac{1}{q^6}\right)\Big\}$$

B : constant η

➢ 2-dim. condensate (absent in QCD since < A² > is not gauge invariant)

➢ 4-dim. gluon condensate : $\boxed{\langle\frac{\alpha_s}{\pi}G^2\rangle = \frac{4\alpha_s}{\pi^3}\left(2\eta_0 - \frac{5}{6}\right)c^4}$

➢ correlator at q² =0 : $\boxed{\Pi_S^{(AdS)}(0) = \frac{R^3}{k}2\eta_0 c^4}$

Low Energy Theorem :

$$\Pi_S^{(QCD)}(0) = -16\beta_0\langle\frac{\alpha_s}{\pi}G^2\rangle$$

⇨ $$\eta_0 = \frac{5}{12}\left(\frac{1}{1+\frac{\alpha_s}{4\pi}\beta_0}\right)$$ and (α = 1.5) $$\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.007\ \mathrm{GeV}^4$$

negative if not η_0

Scalar glueball b.-to-b. prop. :

$$f_\pi^2 m_\pi^2 = 2 m_q\, \sigma$$

(pseudo-scalar 2-point correlator)

- Hard wall model : $v_{h.w.}(z) = \frac{m_q}{R} z + \frac{\sigma}{R^3} z^3 \underset{z\to\infty}{\longrightarrow} \infty$ ⇨ $f(u) = \frac{u^3}{R^2 v^2(u)}$ peaked at $u_c \ll 1$

↳ $\tilde{\pi}(0,z) = -f_\pi^2 m_\pi^2 \int_0^{z\to\infty} du\, f(u) = -\frac{f_\pi^2 m_\pi^2}{2 m_q \sigma}$ **= -1** **:** GMOR relation

- Soft wall model :

b.c. at z → ∞

$$v_{s.w.}(z) = \frac{m_q}{Rc} \Gamma(3/2)\,(cz)\, U(1/2;0;c^2z^2) + \cancel{B\,(cz)^3\,{}_1F_1(3/2;2;c^2z^2)} \underset{z\to\infty}{\longrightarrow} const.$$

↳ f(u) not bounded from above : **NO** GMOR relation (other mechanism ?)

More about the Operator/Field correspondence

- Bulk field X(x,z) : p form (totally antisymmetric tensor with p indices)

$$\begin{cases} \text{0-form : } & \phi & \text{(scalar)} \\ \text{1-form : } & A_M & \text{(vector)} \\ \text{2-form : } & A_{[M,N]} & \text{(strength field } F_{MN} \text{)} \end{cases}$$

- 5d eq. of motion of X(x,z) : mass term $m_{AdS} X(x^M)$

- Superconformal gauge theory : conformal group invariant

Scale transf. : $x^\mu \to \lambda x^\mu$

Field $X_0(x^\mu) \to \lambda^{-\tilde{\Delta}} X_0(x^\mu)$

Operator $O(x^\mu) \to \lambda^{-\Delta} O(x^\mu)$

$\Delta, \tilde{\Delta}$: scaling dim. = canonical dim. (without anomalous dim.)

$$\langle e^{i\int d^4x X_0(x) O(x)} \rangle_{CFT} \to \langle e^{i\int d^4x \lambda^4 \lambda^{-\tilde{\Delta}} X_0(x) \lambda^{-\Delta} O(x)} \rangle_{CFT}$$

$4 - \tilde{\Delta} - \Delta = 0$ or $\tilde{\Delta} = 4 - \Delta$

Bulk

p, m_{AdS}

Boundary

4, Δ

Homogeneous RGE : $$(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + \gamma_m(\lambda)m\frac{\partial}{\partial m})G^{(n)} = 0$$

Scale transf. : $$G^{(n)}(p, m, \lambda, \mu) \to G^{(n)}(e^t p, \underbrace{m}_{0}, \lambda, \mu) = G^{(n)}(p, \underbrace{\bar{\lambda}(t)}_{\lambda}, \underbrace{e^{-t}\bar{m}(t)}_{0}, \mu)$$

Chiral limit **m=0** : **λ(t)** breaks scale invariance

Classical theory or fixed point : β=0 and λ(t) = λ = **const.**

} scale invariant theory

{ Chiral QCD : m=0

IR fixed point λ* : β(λ*)=0

(Brodsky '02; Alkofer et al. '04)

QCD nearly conformal

(asymptotically conformal in UV)

AdS/CFT

AdS/QCD

(String-inspired) Effective bulk field action

$$S_5^{eff}[A(x^M)]$$

Deformation of the geometry from AdS $_5$

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

warp factor

AdS/QCD spectrum of ρ meson (Son et al. '05)

QCD in $\mathcal{M}^4$

Chiral symmetry

$(SU(3)_L \times SU(3)_R)_{global}$

Condensate : $\bar{q}q(x)$

Left/right currents :

$j_L^a(x)$ $j_R^a(x)$

operators

Dual theory in AdS_5

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Gauge symmetry

$(SU(3)_L \times SU(3)_R)_{local}$

Dilaton : $\phi(z)$

Scalar field : $X(x,z)$

Left/right gauge fields :

$A_L^a(x,z)$ $A_R^a(x,z)$

z

bulk fields

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

massless

tachyonic

$$S_5^{eff} = \int d^5x\sqrt{-g}e^{-\phi}\left[|DX|^2 + m_5^2 X^2 + \frac{1}{4g_5^2}Tr(F_L^2 + F_R^2)\right]$$

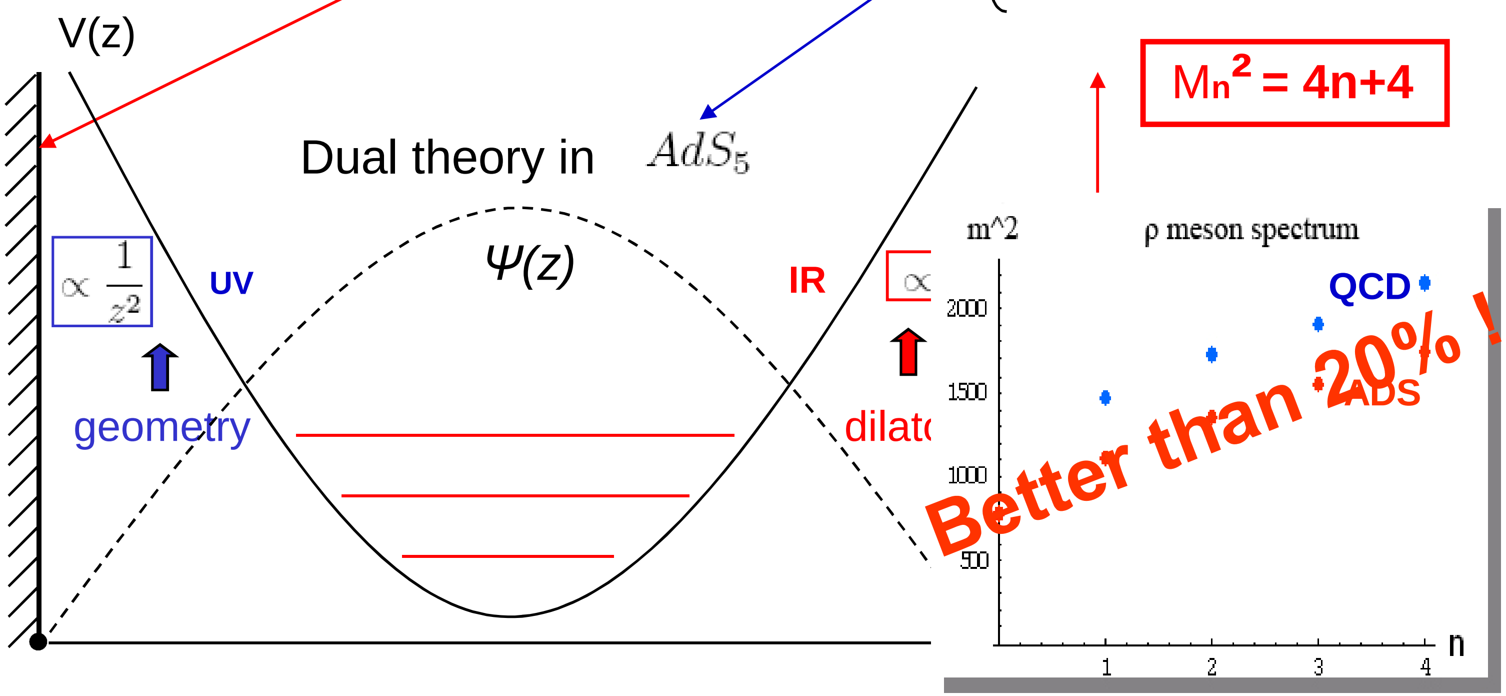
(Classical) eq. of motion : $\partial_M \left(\sqrt{-g}e^{-\phi}\left[\partial^M V^N - \partial^N V^M\right]\right) = 0$

ρ meson vector field : $V = \dfrac{A_R + A_L}{2}$ ⇨ $V_\mu(x,z) = \underbrace{\epsilon_\mu e^{iq\cdot x}}_{\text{plane wave}}\underbrace{\psi(z)}_{\text{holo. wave function}}$

Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2\psi(z)$

Regge behaviour : $m_n^2 \propto n$ ⇨ connection dilaton/geometry
$$\begin{cases} \phi - A \xrightarrow{z\to 0} -\ln\left(\dfrac{z}{R}\right) \\ \phi - A \xrightarrow{z\to\infty} \dfrac{z^2}{R^2} \end{cases}$$

QCD in $\mathcal{M}^4$



AdS/QCD Model of light glueballs (scalar, vector)

Glueballs : Bound-states of gluons (gg...)

QCD in $\mathcal{M}^4$

Dual theory in AdS_5

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

(AdS radius : R=1)

Scalar glueball

TrF^2

Vector glueball

$Tr(F(DF)F)_\mu$

Scalar field : $X(x,z)$

Vector field : $A_M(x,z)$

bulk fields

z

boundary operators

Operators / fields of the model

4D : $\mathcal{O}(x)$	5D : $\phi(x,z)$	p	Δ	m_{Ads}^2	
TrF^2	$X(x,z)$	0	4	0	massless
$Tr(F(DF)F)_\mu$	$A_M(x,z)$	1	7	24	massive

<u>boundary</u> <u>bulk</u>

J^{PC}

Scalar glueball 0^{++} TrF^2 (Δ=4) ⇨ $X(x,z)$ (p=0) $m_5^2=0$

Vector glueball 1^{--} $Tr(F(DF)F)_\mu$ (Δ=7) ⇨ $A_M(x,z)$ (p=1) $m_5^2=24$

AdS/CFT
$$\begin{cases} A(x^M) = \int_{M^4} d^4x' K(x^M, x'^\mu) A_0(x'^\mu) \\ m_5^2 = (\Delta - p)(\Delta + p - 4) \end{cases}$$

⇨ AdS/QCD
$$\begin{cases} A(x^M) \; ? \not\to \; A_0(x^\mu) \\ m_5^2 = m_{AdS}^2 \end{cases}$$

- **Scalar bulk field :**

$$S_5^{eff} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}g^{MN}(\partial_M X)(\partial_N X)$$

- **Vector bulk field :**

$$S_5^{eff} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}\left[\frac{1}{2}g^{MN}g^{ST}F_{MS}F_{NT} + m_{AdS}^2 g^{ST}A_S A_T\right]$$

5-dim. bulk **Dilaton** $\phi(z) = a^2 z^2$ Bulk field mass

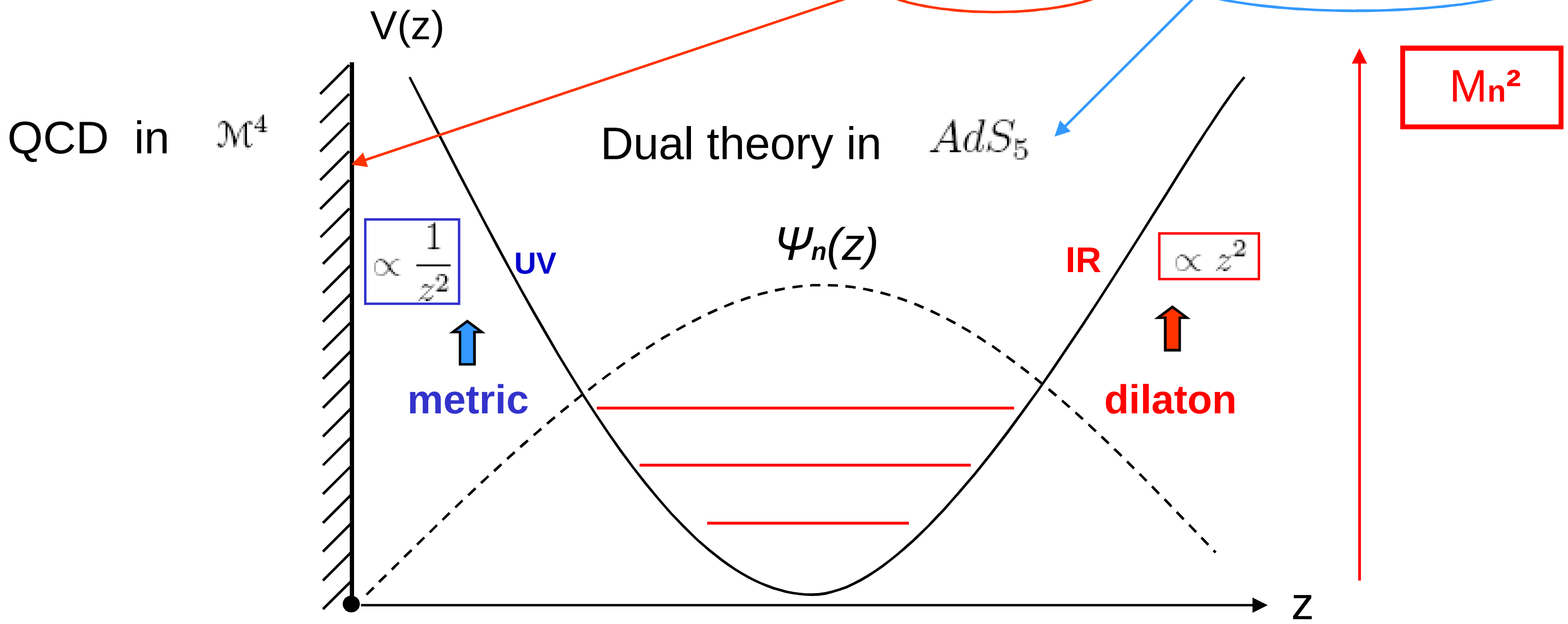
$F_{MS} = \partial_M A_S - \partial_S A_M$

⇩

- Broken AdS isometries/conformal sym. (energy scale [a]=1)
- Regge behaviour of the mass spectrum

- (Classical) eq. of motion : $\partial_N\left(\sqrt{-g}e^{-\phi}F^{NM}\right) - \sqrt{-g}e^{-\phi}m_5^2 A^M = 0$

- Bulk field decomposition (mode) : $A_\mu(x,z) = \underbrace{\epsilon_\mu e^{iq\cdot x}}_{\text{plane wave}}\underbrace{\psi(z)}_{\text{holo. wave function}}$



- Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2\psi(z)$ with $V(z) = \underbrace{a^4 z^2}\, + \underbrace{\frac{4m_5^2 + (c+2)c}{4z^2}} + \underbrace{(c-1)a^2}$

$\begin{cases} c = 1 : & A_M(x,z) \\ c = 3 : & X(x,z) \end{cases}$

dilaton $\phi(z) = a^2 z^2$ (**IR** : $z \to \infty$)

metric $g_{MN} = \frac{1}{z^2}\eta_{MN}$ (**UV** : $z \to 0$)

- **Mass spectrum :** $$m_n^2 = \left(4n + 1 + c + \sqrt{(c+1)^2 + 4m_5^2}\,\right) a^2$$

- **Holo. wave function :** $$\psi_n(z) = A_n\, e^{-a^2 z^2/2} z^{g(c,m_5^2)+1/2}\; {}_1F_1\left(-n, g(c,m_5^2)+1, a^2 z^2\right) \longrightarrow 0 \begin{cases} z \to \infty \\ z \to 0 \end{cases}$$

$$g(m_5, c) = \sqrt{m_5^2 + \frac{(c+1)^2}{4}}$$

Kummer confluent hypergeometric function (-n < 0 : polynomial)

		Scalar glueball	**Vector glueball**	**Vector ρ meson** (Son et al. '05)
J^{PC}	:	0^{++}	1^{--}	1^{--}
Boundary		TrF^2	$Tr(F(DF)F)_\mu$	$j_L^a(x)$ $j_R^a(x)$
		(Δ=4)	(Δ=7)	(Δ=3)
Bulk		$X(x,z)$	$A_M(x,z)$	$A_L^a(x,z)$ $A_R^a(x,z)$
		(p=0)	(p=1)	(p=0)
		$m_5^2 = 0$	$m_5^2 = 24$	$m_5^2 = 0$
Spectra		$\boxed{m_n^2 = \left(4n+8\right)a^2}$	$\boxed{m_n^2 = \left(4n+12\right)a^2}$	$\boxed{m_n^2 = \left(4n+4\right)a^2}$

Perturbed background

Background :
- AdS dual spacetime : $ds^2 = e^{2A(z)}\eta_{MN}ds^M dx^N = \frac{1}{z^2}(dx^2 + dz^2)$
- Dilaton : $\phi(z) = c^2 z^2$

Regge behaviour : $m_n^2 \propto n$ ⇒ connection dilaton/metric

- z → 0 : asymptotic AdS $\phi - A \xrightarrow{z \to 0} -\ln(z)$
- z → ∞ : harmonic-like potential $\phi - A \xrightarrow{z \to \infty} z^2$
- Higher spin meson spectrum $A(z) \not\ni z^{2+\beta}$ β > 0

Perturbation :

$\phi - A \sim z^{\alpha}$

0 ≤ α < 2

$\boxed{\alpha = 1}$

Decay constants of glueballs

Operator/field correspondence : $e^{iS_5^{eff}[X(x,z)]} = \langle e^{i\int d^4x X_0(x)\mathcal{O}(x)} \rangle_{CFT}$

2-points correlator function $\Pi(q^2)$ $\Longrightarrow$ Decay constant $f_n = \langle 0|\mathcal{O}(0)|n\rangle$

$\boxed{\Pi_{QCD}(q^2) = \Pi_{AdS}(q^2)}$

- **QCD :** $\Pi_{QCD}(q^2) \equiv i \int d^4x\, e^{iq.x} \langle 0|T[\mathcal{O}(x)\mathcal{O}(0)]|0\rangle$

Completeness in the 2 chronological order : $\Pi_{QCD}(q^2) = \Sigma_n \frac{f_n^2}{q^2 + m_n^2}$

- **AdS :** $\Pi_{AdS}(q^2) = \left(\tilde{X}(q,z), \partial_z \tilde{X}(q,z)\right)\Big|_{z\to 0}$ $\Longrightarrow$ Bulk-to-boundary propagator

Fourier transf. of X(x,z)

Bulk-to-boundary propagator (massless scalar bulk field) :

$$X(x,z) = \int_{M^4} d^4x' \underbrace{K(x,z;x',0)}X_0(x')$$

Boundary translation invariance : $K(x-x';z,0) \xrightarrow{z \to 0} \delta^4(x-x')$

$\tilde{X}(q,z) = \tilde{K}(q,z)\tilde{X}_0(q)$ **with** $\tilde{K}(q,z) \xrightarrow{z \to 0} 1$ (massless scalar)

$$\Rightarrow \quad \boxed{\Pi_{AdS}(q^2) = \tilde{K}(q,z)\left(\frac{e^{-\phi(z)}}{z^3}\right)\partial_z \tilde{K}(q,z)\Big|_{z \to 0}}$$

- $q^2 = -m_n^2$ normalizable bulk mode $\tilde{K}_n(z)$ $\Rightarrow$ dual to particle states

  z → 0 $\boxed{\tilde{K}_n(z) \sim A_n\, z^4}$

- $q^2 > 0$ non-normalizable bulk mode $\tilde{K}(q,z)$ $\Rightarrow$ dual to currents (virtuality)

  (deep inelastic limit : $q^2 \to \infty$)

  z → 0 $\boxed{\tilde{K}(q,z) \sim 1}$

eq. of motion : $\mathcal{D}\tilde{K}_n(z) = \left[\partial_z\left(\frac{e^{-\phi}}{z^3}\partial_z\right) + m_n^2\frac{e^{-\phi}}{z^3}\right]\tilde{K}_n(z) = 0$ $\qquad q^2 = -m_n^2$

Sturm-Liouville operator

completeness

Green's function : $\mathcal{D}G(q^2; z, z') = -\delta(z - z')$

$$G(q^2; z, z') = \Sigma_n \frac{\tilde{K}_n(z)\tilde{K}_n(z')}{q^2 + m_n^2}$$

Green's theorem : $\tilde{K}(q, z) = \tilde{K}(q, z')\left(\frac{e^{-\phi(z')}}{z'^3}\right)\partial_{z'}G(q^2, z', z)\Big|_{z'\to 0}$

$$\boxed{\Pi_{AdS}(q^2) = \Sigma_n \frac{1}{q^2 + m_n^2}\Big[\underbrace{\tilde{K}(q, z)}_{1}\underbrace{\frac{e^{-\phi(z)}}{z^3}}_{1/z^3}\underbrace{\partial_z\tilde{K}_n(z)}_{4A_n z^3}\Big]^2\Big|_{z\to 0}}$$

$$\boxed{f_n = 4A_n \sim \sqrt{8(n+1)(n+2)}}$$

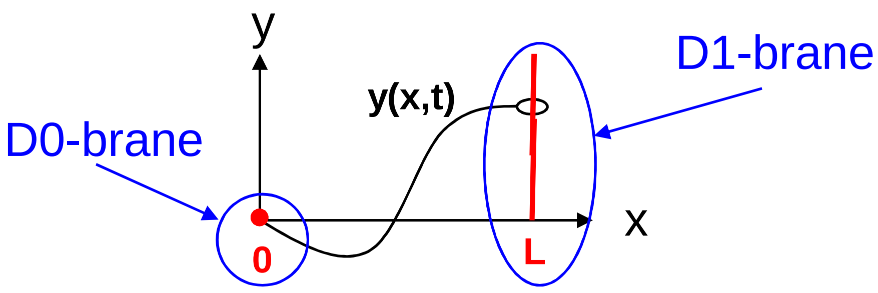
Heavy-light meson spectum (Evans et al. '06)

$Q\bar{q}$ mesons $\begin{cases} D = c\bar{q} \\ B = b\bar{q} \end{cases}$ (q=u,d,s)

D(irichlet) p-brane model of spacetime :

- p spatial-dim. object
- (p+1)-dim. spacetime

D3-brane in 4-dim. Spacetime : $\mathcal{M}^4$



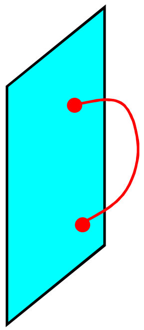
$\underbrace{y(0,t)=0 \qquad y'(L,t)=0}$

Dp-branes : boundary conditions

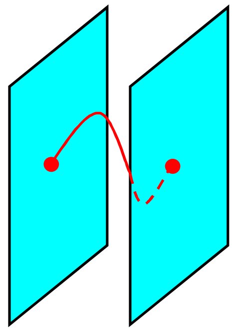
Open string endpoints attached to Dp-branes

Open string spectrum

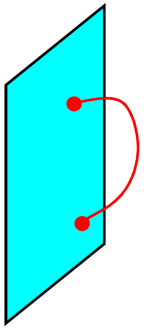
D3-brane :



D3-D3-branes :



D3-brane :

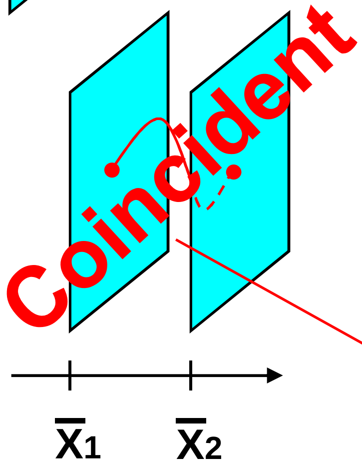


$$M^2 = \frac{1}{\alpha'}(N-1)$$

(harm. osc. E=ħω(N+1/2))

1 massless vector
(tachyon, massless scalars)

D3-D3-branes :



$$M^2 = \underbrace{\frac{1}{\alpha'}(N-1)}_{\text{quantum osc.}} + \underbrace{\left[T_0(\bar{x}_2 - \bar{x}_1)\right]^2}_{\text{classical energy of the stretched string}}$$

: (energy/length) x (length)

1 massive vector
(tachyon, massive scalars)

$$M^2 = \left[T_0(\bar{x}_2 - \bar{x}_1)\right]^2$$

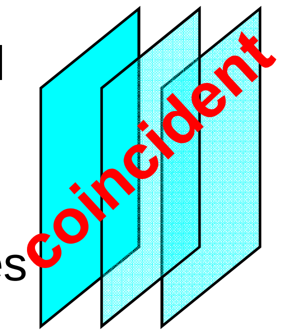
$\bar{x}_1 \longrightarrow \bar{x}_2$

1 massless vector

$$M^2 = 0$$

Standard Model
(QCD)

$\mathcal{M}^4$ 3 D3-branes



3 x 3 massless vectors : 9 gauge fields : SU(3) x U(1)
in (3+1) spacetime

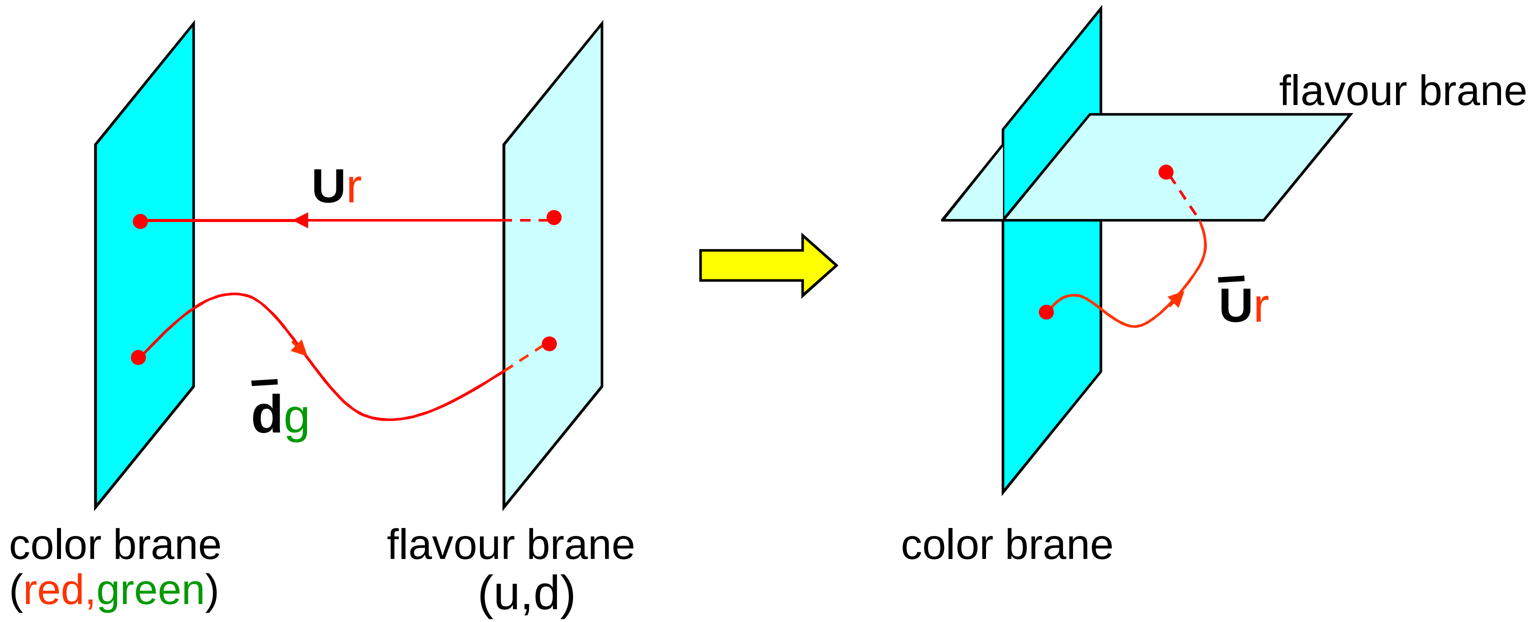
SU(3)c

- N superposed Dp-branes ⟹ Gauge theory SU(N) in (p+1) spacetime

  3 D3-branes ⟹ SU(3) in (3+1) spacetime

  ⇓ Boundary of the bulk ⇓ $\mathcal{M}^4$

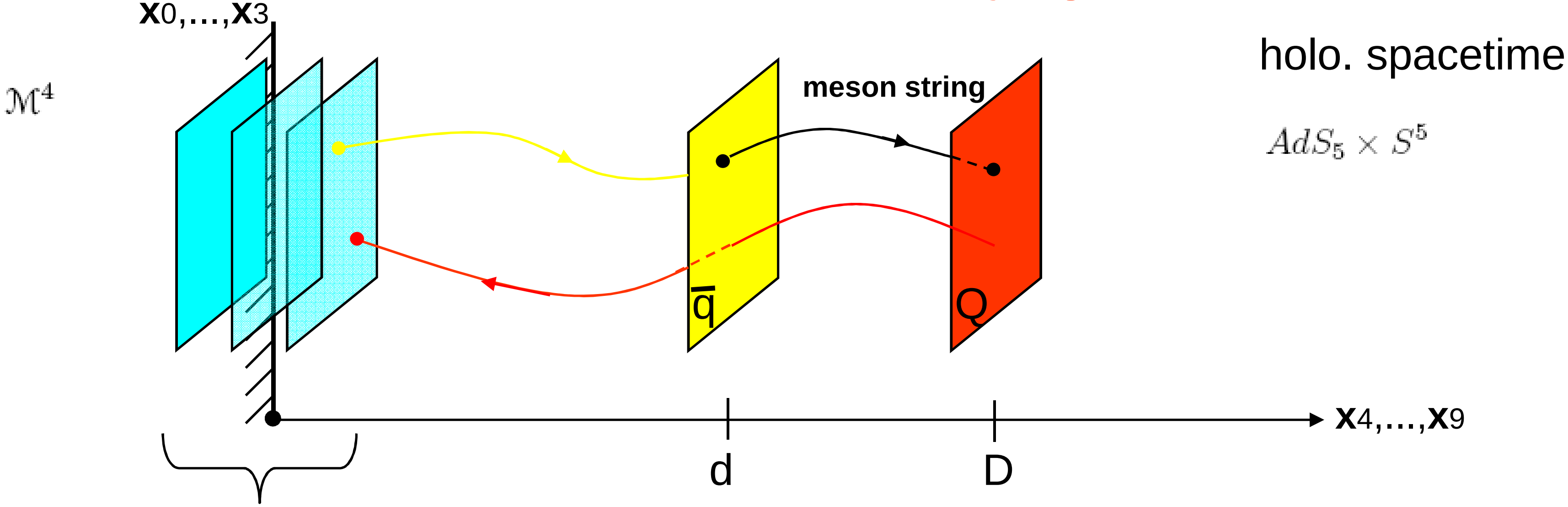
- Gluons : open strings with the 2 endpoints attached on the 3 (colored) D3-branes
- Quarks : open strings with
  - 1 endpoint attached on the 3 (colored) D3-brane
  - 1 endpoint attached to a flavour Dp-brane (D7-brane)



Massive quarks $M^2 = \left[T_0(\bar{x}_2 - \bar{x}_1)\right]^2$

Massless (chiral) quarks

D3-D7-brane model of heavy-light mesons



3 D3-baryonic branes (r,b,g)

SU(3) : QCD

2 D7-flavour branes (u,d,s) and (c,b)

D7-D3 open string spectrum : $M^2 = \frac{1}{\alpha'}(N - 1 + \frac{1}{4}) + [T_0(\bar{x}_2 - \bar{x}_1)]^2$

semi-classical string limit $\longrightarrow$ D>>d (B meson)

Heavy-light meson spectrum : $M^2 = [T_0(D - d)]^2$

M_ρ = 770 MeV : d

M_Υ = 9.4 GeV : D

B meson : M_B = 6529 MeV (5279 MeV)

better than 20% !