INSTITUT D'ASTROPHYSIQUE DE PARIS
Unité mixte de recherche 7095 *CNRS - Université Pierre et Marie Curie*

First Einstein Telescope France Workshop

Tests of General Relativity (GR)

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Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
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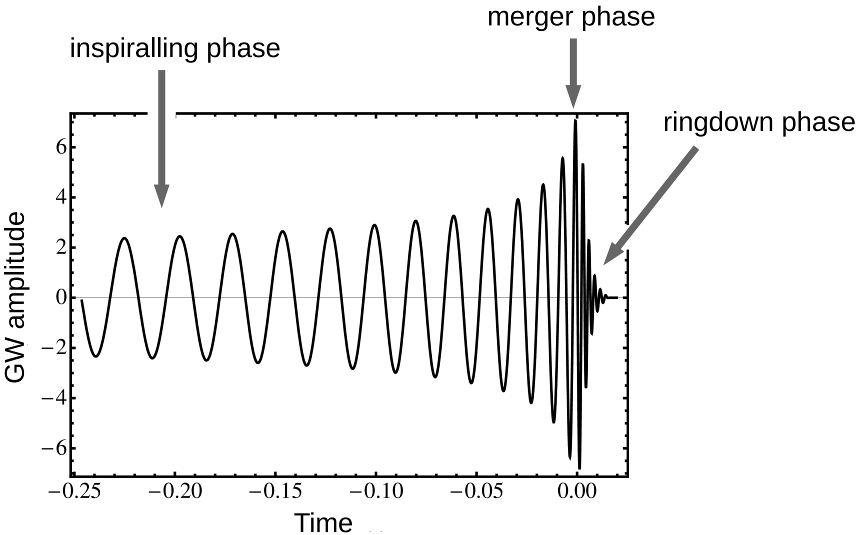
Plan of the talk

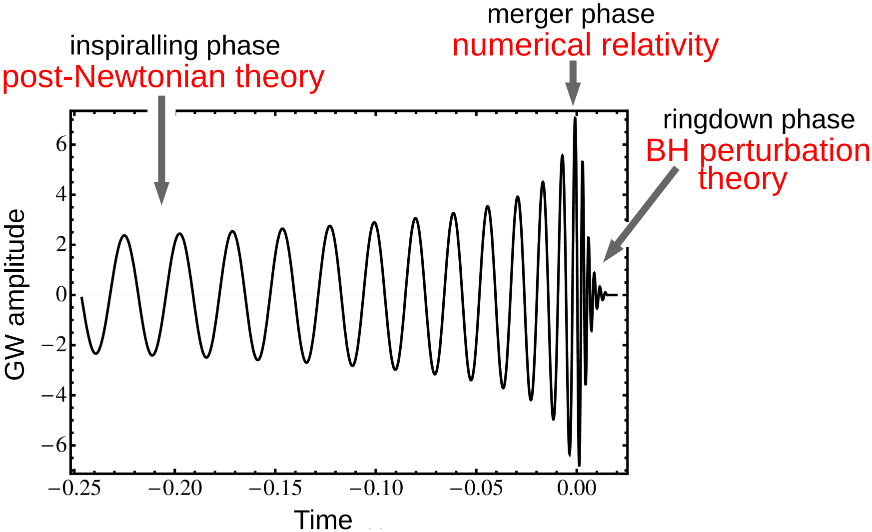
1. Consistency tests within GR
2. Tests of BHs and exotic compact objects
3. Tests of alternative theories
4. Tests with the speed of GWs

CONSISTENCY TESTS WITHIN GR

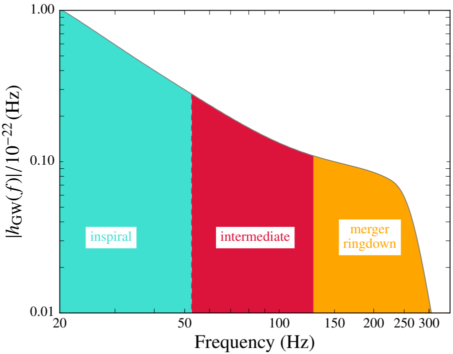
The gravitational chirp of binary black holes



The gravitational chirp of binary black holes



The inspiral-merger-ringdown (IMR) model

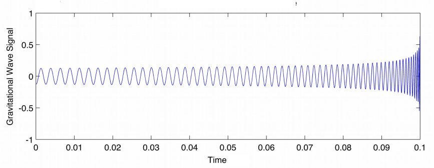


Effective methods that interpolate between the different phases play a crucial role

- The effective-one-body (EOB) approach [Buonanno & Damour 1999]
- The inspiral-merger-ringdown (IMR) [Ajith *et al.* 2008]

$$\{\underbrace{\text{PN parameters};}_{\text{inspiral}}\ \underbrace{\beta_2, \beta_3}_{\text{intermediate}}\ ;\ \underbrace{\alpha_2, \alpha_3, \alpha_4}_{\text{merger-ringdown}}\ \}$$

PN parameters in the orbital phase evolution



- The PN parameters come from a mixture of conservative and dissipative effects through the energy balance equation

$$\frac{\mathrm{d}\overbrace{E}^{\text{conservative energy}}}{\mathrm{d}t} = - \underbrace{\mathcal{F}^{\mathsf{GW}}}_{\text{dissipative energy flux}}$$

- The orbital phase $\phi = \int \omega \, \mathrm{d}t$ is obtained as a function of $x = \left(\frac{GM\omega}{c^3}\right)^{2/3}$ and the mass ratio $\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$

$$\boxed{\phi(x) = \phi_0 - \frac{x^{-5/2}}{32\nu} \sum_p \Big(\varphi_{p\mathsf{PN}}(\nu) + \varphi^{(l)}_{p\mathsf{PN}}(\nu) \, \log x \Big) \, x^p + \mathcal{O}\big[(\log x)^2\big]}$$

The known 3.5PN parameters [see Blanchet 2014 for a review]

$$\varphi_{\text{0PN}} = 1 \qquad \Longleftarrow \quad \text{Einstein quadrupole formula}$$

$$\varphi_{\text{1PN}} = \frac{3715}{1008} + \frac{55}{12}\nu$$

$$\varphi_{\text{1.5PN}} = -10\pi$$

$$\varphi_{\text{2PN}} = \frac{15293365}{1016064} + \frac{27145}{1008}\nu + \frac{3085}{144}\nu^2$$

$$\varphi^{(l)}_{\text{2.5PN}} = \left(\frac{38645}{1344} - \frac{65}{16}\nu\right)\pi$$

$$\begin{aligned}\varphi_{\text{3PN}} &= \frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}\gamma_{\text{E}} - \frac{3424}{21}\ln 2 \\ &\quad + \left(-\frac{15737765635}{12192768} + \frac{2255}{48}\pi^2\right)\nu + \frac{76055}{6912}\nu^2 - \frac{127825}{5184}\nu^3\end{aligned}$$

$$\varphi^{(l)}_{\text{3PN}} = -\frac{856}{21}$$

$$\varphi_{\text{3.5PN}} = \left(\frac{77096675}{2032128} + \frac{378515}{12096}\nu - \frac{74045}{6048}\nu^2\right)\pi$$

The known 3.5PN parameters [see Blanchet 2014 for a review]

tail terms

$$\varphi_{0\text{PN}} = 1$$

$$\varphi_{1\text{PN}} = \frac{3715}{1008} + \frac{55}{12}\nu$$

$$\varphi_{1.5\text{PN}} = \boxed{-10\pi}$$

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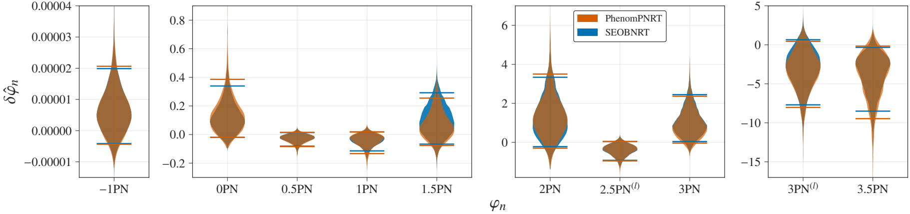
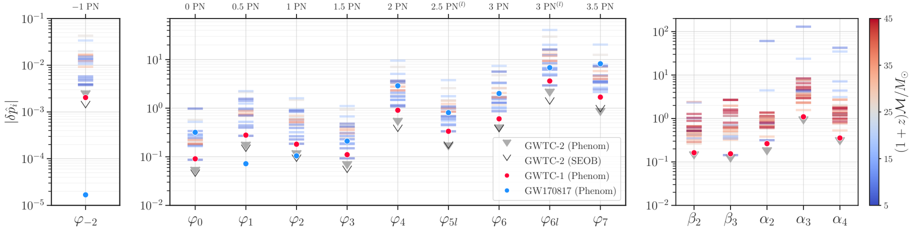
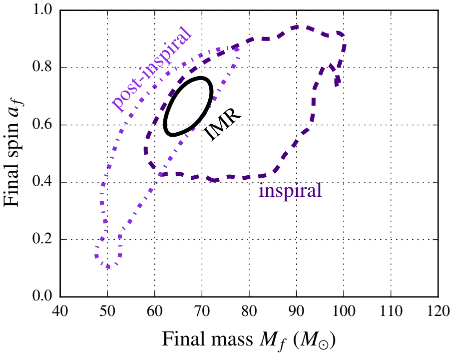
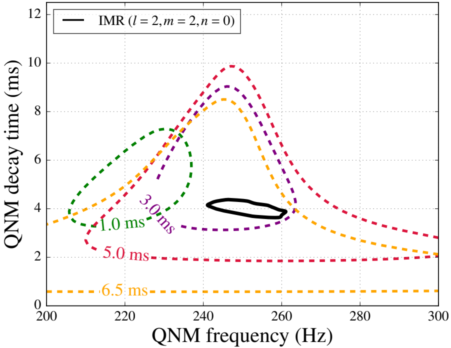
Measurement of PN parameters [LIGO/Virgo 2017, 2020]





Inspiral-Merger-Ringdown consistency test [LIGO/Virgo 2016]



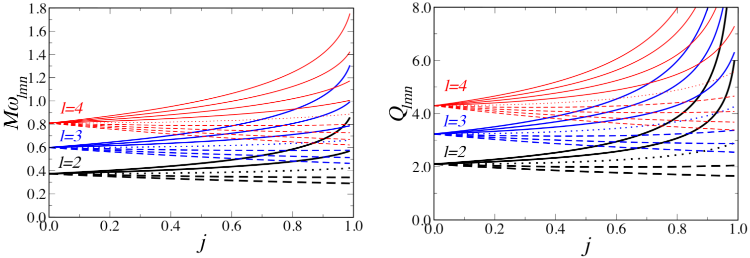


TESTS OF BHs AND EXOTIC COMPACT OBJECTS

Testing the no-hair theorem with the ringdown

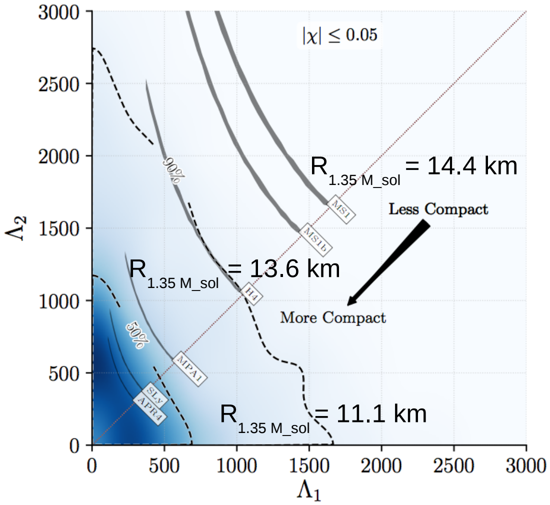
- The merger of two black holes produces a distorted BH who emits ringdown radiation to shed hair
- The frequency modes of ringdown radiation are [e.g. Berti, Cardoso & Will 2006]

$$\omega = \omega_{\ell mn}\left[1+\frac{\mathrm{i}\pi}{2Q_{\ell mn}}\right] \qquad (n = \text{overtone index})$$



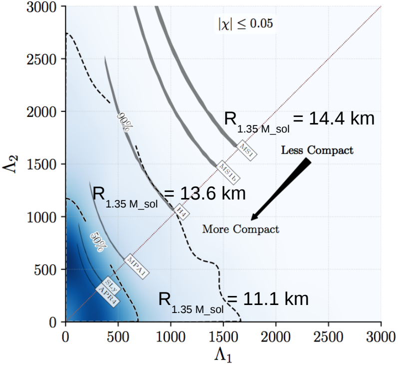
$$j = \frac{J}{M^2} = \frac{a}{M}$$

Test of the NS equation of state [LIGO/Virgo 2017]



$$\boxed{\Lambda_a = \frac{2}{3} k_a \left(\frac{c^2 R_a}{G m_a} \right)^5}$$

Test of the NS equation of state [LIGO/Virgo 2017]



- Contribution to the GW chirp

$$\boxed{\begin{aligned} x &= \frac{1}{4}\theta^{-1/4}\left[1 + \frac{39}{8192}\tilde{\Lambda}\,\theta^{-5/4}\right] \\ \varphi &= \varphi_0 - \frac{x^{-5/2}}{32\nu}\left[1 + \underbrace{\frac{39}{8}\tilde{\Lambda}\,x^5}_{\text{5PN effect}}\right] \end{aligned}}$$

- Measured tidal deformability parameter

$$\tilde{\Lambda} = \frac{16}{13}\left[\left(1 + 11\frac{m_2}{m}\right)\left(\frac{m_1}{m}\right)^4 \Lambda_1 + 1 \leftrightarrow 2\right]$$

[Flanagan & Hinderer 2008]

Postmerger echoes [Cardoso & Pani 2016]

- Suppose that the object formed by the merger of two BHs contains a material surface between the horizon at $2M$ and the photon sphere at $3M$,

$$R_{\text{material}} = 2M + \epsilon \quad \text{with} \quad \epsilon \ll M$$

- In that case the ringdown radiation at frequency $f_{\text{ringdown}} \sim \pi/M$ should be followed by a succession of echoes at frequency $f_{\text{echo}} \sim \pi/\tau_{\text{echo}}$ with

$$\tau_{\text{echo}} \sim M \left| \ln\left(\frac{\epsilon}{M}\right) \right|$$

- Echos may reveal the existence of ultra compact exotic objects with radius between the Buchdahl limit $R_{\text{B}} = \frac{9}{4}M$ and the photon sphere $3M$

Postmerger echoes [Cardoso & Pani 2016]

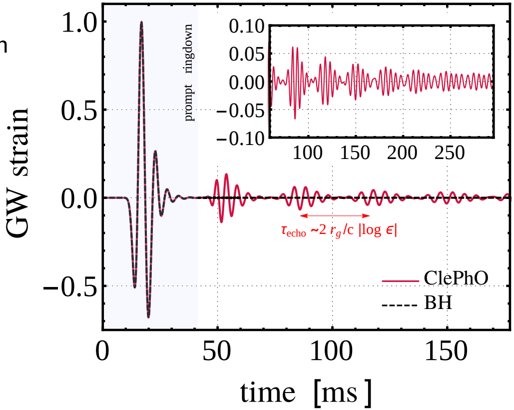
- Speculate that ϵ is the Planck length related to unknown quantum effects at the horizon scale

$$\epsilon \sim 1.6\ 10^{-34}\,\text{m}$$

- The echoes occur with frequency

$$f_{\text{echo}} \sim 10^3 \left(\frac{M_\odot}{M}\right)\ \text{Hz}$$

- No evidence was found for echoes in the LIGO/Virgo data



TESTS OF ALTERNATIVE THEORIES

GW solutions in metric theories of gravity

1. Small perturbation of the metric around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad |h_{\mu\nu}| \ll 1$$

2. Restrict attention to theories admitting GW solutions propagating at the speed of light: $c_g = 1$. Far from the sources the waves are planar hence

$$\Box h_{\mu\nu} = 0 \quad \Longleftrightarrow \quad h_{\mu\nu} = h_{\mu\nu}(t - z)$$

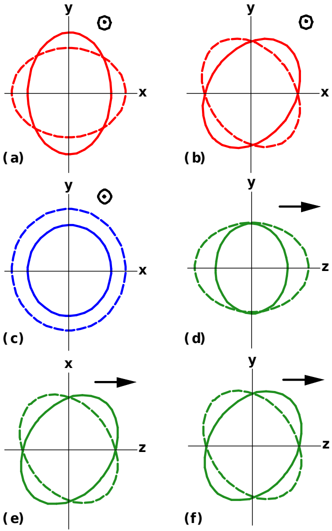
3. From the linearized Bianchi's identity obtain

$$\boxed{\Box R_{\mu\nu\rho\sigma} = 0 \quad \Longleftrightarrow \quad R_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma}(t - z)}$$

showing that GWs have an invariant, coordinate-independent meaning

4. The six components R_{0i0j} (where $i, j = x, y, z$) represent six independent components (polarization modes)

GW polarization modes in metric theories of gravity



- General Relativity
- Scalar-Tensor theory
  [*e.g.* Will 1993]
- Massive Gravity theory
  [*e.g.* de Rham 2014]
- Scalar-Vector-Tensor
  [Sagi 2010]

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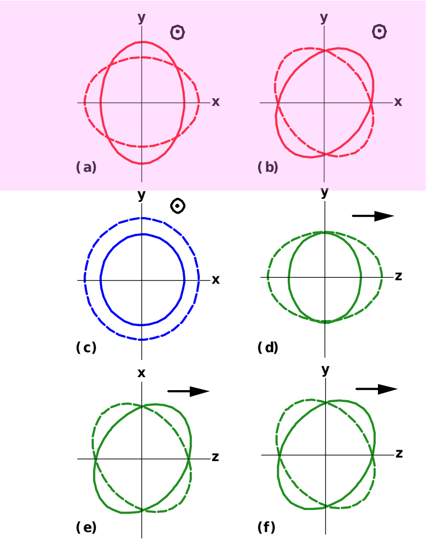

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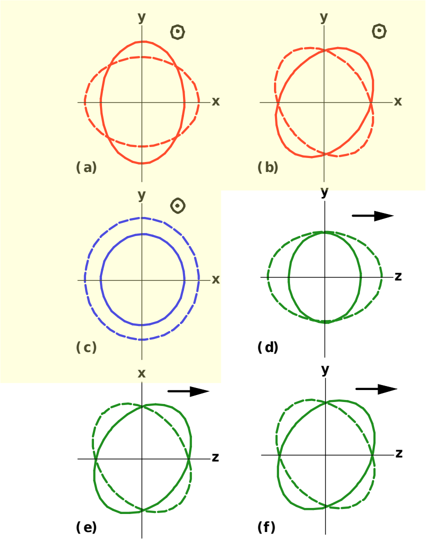

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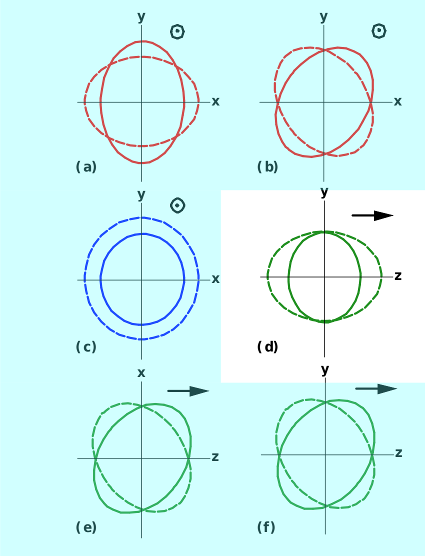

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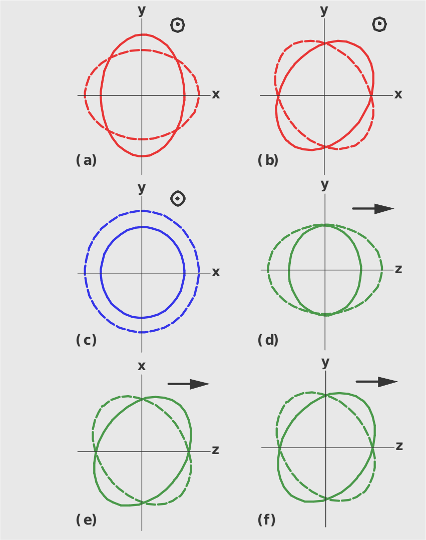

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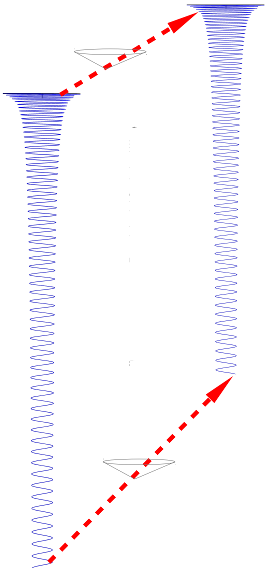
Test on polarizations with GW170817 [LIGO-Virgo 2017]

$$h(t) = \sum_{A=1}^{6} \underbrace{F^A(\Theta, \Phi, \Psi)}_{\text{detector response functions}} \overbrace{h_A[\varphi(t), i(t)]}^{\text{independent polarizations}} \qquad A = (+, \times, x, y, b, \ell)$$

- Coherent Bayesian analysis of the signal properties with the three interferometer outputs LIGO-Virgo
- The phase evolution of the GW is described by GR templates, but the polarization content varies
- The sky location of GW170817 is constrained to the host galaxy NGC4993
- Overwhelming evidence in favor of pure tensor polarization modes in comparison to pure vector and pure scalar modes with a (base ten) logarithm of the Bayes factor

$$\boxed{\log_{10} \mathcal{B}^{\mathsf{T}}_{\mathsf{V}} = +20.81 \pm 0.08 \qquad \log_{10} \mathcal{B}^{\mathsf{T}}_{\mathsf{S}} = +23.09 \pm 0.08}$$

Bounding the mass of the graviton [Will 1998]



- Dispersion relation for a massive graviton

$$\boxed{\frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E_g^2}} \quad \text{with} \quad E_g = \hbar\omega_g$$

- The frequency of GW sweeps from low to high frequency during the inspiral
- The constraint is [LIGO/Virgo 2016]

$$\boxed{m_g \lesssim 10^{-22}\,\text{eV}} \quad \Longleftrightarrow \quad \lambda_g \gtrsim 0.02\ \text{ly}$$

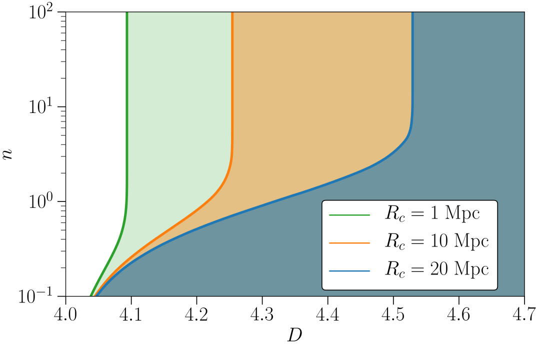
- But too low to be relevant to the problem of the cosmological constant

$$\boxed{\Lambda \sim \frac{m_g^2 c^2}{\hbar^2}} \quad \Longrightarrow \quad m_g \sim 3.7\,10^{-33}\,\text{eV}$$

Constraint on the number of space-time dimensions

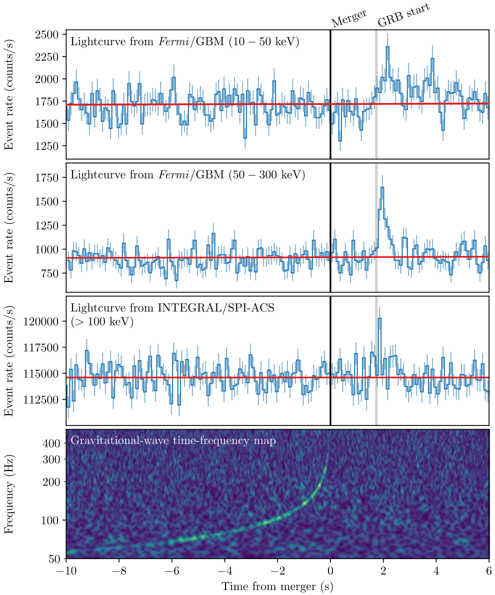
- Theories with non-compact extra-dimensions generically predict a deviation from the law $h^{\text{GW}} \propto 1/d_L$ due to gravitational leakage into extra dimensions
- Those that admit a screening scale R_c exhibit gravitational leakage above this scale [Deffayet & Menou 2007; Pardo, Fishbach, Holz & Spergel 2018]

$$h^{\text{GW}} \propto \frac{1}{d_L}\left[1+\left(\frac{d_L}{R_\text{c}}\right)^{\frac{n(D-4)}{2}}\right]^{-\frac{1}{n}} \propto \frac{1}{d_L^{\frac{D-2}{2}}} \qquad \text{when} \quad d_L \gg R_\text{c}$$



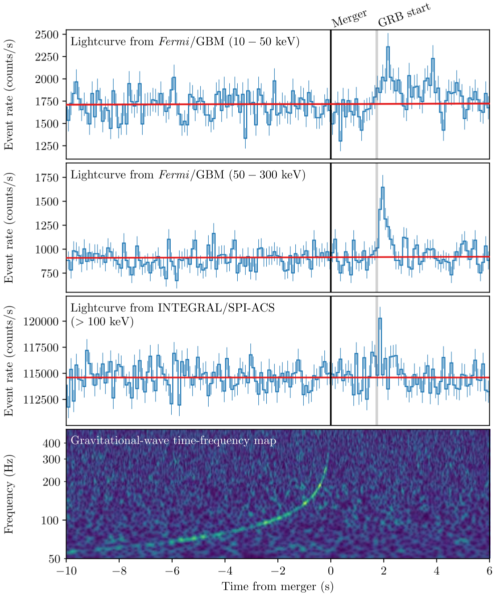
TESTS WITH THE SPEED OF GWs

The advent of multi-messenger astronomy



- The gamma-ray burst has been detected 1.7 second after the instant of merger
- This is the closest gamma-ray burst whose distance is known and is probably seen off-axis with respect to the relativistic jet

Speed of gravitational waves versus speed of light



- The observed time delay between GW170817 and GRB170817A gives a strong constraint

$$\boxed{|c_g - c_{em}| \lesssim 10^{-15} c}$$

- This eliminated a series of alternative theories

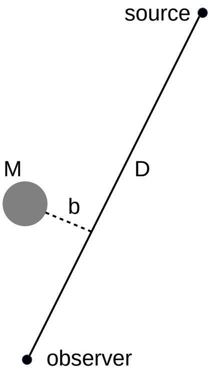
Test of the strong equivalence principle [Desai & Kahya 2016]

- The test involves the cumulative Shapiro time delay due to the gravitational potential of the dark matter distribution
- The violation of the equivalence principle is quantified by a PPN like parameter γ_a depending on the type of radiation $a = \text{GW}, \text{EM}$. For a spherical mass distribution

$$\Delta t^a_{\text{Shapiro}} = (1 + \gamma_a) \frac{GM}{c^3} \ln\left(\frac{D}{b}\right)$$

- The main contributions come from the galaxy NGC4993 and our own Galaxy with mass $M_{\text{MW}} = 5.6\, 10^{11}\, M_\odot$
- Assuming an isothermal density profile for dark matter this yields about 400 days delay in GR
- The observed difference in arrival time $\Delta t = 1.7\,\text{s}$ yields

$$\boxed{|\gamma_{\text{GW}} - \gamma_{\text{EM}}| \lesssim 10^{-7}}$$



Dark energy after GW170817 [Bettoni *et al.* 2017; Creminelli & Vernizzi 2017]

- Consider models of dark energy and modified gravity characterized by a single scalar degree of freedom (with $X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$) [Horndeski 1974]

$$\begin{aligned}L = {} & G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R \\ & - 2G_{4,X}(\phi, X)\left(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right) + G_5(\phi, X)E^{\mu\nu}\phi_{\mu\nu} \\ & + \frac{1}{3}G_{5,X}(\phi, X)\left(\Box\phi^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu\nu}\phi_{\mu\rho}\phi^{\rho}_{\nu}\right)\end{aligned}$$

- Imposing the speed of GWs to be one (*i.e.* $c_\mathrm{g} \equiv c_\mathrm{T} = 1$) drastically reduces the space of allowed theories

$$\boxed{L_{c_\mathrm{g}=1} = G_2(\phi, X) + \overbrace{B_4(\phi)R}^{\text{conformal coupling}} + \overbrace{G_3(\phi, X)\Box\phi}^{\text{cubic galileon}}}$$

- In beyond-Horndeski theory [Gleyzes, Langlois, Piazza & Vernizzi 2015] another type of term is also allowed

Mondian dark matter after GW170817 [Woodard *et al.* 2018]

In TeVeS [Bekenstein 2004] which is a specific version of relativistic MOND theory:

- Gravitational waves couple to the Einstein-frame metric $g_{\mu\nu}$ produced by GR without dark matter
- Ordinary matter couples to the Jordan-frame metric $\tilde{g}_{\mu\nu}$ which is a disformally transformed metric that would be produced by GR with dark matter

$$\tilde{g}_{\mu\nu} = e^{2\phi}(g_{\mu\nu} + U_\mu U_\nu) - e^{-2\phi} U_\mu U_\nu$$

- This theory is excluded by the test of the speed of GWs
- But other MOND-motivated theories (e.g. Khronon theory) survive the test