

# Small-Scale Tests of Inflation

Laura Iacconi

Based on: 1910.12921 & 2008.00452

in collaboration with Matteo Fasiello, David Wands,  
Hooshyar Assadullahi and Ema Dimastrogiovanni



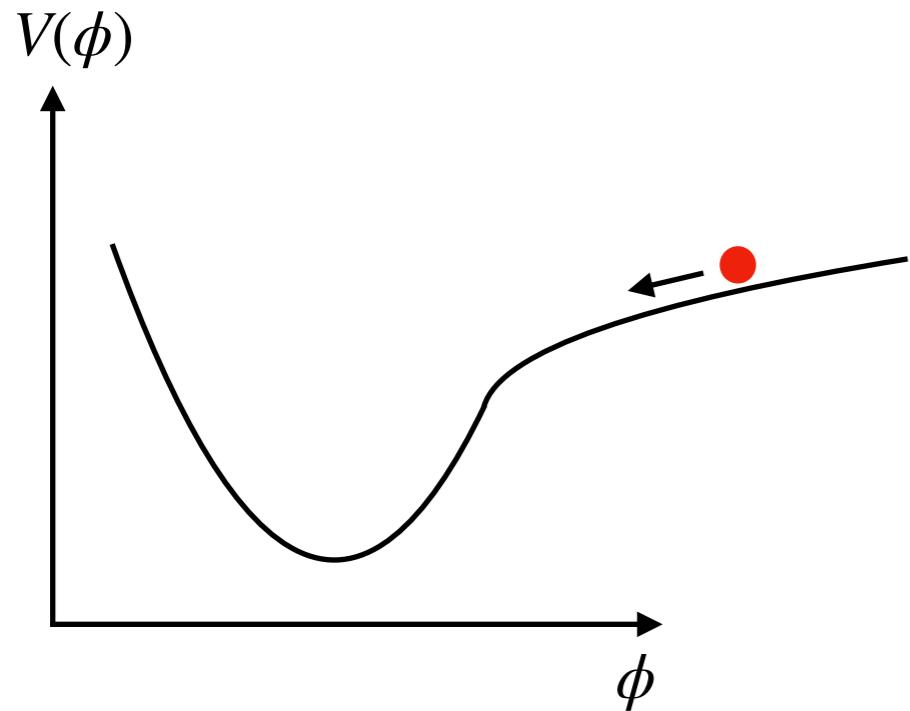
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**INFLATION:** period of accelerated expansion in the early universe

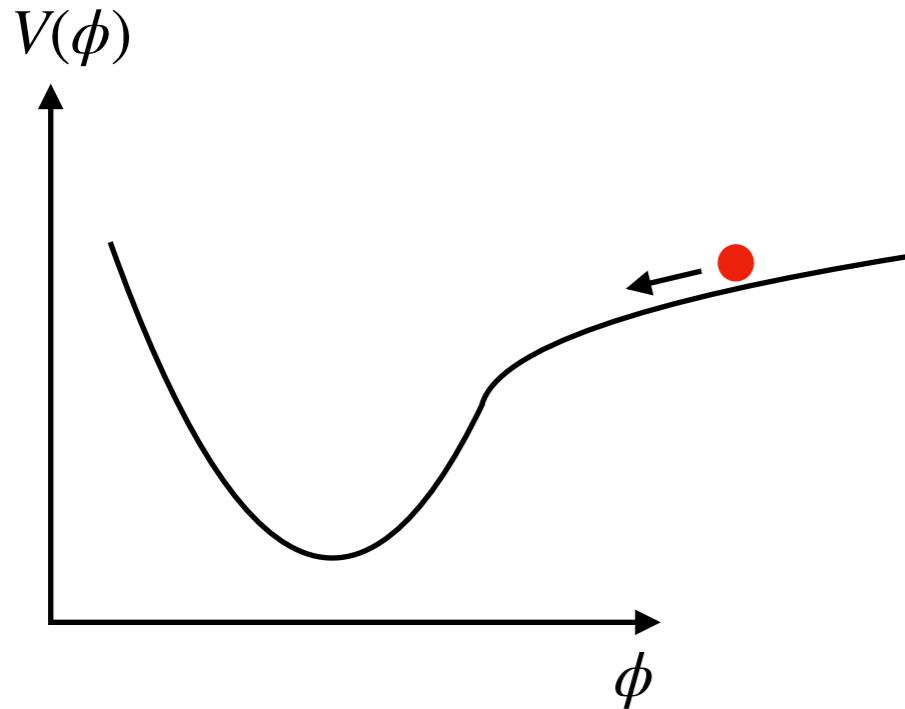


Single field slow roll inflation

$$\mathcal{S}[\phi] = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} \mathcal{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$H^2 = \frac{1}{3M_p^2} \left( V(\phi) + \frac{1}{2} \dot{\phi}^2 \right)$$

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$$H^2 = \frac{1}{3M_p^2} \left( V(\phi) + \cancel{\frac{1}{2} \dot{\phi}^2} \right) \rightarrow H^2 \simeq \frac{1}{3M_p^2} V(\phi)$$

slow roll parameters:

$$\epsilon_1 = -\frac{\dot{H}}{H^2} \ll 1$$

$$\epsilon_2 = \frac{\dot{\epsilon}_1}{H\epsilon_1} \ll 1$$

## Primordial GWs: key prediction of inflation

$$ds^2 = -dt^2 + a^2[(1 - 2\zeta)\delta_{ij} + \underline{\gamma_{ij}}]dx^i dx^j$$

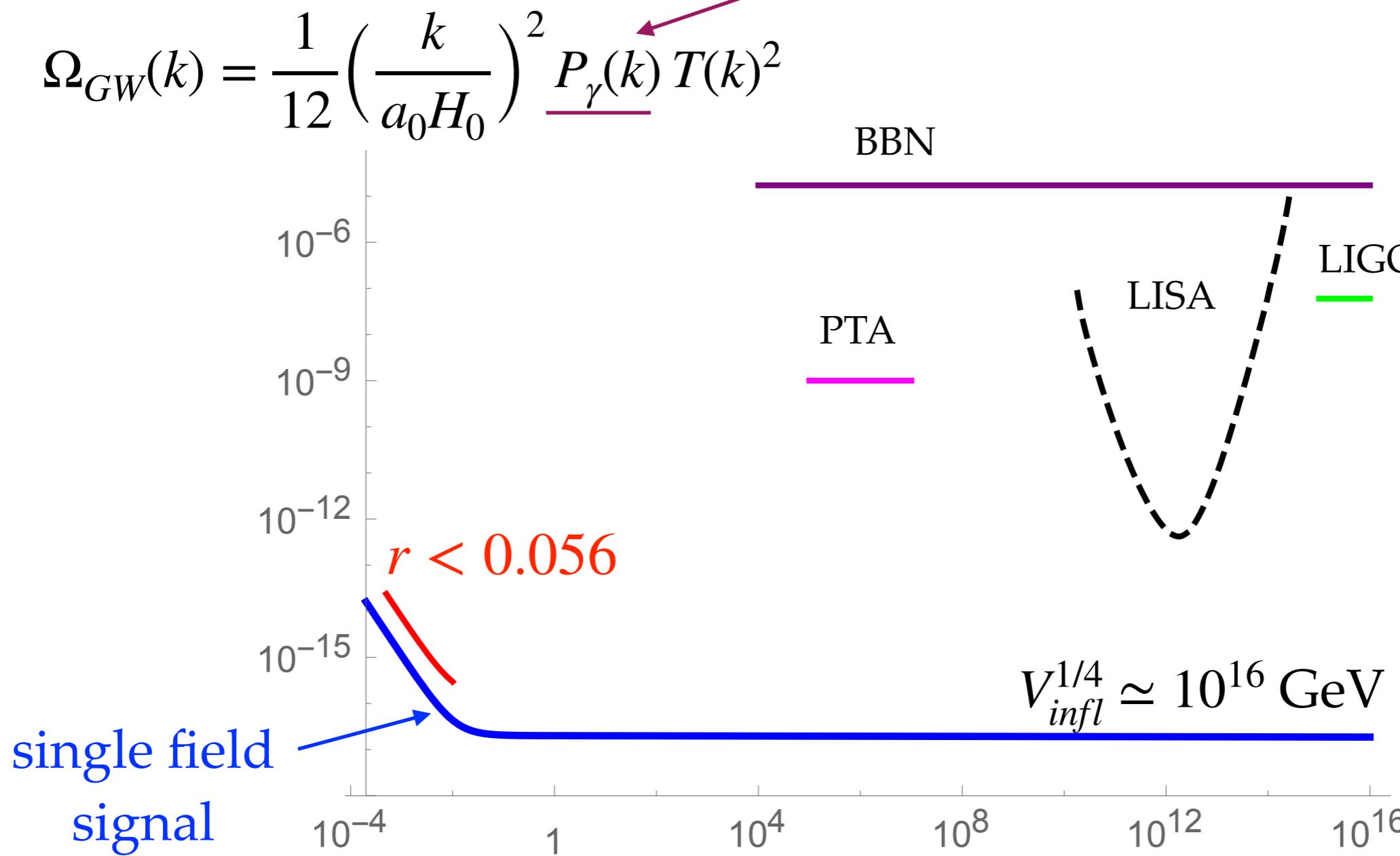
## Primordial GWs: key prediction of inflation

$$ds^2 = -dt^2 + a^2[(1 - 2\zeta)\delta_{ij} + \gamma_{ij}]dx^i dx^j$$

$$\Omega_{GW}(k) = \frac{1}{12} \left( \frac{k}{a_0 H_0} \right)^2 \underline{P_\gamma(k)} T(k)^2$$

## Primordial GWs: key prediction of inflation

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## Beyond SF inflation: non-minimal inflationary field content

- natural: plenty of candidates from string theory
- window on high energy physics, i.e. beyond scalar fields
- interesting: richer phenomenology

scale dep. amplitude  
chiral  
non-Gaussian

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primordial GWs sourced by extra particle content

$$P_\gamma = \boxed{\frac{2H^2}{\pi^2 M_p^2} \left( \frac{k}{k_\star} \right)^{n_t}} + \boxed{\text{source}}$$

primordial GWs sourced by vacuum fluctuations of the metric

Powerful tool to describe inflation:  
**Effective Field Theory**

- description of perturbations around inflating background
- captures phenomenology without committing to a particular model

{0709.0293 - Cheung et al.}  
{1806.10587 - Bordin et al.}

## EFT for spinning fields during inflation:

- de Sitter space: unitarity bounds on massive spinning particles
  - aka Higuchi bound on spin-2 fields:  $m^2 \geq 2H^2$
  - do not survive, may leave imprint on correlations of light fields

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- de Sitter space: unitarity bounds on massive spinning particles  
*aka* Higuchi bound on spin-2 fields:  $m^2 \geq 2H^2$   
→ do not survive, may leave imprint  
on correlations of light fields

- quasi de Sitter space: non-minimal coupling with the inflaton makes the spin-2 effectively lighter

{1806.10587 - Bordin et al.}

Action for the perturbations:

spin-2:  $\sigma_{ij}(\bar{x}, t)$  , scalar:  $\zeta(\bar{x}, t) = -H\pi(\bar{x}, t)$  , tensor:  $\gamma_{ij}(\bar{x}, t)$

$$\mathcal{L}^{(2)} = \mathcal{L}_{free}^{(2)} + a^3 \left[ -\frac{\rho}{\sqrt{2\epsilon_1} H} a^{-2} \partial_i \partial_j \pi_c \sigma^{ij} + \frac{1}{2} \rho \dot{\gamma}_c{}_{ij} \sigma^{ij} \right]$$

$\mathcal{L}_{int}^{(2)}$

$$\mathcal{L}^{(3)} = -a^3 \mu (\sigma_{ij})^3$$

## Quadratic interactions:

$$\mathcal{L}^{(2)} = \mathcal{L}_{free}^{(2)} + a^3 \left[ -\frac{\rho}{\sqrt{2\epsilon_1} H} a^{-2} \partial_i \partial_j \pi_c \sigma^{ij} + \frac{1}{2} \rho \dot{\gamma}_c{}_{ij} \sigma^{ij} \right]$$

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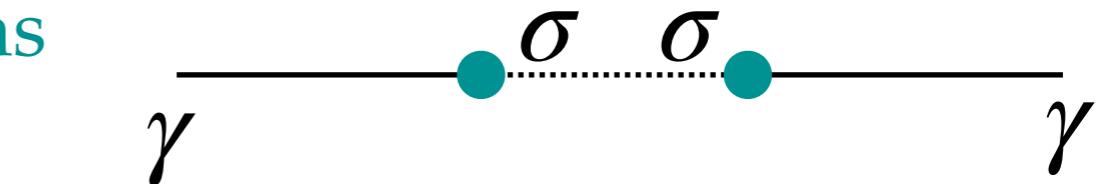
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- sources scalar perturbations
- linearly sources tensor perturbations

$$P_\gamma = \frac{2H^2}{\pi^2 M_p^2} + \frac{2H^2}{\pi^2 M_p^2} \frac{C_\gamma(\nu)}{c_2^{2\nu}} \left( \frac{\rho}{H} \right)^2$$

vacuum                          source





- spin-2 mass,  $\nu = \sqrt{\frac{9}{4} - \left(\frac{m}{H}\right)^2}$

- helicity-2 sound speed  $c_2$

- coupling  $\frac{\rho}{H}$

Our set-up: *time-dependent* sound speeds  $\{c_0, c_1, c_2\}$ ,  $s_i = \frac{\dot{c}_i}{Hc_i}$

{1610.06481 - Bartolo et al.} 

Why? Integrating out heavy fields may result  
into  $c_s < 1$  for the remaining light field(s)

{1201.6342 - Achucarro et al., ...}

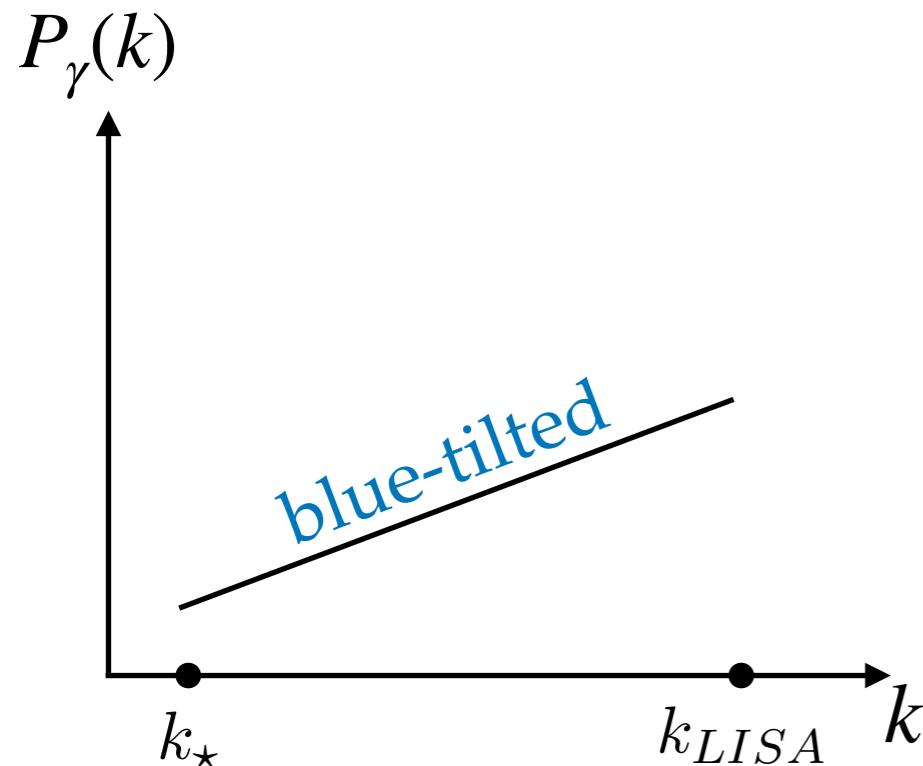
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➡ scale dependent  $P_\gamma$  &/or  $P_\zeta$

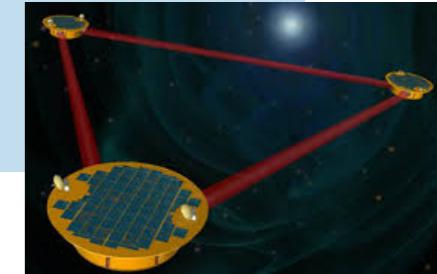


$$P_\gamma(k) \propto \frac{1}{c_2^{2\nu}} \rightarrow \frac{1}{c_2^{2\nu}} \left(\frac{k}{k_\star}\right)^{-2\nu s_2}$$

if  $s_2 < 0$  (decreasing  $c_2$ )  
the sourced contribution is blue-tilted

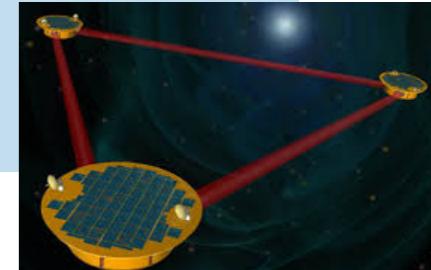
## The recipe of our analysis:

1. Consider a certain time-evolution of  $\{c_0, c_1, c_2\}$
2. Constrain it with theoretical and experimental bounds
3. Can the signal be detected by LISA (2034)?

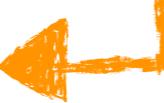


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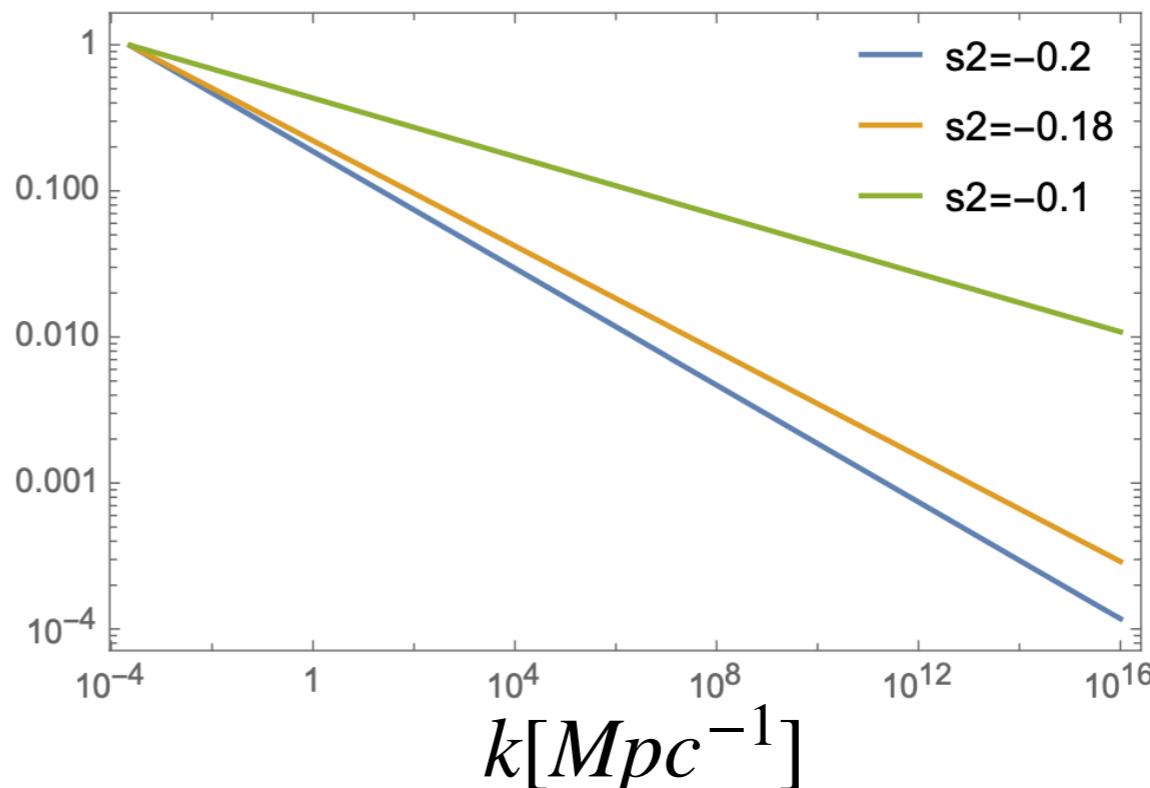


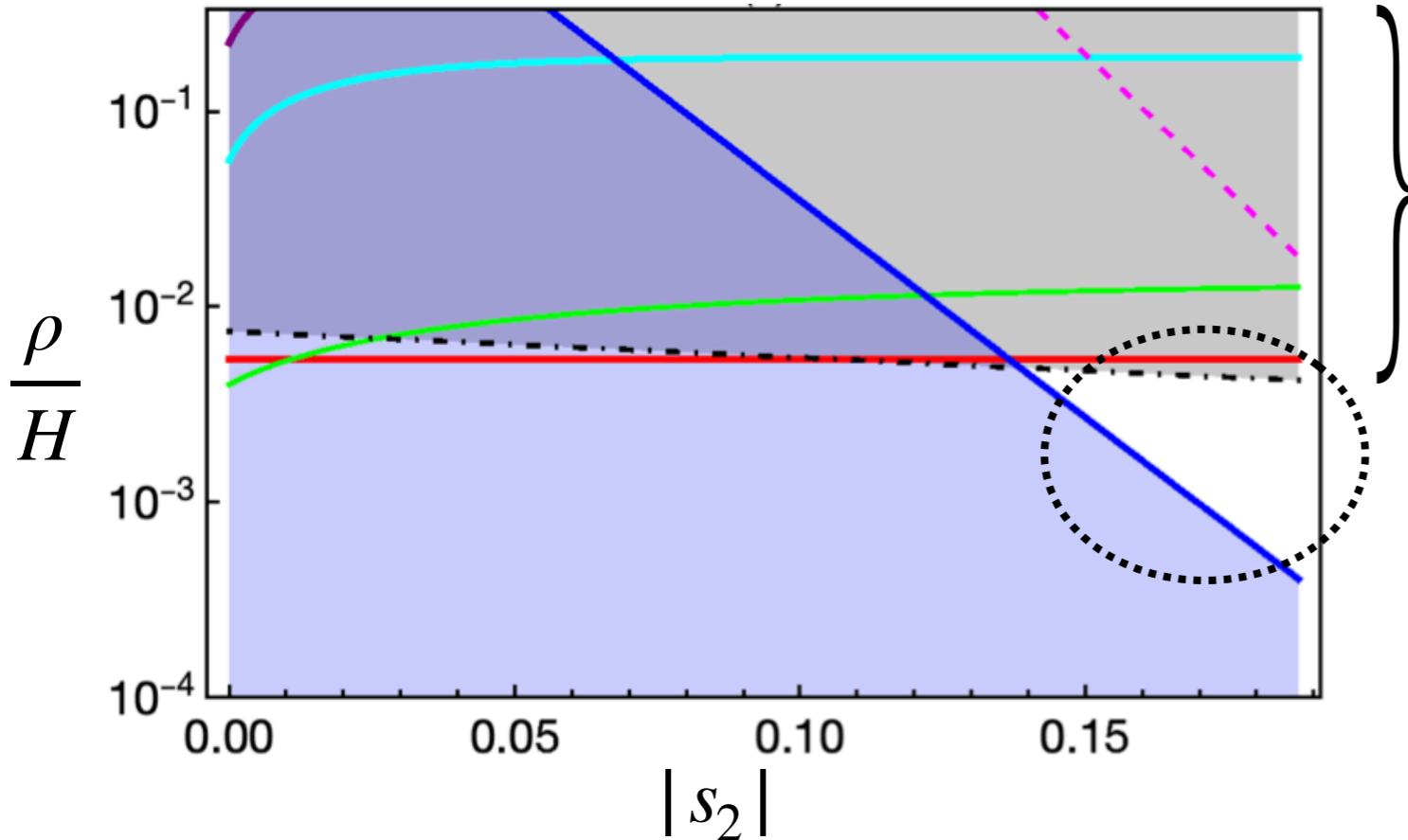
**Example:**  $\{H = 6.1 \times 10^{13} \text{ GeV}, \frac{m}{H} = 0.54, c_{2in} = 1, c_1 = 0.55, \frac{\rho}{H}, s_2\}$

we fix these 

param. space 

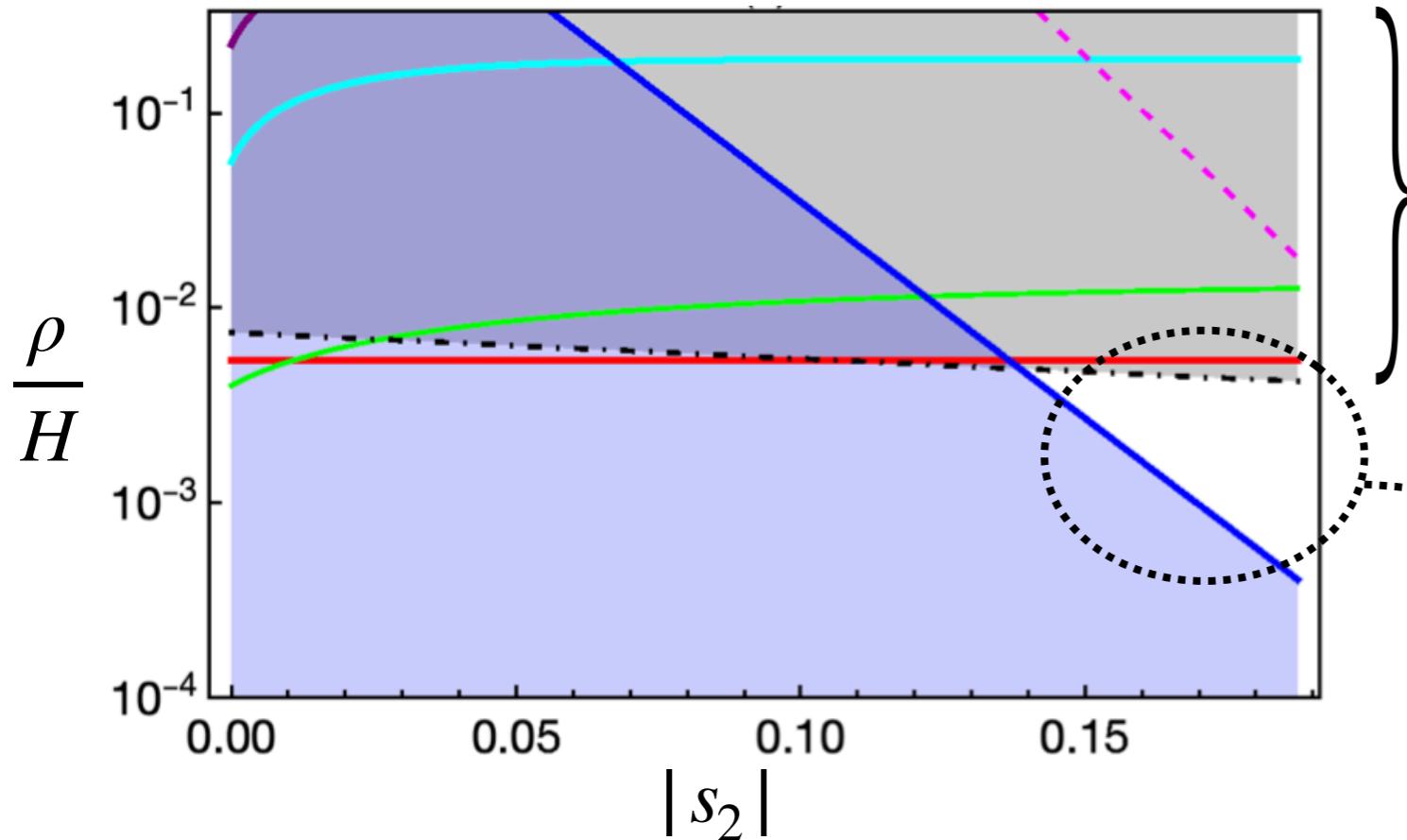
$$c_2(k) = c_{2in} \left( \frac{k}{a_0 H_0} \right)^{s_2}$$





CMB, PBHs, LIGO,  
UCMHs,  
theoretical  
requirements,..

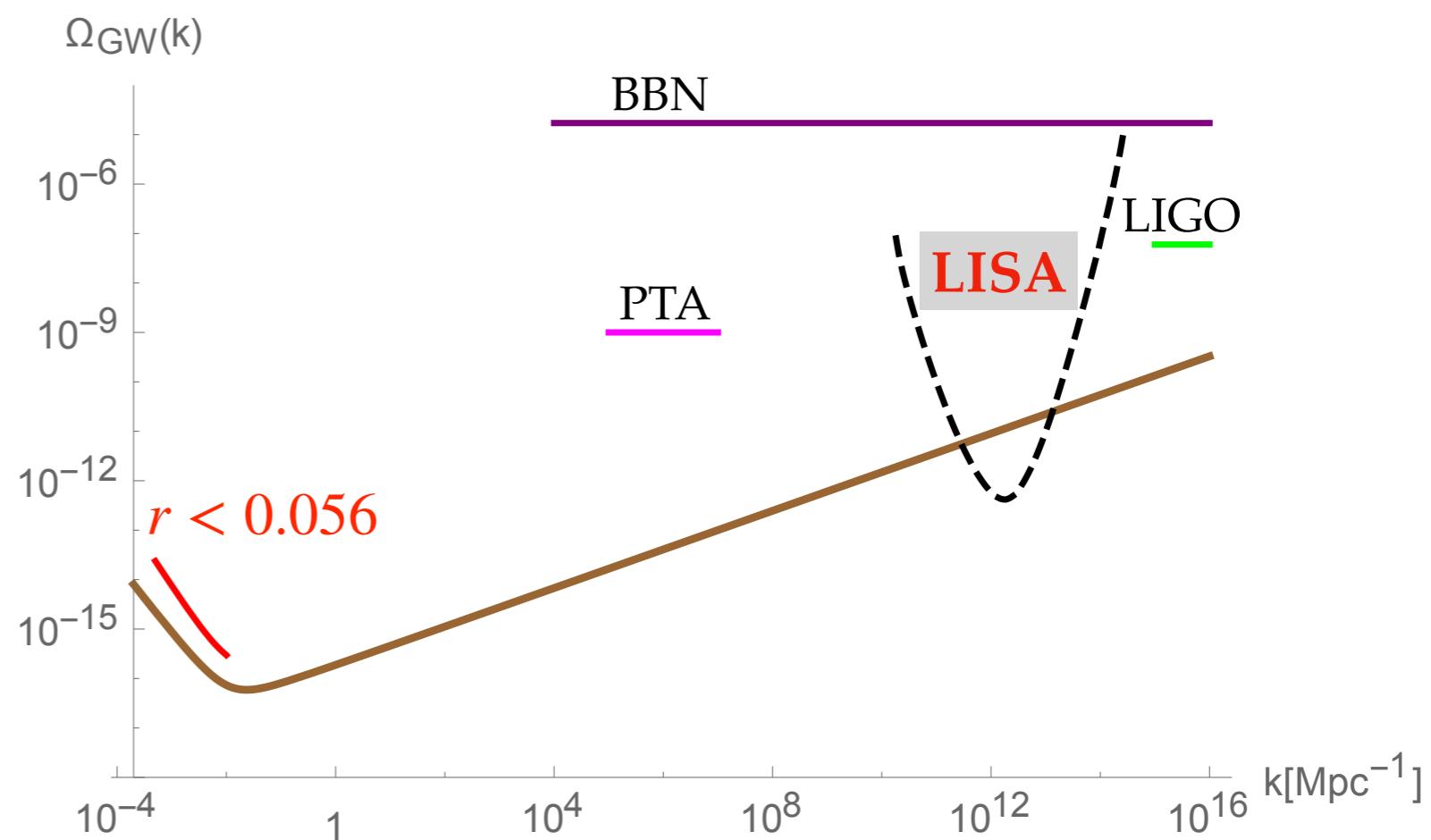
- grey region: excluded by the strongest existing bound (r)
- white region: can be surveyed by LISA
- blue region: can be excluded by LISA in case of detection



CMB, PBHs, LIGO,  
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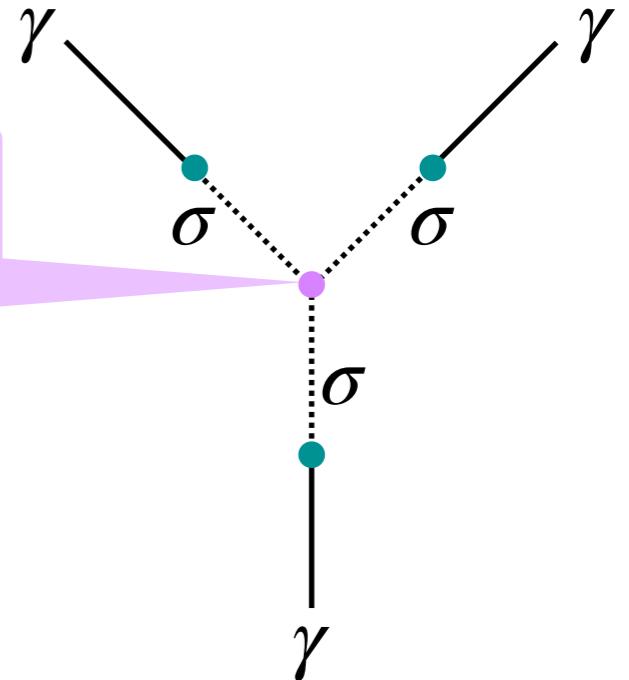
how does the signal  
look like?

LISA can be an efficient  
probe in constraining  
the inflationary field content



Cubic interaction  
& Tensor non-Gaussianities:

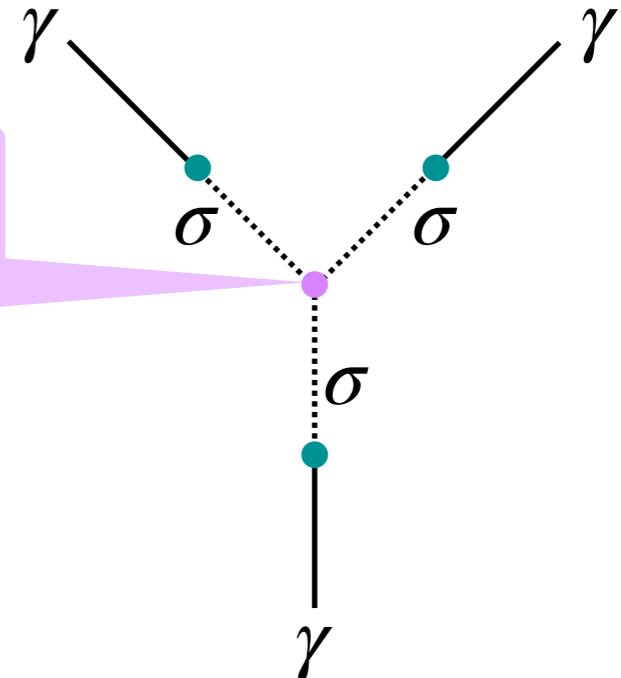
$$\mathcal{L}_{int}^{(3)} \sim \mu(\sigma_{ij})^3$$



$$\langle \gamma_{\mathbf{k}_1}^{\lambda_1} \gamma_{\mathbf{k}_2}^{\lambda_2} \gamma_{\mathbf{k}_3}^{\lambda_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\gamma^{\lambda_1 \lambda_2 \lambda_3}(k_1, k_2, k_3)$$

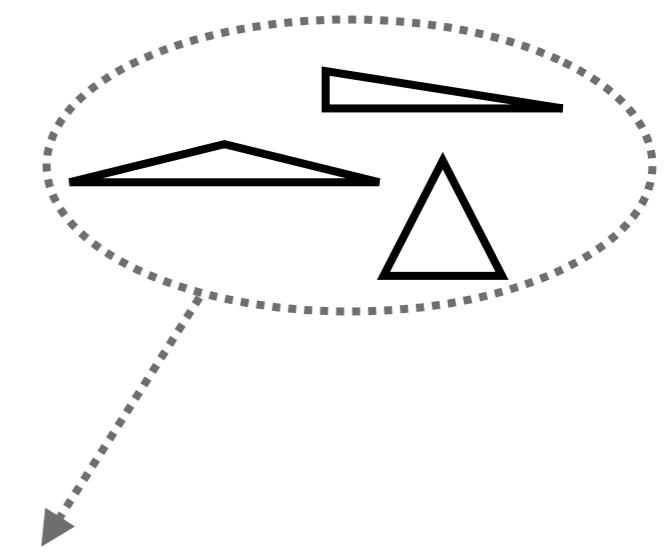
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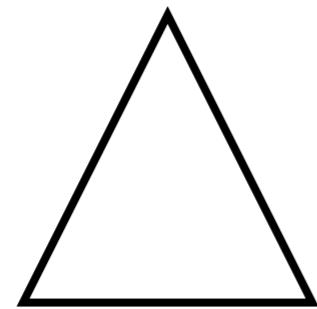
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$$f_{NL} = \frac{B}{P_\gamma^2}$$



- characterised in terms of amplitude and shape
- source of a wealth of information about the Lagrangian

Equilateral configuration



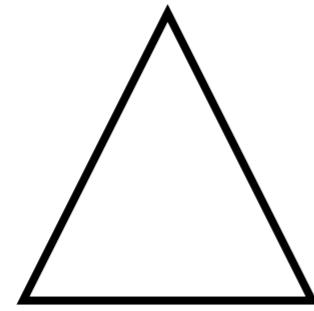
$$k_1 \sim k_2 \sim k_3 \sim k$$

Squeezed configuration



$$k_3 \sim k_L \ll k_1 \sim k_2 \sim k_S$$

## Equilateral configuration



$$k_1 \sim k_2 \sim k_3 \sim k$$

$$B_{eq}(k) \propto \frac{1}{k^6} \frac{\mu}{H} \left( \frac{\rho}{M_p} \right)^3 s_{eq}(c_2(k), \nu)$$

## Squeezed configuration

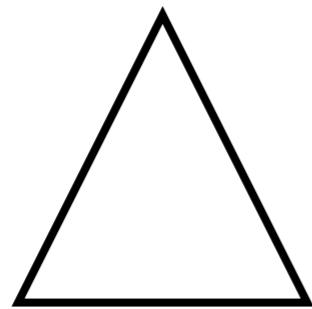


$$k_3 \sim k_L \ll k_1 \sim k_2 \sim k_S$$

$$\begin{aligned} B_{sq}(k_L, k_S) &\propto \frac{1}{k_S^{9/2-\nu} k_L^{3/2+\nu}} \frac{\mu}{H} \left( \frac{\rho}{M_p} \right)^3 \\ &\times s_{eq}(c_2(k_L), c_2(k_S), \nu) \end{aligned}$$

$$\nu = \sqrt{9/4 - (m/H)^2}$$

## Equilateral configuration



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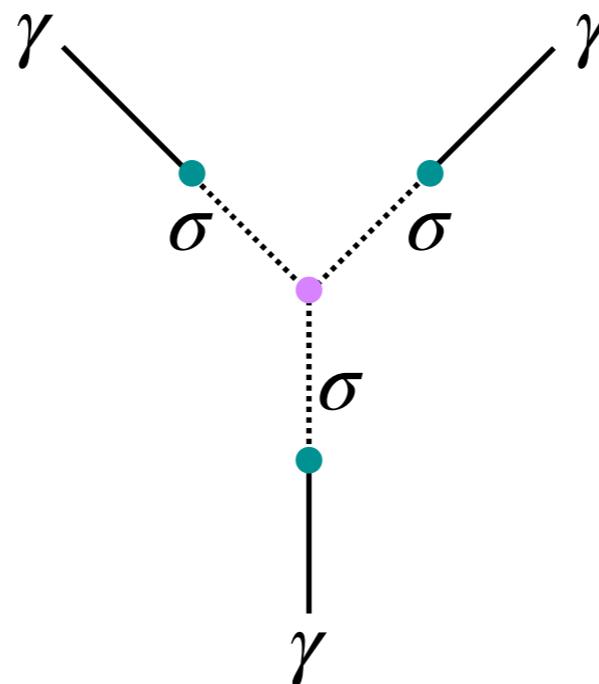
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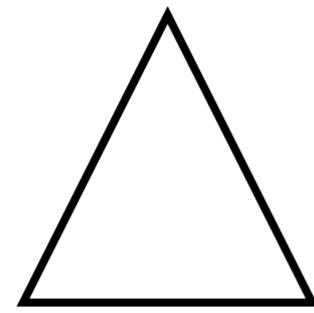


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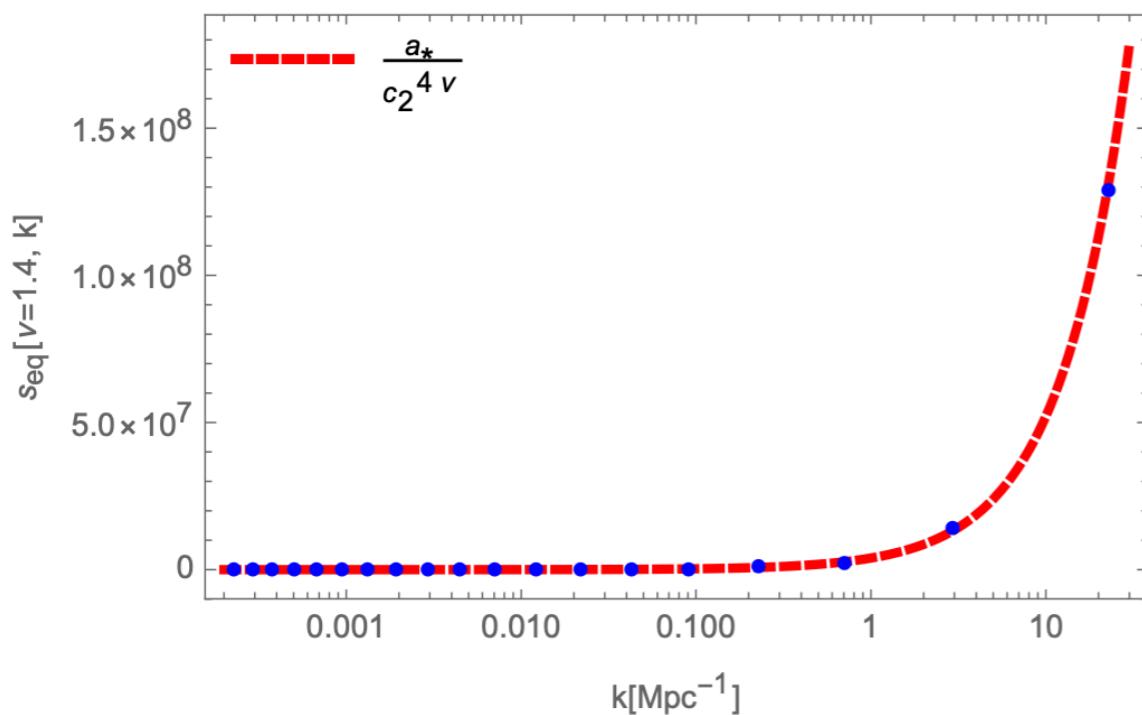


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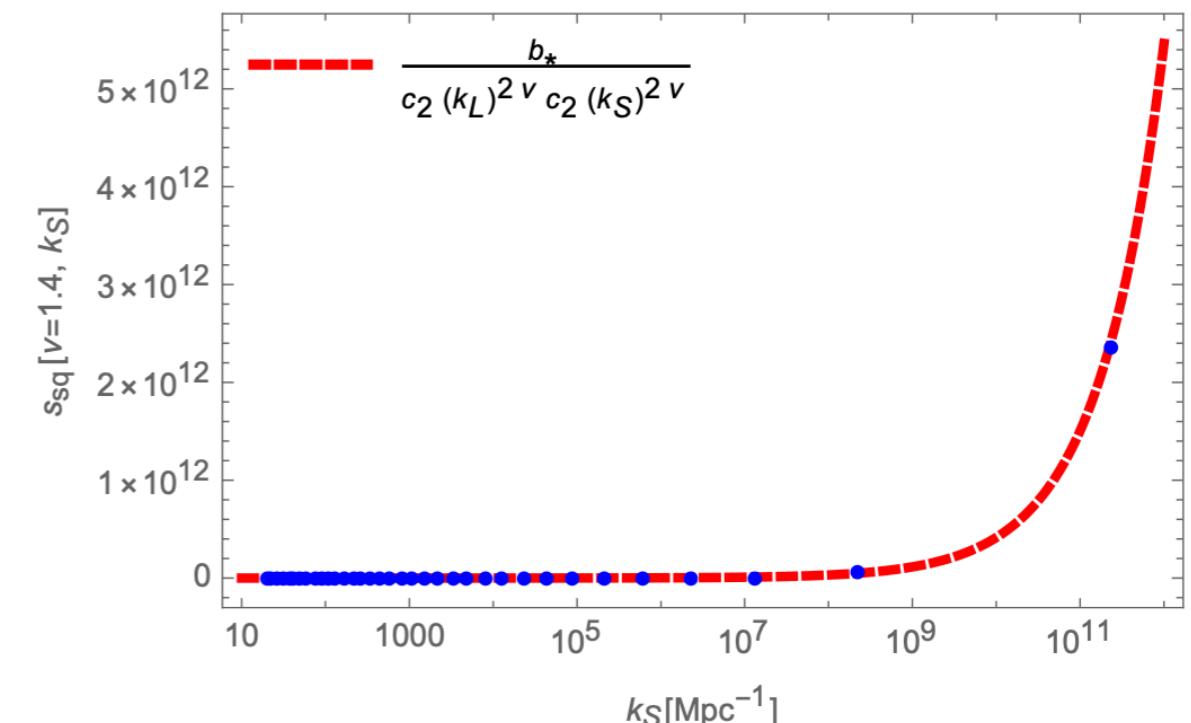


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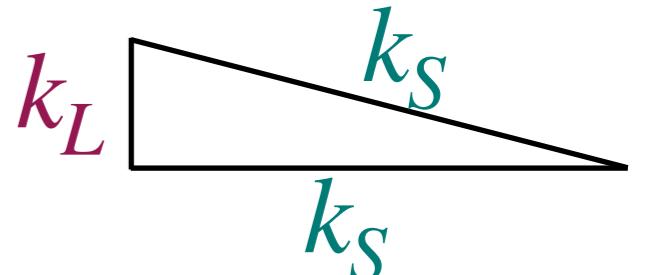
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## Detectability of the squeezed bispectrum

If there is a non-trivial squeezed bispectrum,  
the long wavelength mode  $k_L$  induces a  
quadrupolar anisotropy in the short modes  $P_\gamma(k_S)$



$$P_\gamma(\mathbf{k}_S, \mathbf{x}_c) = P_\gamma(k_S) \left( 1 + \mathcal{Q}_{lm}(\mathbf{k}_S, \mathbf{x}_c) \hat{k}_{Sl} \hat{k}_{Sm} \right)$$

$$\mathcal{Q}_{lm}(\mathbf{k}_S, \mathbf{x}_c) = \int \frac{d^3 k_L}{(2\pi)^3} e^{i \mathbf{k}_L \cdot \mathbf{x}_c} f_{NL}(k_L, k_S) \sum_{\lambda_3} \epsilon_{lm}^{\lambda_3}(-\hat{k}_L) \gamma_{-\mathbf{k}_L}^{*\lambda_3}$$

amplitude of  $B_{sq}$

## Recipe to indirectly test the squeezed bispectrum on small scales:

1. Detect the tensor power spectrum
2. Detect a certain level of quadrupolar modulation of the tensor power spectrum
3. Connect it to the amplitude of the squeezed bispectrum which is induced by

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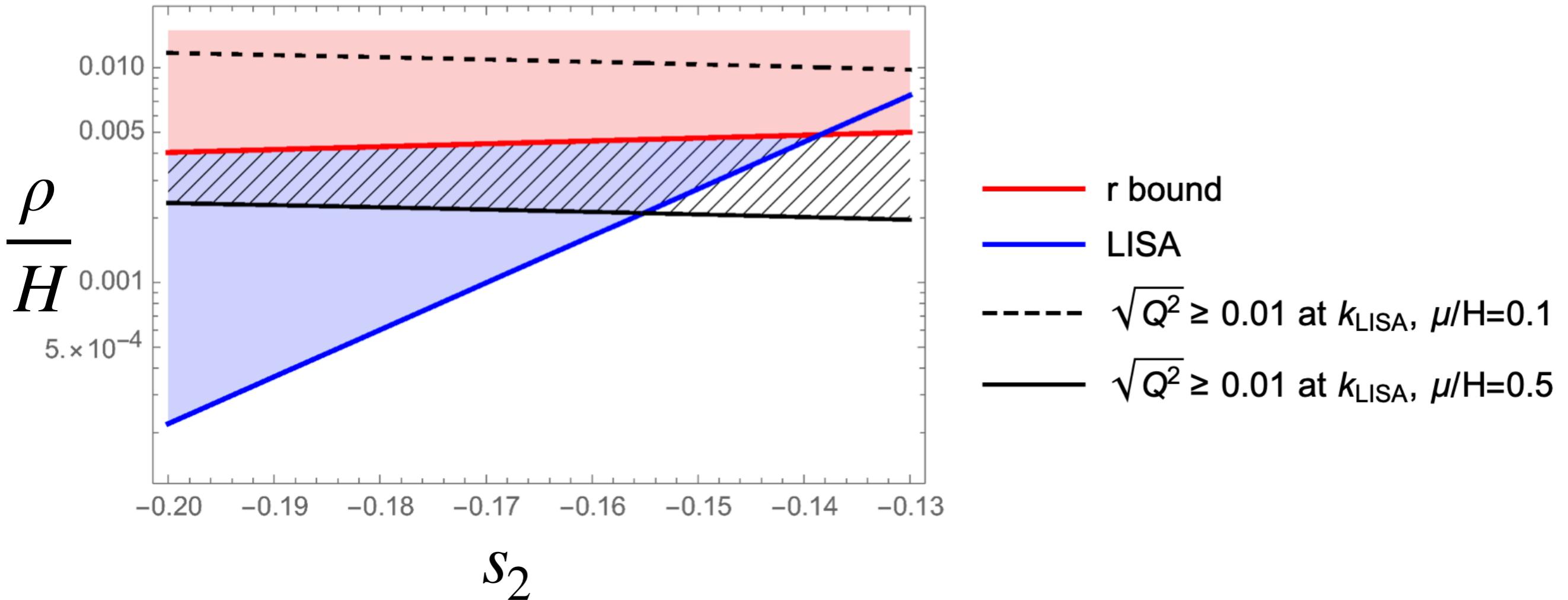
ongoing analysis for LISA  
→ we use  $\sqrt{Q^2} \geq 0.01$



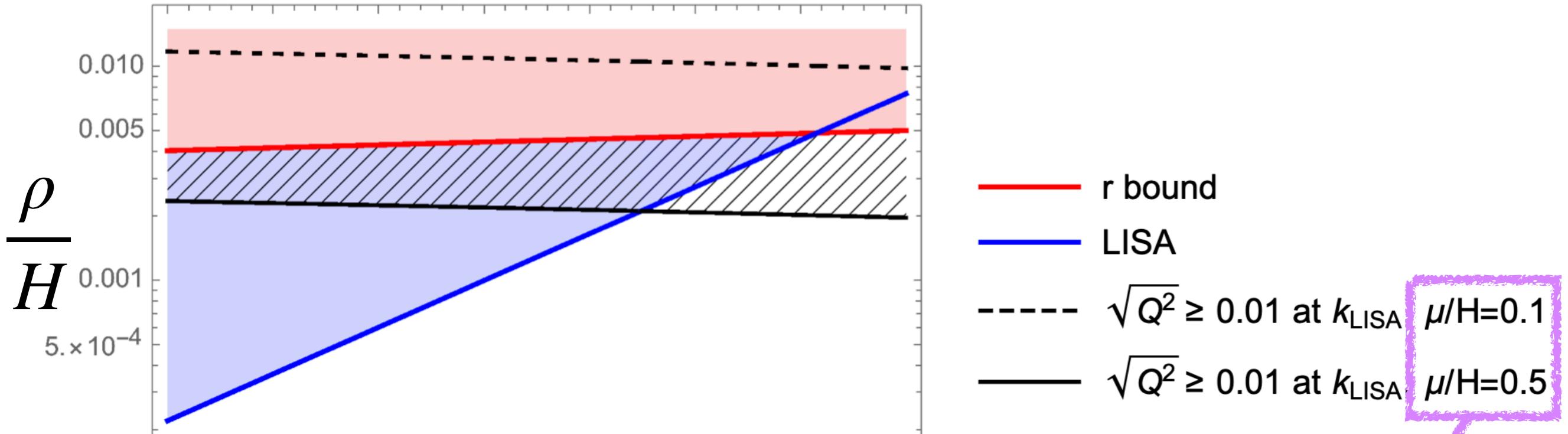
## How to analyse the parameter space:

Find area of parameter space which delivers a detectable tensor power spectrum & quadrupolar modulation

$$\mathcal{Q}_{lm} \text{ standard deviation: } \sqrt{\bar{Q}^2} = 16 \int_0^{k_{Lmax}} \frac{dk_L}{k_L} f_{NL}^2(k_L, k_S) \mathcal{P}_\gamma(k_L)$$

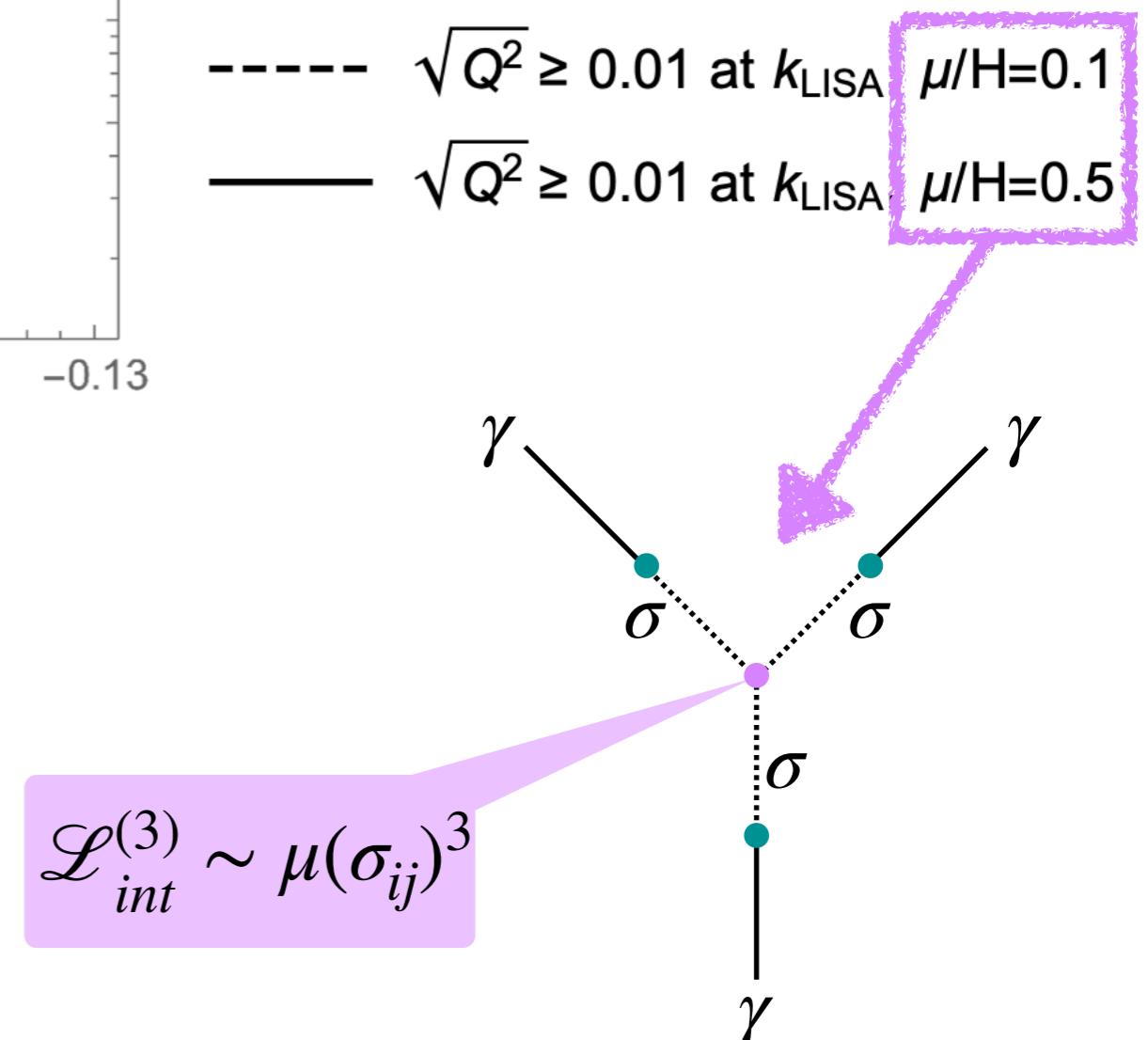


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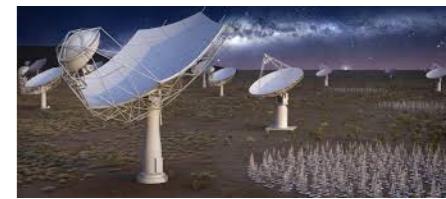
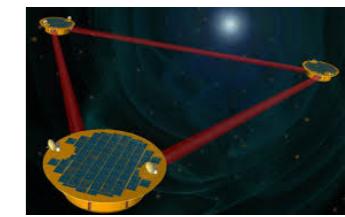


$s_2$

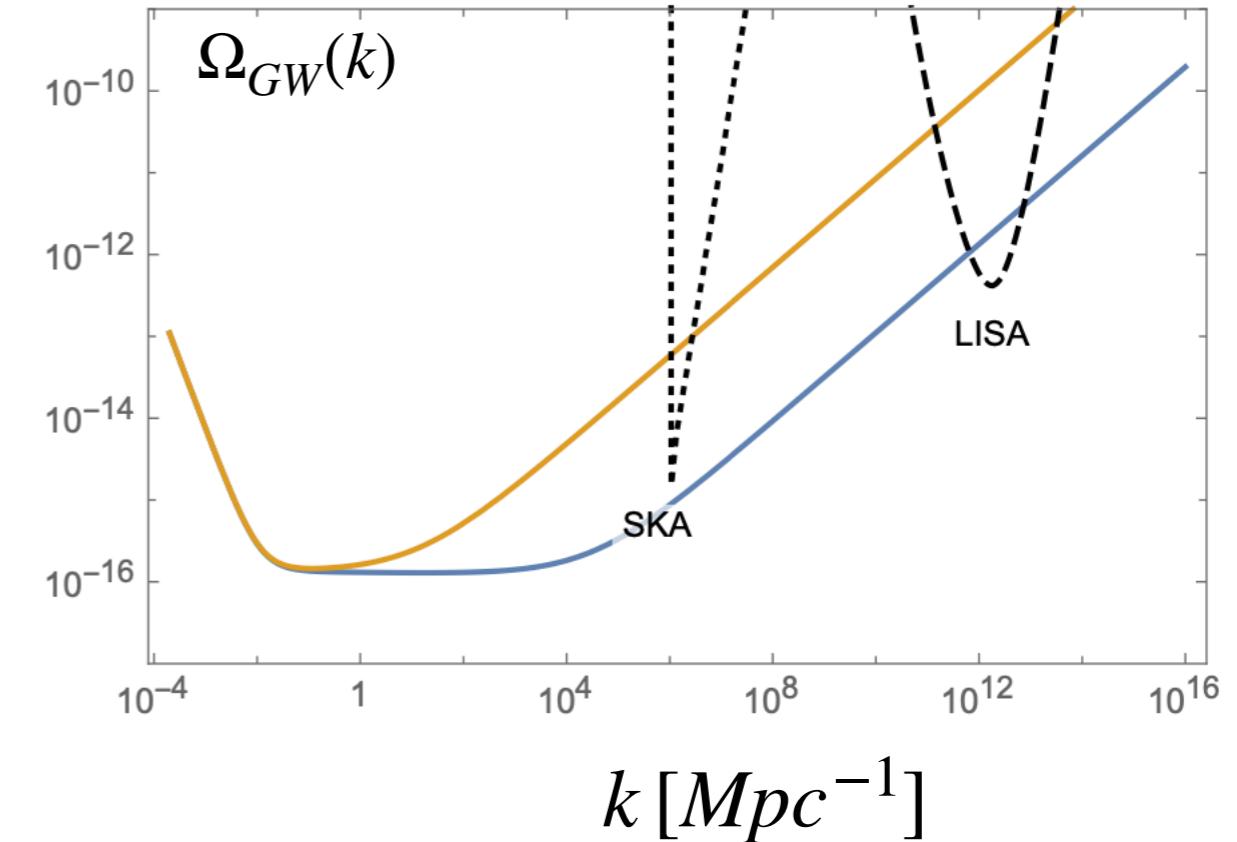
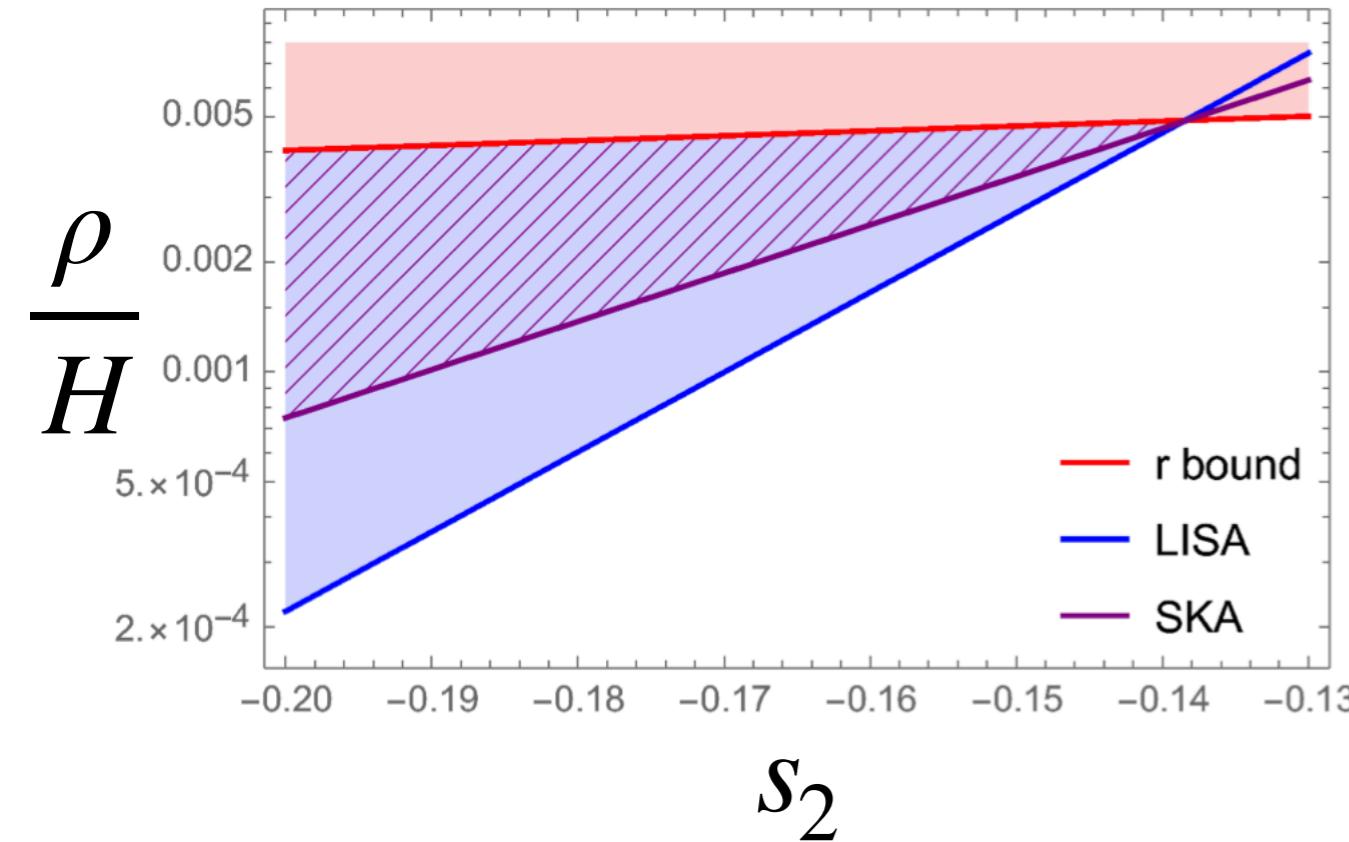
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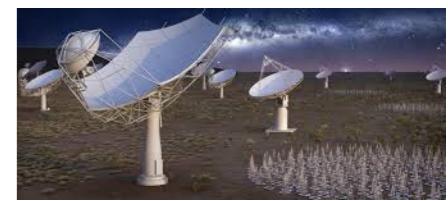
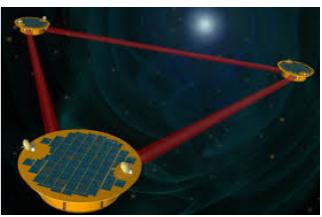
## Multi-probe analysis with LISA and SKA:



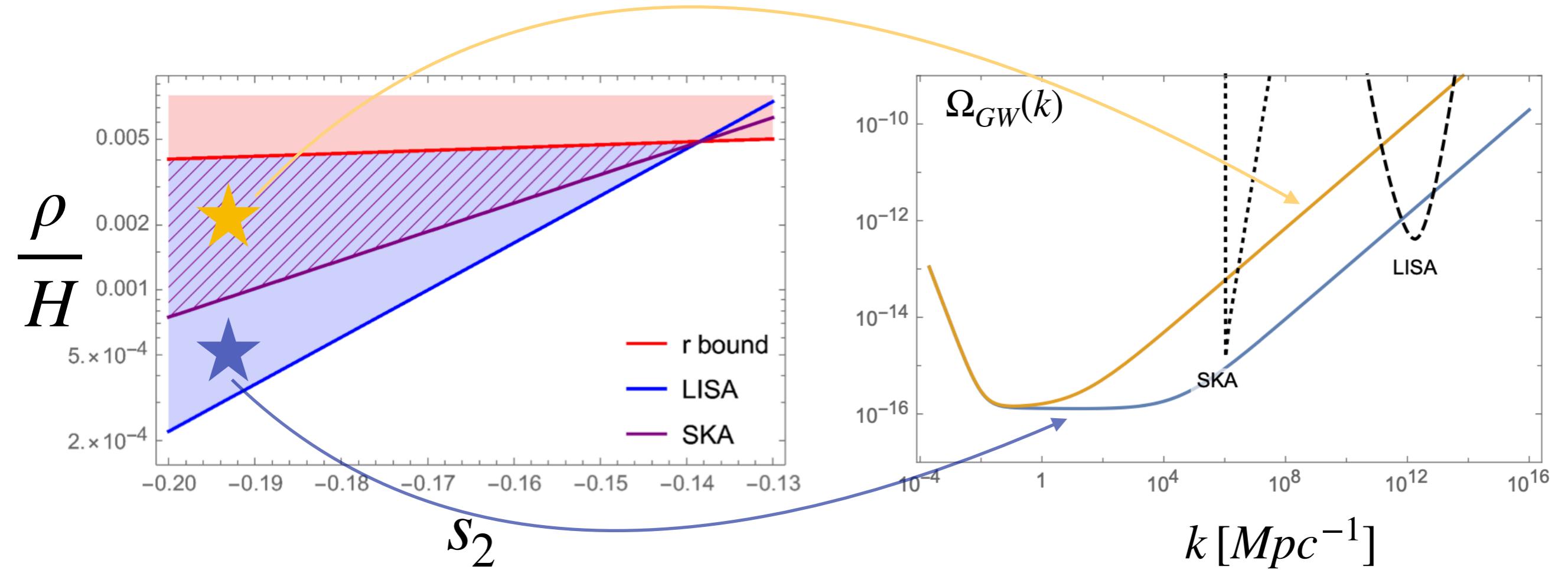
- Area of parameter space delivers a primordial GWs signal which can be probed with LISA & SKA



# Multi-probe analysis with LISA and SKA:

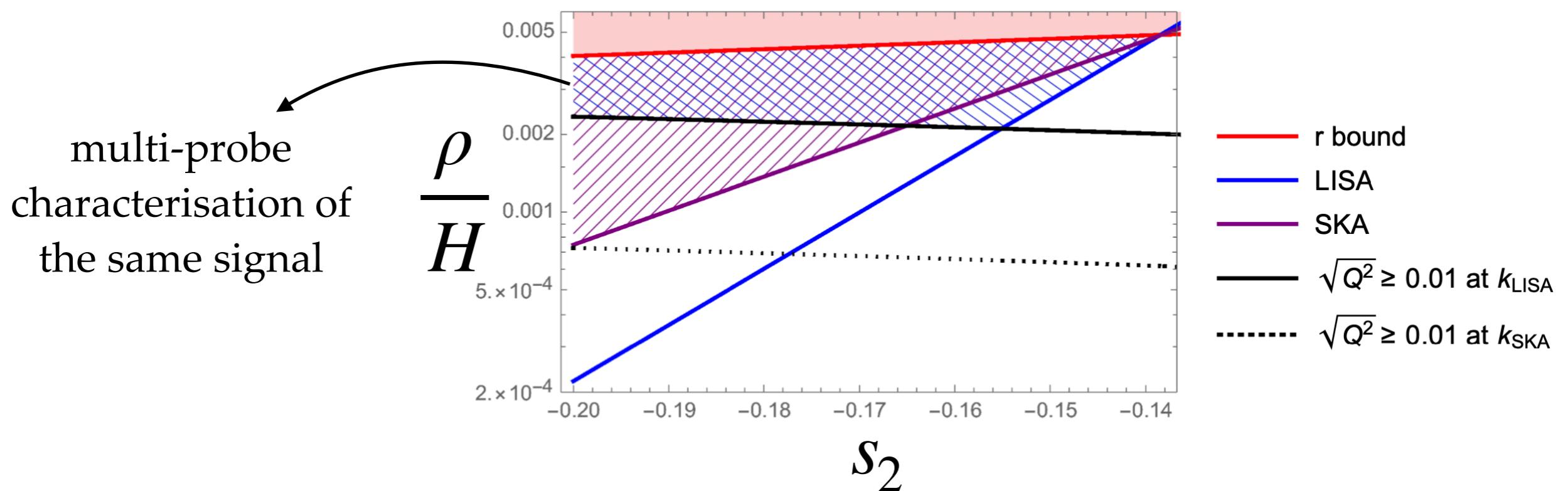


- Area of parameter space delivers a primordial GWs signal which can be probed with LISA & SKA



## Multi-probe analysis with LISA and SKA:

- Multi-probe analysis requires that there is an area of parameter space which delivers in BOTH probes a detectable tensor power spectrum & quadrupolar modulation on top of it
- Perform the same analysis as before for the single probe and then intersect the resulting areas



## Conclusions

- Primordial GWs can be an excellent probe of primordial physics + carry info on extra particle content during inflation
- Non-minimal couplings enhance the effects of massive spinning fields at small scales
- Time-dependent  $c_2$  can deliver detectable signal at small scales
- Squeezed Bispectrum mediated by spin-2 fields testable at small scales

1910.12921  
& 2008.00452

Thank you!



## Sound speed time evolution:

- $c_1^2 = \frac{1}{4}c_2^2 + \frac{3}{4}c_0^2$
- $s_i = \frac{\dot{c}_i}{Hc_i}$
- $s_0 = \frac{4}{3} \frac{c_1^2}{c_0^2} s_1 - \frac{1}{3} \frac{c_2^2}{c_0^2} s_2$
- perturbativity bound:  $c_2 > 10^{-4}$ , sets a bound on  $s_2 = \frac{\dot{c}_2}{Hc_2}$

Non-perturbative treatment of the  $(\gamma - \sigma)$  mixing:

$$P_\gamma = \frac{2H^2}{\pi^2 M_p^2} \frac{1}{c_2^{2\nu}} \left( \frac{6\rho H}{m^2 + 2\rho^2} \right)^2 \left( 1 - e^{-\frac{m^2 + 2\rho^2}{3H^2} N_k} \right)^2$$

- $m^2 \gg \rho^2$
- $N_k \gg \frac{H^2}{m^2}$
- $c_2 \ll 1$

## EFTI Philosophy:

- Inflation has to come to an end:  $\phi(t)$  breaks de Sitter isometries  
→ there is a Goldstone boson  $\pi(\bar{x}, t)$
- On scales smaller than  $H$ : Lorentz group → rotations
- Fields are given as rep. of unbroken symmetries:  
spinning field given as rep. of rotations

*spin-2 case:*

- traceless rank-2 symmetric tensor  $\Sigma^{ij}$
  - “push forward” to 4D:  $\Sigma^{\mu\nu}$
- $$\left\{ \begin{array}{l} \Sigma^{00} = \frac{\partial_i \pi \partial_j \pi}{(1 + \dot{\pi})^2} \Sigma^{ij} \\ \Sigma^{0j} = -\frac{\partial_i \pi}{1 + \dot{\pi}} \Sigma^{ij} \end{array} \right.$$

## The interaction sector

- at leading order
- dictated by the symmetries

$$\mathcal{L}_{int} = M_p \rho \delta K_{\alpha\beta} \Sigma^{\alpha\beta} + M_p \tilde{\rho} \delta g^{00} \delta K_{\alpha\beta} \Sigma^{\alpha\beta} - \mu \Sigma^{\alpha\beta} \Sigma_\alpha^\gamma \Sigma_{\gamma\beta}$$



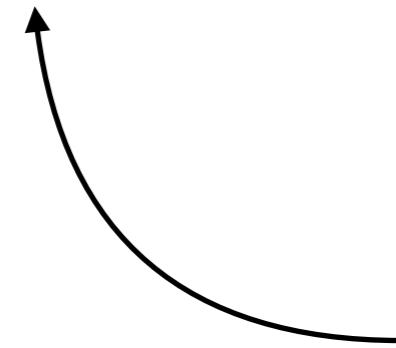
fluctuation of the extrinsic curvature  
of constant inflaton surfaces

$$\delta K_{\alpha\beta} = K_{\alpha\beta} - a^2 H h_{\alpha\beta}$$

**Cut-off** of the EFTI with spin-2 field non-minimally coupled with  $\phi$ :

radiative corrections  
to the mass of the spin-2  
due to coupling with  $\pi$   
must be small

$$\frac{\Lambda^4}{\epsilon H^2 M_{Pl}^2} < 1$$



Explore the area of parameter space which can be surveyed by LISA

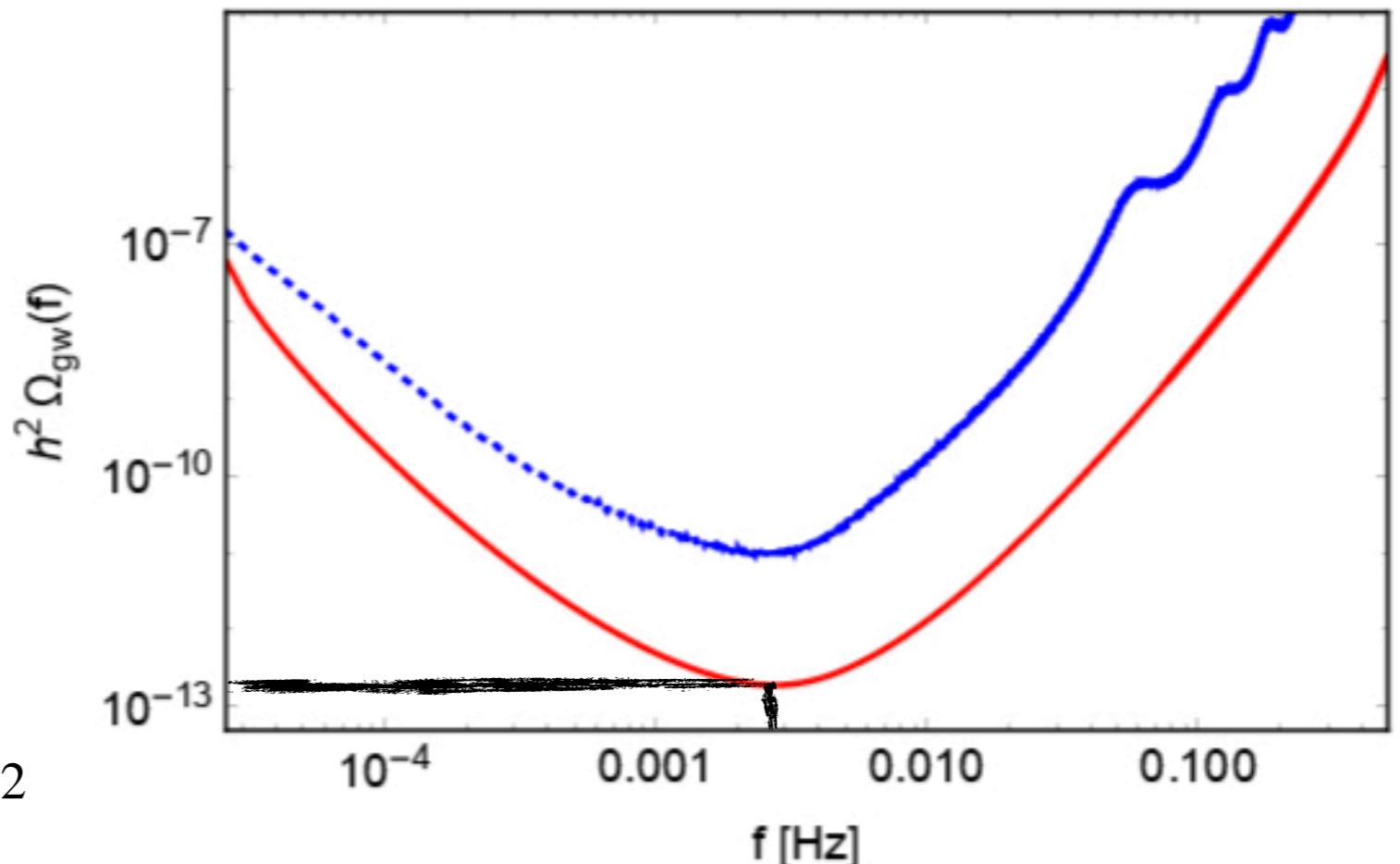
- 4 yrs of observational time
- arms  $2.5 \times 10^6$  kms long
- SNR>10

$$\Omega_{GW}(k) = \frac{1}{12} \left( \frac{k}{a_0 H_0} \right)^2 P_\gamma(k) T(k)^2$$

signal detectable:

$$\boxed{\Omega_{GW}(k) > \Omega_{LISA}(k)}$$

LISA sensitivity curve



{1906.09244 - Caprini et al.}