Multifield stochastic inflation in phase space: a manifestly covariant theory and its first principle derivation

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1806.10126 and 2008.07497
Pinol, Renaux-Petel, Tada

Meeting IEA PBH#2

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Why stochastic inflation?

- Classical background + quantum fluctuations:

  - conceptually not satisfactory

  - breaks down for very light scalar fields

- Late time IR structure of correlators in (near) de Sitter, eternal inflation

- Can be used to compute full pdf of curvature fluctuation (e.g. for some PBHs generation mechanism)

- Stability of Higgs during inflation?

- …

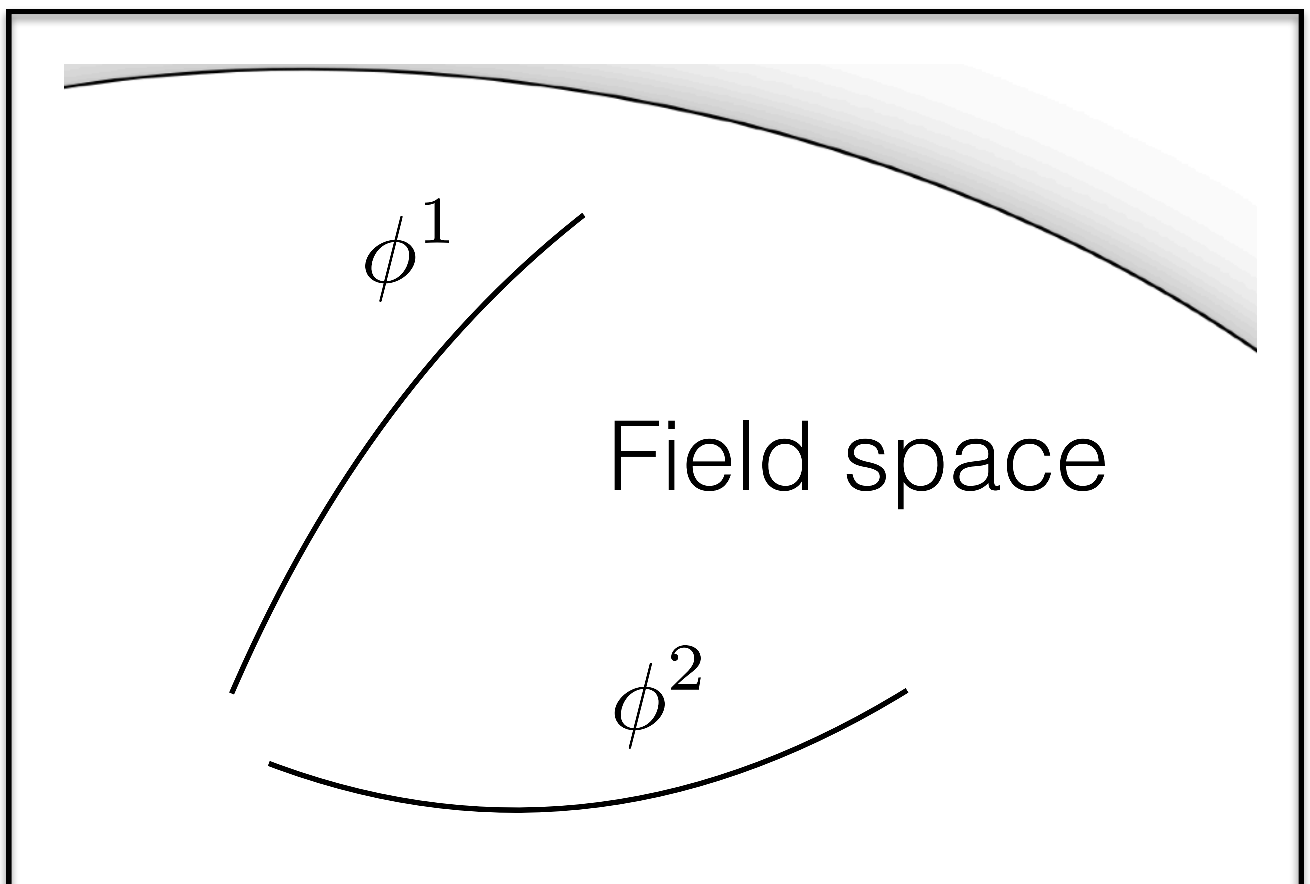
Our work

- Formulate stochastic inflation in a manifestly **covariant** manner under field redefinitions

Our work

- Formulate stochastic inflation in a manifestly **covariant** manner under field redefinitions

$$\mathcal{L} = -\frac{1}{2} G_{IJ}(\phi^K) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi^K)$$



Fields are simply coordinates

Not obvious in general that classical symmetries are maintained at the quantum level (anomalies)

e.g. Vilkovisky-DeWitt variables for quantum effective action

Our work

- Formulate stochastic inflation in a manifestly **covariant** manner under field redefinitions
- Beyond heuristic approach: **derivation** using tools of nonequilibrium quantum field theory
- Markovian approximation: covariant Fokker-Planck eq in phase space, and analytical formulae for noises

I Introduction to stochastic inflation

II Stochastic anomalies and solution

III Path-integral derivation

disclaimer: many references in the paper, not here

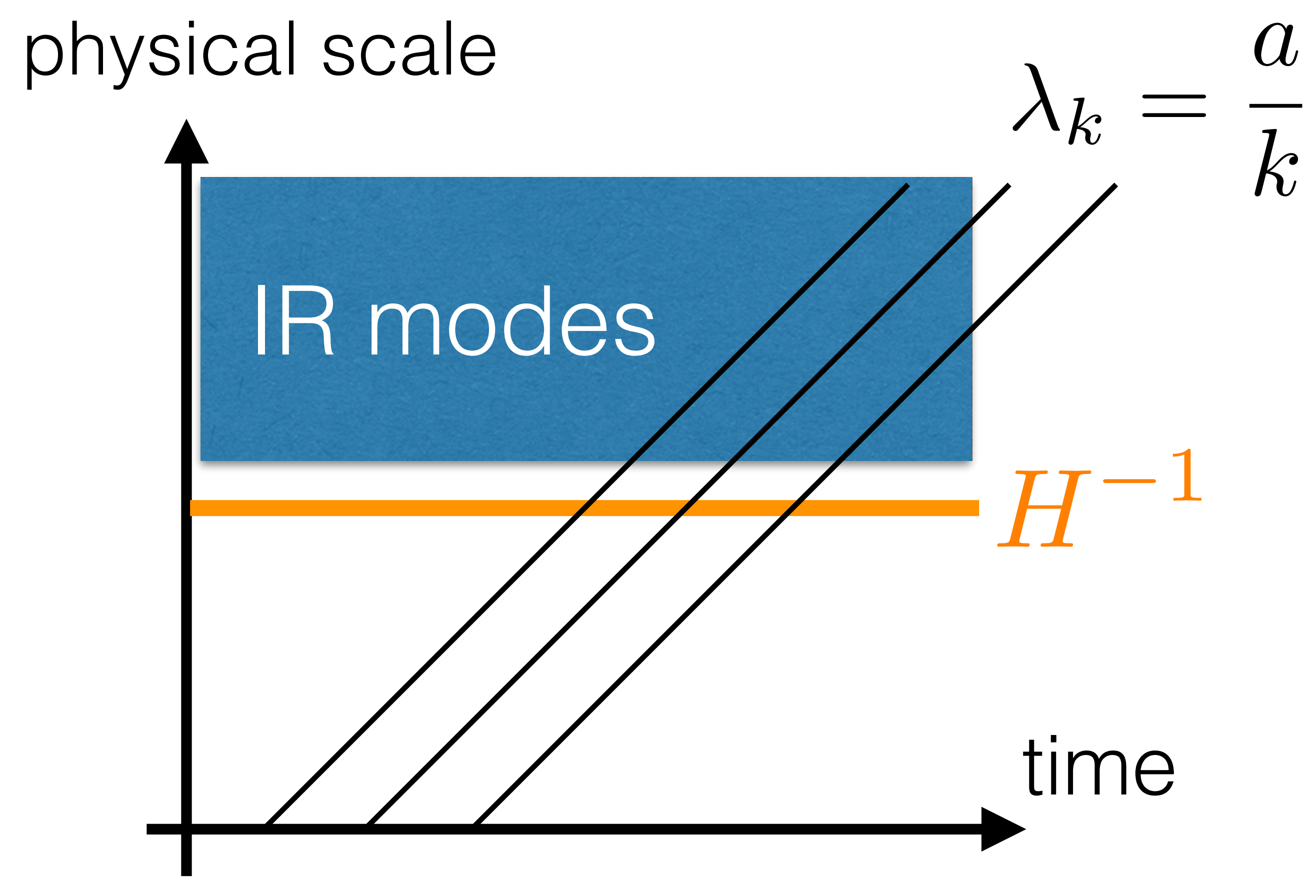
I Introduction to stochastic inflation

Stochastic formalism

Classical stochastic effective theory for coarse-grained fields

Background

super-Hubble modes



Continuous flow of initially sub-Hubble (UV) modes joining the super-Hubble (IR) sector

$$\frac{\mathrm{d}\varphi}{\mathrm{d}N} = -\frac{V'(\varphi)}{3H^2} + \frac{H}{2\pi}\xi$$

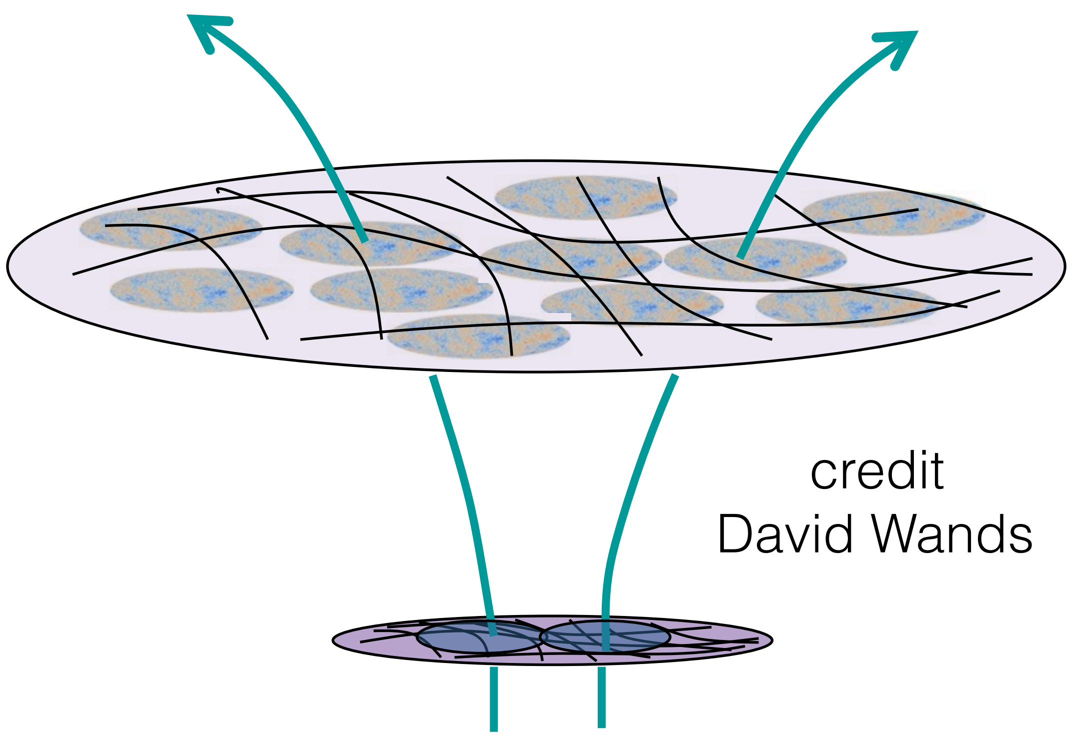
Gaussian white noise

Starobinsky, 86

Open quantum system

Stochastic formalism

Many (super)-Hubble regions evolving like locally separate universes, emerging from same initial conditions



credit
David Wands

classical drift
(slow-roll)

Langevin equation

$$\boxed{\frac{\mathrm{d}\varphi}{\mathrm{d}N} = -\frac{V'(\varphi)}{3H^2} + \frac{H}{2\pi}\xi}$$

quantum diffusion

$$\langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

stochastic dynamics of a representative Hubble region

φ coarse-grained long-wavelength scalar field

Fokker-Planck equation

$$\boxed{\frac{\partial P(\varphi, N)}{\partial N} = \mathcal{L}_{\mathrm{F}P} \cdot P(\varphi, N)}$$

probability density function of field's values at time N

Renaux-Petel, IAP

IR resummation

- Agreement with QFT computations (but much simpler)

Woodard, Starobinsky, Rigopoulos …

- Enables one to resum late time divergences of perturbative QFT

e.g. in $\lambda\varphi^4$ theory in de Sitter, secular effects for $\lambda N^2 > 1$

and derive non-perturbative results, e.g. $P_{\rm eq}(\varphi) \propto e^{-8\pi^2 V(\varphi)/(3H_0^4)}$

- Outstanding questions: limitations, rigorous derivation, corrections

many recent works

Cosmological correlators

Stochastic $\delta\mathcal{N}$ formalism: Fujita, Kawasaki, Tada, Takesako 13
Vennin, Starobinsky 15

$$P(\varphi_{\rm stoc}, N)$$

pact-dependent fields
with deterministic clock

invert
FP equation
$\mathcal{L}^\dagger_{\mathrm{F}P}$

$$P(\mathcal{N}_{\rm stoc}(\varphi))$$

patch-dependent
durations of inflation,
starting from progenitor patch

$\mathcal{N}(\varphi)$ number of e-folds of inflation realized starting from field value φ

stochastic quantity, directly related to observable curvature perturbation

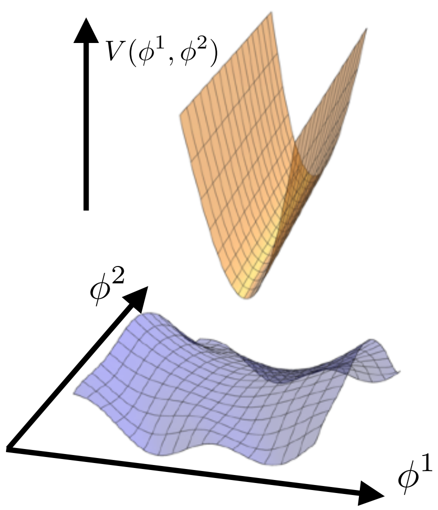
$$\boxed{\zeta = \delta\mathcal{N} = \mathcal{N} - \langle \mathcal{N} \rangle}$$

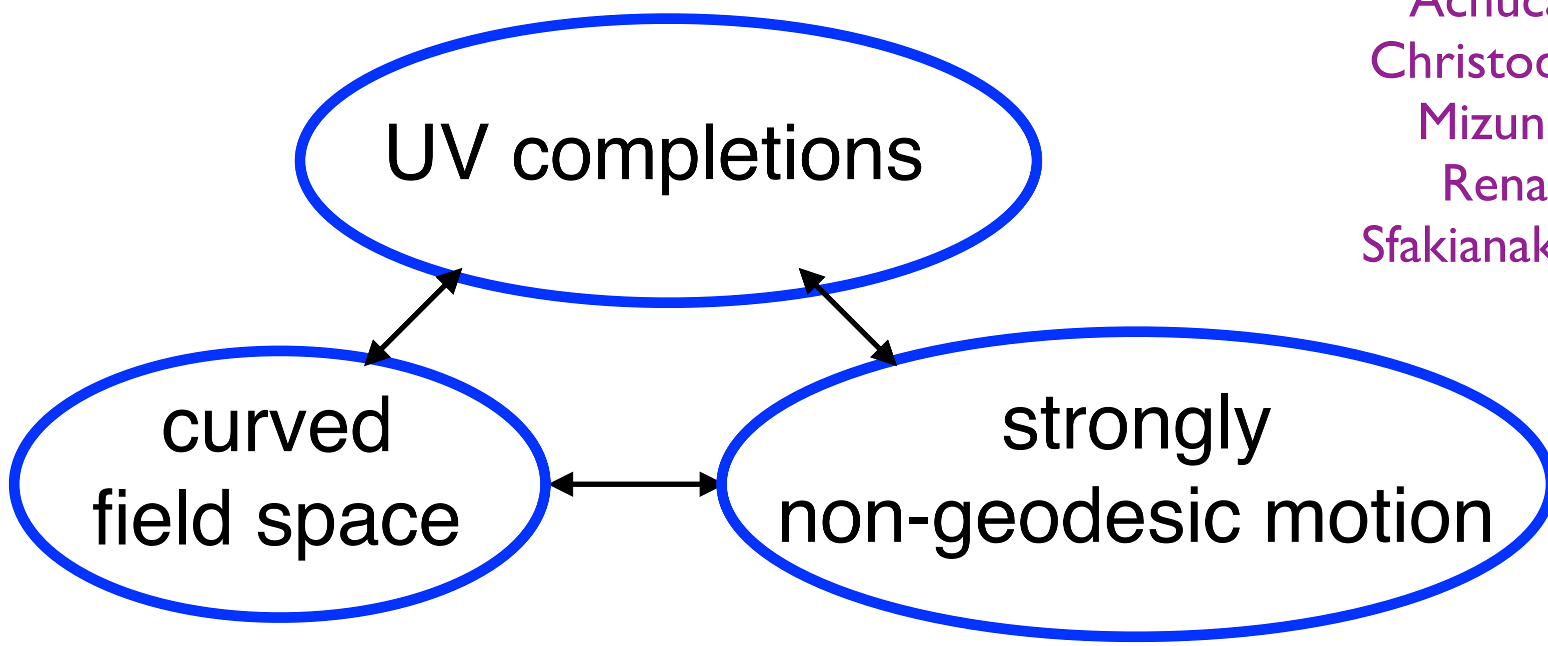
$\longrightarrow$ Full pdf of curvature perturbation

Fujita, Kawasaki, Tada,
Vennin, Starobinsky, Pattison,
Assadullahi, Firouzjahi, Noorbala,
Wands, Pinol, Renaux-Petel ...

Why generic multifield models?

Many recent developments

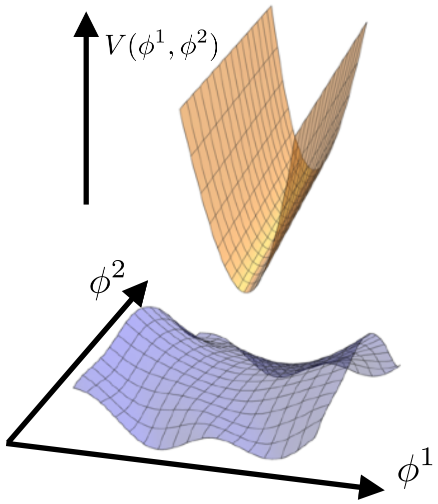


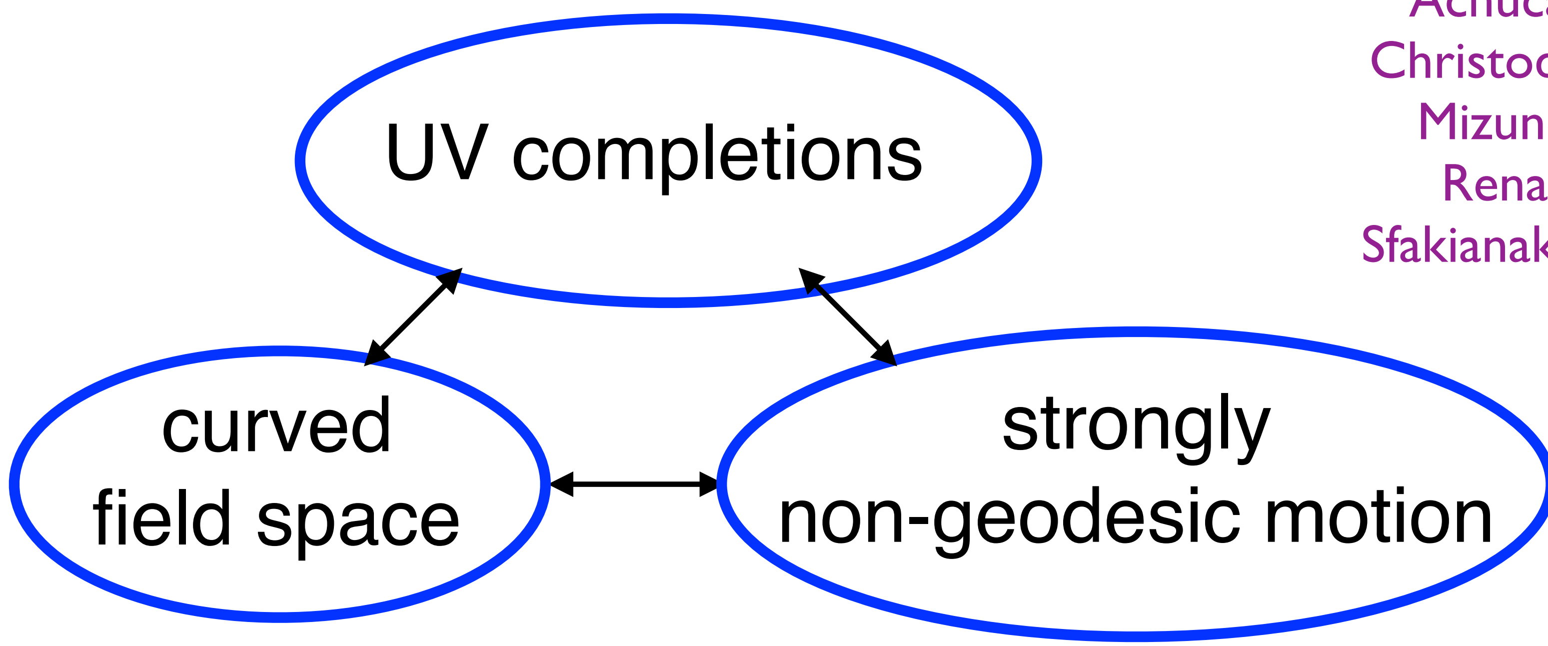


Achucarro, Brown, Bjorkmo,
Christodoulidis, Ferreira, Marsh,
Mizuno, Mukohyama, Palma,
Renaux-Petel et al, Roest,
Sfakianakis, Sypsas, Wang, Welling
…

Why generic multifield models?

Many recent developments





Achucarro, Brown, Bjorkmo, Christodoulidis, Ferreira, Marsh, Mizuno, Mukohyama, Palma, Renaux-Petel et al, Roest, Sfakianakis, Sypsas, Wang, Welling
...

Propaganda for works in my group:

with Fumagalli, Garcia-Saenz, Pinol, Ronayne, Witkowski

- PBHs from strong turns in landscape
- Higgs stability during inflation
- Flattened NGs from strongly non geodesic motion
- Generalization of Maldacena's computation to multifield inflation with curved field space

Heuristic approach

$$\phi = \varphi_{\rm IR} + Q_{\rm UV}$$
$$\pi = \varpi_{\rm IR} + \tilde{P}_{\rm UV}$$

Split in Fourier-space,
with IR fields only containing

$$k < k_\sigma(N) = \sigma a H$$
$$\sigma \ll 1$$

Plug in classical equations of motion

+ linearize in UV fields, work at leading order in gradients for IR fields

+ assume usual quantization of UV fields

+ time-dependence of coarse graining scale

+ super-Hubble squeezed state, effectively classical

see Grain and Vennin 2017
in single-field

(gauge and mixing with gravity also taken into account)

Heuristic approach

$$\phi = \varphi_{\rm IR} + Q_{\rm UV}$$
$$\pi = \varpi_{\rm IR} + \tilde{P}_{\rm UV}$$

Split in Fourier-space,
with IR fields only containing

$$k < k_\sigma(N) = \sigma a H$$
$$\sigma \ll 1$$

$$\boxed{\begin{aligned} \varphi^{I\prime} &= \frac{1}{H} G^{IJ} \varpi_J + \xi^{QI} \\ \mathcal{D}_N \varpi_I &= -3\varpi_I - \frac{V_I}{H} + \xi_I^{\tilde{P}} \end{aligned}}$$

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2} G^{IJ} \varpi_I \varpi_J + V$$

local Friedmann constraint

$$\mathcal{D}_N \varpi_I = \partial_N \varpi_I - \Gamma^K_{IJ} \varphi^{J\prime} \varpi_K$$

covariant derivative

Heuristic approach

$$\phi = \varphi_{\rm IR} + Q_{\rm UV}$$
$$\pi = \varpi_{\rm IR} + \tilde{P}_{\rm UV}$$

Split in Fourier-space,
with IR fields only containing

$$k < k_\sigma(N) = \sigma a H$$
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local Friedmann constraint

$$\mathcal{D}_N \varpi_I = \partial_N \varpi_I - \Gamma^K_{IJ} \varphi^{J\prime} \varpi_K$$

covariant derivative

Autocorrelation of noises:
power spectra of UV modes
in IR background, e.g.

$$\langle \xi^{QI}(N) \xi^{QJ}(N') \rangle \sim \langle Q^I(N, k_\sigma(N))\, Q^J(N, k_\sigma(N)) \rangle\, \delta(N - N')$$

Take real noises by hand,
this is proved in path-integral derivation

non-Markovian

only with Markovian
approximation

$\longrightarrow$ FP equation

II Stochastic anomalies and solution

Stochastic calculus

Langevin equation:

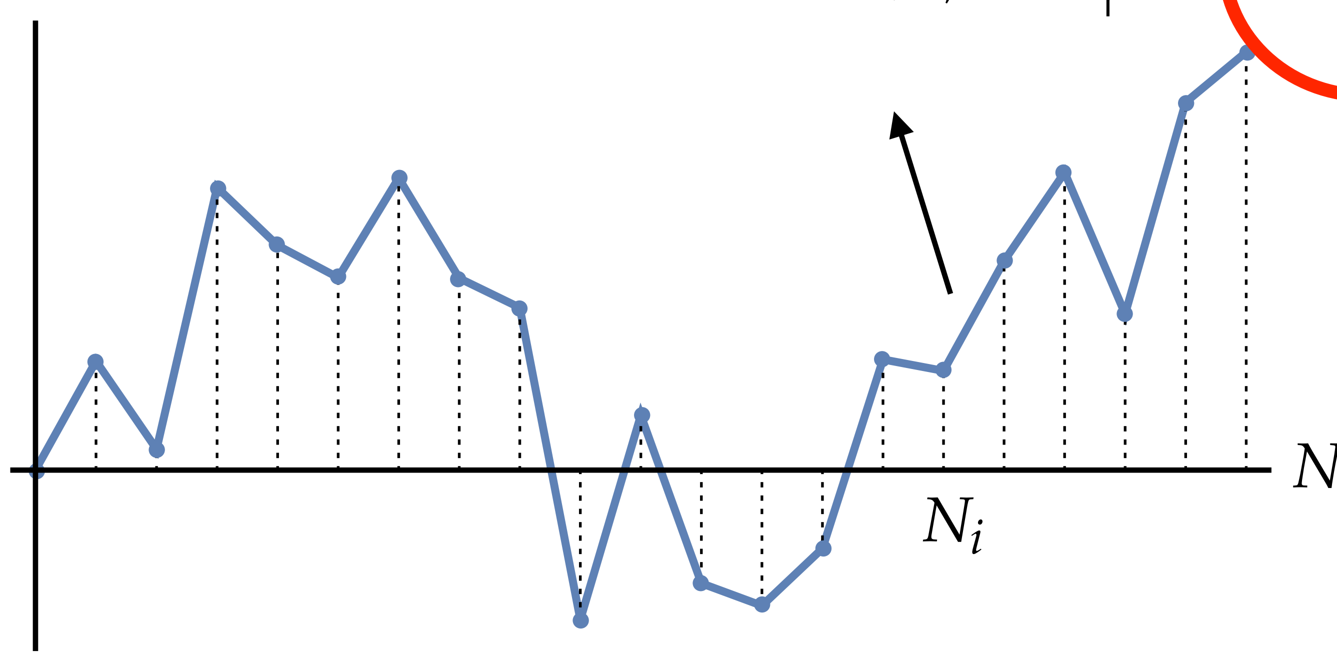
$$\frac{\mathrm{d}X}{\mathrm{d}N} = h(X) + g(X)\,\xi \qquad \langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

Continuous limit of discrete process:

$$\Delta X_i = h(X(N_i^*))\Delta N_i + g\left(X(N_i^*)\right)\Delta W_i$$

$$N_i^* \in [N_i, N_{i+1}]$$

Brownian $W(N)$ $\quad \Delta W_i$: Gaussian $(0, \Delta N_i)$ $\sim \Delta N_i^{1/2}$



X's properties depend on choice of N_i^*

Renaux-Petel, IAP

Stochastic calculus

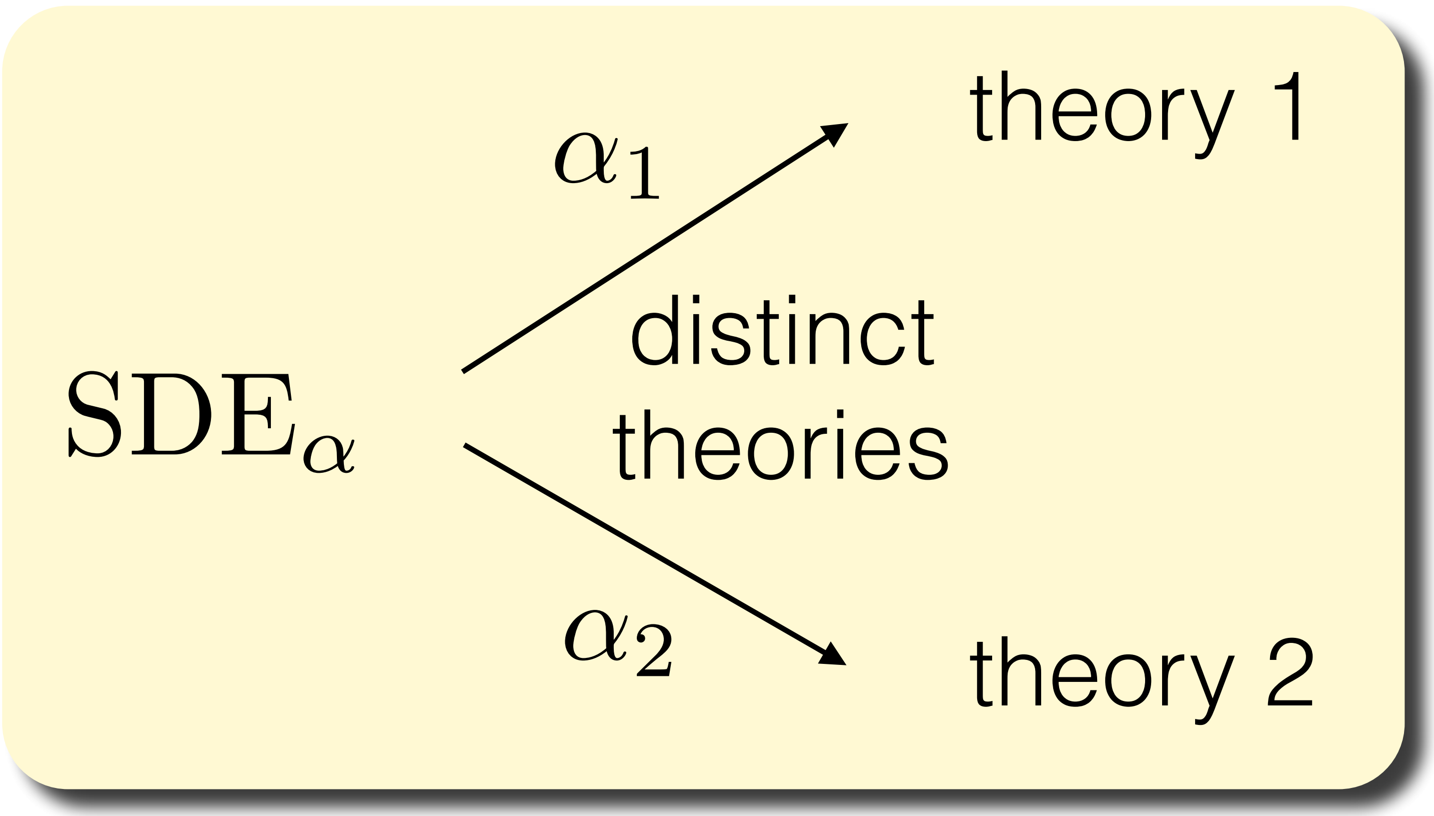
Ito	Stratonovich
$N_i^* = N_i$	$N_i^* = \frac{1}{2}(N_i + N_{i+1})$
$\mathrm{d}X = h\,\mathrm{d}N + g\,\mathrm{d}W$	$\mathrm{d}X = h\,\mathrm{d}N + g \circ \mathrm{d}W$
Ito's lemma:	standard chain rule
$\mathrm{d}f(X) = f_{,X}\mathrm{d}X + \frac{1}{2} f_{,XX}\, g^2 \mathrm{d}N$	$\mathrm{d}f(X) = f_{,X}\mathrm{d}X$

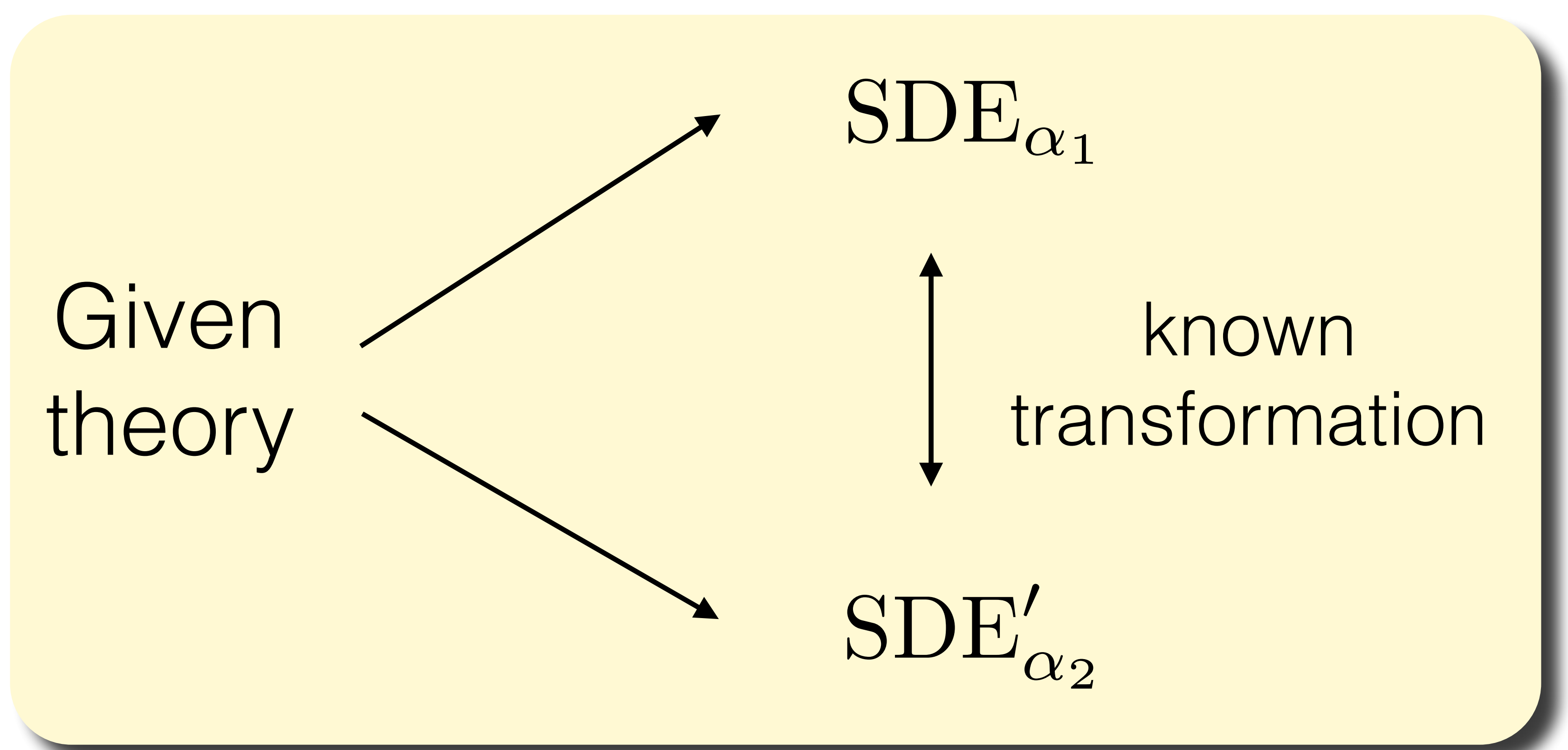
Fokker-Planck equation for $P(X, N)$: pdf of X at time N

$$\frac{\partial P}{\partial N} = \mathcal{L}_{\mathrm{FP}} \cdot P = -\frac{\partial}{\partial X}\left(D(X)P\right) + \frac{1}{2}\frac{\partial^2}{\partial X^2}\left(g^2(X)P\right)$$

$D_{\mathrm{I}} = h$	$D_{\mathrm{S}} = h + \frac{1}{2} g \frac{\partial g}{\partial X}$ noise-induced drift

Stochastic calculus



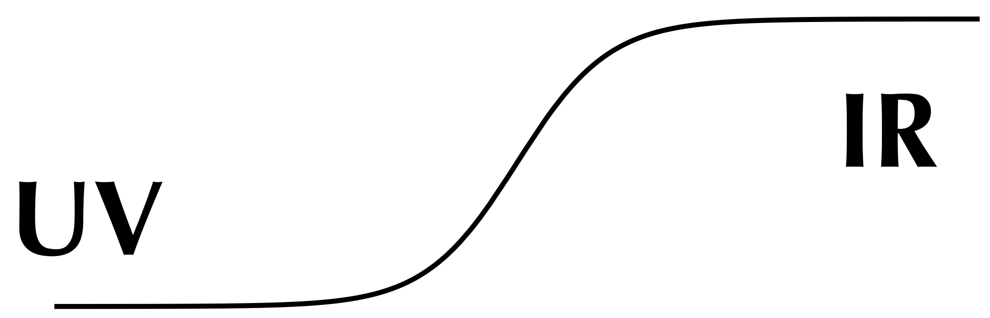


Ito versus Stratonovich for inflation

- Many papers: **Ito** ‘to respect causality’ (no good reason)

  Vilenkin 1999, Fujita, Kawasaki, Tada 2014, Tokuda & Tanaka 2017, …

- **Stratonovich**: white noises are idealizations of colored noises



smooth splitting
between UV and IR modes

Mezhlumian & Starobinsky 1994

- Discrepancy **exceeds the accuracy** of stochastic formalism

  Vennin & Starobinsky 2015

- Here: **new perpective with requirement of covariance**

Single-field slow-roll

$$\frac{\mathrm{d}\varphi}{\mathrm{d}N} = -\frac{V'(\varphi)}{3H^2} + \frac{H}{2\pi}\xi$$

Test scalar fields (e.g. in de Sitter)

$H(\cancel{\varphi})$ → unambiguous

Inflaton: $3H^2(\varphi)M_{\mathrm{Pl}}^2 = V(\varphi)$

→ Ito versus Stratonovich ambiguity of multiplicative noises

Difference between Ito and Stratonovich, in classical or stochastic regimes, suppressed by $V/M_{\mathrm{Pl}}^4 \ll 1$

Pinol, Renaux-Petel, Tada 18

In general, e.g. with multiple fields: real issue

Multivariate calculus

Langevin equations

$$\boxed{\frac{\mathrm{d}X^a}{\mathrm{d}N} = h^a(X) + \sum_A g^a_A(X)\,\xi^A}$$

$$\langle \xi^A(N)\xi^B(N')\rangle = \delta^{AB}\delta(N-N')$$

Multivariate calculus

Langevin equations

$$\boxed{\frac{\mathrm{d}X^a}{\mathrm{d}N} = h^a(X) + \sum_A g^a_A(X)\,\xi^A}$$

$$\langle \xi^A(N)\xi^B(N')\rangle = \delta^{AB}\delta(N-N')$$

Fokker-Planck equation

$$\frac{\partial P}{\partial N} \quad = \quad \mathcal{L}_{\mathrm{FP}}(X^a)\cdot P$$

with $\boxed{\mathcal{L}_{\mathrm{FP}}(X^a) \quad = \quad -\frac{\partial}{\partial X^a}D^a + \frac{1}{2}\frac{\partial^2}{\partial X^a \partial X^b}A^{ab}}$

drift vector

diffusion matrix

$$D^a_I = h^a$$

$$D^a_{\mathrm{S}} = h^a + \frac{1}{2}g^b_A\frac{\partial g^a_A}{\partial X^b}$$

$$A^{ab} = g^a_A g^b_A$$

Renaux-Petel, IAP

The problem in a simple setup

$$\frac{\mathrm{d}\varphi^I}{\mathrm{d}N} = -\frac{G^{IJ}V_{,J}}{3H^2} + \xi^I$$

Slow-roll, overdamped limit

with noise correlations

$$\langle \xi^I(N)\,\xi^J(N')\rangle = \left(\frac{H}{2\pi}\right)^2 G^{IJ}\,\delta(N-N')$$

Not yet well defined stochastic differential equations

$\rightarrow$ $\xi^I = g^I_A\,\xi^A$ with $\boxed{g^I_A = \frac{H}{2\pi} e^I_A}$

and set of vielbeins $e^I_A e^J_A = G^{IJ}$

The problem in a simple setup

- Vielbeins can a priori differ by arbitrary field-dependent rotations $e^I_A \to \Omega^B_A(X) e^I_B$

Arbitrary choice of vielbeins:

- no impact on Ito FP equation
- matters for Stratonovich

- Field space covariance

Physical quantities do not depend on field redefinitions

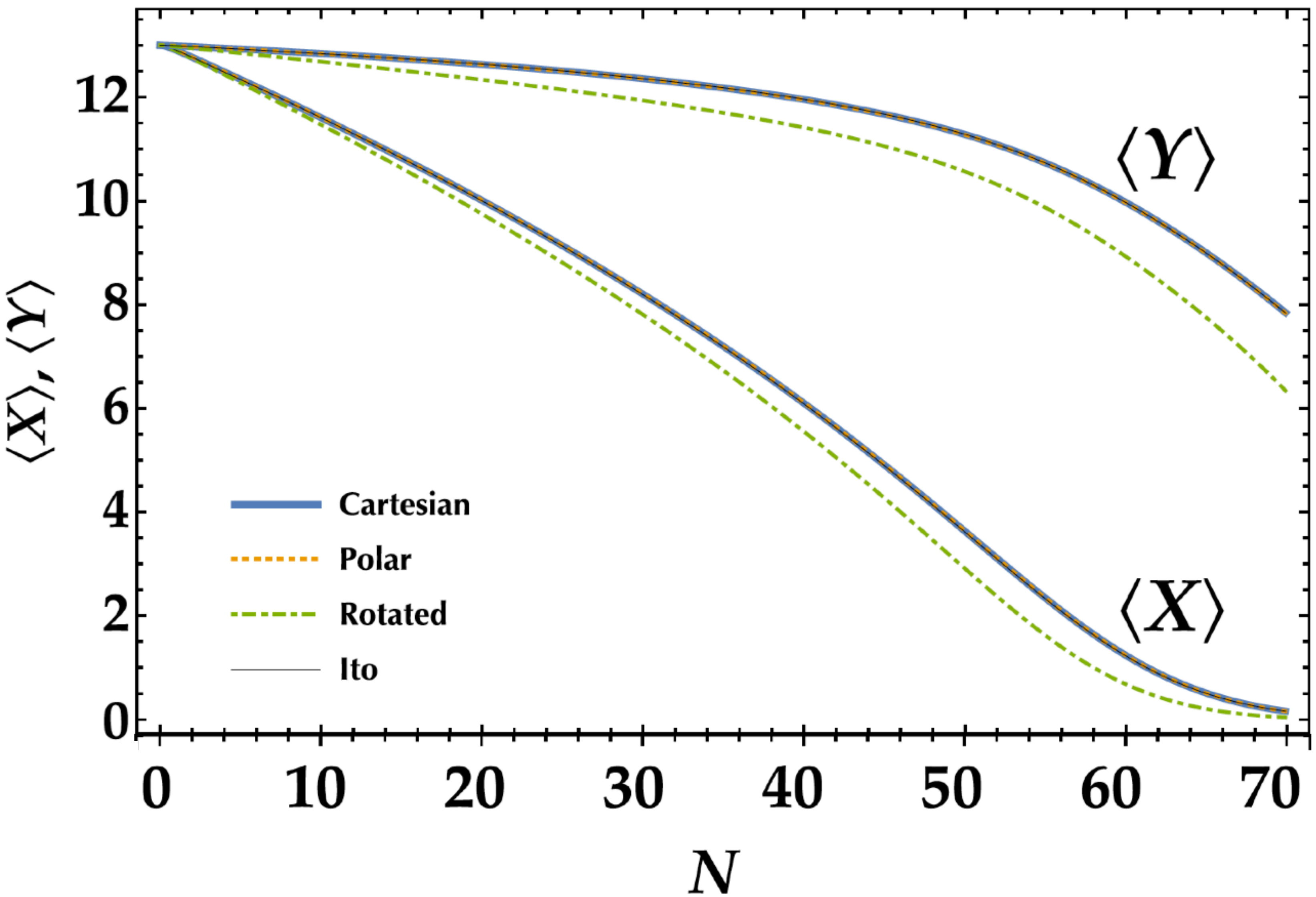
Covariance is respected only in Stratonovich interpretation

Ito lemma vs standard chain rule

Itô-vs Stratonovich dilemma

van Kampen 81, 86

The problem in a simple setup



Statistical averages over Langevin eqs with different vielbeins

Pinol, Renaux-Petel, Tada 18

$$\mathcal{L} = -\frac{1}{2}\partial_\mu X \partial^\mu X - \frac{1}{2}\partial_\mu Y \partial^\mu Y - \frac{1}{2}M_X^2 X^2 - \frac{1}{2}M_Y^2 Y^2$$

The problem in a simple setup

$$\boxed{P_\mathrm{s}(\varphi^I)=\frac{P(\varphi^I)}{\sqrt{\det(G_{IJ})}}}$$

should be a scalar
under field redefinitions

Stratonovich: manifestly covariant FP equation

$$\frac{\partial P_\mathrm{s}}{\partial N} = \nabla_I\left(\frac{V^{,I}}{3H^2}P_\mathrm{s}\right)+\frac{1}{2}\nabla_I\left(\frac{H}{2\pi}\nabla^I\left(\frac{H}{2\pi}P_\mathrm{s}\right)\right) + \frac{1}{2}\nabla_I\left(\left(\frac{H}{2\pi}\right)^2 e^I_A(\nabla_J e^J_A)\,P_\mathrm{s}\right)$$

covariant
derivatives

but…

Spurious dependence on
the arbitrary choice of vielbeins,
in curved or flat field space

Renaux-Petel, IAP

Solution

`Standard manipulations' are implicitly using Stratonovich

+ quantum theory: identification of independent noises.

Multifield quantization:

$$\hat{Q}^I(N,\mathbf{k}) = Q^I_A(N,k)\hat{a}^A_{\mathbf{k}} + \left(Q^I_A(N,k)\right)^* \hat{a}^{A\dagger}_{-\mathbf{k}} \qquad \left[\hat{a}^A_{\mathbf{k}}, \hat{a}^{B\dagger}_{\mathbf{k}'}\right] = (2\pi)^3\delta^{AB}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

Classicalisation (light fields): complex mode functions $(Q^I_A, \tilde{P}_{IA})$ become real outside the horizon to a very good accuracy

(up to an irrelevant constant unitary matrix)

Solution

$$\xi^A(x) \sim \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} \frac{\mathrm{d}\theta(k - k_\sigma(N))}{\mathrm{d}N} \left(\hat{a}^A_{\mathbf{k}} + \hat{a}^{A\dagger}_{-\mathbf{k}}\right)$$

commute with
one another, classical

independent Gaussian white noises
normalized to unity inside Hubble patch

$$\varphi^{I\prime} = \frac{1}{H} G^{IJ} \varpi_J + Q^I_A(N, k_\sigma(N)) \circ \xi^A$$

$$\mathcal{D}_N \varpi_I = -3\varpi_I - \frac{V_I}{H} + \tilde{P}_{IA}(N, k_\sigma(N)) \circ \xi^A$$

$$\mathcal{D}_N \mathcal{V}_I = \mathcal{V}_I{}' - \Gamma^K_{IJ} \mathcal{V}_K \circ \varphi^{J\prime}$$

covariant derivative

Solution

Stochastic anomalies are solved, but still formal.

Conversion from Stratonovich to Itô

with auxiliary variables (eventually disappear):
stochastically-parallel-transported vielbeins

Itô: $$\mathfrak{D}_N \varphi^I = \frac{\varpi^I}{H} + \xi^{QI}, \qquad \mathfrak{D}_N \varpi_I = -3\varpi_I - \frac{V_I}{H} + \xi_I^{\tilde{P}}$$

covariant derivatives compatible with Itô calculus

Solution

$$\mathfrak{D}_N \varphi^I = \frac{\varpi^I}{H} + \xi^{QI}, \qquad \mathfrak{D}_N \varpi_I = -3\varpi_I - \frac{V_I}{H} + \xi_I^{\tilde{P}}$$

Itô

$$\langle \xi^{\tilde{X}I}(N)\xi^{\tilde{Y}J}(N')\rangle \equiv A^{\tilde{X}\tilde{Y}IJ}(N)\delta(N-N') = \mathrm{Re}\mathcal{P}^{\tilde{X}\tilde{Y}IJ}(N;k_\sigma(N))\delta(N-N')$$

Itô-covariant derivatives = standard derivatives

+ (Christoffel) x (autocorrelation of noises)

Solution

$$\mathfrak{D}_N \varphi^I = \frac{\varpi^I}{H} + \xi^{QI}, \qquad \mathfrak{D}_N \varpi_I = -3\varpi_I - \frac{V_I}{H} + \xi_I^{\tilde{P}}$$ Itô

$$\langle \xi^{\tilde{X}I}(N)\xi^{\tilde{Y}J}(N')\rangle \equiv A^{\tilde{X}\tilde{Y}IJ}(N)\delta(N-N') = \mathrm{Re}\mathcal{P}^{\tilde{X}\tilde{Y}IJ}(N;k_\sigma(N))\delta(N-N')$$

Itô-covariant derivatives = standard derivatives

+ (Christoffel) x (autocorrelation of noises)

coordinates on manifold $\mathfrak{D}\mathcal{X}^I = \mathrm{d}\mathcal{X}^I + \frac{1}{2}\Gamma^I_{JK}A^{\mathcal{X}\mathcal{X}JK}\mathrm{d}N$

covectors $\mathfrak{D}\mathcal{V}_I = \mathrm{d}\mathcal{V}_I - \Gamma^J_{IK}\mathcal{V}_J\mathrm{d}\mathcal{X}^K - \frac{1}{2}\left(\Gamma^S_{IJ,K} + \Gamma^M_{IJ}\Gamma^S_{KM}\right)\mathcal{V}_S A^{\mathcal{X}\mathcal{X}JK}\mathrm{d}N - \Gamma^K_{IJ}A^{\mathcal{X}\tilde{\mathcal{V}}J}{}_K\mathrm{d}N$

Renaux-Petel, IAP

Fokker-Planck equation

In Markovian approximation, FP for scalar phase space pdf $P(\phi^I, \pi_I, N)$

$$\partial_N P = -\, D_{\varphi^I}\left[\frac{G^{IJ}}{H}\varpi_J P\right] + \partial_{\varpi_I}\left[\left(3\varpi_I + \frac{V_I}{H}\right)P\right]$$
$$+ \frac{1}{2}D_{\varphi^I}D_{\varphi^J}(A^{QQIJ}P) + D_{\varphi^I}\partial_{\varpi_J}(A^{Q\tilde{P}I}{}_J P) + \frac{1}{2}\partial_{\varpi_I}\partial_{\varpi_J}(A^{\tilde{P}\tilde{P}}{}_{IJ}P)$$

$$D_{\varphi^I} = \nabla_{\varphi^I} + \Gamma^K_{IJ}\varpi_K\partial_{\varpi_J}$$

phase-space
covariant derivative

manifestly covariant

derived from Itô-Langevin equations

nontrivial consistency check

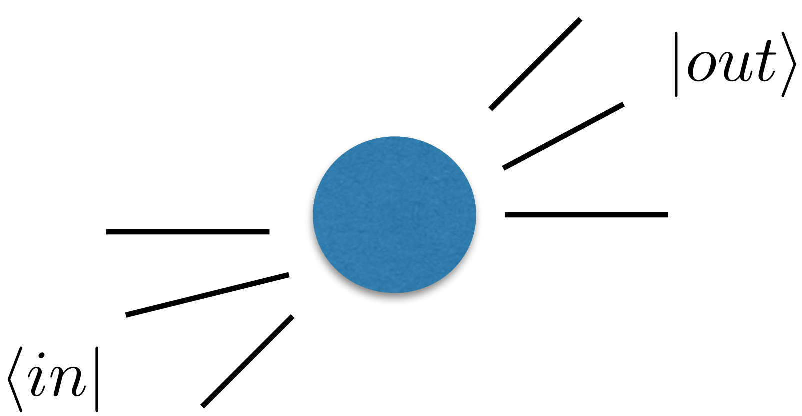
+ analytical approximations for noises autocorrelations (see paper)

III Path-integral derivation

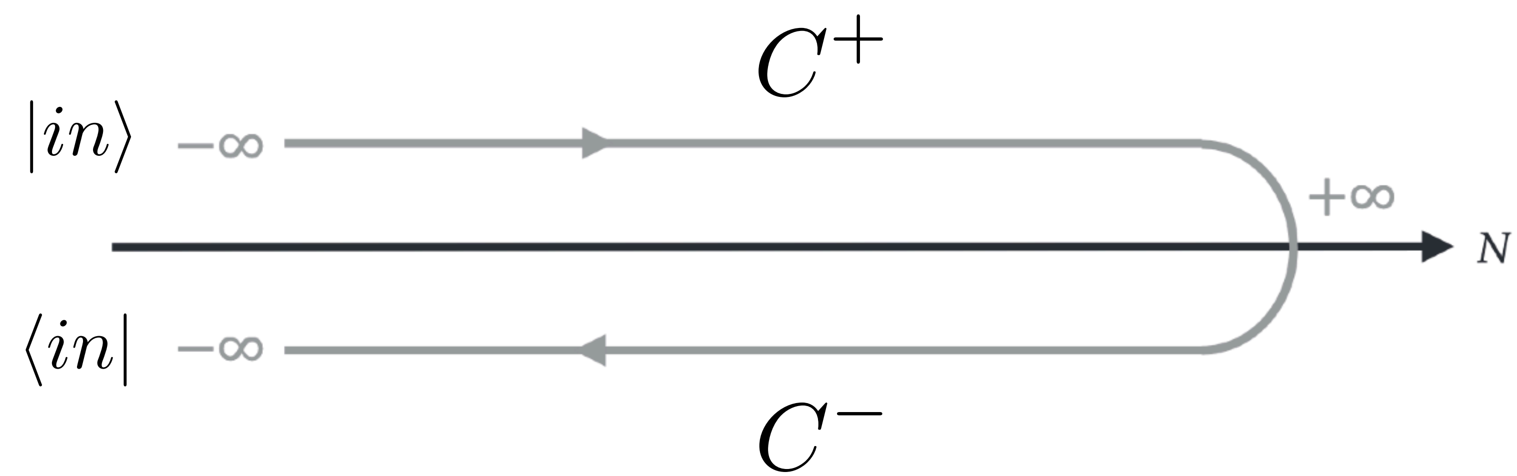
Principles

1)

particle physics:
in-out transition amplitudes



cosmology:
expectations values in in state
and causal equations of motion



generating functional with
closed-time path contour

$$C = C^+ \cup C^-$$

$$Z\left[J_{XI}\right] = \int_C \mathscr{D}\phi^{XI} \exp\left(iS\left[\phi^{XI}\right] + i\int \mathrm{d}^4x J_{XI}\phi^{XI}\right)$$

Schwinger-Keldysh (in-in) formalism

Principles

2) Hamitonian action

$$S[\phi^{XI}] = \int d^4x \left[\pi_I \dot{\phi}^I - \mathcal{H}(\phi^I, \pi_I)\right]$$

X = position or momentum
in phase space

first-principle

simpler for covariance

well-suited to stochastic inflation

(mixing with gravity with ADM)

3) Doubling the dofs along simple forward path

$$Z = \int_{C^+} \mathscr{D}\phi^{XI\pm} \exp\left(iS\left[\phi^{XI+}\right] - iS\left[\phi^{XI-}\right]\right)$$

+- fields considered independent,
except where time path closes:

$$\phi^{I+}(+\infty) = \phi^{I-}(+\infty)$$

momenta unconstrained

Principles

3bis)

Keldysh basis:

$$\phi^{\mathrm{cl}} = \frac{1}{2}\left(\phi^{+} + \phi^{-}\right)$$ ‘classical’ component

$$\phi^{\mathrm{q}} = \phi^{+} - \phi^{-}$$ ‘quantum’ component

Keldysh action

$$S\left[\phi^{XI\mathfrak{a}}\right] = S\left[\phi^{XI\mathrm{cl}} + \phi^{XI\mathrm{q}}/2\right] - S\left[\phi^{XI\mathrm{cl}} - \phi^{XI\mathrm{q}}/2\right]$$

Rationale: there is always a solution to the saddle point eq of the Keldysh action with

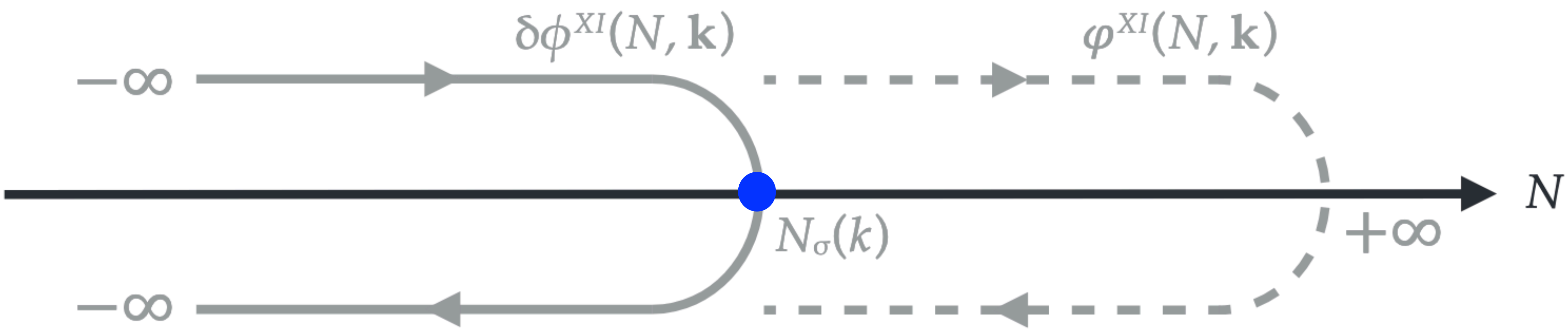
$$\left(\begin{array}{l} \phi^{\mathrm{q}} = 0 \\ \dfrac{\delta S}{\delta \phi^{XI}}_{|\phi^{\mathrm{cl}}} = 0 \end{array}\right.$$

Principles

4) UV/IR splitting

$$\phi^{XI\mathfrak{a}}(x) = \varphi^{XI\mathfrak{a}}(x) + \delta\phi^{XI\mathfrak{a}}(x)$$

IR + UV



Boundary condition at transition time: $k = \sigma a H(N_\sigma(k))$

$$\boxed{\phi^{I\mathfrak{a}}\left(N_\sigma(k),\mathbf{k}\right) = \begin{cases} \delta\phi^{I\mathrm{cl}}\left(N_\sigma(k),\mathbf{k}\right), & \text{if } \mathfrak{a}=\mathrm{cl}, \\ \varphi^{I\mathrm{q}}\left(N_\sigma(k),\mathbf{k}\right), & \text{if } \mathfrak{a}=\mathrm{q}, \end{cases}}$$

see also
Tokuda & Tanaka 17,18

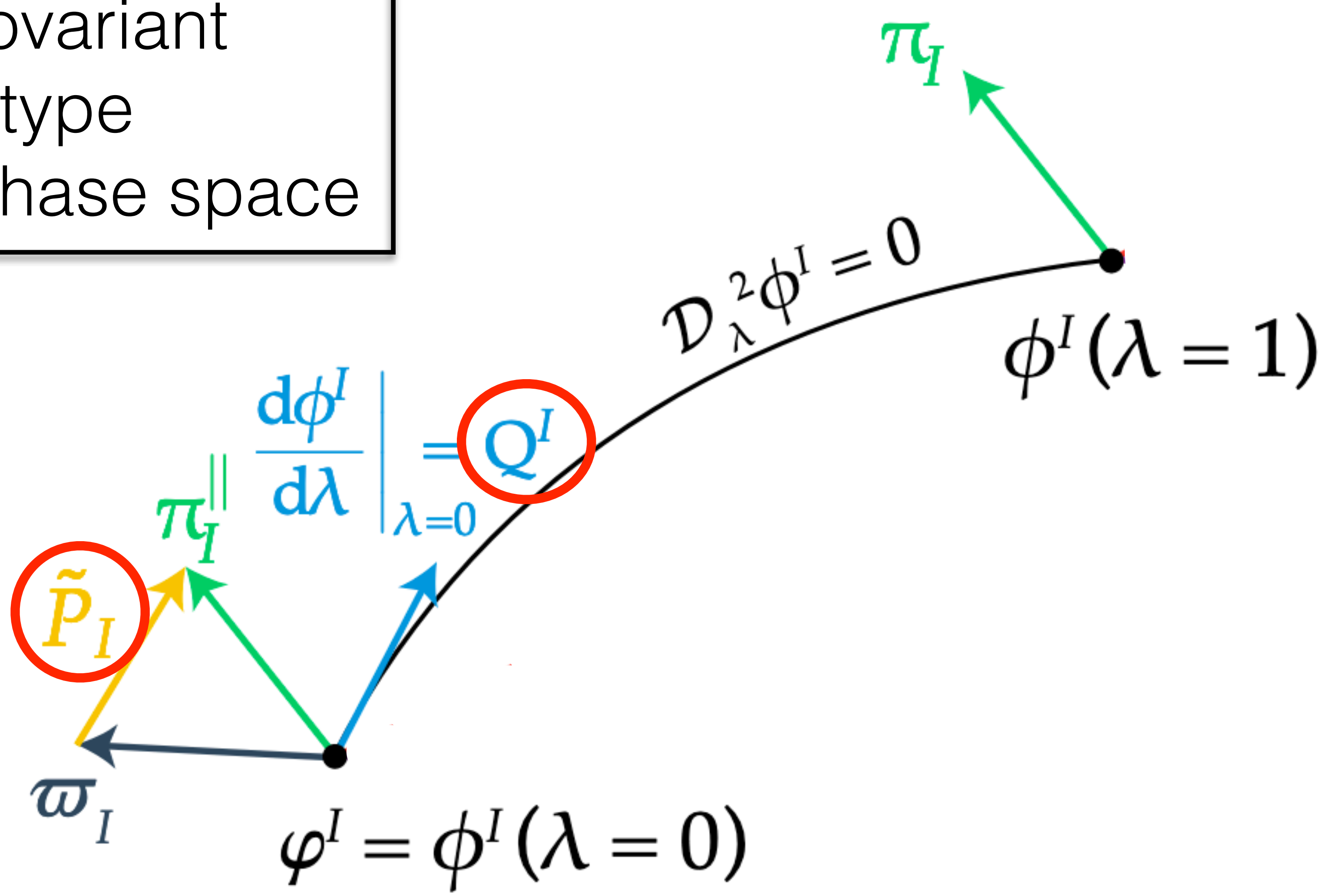
Principles

$$Z = \int \mathscr{D}\varphi^{XI\mathfrak{a}} \exp\left(iS_{\text{eff}}\left[\varphi^{XI\mathfrak{a}}\right]\right), \qquad \text{with}$$

$$\exp\left(iS_{\text{eff}}\left[\varphi^{XI\mathfrak{a}}\right]\right) = \int \mathscr{D}\delta\phi^{XI\mathfrak{a}} \exp\left(iS\left[\varphi^{XI\mathfrak{a}} + \delta\phi^{XI\mathfrak{a}}\right]\right),$$

Too naive: path-integral should be expressed in terms of covariant objects

5) Identification of covariant Vilkovisky-DeWitt-type perturbations in phase space



Renaux-Petel, IAP

Principles

$$Z = \int \mathscr{D}\varphi^{XI\mathfrak{a}} \exp\left(iS_{\rm eff}\left[\varphi^{XI\mathfrak{a}}\right]\right), \qquad \text{with}$$

$$\exp\left(iS_{\rm eff}\left[\varphi^{XI\mathfrak{a}}\right]\right) = \int \mathscr{D}\delta\phi^{XI\mathfrak{a}} \underline{\exp\left(iS\left[\varphi^{XI\mathfrak{a}} + \delta\phi^{XI\mathfrak{a}}\right]\right),}$$

Too naive: path-integral should be expressed in terms of covariant objects

5) Identification of covariant Vilkovisky-DeWitt-type perturbations in phase space $\longrightarrow$ $Q^{\tilde{X}I\mathfrak{a}} = (Q^{I\mathfrak{a}}, \tilde{P}^{I\mathfrak{a}})$

$$\delta\phi^I = Q^I - \frac{1}{2}\Gamma^I_{JK}Q^JQ^K + \dots$$

cf Gong and Tanaka 11

$$\delta\pi_I = \tilde{P}_I + \Gamma^K_{IJ}\varpi_K Q^J + \Gamma^K_{IJ}Q^J\tilde{P}_K + \frac{1}{2}(\Gamma^S_{IJ,K} - \Gamma^S_{IR}\Gamma^R_{JK} + \Gamma^R_{IJ}\Gamma^S_{RK})\varpi_S Q^J Q^K + \cdots$$

Integrating out UV fields

$$\exp\left(iS_{\rm eff}[\varphi^{XI\mathfrak{a}}]\right) = \int \mathscr{D}Q^{\tilde{X}I\mathfrak{a}} \exp\left(iS^{(0)}[\varphi^{XI\mathfrak{a}}] + S^{(1)}[\varphi^{XI\mathfrak{a}}, Q^{\tilde{X}I\mathfrak{a}}] + S^{(2)}[\varphi^{XI\mathfrak{a}}, Q^{\tilde{X}I\mathfrak{a}}]\right)$$

$$S^{(1)} = \int \mathrm{d}^4x\, a^3 \left[\tilde{P}_I \left(\varphi^{I\prime} - \frac{\varpi^I}{H}\right) - Q^I \left(\mathcal{D}_N \varpi_I + 3\varpi_I + \frac{V_I}{H}\right)\right]$$

Integrating out UV fields

$$\exp\left(iS_{\text{eff}}[\varphi^{XI\mathfrak{a}}]\right) = \int \mathscr{D}Q^{\tilde{X}I\mathfrak{a}} \exp\left(iS^{(0)}[\varphi^{XI\mathfrak{a}}] + S^{(1)}[\varphi^{XI\mathfrak{a}}, Q^{\tilde{X}I\mathfrak{a}}] + S^{(2)}[\varphi^{XI\mathfrak{a}}, Q^{\tilde{X}I\mathfrak{a}}]\right)$$

$$S^{(1)} = \int \mathrm{d}^4x\, a^3 \left[\tilde{P}_I \left(\varphi^{I\prime} - \frac{\varpi^I}{H}\right) - Q^I \left(\mathcal{D}_N \varpi_I + 3\varpi_I + \frac{V_I}{H}\right)\right]$$

Absent in standard perturbation theory, because background fields obey classical eoms

Crucial here: governs the UV-IR interactions from time-dependent coarse-graining scale

Integrating out UV fields

$$\exp\left(iS_{\text{eff}}[\varphi^{XI\mathfrak{a}}]\right) = \int \mathscr{D}Q^{\tilde{X}I\mathfrak{a}} \exp\left(iS^{(0)}[\varphi^{XI\mathfrak{a}}] + S^{(1)}[\varphi^{XI\mathfrak{a}}, Q^{\tilde{X}I\mathfrak{a}}] + S^{(2)}[\varphi^{XI\mathfrak{a}}, Q^{\tilde{X}I\mathfrak{a}}]\right)$$

$$S^{(1)} = \int \mathrm{d}^4x\, a^3 \left[\tilde{P}_I \left(\varphi^{I\prime} - \frac{\varpi^I}{H}\right) - Q^I \left(\mathcal{D}_N \varpi_I + 3\varpi_I + \frac{V_I}{H}\right)\right]$$

Geometric nonlinear definitions of UV fields needed to ensure covariance, despite restricting to quadratic action

Configurations with $\varphi^{\mathrm{q}} \neq 0$ are heavily suppressed in the path integral: computation at leading order in quantum components of IR fields

Influence action

$$S_{\rm eff}[\varphi^{XI\mathfrak{a}}] = S^{(0)}[\varphi^{XI\mathfrak{a}}] + S_{\rm ren}[\varphi^{XI\rm cl}] + S_{\rm IA}[\varphi^{XI\mathfrak{a}}]$$

influence of integrated UV fields on IR ones

$$S_{\rm IA}[\varphi^{XI\mathfrak{a}}] \sim i[\varphi^{\rm q}]^2\, \underline{{\rm Re}\mathcal{P}}^{\rm UV} + \mathcal{O}(\varphi^{\rm q})^3$$

Power spectra of UV fields in ‘background’ of IR classical components

Influence action

$$S_{\text{eff}}[\varphi^{XI\mathfrak{a}}] = S^{(0)}[\varphi^{XI\mathfrak{a}}] + S_{\text{ren}}[\varphi^{XI\text{cl}}] + S_{\text{IA}}[\varphi^{XI\mathfrak{a}}]$$

influence of integrated UV fields on IR ones

$$S_{\text{IA}}[\varphi^{XI\mathfrak{a}}] \sim i\,[\varphi^{\text{q}}]^2\, \underline{\text{Re}\mathcal{P}}^{\text{UV}} + \mathcal{O}(\varphi^{\text{q}})^3$$

Power spectra of UV fields in 'background' of IR classical components

In IR path-integral, weight of configurations with non-zero quantum components exponentially suppressed

Feynman Vernon 63

$$\mathrm{e}^{iS_{\text{IA}}} = \int \mathscr{D}\xi^{XI} P\left[\xi^{XI};\varphi^{XI\text{cl}}\right] \mathrm{e}^{i\int \mathrm{d}^4x\, a^3 \xi^{XI}\varphi_{XI}{}^{\text{q}}}$$

$$\sim e^{-\xi^2/(\text{Re}\mathcal{P}^{UV})}$$

auxiliary fields (Hubbard-Stratonovich)

Gaussian weight with variance: UV power spectra

Renaux-Petel, IAP

Langevin equations

$$Z = \int \mathscr{D}\varphi^{XI\mathrm{cl}} \int \mathscr{D}\xi^{XI} P\left[\xi^{XI};\varphi^{XI\mathrm{cl}}\right] \int \mathscr{D}\varphi^{XI\mathrm{q}} \exp\left(iS^{(0)}\left[\varphi^{XI\mathfrak{a}}\right] + i\int \mathrm{d}^4x\, a^3 \xi^{XI} {\varphi_{XI}}^{\mathrm{q}}\right)$$

with
$$S^{(0)}\left[\varphi^{XI\mathfrak{a}}\right] = \int \mathrm{d}^4x \left.\frac{\delta S^{(0)}\left[\varphi^{XI}\right]}{\delta\varphi^{YJ}(x)}\right|_{\varphi^{XI}=\varphi^{XI\mathrm{cl}}} \varphi^{YJ\mathrm{q}}(x) + \mathcal{O}\left(\varphi^{\mathrm{q}}\right)^3$$

⟶ Path integral over quantum components yields delta function

$$\varphi^{I\prime} = \frac{\varpi^I}{H} + \xi^{QI}, \qquad \mathcal{D}_N \varpi_I = -3\varpi_I - \frac{V_I}{H} + \xi_I^{\tilde{P}}$$

truly for classical components

Stratonovich to Ito

$$\boxed{\mathfrak{D}_N \varphi^I = \frac{\varpi^I}{H} + \xi^{QI}, \qquad \mathfrak{D}_N \varpi_I = -3\varpi_I - \frac{V_I}{H} + \xi_I^{\tilde{P}}\,.}$$

Conclusion

- Rigorous path-integral derivation of stochastic inflation using methods of nonequilibrium quantum field theory, solving conceptual issues of heuristic approach.

- Resolution of inflationary stochastic anomalies: covariant Itô-Langevin equations in phase space, ready to be used.

- Markovian approximation: covariant FP eq in phase space, and analytical formulae for noises.

- Many phenomenological and theoretical applications: statistical properties of zeta in concrete models, study of phase space FP operator in multifield contexts …