10th Nov. 2020 @ PBH & stochastic workshop

StocDeltaN: Numerical Approach to Inflation in combination of the Stochastic & δN formalism

Yuichiro TADA Nagoya U.

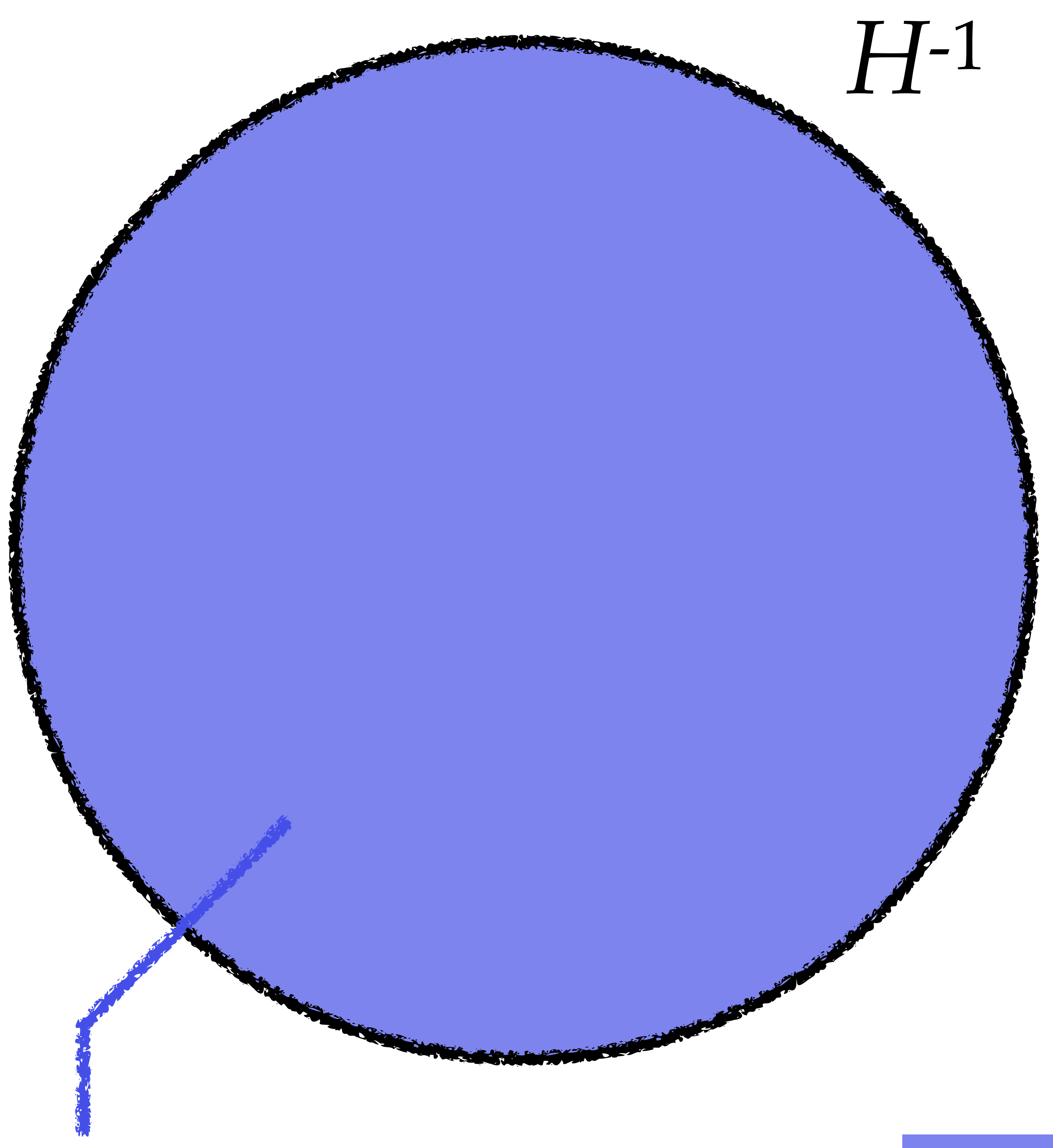
w/ S. Renaux-Petel & V. Vennin in prep.

ref. Pinol, Renaux-Petel, YT '18, '20

Fujita, Kawasaki, YT '14 (+ Takesako '13)

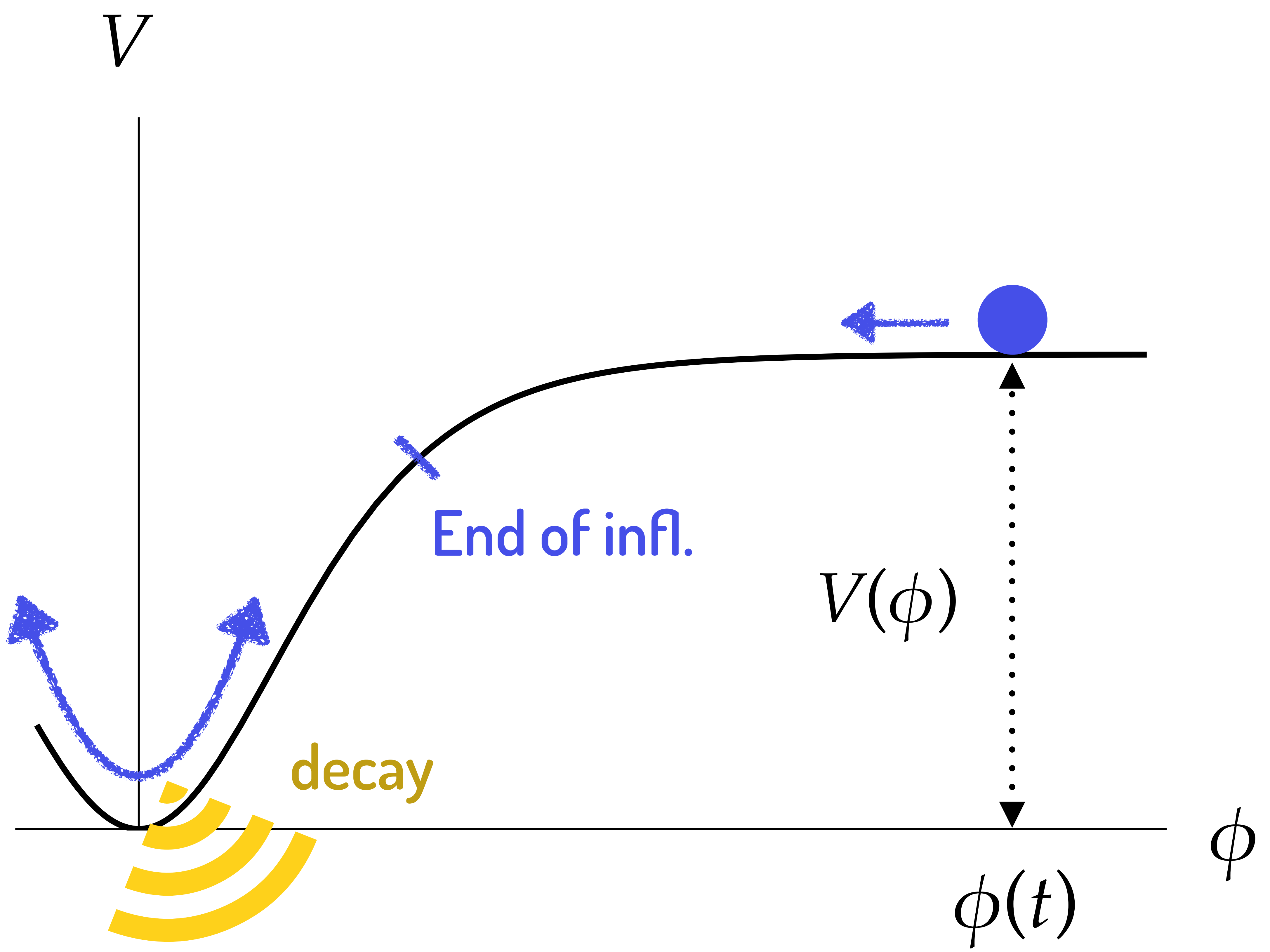
Inflation

pre-Big-Bang accelerated expansion



"Dark Energy": $\rho_{\mathrm{const}} = V(\phi_{\mathrm{const}})$

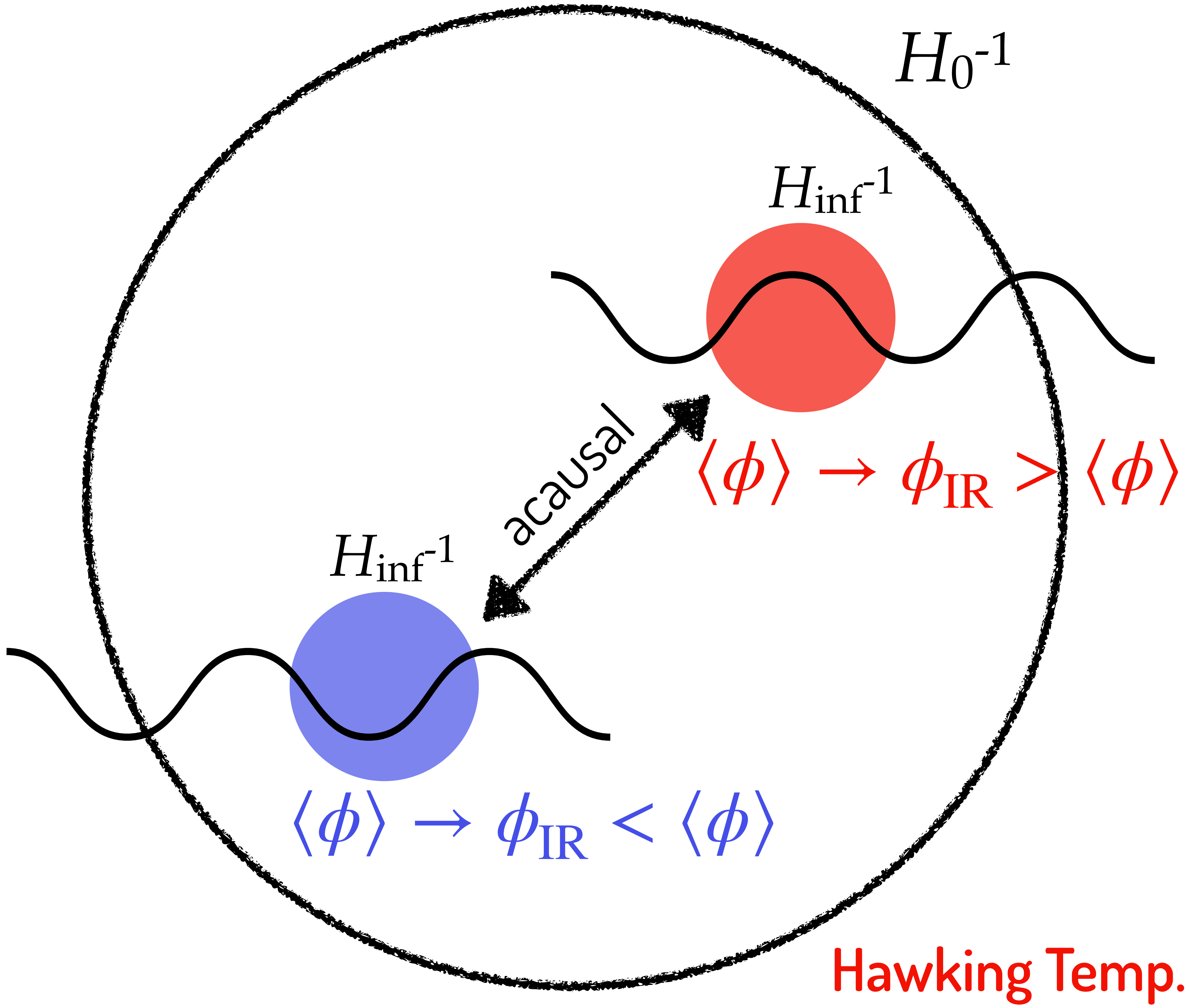
Inflaton: scalar particle

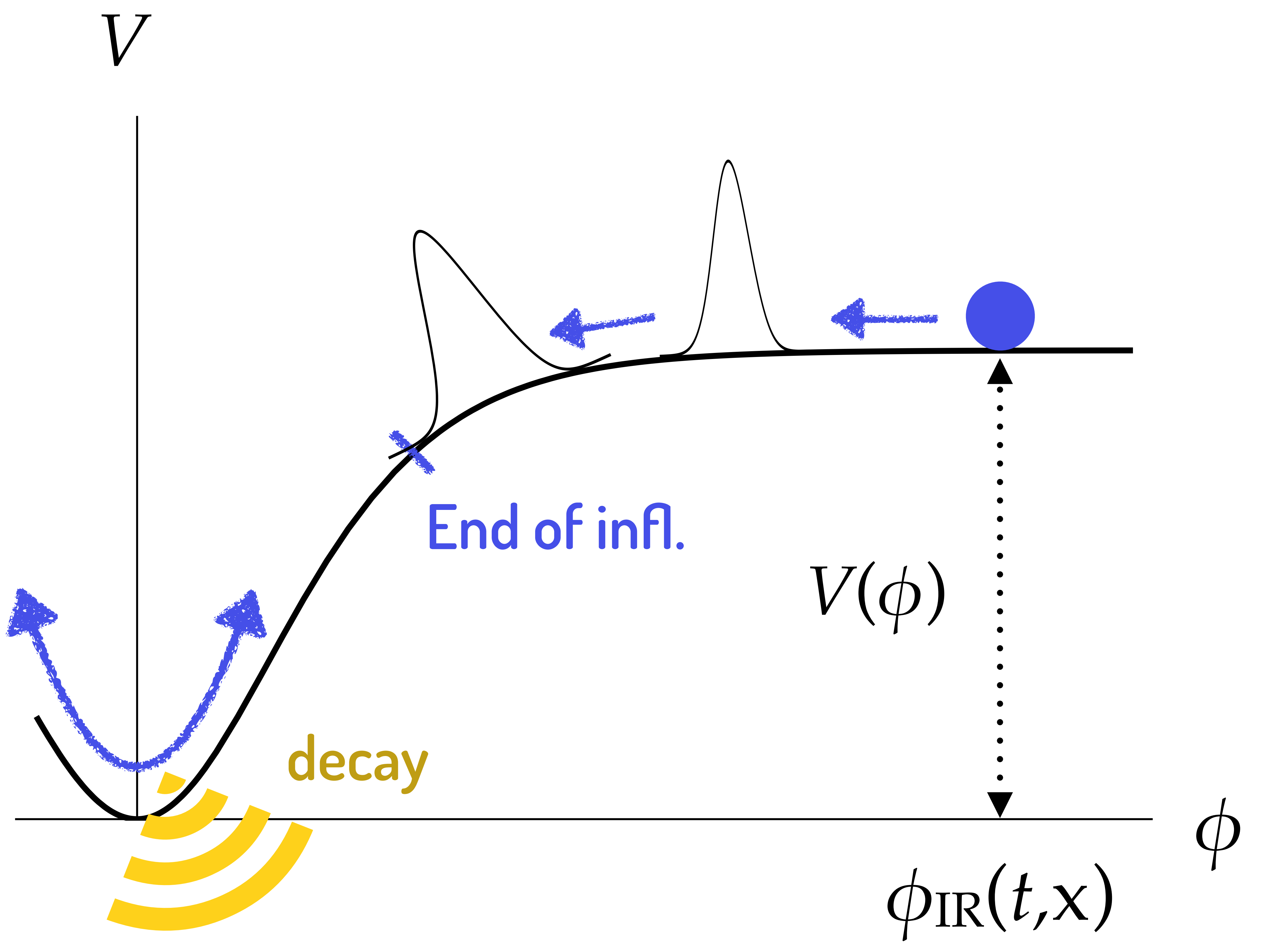


ρ_{r} : Big-Bang universe

Stochastic Form.

Starobinsky '86





$\rho_{\rm r}$: Big-Bang universe

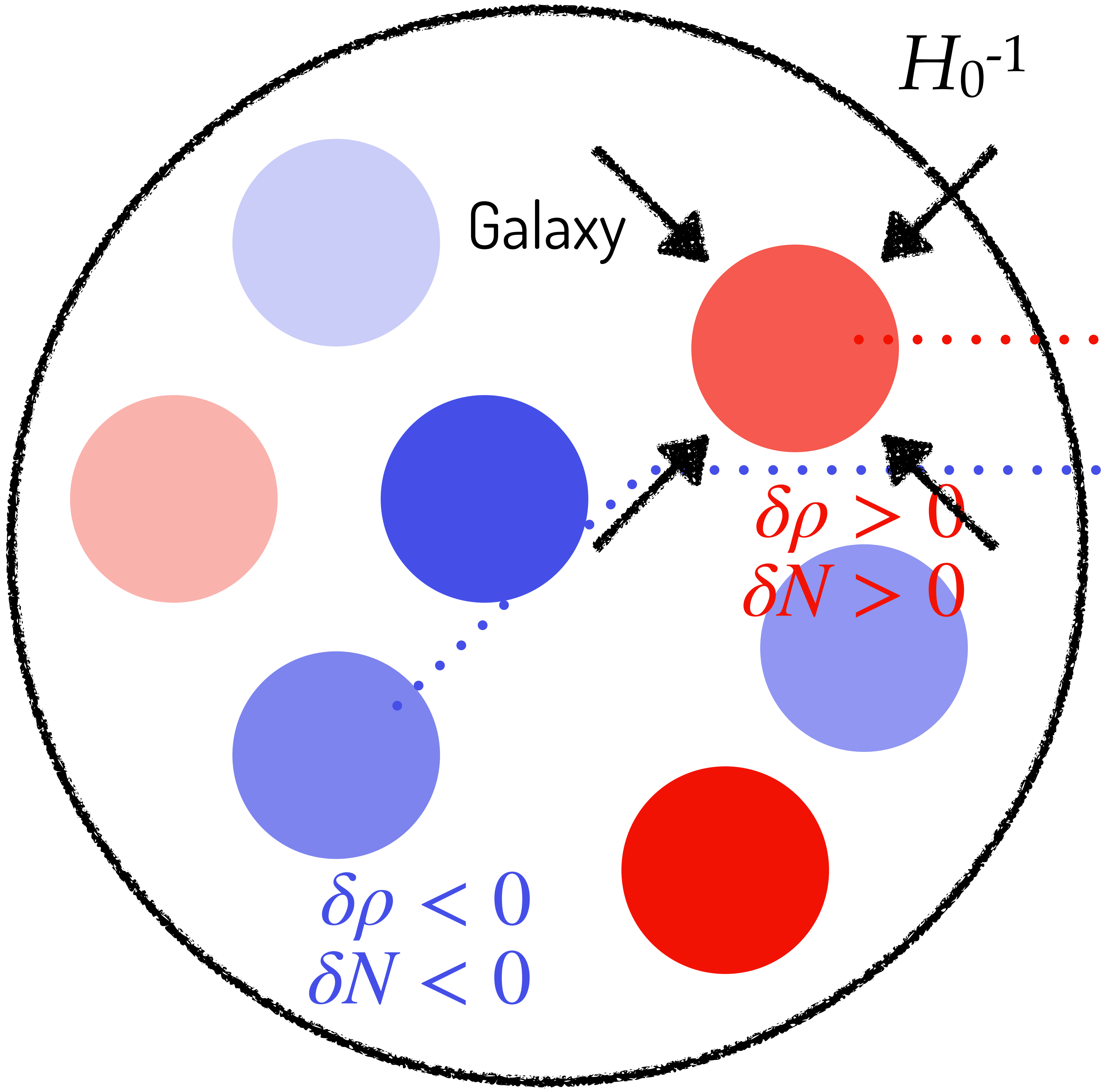
Hawking Temp.

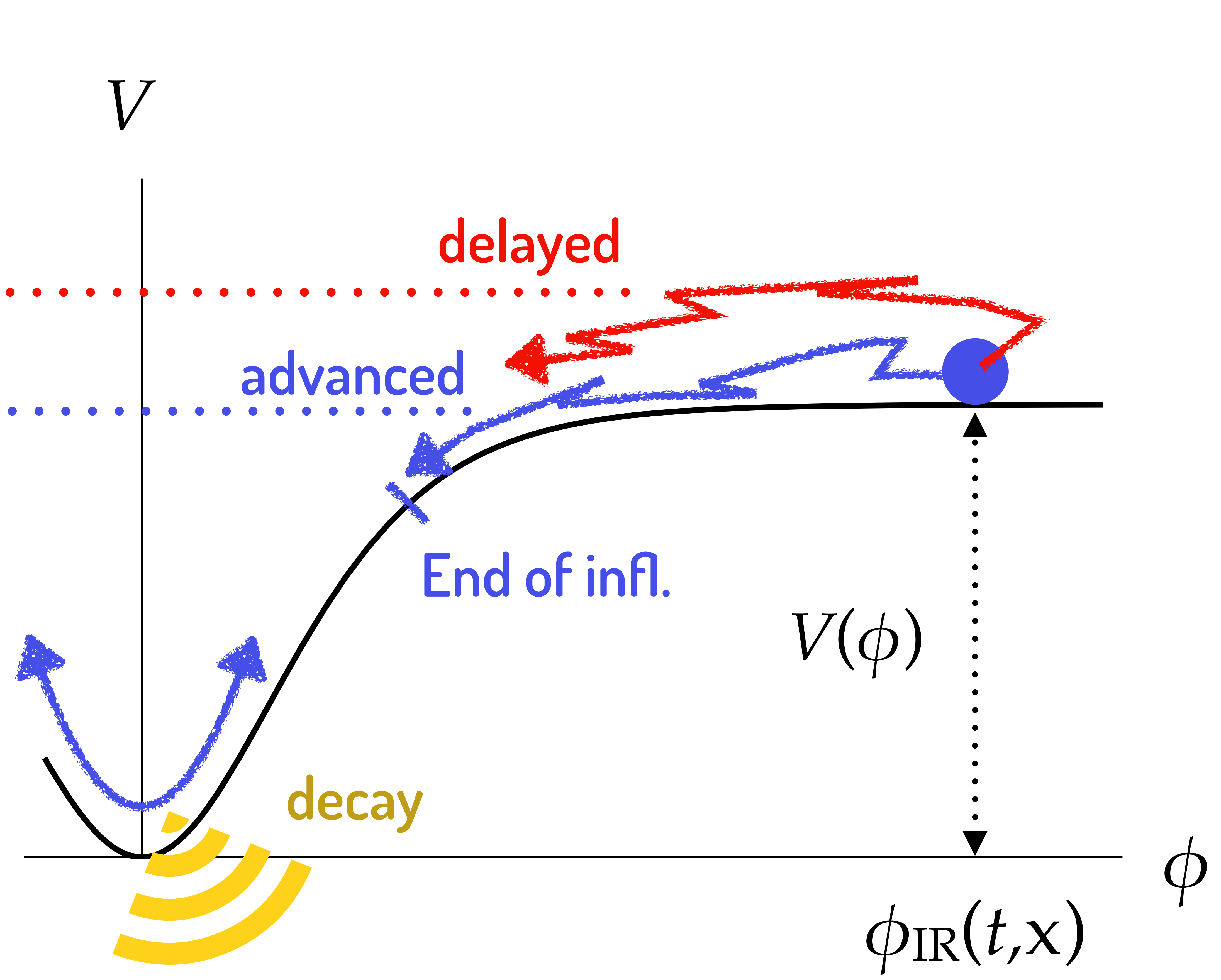
❖ Stochastic EoM: $$\frac{\mathrm{d}\phi_{\rm IR}}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi$$

Gaussian Rand.

(conserved) δN Form.

Starobinsky '85





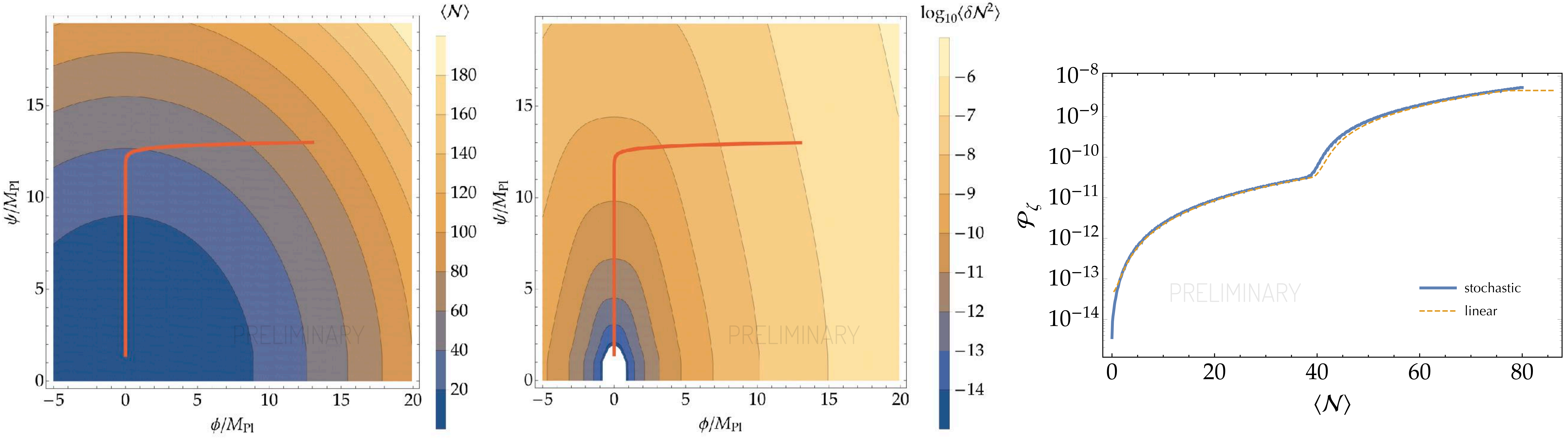
$\rho_{\rm r}$: Big-Bang universe

- Stochastic-δN formalism
  Fujita, Kawasaki, YT, Takesato '13
  Vennin & Starobinsky '15

StocDeltaN

Renaux-Petel, YT, Vennin in prep.

❖ Double Mass-term $V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\psi^2, \qquad M = 9m = 10^{-5}M_{\rm Pl}$

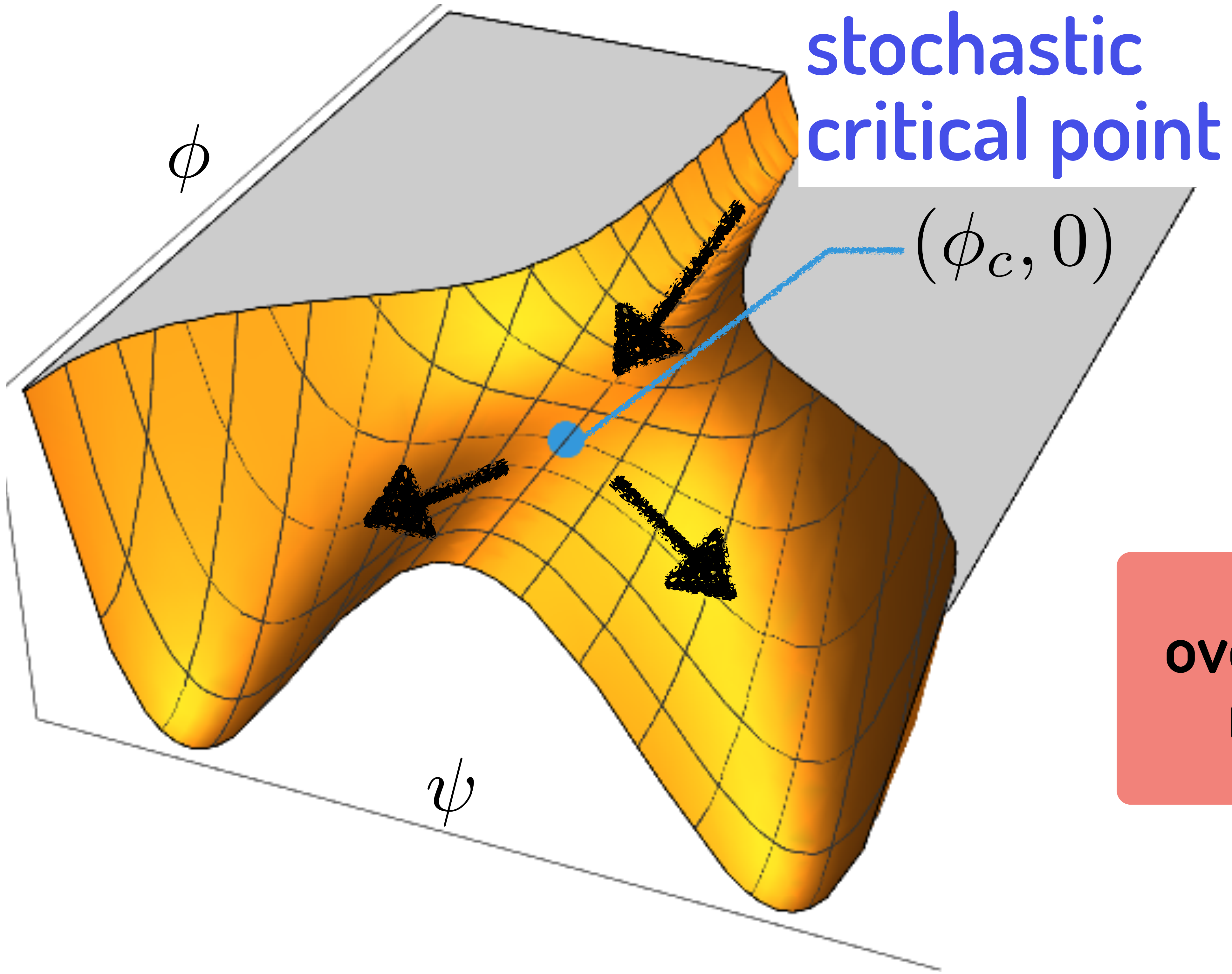


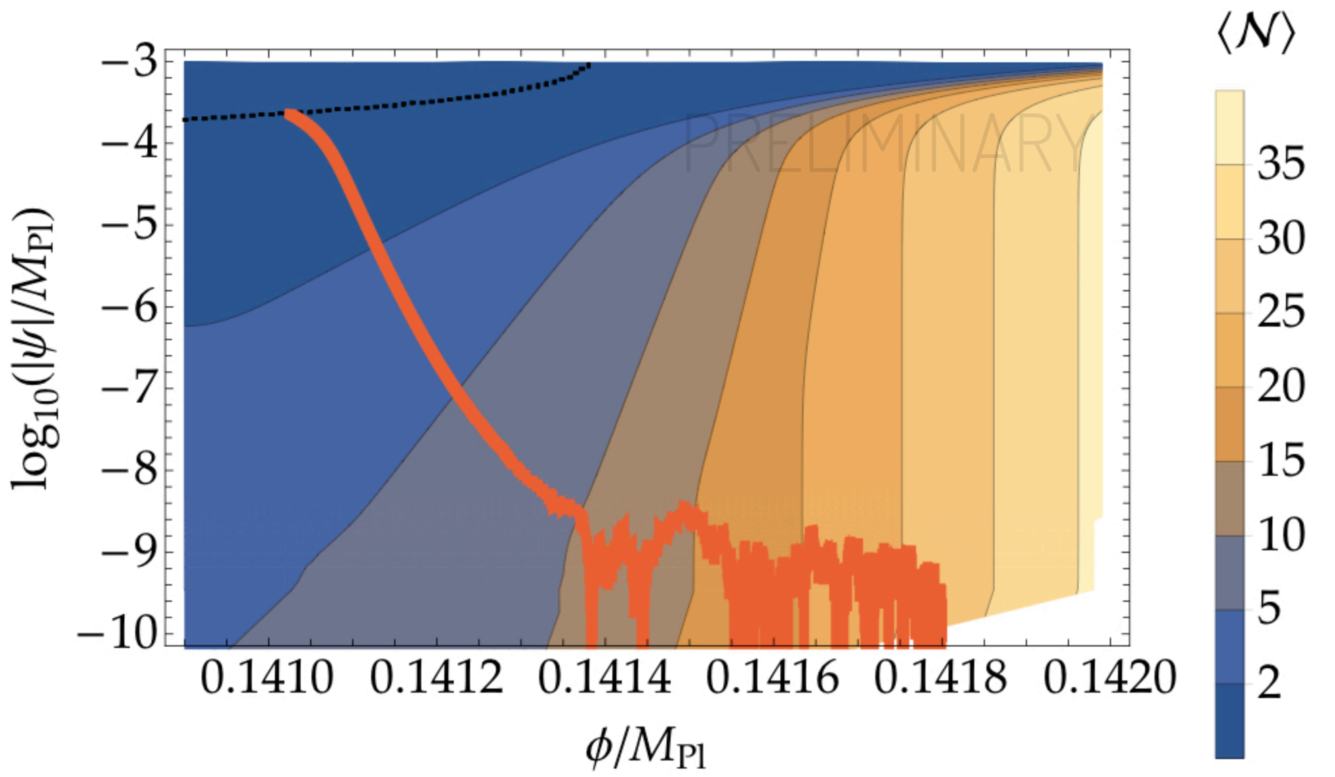
STOCDELTAN

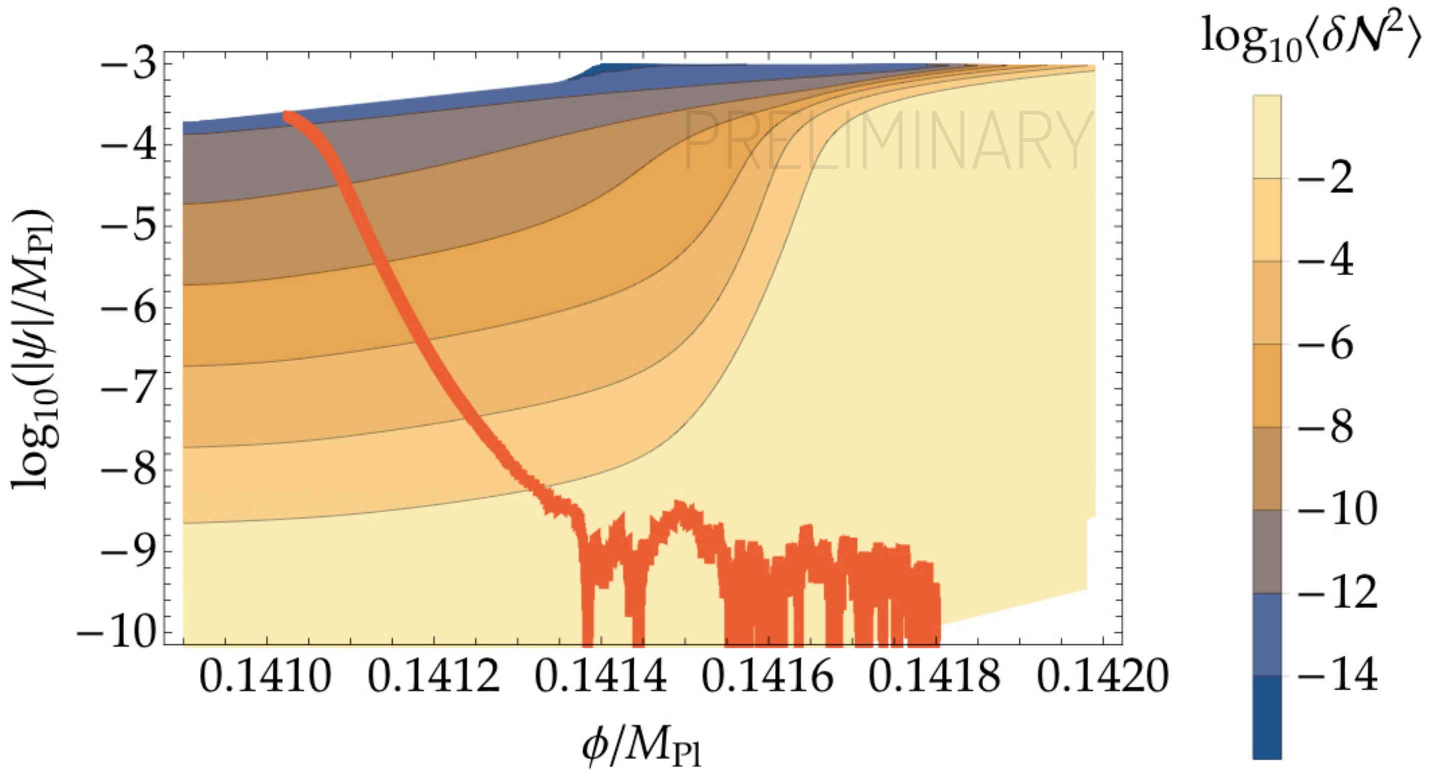
Renaux-Petel, YT, Vennin in prep.

❖ Hybrid Inflation

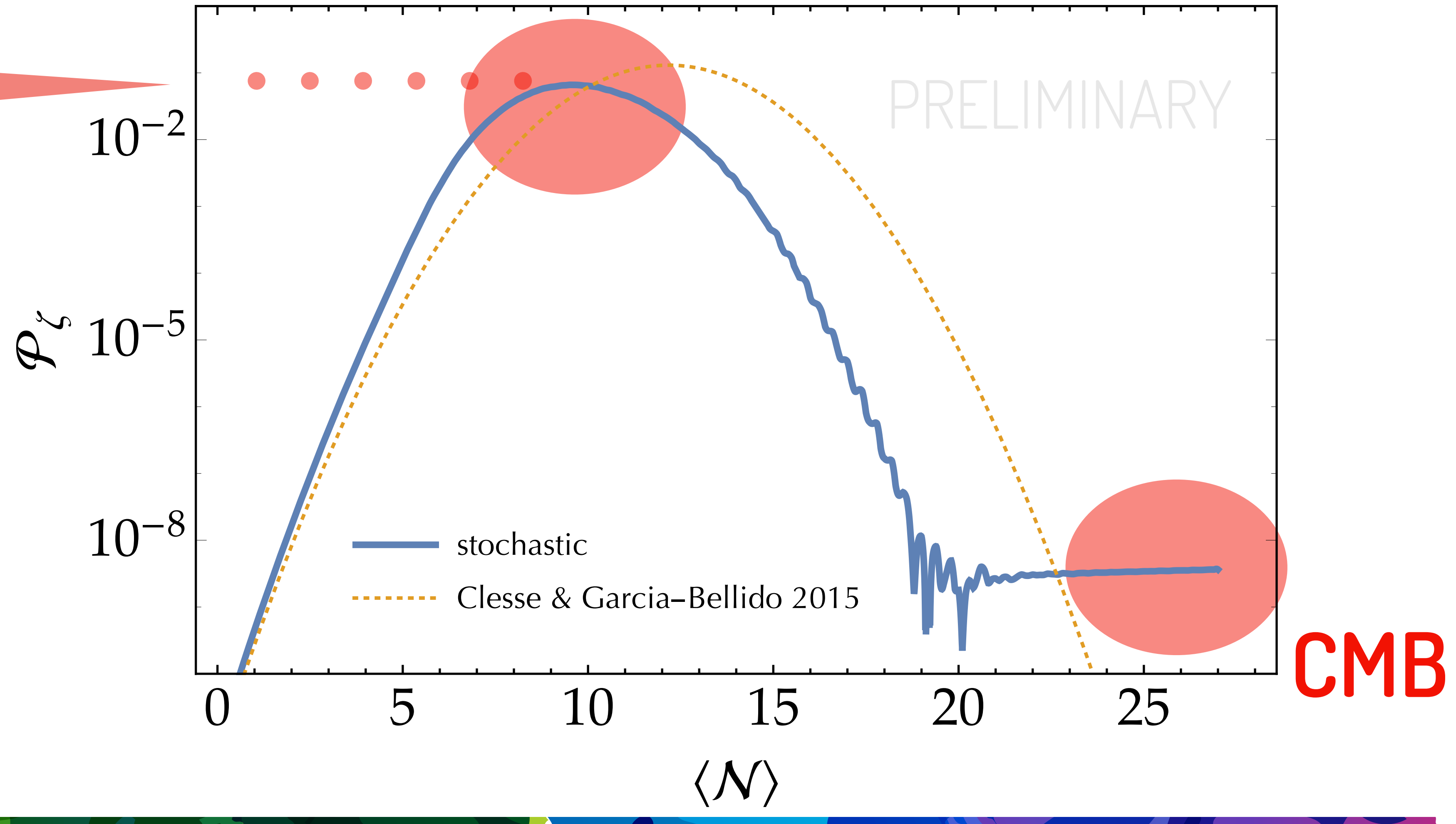
$$V = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2} \right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} + \frac{\phi - \phi_c}{\mu_1} - \frac{(\phi - \phi_c)^2}{\mu_2^2} \right]$$







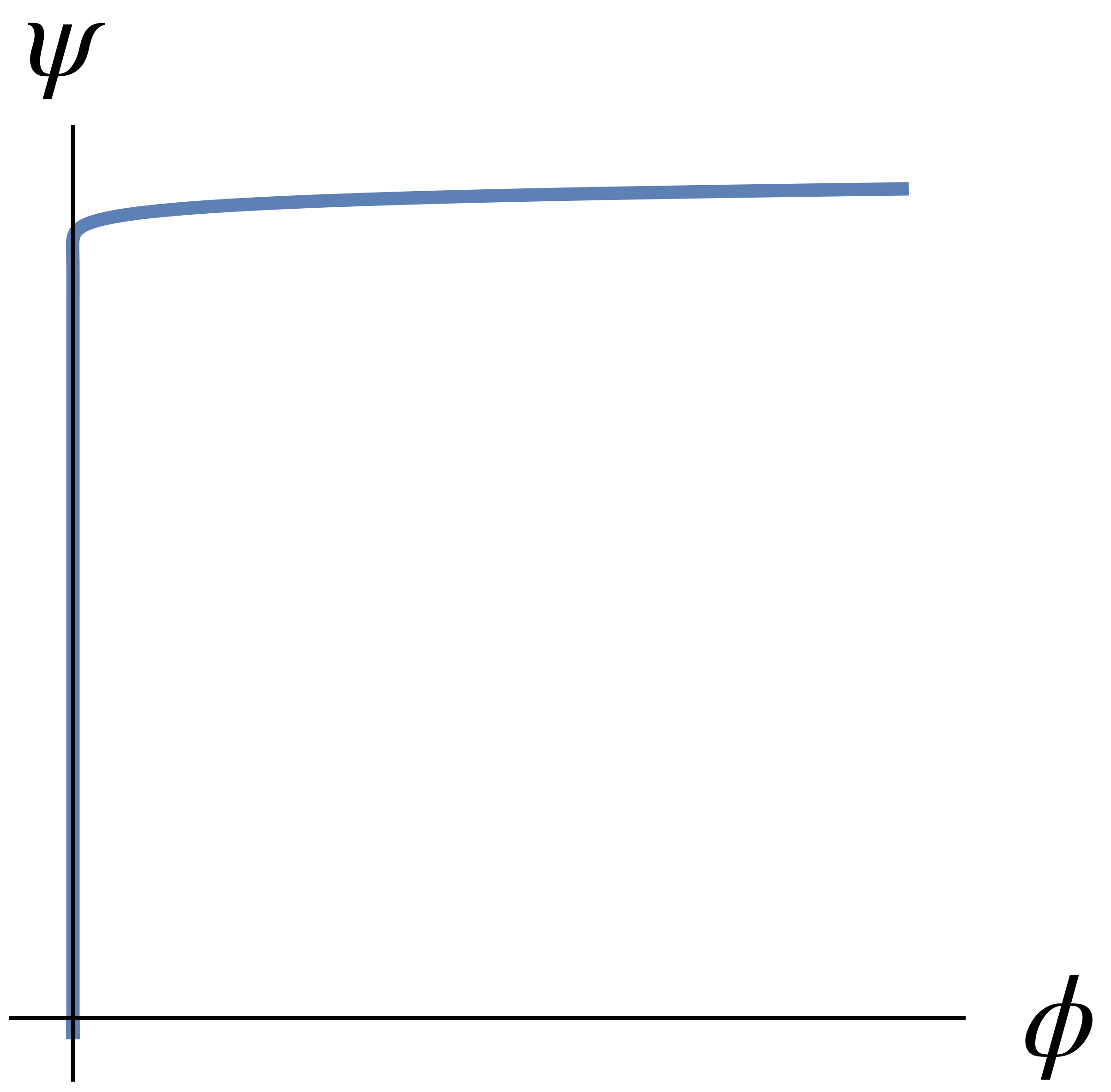
overproduce PBHs
(Kawasaki, YT 2015)



GeoDesI

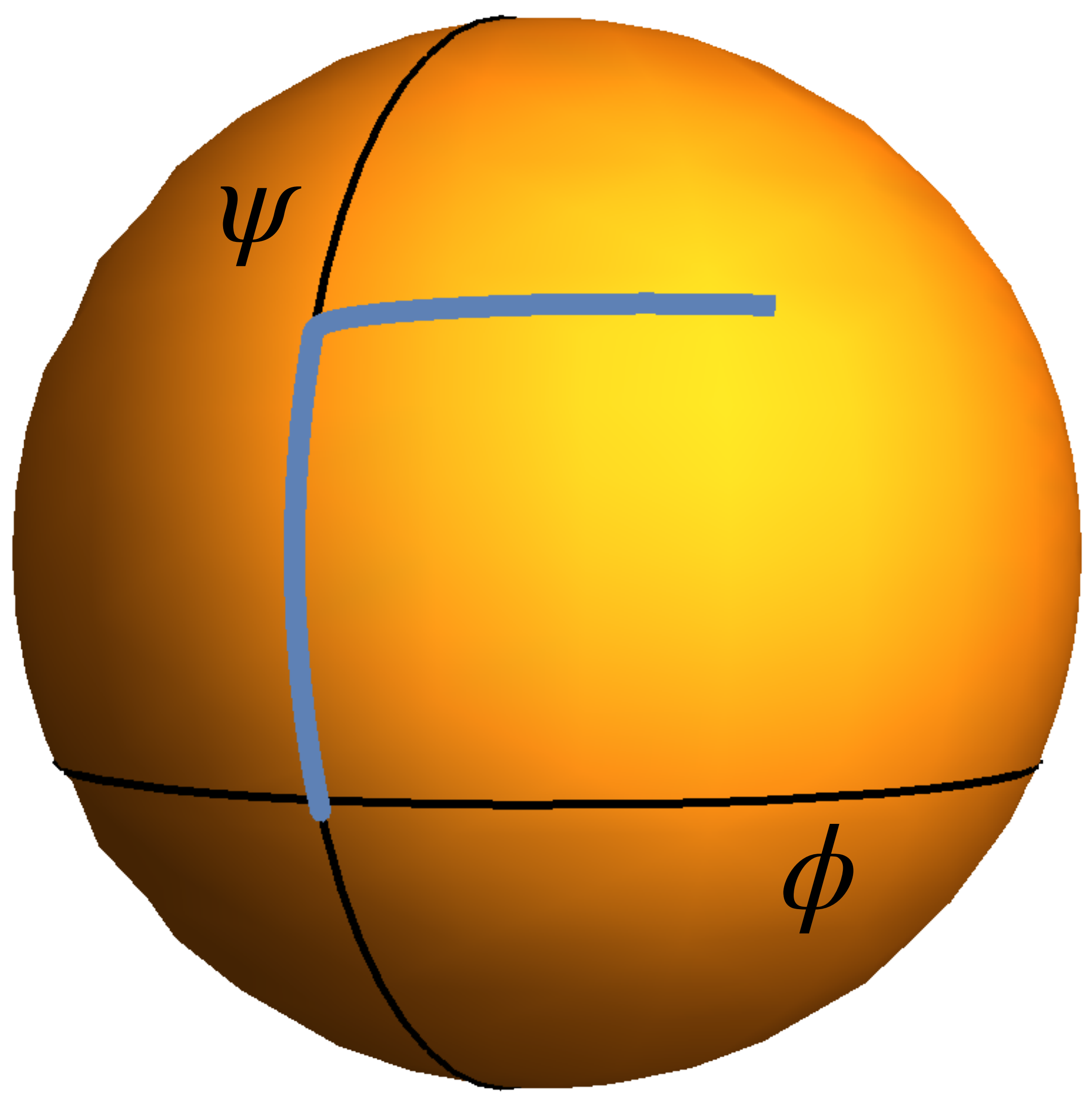
Renaux-Petel & Turzyński '15

❖ Flat Fields



$$\mathcal{L}_{\rm kin} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\psi)^2$$
$$= -\frac{1}{2}\delta_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J$$

❖ Curved Fields

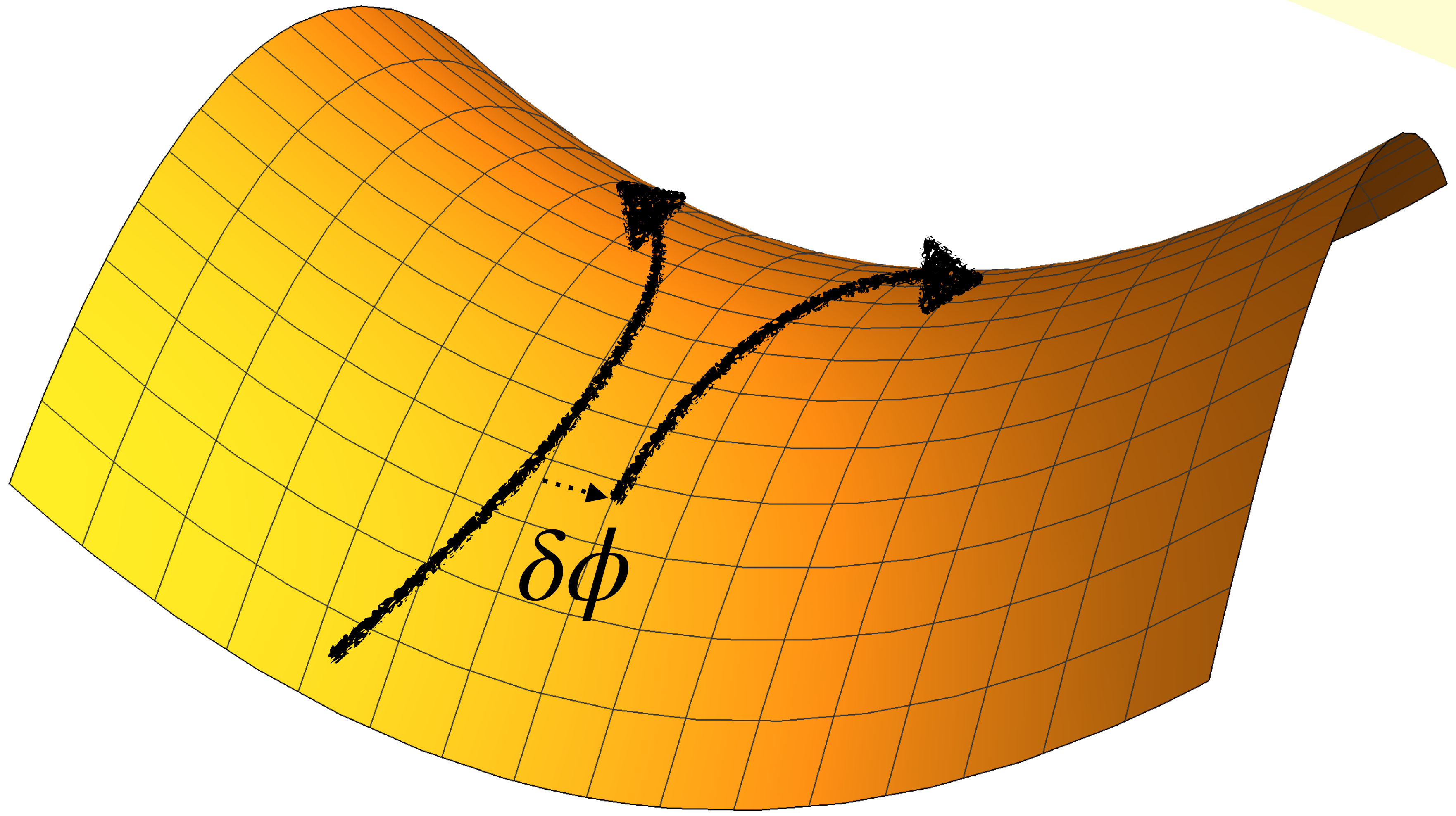


$$\mathcal{L}_{\rm kin} = -\frac{1}{2}G_{IJ}(\phi)\partial_\mu\phi^I\partial^\mu\phi^J$$

GeoDesI

Renaux-Petel & Turzyński '15

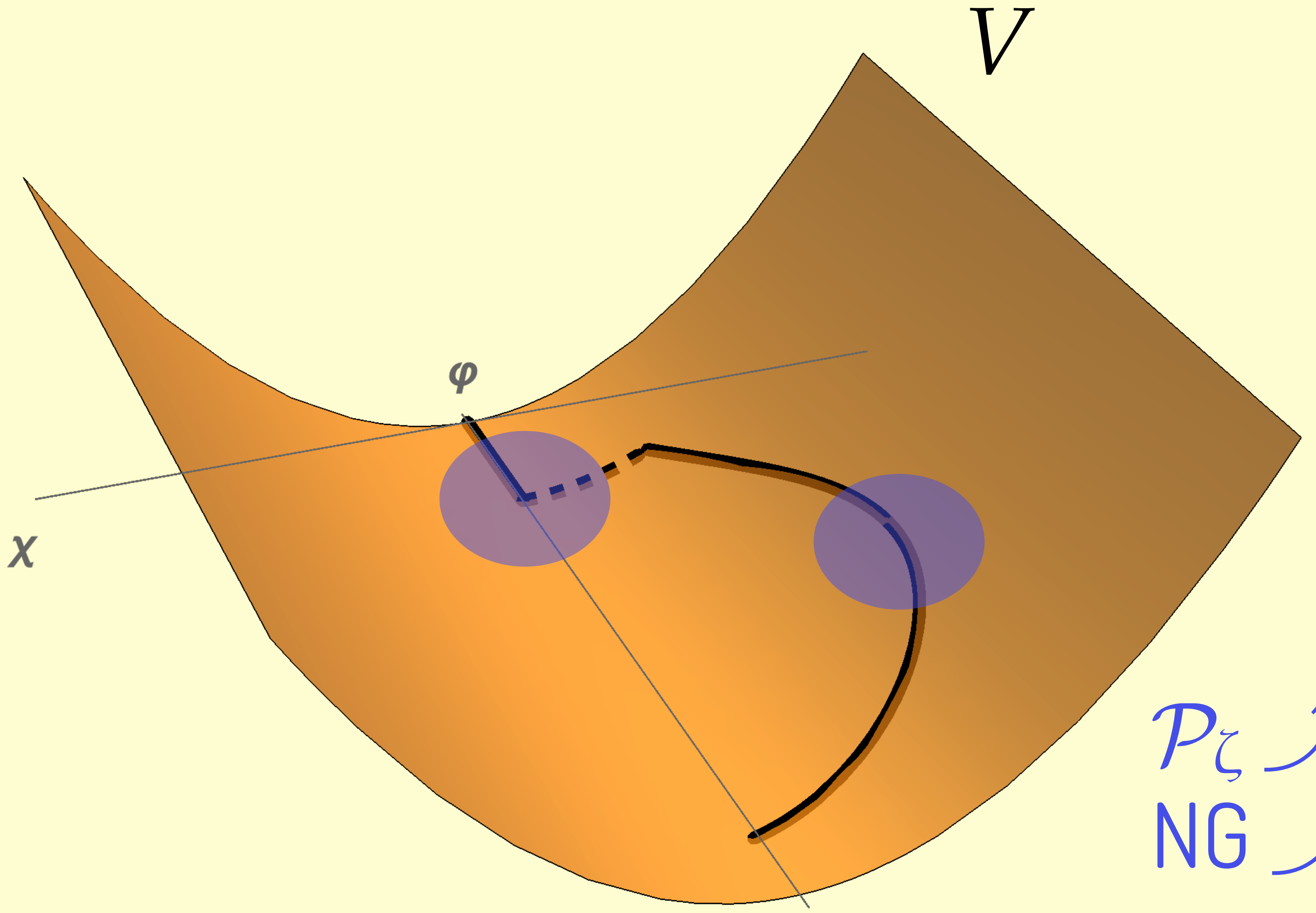
❖ Hyperbolic $R < 0$



unstable

Sidetracked Inflation

Gracia-Saenz, Renaux-Petel, Ronayne '18



$\mathcal{P}_\zeta \nearrow$

NG $\nearrow$

Generalization?

Pinol, Renaux-Petel, YT '18, '20

Inflaton-space metric

- ❖ General Multi-scalar $\mathscr{L} = -\frac{1}{2} g^{\mu\nu} G_{IJ}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi)$

Stochastic EoM ??

$$\frac{\mathrm{d}\phi^I}{\mathrm{d}N} \overset{?}{=} -\frac{G^{IJ}\partial_J V}{3H^2} + \xi^I, \quad \text{with } \langle \xi^I(N) \xi^J(N') \rangle = \left(\frac{H}{2\pi} \right)^2 G^{IJ} \delta(N - N')$$

- ~~Covariance~~ under $\phi^I \to \bar{\phi}^{\bar{I}} = \bar{\phi}^{\bar{I}}(\phi)$

$$\frac{\mathrm{d}\phi^I}{\mathrm{d}N} = -\frac{G^{IJ}\partial_J V}{3H^2} + \xi^I \quad \Rightarrow \quad \frac{\mathrm{d}\bar{\phi}^{\bar{I}}}{\mathrm{d}N} \neq -\frac{G^{\bar{I}\bar{J}}\partial_{\bar{J}} V}{3H^2} + \xi^{\bar{I}}$$

and/or

- Spurious Frame Dependence

$$\frac{\mathrm{d}\phi^I}{\mathrm{d}N} = -\frac{G^{IJ}\partial_J V}{3H^2} + \xi^I \neq -\frac{G^{IJ}\partial_J V}{3H^2} + R^I{}_{\tilde{A}} \xi^{\tilde{A}}$$

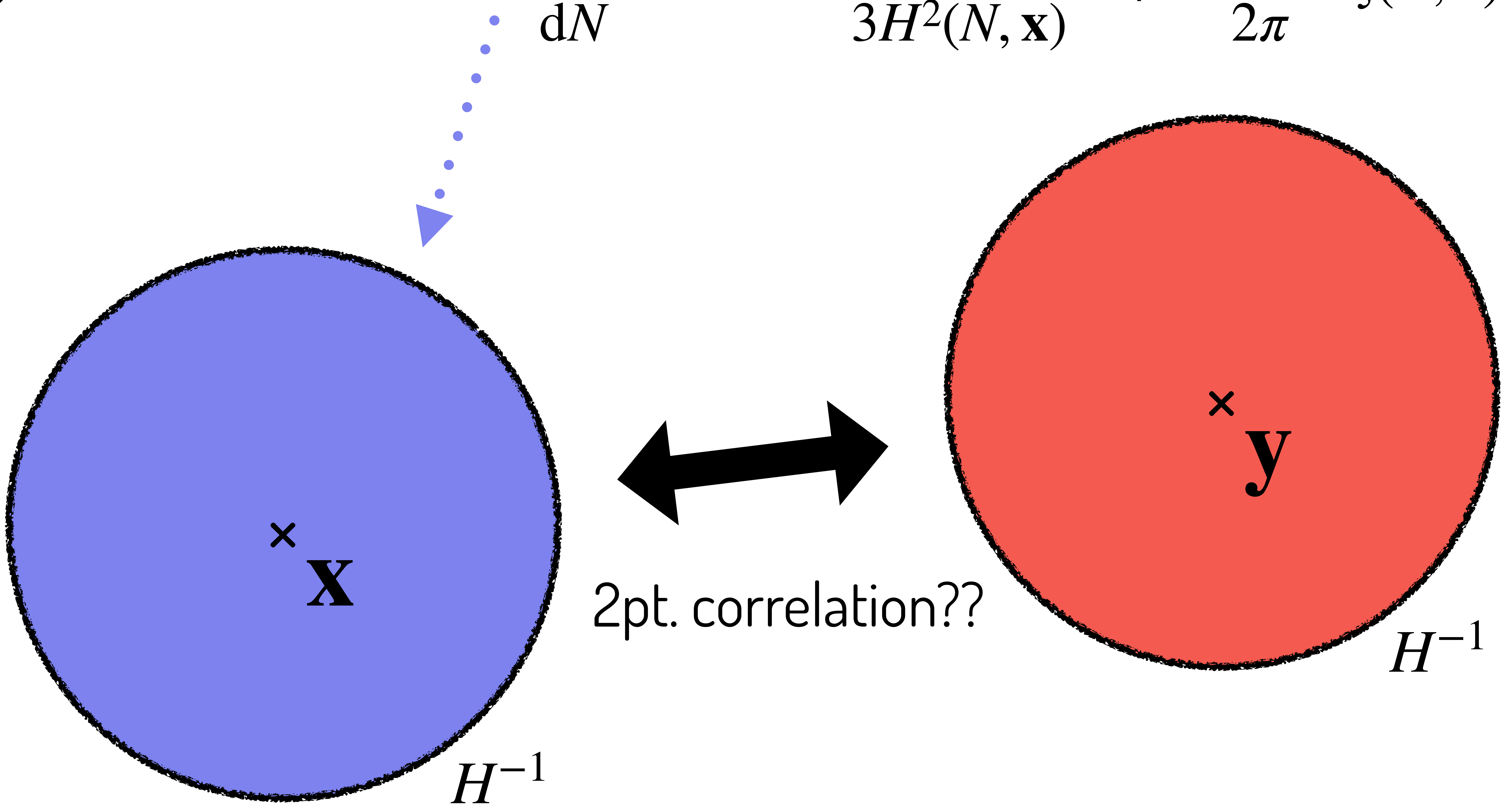
Rotation/Diagonalization

Stochastic-δ N formalism

2pt. func. in stoc.

1pt. dynamics

- Stochastic EoM: $$\frac{\mathrm{d}\phi_{\mathrm{IR}}(N,\mathbf{x})}{\mathrm{d}N} = -\frac{V'(\phi_{\mathrm{IR}}(N,\mathbf{x}))}{3H^2(N,\mathbf{x})} + \frac{H(N,\mathbf{x})}{2\pi}\xi(N,\mathbf{x})$$



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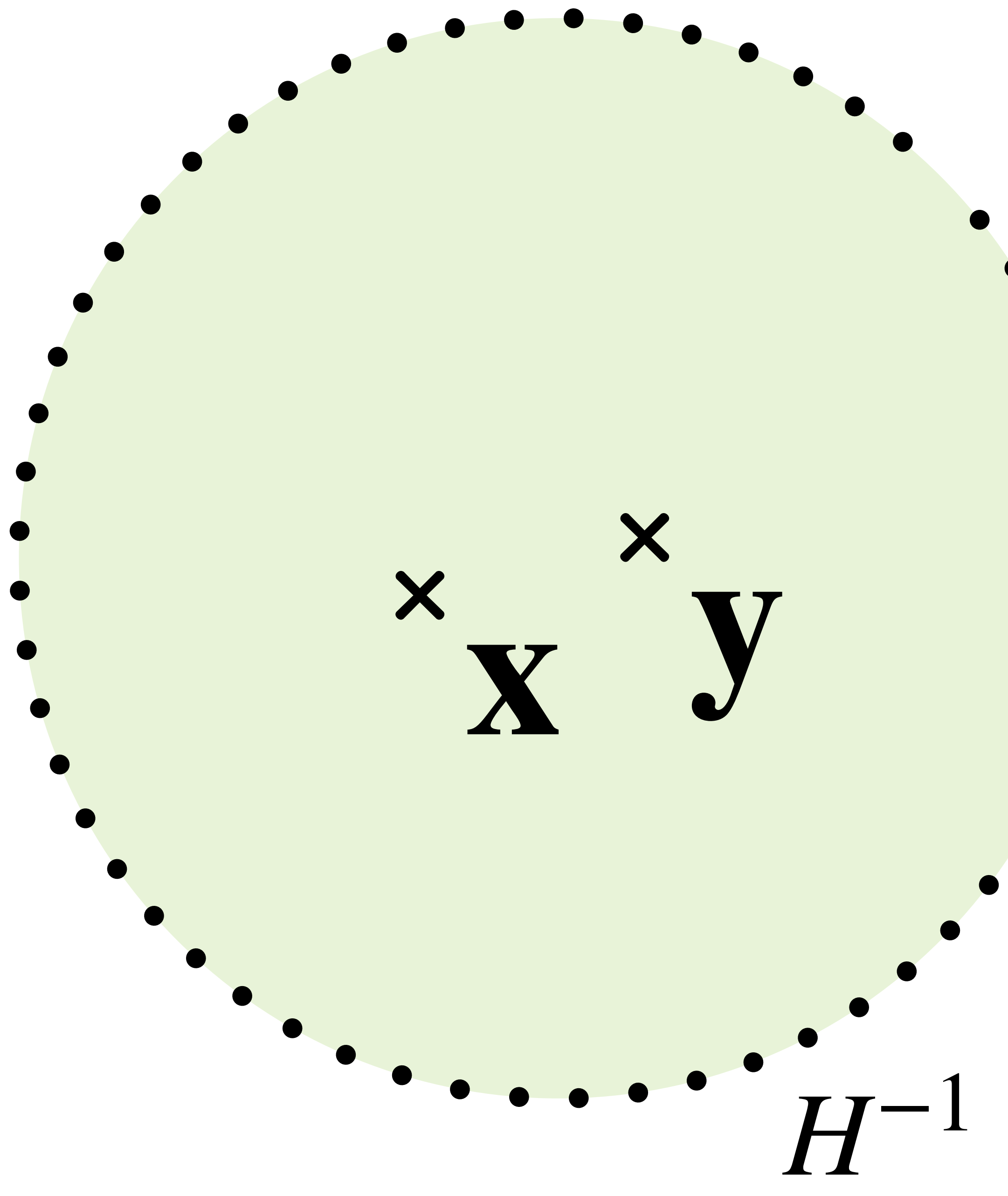

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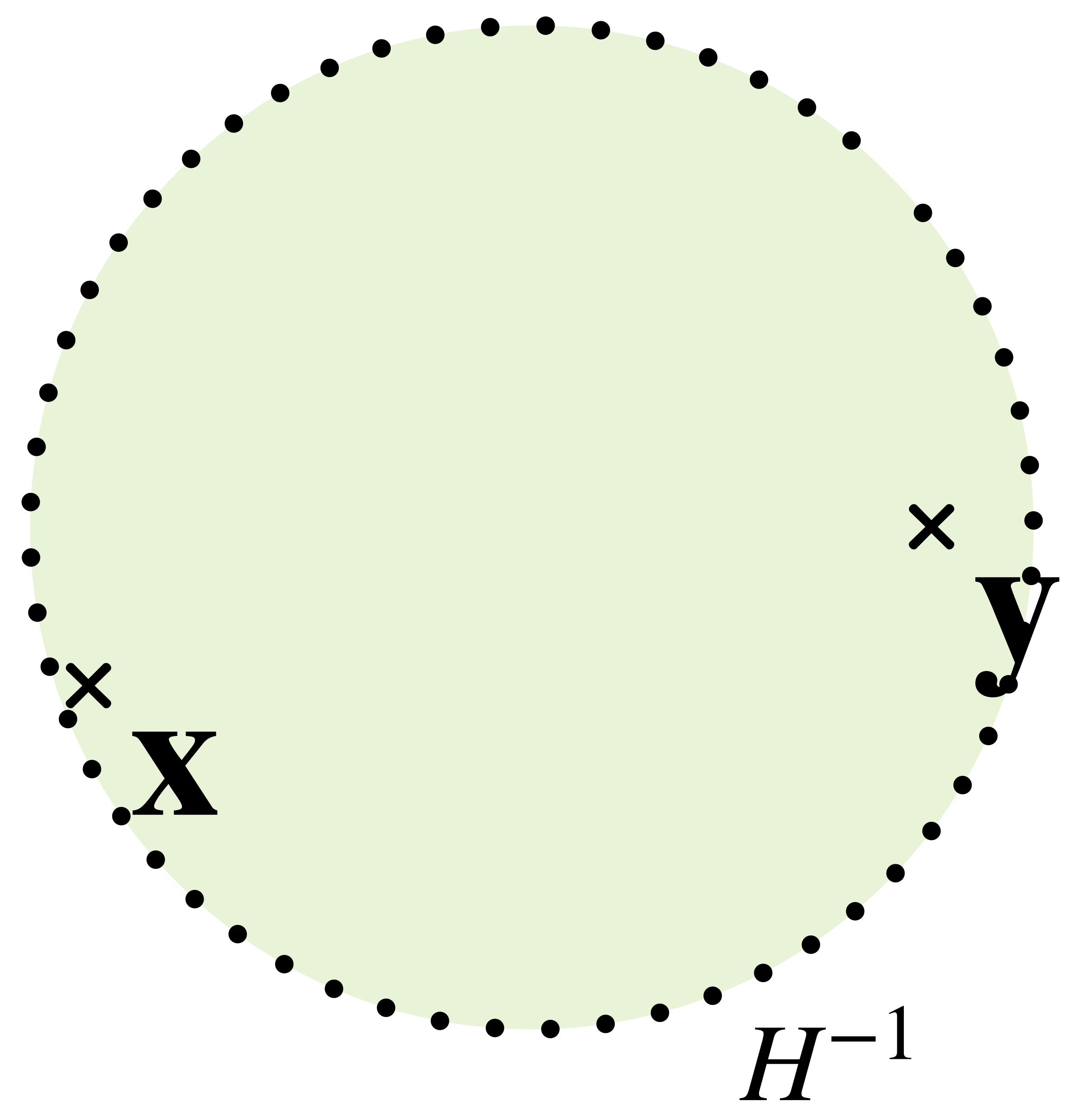

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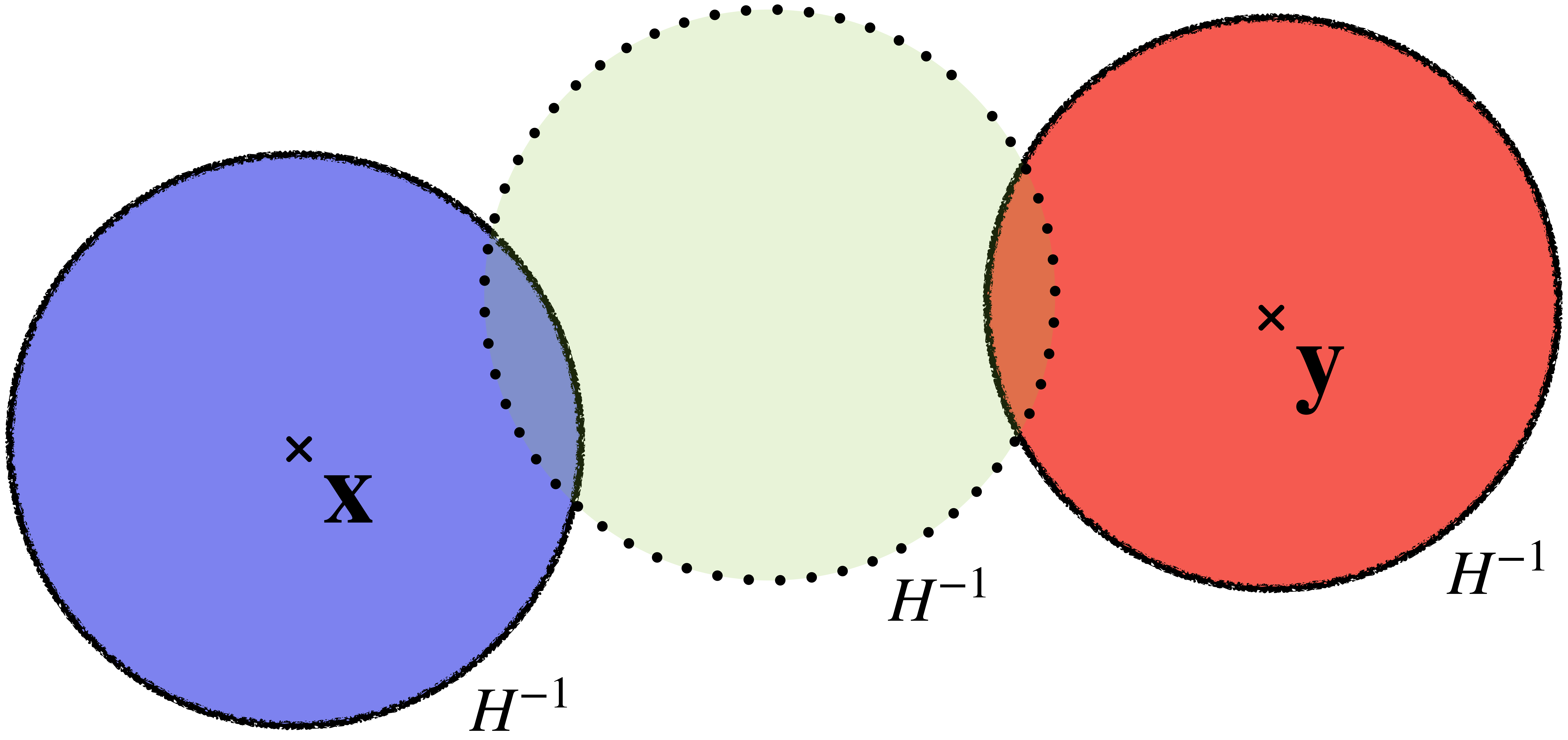


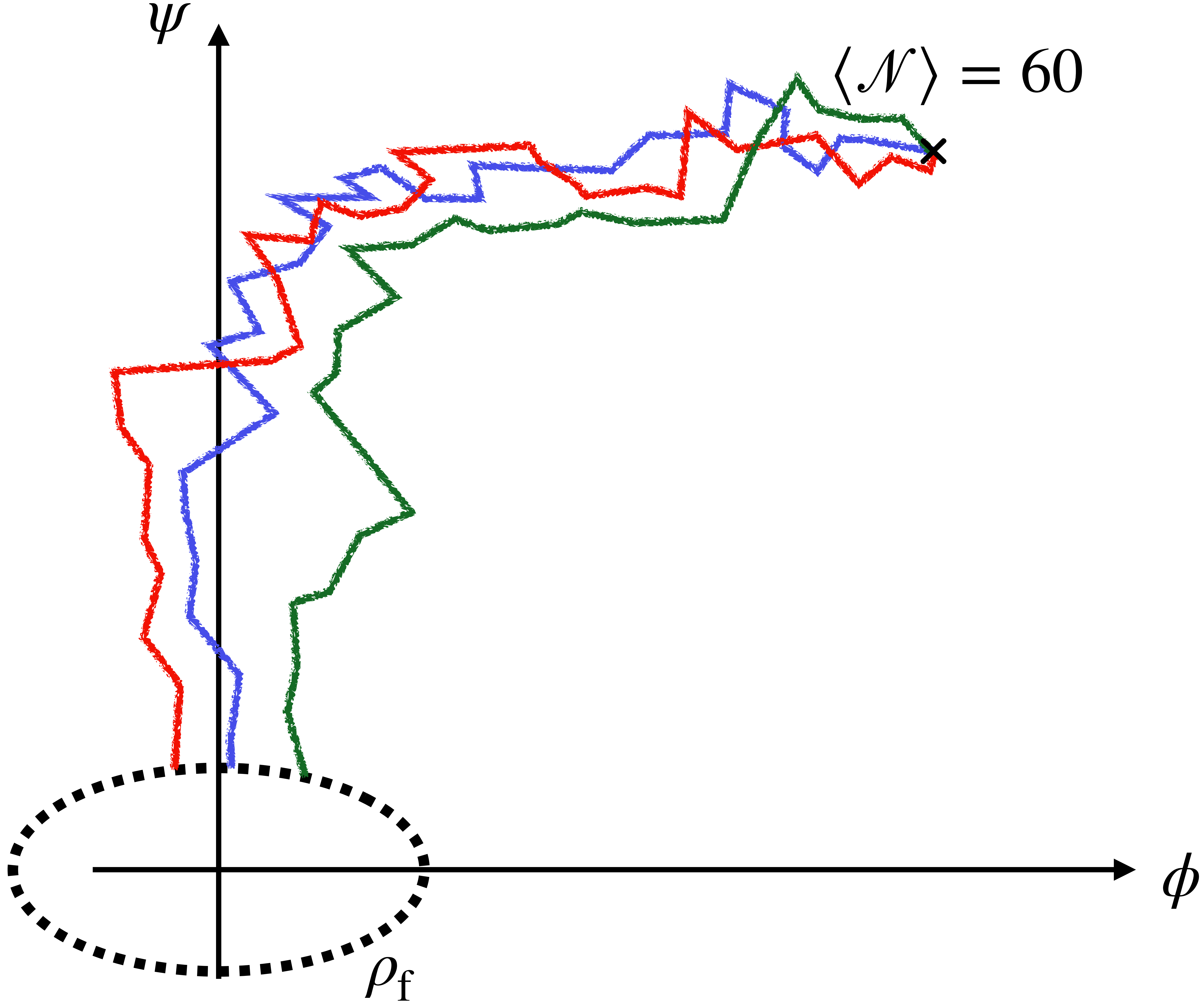
DISTANCE btw. x & y

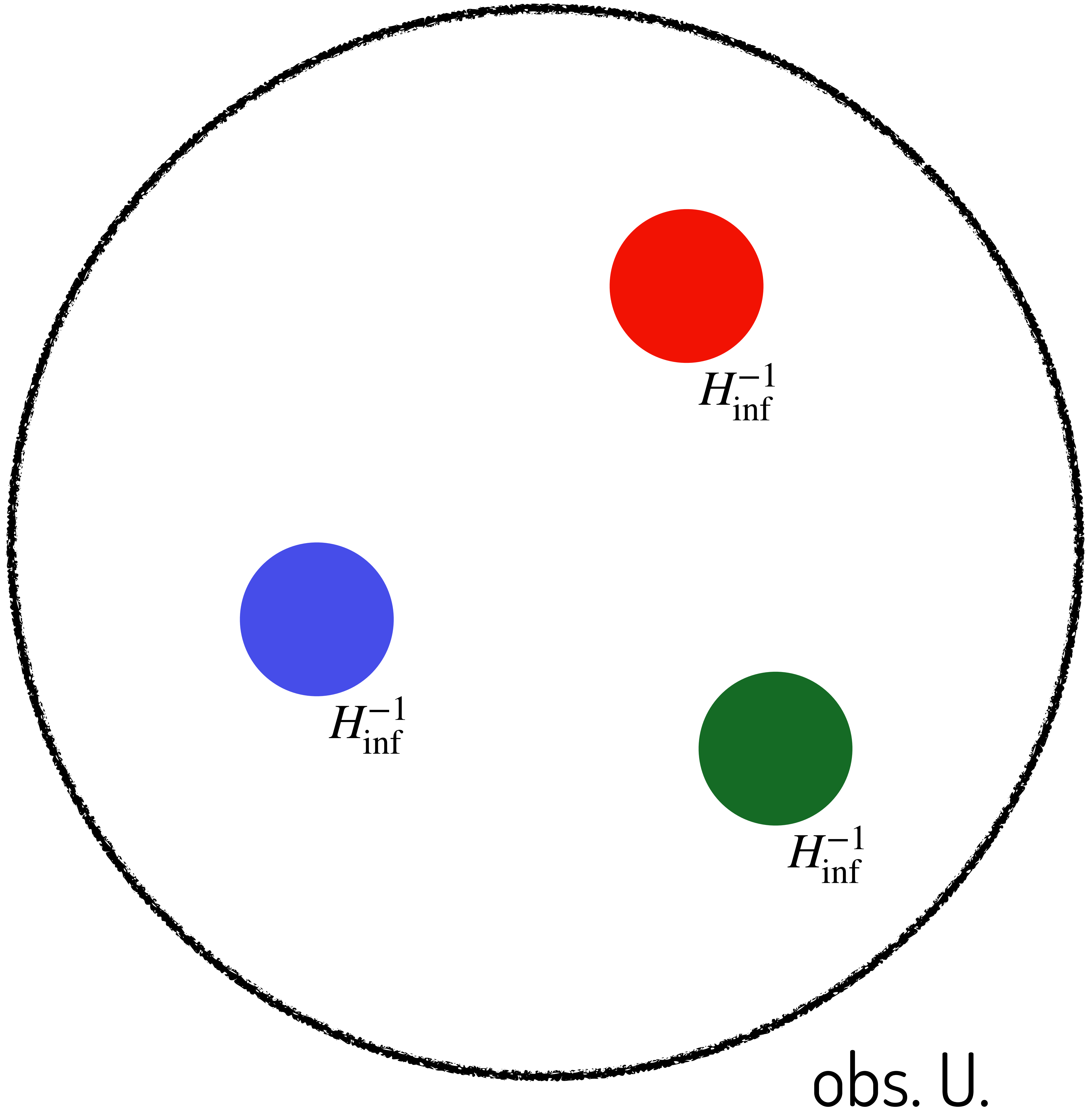
↔ independence TIME

c.f. Starobinsky & Yokoyama '94

Stochastic-δN

Fujita, Kawasaki, YT, Takesako '13
Vennin & Starobinsky '15





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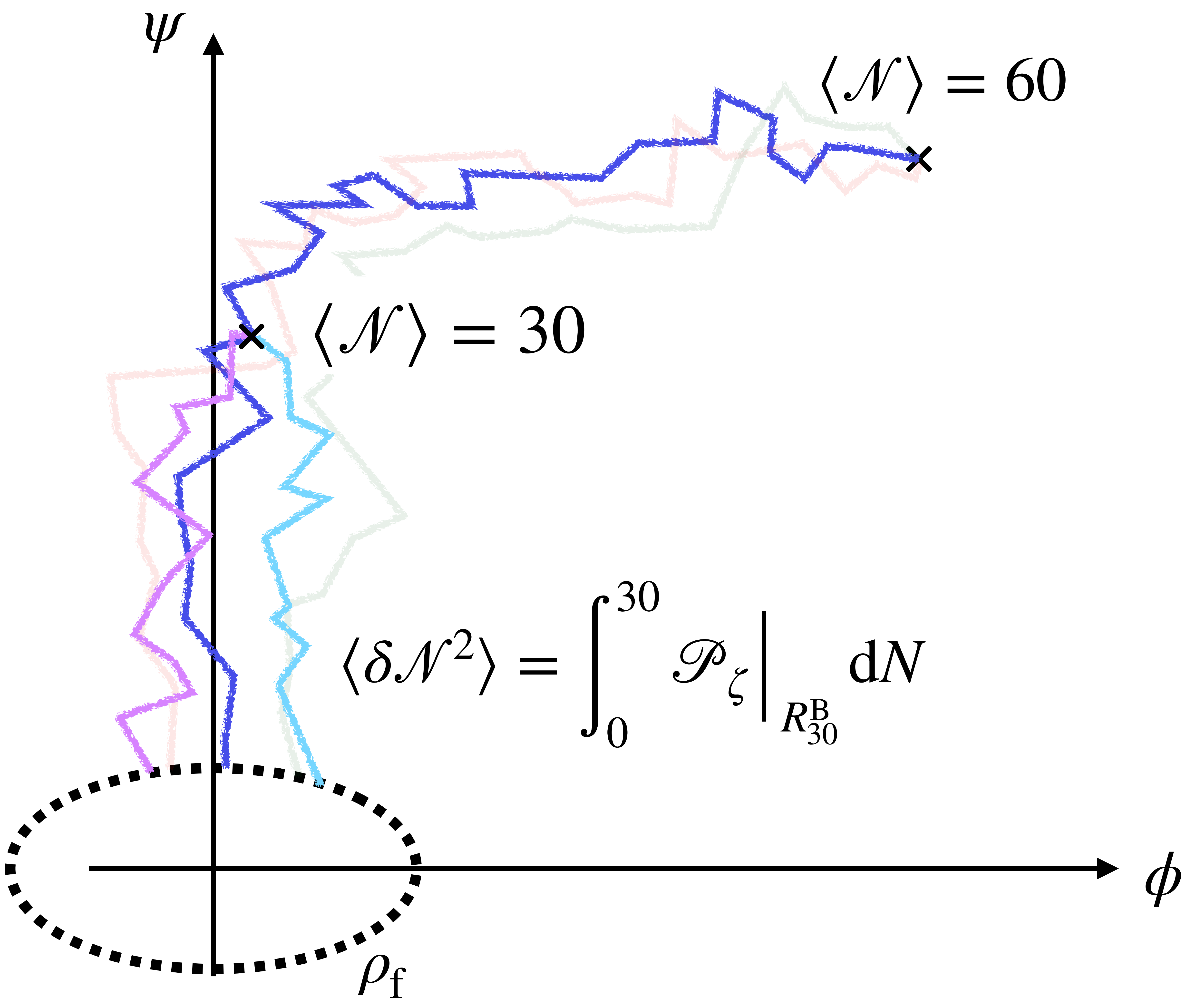

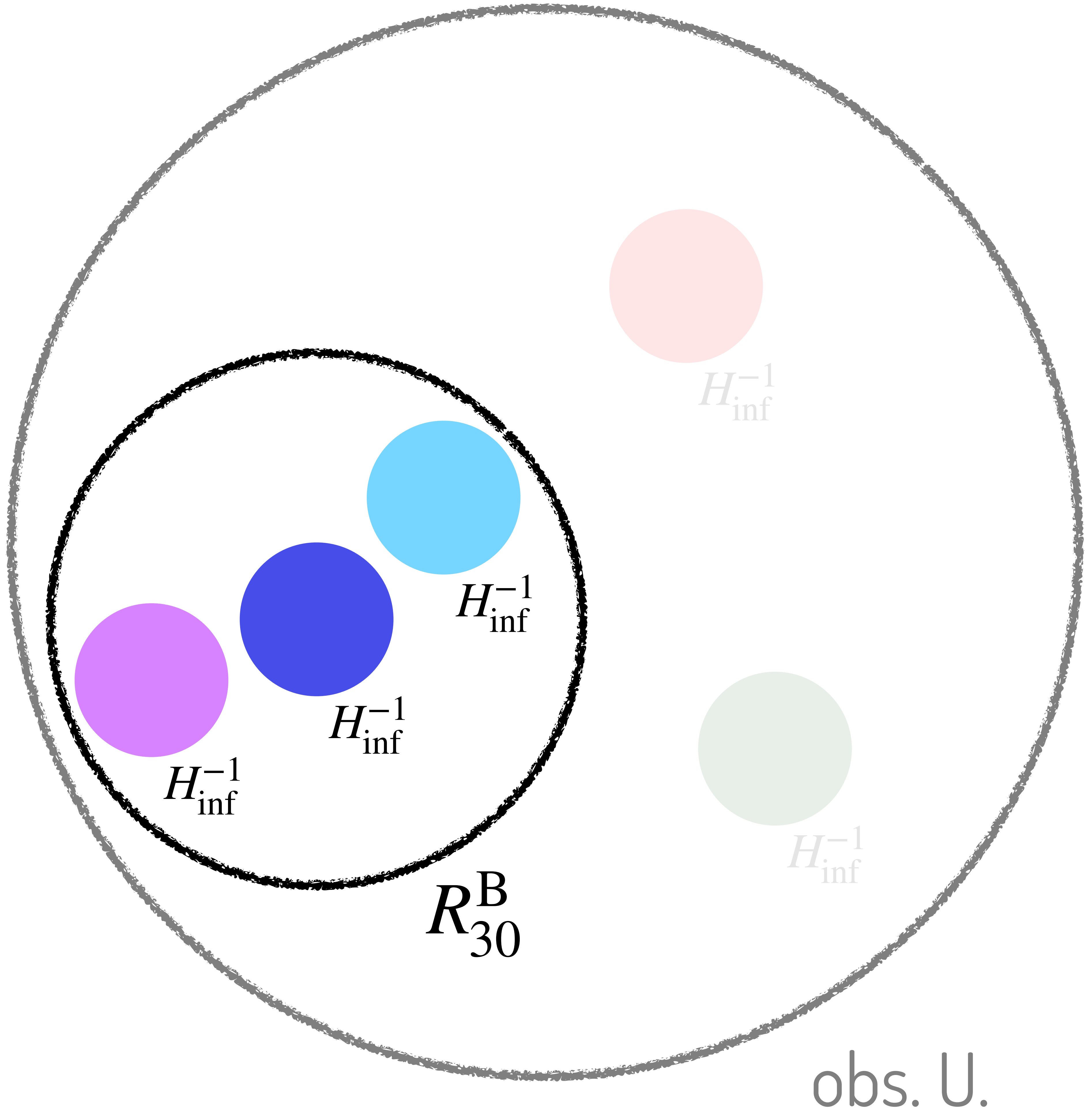

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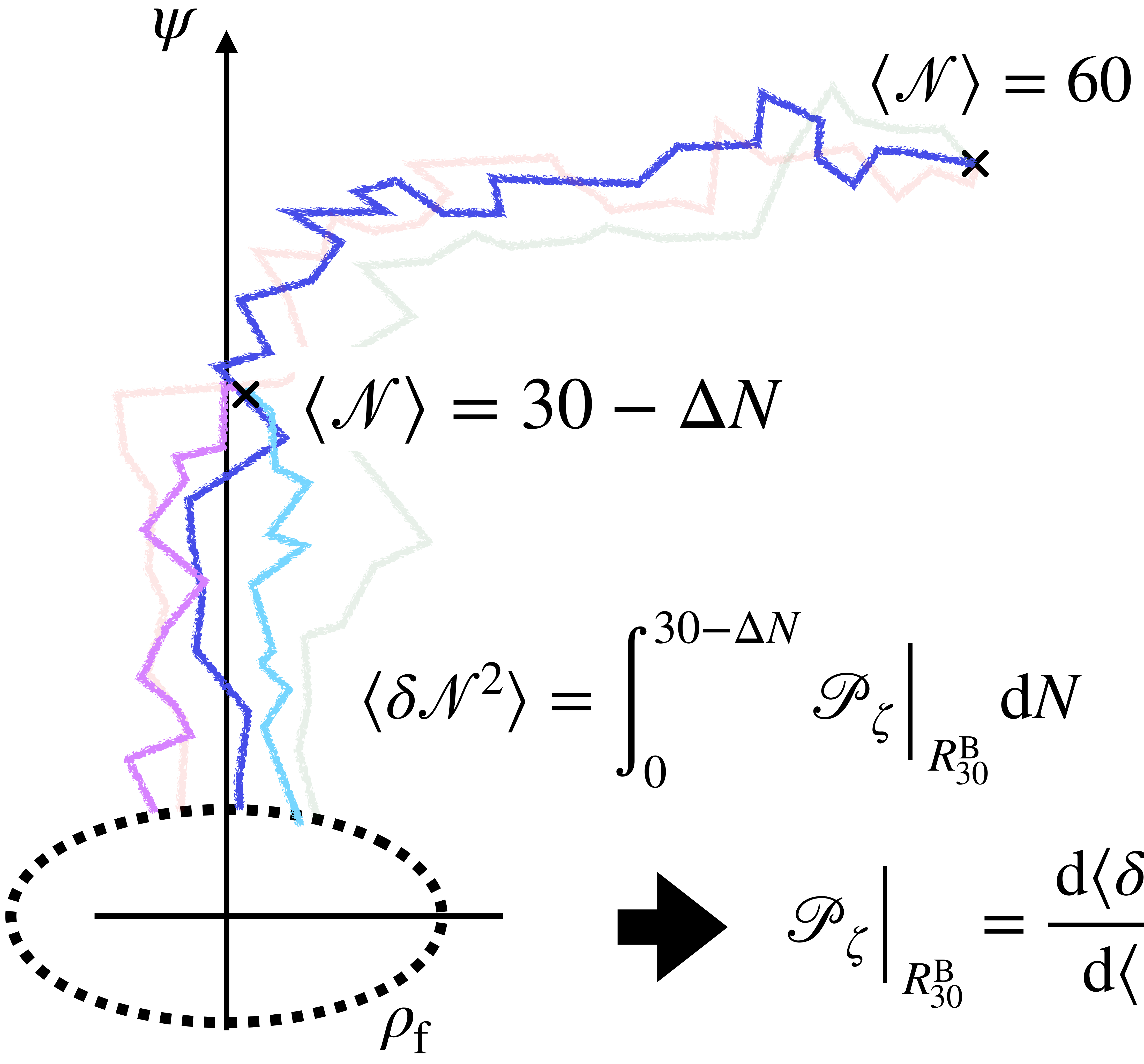
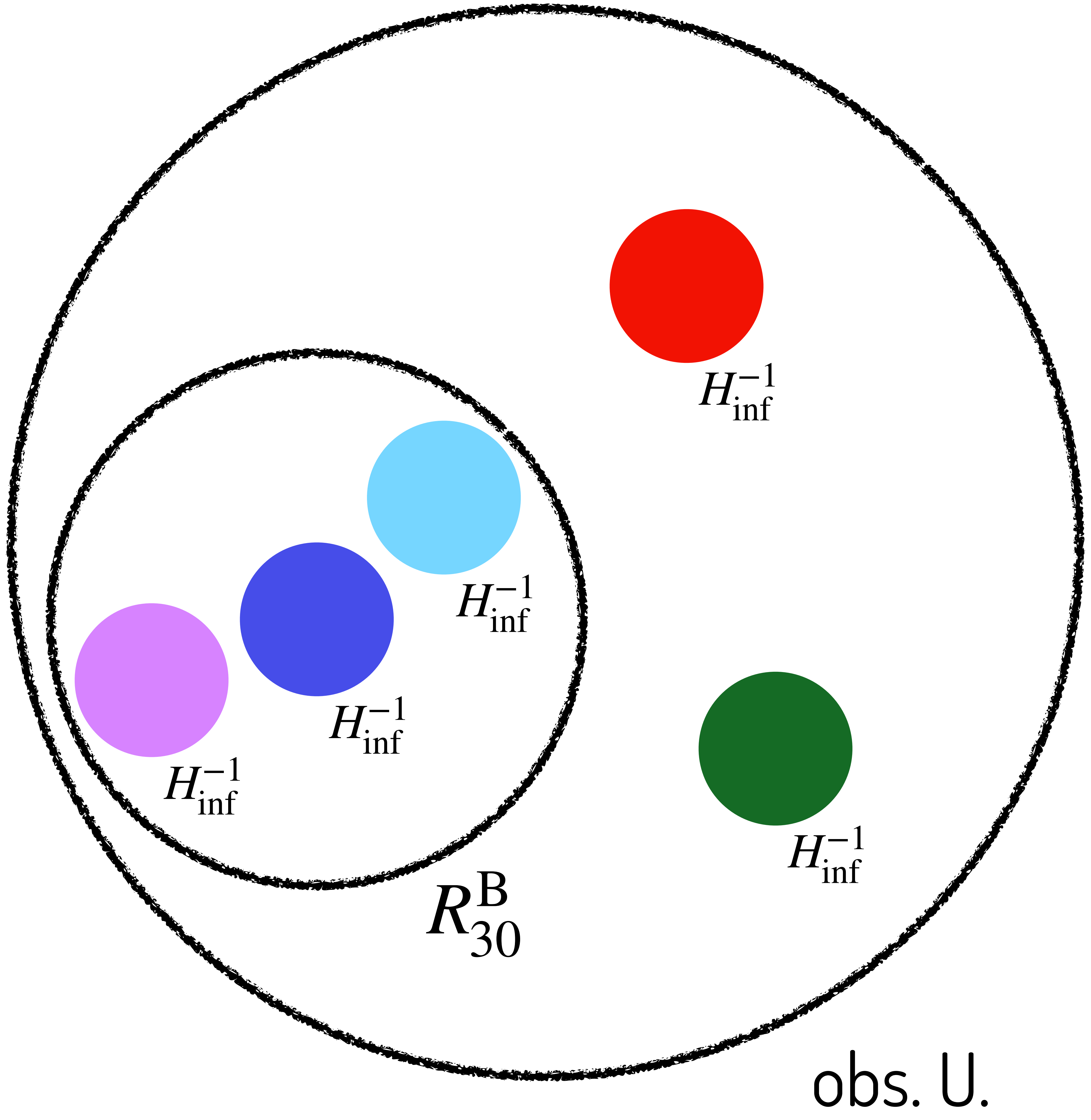


$$\langle \delta\mathcal{N}^2 \rangle = \int_0^{30-\Delta N} \mathcal{P}_\zeta \Big|_{R_{30}^\mathrm{B}} \mathrm{d}N$$



$$\Rightarrow \mathcal{P}_\zeta \Big|_{R_{30}^\mathrm{B}} = \frac{\mathrm{d}\langle \delta\mathcal{N}^2 \rangle}{\mathrm{d}\langle \mathcal{N} \rangle}\Bigg|_\mathrm{blue} \Rightarrow \mathcal{P}_\zeta \Big|_\text{obs. U.} = \underset{i=\mathrm{RGB}}{E}\left[\mathcal{P}_\zeta \Big|_{R_{30}^i} \right]$$

Stochastic-δN

Fujita, Kawasaki, YT, Takesako '13
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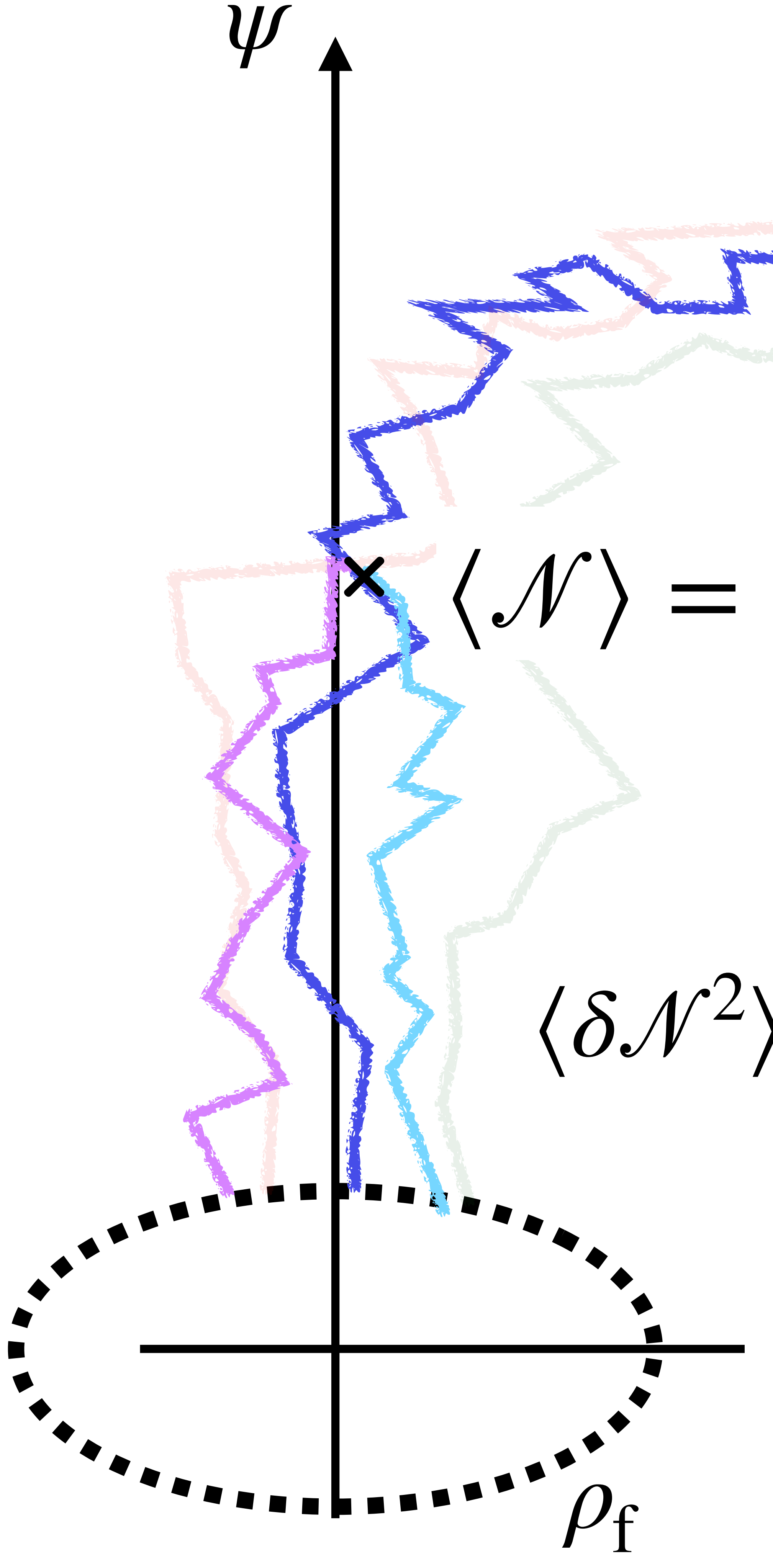

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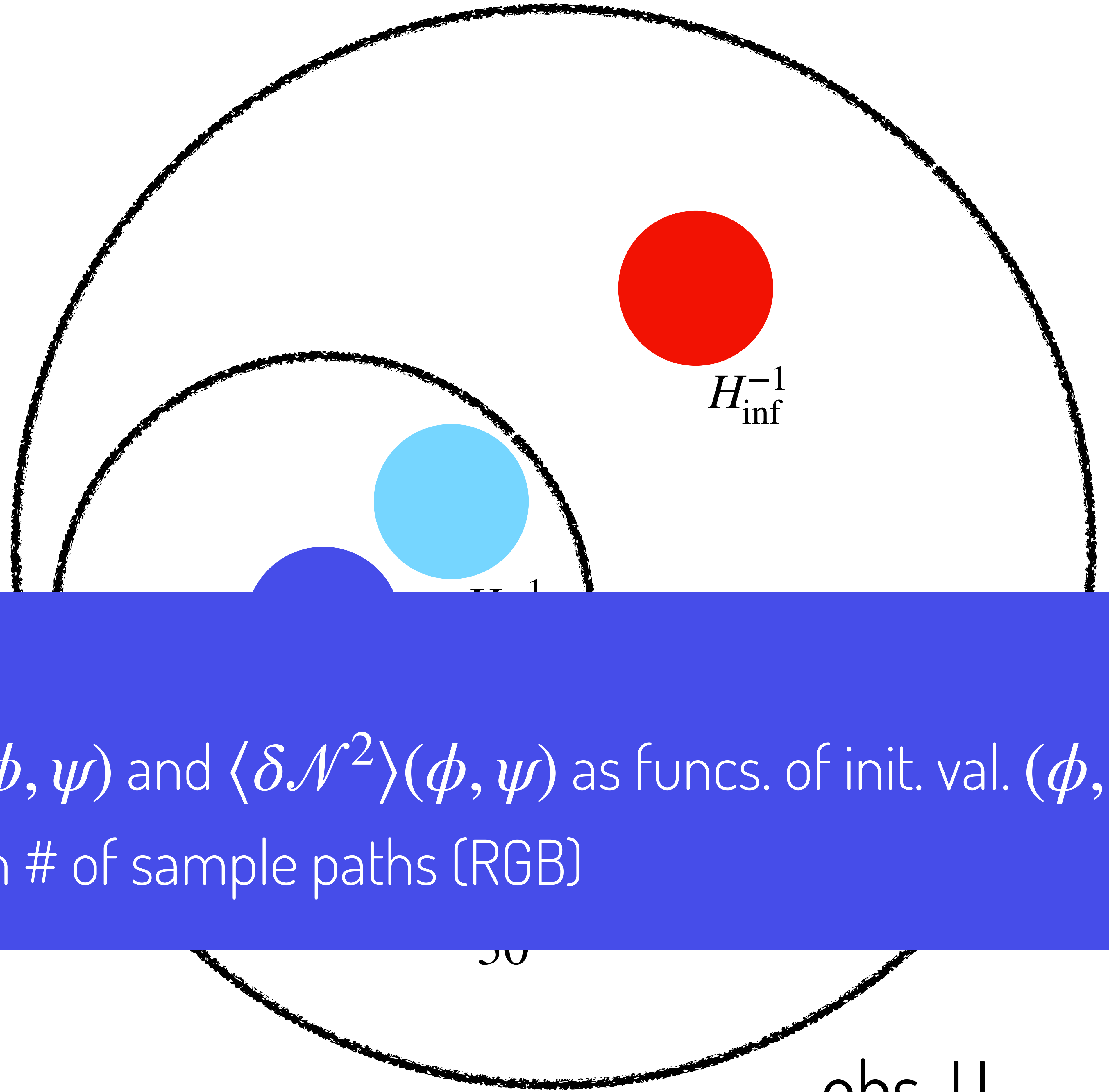


Requisites:

- $\langle \mathcal{N} \rangle(\phi,\psi)$ and $\langle \delta\mathcal{N}^2 \rangle(\phi,\psi)$ as funcs. of init. val. (ϕ,ψ)
- enough # of sample paths (RGB)

$$\Rightarrow \quad \mathcal{P}_\zeta \Big|_{R_{30}^{\mathrm{B}}} = \frac{\mathrm{d}\langle \delta\mathcal{N}^2 \rangle}{\mathrm{d}\langle \mathcal{N} \rangle}\Bigg|_{\mathrm{blue}} \quad \Rightarrow \quad \mathcal{P}_\zeta \Big|_{\text{obs. U.}} = \mathop{E}_{i=\mathrm{RGB}} \left[\mathcal{P}_\zeta \Big|_{R_{30}^{i}} \right]$$

STOCDELTAN: C++ PACKAGE

PDE Approach

Vennin & Starobinsky '15

❖ Fokker-Planck eq. (diffusion)

$$\partial_N P(\phi^I; N) = -\partial_I \left[h^I P(\phi^I; N)\right] + \frac{1}{2}\partial_I \partial_J \left[A^{IJ} P(\phi^I; N)\right]$$

PDF of ϕ^I @ N

e.g. $h^I = -\frac{V^I}{3H^2}, \quad A^{IJ} = \left(\frac{H}{2\pi}\right)^2 \delta^{IJ}$

⇕

❖ adjoint FP eq.

$$\partial_{\mathcal{N}} \bar{P}(\mathcal{N}; \phi^I) = h^I \partial_I \bar{P}(\mathcal{N}; \phi^I) + \frac{1}{2} A^{IJ} \partial_I \partial_J \bar{P}(\mathcal{N}; \phi^I)$$

PDF of 1st. passage time $\mathcal{N}$ from ϕ^I

StocDeltaN

Renaux-Petel, YT, Vennin in prep.

- C++ package applicable to a general Lagrangian $\mathscr{L} = -\frac{1}{2}g^{\mu\nu}G_{IJ}(\phi)\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi)$

  +

  Python automatic plotting support

- slow-roll field-space (ϕ^I) $\leftrightarrow$ full phase-space (ϕ^I, π_J)

- automatic parallelization w/ OpenMP

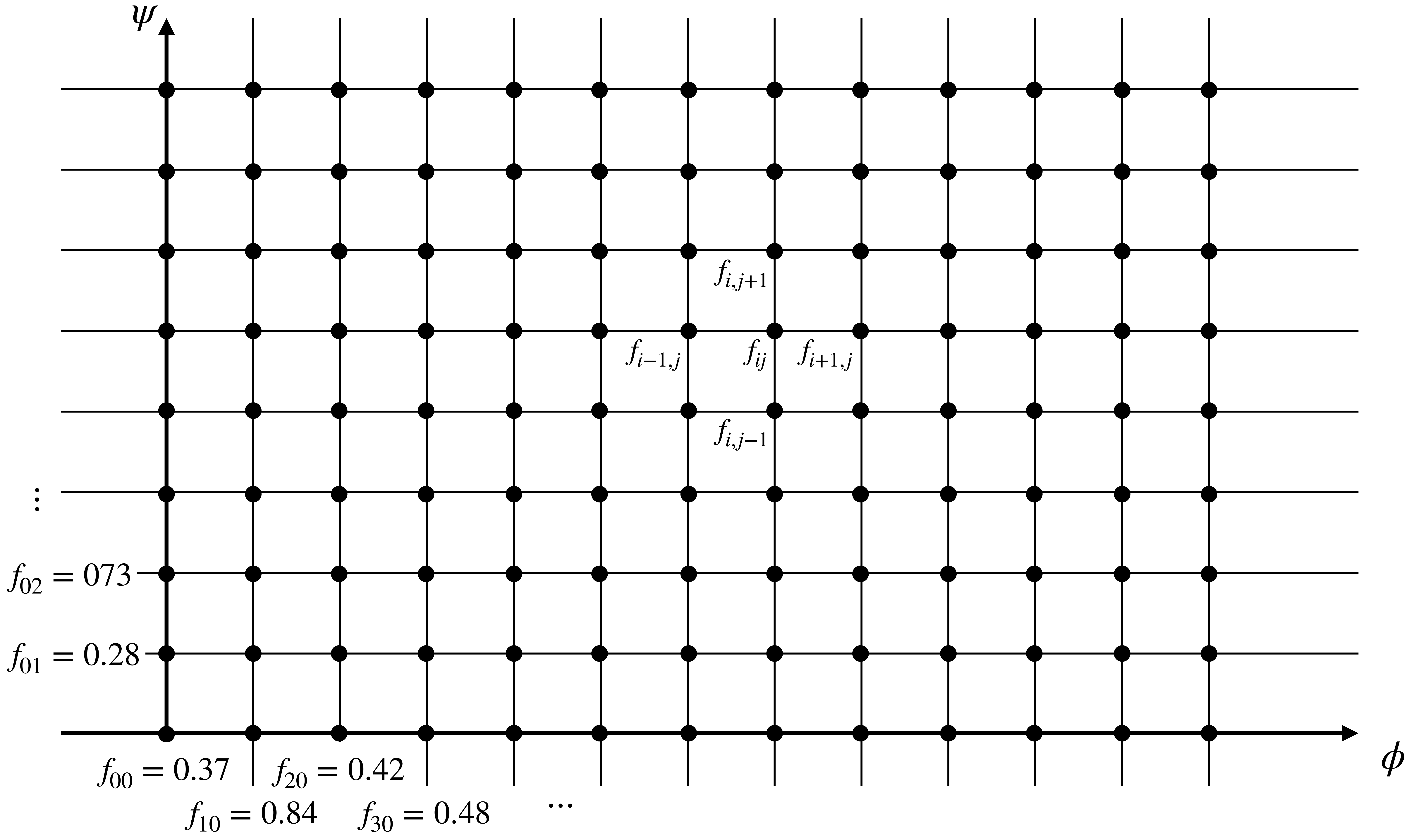
Solve 2 PDE in Jacobi method

$$\begin{cases} \mathscr{L}_{\mathrm{FP}}^\dagger \cdot \langle \mathcal{N} \rangle(\phi^I) = -1, \\ \mathscr{L}_{\mathrm{FP}}^\dagger \cdot \langle \delta\mathcal{N}^2 \rangle(\phi^I) = -D^{IJ}(\partial_I\langle \mathcal{N} \rangle)(\partial_J\langle \mathcal{N} \rangle) \end{cases}$$

+

Generate many sample paths in stochastic Runge-Kutta (3,2)

Jacobi for PDE



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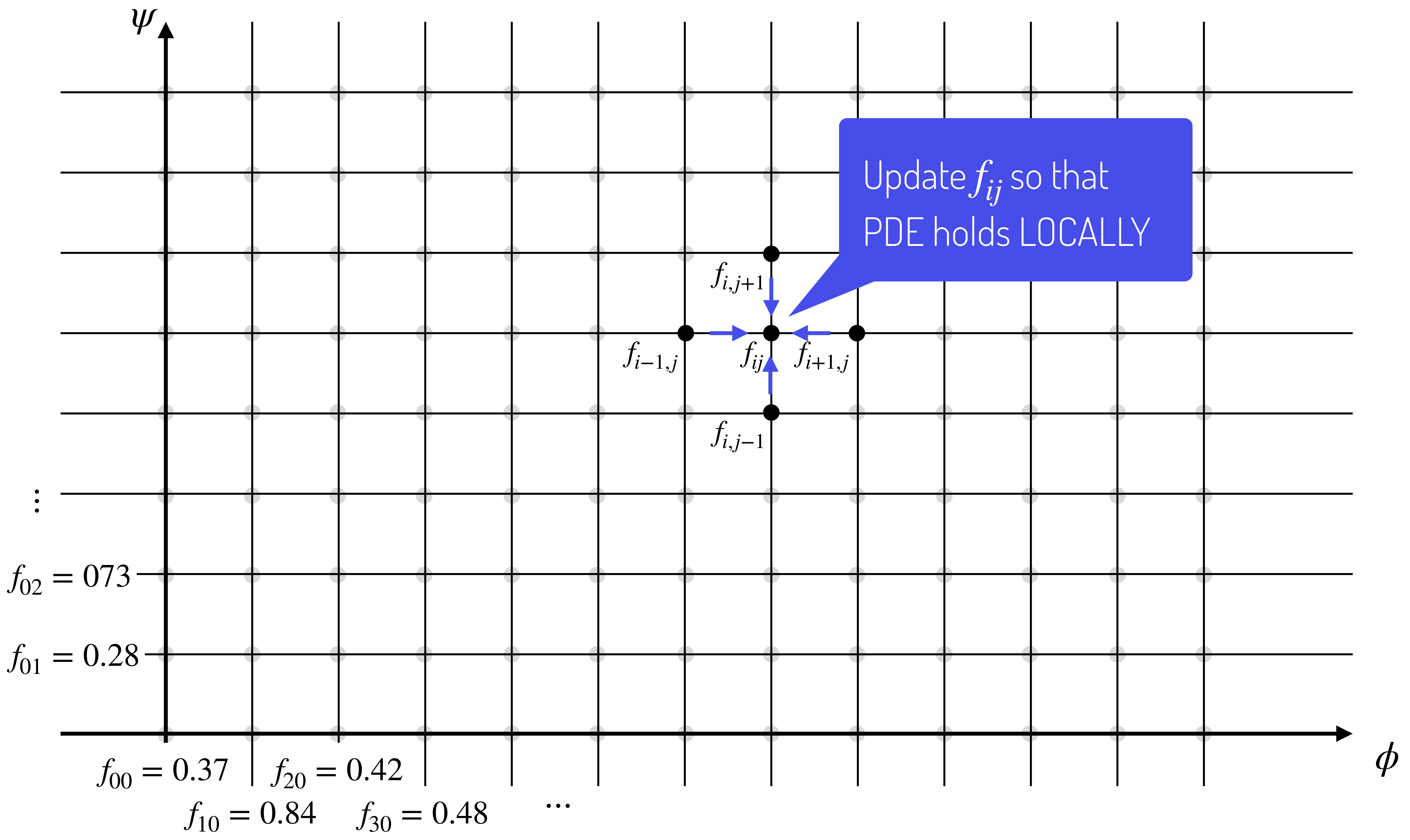

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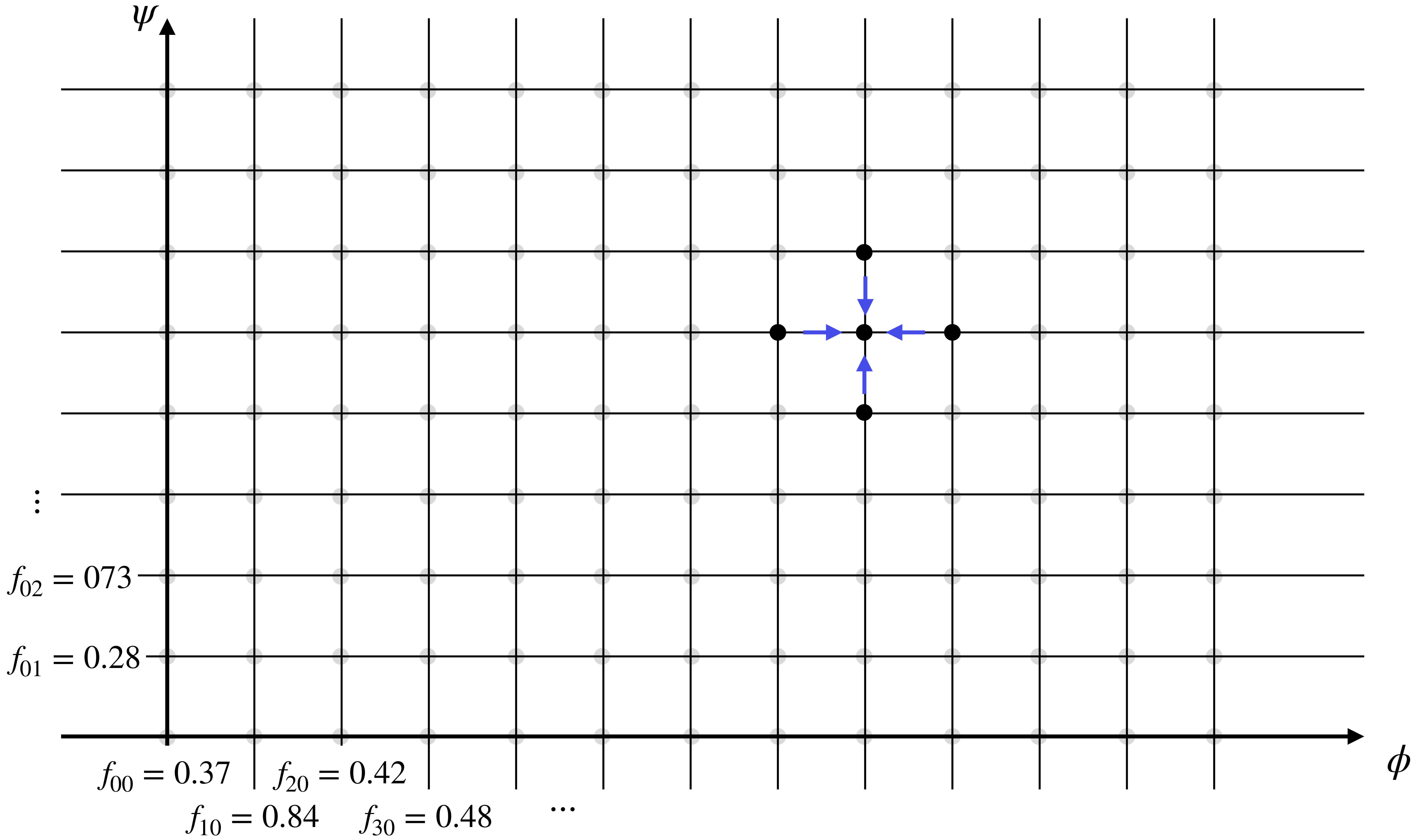

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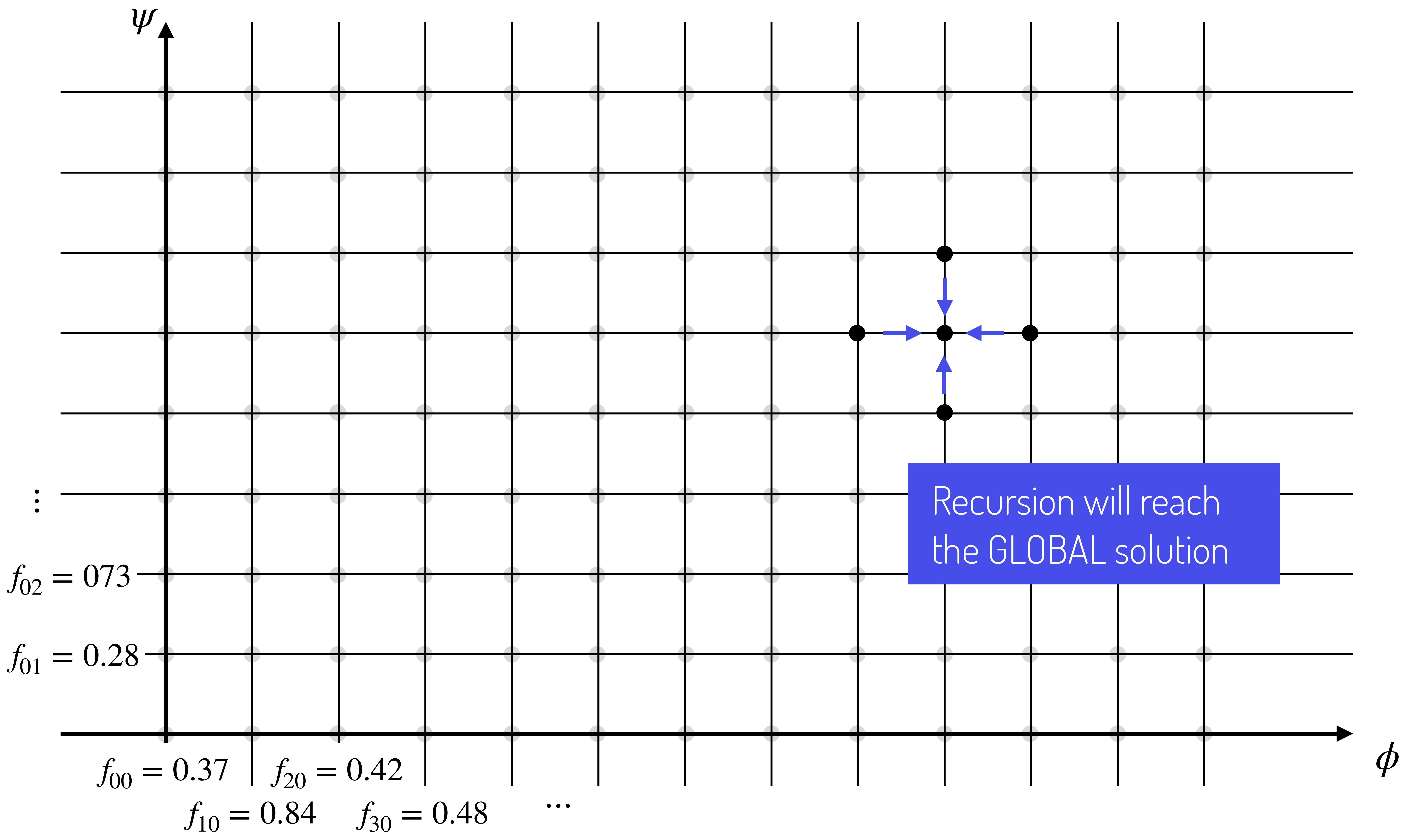

StocDeltaN

Renaux-Petel, YT, Vennin in prep.

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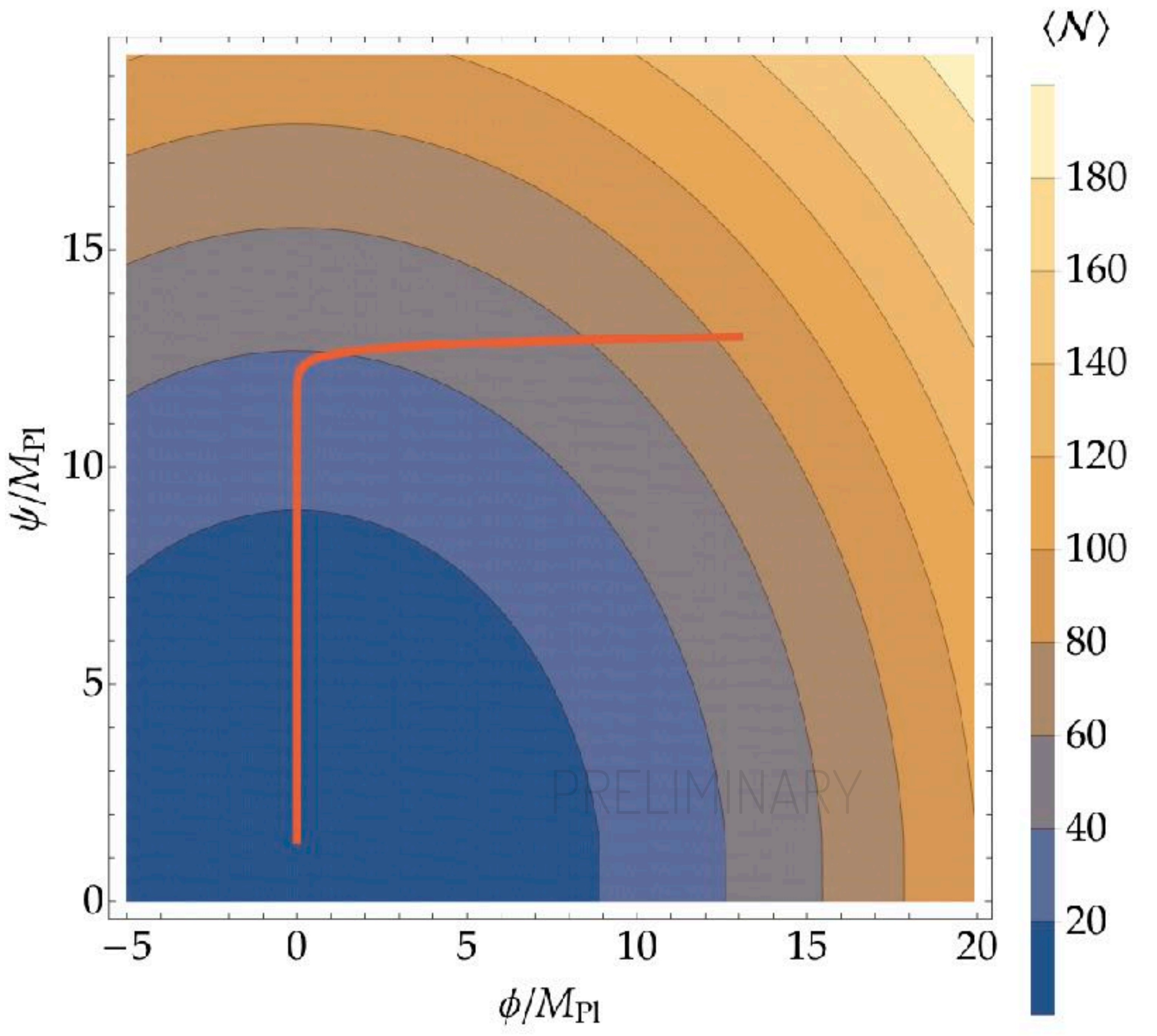

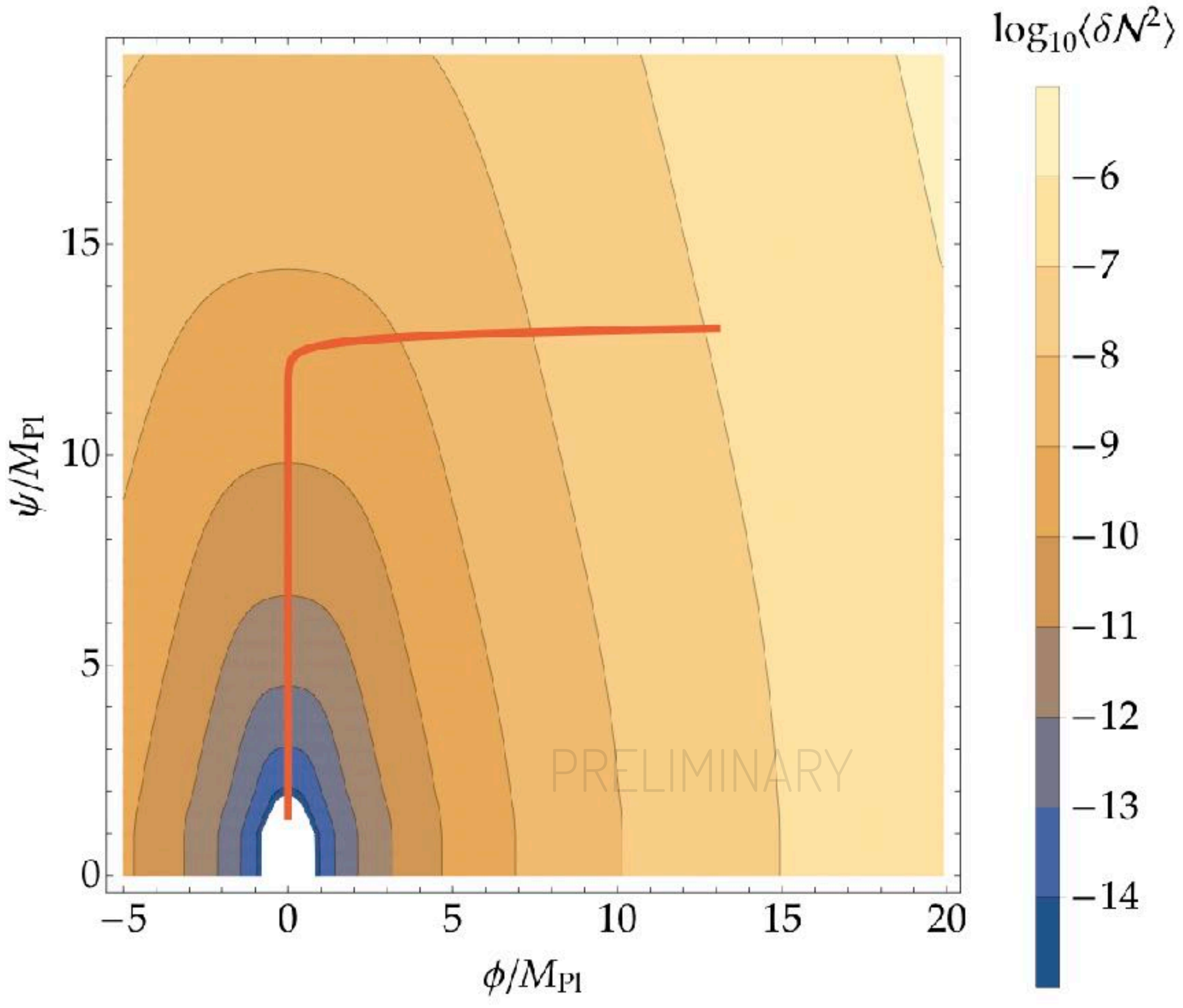

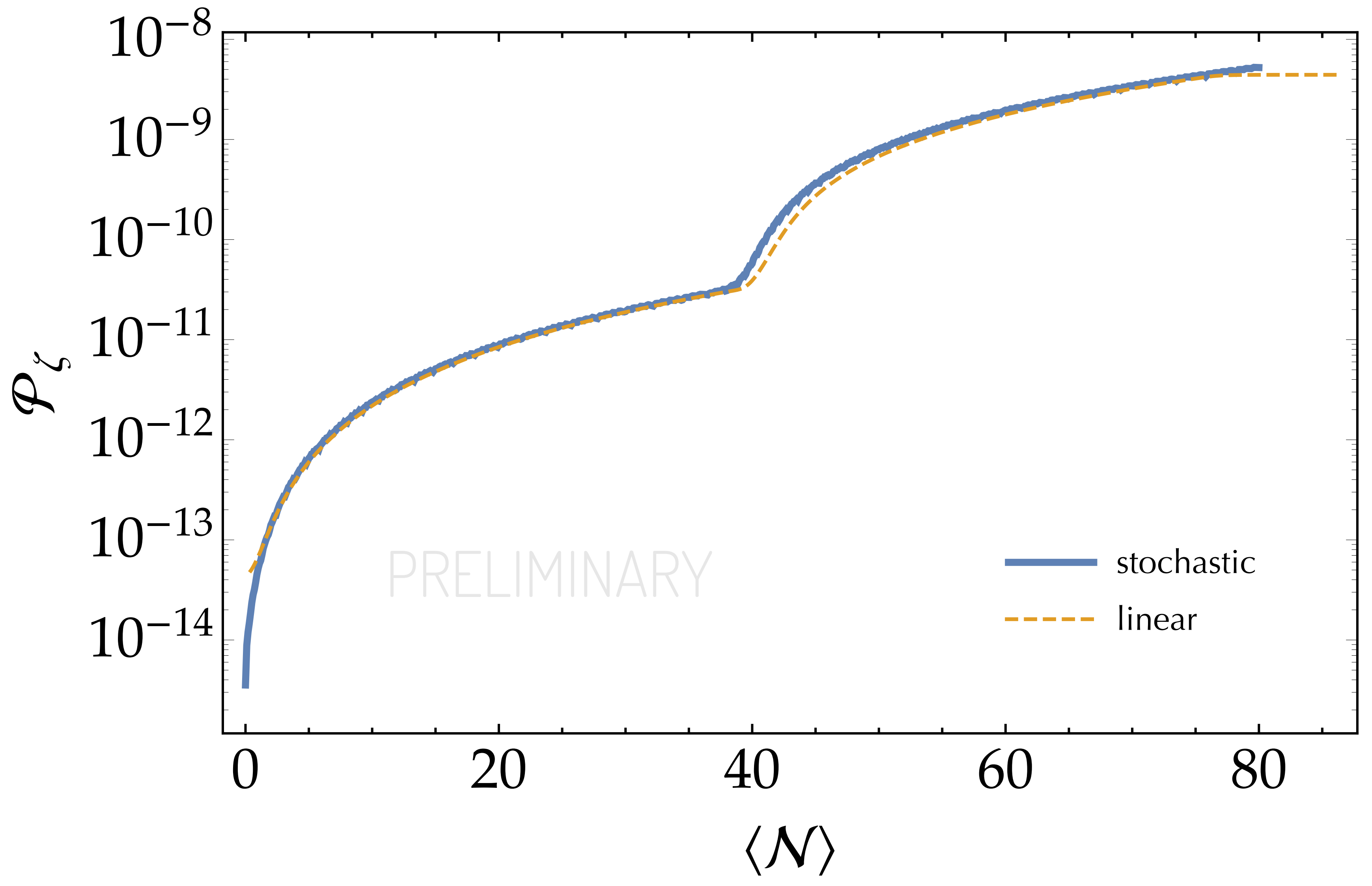

STOCDELTAN

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❖ Hybrid Inflation

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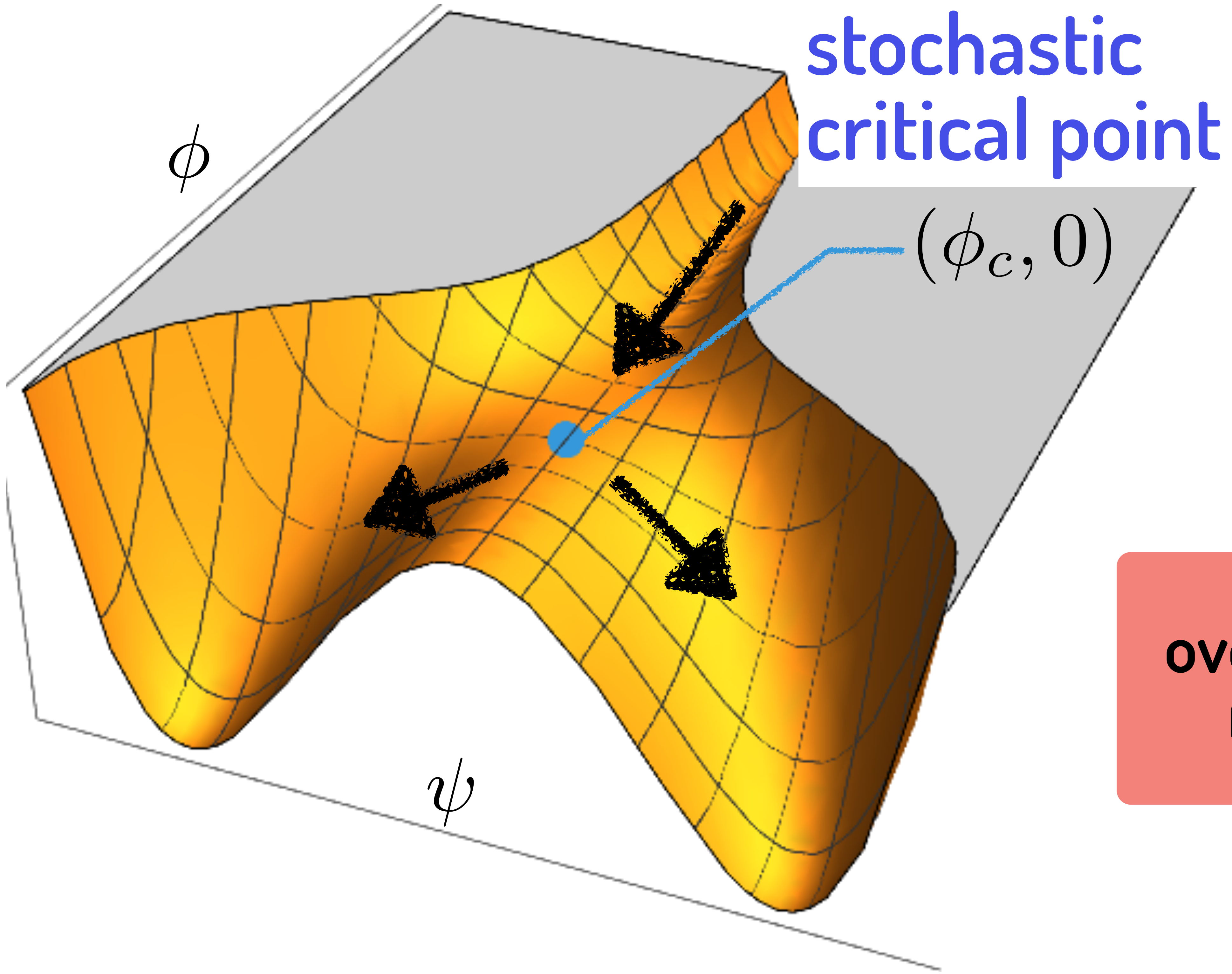

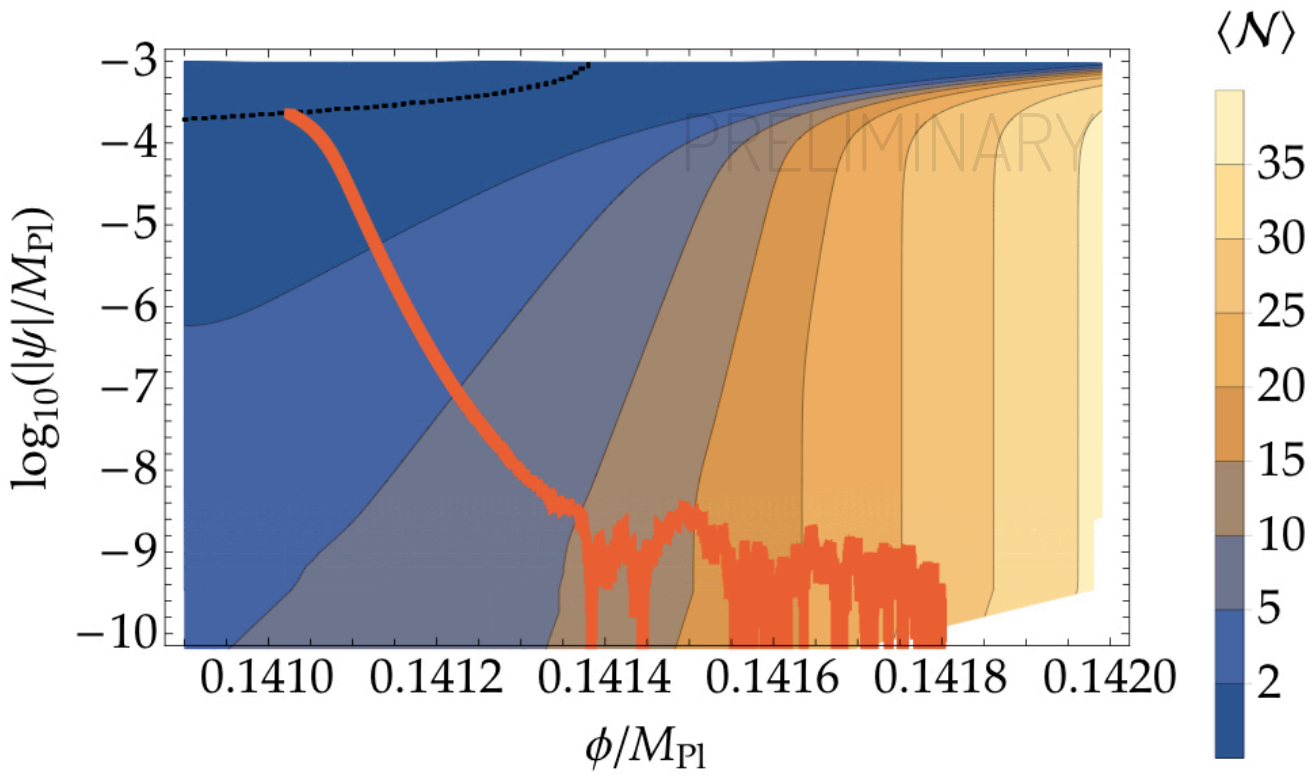

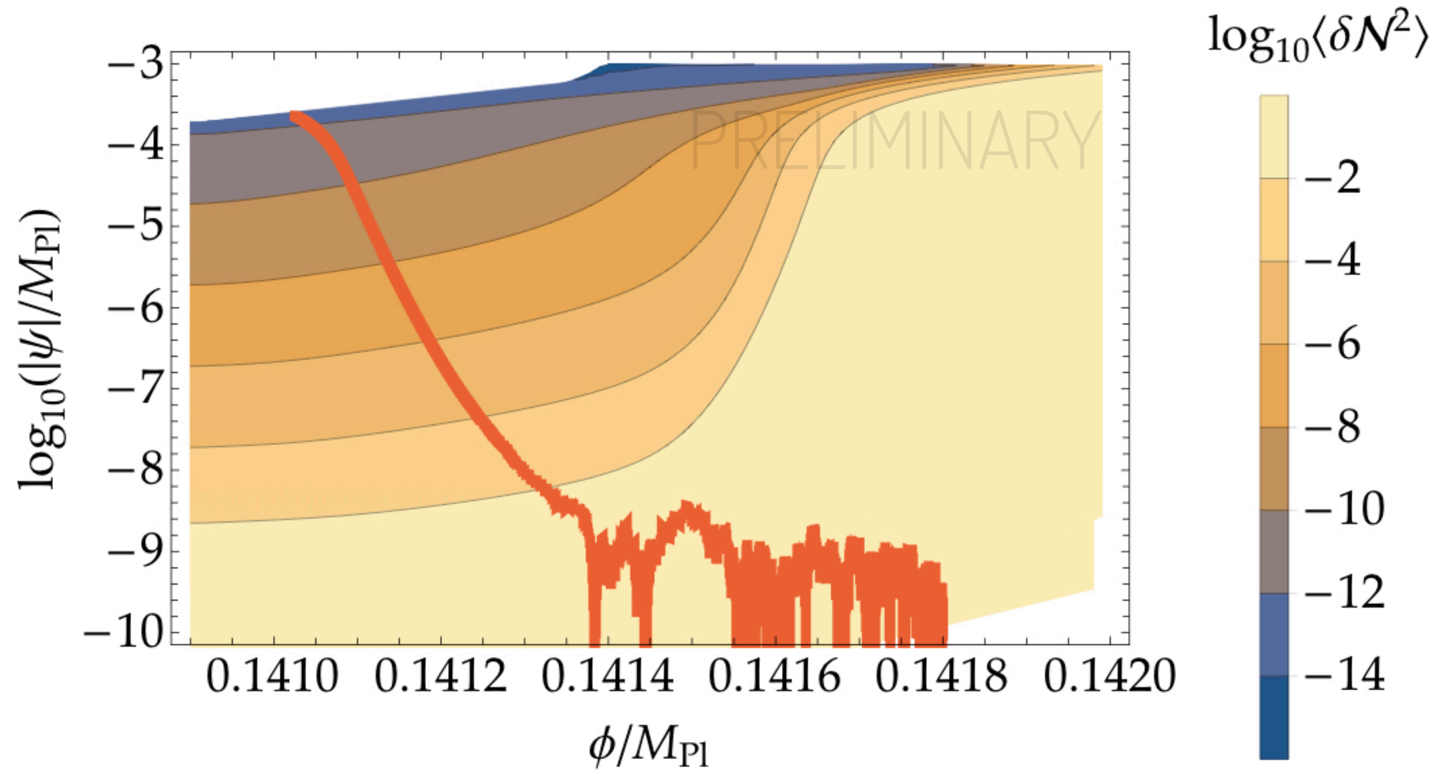

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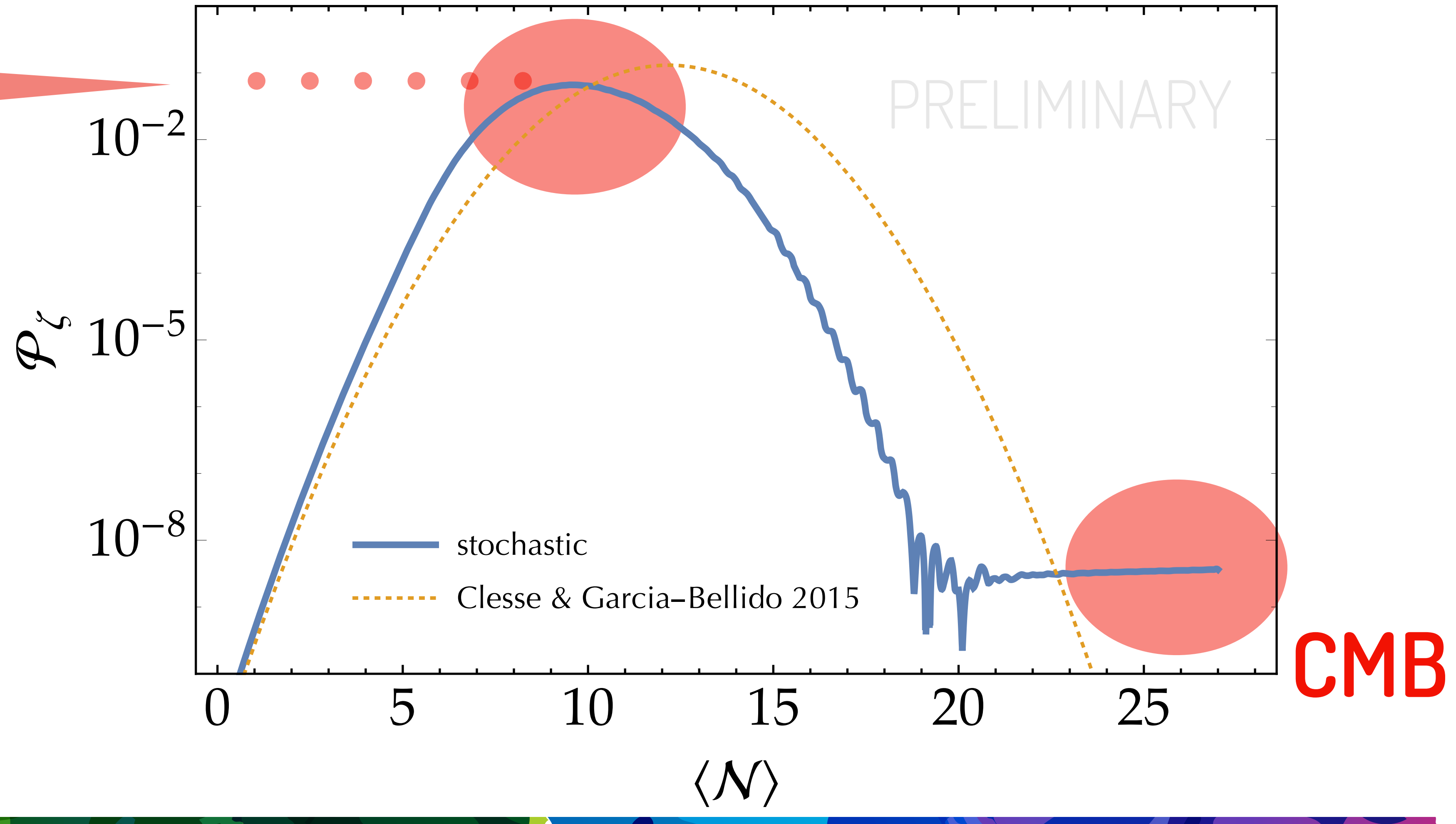

Conclusions

- StocDeltaN: C++ package for stochastic-δN analysis
- available from my GitHub page:

  https://github.com/NekomammaT/StocDeltaN_dist