Stochastic Non-Attractor Inflation

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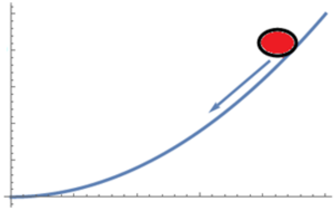
Overview

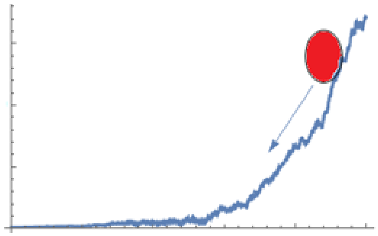
* A review on Ultra slow roll inflation
* Stochastic Ultra slow roll inflation
* Generalization
* Boundary Crossing Probabilities
* Large Diffusion Limit
* Summary

A Brief Review on Stochastic Inflation

* When there is no noise in evolution of the system the field evolves classically:



However the presence of noises which originate from deep inside horizon, change the evolution of field:



Ultra slow roll inflation (USR)

In general the Klein-Gordon equation for inflaton is:

$$\left(\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{\nabla^2}{a^2}\right)\phi\left(\mathbf{x},t\right) + \frac{\partial V}{\partial \phi}(\mathbf{x},t) = 0$$

By putting the potential equal to a constant at the background level we have:

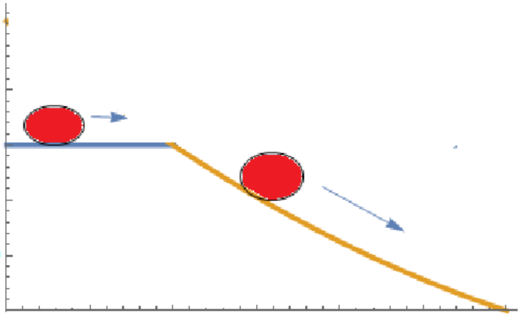
$$\ddot{\phi} + 3H\dot{\phi} = 0 \qquad \epsilon = -\frac{\dot{H}}{H^2} \propto a^{-6}$$

$$H \simeq const$$

USR Consequences

The number of efolds to the end of inflation: (Namjoo, Firouzjahi & Sasaki 2012**):**

$$N(\phi, \dot{\phi}) = \frac{1}{3} \ln \left[\frac{\dot{\phi}}{\dot{\phi} + 3H(\phi - \phi_e)} \right]$$



In this model the power spectrum is given by:

$$\frac{k^3}{2\pi} \langle \mathcal{R}_k \mathcal{R}_{-k} \rangle = \mathcal{P}_e \simeq \frac{H^2}{8\pi^2 M_P^2 \epsilon_e}$$

Also the non Gaussianity is given by

$$f_{NL} \simeq \frac{5}{2}$$

Stochastic ultra slow roll inflation

USR Langevin equations

In ultra slow roll inflation we can not neglect the second derivative of our test field. We should solve these set of equations together:

$$\begin{aligned}\frac{d\phi}{dN} &= \frac{v}{H} + \frac{H}{2\pi}\xi\left(N\right)\\ \frac{dv}{dN} &= -3v\end{aligned}$$

Where ξ(N) is the white noise.

A code for mathematica to generate white noise

```
W=ConstantArray[0,{10^4}];
For[i=1,i<=10000,i++,
   W[[i]]  =RandomVariate[NormalDistribution[]]
]
white=Manipulate[ListPlot[{W[[1;;i]]}],{i,1,Length[W],1,Appearance->"Open"},ContentSize->550]
```

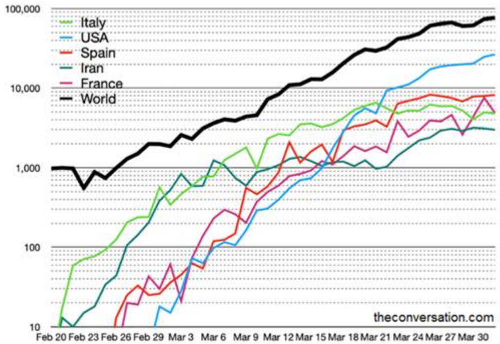
Solving the Langevin Eqautions

Solving the Langevin equations for the field and its conjugate momentum yields:

$$\phi\left(N\right) = \phi_0 + \frac{\dot{\phi}_0}{3H}\left(1 - e^{-3N}\right) + \frac{H}{2\pi} W\left(N\right)$$

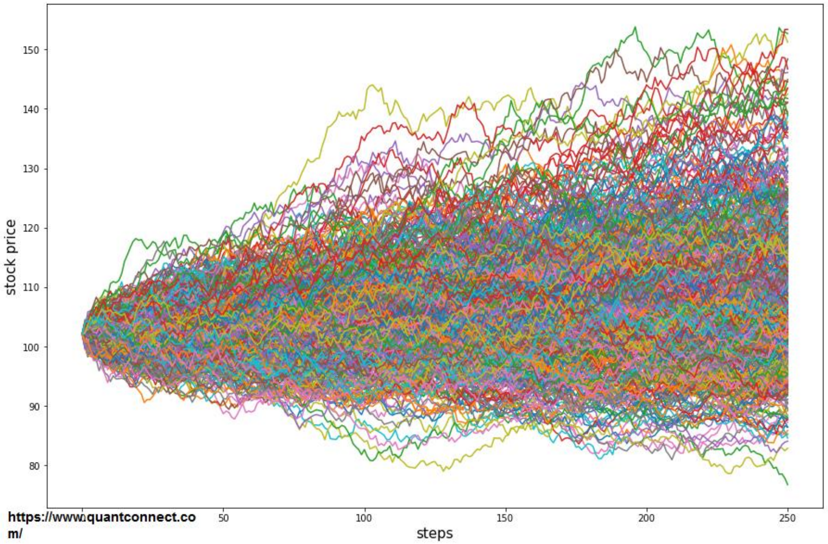
And W(N) is the Wiener process:

$$W\left(N\right) \equiv \int_0^N \xi\left(N\right) dN$$



Moments of W

$$\langle W(\mathcal{N})^2 \rangle = \langle \mathcal{N} \rangle$$



https://www.quantconnect.com/

$$\langle W(\mathcal{N})^3 \rangle = 3\langle \mathcal{N} W(\mathcal{N}) \rangle$$

$$\langle W(\mathcal{N})^4 \rangle = 6\langle \mathcal{N} W(\mathcal{N})^2 \rangle - 3\langle \mathcal{N}^2 \rangle$$

Physical quantities

We rewrite our primary solution at the end of inflation as:

$$e^{-3\mathcal{N}} = e^{-3N_c}\left(1 + \kappa W(\mathcal{N})\right)$$

where

$$\kappa \equiv 3\, e^{3N_c}\left(\frac{H^2}{2\pi\dot{\phi}_0}\right) \equiv 3\, e^{3N_c}\sqrt{\mathcal{P}_0} = 3\sqrt{\mathcal{P}_e}$$

Moments of number of e-folds

By expanding in terms of κ and using the recursive relations we get

$$\langle \mathcal{N} \rangle = N_c\left[1 + \frac{\kappa^2}{6} + \frac{\kappa^4}{36}(5 + 9N_c) + \frac{\kappa^6}{72}(17 + 77N_c + 60N_c^2)\right]$$

$$\langle \delta\mathcal{N}^2 \rangle = \frac{\kappa^2}{9}N_c\left[1 + \frac{\kappa^2}{6}(7 + 15N_c) + \frac{\kappa^4}{12}(28 + 143N_c + 128N_c^2)\right]$$

$$\langle \delta\mathcal{N}^3 \rangle = \frac{\kappa^4}{27}\left(N_c + 3N_c^2\right) + \frac{\kappa^6}{162}\left(19N_c + 120N_c^2 + 132N_c^3\right)$$

Power Spectrum and Non - Gaussianity

Using the Stochastic δN formalism one can show that:

(V. Vennin, A. A. Starobinsky,Correlation Functions in Stochastic Inflation ,Eur. Phys. J. C (2015) 75: 413)

(T. Fujita, M. Kawasaki, Y. Tada, T. Takesako,A new algorithm for calculating the curvature perturbations in stochastic inflation,JCAP 1312 (2013) 036)

$$\mathcal{P}_{\mathcal{R}} = \frac{\mathrm{d}\left\langle \delta\mathcal{N}^2\right\rangle}{\mathrm{d}\left\langle \mathcal{N}\right\rangle} =$$

$$= \frac{\kappa^2}{9}\Big(1+\kappa^2(1+5N_c)\Big) = \mathcal{P}_e\Big(1+9\mathcal{P}_e(1+5N_c)\Big)$$

$$f_{NL} = \frac{5}{36\mathcal{P}_{\mathcal{R}}^2}\frac{\mathrm{d}^2\left\langle \delta\mathcal{N}^3\right\rangle}{\mathrm{d}\left\langle \mathcal{N}\right\rangle^2} = \frac{5}{2}+\kappa^2\left(\frac{65}{6}+30N_c\right)+\mathcal{O}(\kappa^4)$$

Generalization

Let's assume that the matter Lagrangian is an arbitrary function of the kinetic term:
(X. Chen, M. Huang, S. Kachru, G. Shiu,Observational Signatures and Non-Gaussianities of General Single Field Inflation,JCAP 0701:002,2007)

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2} + P(X) \right)$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2X P_{,XX}}$$

The Langevin equation

Similar to stochastic ultraslow roll equation one can show that:
(H. Firouzjahi, A. Nassiri-Rad, M. Noorbala,Stochastic non-attractor inflation, accepeted to publish in PRD)

$$\phi\left(N\right)=\phi_0+\frac{\sqrt{2X_0}}{3Hc_s^2}\left(1-\exp\left(-3Nc_s^2\right)\right)+\frac{H}{2\pi\sqrt{c_s}}\int_0^N\frac{1}{\sqrt{P_{,X}}}\xi\left(N'\right)dN'$$

$$\left\langle\hat{W}^n\left(\mathcal{N}\right)\right\rangle=\frac{n(n-1)}{2}\left\langle\int_0^{\mathcal{N}}\frac{\hat{W}\left(N\right)^{n-2}}{P_X}dN\right\rangle$$

$$\hat{W}(\mathcal{N})\equiv\int_0^{\mathcal{N}}\frac{dW(N)}{\sqrt{P_X(N)}}$$

As a specific example we take our model as follows:
(X. Chen, H. Firouzjahi, E. Komatsu, M. H. Namjoo, M. Sasaki,In-in and δN calculations of the bispectrum from non-attractor single-field inflation,DOI:10.1088/1475-7516/2013/12/039)

$$c_s^2 \simeq 1/(2\alpha - 1)$$

$$P(X) = X + \beta X^{\alpha}$$

The numeric calculations which will be seen later are done by using this specific example and setting cs^2=0.5 .

Power Spectrum

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}^{(0)}\left[1 + \mathcal{P}_{\mathcal{R}}^{(0)}\left(\frac{3}{2}(1+5c_s^2) + 45c_s^4 P_X I(N_c)\right)\right]$$

$$I(N) \equiv \int_0^N \frac{dN'}{P_{,X}(N')}$$

Boundary Crossing Probabilities

Standard Brownian motion

In this case the initial velocity of the field is zero and the field evolves as follows:

$$\phi\left(\mathcal{N}\right) = \frac{H}{2\pi} W\left(\mathcal{N}\right)$$

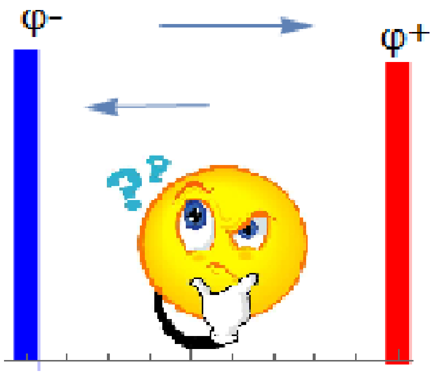
A code for Mathematica for random walk

- proc=ItoProcess[□x[t]==-0*x[t] □t+ □w[t],x[t],{x,1},t,w□WienerProcess[]];
- A=RandomFunction[proc,{0.,1,0.001}]["PathComponents"][[1,2,1]][[1]];
- B=RandomFunction[proc,{0.,1,0.001}]["PathComponents"][[1,2,1]][[1]];
- RAN=Table[{A[[i]],B[[i]]},{i,1,Length[A]}];
- brownian=Manipulate[ListLinePlot[{RAN[[1;;i]]}],{i,1,Length[A] ,1,Appearance->"Open"},ContentSize->550]

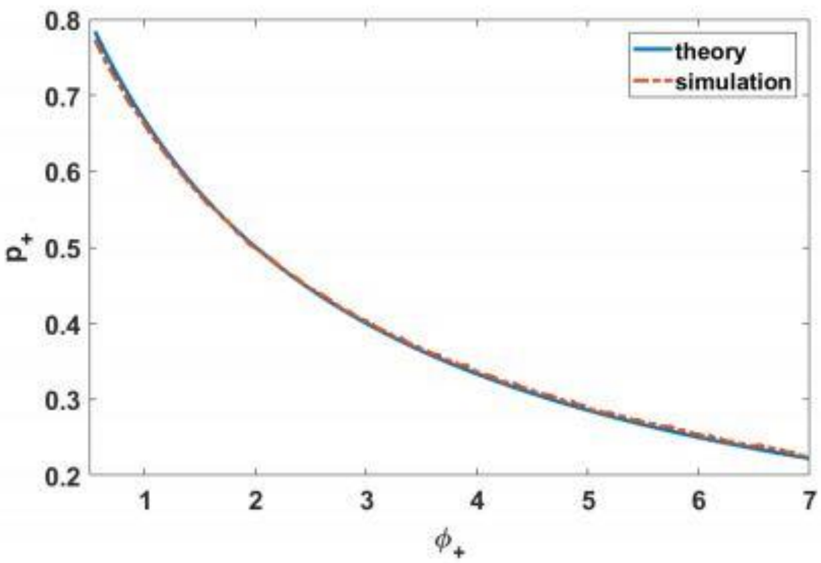
Standard Brownian motion

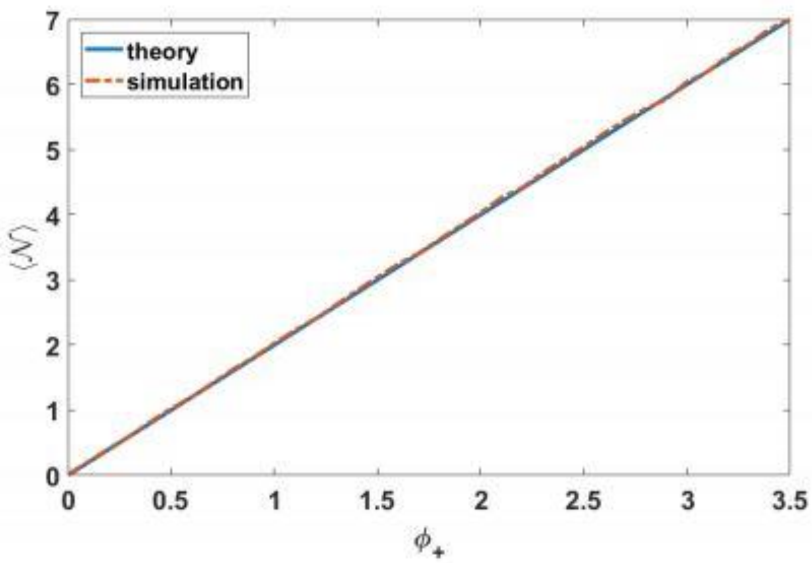
$$p_+ = \frac{-\phi_-}{\phi_+ - \phi_-}, \qquad p_- = \frac{\phi_+}{\phi_+ - \phi_-}$$

$$\langle \mathcal{N} \rangle = \frac{-\phi_-\phi_+}{\left(\frac{H}{2\pi}\right)^2} = \left(\frac{\phi_+}{\frac{H}{2\pi}}\right)\left(\frac{-\phi_-}{\frac{H}{2\pi}}\right)$$



Simulation of Standard Brownian motion



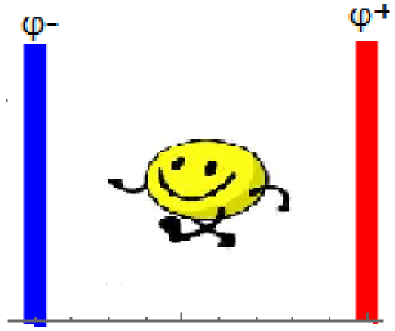


Brownian motion with non- zero drift

$$\phi(N) = \phi_0 + \frac{\dot{\phi}_0}{3H}\left(1 - e^{-3N}\right) + \frac{H}{2\pi} W(N)$$

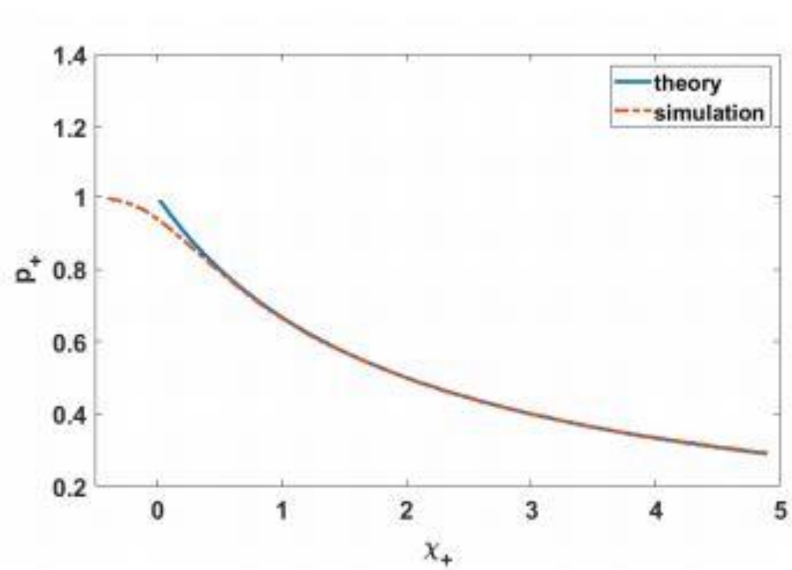
For large values of barriers we have:

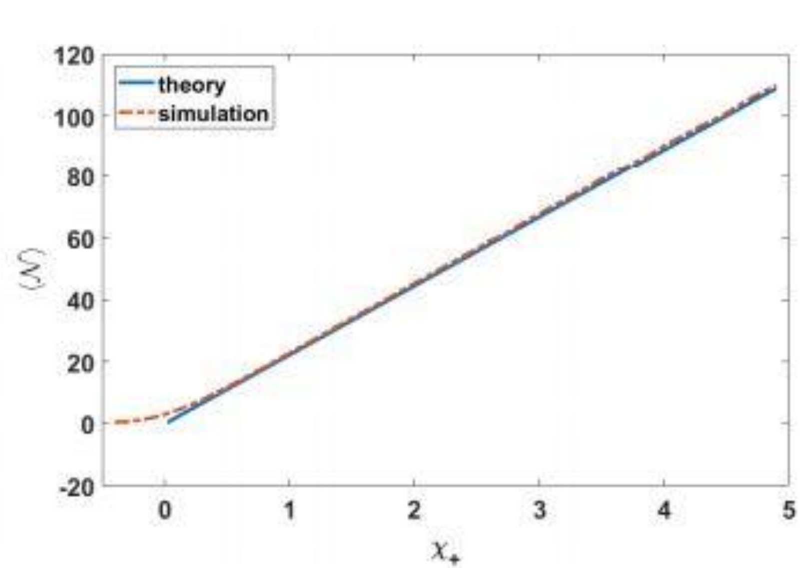
$$p_+ = \frac{\dot{\phi}_0 - 3H\phi_-}{3H(\phi_+ - \phi_-)} \qquad p_- = \frac{-\dot{\phi}_0 + 3H\phi_+}{3H(\phi_+ - \phi_-)}$$



$$\chi \equiv \frac{\phi(\mathcal{N})}{\phi_{\max}} - 1 \qquad \langle \mathcal{N} \rangle = -\frac{\chi_+ \chi_-}{9\mathcal{P}_{\mathcal{R}}}$$

Simulation of Standard Brownian motion with non zero drift





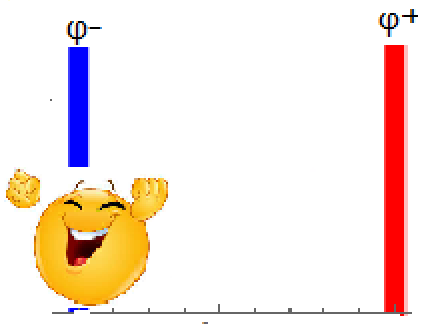
Time dependent Diffusion with Drift

$$\frac{H}{2\pi\sqrt{c_s}\phi_{max}}\hat{W}(\mathcal{N}) = \chi(\mathcal{N}) + e^{-3c_s^2\mathcal{N}}$$

$$\chi \equiv \frac{\phi(\mathcal{N})}{\phi_{\max}} - 1 \qquad \phi_{\max} \equiv \phi_0 + \frac{\dot{\phi}_0}{3Hc_s^2}$$

In the limit that χ>>1 we have the following expressions for the probability:

$$
\begin{aligned}
p_+ &\simeq \frac{-\chi_-}{\chi_+ - \chi_-} = \frac{\dot{\phi}_0 - 3Hc_s^2\phi_-}{3Hc_s^2(\phi_+ - \phi_-)} \\
p_- &\simeq \frac{\chi_+}{\chi_+ - \chi_-} = \frac{-\dot{\phi}_0 + 3Hc_s^2\phi_+}{3Hc_s^2(\phi_+ - \phi_-)}
\end{aligned}
$$

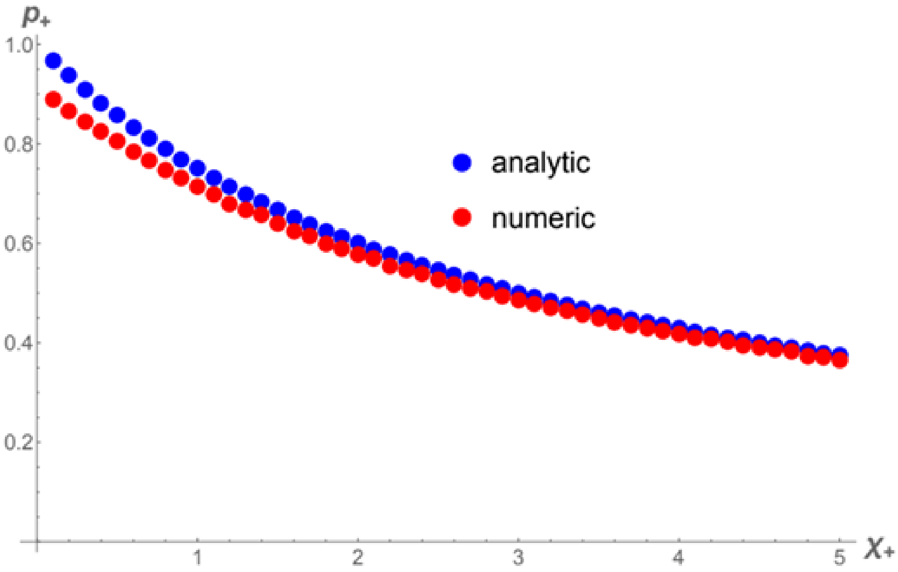


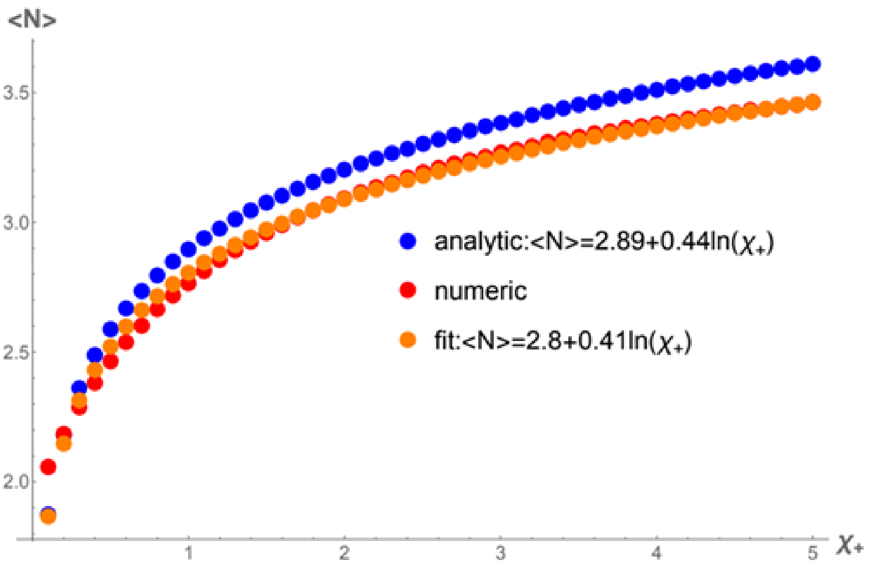
We can obtain the following order of magnitude for the number of efolds to the end of inflation

$$\langle \mathcal{N} \rangle \sim \frac{1}{3(1-c_s^2)} \ln \left[\frac{-\chi_- \chi_+}{3 c_s^4 P_X \mathcal{P}_\mathcal{R}} (1-c_s^2) \right] \qquad \mathcal{N} \gg 1$$

$$\langle \mathcal{N} \rangle = \frac{1+\chi_+}{3 c_s^2} \qquad \mathcal{N} \ll 1$$

Simulation of probability and time average





Large Diffusion Limit

Let's assume the case that the diffusion is much larger than drift:

(H. Firouzjahi, A. Nassiri-Rad, M. Noorbala,Stochastic non-attractor inflation, accepted to publish in PRD)

$$\phi(N) - \phi_0 = \mu F(N) + hW(N)$$

$$\mu = \frac{\dot{\phi}_0}{3Hc_s^2} \qquad h = \frac{H}{2\pi\sqrt{c_s}}$$

$$F(N) = 1 - \exp\left(-3Nc_s^2\right)$$

Power Spectrum(Absorbing Condition)

In the case that both of barriers can play the role of end of inflation, at leading order we have:

(C. Pattison, V. Vennin, H. Assadullahi, D. Wands,Quantum diffusion during inflation and primordial black holes

,DOI:10.1088/1475-7516/2017/10/046)

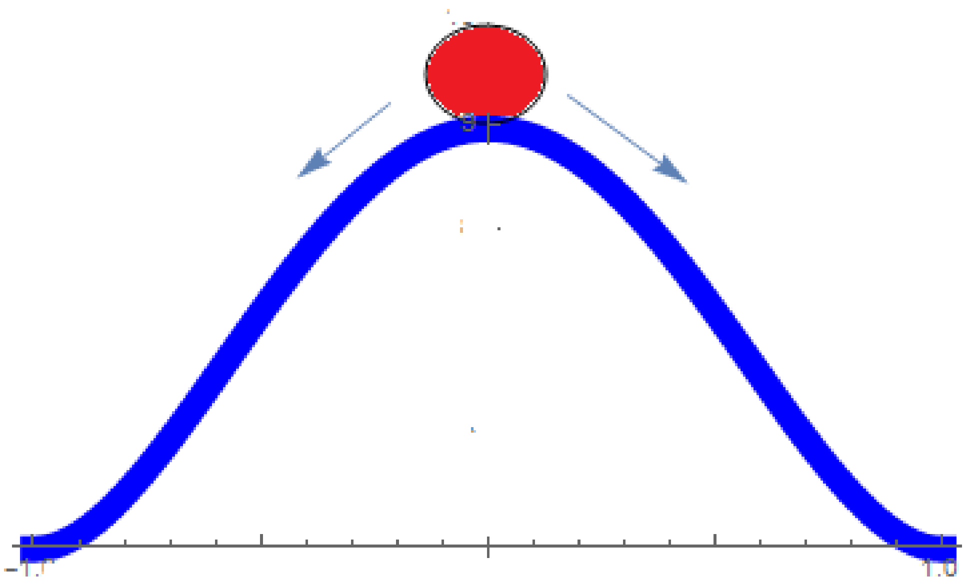
(H. Firouzjahi, A. Nassiri-Rad, M. Noorbala,Stochastic non-attractor inflation, accepted to publish in PRD)

$$\langle \mathcal{N} \rangle = -\frac{(\phi_+ - \phi_0)(\phi_- - \phi_0)}{h^2}$$

$$\langle \mathcal{N}^2 \rangle = -\frac{(\phi_- - \phi_0)(\phi_+ - \phi_0)\left[(\phi_- - \phi_0)^2 - 3(\phi_+ - \phi_0)(\phi_- - \phi_0) + (\phi_+ - \phi_0)^2\right]}{3h^4}$$

$$\mathcal{P}_\zeta = \frac{\left(2\phi_0 - \phi_- - \phi_+\right)^2}{3h^2}$$

Absorbing by image



Power Spectrum(Reflective Condition)

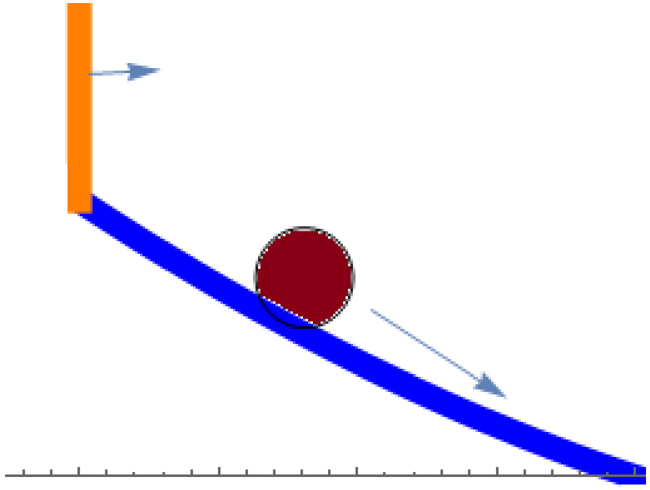
In the case that ϕ- plays the role of a reflecting barrier we have:

$$\langle \mathcal{N}_+ \rangle = \frac{(\phi_+ - \phi_0)(\phi_+ - 2\phi_- + \phi_0)}{3h^2}$$

$$\langle \mathcal{N}_+^2 \rangle = -\frac{(2\phi_- - \phi_+ - \phi_0)\,[\phi_+ - \phi_0)\left(4\phi_-^2 + (6\phi_0 - 14\phi_+)\,\phi_- + 7\phi_+^2 - 3\phi_0^2\right]}{45h^4}$$

$$\mathcal{P}_\zeta = \frac{4\,(\phi_0 - \phi_-)^2}{15h^2}$$

Reflective by image



Summary

* We have studied the quantum jumps to the power spectrum by the stochastic methods in non attractor inflationary models.
* The order of corrections is as small as the squared power spectrum and so negligible.
* We studied the probability and expected time averages to hit the absorbing barriers in these models.
* We studied the power spectrum in case that the coefficient of diffusion is much larger that the drift coefficient. As we see in this case the power spectrum is proportional to the inverse of square of diffusion.


Thank you
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