Numerically modeling stochastic inflation

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FRAMEWORK

- Cosmic inflation is a phase of accelerated in the very early history of the universe.
- Quantum fluctuations of the inflaton get stretched to cosmological scales and give rise to metric and density perturbations which seed structure formation that are manifest in the CMB temperature fluctuations.
- We split the inflaton into a back- ground field, comprising of long wavelength modes, and small perturbative corrections which are comprised of short wavelength modes that initially start out inside the horizon and are later stretched out of causal contact during inflation.

$$\phi(t, \boldsymbol{x}) = \bar{\phi}(t) + \hat{\phi}(t, \boldsymbol{x})$$

  where $\bar{\phi}$ refers to the classical, superhorizon inflaton field and $\hat{\phi}$ is the subhorizon, quantum part of the field that has not become classical.
- Quantum fluctuations backreact and modify the inflaton trajectory as stochastic noise. This is precisely the physics that is captured by stochastic inflation. [**Starobinsky (1986)**]

- The equation that describes the inflaton evolution:

$$\frac{d^2\phi}{dN^2} + (3-\epsilon_1)\frac{d\phi}{dN} + (3-\epsilon_1)\frac{\partial_\phi V}{V} = 0$$

- The inflaton evolution has been expressed in the e-fold time N with $dN = Hdt$.
- H is the Hubble parameter defined by

$$H^2 = \frac{V(\phi)}{3 - \frac{1}{2}\left(\frac{d\phi}{dN}\right)^2}$$

- The Hubble flow parameters are defined as follows:

$$\epsilon_1 = -\frac{1}{H}\frac{dH}{dN} \qquad \epsilon_n = \frac{d\ln\epsilon_{n-1}}{dN}$$

Mukhanov Sasaki

- Now the evolution of the Fourier modes (Mukhanov-Sasaki Equation):

$$\frac{d^2\delta\phi_k}{dN^2} + (3-\epsilon_1)\frac{d\delta\phi_k}{dN} + \left[\left(\frac{k}{aH}\right)^2 + (3-\epsilon_1)\frac{\partial_{\phi\phi}V}{V} - 2\epsilon_1(3-\epsilon_1+\epsilon_2)\right]\delta\phi_k = 0$$

- This describes the evolution of the Fourier modes of the inflaton quantum fluctuations from an initially subhorizon regime ($k \gg aH$) to a superhorizon regime ($k \ll aH$).
- We considered $k = 100a(N_i)H(N_i)$ and $k = 0.01a(N_f)H(N_f)$.
- Deep inside the horizon the quantum modes do not feel the curvature of spacetime and the Bunch-Davies initial condition is imposed.

$$\delta\phi_k = \left.\frac{1}{a\sqrt{2k}}\right|_{N=N_i} \qquad \frac{d\delta\phi_k}{dN} = -\left.\left(\frac{1}{a\sqrt{2k}} + i\frac{k}{aH}\frac{1}{a\sqrt{2k}}\right)\right|_{N=N_i}$$

- The quantum fluctuations of the inflaton can then be used to define the gauge-invariant curvature perturbations.

$$\zeta_k = \Psi_k + \frac{\delta\phi_k}{d\bar{\phi}/dN}$$

  Here Ψ_k is a metric scalar perturbation.
- Power spectrum of curvature perturbations are defined in this regime as follows

$$\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2}|\zeta_k|^2_{k\ll aH} = \frac{k^3}{2\pi^2}\left|\frac{\delta\phi_k}{\sqrt{2\epsilon_1}}\right|^2_{k\ll aH}$$

Window function

- The subhorizon fluctuations can be decomposed into a mode expansion.

$$\delta\hat{\phi}(N,\boldsymbol{x}) = \int_{k>0} \frac{d^3k}{(2\pi)^{3/2}} W\left(\frac{k}{\sigma a H}\right) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \hat{a}_{\boldsymbol{k}} \delta\phi_{\boldsymbol{k}}(N) + \text{h.c.}$$

  where "h.c." stands for the Hermitian conjugate of the mode expansion.
- $W(k/\sigma a H)$ is a suitably defined window function that picks out modes smaller than the horizon. The nature of the stochastic process depends on the type of window function that has been used.
- The simplest and most commonly employed one is a sharp cut-off in momentum space

$$W\left(\frac{k}{\sigma a H}\right) = \Theta\left(\frac{k}{\sigma a H} - 1\right)$$

  This type of window function produces noise that is uncorrelated in time (white noise). This is because in the definition of the noise terms the window function appears as a time derivative

$$\frac{\partial}{\partial N} W\left(\frac{k}{\sigma a H}\right) = \frac{k}{\sigma a H}(\epsilon_1 - 1) W'\left(\frac{k}{\sigma a H}\right) = k(\epsilon_1 - 1)\delta(k - \sigma a H)$$

- A more physically motivated choice for a window function may be a Gaussian one. However, this leads to colored noise which is correlated in time. This will be addressed in a future work.

- The SDEs in focus in our work is as follows where canonical momentum field $\bar{\pi}_\phi = d\bar{\phi}/dN$.

$$\begin{aligned} \frac{d\bar{\phi}}{dN} &= \bar{\pi}_\phi + \xi_\phi \\ \frac{d\bar{\pi}_\phi}{dN} &= -(3-\epsilon_1)\left(\bar{\pi}_\phi + \frac{\partial_\phi V}{V}\right) + \xi_\pi \end{aligned}$$

- The correlation functions simplify to terms proportional to Dirac δ-functions. It can be shown that the correlation functions reduce to [**Grain, Vennin (2017)**]

$$\begin{aligned} \Xi_{fg}(\mathbf{x}_1 - \mathbf{x}_2; N_1 - N_2) &= \frac{k_\sigma^3(N_1)}{2\pi^2}(1-\epsilon_1(N)) f_{k_\sigma(N_1)} g^*_{k_\sigma(N_1)} \frac{\sin\left[k_\sigma(N_1)|\mathbf{x}_2 - \mathbf{x}_1|\right]}{k_\sigma(N_1)|\mathbf{x}_2 - \mathbf{x}_1|}\delta(N_1 - N_2) \\ &= (1-\epsilon_1(N))\mathcal{P}_{fg}(k_\sigma)\frac{\sin\left[k_\sigma(N_1)|\mathbf{x}_2 - \mathbf{x}_1|\right]}{k_\sigma(N_1)|\mathbf{x}_2 - \mathbf{x}_1|}\delta(N_1 - N_2) \end{aligned}$$

where $\mathcal{P}_{fg}$ is the dimensionless power spectrum of the form fg^* evaluated at k_σ.

- In slow-roll approximation, the mode functions (in the superhorizon regime) are given by

$$\delta\phi_k = \frac{H}{\sqrt{2k^3}}\left(\frac{k}{aH}\right)^{\frac{3}{2}-\nu} \qquad \delta\pi_k = \frac{H}{\sqrt{2k^3}}\left(\nu - \frac{3}{2}\right)\left(\frac{k}{aH}\right)^{\frac{3}{2}-\nu} \quad \text{where} \quad \nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$$

- At equal spatial points,

$$\Xi_{\phi\phi} = \frac{k_\sigma^3}{2\pi^2}\frac{H^2}{2k_\sigma^3}\left(\frac{k_\sigma}{aH}\right)^{\frac{3}{2}-\nu} = \frac{H^2}{4\pi^2}\sigma^{3-2\nu}$$

$$\Xi_{\pi\pi} = \frac{k_\sigma^3}{2\pi^2}\frac{H^2}{2k_\sigma^3}\left(\nu-\frac{3}{2}\right)^2\left(\frac{k_\sigma}{aH}\right)^{\frac{3}{2}-\nu} = \frac{H^2}{4\pi^2}\left(\nu-\frac{3}{2}\right)^2\sigma^{3-2\nu}$$

$$\langle\xi_f(\boldsymbol{x}_1, N_1)\xi_g(\boldsymbol{x}_2, N_2)\rangle = \Xi_{fg}(\boldsymbol{x}_1-\boldsymbol{x}_2; N_1)\quad\delta(N_1-N_2)$$

- In the massless de Sitter limit, $\Xi_{\phi\phi}\simeq H^2/4\pi^2$ and $\Xi_{\pi\phi}\simeq 0$. This is a usable approximation but not perfect.
- In the spatially flat gauge, the curvature perturbations are defined as

$$\zeta_k = \frac{\delta\phi_k}{\sqrt{2\epsilon_1}}$$

  where $\delta\phi_k$ are the Fourier modes of the inflaton fluctuations.
- The definition of $\mathcal{P}_\zeta$ comes from the two-point function of $\zeta(\boldsymbol{x})$

$$\langle\zeta(\boldsymbol{x})^2\rangle = \int\frac{dk}{k}\mathcal{P}_\zeta(k)\Rightarrow\mathcal{P}_\zeta(k) = \frac{d\langle\zeta^2\rangle}{d\ln k} = \frac{d}{d\ln k}\left(\frac{\langle\delta\phi_k^2\rangle}{2\epsilon_1}\right)$$

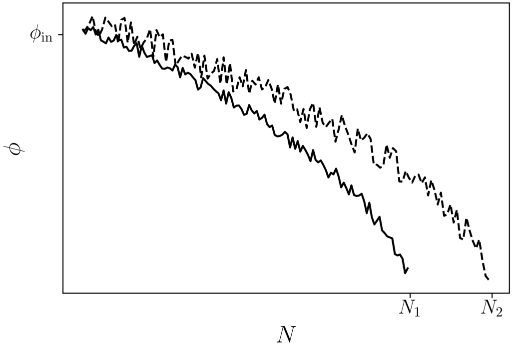


FIGURE: An illustration showing the variation of the coarse-grained inflaton field during two different realizations. We have not given any units because this figure is only for illustrative purposes. The fluctuations have been amplified to make them more visible.

If the SDEs are solved enough times, the $\langle \delta\phi^2 \rangle$ can be interpreted as a stochastic average over all the realizations

$$\langle \delta\phi^2_{\text{stochastic}} \rangle = \frac{1}{n_{\text{simulations}}} \sum_{i=1}^{n_{\text{sim}}} (\bar{\phi} - \phi_{\text{bg}})_i^2$$

- The two-point function is calculated as follows.

$$\langle f(x)f(x')\rangle = \frac{\displaystyle\sum_{\text{all random events}} f(x)f(x')}{\text{number of random events}}$$

- A white noise random function is defined as :
$\langle f(x)f(x')\rangle = M(x)\delta(x-x')$ and $\langle f(x)\rangle = 0$. with $M(x)$ as the normalization factor and all higher-order cumulants are required to vanish. We will fix $M(x) = 1$ here.
-
$$\langle f(x_i)f(x_{i'})\rangle = \frac{\delta_{ii'}}{\Delta x}$$
- The $\delta_{ii'}/\Delta x$ becomes a Dirac δ-function in the limit $\Delta x \to 0$.
- We use a random number generator for a large number of instances (e.g. 10^6) to overcome the statistical noise.
- These are some of the properties one can deduce: [**De, Plumberg, Kapusta (2020)**]

$$\begin{aligned}
\langle f(x_i)f'(x_{i'})\rangle &= \frac{\delta_{i+1,i'} - \delta_{i,i'}}{\Delta x^2} \\
\langle f'(x_i)f'(x_{i'})\rangle &= -\frac{\delta_{i,i'+1} + \delta_{i,i'-1} - 2\delta_{i,i'}}{\Delta x^3} \\
\left\langle \int_{x_i}^{x_1} f(x')dx' \int_{x_i}^{x_2} f(x')dx' \right\rangle &= \min(x_1, x_2) - x_i
\end{aligned}$$

Algorithm: Coarse-grained inflaton evolution algorithm

Solve background evolution;
Set ΔN, a, H, ϵ_1 and ϵ_2;
Set $\sigma = 0.01$;
for $N \in [N_{initial}, N_{final}]$ **do**
  $k_N = \sigma a_N H_N$;
  Solve Mukhanov-Sasaki Equation for each k_N, N;
  return $\delta\phi_{k_N}, \delta\pi_{k_N}$;
end
Calculate $\Xi_{\phi\phi}, \Xi_{\pi\pi}$ for each N
for $j \in \{1, 2, 3, \cdots, n_{sim}\}$ **do**
  Generate random normal event $\mathcal{N}(0, 1)$ for each N-step.
  Multiply the random event with the amplitude.
  for *each N-step* **do**
    Solve SDE.
  end
end

- One of the common ways to reduce bias and estimate errors in stochastic modeling is the jackknife method.
- It estimates the error of statistics without making any assumptions about the distribution that generated the data.
- We create jackknife samples by sequentially deleting a single observation from the sample, or in other words, creating "leave-one-out" data sets. In our case, we consider the two-point correlation statistic S.
- We leave out the i_{th} event to create the i_{th} jackknife statistic S_i. The average of the jackknife samples is $S_{\text{avg}} = \sum_i S_i/n$. The jackknife error is then estimated as

$$\sigma_{\text{jack}} = \sqrt{\frac{n-1}{n}\sum_i (S_i - S_{\text{avg}})^2}$$

- The error plots are provided in the paper.

RESULTS FROM TEST POTENTIALS

- We start with the quadratic potential.

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

- We choose $N = 64$ for the fiducial run and set the observable scale at $N_\star = 10$ producing a total of $\Delta N = 54$ e-folds of observable inflation. Using the pivot scale set at $N_\star$ where $k_\star = Mpc^{-1}$, the parameter m^2 is set to $4.42 \times 10^{-11} M_{\rm pl}^2$. Here we use the fact that $\mathcal{P}_\zeta$, under the slow-roll approximation at CMB scales, is given by [**Planck (2018)**]

$$\mathcal{P}_\zeta(k_\star) = \frac{H^2(k_\star)}{8\pi^2 \epsilon_1(k_\star)} \simeq 2.2 \times 10^{-9}$$

- We check how the two correlation functions $\Xi_{\phi\phi}$ and $\Xi_{\pi\pi}$ evolve with time in the next slide.

Noise Amplitude

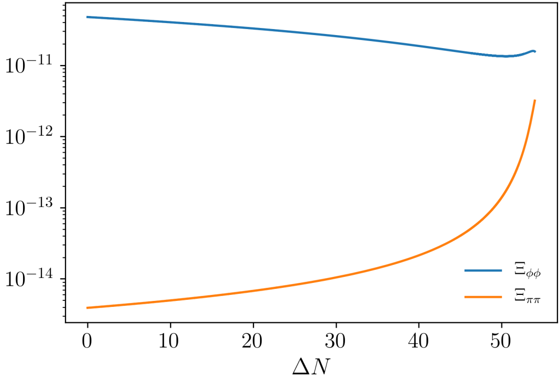


Figure: Evolution of correlation functions $\Xi_{\phi\phi}$ and $\Xi_{\pi\pi}$ for $\sigma = 0.01$.

$\Xi_{\phi\phi}$ is the dominant contributor to the stochastic noise and $\Xi_{\pi\pi}$ is suppressed by a few orders of magnitude.

We can compute the curvature power spectrum $\mathcal{P}_\zeta$ for a large number of realizations and compare with the result obtained by solving the Mukhanov-Sasaki equation.

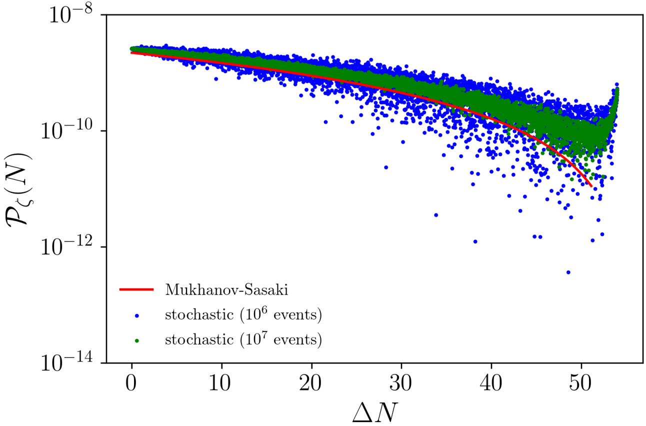


Figure: Power spectrum of curvature perturbations for the chaotic potential using $\sigma = 0.01$. The blue and green dotted curves represent results for 10^6 and 10^7 realizations respectively while the solid red curve is the solution obtained from solving the Mukhanov-Sasaki equation.

- $$\frac{d\ln k}{dN} = \frac{da/dN}{a} + \frac{dH/dN}{H} = 1 - \epsilon_1$$

  Hence
  $$\mathcal{P}_\zeta(N) = \frac{1}{1-\epsilon_1}\frac{d}{dN}\left(\frac{\langle\delta\phi_{\rm st}^2\rangle}{2\epsilon_1}\right) = \frac{1}{1-\epsilon_1}\frac{1}{2\epsilon_1}\left(\frac{d}{dN}\langle\delta\phi_{\rm st}^2\rangle - \epsilon_2\langle\delta\phi_{\rm st}^2\rangle\right)$$

- We can use the following identity to reduce the randomness of the noise.(Derivation is given in the paper). The time evolution of any *n*-point correlation function is derived using Fokker-Planck equation. [**Ezquiaga, Garcia-Bellido (2018)**]

  $$\frac{d}{dN}\langle\delta\phi_{\rm st}^2\rangle = \Xi_{\phi\phi} + 2\langle\delta\phi_{\rm st}\delta\pi_{\rm st}\rangle$$

- The power spectrum can be expressed in the form

  $$\mathcal{P}_\zeta = \frac{1}{1-\epsilon_1}\frac{1}{2\epsilon_1}\left(\Xi_{\phi\phi} + 2\langle\delta\phi_{\rm st}\delta\pi_{\rm st}\rangle - \epsilon_2\langle\delta\phi_{\rm st}^2\rangle\right)$$

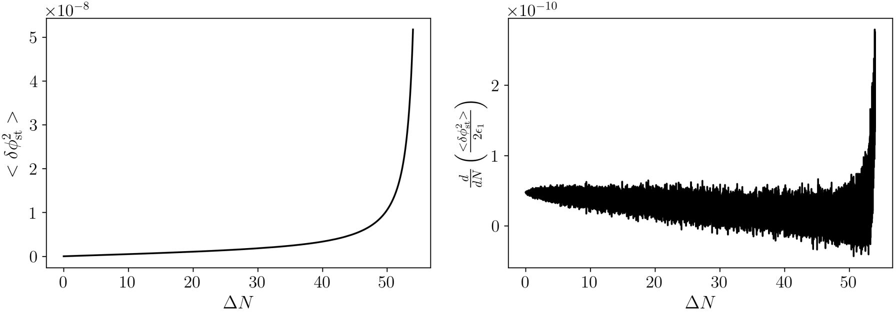


Figure: Comparison of $\langle \delta\phi^2_{\rm st} \rangle$ and its derivative. They are in units of $M^2_{\rm pl}$. We see that while the former appears smooth, the latter is not.

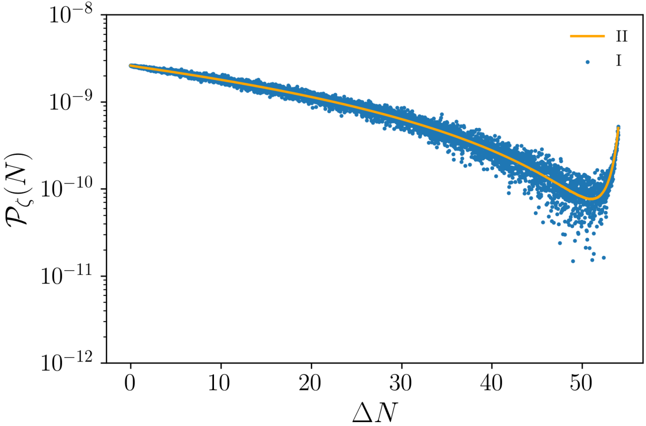


Figure: Comparison of $\mathcal{P}_\zeta$ with I with the original expression and II with using the identity. The orange curve passes through the points and has no noisy features.

Deformed Starobinsky potential

- USR inflation models typically possess some peculiar features in their potentials that create departures from slow-roll behaviour. The presence of an inflection point in the potential can slow down the inflaton and give rise to amplifications in the curvature power spectrum.
- We consider the following potential: [**Ketov, Khlopov (2018)**]

$$V(\phi) = V_0 \left(1 + \xi - e^{-\alpha\phi} - \xi e^{-\beta\phi^2}\right)^2$$

In the limit $\xi \to 0$, the potential reduces to the $R + R^2$ modification of Einstein gravity, which gives rise to Starobinsky inflation. There are three free parameters in this potential since V_0 is fixed by the CMB normalization of the power spectrum. We impose the condition that there is an inflection point of $V(\phi)$ at $\tilde{\phi}$.

$$\xi = -\frac{\alpha}{2\beta\tilde{\phi}} e^{-\alpha\tilde{\phi} + \beta\tilde{\phi}^2}$$

$$\beta = \frac{\beta\tilde{\phi} + 1}{2\tilde{\phi}^2}$$

- We chose the following parameter set: $V_0 = 1.27 \times 10^{-9} M_{\text{pl}}^4$, $\alpha = \sqrt{2/3}$, $\beta = 1.114905$ and $\xi = -0.480964$. For the numerical simulations, we set $\phi_{\text{in}} = 5.82 M_{\text{pl}}$ which produces approximately $N = 70$ e-folds of inflation. Like the chaotic potential case, we fix the observable scale at $N_\star = 10$ where we start adding the noise terms to the SDEs.

Deformed Starobinsky potential

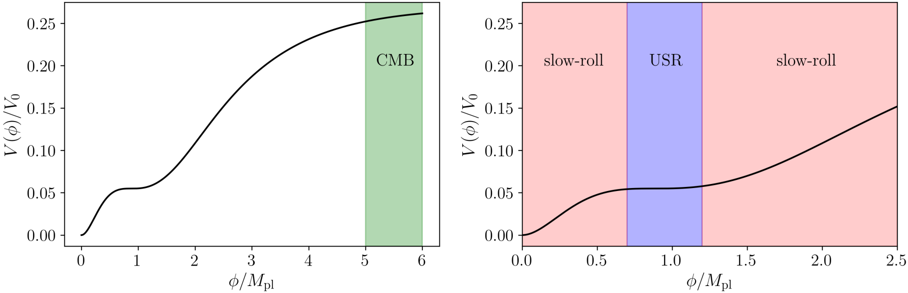


Figure: The deformed Starobinsky potential for super-Planckian field excursions. The shaded regions show the different periods in the inflationary stage.

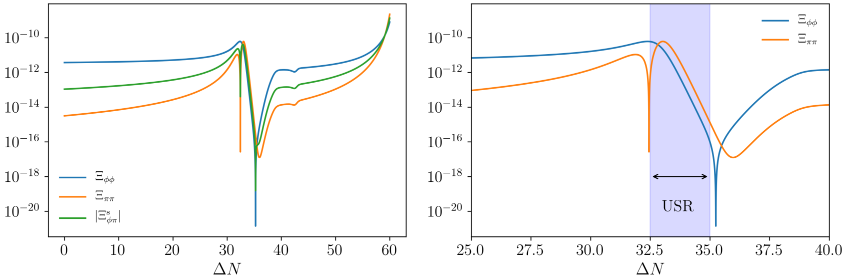


Figure: Evolution of the correlation functions $\Xi_{\phi\phi}$, $\Xi_{\pi\pi}$ and $\Xi^{s}_{\phi\pi}$ for $\sigma = 0.01$. It is clear that the $\pi - \pi$ noise becomes significant in the USR phase.

- During the early stages of the inflation, much like slow-roll, the $\Xi_{\pi\pi}$ term is subdominant. Once the inflation enters the USR phase, $\Xi_{\pi\pi}$ becomes comparable to $\Xi_{\phi\phi}$ and can no longer be ignored. As a result, one should expect significant difference between the behavior of background fields and noise-incorporated fields.
- We already know that curvature power spectrum is enhanced near an inflection point. We can see this semi-quantitatively in the following way: near an inflection point $\partial_\phi V \simeq \partial_{\phi\phi} V = 0$ and the inflaton evolution simplifies to

$$\frac{d^2\phi}{dN^2} + (3 - \epsilon_1)\frac{d\phi}{dN} \simeq 0$$

the solution of which can be expressed as

$$\phi(N) \sim \int^N e^{-\int^{N'}(3-\epsilon_1)dN''} dN'$$

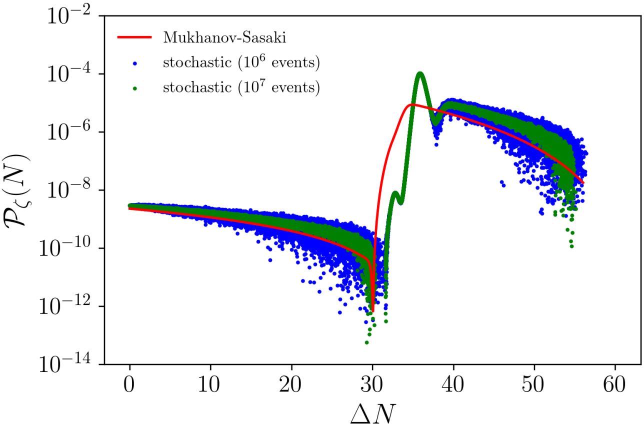
Then the curvature power spectrum behaves in the following way near the inflection point

$$\mathcal{P}_\zeta^{1/2} = \frac{H^2}{2\pi\dot{\phi}} = \frac{H}{2\pi d\phi/dN} \sim \frac{H}{2\pi}\left[\int e^{-\int(3-\epsilon_1)dN''} dN'\right]^{-1}$$

As long as $\epsilon_1 < 3$, there is an exponential amplification of the curvature power spectrum near the vicinity of the inflection point. If we disregard ϵ_1 for a moment and consider that the USR phase lasts for δN e-folds, the power spectrum scales as $\mathcal{P}_\zeta \sim e^{6\delta N}$. Now we can compare the standard result of $\mathcal{P}_\zeta$ computed by solving the Mukhanov-Sasaki equation with that of the stochastic procedure. [**Dimopoulos (2017)**]

Power spectrum for Deformed Starobinsky

There is an $\mathcal{O}(1)$ enhancement in $\mathcal{P}_\zeta$ relative to the Mukhanov-Sasaki result. The peak occurs at $\Delta N_{\text{peak}} = 35.9$ which, in terms of the comoving wavenumber, is around $k_{\text{peak}} \sim 8.63 \times 10^{13} Mpc^{-1}$. Concerning PBH formation, comoving scales of this size would collapse to form PBHs of mass close to $6.6 \times 10^{17} g$. The peak in the curvature power spectrum is approximately $\mathcal{P}_\zeta^{\text{peak}} \simeq 10^{-4}$. Although this is not nearly large enough to collapse to produce PBHs in sufficient abundances, it indicates that there are parameter sets which can work in favor of PBH formation. Due to the added amplification in the power spectrum, less finely tuned parameter sets can be used to explain PBH formation.



- Although the parameters in both potentials have been chosen such that $\mathcal{P}_\zeta \sim 2.2 \times 10^{-9}$ at the pivot scale $k_\star = 0.05\, Mpc^{-1}$, the stochastic results predict slighly larger values. In our computations, we chose $N_\star = 10$ into the fiducial run as corresponding to $k_\star$. The results obtained in these computations were
    - Chaotic: $\mathcal{P}_\zeta(k_\star) = 2.61 \times 10^{-9}$
    - Deformed Starobinsky: $\mathcal{P}_\zeta(k_\star) = 2.95 \times 10^{-9}$
- These values are larger than the observable obtained from CMB measurements. However, we see that these discrepancies are not present when the slow-roll expressions for the noise terms are used. As stated in a previous section, under the slow-roll approximation the noise terms take the following forms

$$\langle \xi_\phi(N_1)\xi_\phi(N_2)\rangle \simeq \frac{H^2}{4\pi^2}\delta(N_1 - N_2)$$
$$\langle \xi_\pi(N_1)\xi_\pi(N_2)\rangle \simeq 0$$

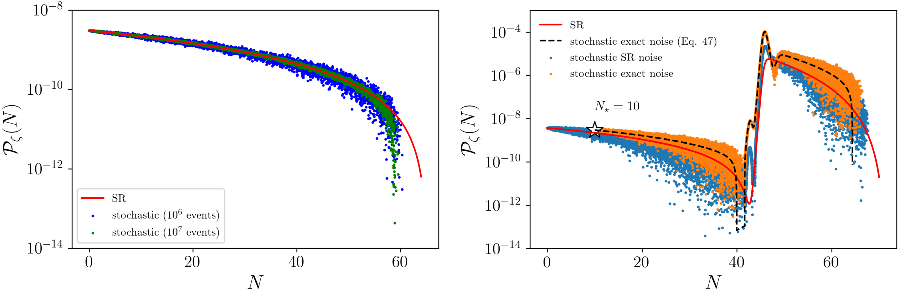


FIGURE: Power spectrum of curvature perturbations computed using slow-roll noise from the chaotic (left panel) and deformed Starobinsky potentials (right panel) respectively. The power spectrum has been computed for 10^6 realizations of the stochastic process using $\sigma = 0.01$. The bottom panel contains both slow-roll and exact noise, along with the data obtained from.

Even for complete slow-roll, the $\Xi_{\phi\phi}$ noise does not exactly correspond to $H^2/4\pi^2$. In following figure, we plot the exact numerical calculation of the noise along with $H^2/4\pi^2$. Therefore, at each time step in the SDEs, the amount of noise being added is slightly different than $H^2/4\pi^2$, which is then reflected in the calculation of $\mathcal{P}_\zeta$.

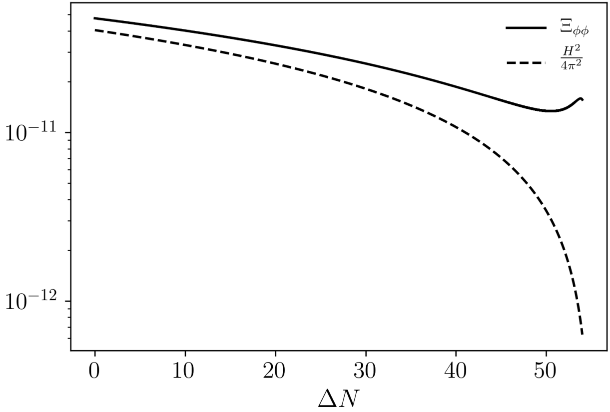


Figure: Comparison of the exact $\phi - \phi$ noise (solid black) with the slow-roll approximation (dotted black).

ROLE OF σ

A different choice of σ does not produce any changes for the quadratic potential, as can be expected. But not for the deformed starobinsky potential. We carry out computations for the same parameter set with $N = 70$ for $\sigma = 5 \times 10^{-3}$. Coincidentally, for this value of σ, the k_σ would correspond to the smallest wavenumber for which the evolution can be numerically computed since, for anything smaller, there would not be enough background evolution information. We plot the P_ζ in Fig. 11 for $\sigma = 10^{-2}, 7.5 \times 10^{-3}$ and 5×10^{-3} for 10^6 realizations of the SDEs. We observe that, although the shape of $\mathcal{P}_\zeta$ stays similar, there is an increase in the size of the peak, the largest of which is of the order $\mathcal{P}_\zeta^{\mathrm{max}} \sim 5 \times 10^{-4}$ for $\sigma = 5 \times 10^{-3}$.

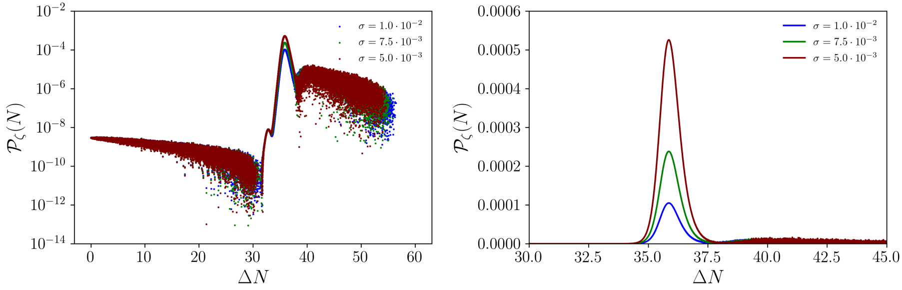


FIGURE: Power spectrum of curvature perturbations for the deformed Starobinsky potential using $\sigma = 1.0 \times 10^{-2}$ (blue), $\sigma = 7.5 \times 10^{-3}$ (green) and $\sigma = 5.0 \times 10^{-3}$ (magenta). The bottom panel is a magnified version of the peaks in the top panel. There is a relative increase in the size of the peak going from the first to last.

Summary of the talk

- We numerically modeled stochastic inflationary dynamics under the influence of Gaussian white noise without any slow-roll simplifications.
- We studied two potentials: the quadratic potential and the deformed Starobinsky potential.
- Deformed Starobinsky potential has an inflection point and the inflationary dynamics around such an inflection point is of interest to PBH formation.
- In the case of the deformed Starobinsky potential there is an amplification of the curvature power spectrum $\mathcal{P}_\zeta$ for modes that cross the horizon near the plateau region due to the interplay between the $\Xi_{\phi\phi}$ and $\Xi_{\pi\pi}$ noise terms.
- We conclude that the exact form of the stochastic noise terms have implications for the stochastic dynamics and subsequent computation of the curvature power spectrum. A slow-roll approximation of $\Xi_{\phi\phi} \simeq H^2/4\pi^2$ does not exactly match numerical results even for completely slow-roll inflation models like ϕ^2.
- The stochastic calculations should potentially help in alleviating some of the difficulties associated with this extreme fine-tuning issue plaguing USR inflation models.
- We also note that there were small changes in the height of the peak for the deformed Starobinsky potential for different values of the coarse-graining scale.

Future work:

- Colored Noise.
- Investigation of PBH formation.

We thank Prof. Joseph Kapusta (University of Minnesota, Twin-Cities) for his guidance during this project.

Thank You for listening!