Stochastic Trailing Strings as Dual Probes to the QGP ?

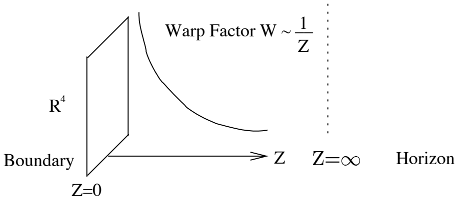
Gregory Giecold

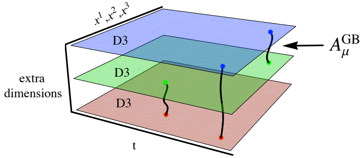
IPhT
CEA–Saclay

Outline

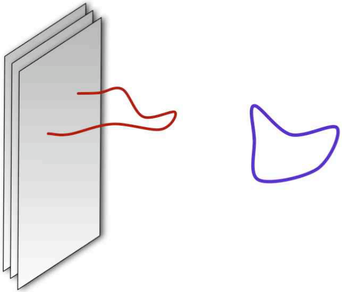
1. Crash review of AdS/CFT
2. Black hole thermodynamics vs. QCD data
3. Heavy probes and trailing strings
4. Summary and outlook

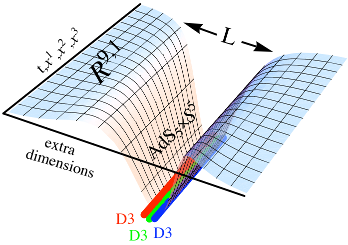
- ’t Hooft genus expansion
- D-brane constructions of gauge theories
- Black hole microstate counting ; cross–sections
- Polyakov’s insight: warped 5d geometry
- Maldacena’s breakthrough
- GKP’s recipe
- Wilson lines from strings
- Witten on confinement
- Probes to a strongly–coupled plasma


Warp Factor W ~ 1/Z
R4
Boundary
Z=0
Z
Z=∞
Horizon

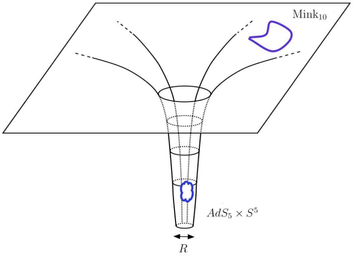


weak coupling





strong coupling



- Near–extremal D3–branes describe $\mathcal{N} = 4$ gauge theory at finite temperature
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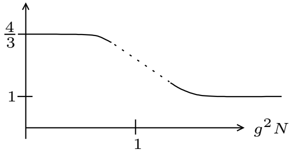
$$ds^2 = \frac{R^2}{z_H^2 z^2}\left(-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)} + z_H^2 z^2 d\Omega_5^2\right)$$

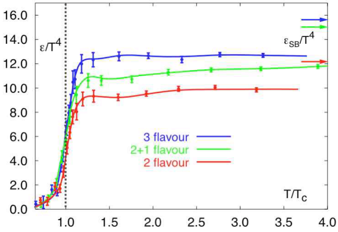
$$f(z) = 1 - z^4,\, T = \frac{1}{\pi z_H}$$

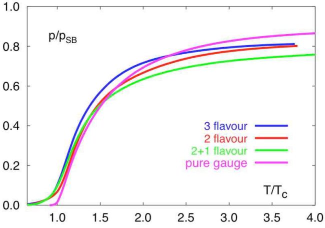
- $\frac{R^4}{\alpha'^2} = 4\pi g_s N = 4\pi g_{YM}^2 N >> 1, \quad N >> 1$
- Entropy density from Hawking–Bekenstein formula

$$\frac{S}{\frac{\pi^2}{2} N^2 T^3 V}$$

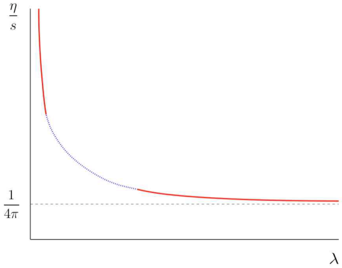


Energy density and pressure lattice calculations





- SUGRA computations suggest $\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$



- Hard work for lattice QCD

- Jet quenching in pQCD :

$$\Delta E = \frac{1}{2}\alpha_s C_R \hat{q}\,(\Delta x)^2\,,$$

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{\Delta x}$$

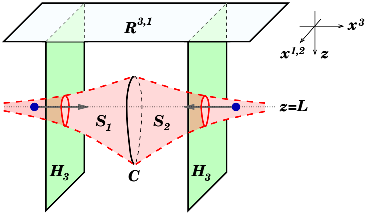
- Liu, Rajagopal, Wiedemann prescription :

$$\langle W^{adjoint}(\mathcal{C})\rangle \simeq \exp\left(-\frac{1}{4}\hat{q}L^2\Delta x\right)\,,$$

$$\hat{q} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)}\sqrt{\lambda}T^3$$

- Gubser et al.'s approach : falling strings

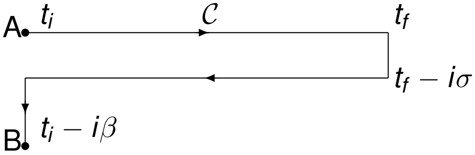
- Total multiplicity from point–sourced gravitational shock wave



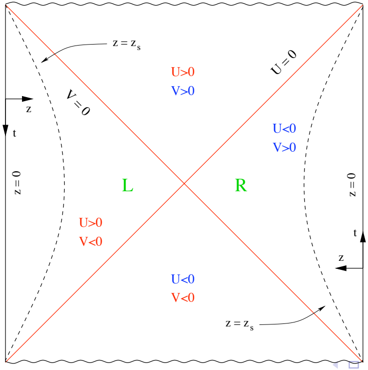
- Energy loss, momentum broadening from string hanging in AdS

- Nambu–Goto string action : $S = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det\left(h_{\alpha\beta}\right)}$
- $x_0^3 = vt + \frac{vz_H}{2} \left(\arctan z - \text{arctanh} z\right)$
- Drag force $\frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}}{2} T^2 \frac{v}{\sqrt{1-v^2}}$
- Calculate stress–tensor gauge theory correlators from linearized bulk Einstein equations $\rightarrow$ jet–splitting

- Stochastic forces on heavy quarks from fluctuations around the trailing string
- $$S = -\frac{\sqrt{\lambda}Tz_s^2}{2}\int dtdz\frac{1}{z^2} + \int dtdz P^\alpha \partial_\alpha \delta x_\ell - \frac{1}{2}\int dtdz\left[T_\ell^{\alpha\beta}\partial_\alpha\delta x_\ell \partial_\beta \delta x_\ell + T_\perp^{\alpha\beta}\partial_\alpha \delta x_\perp^i \partial_\beta \delta x_\perp^i\right]$$
- $$P^\alpha = \frac{\pi v\sqrt{\lambda}T^2}{2z_s^2}\begin{pmatrix} \frac{z_H}{z^2(1-z^4)} \\ 1 \end{pmatrix}$$
- $$T_\perp^{\alpha\beta} = z_s^4 T_\ell^{\alpha\beta} = -\frac{\pi\sqrt{\lambda}T^2}{2z_s^2}\begin{pmatrix} \frac{z_H}{z^2}\frac{1-(zz_s)^4}{(1-z^4)^2} & \frac{v^2}{1-z^4} \\ \frac{v^2}{1-z^4} & \frac{z^4-z_s^4}{z_H z^2} \end{pmatrix}$$
- EOM and induced metric : world–sheet causal horizon at $z_s = z_H/\sqrt{\gamma}$



Interlude : Schwinger–Keldysh / Penrose diagram



$$iG(j,k) = \frac{1}{i^2}\frac{\delta^2 \ln Z\left[\eta_{1,2},\bar{\eta}_{1,2}\right]}{\delta\eta_j\delta\eta_k^\dagger} = i\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22}\end{pmatrix}.$$

$$\begin{cases} G_{11}(t,\mathbf{x}) = -i\langle T\Upsilon(t,\mathbf{x})\Upsilon^\dagger(0)\rangle, & G_{12}(t,\mathbf{x}) = \pm i\langle \Upsilon^\dagger(0)\Upsilon(t,\mathbf{x})\rangle, \\ G_{21}(t,\mathbf{x}) = -i\langle \Upsilon(t,\mathbf{x})\Upsilon^\dagger(0)\rangle, & G_{22}(t,\mathbf{x}) = -i\langle \hat{T}\Upsilon(t,\mathbf{x})\Upsilon^\dagger(0)\rangle, \end{cases}$$

The boundary SUGRA action yields the expected structure for real–time correlators (written here for scalar operator / bulk field):

$$\begin{cases} G_{11}(k) = \mathrm{Re}G_R(k) + i\coth(\frac{\omega}{2T})\mathrm{Im}G_R(k), \\ G_{12}(k) = \frac{2ie^{-(\beta-\sigma)\omega}}{1-e^{-\beta\omega}}\mathrm{Im}G_R(k), \\ G_{21}(k) = \frac{2ie^{-\sigma\omega}}{1-e^{-\beta\omega}}\mathrm{Im}G_R(k), \\ G_{22}(k) = -\mathrm{Re}G_R(k) + i\coth(\frac{\omega}{2T})\mathrm{Im}G_R(k). \end{cases}$$

- Kruskal coordinates :

$$UV = -\frac{1 - z/z_H}{1 + z/z_H} e^{-2\arctan(z/z_H)}, \quad \frac{V}{U} = -e^{4t/z_H}$$

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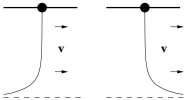
$$x_0^3 = \frac{v z_H}{2} \log V + v z_H \arctan(z/z_H)$$



- Expand fluctutions in retarded / advanced basis :

$$\psi_R(\omega, z) = A(\omega)\psi_{ret}(\omega, z) + B(\omega)\psi_{adv}(\omega, z),$$
$$\psi_L(\omega, z) = C(\omega)\psi_{ret}(\omega, z) + D(\omega)\psi_{adv}(\omega, z)$$

-

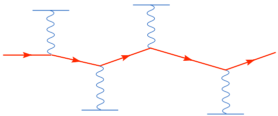
$$\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{\omega}{z_s T}} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

- $$iS_{bndry} = -\, i \int \frac{d\omega}{2\pi}\, \psi_a^0(-\omega) G_R^0(\omega) \psi_r^0(\omega) - \frac{1}{2} \int \frac{d\omega}{2\pi}\, \psi_a^0(-\omega) G_{\rm sym}(\omega) \psi_a^0(\omega)$$
- $\psi_r = \frac{1}{2}(\psi_R + \psi_L), \quad \psi_a = \psi_L - \psi_R$
- $G_{sym}(\omega) = -(1+2n)(\omega)\mathrm{Im}G_R^0(\omega)$, with the thermal distribution at $T_{eff} = T_{plasma}/\sqrt{\gamma}$
- Hubbard–Stratonovitch transform to linearize the quadratic term of the advanced field results in a Langevin equation for the heavy quark :

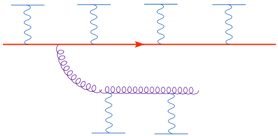
$$\int dt'\, G_R^0(t,t')\psi_r^0(t') + P^z - \xi(t) = 0, \quad \langle \xi(t)\xi(t') \rangle = G_{sym}(t,t')$$

- Einstein relation breaks down in the relativistic case. Seen in experimental data.

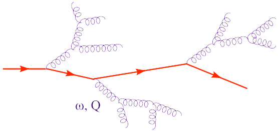
Dominant pQCD mechanism for broadening :



Energy loss at weak coupling :



At strong coupling :



- Extensions to other aAdS backgrounds, possibly with applications to 'latest game in town' AdS/CM [GCG, 0904.1874 (JHEP 0906:002), C. Hoyos-Badajoz, arXiv:0907.5036 (JHEP 0909:068), S. Gubser, A. Yarom, 0908.1392]
- The real–time formalism has been developped by Leigh, Hoang for Schr/nrCFT.
- Extensions for fermionic operators