Exact Black Hole Solutions in Einstein-Maxwell-Dilaton Theories with a Liouville potential, arXiv:0905.3337v1 [gr-qc]

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LAPTH

Outline

1. Introduction

2. Formalism
   - Metrics
   - Electromagnetic duality
   - Integrability

3. Black Hole Solutions

4. Conclusions

Action and Contents of the theory

$$S_4 = \int \mathrm{d}^4x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\left(\nabla\Phi\right)^2 - \frac{1}{4}\mathcal{F}^2 e^{\gamma\Phi} - 2\Lambda e^{-\delta\Phi}\right]$$

- To Einstein's pure gravity, we add electromagnetism and scalar matter (Dilaton).
- The dilaton couples both to electromagnetism and the cosmological constant, and has an exponential (Liouville) potential.

Equations of motion

- Einstein equation : $G_{\mu\nu} = T_{\mu\nu}\left(\Phi, \mathcal{F}\right)$
- Modified Maxwell equation : $0 = \partial_\mu\left(\sqrt{-g}e^{\gamma\Phi}\mathcal{F}^{\mu\nu}\right)$
- Dilaton eom : $\Box\Phi = \frac{\gamma}{4}e^{\gamma\Phi}\mathcal{F}^2 - 2\delta\Lambda e^{-\delta\Phi}$

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Interesting limits and Motivations

$$S_4 = \int \mathrm{d}^4 x \sqrt{-g_4} \left[\mathcal{R}_4 - \frac{1}{2} (\nabla \Phi)^2 - \frac{1}{4} \mathcal{F}^2 e^{\gamma \Phi} - 2\Lambda e^{-\delta \Phi} \right]$$

Two interesting limits :

- String theory low-energy effective actions (*Zwiebach86'*) $\longrightarrow \gamma = \delta = 1$.
- Dimensionally reduced 5D Kaluza-Klein theories (*Kaluza21'*, *Klein26'*) $\longrightarrow \gamma = \sqrt{3}$, $\delta = \frac{1}{\sqrt{3}}$.

Motivations :

- Study the 4-dimensional solutions of the general equations of motion.
- Find the 5-dimensional solutions, in the case where $\gamma = \sqrt{3}$, $\delta = \frac{1}{\sqrt{3}}$.
- Previous work by *Gibbons&Maeda88'*, *Garfinkle&Horowitz&Strominger91'*, *Wiltshire&al.94'*, *Mann&al.95'*,

Metric Ansatz

- Cylindrical Symmetry $\supset$ Planar Maximal Symmetry $\kappa = 0$

$$\begin{aligned} \mathrm{d}s^2 &= -e^{2\hat{U}_t}\mathrm{d}t^2 + e^{2\hat{U}_\varphi}\mathrm{d}\varphi^2 + e^{2V}\left(\mathrm{d}r^2 + \mathrm{d}z^2\right) \qquad \mathit{CLSZ07', CGS09'} \\ &\Updownarrow \\ \mathrm{d}s^2 &= -\alpha e^{2U}\mathrm{d}t^2 + \alpha e^{-2U}\mathrm{d}\varphi^2 + \alpha^{-\frac{1}{2}} e^{2\chi}\left(\mathrm{d}r^2 + \mathrm{d}z^2\right) \end{aligned}$$

- Maximal Symmetry :
  Set $\alpha^{\frac{3}{2}} = e^{2\chi+2U}$, change coordinates $\mathrm{d}x = \alpha \mathrm{d}r$ and generalize to constant curvature subspaces :

$$\mathrm{d}s^2 = -\alpha e^{2U}\mathrm{d}t^2 + \frac{\mathrm{d}x^2}{\alpha e^{2U}} + \alpha e^{-2U}\mathrm{d}\Omega^\kappa_{\mathrm{II}}$$

$\Rightarrow$ Flat Maximally Symmetric Subspace ! This is just the usual Black hole metric

$$\mathrm{d}s^2 = -V(r)\mathrm{d}t^2 + \frac{\mathrm{d}x^2}{} + R(r)^2 \qquad (1)$$

Cylindrical spacetimes : choice of coordinates

$$\begin{aligned} \mathrm{d}s^2_{\mathrm{cyl}} &= -\alpha e^{2U}\mathrm{d}t^2 + \alpha e^{-2U}\mathrm{d}\varphi^2 + \alpha^{-\frac{1}{2}}e^{2\chi}\left(\mathrm{d}r^2 + \mathrm{d}z^2\right) \\ \mathrm{d}s^2_{\mathrm{max}} &= -\alpha e^{2U}\mathrm{d}t^2 + \frac{e^{-2U}}{\alpha}\mathrm{d}x^2 + \alpha e^{-2U}\left(\frac{\mathrm{d}z^2}{1-\kappa z^2} + z^2\mathrm{d}\varphi^2\right) \end{aligned}$$

- To simplify the integrability analysis of the EOM difficult, we apply the coordinate change : $p = \frac{\mathrm{d}\alpha}{\mathrm{d}r}$, and set $X(p) \equiv \frac{p\mathrm{d}p}{\mathrm{d}\ln\alpha}$.
- Harmonic EOM :

$$\begin{array}{ccccc} \alpha'' & = & \dfrac{X(p)}{\alpha} & = -2\Lambda\sqrt{\alpha}e^{2\chi-\delta\Phi} \\ \text{2nd order} & \longrightarrow & \text{0th order} & \end{array}$$

- All quantities : Φ, χ, U and $A(p)$ can now be expressed in terms of $X(p)$: “1-dimensional” problem !

Electromagnetic duality

Two non-trivial cases :

- Electric case $\mathcal{A} = A(r)\mathrm{d}t$.
- Magnetic case $\mathcal{A} = \omega(z)\mathrm{d}\varphi$

There is a duality between these cases :

1. Determine an electric solution to the eom for a given γ and δ
2. Define a dual potential ω by $\partial_z\omega = e^{\gamma\Phi}\sqrt{-g}g^{rr}g^{tt}\partial_r A$
3. Replace A by ω in eom, then change $\gamma \longrightarrow -\gamma$: the eom are back to their original form.

To sum up

From a particular electric solution for a given (γ, δ) theory, we easily obtain the dual magnetic solution in the theory $(-\gamma, \delta)$

From here on, we restrict the analysis to the electric case.

A criterium for $\Lambda \neq 0$ solutions

Question :

How to make sure we discriminate $\Lambda = 0$ solutions, which are fully integrable (*Gibbons&Maeda88'*) ?

Answer : Harmonic equation

- Cylindrical coordinates :

$$\frac{X(p)}{\alpha} = -2\Lambda\sqrt{\alpha}e^{2\chi-\delta\Phi}$$

- Black Hole coordinates :

$$\ddot{\alpha}^2 - \kappa = -4\Lambda\alpha^{-\frac{1}{2}}e^{2\chi-\delta\Phi}$$

Theorem ($\Lambda \neq 0$ solutions)

To obtain $\Lambda \neq 0$ solutions, it is necessary and sufficient to impose

- *if $\kappa = 0$: $X(p) \neq 0$*

Resolution for cylindrical symmetry : Master equation

$$k - \frac{s\epsilon Q}{4}\dot{B}^2 - a\dot{B} + (1-\gamma\delta)p\dot{B} + (\gamma\delta - 1)B =$$
$$\ddot{B}\left(\frac{3-\delta^2}{2}p^2 + (1-\gamma\delta)\frac{a}{s}p - hp + \frac{Q}{2}k - \frac{c^2}{s} + \frac{Q}{2}(\gamma\delta - 1)B\right).$$

- k is an extra dof due to integration of $A(p)$.
- a is a "gauge" dof linked to $A(p)$ (to be fixed upon integration).
- Q is the electric charge defined through integration of Maxwell equation.
- h is an integration constant related to the mass of the solution.
- c is an integration constant which can be interpreted as the dilaton "charge". To be fixed to obtain black hole solutions

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- c is an integration constant which can be interpreted as the dilaton "charge". To be fixed to obtain black hole solutions $\sqrt{2}c$ γ^2+1 h No hair theorem

Resolution scheme : Summary

Cylindrical symmetry

↓

Coordinates : $r \to p$

↓

$[\Phi,\, \chi,\, U,\, A]\,(X(p))$

↓

Solve Master Equation

↓

$X(p)$

↓

$ds^2 =$

$X(p) \neq 0 \Leftrightarrow \Lambda \neq 0 \Leftrightarrow \alpha_2 \neq \kappa$

Maximal symmetry

↓

Coordinates : $r \to x$

↓

$[\Phi,\, \chi,\, U]\,(\alpha(x),\, A(x))$

↓

Solve 2 coupled ODEs

↓

$\alpha(x),\, A(x)$

↓

$ds^2 =$

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Black hole solution : general solution $\kappa = 0,\ \gamma\delta = 1$

Metric

$$\mathrm{d}s^2 = -(p-2)\frac{p^{\frac{-\delta^4+6\delta^2-1}{(1+\delta^2)(3-\delta^2)}}}{(\eta p+1-\eta)^{\frac{2\delta^2}{1+\delta^2}}}\mathrm{d}t^2 - \frac{e^{\delta\Phi_0}}{\Lambda(3-\delta^2)}\frac{p^{\frac{5\delta^4-6\delta^2-3}{(1+\delta^2)(3-\delta^2)}}}{p-2}(\eta p+1-\eta)^{\frac{2\delta^2}{1+\delta^2}}\mathrm{d}p^2$$

$$+\, p^{\frac{2(\delta^2-1)^2}{(\delta^2+1)(3-\delta^2)}}(\eta p+1-\eta)^{\frac{2\delta^2}{1+\delta^2}}\left(\mathrm{d}z^2+\mathrm{d}\varphi^2\right),$$

$$e^{\Phi} = e^{\Phi_0}(\eta p+1-\eta)^{\frac{2\delta}{\delta^2+1}}p^{\frac{4\delta(\delta^2-1)}{(\delta^2+1)(3-\delta^2)}}, \quad \dot{A}(p) = 2e^{-\frac{\Phi_0}{2\delta}}\sqrt{\frac{\delta^2(1-\eta^2)}{1+\delta^2}}\frac{1}{(\eta p+1-\eta)^2}$$

$$\mathcal{R} \sim P_1(p)(\eta p+1-\eta)^{-2\frac{2\delta^2+1}{1+\delta^2}}p^{\frac{3\delta^4-2\delta^2+3}{(\delta^2+1)(\delta^2-3)}} \sim_\infty p^{\frac{2\delta^2}{\delta^2-3}}$$

- Coordinate singularity at $p = 2$.
- Curvature singularities at $p = 0$ and $p_\eta = 1 - \frac{1}{\eta}$, $p_\eta < 0$.
- Staticity outside the hole $\Rightarrow \quad \Lambda(3 \quad \delta^2) > 0$

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Exotic asymptotics : general solution $\kappa = 0$, $\gamma\delta = 1$

Metric

$$q = p^{\frac{1}{3-\delta^2}} \quad \Longrightarrow \quad \mathrm{d}s^2 = q^2\Big(- \mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}\varphi^2\Big) - \frac{3-\delta^2}{\Lambda} e^{\delta\Phi_0} q^{2(\delta^2-1)} \mathrm{d}q^2$$

$$\mathcal{R}_\infty \sim q^{-2\delta^2} \longrightarrow 0\,, \quad e^{\Phi} \sim q^{\delta} \longrightarrow 0 \quad \text{if} \quad \delta < 0$$

- Ricci scalar and dilaton regular at spatial infinity if $\delta^2 < 3$: black hole.
- If $\delta^2 > 3$ ($\Lambda > 0$), q undefined at spatial infinity $\Rightarrow p = \frac{1}{q}$, which then gives a cosmological spacetime regular at spatial infinity.
- Funny asymptotics for generic δ. For $\delta = 0$: adS-space at spatial infinity, Schwarzschild-adS with a constant dilaton elsewhere (no electromagnetism).

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General solution $\gamma\delta = 1$: String case

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Polynomial solutions for generic couplings

For generic γ and δ, one can look for polynomial solutions for $B(p)$. The general solution then has :

- $A(p) \sim p$: no singularity in the Maxwell field $\rightarrow$ equivalent of $\eta = 0$ case of the general $\gamma\delta = 1$ solution $\left(A = a + \frac{p-1+\eta}{\eta p-\eta+1}\right)$.
- $e^{\Phi} = e^{\Phi_0} p^{\frac{4(\delta-\gamma)}{3\gamma^2-\delta^2-2\gamma\delta+4}}$: non-trivial dilaton for $\delta = 0$.
- One coordinate singularity (horizon) and one curvature singularity.
- Funny asymptotics for every non-zero value of the couplings.

This is the only non-trivial solution found for $\delta = 0 \longrightarrow$ pure cosmological constant case.

Classes of solutions, by asymptotic behaviour (*Wiltshire94'*)

Asymptotically, go to "black hole" coordinates :

$$\mathrm{d}s^2 = -g_{tt}\mathrm{d}t^2 + g_{qq}\mathrm{d}q^2 + q^2\mathrm{d}\Omega^{\kappa}_{\mathrm{II}}$$

Class	Parameters	g_{tt}	g_{qq}	e^{Φ}	Solutions found
K	$\sigma = 1,\ \kappa > 0$	$q^{\frac{2}{\gamma^2}}$	1	$q^{\pm\frac{2}{\gamma}}$	Mann95', CSG09'
M	$\sigma\kappa > 0$	1	1	1	
N	$\delta^2 < 3,\ \sigma\Lambda < 0,$ $\kappa = 0$	q^2	$q^{2(\delta^2-1)}$	$q^{2\delta}$	Mann95, Cai97', CGS09'
P	$\sigma\Lambda < 0,$ $sign(\kappa) = sign(\sigma(\delta^2-1))$	$q^{\frac{2}{\delta^2}}$	1	$q^{\frac{2}{\delta}}$	Mann95', Cai97', CSG09'
T	$\sigma\Lambda w < 0,\ \kappa = 0,$ $\sigma uv < 0$	$q^{2\frac{\gamma^2-\delta^2+4}{(\gamma-\delta)^2}}$	$q^{2\frac{\delta+\gamma}{\delta-\gamma}}$	$q^{\frac{4}{\delta-\gamma}}$	⊃Cai97', CGS09'

Conclusions and Perspectives

- Integrability of the EMD system has been studied in detail : much more constrained for non-planar 2-dimensional spatial sections.
- Electro-magnetic duality for non-zero cosmological constant.
- No regular asymptotics solutions for generic δ.
- For $\delta = 0$, regular asymptotics but with $\gamma \to \infty$: constant scalar field and no Maxwell field.
- Maximal spacetimes : generically one extra horizon ($\kappa \neq 0$).

  $$\Updownarrow$$

  Cylindrical subspaces : generically one extra singularity ($\kappa = 0$).
- Inclusion of expected higher-order curvature/scalar corrections (Gauss-Bonnet term) in string action/Kaluza-Klein decomposition ? Work in progress . . .