

# MSSM Parameters from Orbifold GUTs

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partly based on arXiv:0906.2957 (“Phenomenology of SUSY Gauge-Higgs Unification”) with Sylvain Fichet, Sabine Kraml (LPSC Grenoble) and Arthur Hebecker (Heidelberg)

# Beyond the Standard Model: the MSSM

- minimal SUSY extension of Standard Model
- $\mathcal{O}(100)$  additional free parameters, mostly parameterizing soft SUSY breaking
- for phenomenological studies: restrict parameter space
  - by pheno arguments (e.g. no excessive flavour/CP violation observed. . . )
  - by imposing ad-hoc simplicity constraints, universality relations. . .
  - **by assuming an underlying model of UV-scale physics**

# Beyond the MSSM: SUSY orbifold GUTs

- Models with extra dimensions compactified at GUT scale
- Well-motivated from the top down: heterotic string compactifications
- Well-motivated from the bottom up: unification of MSSM gauge couplings
- The models we will consider are mostly field-theoretic, but can be found in stringy models in certain limits
- **Interesting possibilities to generate MSSM parameters:**  
depending on model details and assumed SUSY breaking mechanism
- **Work out spectra and signatures at low energies!**

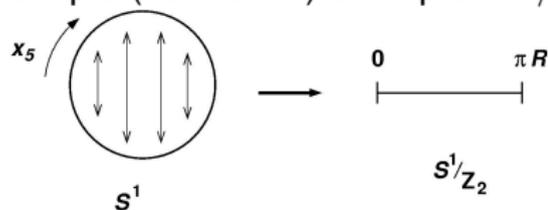
# Outline

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  - Orbifolds in field theory
  - Example: The Burdman-Nomura model
  - Heterotic orbifolds
  - The GUT/string-scale problem and anisotropic limits
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  - Example: MSSM parameters for the Burdman-Nomura model
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  - Gauge-Higgs unification relations for Higgs mass parameters
  - Renormalization group analysis
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# Orbifold GUTs in field and string theory

# Orbifolds as extra dimensions

- Orbifold = quotient of torus / (discrete group action with fixed points)
- everywhere smooth except at the fixed points: “branes”
- simple (but useful) example:  $S^1/\mathbb{Z}_2$



- identify  $x_5 \sim -x_5$  (and  $x_5 \sim x_5 + 2\pi R$  on circle)
- Spacetime is  $\mathbb{R}^{3,1} \times S^1/\mathbb{Z}_2$ : bulk is 5d, branes are 4d
- On  $S^1$ : 4d low-energy d.o.f. are Kaluza-Klein zero modes of bulk fields
- On orbifold: can also have fields localized on the branes
- On orbifold: discrete group acts on field space  
→ part of the bulk zero modes projected out

# Orbifold action on field space

$S^1/\mathbb{Z}_2$  example:

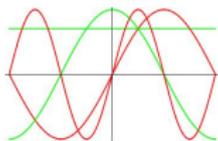
Consider scalar field  $\phi$  on  $\mathbb{R}^{3,1} \times S^1$ . KK mode expansion:

$$\phi(x_\mu, x_5) = \sum_{n=0}^{\infty} \phi_+^{(n)}(x_\mu) \cos \frac{nx_5}{R} + \sum_{n=1}^{\infty} \phi_-^{(n)}(x_\mu) \sin \frac{nx_5}{R}$$

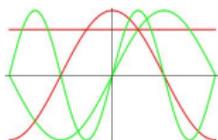
Zero mode is  $\phi_+^{(0)}(x_\mu)$

Now divide by  $\mathbb{Z}_2$  acting as  $\phi(x_\mu, -x_5) = P\phi(x_\mu, x_5)$  ( $P = +1$  or  $-1$ ):

- If  $P = +1$ , only even modes  $\phi_+^{(n)}$  survive



- If  $P = -1$ , only odd modes  $\phi_-^{(n)}$  survive: **zero mode projected out**



# Symmetry breaking by orbifolds

For illustration: Consider  $SU(3)$  gauge theory on  $S^1$ .  
Divide by  $\mathbb{Z}_2$  acting on field space as

$$A_\mu(x_\mu, -x_5) = \mathcal{P} A_\mu(x_\mu, x_5) \mathcal{P}^{-1}, \quad \text{where } \mathcal{P} = \text{diag}(1, -1, -1)$$

This breaks  $SU(3) \rightarrow SU(2) \times U(1)$  in the 4d effective theory.

- Gauge symmetry in the 5d bulk is still  $SU(3)$
- An  $SU(3)$  triplet  $\phi$  with  $\mathbb{Z}_2$  acting as

$$\phi(x_\mu, -x_5) = \mathcal{P} \phi(x_\mu, x_5)$$

will only have a 4d gauge singlet zero mode.

Generally: Bulk fields transforming under the bulk gauge group may lead to **split representations** in 4d effective theory.

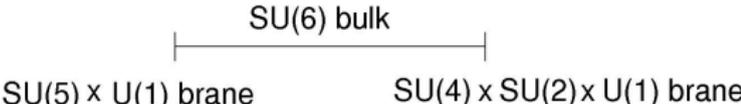
- Can apply these mechanisms to GUT models ( $\rightarrow$  Kawamura '00, ...)
    - break GUT group  $\rightarrow$  (MS)SM gauge group
    - project out unwanted components of Higgs fields: **Doublet-triplet splitting**
- and in particular to **5d SUSY GUTs**

# Example: the Burdman-Nomura model

(→ Burdman/Nomura '02)

The MSSM from a 5d SU(6) SUSY gauge theory:

- 5d gauge supermultiplet splits into
  - 4d gauge supermultiplet  $V \supset A_\mu$  ( $\mu = 0 \dots 3$ )
  - 4d chiral adjoint  $\Phi \supset \Sigma + iA_5$
- Orbifold in this case is  $S^1 / (\mathbb{Z}_2 \times \mathbb{Z}'_2)$
- Choose suitable orbifold boundary action on  $V$  and  $\Phi$  to obtain

gauge structure: A diagram showing a horizontal line with vertical end caps. Above the line is the text "SU(6) bulk". Below the line, on the left side, is "SU(5) x U(1) brane" and on the right side is "SU(4) x SU(2) x U(1) brane".

- In 4d:  $SU(6) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$   
All other  $V$  zero modes projected out  
All  $\Phi$  zero modes projected out except two  $SU(2)_L$  doublets  $H_1$  and  $H_2$   
These become the MSSM Higgs fields: **Gauge–Higgs unification**

# The Burdman-Nomura model

Add charged matter and choose appropriate orbifold projections:

Hypermultiplet	SU(6) rep	zero mode 4d quantum numbers
$\mathcal{U}$	<b>20</b>	$(\mathbf{3}, \mathbf{2})_{1/6, -3} + (\mathbf{\bar{3}}, \mathbf{1})_{-2/3, -3} + (\mathbf{1}, \mathbf{1})_{1, -3}$
$\mathcal{D}$	<b>15</b>	$(\mathbf{3}, \mathbf{2})_{1/6, 2} + (\mathbf{\bar{3}}, \mathbf{1})_{1/3, 4}$
$\mathcal{E}$	<b>15</b>	$(\mathbf{\bar{3}}, \mathbf{1})_{-2/3, 2} + (\mathbf{1}, \mathbf{1})_{1, 2} + (\mathbf{1}, \mathbf{2})_{-1/2, 2}$
$\mathcal{N}$	<b>6</b>	$(\mathbf{1}, \mathbf{1})_{0, 5} + (\mathbf{1}, \mathbf{2})_{-1/2, 1}$

Nearly MSSM — but **two quark and lepton doublets each** and other **exotics**

Also, Yukawas (from 5d gauge couplings) have wrong structure:

- up-type RH quark couples only to “up-type quark doublet”
- down-type RH quark only to “down-type doublet”
- Leptons similarly

Introduce **brane-localized superfields** with appropriate superpotentials:

- unwanted fields decouple, only one diagonal combination of each quark doublets and lepton doublets remains massless.
- Also breaks  $U(1)_X$  spontaneously on the  $x_5 = 0$  brane.

# Heterotic strings on orbifolds

( $\rightarrow$  Dixon et al. '85-'86, Hamidi/Vafa '86, Narain et al. '86, Ibáñez/Nilles/Quevedo '87,...)

Related models found in orbifold compactifications of the

$E_8 \times E_8$  heterotic superstring

Superstring lives in 10 dimensions: compactify 6 of them

- Torus compactification on  $T^6$  gives simple, calculable 4d effective theory, but too much supersymmetry
- To get 4d  $\mathcal{N} = 1$  SUSY, compactification manifold must be “Calabi-Yau”: very complicated objects
- Special, calculable, singular limit of Calabi-Yau manifolds: **Toroidal orbifolds**  
 $T^6/\mathbb{Z}_N$



# Heterotic strings on orbifolds

String theory on orbifolds well-behaved despite singularities at fixed points

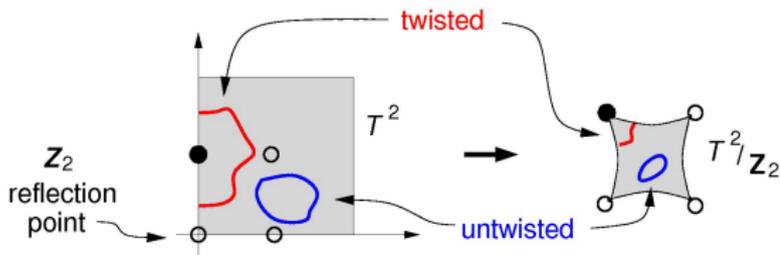
2 kinds of closed strings:

- closed on  $T^6$  already: “**untwisted**” sector  $\rightarrow$  zero modes = **bulk fields**
- closed not on  $T^6$  but on  $T^6/\mathbb{Z}_N$ :  
“**twisted**” sector  $\rightarrow$  states live on fixed points (or fixed planes), **brane fields**

2-dimensional visualization:

$$T^2/\mathbb{Z}_2$$

(opposite edges identified,  
 $\mathbb{Z}_2$  acts as reflection)



- Massless string states carry  $E_8 \times E_8$  gauge symmetry
- Partly broken by orbifold compactification; breaking pattern depends on details of group action, Wilson line backgrounds...
- Can get SM gauge group:  $E_8 \supset E_6 \supset SO(10) \supset SU(3) \times SU(2) \times U(1)$
- Obtaining realistic MSSMs nontrivial: Many parameters, many constraints (from stringy consistency)



## Example: the BHLR model

( $\rightarrow$  Buchmüller/Hamaguchi/Lebedev/Ratz '05, '06; Kobayashi et al. '04, Lebedev et al. '06-'08)

Construction:

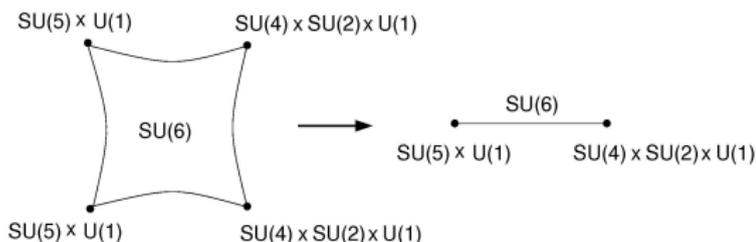
- $E_8 \times E_8$  heterotic on  $T^6/\mathbb{Z}_6$  orbifold
- $T^6$  obtained as  $\mathbb{R}^6/\Lambda$ , where  $\Lambda$  is a particular 6d lattice (root lattice of Lie algebra  $\mathfrak{g}_2 \times \mathfrak{su}_3 \times \mathfrak{so}_4$ )
- $T^6 \simeq T^2 \times T^2 \times T^2$ , with  $\mathbb{Z}_6$  acting by rotation on three 2-tori separately
- Wilson lines (gauge potential vevs) chosen appropriately

Some properties:

- gauge group is  $SU(3)_C \times SU(2)_L \times U(1)_Y \times G_{\text{hidden}}$
- chiral matter spectrum is precisely the MSSM
- model admits SUSY MSSM vacua with all exotics decoupled

Possible **5d limit**: shrink  $\mathfrak{g}_2$  and  $\mathfrak{su}_3$  tori to zero. Obtain 6d orbifold GUT on  $T^2/\mathbb{Z}_2$ .

Further shrinking vertical  $S^1$  gives model resembling Burdman/Nomura.



# MSSM parameters from orbifold GUTs

# Higgs sector and $\mu$ problem

The  $\mu$  problem: SUSY allows for a superpotential term

$$W = \mu H_1 H_2$$

Need  $\mu \approx m_{3/2} \approx \text{TeV}$  because Higgs is light — but  $\mu$  is a SUSY parameter.

**Why should  $\mu$  be connected to the scale of SUSY breaking?**

Need to forbid tree-level, renormalizable  $\mu$  term and generate effective  $\mu$  term through SUSY breaking mechanism

Two ideas of interest for orbifold GUT model building:

- $\mu$  from superpotential  $W$  ( $\rightarrow$  Kim/Nilles '83, Casas/Muñoz '93)
- $\mu$  from Kähler potential  $K$  ( $\rightarrow$  Giudice/Masiero '88)

# $\mu$ from $W$ with accidental R-symmetries

( $\rightarrow$  Kappl et al. '08; FB/Kappl/Ratz/Schmidt-Hoberg, work in progress)

- R-symmetry = symmetry which does not commute with SUSY  
Superpotential carries R-charge:  $R[W] = 2$
- Many heterotic orbifold models come with **discrete** R-symmetries acting on zero modes  
E.g.  $\mathbb{Z}_N$ : For term  $M(\phi_i)$  to be allowed in  $W$ , require  $R[M] = 2 \bmod N$
- Taking only the lowest-order terms in  $W$  into account, this “looks like” a U(1) R-symmetry.  
Toy example: For  $N = 10$ ,  $R[\phi_1] = 2$ ,  $R[\phi_2] = -1$  get

$$W = \phi_1 + \phi_1^2 \phi_2^2 + \phi_1^6 + \text{higher powers}$$

**Higher order terms** of order  $\geq K$  (here  $K = 6$ ) break  $U(1)_R$  explicitly to  $\mathbb{Z}_N$

## $\mu$ from $W$ with accidental R-symmetries

If  $U(1)_R$  were exact, would get  $\langle W \rangle = 0$  in SUSY vacua by

$$2W = R[W]W = \sum_i R[\phi_i] \frac{\partial W}{\partial \phi_i} \phi_i = -\frac{1}{2\sqrt{2}} \sum_i R[\phi_i] \{ \bar{Q}_{\dot{\alpha}}, \bar{\psi}_i^{\dot{\alpha}} \phi_i \}$$

Generically  $\langle W \rangle = 0$  even **term by term** (not so easy to see)

But since  $U(1)_R$  is broken by powers  $\sim \phi^K$ , get  $\langle M_a \rangle \lesssim \langle \phi \rangle^K$  for terms  $M_a$  in  $W$

- If field vevs  $\langle \phi \rangle$  mildly suppressed w.r.t.  $M_{\text{Planck}}$  (typically by a loop factor) then  $\langle W \rangle$  is suppressed by several powers:  $\langle W \rangle \sim \langle \phi \rangle^K$
- In SUSY breaking Minkowski vacua,  $\langle W \rangle \sim m_{3/2} \dots$
- ... thus obtain **hierarchically small**  $m_{3/2} \sim \langle \phi \rangle^K$ .
- Furthermore: In models like BHLR, Higgs bilinear  $H_1 H_2$  is singlet w.r.t. all selection rules. Thus

$$W = \sum_a M_a(\phi_i) + \sum_a M_a(\phi_i) H_1 H_2$$

and since  $\langle M_a \rangle \lesssim m_{3/2}$ , integrating out the  $\phi_i$  gives effective  $\mu$

$$\mu \sim m_{3/2}$$

# $\mu$ from $K$ with modulus-dominated SUSY breaking

( $\rightarrow$  Antoniadis et al. '94, Brignole/Ibáñez/Muñoz '96)

General idea (Giudice-Masiero mechanism): Operators such as

$$\int d^4\theta (Z + \bar{Z}) H_1 H_2$$

induce an **effective  $\mu$  term** if  $Z$  breaks SUSY,  $\mu \sim \langle \bar{F}_Z \rangle$

In string models typical Kähler potentials take the form

$$K = -3 \log [(T + \bar{T}) + (H_1 + \bar{H}_2)(\bar{H}_1 + H_2)] + \dots$$

$T$  = modulus = field controlling compactification geometry, massless at tree-level, must be stabilized by non-perturbative effects when breaking SUSY

- Generically  $\langle F_T \rangle \neq 0$
- Assume this term is dominant for SUSY breaking mediation:  
Expanding  $T = R + F_T \theta^2 + \dots$  in  $K$  gives effective  $\mu$  term  $\sim F_T/2R$   
(as well as  $B_\mu$  term and soft masses  $m_{H_1}^2, m_{H_2}^2$ )

## $\mu$ from $K$ + soft terms in a concrete model

In simplified setting of **5d orbifold model**: only one modulus  $T$   
 $T$  = “radion”, controlling size of extra dimension:  $\langle T \rangle = R$  = interval length  
**Radion-mediated SUSY breaking** ( $\rightarrow$  Chacko/Luty '00)

In effective 4d SUGRA: also include chiral compensator field  $\varphi = 1 + F_\varphi \theta^2$   
= non-dynamical, contains auxiliary field  $F_\varphi$  of 4d gravity multiplet

Parameterize SUSY breaking by two sources:

- $\langle F_T \rangle \neq 0$ , SUSY breaking in radion multiplet
- $\langle F_\varphi \rangle \neq 0$ , SUSY breaking in 4d gravity multiplet

Regard both as background fields

Also assume radion stabilized, cosmological constant tuned to zero etc. by unspecified mechanisms

With these assumptions, can calculate soft terms e.g. for 5d  
Burdman-Nomura model

$\rightarrow$  Choi et al. '03, Hebecker/March-Russell/Ziegler '08, FB/Fichet/Hebecker/Kraml '09

## MSSM parameters: Gaugino mass

5d gauge-kinetic + Chern-Simons action contains term

$$S_5 \supset \text{tr} \int d^5x \int d^2\theta \left( \frac{1}{2g_5^2} + c\Phi \right) W^\alpha W_\alpha + \text{h.c.}$$

where  $\Phi$  = chiral adjoint from 5d gauge multiplet containing  $H_1, H_2$ ;  
 $c$  = coefficient of Chern-Simons term in 5d (free parameter)

In terms of dimensionless combination  $c' = 4cv g_5^2$  (where  $v \sim |\langle \Phi \rangle|$ ,  
 $g_4^2 = \pi R g_5^2$ ) obtain 4d Lagrangian

$$\mathcal{L}_4 \supset \frac{1}{g_4^2} \text{tr} \int d^2\theta \left( \frac{T}{R} + c' \right) W^\alpha W_\alpha + \text{h.c.}$$

Replacing  $T$  with its vev,  $T = R + F_T \theta^2$ , gives universal gaugino mass

$$M_{1/2} = \frac{\overline{F_T}}{2R} \frac{1}{1 + c'/2}$$

# MSSM parameters: Higgs sector

Notation:

$$V = (|\mu|^2 + m_{H_1}^2)|H_1|^2 + (|\mu|^2 + m_{H_2}^2)|H_2|^2 + B\mu(H_2H_1 + \text{h.c.})$$

From 5d gauge-kinetic and Chern-Simons action obtain 4d Kähler potential and thus

$$\mu = \bar{F}_\varphi - \frac{\bar{F}_T}{2R} \frac{1 + 2c'}{1 + c'}$$

$$\begin{aligned} |B\mu| &= |\mu|^2 + m_{H_1}^2 = |\mu|^2 + m_{H_2}^2 \\ &= |F_\varphi|^2 - \frac{F_\varphi \bar{F}_T + \text{h.c.}}{2R} \frac{1 + 2c'}{1 + c'} + \frac{|F_T|^2}{(2R)^2} \frac{2c'^2}{(1 + c')^2} \end{aligned}$$

Notice “**Gauge-Higgs unification relations**” for MSSM Higgs mass parameters  
(→ Brignole et al. '95)

Reason: For models with gauge-Higgs unification, Kähler potential only depends on combination  $|\bar{H}_1 + H_2|^2$

# MSSM parameters: Sfermion soft terms

Matter soft terms: more model dependence, additional parameters

For definiteness follow Burdman-Nomura model: 3rd generation matter from 5d bulk hypermultiplets, Yukawa couplings from 5d gauge couplings.

Recall e.g. quark sector:

Hypermultiplet	SU(6) rep	zero mode 4d quantum numbers
$\mathcal{U}$	<b>20</b>	$(\mathbf{3}, \mathbf{2})_{1/6, -3} + (\bar{\mathbf{3}}, \mathbf{1})_{-2/3, -3} + (\mathbf{1}, \mathbf{1})_{1, -3}$
$\mathcal{D}$	<b>15</b>	$(\mathbf{3}, \mathbf{2})_{1/6, 2} + (\bar{\mathbf{3}}, \mathbf{1})_{1/3, 4}$

Two quark doublets need to mix via brane field for one diagonal combination to decouple  $\rightarrow$  additional free parameter: mixing angle  $\phi_Q$

- In total, 3 more parameters:  $\phi_Q$  and 5d bulk masses  $M_u$  and  $M_d$
- subject to 2 constraints: proper  $y_t$  and  $y_b$  must be reproduced (for fixed ratio  $\tan \beta$  of Higgs vevs)
- determines squark soft masses and trilinear couplings together with  $F_T$
- 3rd generation leptons analogous

# Phenomenology

# Implications of gauge-Higgs unification mass relations

Notation:  $m_1^2 \equiv |\mu|^2 + m_{H_1}^2$ ,  $m_2^2 \equiv |\mu|^2 + m_{H_2}^2$ ,  $m_3^2 \equiv B\mu$ .

Gauge-Higgs unification relations

$$m_1^2 = m_2^2 = |m_3^2|$$

hold at GUT/compactification scale  $\approx 10^{16}$  GeV.

For stable Higgs potential with EWSB, need

$$m_1^2 m_2^2 - m_3^4 < 0 \quad (\text{EWSB})$$

$$m_1^2 + m_2^2 - 2m_3^2 > 0 \quad (\text{D-flat directions stabilized})$$

at minimization scale  $M_S =$  soft mass scale  $\approx 1$  TeV.

RG running will have to do this if our models are to be realistic.

## Some remarks on RG evolution

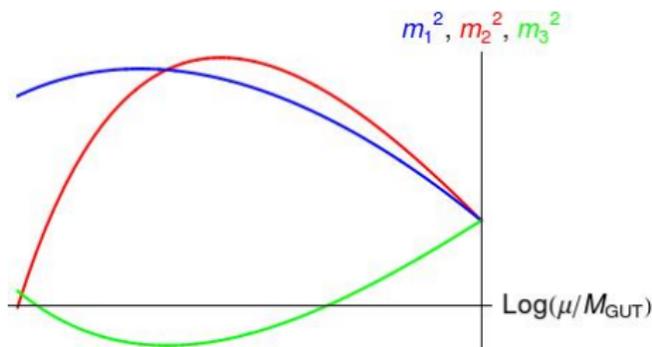
Needed for fully realistic MSSM vacua (Higgs mass above LEP bound):

- At least moderately large  $\tan \beta$   
(to be close to saturating tree-level Higgs mass bound)
- Small  $M_Z/M_S$   
(to allow for large stop loop corrections to Higgs mass)

This translates *very roughly* into  $m_2^2 \ll m_1^2$  and  $m_3^2 \ll m_1^2$  at  $M_S$   
(assuming  $m_1^2(M_S)$  is of its natural order of magnitude  $\approx M_S^2$ ), since

$$\tan \beta + \cot \beta = \frac{m_1^2 + m_2^2}{2m_3^2}, \quad \frac{M_Z^2}{2} = (m_2^2 - m_3^2 \cot \beta) / \cos 2\beta$$

So should expect a running pattern similar to this:



# Some remarks on RG evolution

One-loop RGEs:

$$16\pi^2 \frac{d}{dt} \mu = \mu \left( 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right)$$

$$16\pi^2 \frac{d}{dt} B_\mu = B_\mu \left( 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right) \\ + \mu \left( 6a_t \bar{y}_t + 6a_b \bar{y}_b + 2a_\tau \bar{y}_\tau + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1 \right)$$

$$16\pi^2 \frac{d}{dt} m_{H_1}^2 = 3X_b + X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2$$

$$16\pi^2 \frac{d}{dt} m_{H_2}^2 = 3X_t - 6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2$$

where

$$X_{t,b,\tau} = 2|y_{t,b,\tau}|^2 \left( m_{H_2,1,1}^2 + m_{Q,Q,L}^2 + m_{t,b,\tau}^2 \right) + 2|a_{t,b,\tau}|^2$$

Detailed numerical analysis needed

# RG analysis of gauge-Higgs unified models with radion mediation

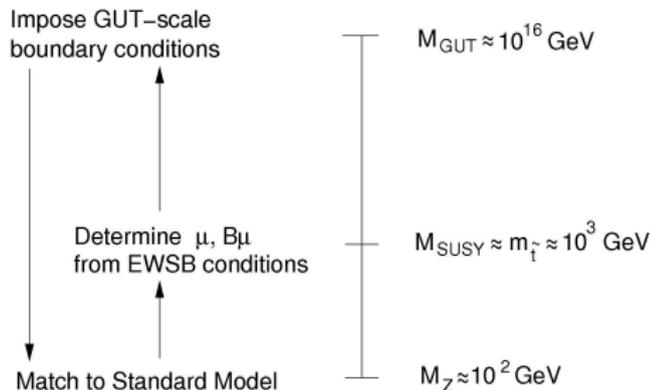
- Previous analysis (→ Choi et al. '03) pessimistic: radion-mediated SUSY breaking seems too restrictive?  
But: not based on realistic models,
  - too strict assumptions on sfermion masses and trilinears
  - no Chern-Simons term included (this will turn out to be crucial!)and not using state-of-the-art spectrum generators
- Now improved analysis with realistic constraints on parameters, (e.g. as in Burdman-Nomura model), and allowing for CS contribution (→ FB/Fichet/Hebecker/Kraml '09)
- Using modified version of SuSpect code (→ Djouadi/Kneur/Moultaka '02)
- Relic densities and rare decay branching rates computed with micrOMEGAs 2.2 (→ Bélanger/Boudjema/Pukhov/Semenov '08)

# How a SUSY spectrum generator works

- important dimensionful MSSM parameters:
  - Higgs sector:  $m_{H_1}^2, m_{H_2}^2, B\mu, \mu$
  - gaugino masses:  $M_{1,2,3}$
  - 3rd generation sfermion soft masses:  $m_u^2, m_d^2, m_q^2, m_\tau^2$
  - trilinear terms:  $A_t, A_b, A_\tau$

Evolve to electroweak scale  $\rightarrow$  generically predicts wrong  $M_Z$ , cannot match to Standard Model

- usual procedure: exchange  $\mu$  and  $B\mu$  at high scale for  $M_Z$  and  $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$  at low scale
- determine spectrum iteratively:



# RG analysis of gauge-Higgs unified models

Challenge for implementing gauge-Higgs unification relations:

- GHU relation holds between  $\mu$ ,  $B\mu$ ,  $m_{H_i}^2$  at GUT scale, but  $\mu$  and  $B\mu$  are **outputs** in standard spectrum generators  
→ not straightforward to restrict parameter space scans to points with GHU relations!
- Ideally would like  $\mu$  and  $B\mu$  to be inputs and predict  $\tan \beta$   
→ difficult to implement, numerical problems especially at large  $\tan \beta$
- Next best thing: still use  $\tan \beta$  and  $M_Z$  as inputs, **adjust  $m_{H_i}^2$  iteratively to satisfy GHU relations**  
This turns out to be numerically stable

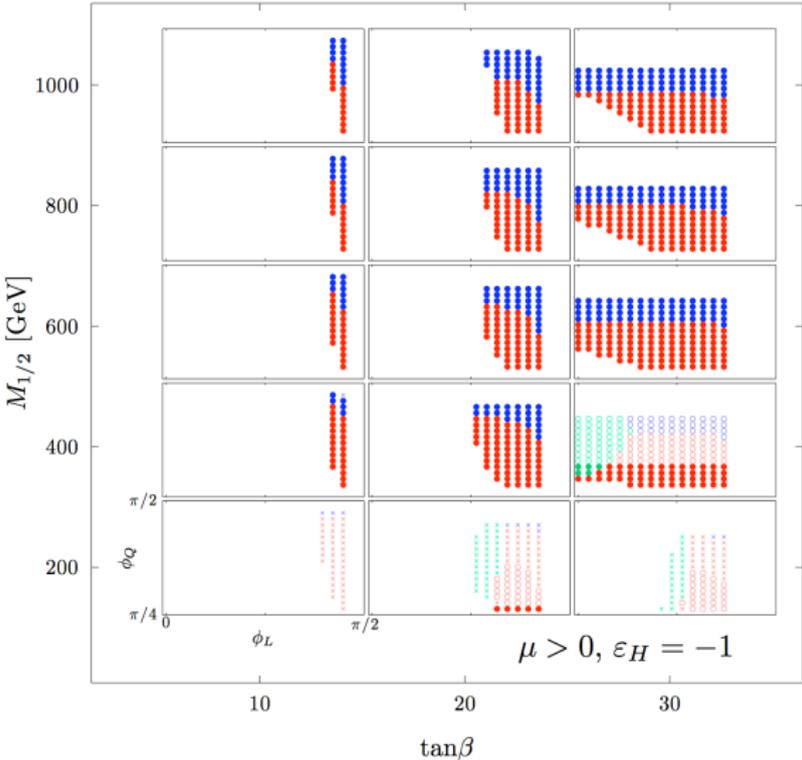
Another challenge for implementing Burdman-Nomura model:

- 3rd generation matter soft terms depend on bulk-brane mixing angles  $\phi_Q$ ,  $\phi_L$  and on explicit 5d masses  $M_U$ ,  $M_D$ ,  $M_E$ : **5 parameters**
- But these also fix Yukawa couplings  $y_t$ ,  $y_b$ ,  $y_\tau$ : **3 constraints**
- Choose  $\phi_Q$ ,  $\phi_L$  as **independent parameters**; add another iteration level

# Parameter scan summary

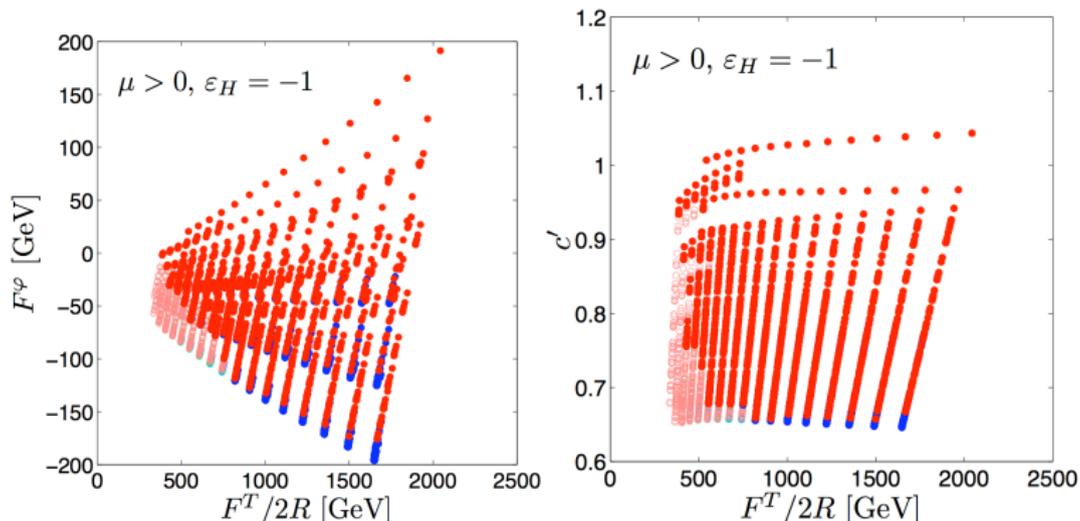
- Gauge-Higgs sector soft terms determined by radion F-term  $F^T/2R$ , compensator F-term  $F^\varphi$  and Chern-Simons parameter  $c'$
- Scan  $M_{1/2}$  between 100 and 1000 GeV and  $\tan\beta = 10, 20, 30$ .  
Reconstruct fundamental model parameters afterwards
- 3rd generation matter soft terms also depend on bulk-brane mixing angles  $\phi_Q$  and  $\phi_L$
- Vary  $\phi_Q$  between  $\pi/4$  and  $\pi/2$ ,  $\phi_L$  between 0 and  $\pi/2$
- First two generation soft terms taken to be zero (effectively brane-localized)
- Two discrete parameters:  $\text{sign}(\mu)$  and  $\epsilon_H = \text{sign}(B\mu)$  at  $M_{\text{GUT}}$

# Results



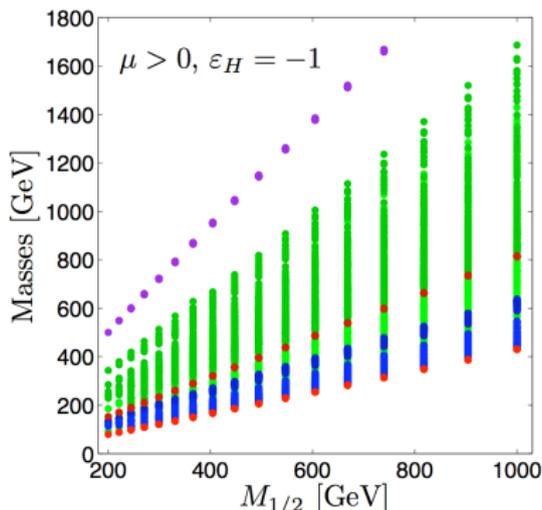
Neutralino, stau, selectron LSP. Small points excluded by LEP or B-physics

# Results



Neutralino, stau, selectron LSP. Open circles excluded by B-physics  
Recall  $F^T/2R$  = radion contribution,  $F^\varphi$  = compensator contribution,  
 $c'$  = Chern-Simons parameter — note  $c' \neq 0$  excluded

# Results: Sparticle masses in neutralino LSP region



Neutralinos, staus, selectrons, gluino

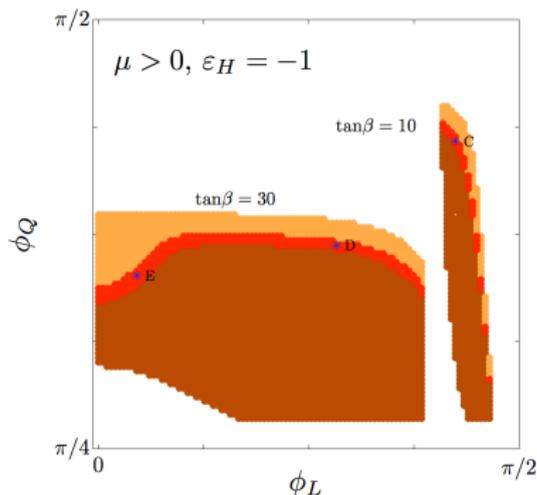
Note small NLSP-LSP mass difference

Also  $\tilde{\chi}_2^0$  heavier than selectrons (sometimes also heavier than stau):

decay  $\tilde{\chi}_2^0 \rightarrow \ell^\pm \tilde{\ell}^\mp \rightarrow \ell^\pm \ell^\mp \tilde{\chi}_1^0$  kinematically allowed, large BR

“Same-flavour-opposite-sign” dilepton signature at LHC

# Results: Neutralino relic density



**Red band** = relic density lies within  $3\sigma$  of WMAP5 observation.

Orange region:  $\Omega h^2$  too low (other DM components besides  $\tilde{\chi}_0$  required)

Brown region:  $\Omega h^2$  too high (with standard cosmology)

# Conclusions

- SUSY orbifold GUTs are viable models for UV-completing the MSSM
- Simple field-theoretic examples exist (e.g. the Burdman-Nomura model)
- More elaborate models can be found in heterotic string theory, with field-theoretic 5d models as well-motivated limiting cases
- They offer interesting mechanisms to generate MSSM parameters, e.g. the  $\mu$  parameter
- Models with gauge-Higgs unification give special Higgs mass relations
- Assuming radion-mediated SUSY breaking, many MSSM soft terms are calculable from a few fundamental parameters
- Effects of Chern-Simons term must be included to obtain viable spectrum
- A simple example model (the Burdman-Nomura model) can give realistic phenomenology for a wide range of parameters
- Predictions:
  - SFOS dilepton signature
  - rather light NLSP
  - (mainly due to no-scale boundary conditions in first two generations)

# Outlook

- More needs to be done in understanding moduli stabilization and SUSY breaking in stringy models (work in progress with TU Munich group)
- “Gauge-Higgs unification relations” interesting because they not only appear in GHU models. Compare “holographic GUT” model by Nomura/Poland/Tweedie '06. MCMC scan of parameter space underway (work in progress with Fichet/Kraml/Singh)
- Also potentially helpful to distinguish from other models with similar collider signatures
- Another promising project: Work out implications for flavour physics in a concrete model
- Probably no “smoking gun signature” for orbifold GUTs exists.  
But LHC may at least find lots of evidence for (or against) this class of models.