New Results About The "Universal One Loop Effective Action"

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Based on: S. A. Ellis, J. Quevillon, P. N. H. V, T. You, Z. Zhang [2006.16260]
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Outline of This Talk

I. Introduction

- EFT from the UV point-of-view (matching - running - mapping)

II. Review functional method & previous results in literatures

- Effective action at tree and one-loop level
- Expansion by regions: Hard vs Soft
- Covariant Derivative Expansion (CDE)
- The "Universal One-Loop Effective Action" (UOLEA)
- Previous literatures and missing pieces

III. UOLEA: integrating out heavy-fermions

- Methodology & Results

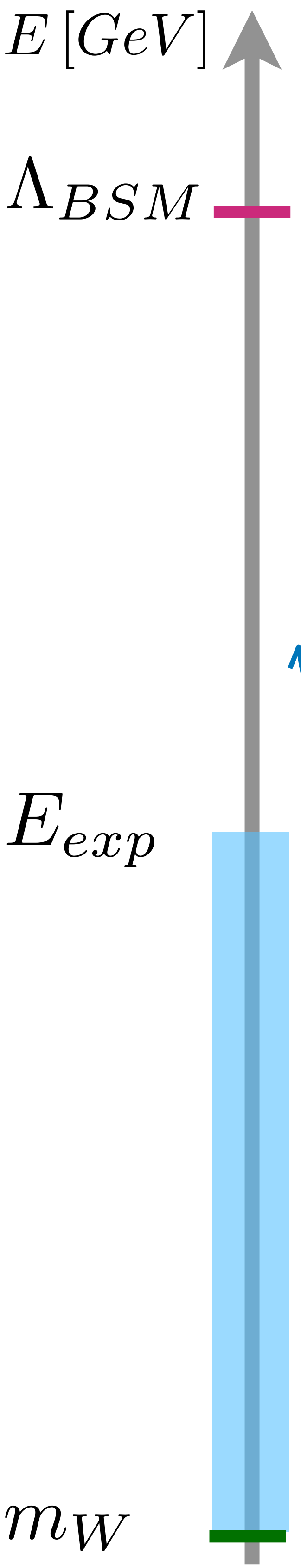
IV. Examples

- Effective couplings $A\gamma\gamma, AZZ, AZ\gamma$

Summary

Backup slides (full new results in degenerate case, covariant diagrams)

Intro: EFT from the UV point-of-view



Scale of New Physics
(New physics should be rather heavy or weakly coupled to the SM)

How to approach new physics with the current and future experimental data?

Direct experimental reach
(Lack of direct discoveries)

SM electroweak scale

Intro: EFT from the UV point-of-view

$E\,[GeV]$

Λ_{BSM} — $\mathcal{L}_{BSM}\left[\psi^{H}_{BSM}, \psi^{L}_{SM}\right]$

$\int$ new heavy paticles

Matching
(calculate Wilson coefficients,
to do for each theory)

← **This talk**

(new physics effects are encoded inside c_i)

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim \Lambda_{BSM}\right)\mathcal{O}_i$$

RGEs running (resum large logs, done once for all)

E_{exp}

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim m_W\right)\mathcal{O}_i$$

Mapping (calculate observables, done once for all)

m_W — EW precision test

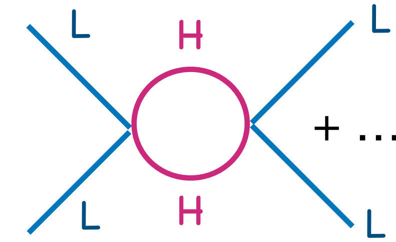
Intro: EFT from the UV point-of-view

$E\,[GeV]$

Λ_{BSM}

$$\mathcal{L}_{BSM}\left[\psi^{H}_{BSM}, \psi^{L}_{SM}\right]$$

Feynman diagrams

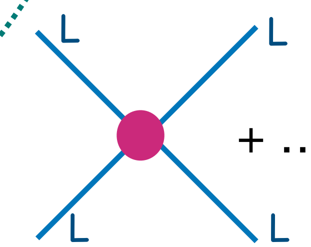


$\int$ new heavy paticles

$$< \psi^{L}_{SM}\psi^{L}_{SM} \cdots \psi^{L}_{SM} >$$

(Decoupled heavy particles, extract Wilson coefficients)

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim \Lambda_{BSM}\right)\mathcal{O}_i$$



RGEs running (resum large logs, done once for all)

E_{exp}

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim m_W\right)\mathcal{O}_i$$

Mapping (calculate observables, done once for all)

m_W

EW precision test

Adapted from Z. Zhang's talk at HEFT 2017

Intro: EFT from the UV point-of-view

$E\,[GeV]$

Λ_{BSM}

$$\mathcal{L}_{BSM}\left[\psi^{H}_{BSM}, \psi^{L}_{SM}\right]$$

$\int$ new heavy paticles

Functional path integral evaluation*

(*) Core techniques:
Covariant Derivative Expansion,
various log expansion,
integration by regions,
Covariant Diagrams, etc.

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim \Lambda_{BSM}\right)\mathcal{O}_i$$

RGEs running (resum large logs, done once for all)

E_{exp}

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim m_W\right)\mathcal{O}_i$$

Mapping (calculate observables, done once for all)

m_W

EW precision test

Outline of section 2

II. Functional method & previous results in literatures

- Effective action at tree and one-loop level
- Expansion by regions: Hard vs Soft
- Covariant Derivative Expansion (CDE)
- The "Universal One-Loop Effective Action" (UOLEA)
- Previous literatures and missing pieces

II. Preliminary: Effective Action by Functional Method

Path integral formalism: $$e^{iS_{eff}\left[\psi^{L}_{SM}\right](\mu)} = \int \mathcal{D}\psi^{H}_{BSM}\, e^{iS\left[\psi^{H}_{BSM},\psi^{L}_{SM}\right](\mu)}$$

Find classical solution by solving EOM:

$$\left.\frac{\delta S\left[\psi^{H}_{BSM},\psi^{L}_{SM}\right]}{\delta\psi^{H}_{BSM}}\right|_{\psi^{H}_{BSM}=\psi^{H}_{BSM,c}} = 0 \Rightarrow \psi^{H}_{BSM,c}(\psi^{L}_{SM})$$

Expand action around minimum:

$$S\left[\psi^{H}_{BSM}\right] = S\left[\psi^{H}_{BSM,c}+\eta\right] = S\left[\psi^{H}_{BSM,c}\right] + \frac{1}{2}\left.\frac{\delta^2 S}{\delta(\psi^{H}_{BSM})^2}\right|_{\psi^{H}_{BSM,c}}\eta^2 + \mathcal{O}(\eta^3)$$

Integrate over quantum fluctuation η :

$$e^{iS_{eff}\left[\psi^{L}_{SM}\right]} = e^{iS\left[\psi^{H}_{BSM,c}\right]}\left[\det\left(-\left.\frac{\delta^2 S}{\delta(\psi^{H}_{BSM})^2}\right|_{\psi^{H}_{BSM,c}}\right)\right]^{-c_s}$$

c_s is spin factor ($c_s = +1/2$ for real scalar, -1 for Dirac fermion)

Re-write the determinant, $\det(A) = e^{\mathrm{Tr}\log A}$:

$$S_{eff}\left[\psi^{L}_{SM}\right] = S\left[\psi^{H}_{BSM,c}\left(\psi^{L}_{SM}\right),\psi^{L}_{SM}\right] + ic_s \mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta(\psi^{H}_{BSM})^2}\right|_{\psi^{H}_{BSM,c}}\right)$$

Tree-level — One-loop level

II. Preliminary: One-Loop Effective Action

We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{UV} = \mathcal{L}_{SM} + \left[\Phi_H^\dagger F(\phi_{SM}) + h.c\right] + \Phi_H^\dagger \left[P^2 - m_{\Phi_H}^2 - U(\phi_{SM})\right] \Phi_H$$

Linear coupling, contribute to tree-level

Quadratic coupling, contribute to heavy-only 1-loop

Notations: $P_\mu = iD_\mu$ (kinetic momentum operator, hermitian)
Φ_H (heavy fields can be bosons or fermions)

Extract the one-loop (heavy-only) piece:

$$S_{eff}^{1-loop} = ic_s \mathrm{Tr}\log\left(-\frac{\delta^2 S}{\delta \Phi_H^2}\bigg|_{\Phi_{H,c}}\right) = ic_s \mathrm{Tr}\log\left[-P^2 + m_{\Phi_H}^2 + U(\phi_{SM})\right] \equiv ic_s \mathrm{Tr}\log \Delta_H$$

Evaluate the Trace by inserting complete set of spatial and momentum states:

$$S_{eff}^{1-loop} = ic_s \int d^dx \int \frac{d^dq}{(2\pi)^d} \mathrm{tr}\log\left(e^{iq\cdot x}\Delta_H e^{-iq\cdot x}\right) = ic_s \int d^dx \int \frac{d^dq}{(2\pi)^d} \mathrm{tr}\log\left(\Delta_H\right)_{P_\mu \to P_\mu - q_\mu}$$

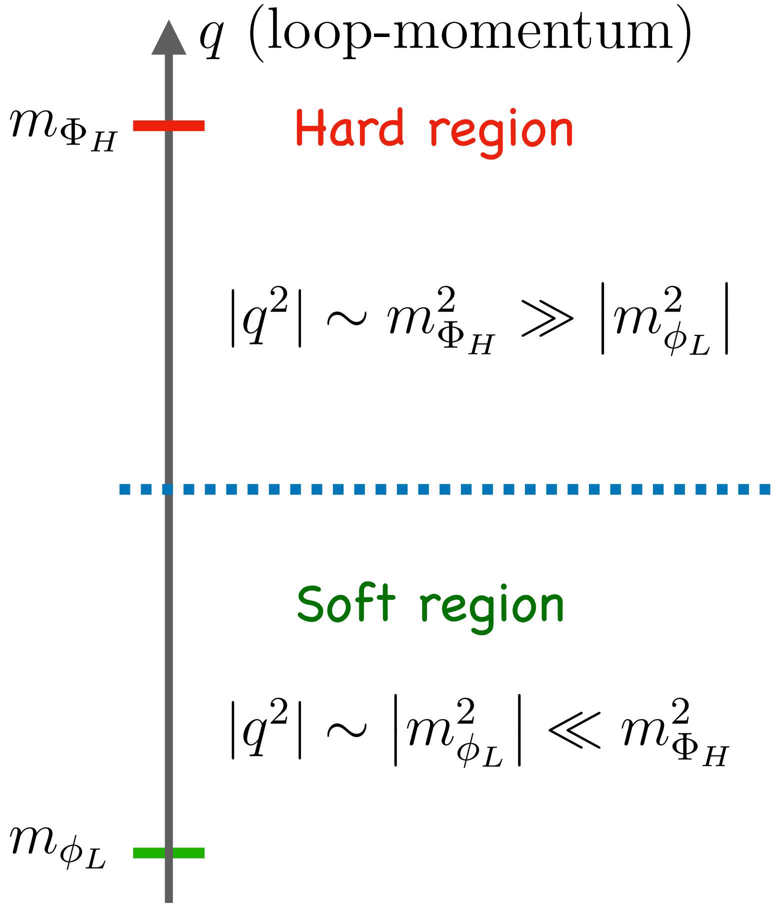
Core techniques to proceed the matching computations (quick overview):

- Expansion by regions => Extract short-distance fluctuation which contribute to the local EFT operators (Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142)
- Covariant Derivative Expansion => Manifestly gauge-invariant in each step of the computation (B. Henning, X. Lu and H. Murayama, arXiv:1412.1837)
- Covariant Diagrams => Keep track of the series expansion (Z. Zhang, arXiv:1610.00710)

II. Core techniques #1: Expansion by regions (1)

In Dim.Reg. with MS-bar scheme, each "log det X" can be separated into "hard" and "soft" region contributions:

$$\log\det X = \log\det X|_{hard} + \log\det X|_{soft}$$



Basis idea:

- **1PI effective action** include quantum fluctuation at **all scales**

$$\int d^d x \, \mathcal{L}^{1-loop}_{EFT}[\phi_{SM}] \neq S^{1-loop}_{eff}[\phi_{SM}]$$

- Extract short-distance fluctuations
  => Local operators in EFT Lagrangian

$$\int d^d x \, \mathcal{L}^{1-loop}_{EFT}[\phi_{SM}] = S^{1-loop}_{eff}[\phi_{SM}]\Big|_{hard-region}$$

Technically speaking:

- Taylor expand the integral in "hard" region, then integrate over the loop momenta

Making use of expansion by regions:

$$\mathcal{L}^{1-loop}_{EFT} = -ic_s \mathrm{tr} \int \frac{d^d q}{(2\pi)^d} \sum_{i=1}^{\infty} \frac{1}{n} \left[\frac{1}{q^2 - m^2_{\Phi_H}} \left(-P^2 + 2q \cdot P + U[\phi_{SM}] \right) \right]^n$$

Adapted from Z. Zhang's talk at HEFT 2017

Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142
Z. Zhang, arXiv:1610.00710

II. Core techniques #2: Covariant Derivative Expansion

$$\mathcal{L}_{EFT}^{1-loop} = -ic_s \mathrm{tr} \int \frac{d^dq}{(2\pi)^d} \sum_{i=1}^{\infty} \frac{1}{n} \left[\frac{1}{q^2 - m_{\Phi_H}^2} \left(-P^2 + 2q \cdot P + U[\phi_{SM}] \right) \right]^n$$

notation: $P_\mu = iD_\mu$

- Basic idea: never separate D_μ into ∂_μ and $-igA_\mu$
- Key points:
  - Expand the series up to n=6
  - Power counting for EFT operator is transparent
  - Each term factorises into a loop integral over momentum-q and a trace involving P, U.
  - 1-loop integrals can be done once for all => Universal results
  - The universal coefficients of operators expressed in terms of the Master Integral:

$$\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1} \cdots q^{\mu_{2n_c}}}{(q^2 - m_i^2)^{n_i} (q^2 - m_j^2)^{n_j} \cdots (q^2)^{n_L}} \equiv g^{\mu_1 \cdots \mu_{2n_c}} \mathcal{I} \left[q^{2n_c} \right]_{ij \cdots 0}^{n_i n_j \cdots n_L}$$

  - Brute force the expansion and obtain the final results ?
    Yes, but tedious and easy to make mistakes !
  - Simpler approach ? => Use Covariant Diagrams to keep track the expansion (see in Backup slides)
  - No matter how we deal with the expansion, the final results are universal !!!

II. Results: Universal One-Loop Effective Action (Heavy-only)

Write down the master formula of Universal One-Loop Effective Action (UOLEA):

$$
\begin{aligned}
\mathcal{L}_{\text{UOLEA}} = -ic_s\,\text{tr}\Big\{ & f_2^i\,U_{ii} + f_3^i\,G_i^{\prime\mu\nu}G'_{\mu\nu,i} + f_4^{ij}\,U_{ij}U_{ji} \\
& + f_5^i\,[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i^{\prime\rho\nu}] + f_6^i\,G'^{\mu}{}_{\nu,i}G'^{\nu}{}_{\rho,i}G'^{\rho}{}_{\mu,i} \\
& + f_7^{ij}\,[P^\mu, U_{ij}][P_\mu, U_{ji}] + f_8^{ijk}\,U_{ij}U_{jk}U_{ki} + f_9^i\,U_{ii}G_i^{\prime\mu\nu}G'_{\mu\nu,i} \\
& + f_{10}^{ijkl}\,U_{ij}U_{jk}U_{kl}U_{li} + f_{11}^{ijk}\,U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12}^{ij}\,[P^\mu,[P_\mu, U_{ij}]]\,[P^\nu,[P_\nu, U_{ji}]] + f_{13}^{ij}\,U_{ij}U_{ji}G_i^{\prime\mu\nu}G'_{\mu\nu,i} \\
& + f_{14}^{ij}\,[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i} + f_{15}^{ij}\,\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}] \\
& + f_{16}^{ijklm}\,U_{ij}U_{jk}U_{kl}U_{lm}U_{mi} \\
& + f_{17}^{ijkl}\,U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl}\,U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}] \\
& + f_{19}^{ijklmn}\,U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}\Big\},
\end{aligned}
$$

notation: $G'_{\mu\nu} = -[P_\mu, P_\nu]$

Trace over internal indices, e.g. spin, color, flavour,...

Model-independent

Universal coefficient as a Master Integral in q, for example: $\frac{1}{6}\mathcal{I}^{111111}_{ijklmn}$

Depend on the UV theories

Operator in EFT Lagrangian, up to d=6

Limit of the master formula:

- Available for heavy-only with non-degenerate mass spectrum
- Available to integrate out heavy fields: scalar, gauge boson, vector-like fermions

A. Drozd, J. Ellis, J. Quevillon and T. You, arXiv:1504.02409

II. Results: What has been done in the literature?

Heavy-only:

- Degenerate in the mass spectrum (B. Henning, X. Lu and H. Murayama, arXiv:1412.1837)
- Non-Degenerate in the mass spectrum (A. Drozd, J. Ellis, J. Quevillon and T. You, arXiv:1504.02409)

Technical improvements:

- Expansion by regions (Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142)
- Covariant Diagrams (Z. Zhang, arXiv:1610.00710)

Mixed Heavy-Light:

- General results in both degenerate and non-degenerate mass spectrum

  (S.A.R. Ellis, J. Quevillon, T. You, Z. Zhang arXiv:1706.07765)

Extend the Universal One-Loop Effective Action (arXiv:2006.16260):

- Integrate out heavy fermions with scalar, vector, pseudo scalar and vector-axial couplings

scalar, vector → Implemented in the UOLEA, but not straight forward in practice

pseudo scalar and vector-axial couplings → Missing pieces in the UOLEA

Outline of sections 3 & 4

III. UOLEA: integrating out heavy-fermions

- Methodology & Results

IV. Examples:

- Effective couplings $A\gamma\gamma, AZZ, AZ\gamma$

Summary

III. UOLEA: Integrate out heavy fermions (1)

Starting point: Let's write down the UV Lagrangian for fermions

$$\mathcal{L}_{UV}\left[\Psi_H, \phi_L\right] = \mathcal{L}_0\left[\phi_L\right] + \overline{\Psi}_H\left(\gamma_\mu P^\mu - m_H - X_H\left[\phi_L\right]\right)\Psi_H$$

general coupling with background fields

The effective action resulting from integrating out heavy-only fermions,

$$S_{eff}^{1-loop} = -\,i\,\mathrm{Tr}\log\left(\gamma_\mu P^\mu - m_H - X_H\left[\phi_L\right]\right)$$

Do not square the quadratic operator! Following the standard manipulation for the functional traces,

$$\mathcal{L}_{EFT}^{1-loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{q_\mu\gamma^\mu + m_H}\left(-P_\mu\gamma^\mu + X_H[\phi_L]\right)\right]^n$$

fermion propagator

We need to fix the form of background function X

We parameterise all possible UV renormalisation structures in the background function:

$$X_H[\phi] \supset W_0 + i\,W_1\gamma^5 + V_\mu\gamma^\mu + A_\mu\gamma^\mu\gamma^5$$

couplings with scalar, pseudo-scalar structures

couplings with vectorial or vector-axial structures

(note: W_0, W_1, V_μ, A_μ independent with the Dirac matrices)

=> Expand order-by-order up to n=6, integrate over the momentum-q, evaluate the trace of Dirac matrices in Breitenlohner-Maison-t'Hooft-Veltman (BMHV) scheme.

III. UOLEA: Integrate out heavy fermions (2)

Master formula for integrating out heavy fermions with chiral interactions (heavy-only):

$$\mathcal{L}_{EFT}^{1-loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{q_\mu\gamma^\mu + m_H}\left(-\not{P} + W_0[\phi_L] + i\,W_1[\phi_L]\gamma^5 + V_\mu[\phi_L]\gamma^\mu + A_\mu[\phi_L]\gamma^\mu\gamma^5\right)\right]^n$$

Main results:

- For the couplings with scalar and pseudo-scalar fields:
  Completely done in both Degenerate and Non-degenerate mass spectrum.
  => 52 universal coefficients (see backup slides)
  (general results where P_μ, W_0, W_1 are considered as Non-commutative objects)

- For the general couplings with scalar, pseudo-scalar, vector and vector-axial structures:
  Completely done in Degenerate mass spectrum.
  => Universal coefficients are available in a Mathematica notebook.

III. UOLEA: Integrate out heavy fermions (3)

$$\mathcal{L}_{EFT}^{1-loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{q_\mu\gamma^\mu + m_H}\left(-\not{P} + W_0[\phi_L] + i\,W_1[\phi_L]\gamma^5 + V_\mu[\phi_L]\gamma^\mu + A_\mu[\phi_L]\gamma^\mu\gamma^5\right)\right]^n$$

Example: Operators involving pseudo-scalar couplings (heavy-only, degenerate mass spectrum)
(full results in the backup slides)

$\mathbb{O}^{(W_1)}$ terms	
$2(\epsilon+4)\mathcal{I}[q^2]_i^2 - 2\mathcal{I}_i^2 m_i^2$	W_1^2
$\mathcal{I}_i^4 m_i^4 - 8m_i^2\mathcal{I}[q^2]_i^4 + 2(11\epsilon+12)\mathcal{I}[q^4]_i^4$	W_1^4
$-\frac{2}{3}\mathcal{I}_i^6 m_i^6 - 48m_i^2\mathcal{I}[q^4]_i^6 + 8m_i^4\mathcal{I}[q^2]_i^6 + 128\mathcal{I}[q^6]_i^6$	W_1^6

Result in BMHV's scheme

Trace over internal indices:
e.g. color, flavour, su(2),… Dirac algebra already computed.

$\mathbb{O}^{(PW_1)}$ terms	
$\mathcal{I}_i^4 m_i^4 - 8m_i^2\mathcal{I}[q^2]_i^4 - 2(\epsilon-12)\mathcal{I}[q^4]_i^4$	$[P_\mu, W_1][P_\mu, W_1]$
$24m_i\mathcal{I}[q^4]_i^5 - 8m_i^3\mathcal{I}[q^2]_i^5 + \mathcal{I}_i^5 m_i^5$	$\epsilon_{\mu\nu\rho\sigma}\,W_1[P_\mu, P_\nu][P_\rho, P_\sigma]$
$-48m_i^2\mathcal{I}[q^4]_i^6 + 4m_i^4\mathcal{I}[q^2]_i^6 + 192\mathcal{I}[q^6]_i^6$	$W_1[P_\mu, W_1]W_1[P_\mu, W_1]$
$-2\mathcal{I}_i^6 m_i^6 - 192m_i^2\mathcal{I}[q^4]_i^6 + 28m_i^4\mathcal{I}[q^2]_i^6 + 576\mathcal{I}[q^6]_i^6$	$W_1^2[P_\mu, W_1][P_\mu, W_1]$
$-2\mathcal{I}_i^6 m_i^6 - 48m_i^2\mathcal{I}[q^4]_i^6 + 16m_i^4\mathcal{I}[q^2]_i^6$	$[P_\mu, W_1][P_\nu, W_1][P_\mu, P_\nu]$
$\mathcal{I}_i^6 m_i^6 + 56m_i^2\mathcal{I}[q^4]_i^6 - 12m_i^4\mathcal{I}[q^2]_i^6 - 64\mathcal{I}[q^6]_i^6$	$W_1^2[P_\mu, P_\nu][P_\mu, P_\nu]$
$-24m_i^2\mathcal{I}[q^4]_i^6 + 2m_i^4\mathcal{I}[q^2]_i^6 + 96\mathcal{I}[q^6]_i^6$	$[P_\mu, [P_\mu, W_1]][P_\nu, [P_\nu, W_1]]$
$-24m_i^2\mathcal{I}[q^4]_i^6 + 2m_i^4\mathcal{I}[q^2]_i^6 + 96\mathcal{I}[q^6]_i^6$	$(W_1[P_\mu, W_1] - [P_\mu, W_1]W_1)\,[P_\nu, [P_\mu, P_\nu]]$

Table 6: *Operator structures in the degenerate fermionic UOLEA involving the pseudoscalar coupling* W_1.

S. A. R. Ellis, J. Quevillon, P. N. H. Vuong, T. You, Z. Zhang (2006.16260)

IV. Examples: Effective couplings $A\gamma\gamma, AZZ, AZ\gamma$

Goal: Test universal coefficients of the operators made by pseudo-scalar, vector-axial structures

Examples: Consider a pseudo-scalar (as A in the MSSM), decays into $\gamma\gamma, ZZ, Z\gamma$ with a top-quark in the loops. Decouple the top-quark and find the EFT Lagrangian for $A \to \gamma\gamma, A \to ZZ, A \to Z\gamma$

- EFT Lagrangian for $A \to \gamma\gamma$:

After the broken phase, write down the relevant terms for $A \to \gamma\gamma$ induced by top-quark in the loops:

$$\mathcal{L}_{\rm MSSM} \supset \bar{u}\left[\left(i\partial_\mu - eQ_f F_\mu\right)\gamma^\mu - m_t + i\frac{m_t}{v}\cot\beta A\gamma^5\right]u$$

After integrating out the top-quark, we could write down the EFT Lagrangian as follows:

$$\mathcal{L}_{\rm EFT}(A\gamma\gamma) = C_{A\gamma\gamma}\, AF_{\mu\nu}\tilde{F}^{\mu\nu}\,,\ \text{with}\ \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

Main exercise: compute the Wilson coefficients $C_{A\gamma\gamma}$ by using the Functional approach and cross-check with the Feynman diagrams computation or previous literatures

=> Let's see how UOLEA works in these examples...

(see backup slides for $A \to ZZ, A \to Z\gamma$)

IV. Examples: Effective couplings $A\gamma\gamma, AZZ, AZ\gamma$

- EFT Lagrangian for $A \to \gamma\gamma$:

Step 1: Re-write the UV-Lagrangian into UOLEA canonical form

$$\mathcal{L}_{UV}(\text{UOLEA form}) = \bar{u}\left[P_\mu\gamma^\mu - m_t - W_1\gamma^5\right]u$$

$$P_\mu = i\partial_\mu - eQ_f F_\mu\,,\; W_1 = -i\frac{m_t}{v}\cot\beta A$$

Step 2: Select relevant operators in the UOLEA

$$AF_{\mu\nu}\tilde{F}_{\mu\nu} = A\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F^{\rho\sigma}$$

included in W_1 included in $[P_\mu, P_\nu]$

=> Select operators in $\mathcal{O}(P^4 W_1)$ terms

Step 3: Take the value of the universal coefficients and evaluate the trace over internal indices

universal coefficient Substituting P, W1, and trace over colour index

$$\mathcal{L}^{\text{1loop}}_{\text{eft}}(A\gamma\gamma) = -\frac{i}{32\pi^2 m_t}\,\text{tr}\left(\epsilon^{\mu\nu\rho\sigma}\,W_1[P_\mu,P_\nu][P_\rho,P_\sigma]\right)$$

$$= \frac{1}{16\pi^2 v}\,e^2 Q_f^2 N_c \cot\beta\, AF_{\mu\nu}\tilde{F}^{\mu\nu}$$

Agree with B. A. Kniehl, M. Spira, Low-energy theorems in Higgs physics, Z. Phys. C 69 (1995) 77 [arXiv:hep-ph/9505225]

Summary

Functional matching method:

- Allow a modelling-independent way to perform the matching calculations
- Universal coefficients can be computed once and for all

The spirit of Universal One-Loop Effective Action (UOLEA):

$$\mathcal{L}_{BSM}\left[\psi^H_{BSM}, \psi^L_{SM}\right]$$

Path integral manipulations
Expansion by regions
Covariant Derivatives Expansion
(Covariant Diagrams)

UOLEA *

Matrix trace evaluation

$$\mathcal{L}_{EFT}\left[\psi^L_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i \left(E \sim \Lambda_{BSM}\right) \mathcal{O}_i$$

=> Start directly from UOLEA !!!

(*) Results:

- Tree-level and Heavy-only ✓ (arXiv:1412.1837, arXiv:1504.02409)
- Mixed Heavy-Light ✓ (arXiv:1706.07765)
- Heavy Fermions with chiral interactions ✓ (arXiv:2006.16260)

Outlook:

- Mixed statistics
- Open covariant derivatives

Backup slides: Full results for scalar, pseudo-scalar (heavy-only, degenerate)

Pure gauge operators:

$\mathbb{O}^{(P)}$ terms	
$-\frac{1}{2}\mathcal{I}_i^4 m_i^4 + 4m_i^2\mathcal{I}[q^2]_i^4 + (5\epsilon - 8)\mathcal{I}[q^4]_i^4$	$[P_\mu, P_\nu][P_\mu, P_\nu]$
$24m_i^2\mathcal{I}[q^4]_i^6 - 2m_i^4\mathcal{I}[q^2]_i^6 - 64\mathcal{I}[q^6]_i^6$	$[P_\mu,[P_\mu, P_\nu]][P_\rho,[P_\rho, P_\nu]]$
$-\frac{2}{3}\mathcal{I}_i^6 m_i^6 + 4m_i^4\mathcal{I}[q^2]_i^6 - \frac{128}{3}\mathcal{I}[q^6]_i^6$	$[P_\mu, P_\nu][P_\nu, P_\rho][P_\rho, P_\mu]$

Operators involving scalar couplings:

$\mathbb{O}^{(PW_0)}$ terms	
$\mathcal{I}_i^4 m_i^4 + (24 - 10\epsilon)\mathcal{I}[q^4]_i^4$	$[P_\mu, W_0][P_\mu, W_0]$
$4\mathcal{I}_i^5 m_i^5 + 192m_i\mathcal{I}[q^4]_i^5 + 16m_i^3\mathcal{I}[q^2]_i^5$	$W_0[P_\mu, W_0][P_\mu, W_0]$
$-2\mathcal{I}_i^5 m_i^5 - 16m_i\mathcal{I}[q^4]_i^5 + 16m_i^3\mathcal{I}[q^2]_i^5$	$W_0[P_\mu, P_\nu][P_\mu, P_\nu]$
$4\mathcal{I}_i^6 m_i^6 + 432m_i^2\mathcal{I}[q^4]_i^6 + 36m_i^4\mathcal{I}[q^2]_i^6 + 192\mathcal{I}[q^6]_i^6$	$W_0[P_\mu, W_0]W_0[P_\mu, W_0]$
$6\mathcal{I}_i^6 m_i^6 + 576m_i^2\mathcal{I}[q^4]_i^6 + 60m_i^4\mathcal{I}[q^2]_i^6 + 576\mathcal{I}[q^6]_i^6$	$W_0^2[P_\mu, W_0][P_\mu, W_0]$
$2\mathcal{I}_i^6 m_i^6 - 16m_i^2\mathcal{I}[q^4]_i^6 - 16m_i^4\mathcal{I}[q^2]_i^6$	$[P_\mu, W_0][P_\nu, W_0][P_\mu, P_\nu]$
$-5\mathcal{I}_i^6 m_i^6 + 72m_i^2\mathcal{I}[q^4]_i^6 + 36m_i^4\mathcal{I}[q^2]_i^6 - 64\mathcal{I}[q^6]_i^6$	$W_0^2[P_\mu, P_\nu][P_\mu, P_\nu]$
$-2\mathcal{I}_i^6 m_i^6 - 8m_i^2\mathcal{I}[q^4]_i^6 + 18m_i^4\mathcal{I}[q^2]_i^6 + 96\mathcal{I}[q^6]_i^6$	$(W_0[P_\mu, W_0] - [P_\mu, W_0]W_0)\,[P_\nu,[P_\mu, P_\nu]]$
$8m_i^2\mathcal{I}[q^4]_i^6 + 2m_i^4\mathcal{I}[q^2]_i^6 + 96\mathcal{I}[q^6]_i^6$	$[P_\mu,[P_\mu, W_0]][P_\nu,[P_\nu, W_0]]$

$\mathbb{O}^{(W_0)}$ terms	
$4m_i\mathcal{I}_i$	W_0
$2\mathcal{I}_i^2 m_i^2 + (8 - 2\epsilon)\mathcal{I}[q^2]_i^2$	W_0^2
$\frac{4}{3}\mathcal{I}_i^3 m_i^3 + (\,16m_i - 4\epsilon m_i\,)\mathcal{I}[q^2]_i^3$	W_0^3
$\mathcal{I}_i^4 m_i^4 + 24m_i^2\mathcal{I}[q^2]_i^4 + (24 - 10\epsilon)\mathcal{I}[q^4]_i^4$	W_0^4
$\frac{4}{5}\mathcal{I}_i^5 m_i^5 + 96m_i\mathcal{I}[q^4]_i^5 + 32m_i^3\mathcal{I}[q^2]_i^5$	W_0^5
$\frac{2}{3}\mathcal{I}_i^6 m_i^6 + 240m_i^2\mathcal{I}[q^4]_i^6 + 40m_i^4\mathcal{I}[q^2]_i^6 + 128\mathcal{I}[q^6]_i^6$	W_0^6

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Backup slides: Full results for scalar, pseudo-scalar (heavy-only, degenerate)

Operators involving pseudo-scalar couplings:

$\mathbb{O}^{(PW_1)}$ terms	
$\mathcal{I}_i^4 m_i^4 - 8m_i^2\mathcal{I}[q^2]_i^4 - 2(\epsilon-12)\mathcal{I}[q^4]_i^4$	$[P_\mu,W_1][P_\mu,W_1]$
$24m_i\mathcal{I}[q^4]_i^5 - 8m_i^3\mathcal{I}[q^2]_i^5 + \mathcal{I}_i^5 m_i^5$	$\epsilon_{\mu\nu\rho\sigma}W_1[P_\mu,P_\nu][P_\rho,P_\sigma]$
$-48m_i^2\mathcal{I}[q^4]_i^6 + 4m_i^4\mathcal{I}[q^2]_i^6 + 192\mathcal{I}[q^6]_i^6$	$W_1[P_\mu,W_1]W_1[P_\mu,W_1]$
$-2\mathcal{I}_i^6 m_i^6 - 192m_i^2\mathcal{I}[q^4]_i^6 + 28m_i^4\mathcal{I}[q^2]_i^6 + 576\mathcal{I}[q^6]_i^6$	$W_1^2[P_\mu,W_1][P_\mu,W_1]$
$-2\mathcal{I}_i^6 m_i^6 - 48m_i^2\mathcal{I}[q^4]_i^6 + 16m_i^4\mathcal{I}[q^2]_i^6$	$[P_\mu,W_1][P_\nu,W_1][P_\mu,P_\nu]$
$\mathcal{I}_i^6 m_i^6 + 56m_i^2\mathcal{I}[q^4]_i^6 - 12m_i^4\mathcal{I}[q^2]_i^6 - 64\mathcal{I}[q^6]_i^6$	$W_1^2[P_\mu,P_\nu][P_\mu,P_\nu]$
$-24m_i^2\mathcal{I}[q^4]_i^6 + 2m_i^4\mathcal{I}[q^2]_i^6 + 96\mathcal{I}[q^6]_i^6$	$[P_\mu,[P_\mu,W_1]][P_\nu,[P_\nu,W_1]]$
$-24m_i^2\mathcal{I}[q^4]_i^6 + 2m_i^4\mathcal{I}[q^2]_i^6 + 96\mathcal{I}[q^6]_i^6$	$(W_1[P_\mu,W_1]-[P_\mu,W_1]W_1)[P_\nu,[P_\mu,P_\nu]]$

$\mathbb{O}^{(W_1)}$ terms	
$2(\epsilon+4)\mathcal{I}[q^2]_i^2 - 2\mathcal{I}_i^2 m_i^2$	W_1^2
$\mathcal{I}_i^4 m_i^4 - 8m_i^2\mathcal{I}[q^2]_i^4 + 2(11\epsilon+12)\mathcal{I}[q^4]_i^4$	W_1^4
$-\frac{2}{3}\mathcal{I}_i^6 m_i^6 - 48m_i^2\mathcal{I}[q^4]_i^6 + 8m_i^4\mathcal{I}[q^2]_i^6 + 128\mathcal{I}[q^6]_i^6$	W_1^6

Operators involving the mixed of scalar and pseudo-scalar couplings:

$\mathbb{O}^{(PW_0W_1)}$ terms	
$48m_i\mathcal{I}[q^4]_i^5 - 8m_i^3\mathcal{I}[q^2]_i^5$	$W_1[P_\mu,W_0][P_\mu,W_1] + \text{h.c.}$
$4\mathcal{I}_i^5 m_i^5 + 96m_i\mathcal{I}[q^4]_i^5 - 32m_i^3\mathcal{I}[q^2]_i^5$	$W_0[P_\mu,W_1][P_\mu,W_1]$
$24m_i^2\mathcal{I}[q^4]_i^6 - 8m_i^4\mathcal{I}[q^2]_i^6 + \mathcal{I}_i^6 m_i^6$	$\epsilon_{\mu\nu\rho\sigma}W_0W_1[P_\mu,P_\nu][P_\rho,P_\sigma] + \text{h.c.}$
$24m_i^2\mathcal{I}[q^4]_i^6 - 8m_i^4\mathcal{I}[q^2]_i^6 + \mathcal{I}_i^6 m_i^6$	$\epsilon_{\mu\nu\rho\sigma}W_0[P_\mu,P_\nu]W_1[P_\rho,P_\sigma]$
$2\mathcal{I}_i^6 m_i^6 + 192m_i^2\mathcal{I}[q^4]_i^6 - 28m_i^4\mathcal{I}[q^2]_i^6 - 576\mathcal{I}[q^6]_i^6$	$W_1W_0[P_\mu,W_1][P_\mu,W_0] + \text{h.c.}$
$48m_i^2\mathcal{I}[q^4]_i^6 - 4m_i^4\mathcal{I}[q^2]_i^6 - 192\mathcal{I}[q^6]_i^6$	$W_1[P_\mu,W_0]W_1[P_\mu,W_0]$
$4\mathcal{I}_i^6 m_i^6 + 144m_i^2\mathcal{I}[q^4]_i^6 - 36m_i^4\mathcal{I}[q^2]_i^6 - 192\mathcal{I}[q^6]_i^6$	$W_0[P_\mu,W_1]W_0[P_\mu,W_1]$
$96m_i^2\mathcal{I}[q^4]_i^6 - 24m_i^4\mathcal{I}[q^2]_i^6 + 384\mathcal{I}[q^6]_i^6$	$W_1[P_\mu,W_1]W_0[P_\mu,W_0] + \text{h.c.}$
$6\mathcal{I}_i^6 m_i^6 - 36m_i^4\mathcal{I}[q^2]_i^6 + 576\mathcal{I}[q^6]_i^6$	$W_0^2[P_\mu,W_1][P_\mu,W_1]$
$-2\mathcal{I}_i^6 m_i^6 - 4m_i^4\mathcal{I}[q^2]_i^6 + 576\mathcal{I}[q^6]_i^6$	$W_0W_1[P_\mu,W_1][P_\mu,W_0] + \text{h.c.}$
$-2\mathcal{I}_i^6 m_i^6 - 4m_i^4\mathcal{I}[q^2]_i^6 + 576\mathcal{I}[q^6]_i^6$	$W_1^2[P_\mu,W_0][P_\mu,W_0]$

$\mathbb{O}^{(W_0W_1)}$ terms	
$4(3\epsilon+4)m_i\mathcal{I}[q^2]_i^3 - 4\mathcal{I}_i^3 m_i^3$	$W_0W_1^2$
$8(\epsilon+12)\mathcal{I}[q^4]_i^4 - 4\mathcal{I}_i^4 m_i^4$	$W_0^2W_1^2$
$-2\mathcal{I}_i^4 m_i^4 + 16m_i^2\mathcal{I}[q^2]_i^4 + 4(5\epsilon-12)\mathcal{I}[q^4]_i^4$	$W_0W_1W_0W_1$
$-4\mathcal{I}_i^5 m_i^5 + 288m_i\mathcal{I}[q^4]_i^5 - 32m_i^3\mathcal{I}[q^2]_i^5$	$W_0^3W_1^2$
$-4\mathcal{I}_i^5 m_i^5 - 96m_i\mathcal{I}[q^4]_i^5 + 32m_i^3\mathcal{I}[q^2]_i^5$	$W_0^2W_1W_0W_1$
$4\mathcal{I}_i^5 m_i^5 + 96m_i\mathcal{I}[q^4]_i^5 - 32m_i^3\mathcal{I}[q^2]_i^5$	$W_0W_1^4$
$-4\mathcal{I}_i^6 m_i^6 + 96m_i^2\mathcal{I}[q^4]_i^6 + 16m_i^4\mathcal{I}[q^2]_i^6 - 768\mathcal{I}[q^6]_i^6$	$W_0^3W_1W_0W_1$
$-2\mathcal{I}_i^6 m_i^6 - 144m_i^2\mathcal{I}[q^4]_i^6 + 24m_i^4\mathcal{I}[q^2]_i^6 + 384\mathcal{I}[q^6]_i^6$	$W_0^2W_1W_0^2W_1$
$-4\mathcal{I}_i^6 m_i^6 + 480m_i^2\mathcal{I}[q^4]_i^6 - 80m_i^4\mathcal{I}[q^2]_i^6 + 768\mathcal{I}[q^6]_i^6$	$W_0^4W_1^2$
$4\mathcal{I}_i^6 m_i^6 + 288m_i^2\mathcal{I}[q^4]_i^6 - 48m_i^4\mathcal{I}[q^2]_i^6 - 768\mathcal{I}[q^6]_i^6$	$W_0W_1W_0W_1^3$
$2\mathcal{I}_i^6 m_i^6 - 48m_i^2\mathcal{I}[q^4]_i^6 - 8m_i^4\mathcal{I}[q^2]_i^6 + 384\mathcal{I}[q^6]_i^6$	$W_0W_1^2W_0W_1^2$
$4\mathcal{I}_i^6 m_i^6 - 96m_i^2\mathcal{I}[q^4]_i^6 - 16m_i^4\mathcal{I}[q^2]_i^6 + 768\mathcal{I}[q^6]_i^6$	$W_0^2W_1^4$

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Backup slides: Some operators involving pseudo-scalar, vector and axial-vector structures

Operators involving pseudo-scalar and vector:

$\mathbb{O}^{(P^2V^2W_1)}$ terms	
$-4m_i^5\mathcal{I}_i^5 + 32m_i^3\mathcal{I}[q^2]_i^5 - 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu W_1 P_\nu V_\rho V_\sigma$
$-4m_i^5\mathcal{I}_i^5 + 32m_i^3\mathcal{I}[q^2]_i^5 - 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu P_\nu V_\rho W_1 V_\sigma$
$4m_i^5\mathcal{I}_i^5 - 32m_i^3\mathcal{I}[q^2]_i^5 + 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu P_\nu V_\rho V_\sigma W_1 + \text{h.c.}$
$4m_i^5\mathcal{I}_i^5 - 32m_i^3\mathcal{I}[q^2]_i^5 + 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu V_\nu P_\rho V_\sigma W_1 + \text{h.c.}$

$\mathbb{O}^{(P^3VW_1)}$ terms	
$-4m_i^5\mathcal{I}_i^5 + 32m_i^3\mathcal{I}[q^2]_i^5 - 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu P_\nu P_\rho V_\sigma W_1 + \text{h.c.}$
$-4m_i^5\mathcal{I}_i^5 + 32m_i^3\mathcal{I}[q^2]_i^5 - 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu P_\nu V_\rho P_\sigma W_1 + \text{h.c.}$

Operators involving pseudo-scalar and axial-vector:

$\mathbb{O}^{(P^2A^2W_1)}$ terms	
$4m_i^5\mathcal{I}_i^5 - 16m_i^3\mathcal{I}[q^2]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu A_\nu P_\rho A_\sigma W_1 + \text{h.c.}$
$-4m_i^5\mathcal{I}_i^5 + 16m_i^3\mathcal{I}[q^2]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu P_\nu A_\rho A_\sigma W_1 + \text{h.c.}$
$4m_i^5\mathcal{I}_i^5 - 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu W_1 P_\nu A_\rho A_\sigma$
$4m_i^5\mathcal{I}_i^5 - 32m_i^3\mathcal{I}[q^2]_i^5 + 96m_i\mathcal{I}[q^4]_i^5$	$\epsilon^{\mu\nu\rho\sigma}P_\mu P_\nu A_\rho W_1 A_\sigma$

(Useful operators for $AZZ, AZ\gamma$ example)

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Backup slides: Effective couplings $A\gamma\gamma, AZZ, AZ\gamma$

- EFT Lagrangian for $A \to ZZ$:

Write down the UV Lagrangian for A->ZZ induced by top-quark in the loops:

$$\mathcal{L}_{\text{MSSM}} \supset \bar{u}\Big[(i\partial_\mu)\,\gamma^\mu - m_t + \left(i\frac{m_t}{v}\cot\beta A\right)\gamma^5 \\ -\frac{g}{\cos\theta_w}\left(\frac{T_3}{2} - Q_f\sin^2\theta\right)Z_\mu\gamma^\mu + \left(\frac{g}{\cos\theta_w}\frac{T_3}{2}\right)Z_\mu\gamma^\mu\gamma^5\Big]\,u$$

Remark: we are in the broken phase => Z-field will be factorised out of the covariant derivative !

$$\mathcal{L}_{\text{UV}}(\text{UOLEA form}) \supset \bar{u}\left[P_\mu\gamma^\mu - m_t - W_1\gamma^5 - V_\mu\gamma^\mu - A_\mu\gamma^\mu\gamma^5\right]u,$$

Where P, W1, V, A are defined as follows:

$$\begin{cases} P_\mu = i\partial_\mu\,;\;\; W_1 = -i\dfrac{m_t}{v}\cot\beta A \\ V_\mu = \dfrac{g}{\cos\theta_w}\left(\dfrac{T_3}{2} - Q_f\sin^2\theta\right)Z_\mu\,;\;\; A_\mu = -\left(\dfrac{g}{\cos\theta_w}\dfrac{T_3}{2}\right)Z_\mu \end{cases}$$

Write down the EFT Lagrangian for A->ZZ: $\mathcal{L}_{\text{EFT}}(AZZ) = C_{AZZ}AZ_{\mu\nu}\tilde{Z}^{\mu\nu}$ need 2 derivatives, 2 gauge fields and 1 pseudo-scalar

Relevant operators in UOLEA: $\mathcal{L}_{EFT}(AZZ) \supset \mathcal{O}(P^2V^2W_1) + \mathcal{O}(P^2A^2W_1)$

Operators: [P, V][P, V]W1 and [P, A][P, A]W1

$$\mathcal{L}^{1-loop}_{\text{EFT}}(AZZ) = \frac{1}{48\pi^2 v}N_c\cot\beta\frac{g^2}{c_W^2}\left(T_3^2 + 3Q_f s_W^2[Q_f s_W^2 - T_3]\right)AZ_{\mu\nu}\tilde{Z}_{\mu\nu}.$$

Agree with B. A. Kniehl, M. Spira, Low-energy theorems in Higgs physics, Z. Phys. C 69 (1995) 77 [arXiv:hep-ph/9505225]

Backup slides: Effective couplings $A\gamma\gamma, AZZ, AZ\gamma$

- EFT Lagrangian for $A \to Z\gamma$:

Write down the UV Lagrangian for A->Z+gamma induced by top-quark in the loops:

$$\mathcal{L}_{\text{MSSM}} \supset \bar{u}\left[(i\partial_\mu - eQ_f F_\mu)\gamma^\mu - m_t + \left(i\frac{m_t}{v}\cot\beta A\right)\gamma^5 - \frac{g}{\cos\theta_w}\left(\frac{T_3}{2} - Q_f \sin^2\theta\right) Z_\mu\gamma^\mu + \left(\frac{g}{\cos\theta_w}\frac{T_3}{2}\right) Z_\mu\gamma^\mu\gamma^5\right] u$$

Remark: Only photon-field can live inside the covariant derivative, the Z-field will be split into vector current and axial current.

$$\mathcal{L}_{\text{UV}}(\text{UOLEA form}) = \bar{u}\left[P_\mu\gamma^\mu - m_t - W_1\gamma^5 - V_\mu\gamma^\mu - A_\mu\gamma^\mu\gamma^5\right] u$$

=> Re-write in terms of the UOLEA canonical form

$$\begin{cases} P_\mu = i\partial_\mu - eQ_f F_\mu\,; \; W_1 = -i\frac{m_t}{v}\cot\beta A \\ V_\mu = \frac{g}{c_W}\left(\frac{T_3}{2} - Q_f s_W^2\right) Z_\mu\,; \; A_\mu = -\left(\frac{g}{c_W}\frac{T_3}{2}\right) Z_\mu \end{cases}$$

Write down the EFT Lagrangian for A->Z+gamma: $\mathcal{L}_{EFT}(AZ\gamma) = C_{AZ\gamma} A Z_{\mu\nu}\tilde{F}_{\mu\nu}$

Relevant operators in UOLEA: $\mathcal{L}_{EFT}(AZ\gamma) \supset \mathcal{O}(P^3 V W_1)$

Operators: [P, V][P, P]W1

$$\mathcal{L}_{EFT}^{1-loop}(AZ\gamma) = \frac{1}{16\pi^2 v} N_c \cot\beta (eQ_f)\frac{g}{c_W}\left(T_3 - 2Q_f s_W^2\right) A Z_{\mu\nu}\tilde{F}_{\mu\nu}$$

Agree with B. A. Kniehl, M. Spira, Low-energy theorems in Higgs physics, Z. Phys. C 69 (1995) 77 [arXiv:hep-ph/9505225]

Backup slides: Covariant Diagrams (Z. Zhang, arXiv:1610.00710)

Main idea: Due to the trace cyclicity, any terms in the expansion can be presented diagrammatically !!!
Power counting is transparent => classify diagrams very easy !

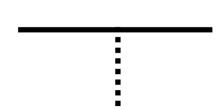
Key points: Define building blocks + readout rules => Generate all possible diagrams at each order, evaluate the prefactor and get the EFT operators (able to automatise easily)

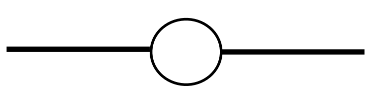
$$\mathcal{L}_{EFT}^{1-loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{q_\mu\gamma^\mu + m_H}\left(-P_\mu\gamma^\mu + W_0[\phi_L] + W_1[\phi_L]\gamma^5 + V_\mu[\phi_L]\gamma^\mu + A_\mu[\phi_L]\gamma^\mu\gamma^5\right)\right]^n$$

Decompose the fermion propagator $\quad \dfrac{-1}{q_\mu\gamma^\mu + m_H} = \dfrac{m_H}{q^2 - m_H^2} + \dfrac{-q_\mu\gamma^\mu}{q^2 - m_H^2}$

Example:

Building blocks:

Fermion propagators: bosonic part $= m_H$; fermionic part  $= -\gamma^\mu$

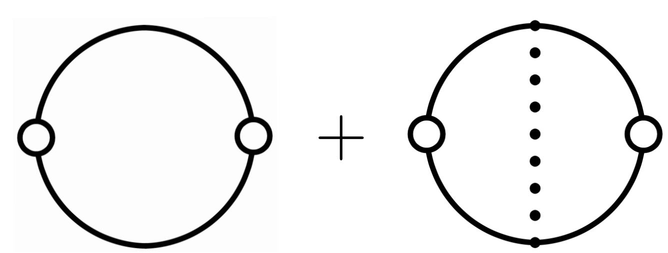
W1 insertion:  $= W_1\,[\phi_L]\,\gamma^5$

Readout rules:

Diagram value = prefactor x trace (building blocks)

Prefactor: $i\,\dfrac{1}{S}\,\mathcal{I}[q^{2n_c}]^{n_i\,n_j\cdots}_{i\,j\cdots}$

if the digram have Z_s symmetry

Let's compute W1^2 term: $\quad (W_1)^2 =$ 

$$= i\,\frac{1}{2}\,m_H^2\mathcal{I}_i^2\,\mathrm{tr}\left(W_1\gamma^5\,W_1\gamma^5\right) + i\,\frac{1}{2}\,\mathcal{I}[q^2]_i^2\,\mathrm{tr}\left(W_1\gamma^5\gamma^\mu\,W_1\gamma^5\gamma_\mu\right)$$

The diagram is symmetry if we rotate 180 degree => symmetry factor = 1/2

Backup slides: Expansion by regions

Making use of expansion by regions:

$$\mathcal{L}_{EFT}^{1-loop} = ic_s \int \frac{d^dq}{(2\pi)^d} \text{tr} \log \left(\Delta_H\right)_{P_\mu \to P_\mu - q_\mu} \Big|_{hard-region}$$

$$= ic_s \int \frac{d^dq}{(2\pi)^d} \text{tr} \log \left(-[P_\mu - q_\mu]^2 + m_{\Phi_H} + U[\phi_{SM}]\right) \Big|_{hard-region}$$

Expanding the Log into the Taylor series with $|q^2| \sim m^2_{\Phi_H} \gg |m^2_{\phi_L}|$

$$\log \left(-[P_\mu - q_\mu]^2 + m_{\Phi_H} + U[\phi_{SM}]\right) \big|_{hard-region} = \log(-q^2 + m^2_{\Phi_H})$$

$$- \sum_{i=1}^{\infty} \frac{1}{n} \left[\frac{1}{q^2 - m^2_{\Phi_H}} \left(-P^2 + 2q \cdot P + U[\phi_{SM}]\right) \right]^n$$

Finally, we obtain the EFT Lagrangian as follows:

$$\mathcal{L}_{EFT}^{1-loop} = -ic_s \text{tr} \int \frac{d^dq}{(2\pi)^d} \sum_{i=1}^{\infty} \frac{1}{n} \left[\frac{1}{q^2 - m^2_{\Phi_H}} \left(-P^2 + 2q \cdot P + U[\phi_{SM}]\right) \right]^n$$

Next step:

- Expand the series order-by-order, up to n=6 (we consider EFT operators up to dimension 6)

Z. Zhang, arXiv:1610.00710

Backup slides: Integrate out heavy fermions (1)

Starting point: Let's write down the UV Lagrangian for fermions

$$\mathcal{L}_{UV}\left[\Psi_H, \phi_L\right] = \mathcal{L}_0\left[\phi_L\right] + \overline{\Psi}_H\left(\gamma_\mu P^\mu - m_H - X_H\left[\phi_L\right]\right)\Psi_H$$

general coupling with background fields

The effective action resulting from integrating out heavy-only fermions,

$$S_{eff}^{1-loop} = -\,i\,\mathrm{Tr}\log\left(\gamma_\mu P^\mu - m_H - X_H\left[\phi_L\right]\right)$$

Two way of proceeding:

1. Squaring the quadratic operators, using the trick $\mathrm{Tr}\log(AB) = \mathrm{Tr}\log A + \mathrm{Tr}\log B$

$$S_{eff}^{1-loop} = -\frac{i}{2}\mathrm{Tr}\log\left(-P^2 + m_H^2 + U_{fermion}\right)\,,$$

$$\text{where } U_{fermion} = -\frac{i}{2}\sigma^{\mu\nu}G'_{\mu\nu} + 2m_H X_H[\phi_L] + X_H^2 + \left[\not{P}, X_H[\phi_L]\right]$$

=> Then we can use the master formula in UOLEA as mentioned before

Disadvantages:

- Not straight forward to derive EFT operators due to the complicated of the background function $U_{fermion}$
- If $X_H[\phi_L]$ contains Dirac matrices, the quantity $\left[\not{P}, X_H[\phi_L]\right]\big|_{P_\mu \to P_\mu - q_\mu}$, will lead to non-trivial terms which are not implemented in the UOLEA before

Backup slides: Integrate out heavy fermions (2)

$$\mathcal{L}_{EFT}^{1-loop} = i\,\mathrm{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d q}{(2\pi)^d} \left[\frac{-1}{q_\mu \gamma^\mu + m_H} \left(-\not{P} + W_0[\phi_L] + i\, W_1[\phi_L]\gamma^5 + V_\mu[\phi_L]\gamma^\mu + A_\mu[\phi_L]\gamma^\mu \gamma^5 \right) \right]^n$$

Any difficulties in this computations ? YES, we have γ^5 in D-dimension !!!

Let's do an example and see...

$$\mathcal{O}(W_1^2) = -\frac{i}{2} m_i^2 \mathcal{I}_i^2\, \mathrm{tr}\left(W_1^2 \gamma^5 \gamma^5\right) - \frac{i}{2} \mathcal{I}[q^2]_i^2\, \mathrm{tr}\left(W_1^2 \gamma^5 \gamma^\mu \gamma^5 \gamma_\mu\right)$$

The 1-loop integral is divergence, using Dim.Reg. to evaluate the integral

Evaluate the Dirac trace in D-dimension

$$\mathcal{I}[q^2]_i^2 = \frac{m_i^2}{2} \left[1 - \log \frac{m_i^2}{\mu^2} + \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi \right) \right]$$

Key points:

- Due to the issue of γ^5 in D-dimension, we used Breitenlohner-Maison- t'Hooft Veltman scheme (BMHV)
- We must keep the terms $\mathcal{O}(\epsilon)$ in the Dirac traces, since they will cancel out the divergence term $\frac{2}{\epsilon}$ of the 1-loop integrals

divergence is cancelled => extra finite term

$$\mathcal{O}(W_1^2) = i \left\{ -2\, m_i^2 \mathcal{I}_i^2 + (8 + 2\epsilon) \frac{m_i^2}{2} \left[1 - \log \frac{m_i^2}{\mu^2} + \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi \right) \right] \right\} \mathrm{tr}\left(W_1^2\right)$$

result of Dirac trace in BMHV-scheme

No need to evaluate Dirac algebra

Backup slides: Loop integrals

Definition of the master integrals:

$$\mathcal{I}[q^{2n_c}]_i^{n_i} = \frac{i}{16\pi^2}\left(-M_i^2\right)^{2+n_c-n_i}\frac{1}{2^{n_c}(n_i-1)!}\frac{\Gamma(\frac{\epsilon}{2}-2-n_c+n_i)}{\Gamma(\frac{\epsilon}{2})}\left(\frac{2}{\bar{\epsilon}}-\log\frac{M_i^2}{\mu^2}\right)$$

The value of some master integrals:

$\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i}$	$n_c=0$	$n_c=1$	$n_c=2$	$n_c=3$
$n_i=1$	$M_i^2\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{4}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{24}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^8}{192}\left(\frac{25}{12}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=2$	$-\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{8}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{48}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{32}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=4$	$\frac{1}{6M_i^4}$	$-\frac{1}{12M_i^2}$	$-\frac{1}{24}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48}\left(1-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192}\log\frac{M_i^2}{\mu^2}$
$n_i=6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

Table 7. Commonly-used degenerate master integrals $\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i} \equiv \mathcal{I}[q^{2n_c}]_i^{n_i}/\frac{i}{16\pi^2}$, with $\frac{2}{\bar{\epsilon}} = \frac{2}{\epsilon}-\gamma+\log 4\pi$ dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).