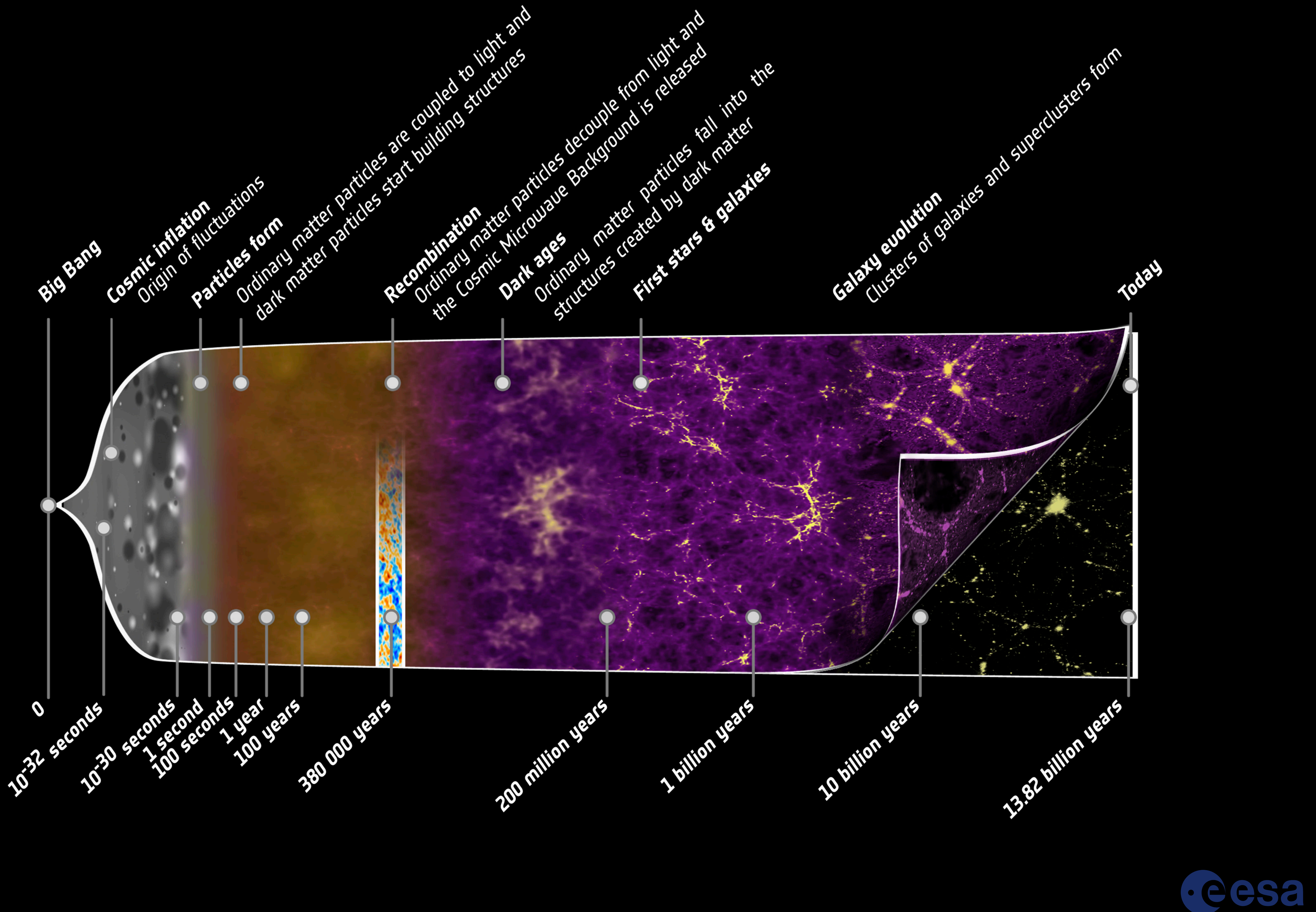
A precision calculation of neutrino decoupling

Julien Froustey

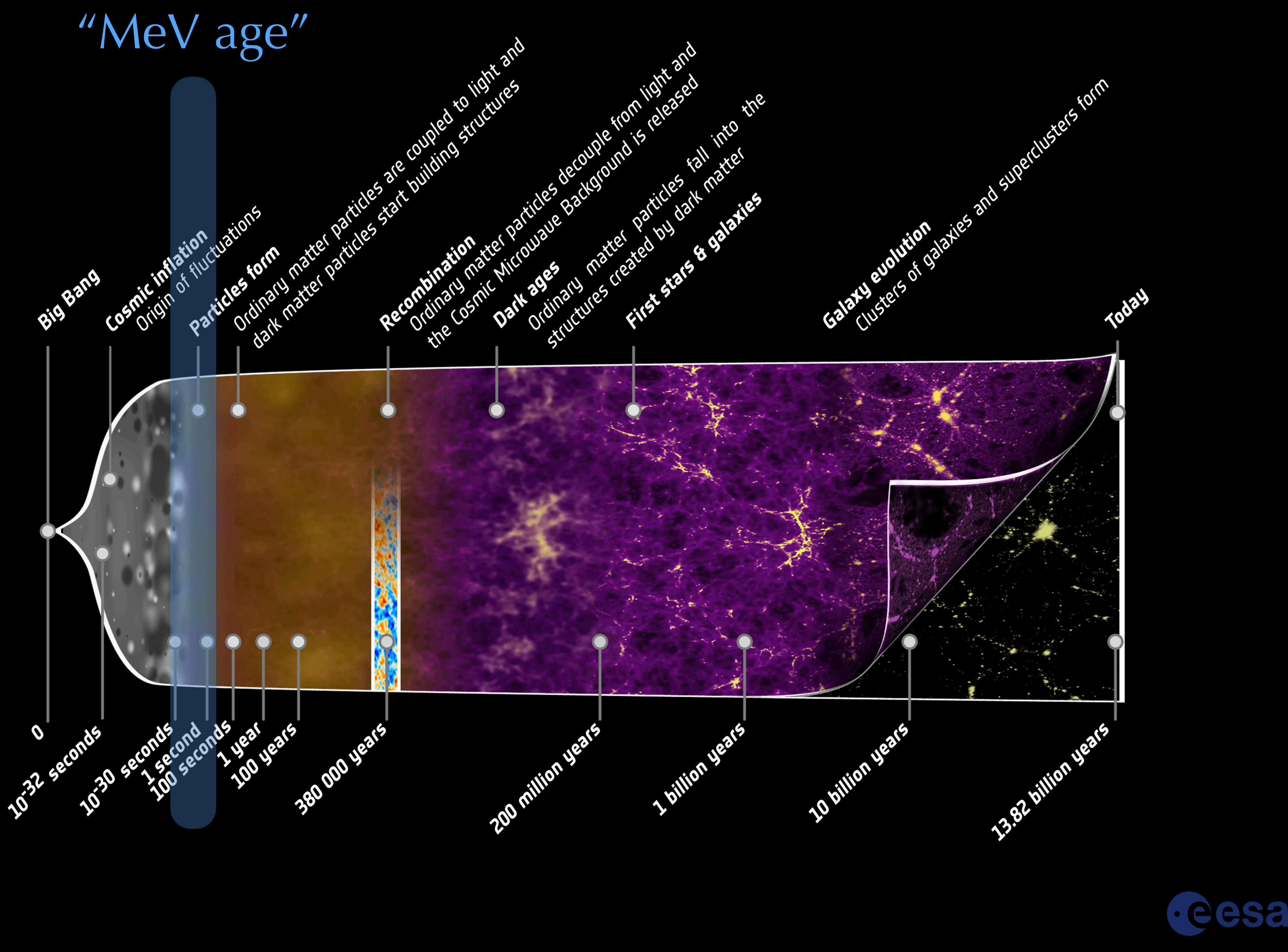
with Cyril Pitrou (IAP), Maria Cristina Volpe (APC)

GDR Neutrino meeting ○ WG 3 ○ 24/11/2020

[1912.09378] **JF**, C. Pitrou, *Phys. Rev. D* 101, 043524 (2020)
[2008.01074] **JF**, C. Pitrou, M.C. Volpe, *to appear in JCAP*



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years
esa


"MeV age"
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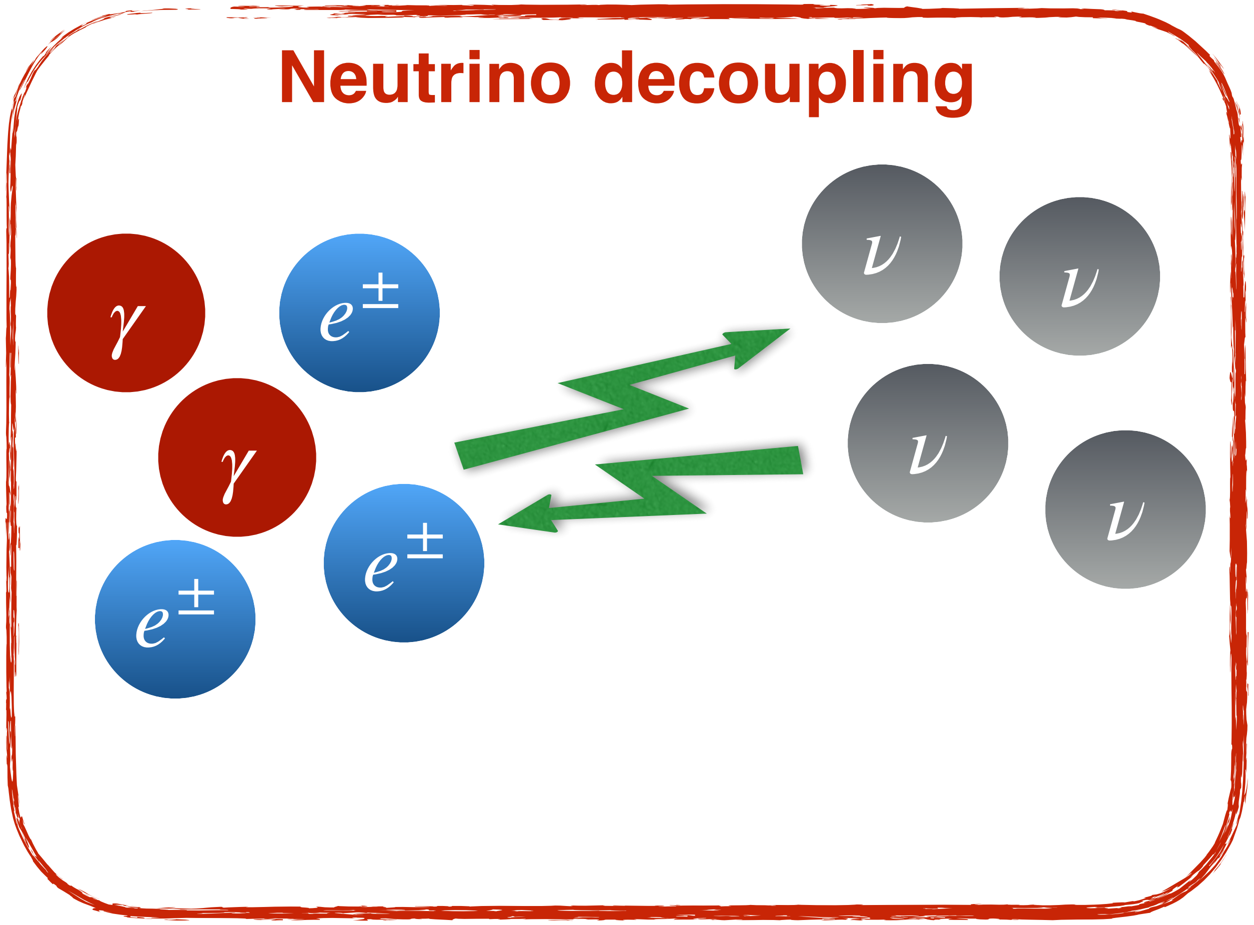
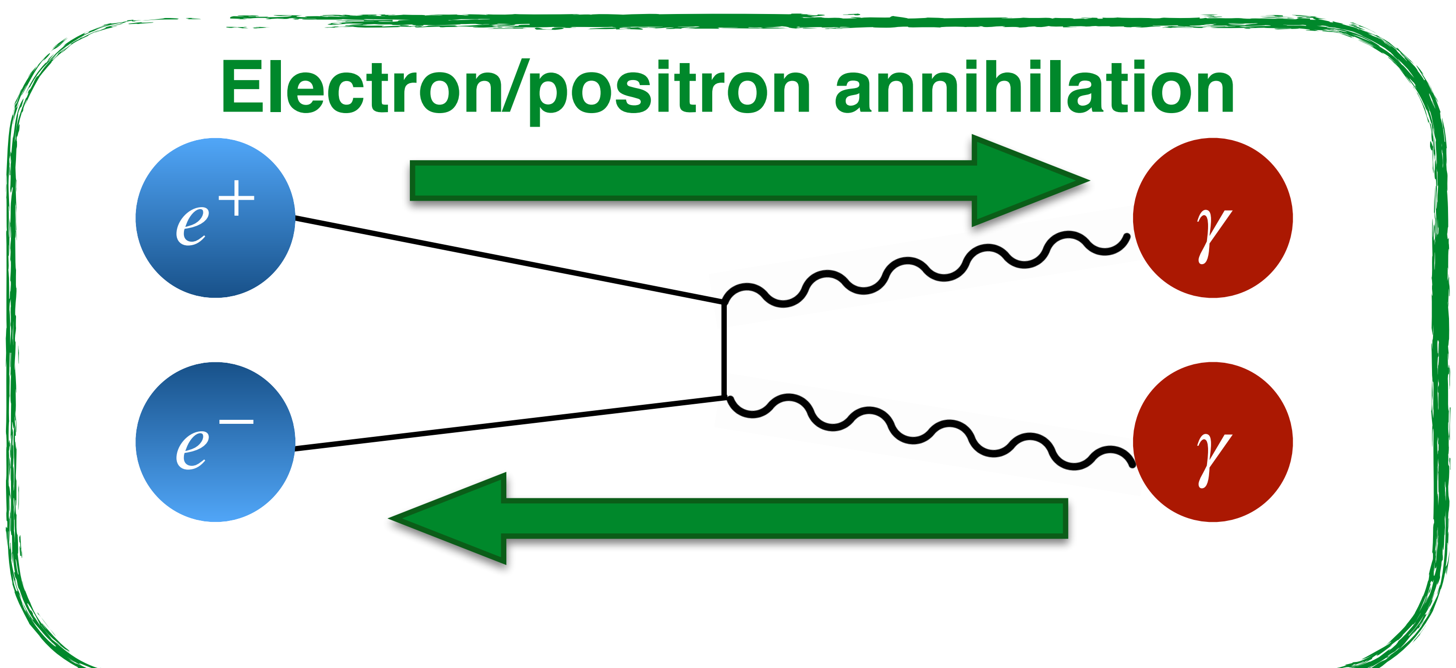
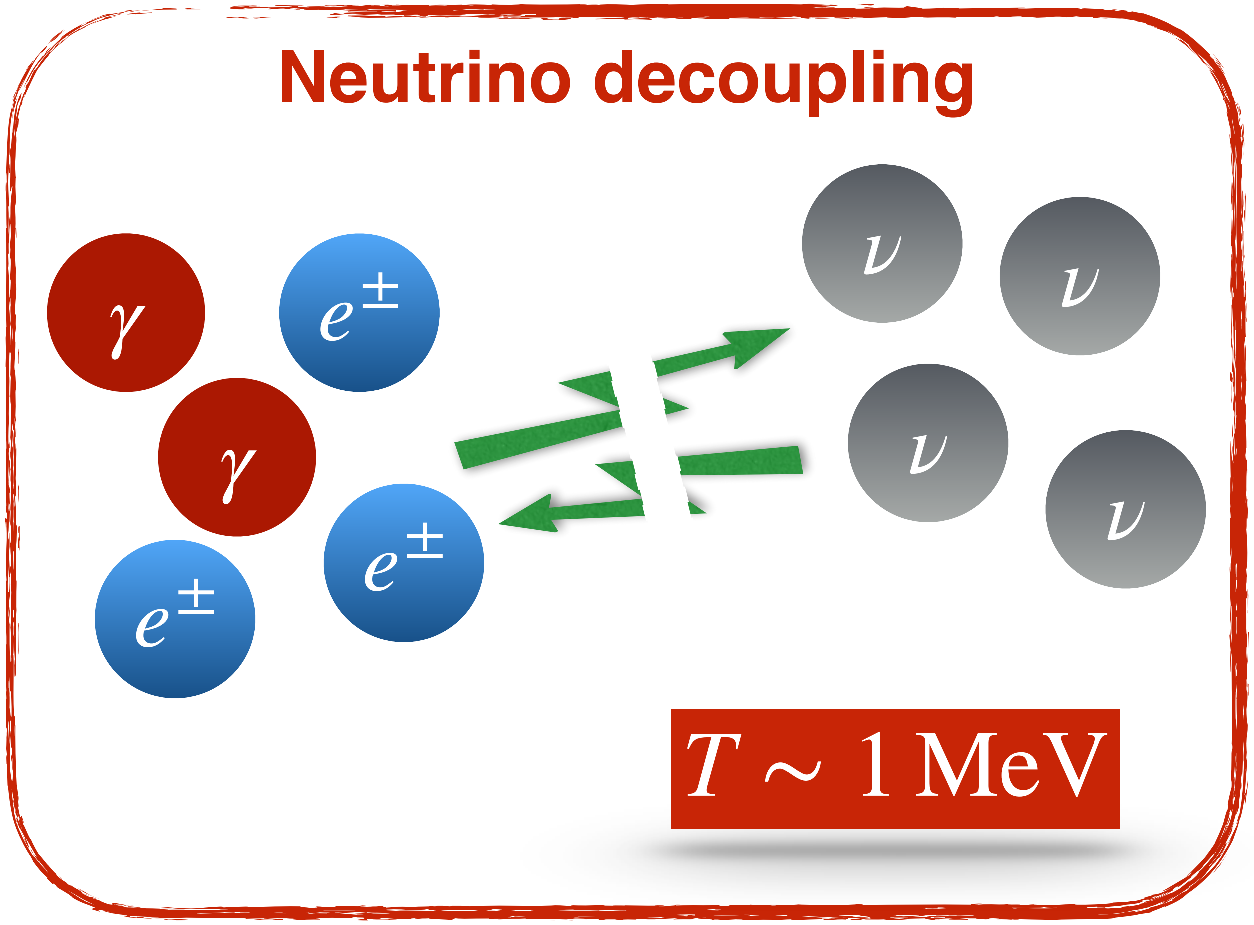
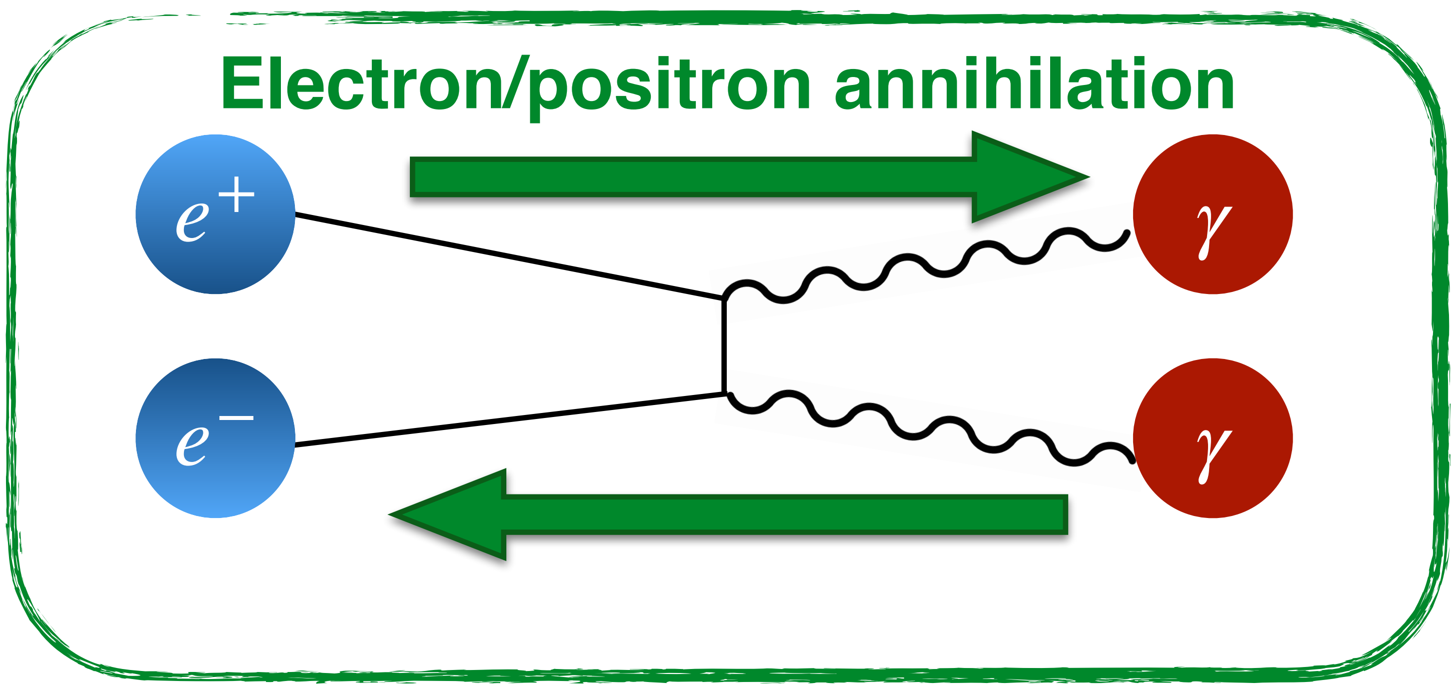
The MeV age


Neutrino decoupling
γ
$e^\pm$
γ
$e^\pm$
$e^\pm$
ν
ν
ν
ν


Electron/positron annihilation
e^+
e^-
γ
γ

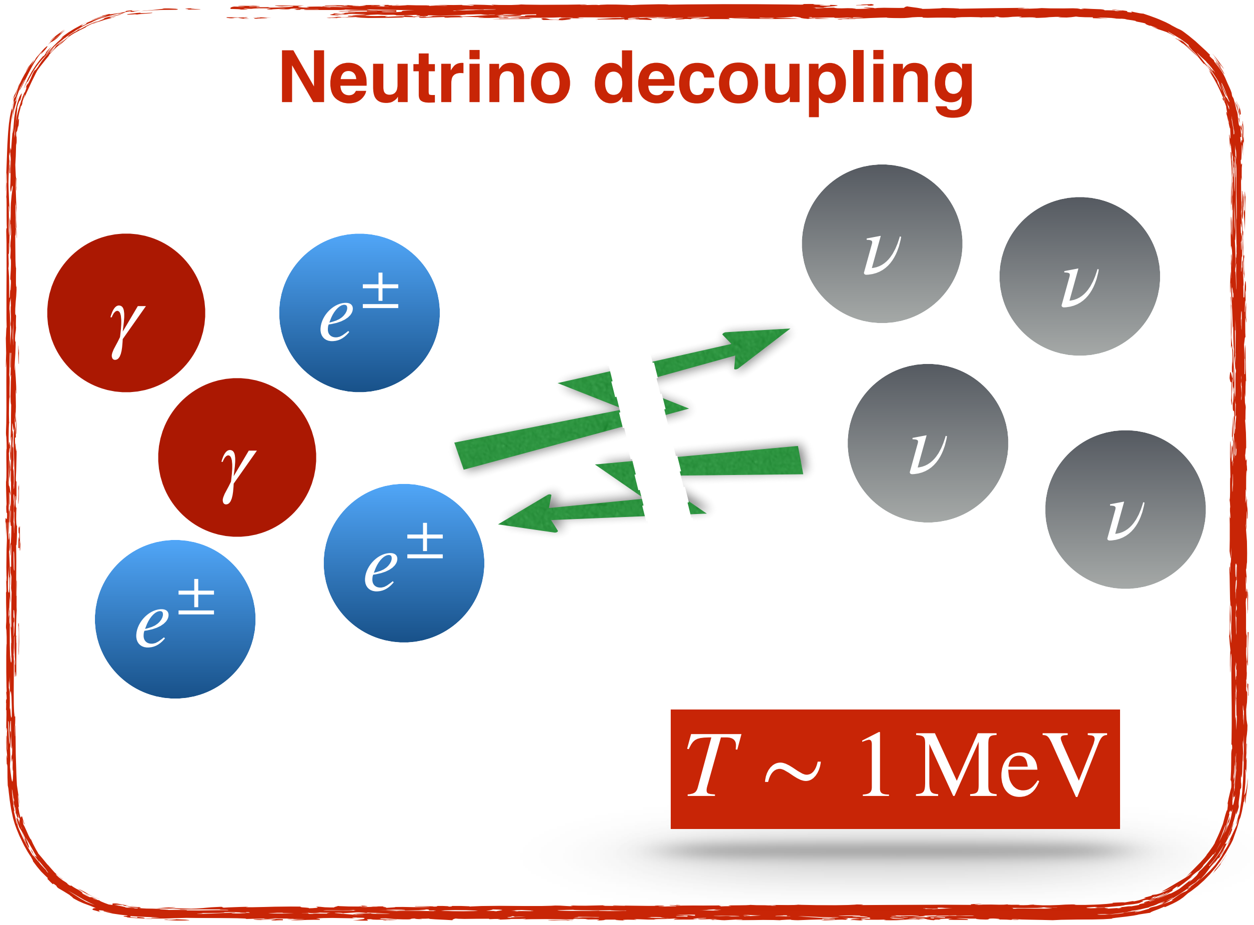
The MeV age





Below this temperature
$T_{\nu_e} = T_{\nu_\mu} = T_{\nu_\tau} \propto a^{-1}$

The MeV age



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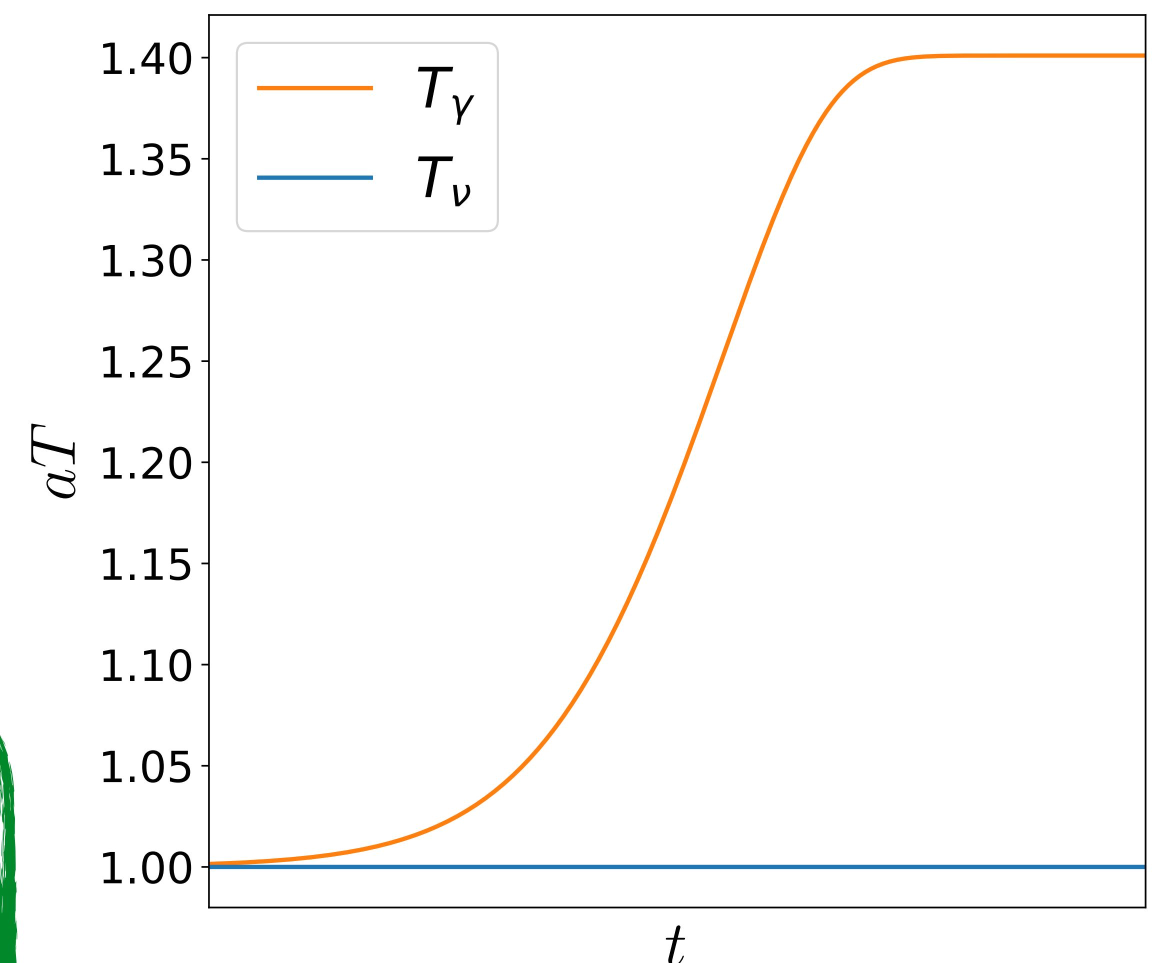
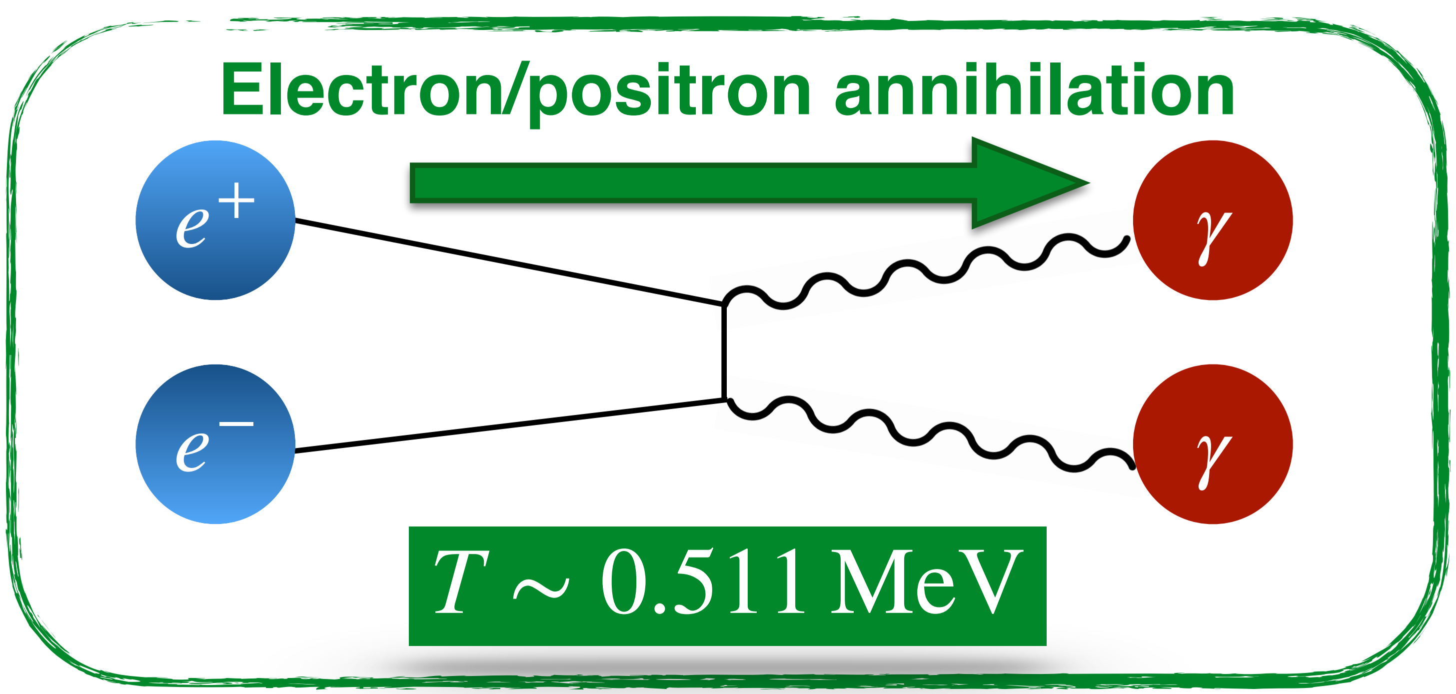
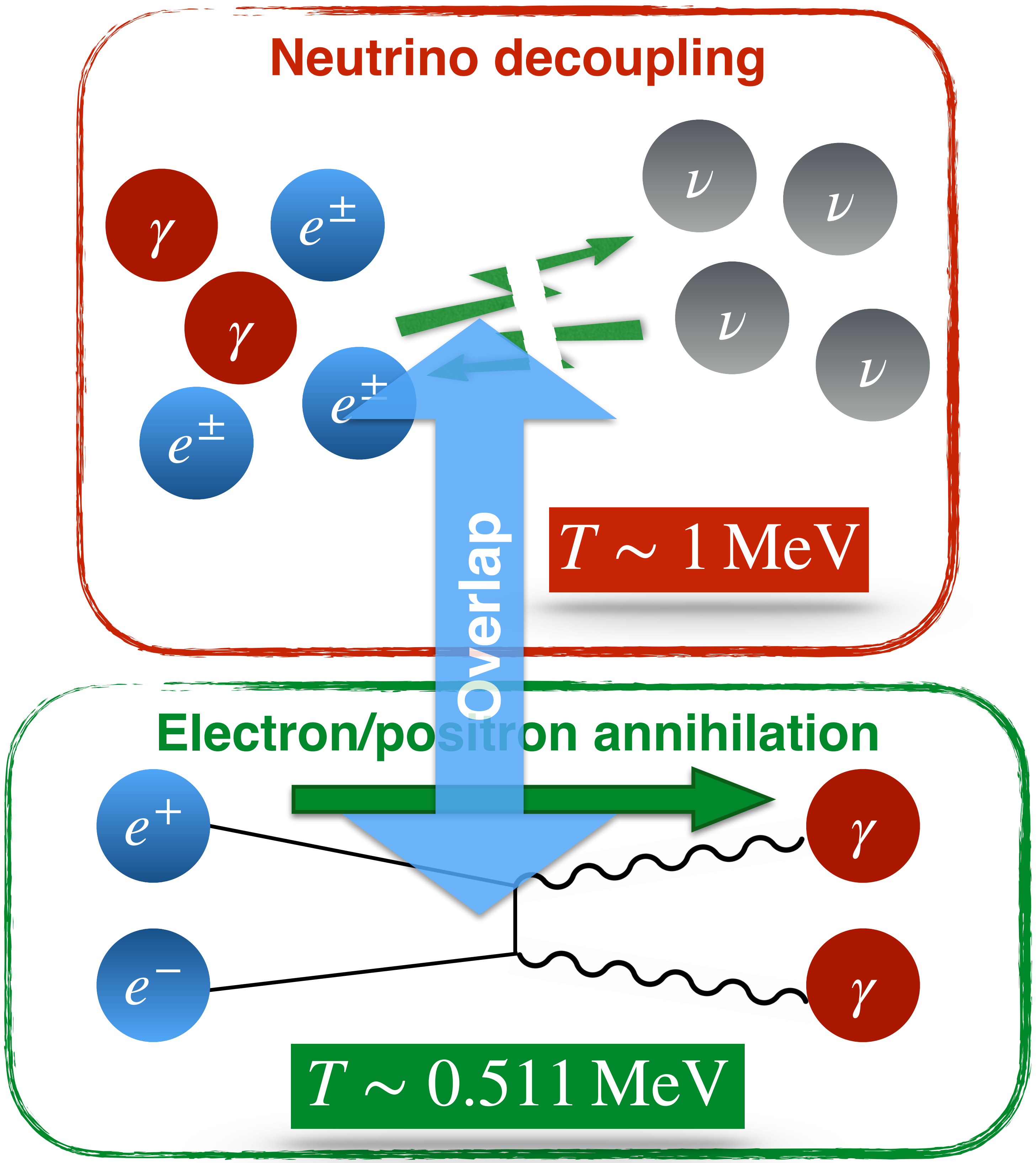




Entropy transfer from $e^{\pm}$

$T_\gamma \searrow$ slower than a^{-1}

The MeV age



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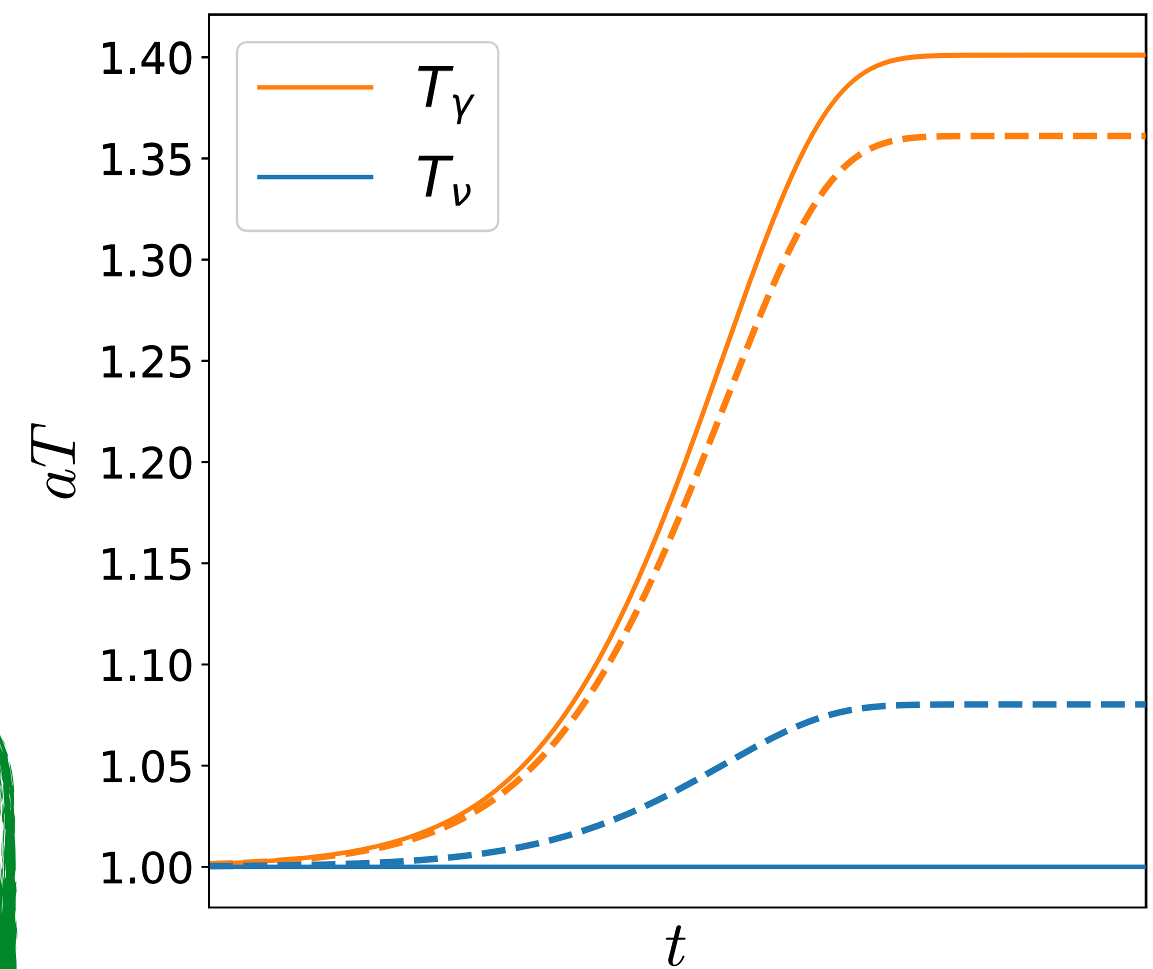

Entropy transfer from $e^\pm$

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Neutrino decoupling

- Instantaneous decoupling approximation

$$\left(\frac{T_\gamma}{T_\nu}\right)_{\rm today} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.40102$$

How to go beyond this approximation?

- Follow the distribution functions $f_{\nu_\alpha}(p,t)$: *Boltzmann equation*

$$f_{\nu_\alpha}(p,t) \equiv \frac{1}{e^{p/T_{\nu_\alpha}}+1}\left[1+\delta g_{\nu_\alpha}(p,t)\right]$$

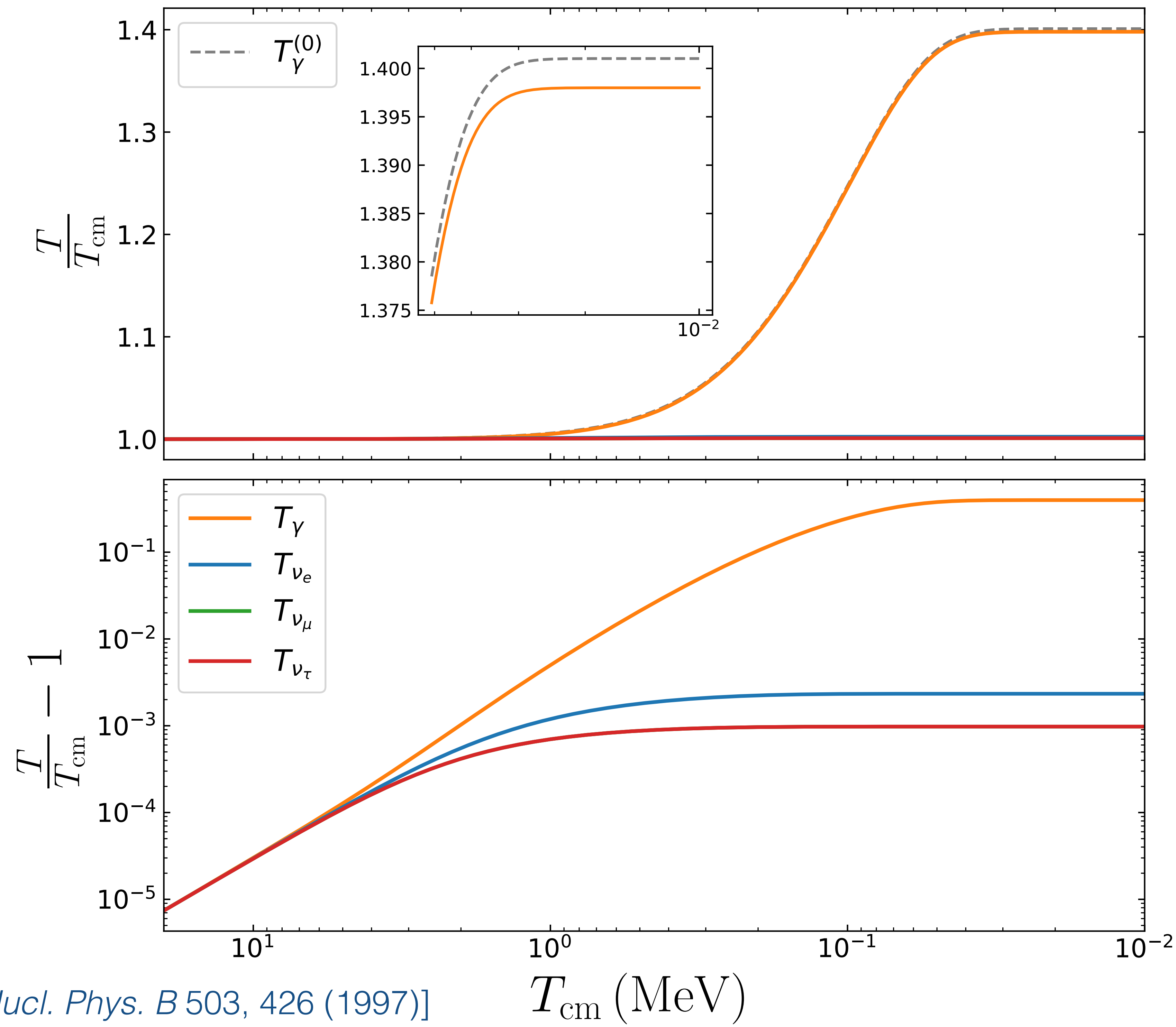
non-thermal distortions

$$\rho_{\nu_\alpha} \equiv \frac{7}{8}\frac{\pi^2}{30}T_{\nu_\alpha}^4$$

effective temperature

We use $T_{\rm cm} = T_\nu^{(0)} \propto a^{-1}$ as the integration variable.

Neutrino decoupling - standard calculations



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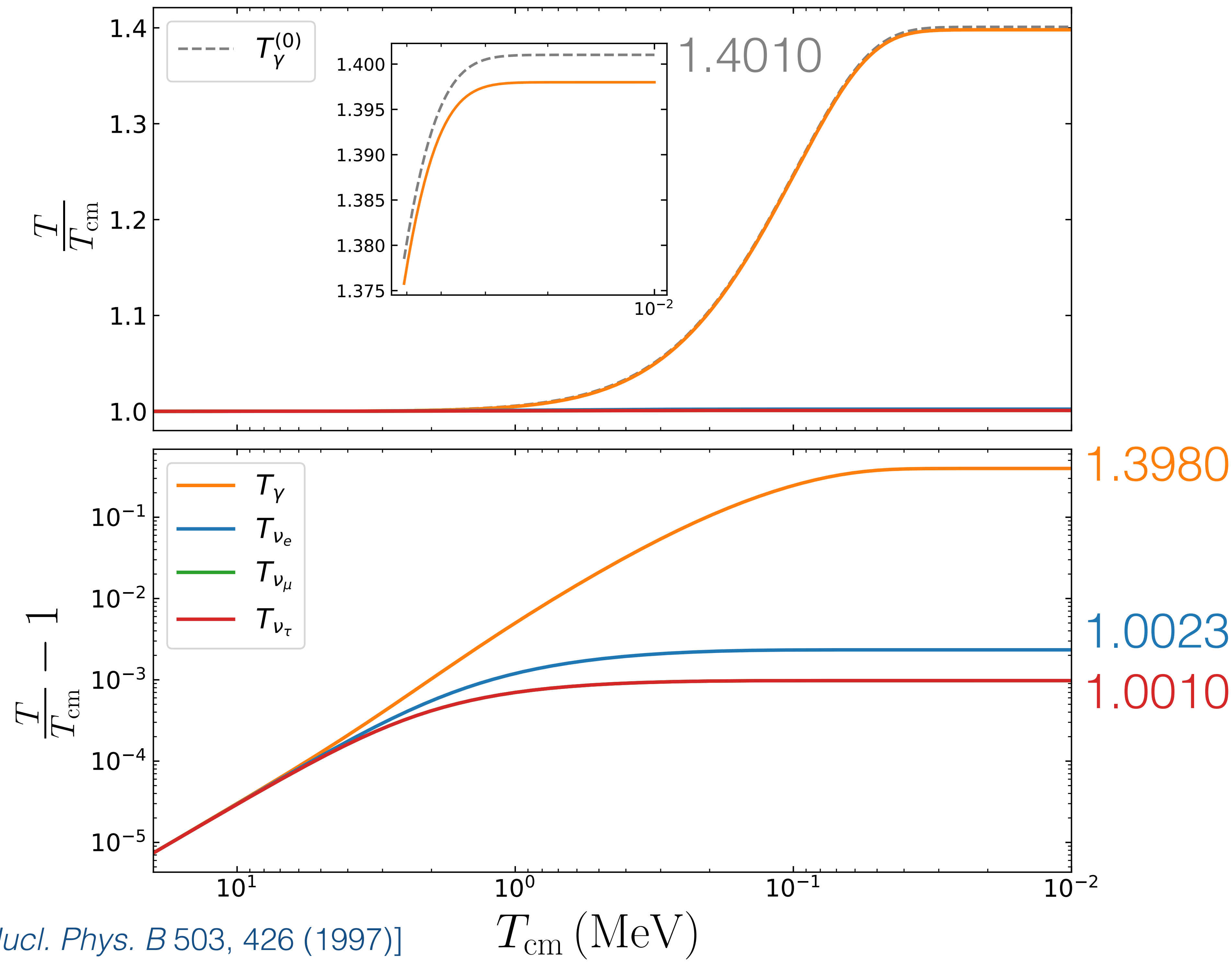

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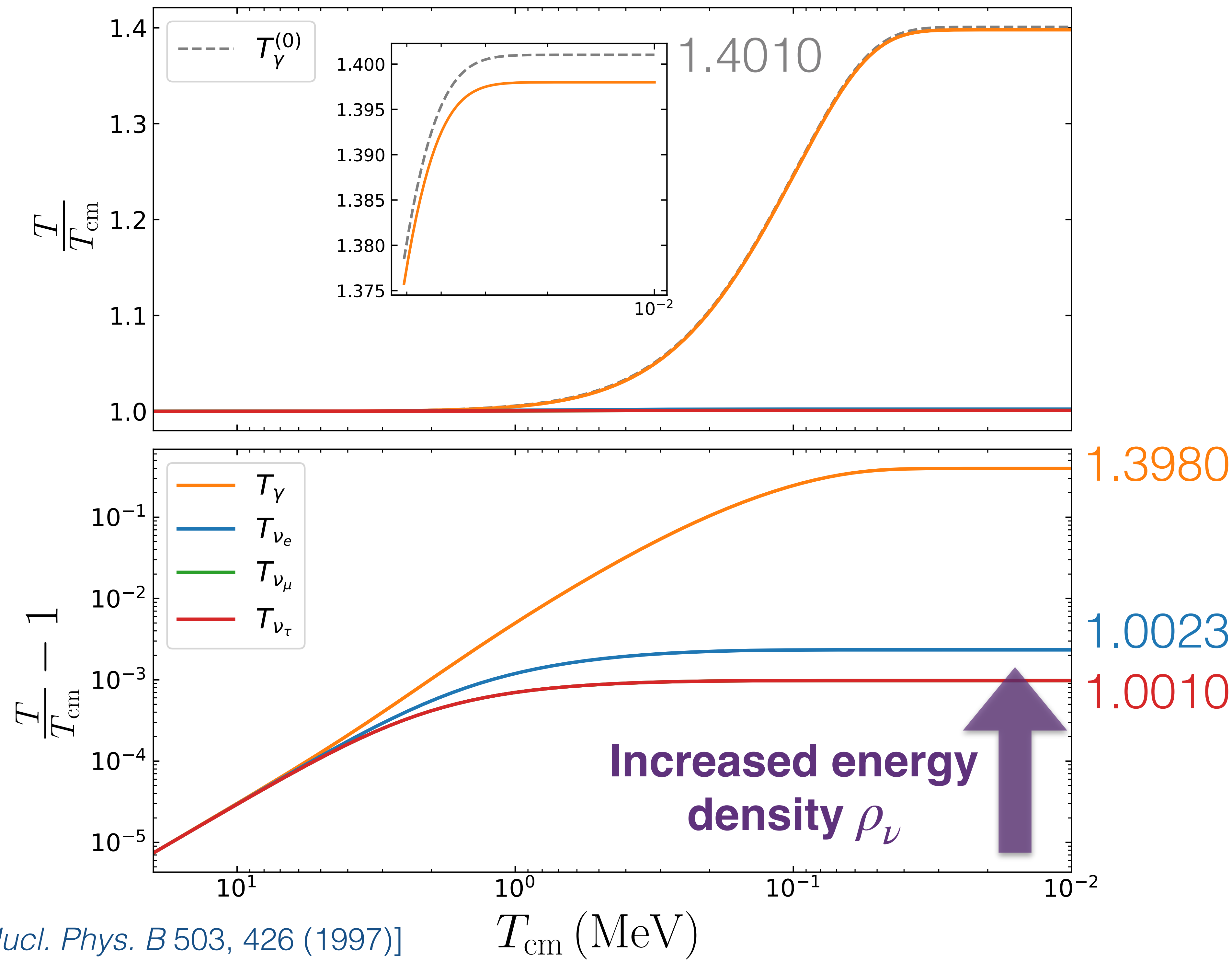

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Towards a precision calculation

- Important parameter: effective number of neutrino species $N_{\rm eff}$

$$\rho_\nu^{(0)} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \times \quad 3 \quad \times \rho_\gamma$$

Towards a precision calculation

- Important parameter: effective number of neutrino species N_{eff}

$$\rho_\nu = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \times N_{\mathrm{eff}} \times \rho_\gamma$$

$$N_{\mathrm{eff}} \simeq 3.0434$$

Planck
$N_{\mathrm{eff}} = 2.99 \pm 0.17 \ (68\%)$

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- Goal: $< 10^{-3}$ precision on N_{eff}

$\Longrightarrow$ **neutrino masses and mixings**

Previous numerical works:
[G. Mangano et al., *Nucl. Phys. B* 729, 221 (2005)]
[P.F. de Salas, S. Pastor, *JCAP* 07, 051 (2016)]
[K. Akita, M. Yamaguchi, *JCAP* 08, 012 (2020)]

Statistical description of neutrinos

- Due to mixing, the three distribution functions $f_{\nu_e}, f_{\nu_\mu}, f_{\nu_\tau}$ are not sufficient to describe the ensemble of neutrinos.

$$\langle \hat{a}^\dagger_{\nu_\beta}(\vec{p}\,', h')\hat{a}_{\nu_\alpha}(\vec{p}, h)\rangle = (2\pi)^3\, 2E_p\, \delta^{(3)}(\vec{p} - \vec{p}\,')\delta_{hh'}\, \varrho^\alpha_\beta(p, t)\, \delta_{h-}$$

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homogeneity

$$\begin{pmatrix} \varrho^e_e & \varrho^e_\mu & \varrho^e_\tau \\ \varrho^\mu_e & \varrho^\mu_\mu & \varrho^\mu_\tau \\ \varrho^\tau_e & \varrho^\tau_\mu & \varrho^\tau_\tau \end{pmatrix} = \begin{pmatrix} f_{\nu_e} & \varrho^e_\mu & \varrho^e_\tau \\ \varrho^\mu_e & f_{\nu_\mu} & \varrho^\mu_\tau \\ \varrho^\tau_e & \varrho^\tau_\mu & f_{\nu_\tau} \end{pmatrix}$$

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homogeneity

One-body density matrix

↓

Evolution equation?

$$\begin{pmatrix} \varrho^e_e & \varrho^e_\mu & \varrho^e_\tau \\ \varrho^\mu_e & \varrho^\mu_\mu & \varrho^\mu_\tau \\ \varrho^\tau_e & \varrho^\tau_\mu & \varrho^\tau_\tau \end{pmatrix} = \begin{pmatrix} f_{\nu_e} & \varrho^e_\mu & \varrho^e_\tau \\ \varrho^\mu_e & f_{\nu_\mu} & \varrho^\mu_\tau \\ \varrho^\tau_e & \varrho^\tau_\mu & f_{\nu_\tau} \end{pmatrix}$$

- Perturbative expansion [G. Sigl, G. Raffelt, *Nucl. Phys. B* 406, 423 (1993)]
- Close Time Path formalism [D. Blaschke, V. Cirigliano, *Phys. Rev. D* 94, 033009 (2016)]
- Extended BBGKY hierarchy [**JF**, C. Pitrou, M.C. Volpe, 2008.01074]

Quantum Kinetic Equations

$$i\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right]\varrho(p,t) = \left[U\frac{\mathbb{M}^2}{2p}U^\dagger, \varrho\right] - 2\sqrt{2}G_F p\left[\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2}, \varrho\right] + i\,\mathcal{C}[\varrho, \bar{\varrho}]$$

Vacuum Mean-field Collisions

$$\mathbb{E}_e + \mathbb{P}_e = \begin{pmatrix} \rho_e + P_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Matter effects (effective potential due to the background)

First calculation with the full collision term

Not the usual $\sqrt{2}G_F n_e$ since $n_e \simeq n_{\bar{e}}$

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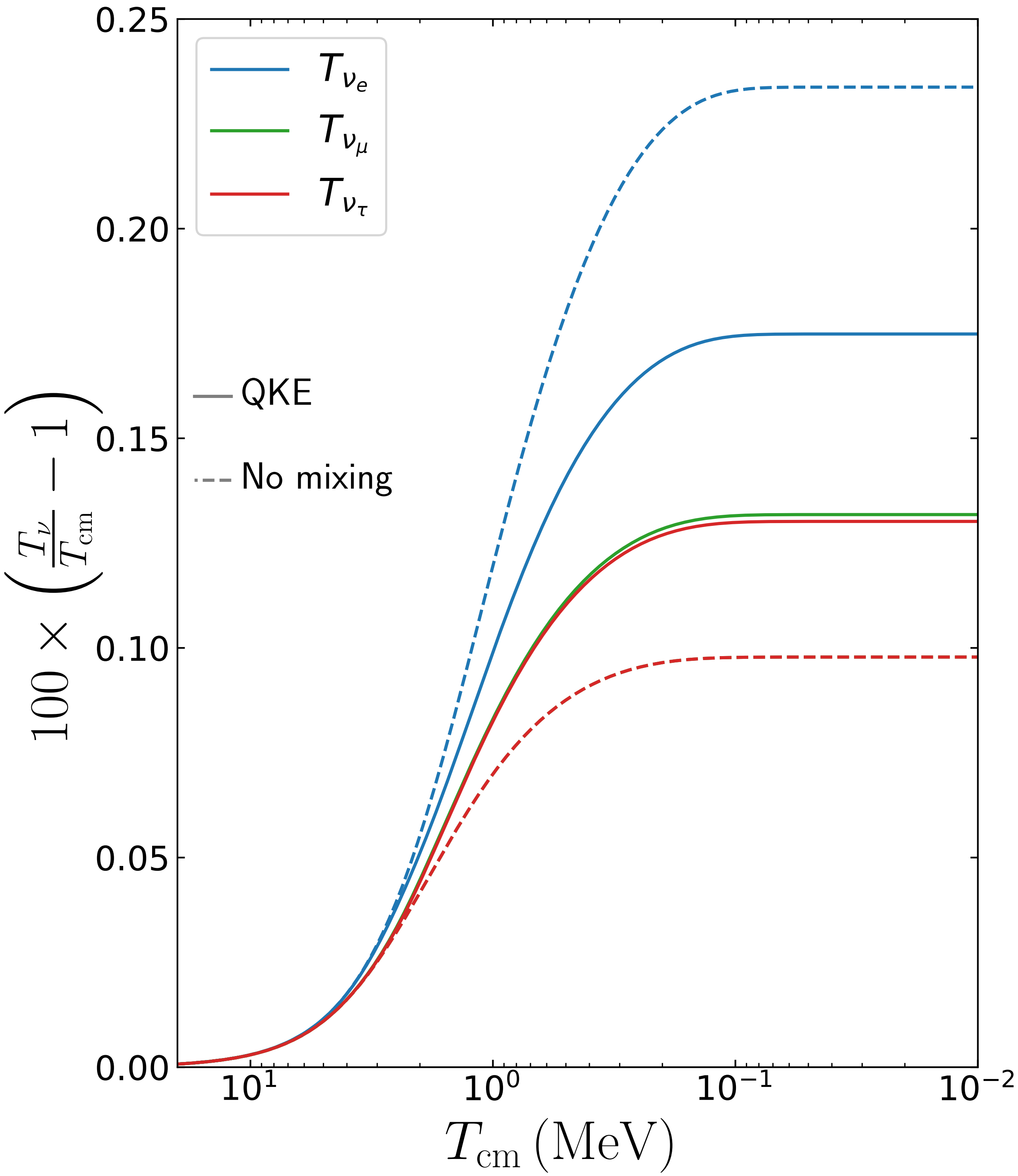
Approximation scheme for neutrino oscillations



"No visible oscillations"

$\Longrightarrow$ averaged oscillations?

$\Longrightarrow$ approximate scheme?

Approximation scheme for neutrino oscillations

- Simplified argument: 2-neutrino mixing, no mean-field

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = -i\left[U\frac{\mathbb{M}^2}{2p}U^\dagger, \varrho\right] + \mathcal{C} \quad \Longleftrightarrow \quad \frac{\mathrm{d}\varrho_m}{\mathrm{d}t} = -i\left[\frac{\mathbb{M}^2}{2p}, \varrho_m\right] + U^\dagger \mathcal{C} U$$

$$\varrho_m \equiv U^\dagger \varrho U$$

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$$\varrho_m = \begin{pmatrix} f_1 & a\, e^{i\frac{\Delta m^2}{2p}t} \\ a\, e^{-i\frac{\Delta m^2}{2p}t} & f_2 \end{pmatrix}$$

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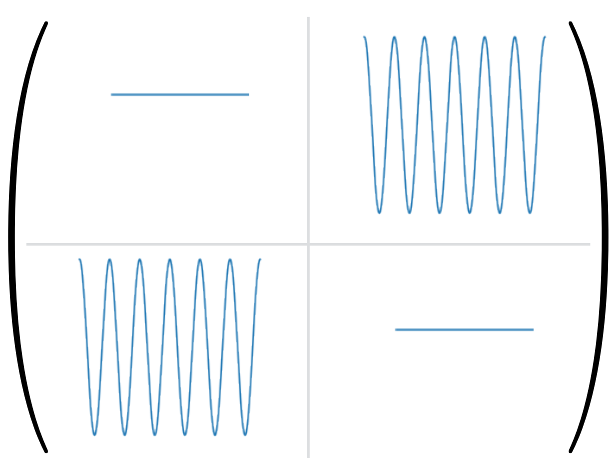
Schematically,

$$\varrho_m =$$



Localized neutrino injection
$(U^\dagger \mathcal{C} U \sim K \times \delta(0))$

Approximation scheme for neutrino oscillations

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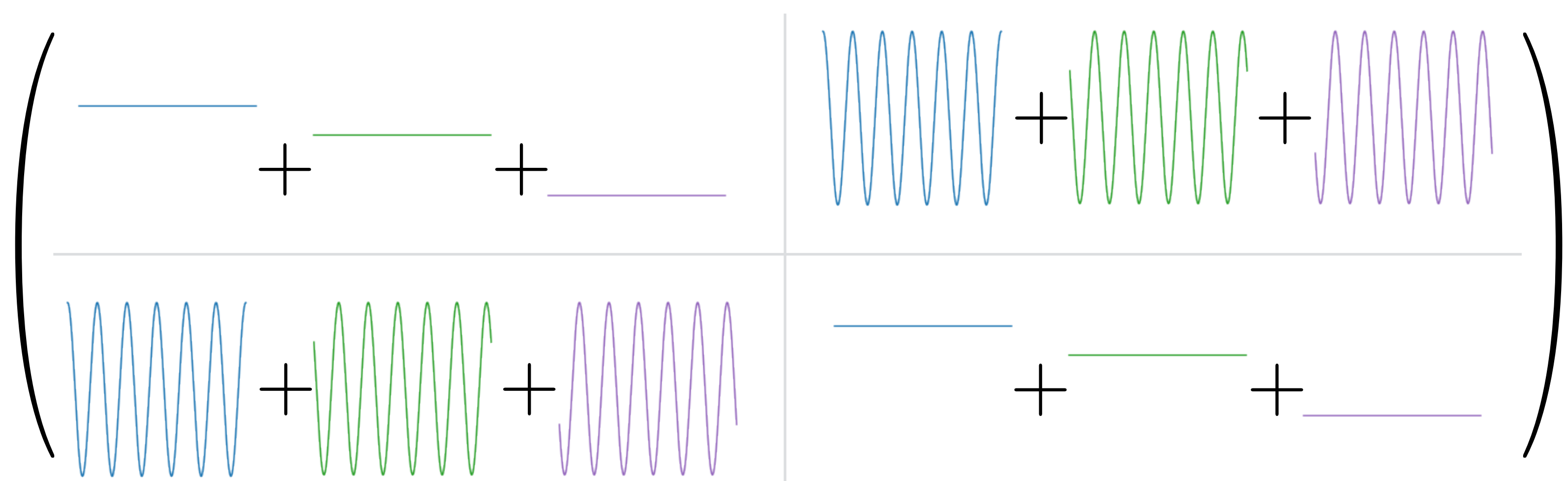

Random neutrino injection

Approximation scheme for neutrino oscillations

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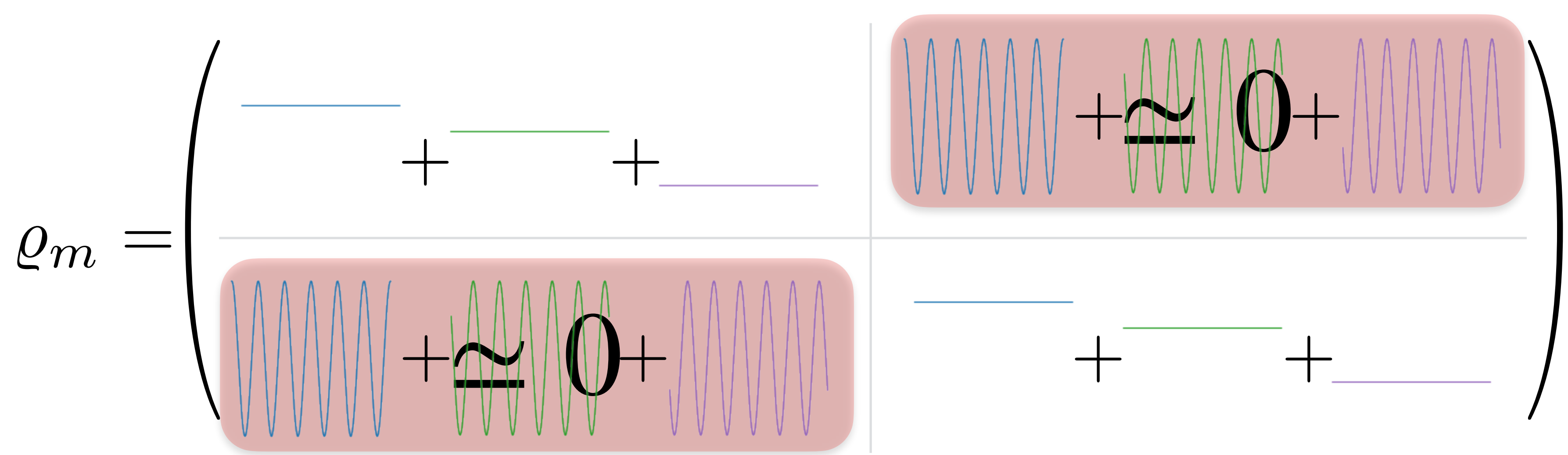

Random neutrino injection

Adiabatic Transfer of Averaged Oscillations

$$i\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right]\varrho(p,t) = \Big[U\frac{\mathbb{M}^2}{2p}U^\dagger, \varrho\Big] - 2\sqrt{2}G_F p\Big[\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2}, \varrho\Big] + i\,\mathcal{C}[\varrho, \bar{\varrho}]$$

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→ diagonal in the matter basis,
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- Non-diagonal components of the density matrix in matter basis are *averaged out*

$$\varrho_m = \begin{pmatrix} * & \sim & \sim \\ \sim & * & \sim \\ \sim & \sim & * \end{pmatrix} \longrightarrow \tilde{\varrho}_m = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

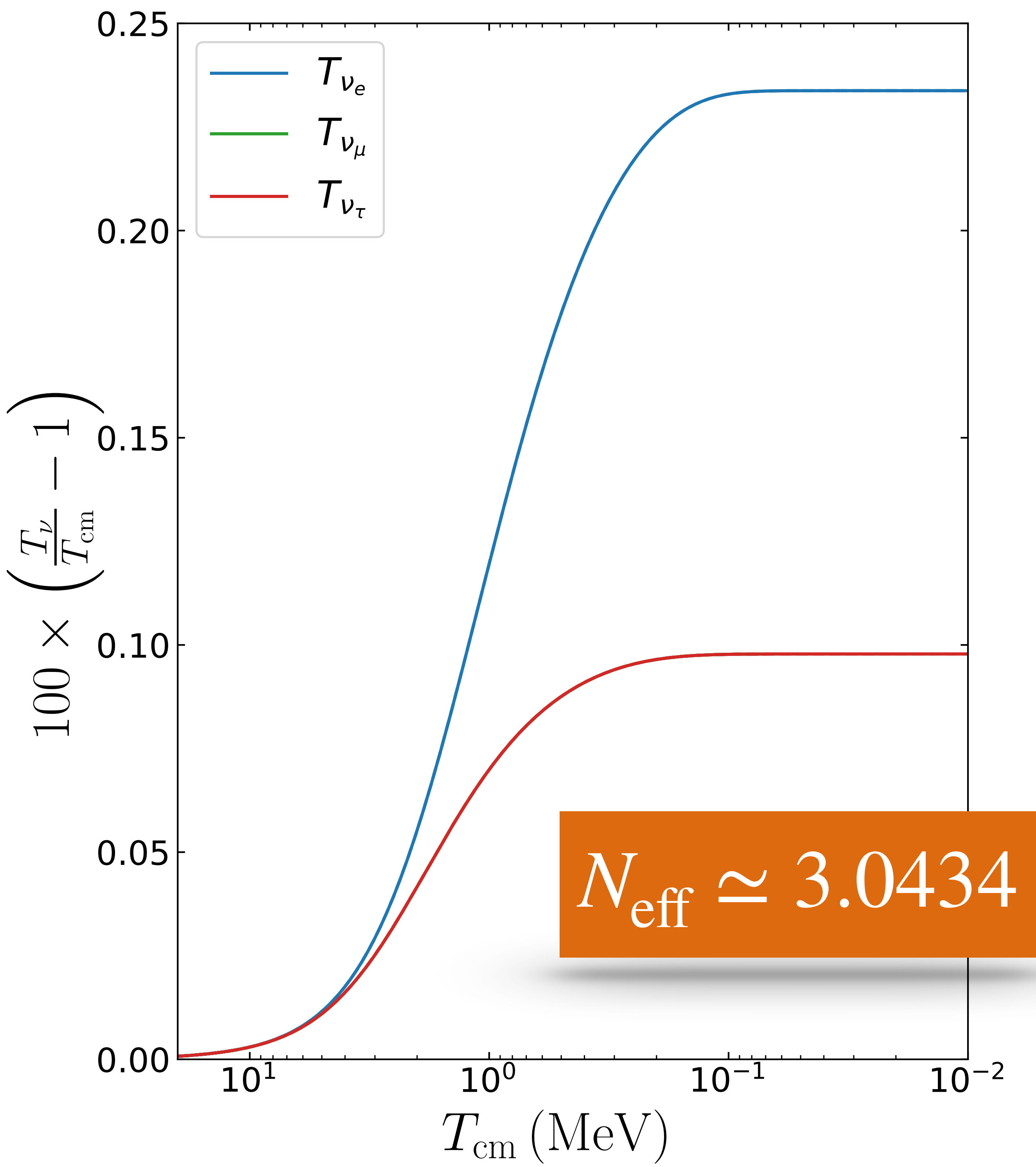
- Effective "ATAO" equation

$$\boxed{\frac{\partial \tilde{\varrho}_m}{\partial x} = \widetilde{U_m^\dagger \mathcal{K} U_m}}$$

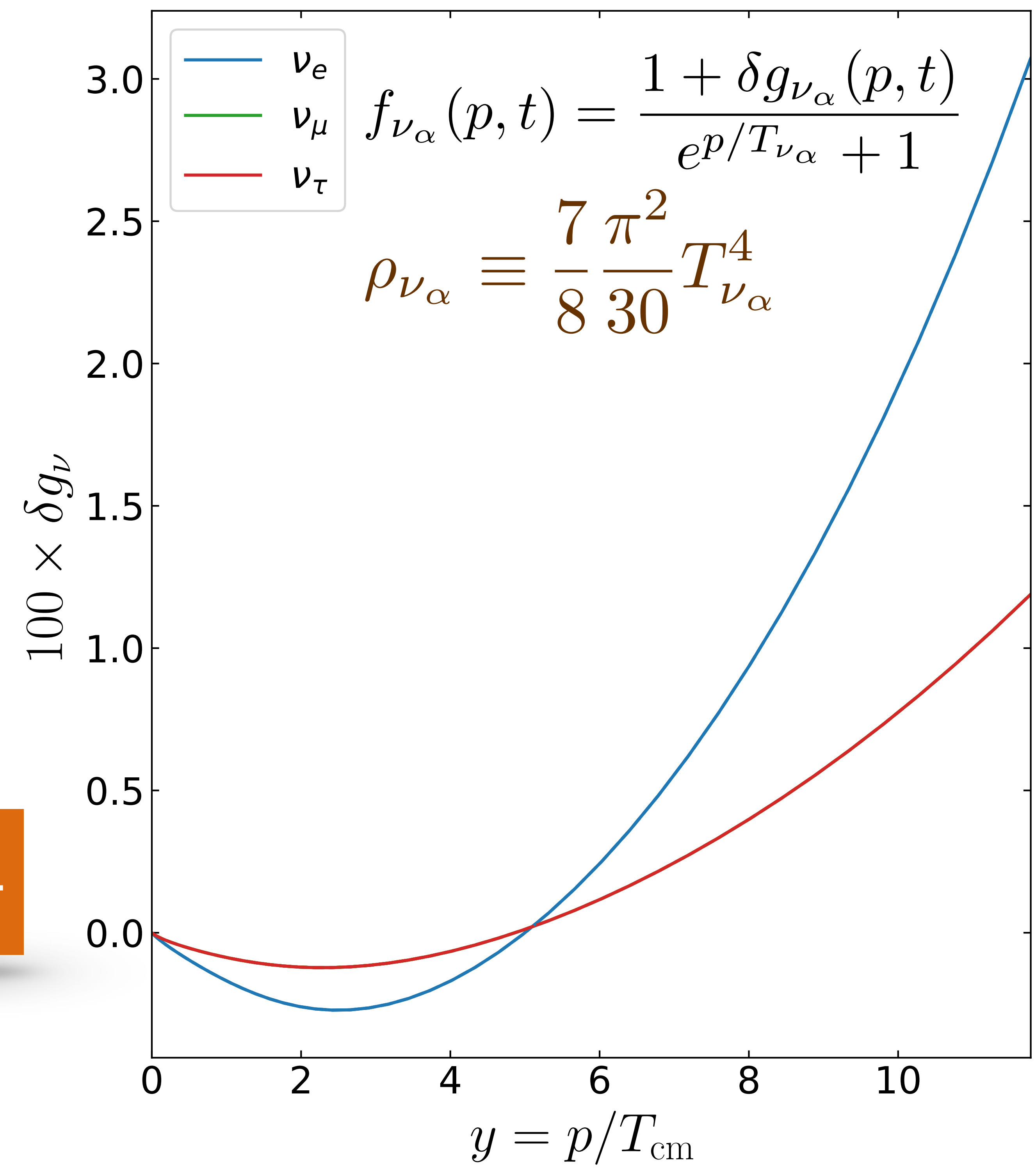
keep only the diagonal

comoving variable $x \propto a$

Neutrino decoupling without flavor oscillations

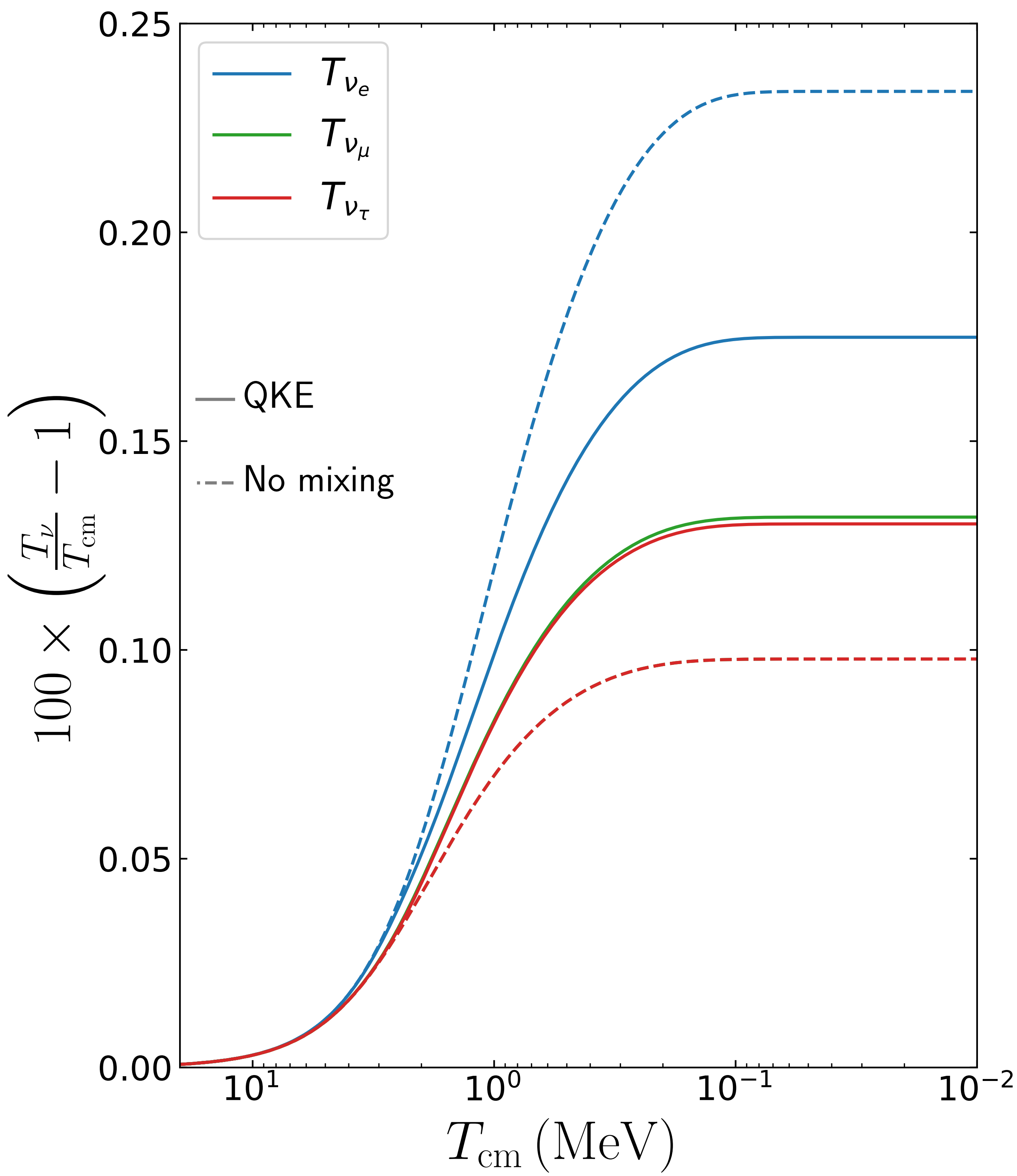


Effective temperatures



Effective distortions

Neutrino decoupling with flavor oscillations



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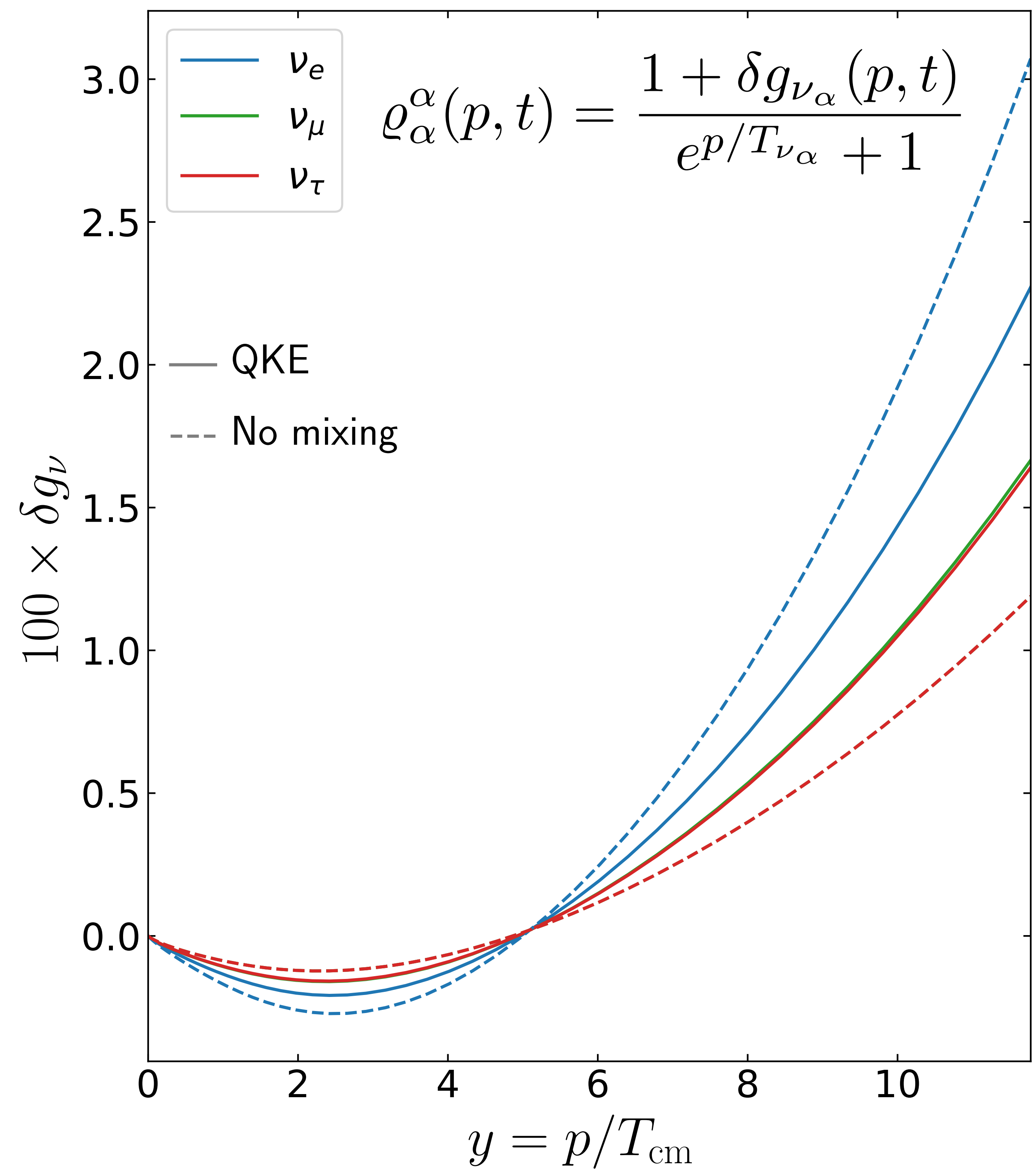

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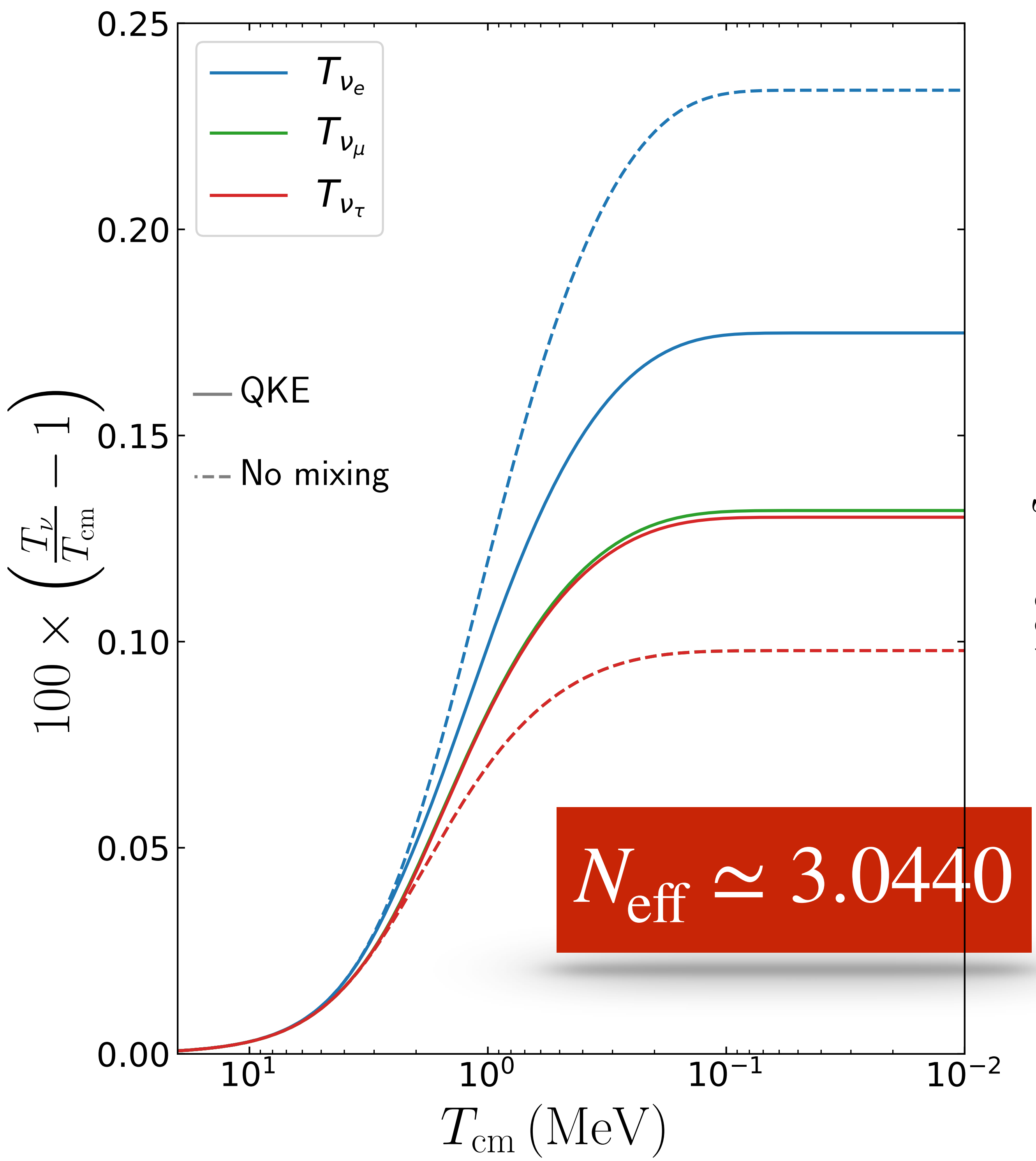
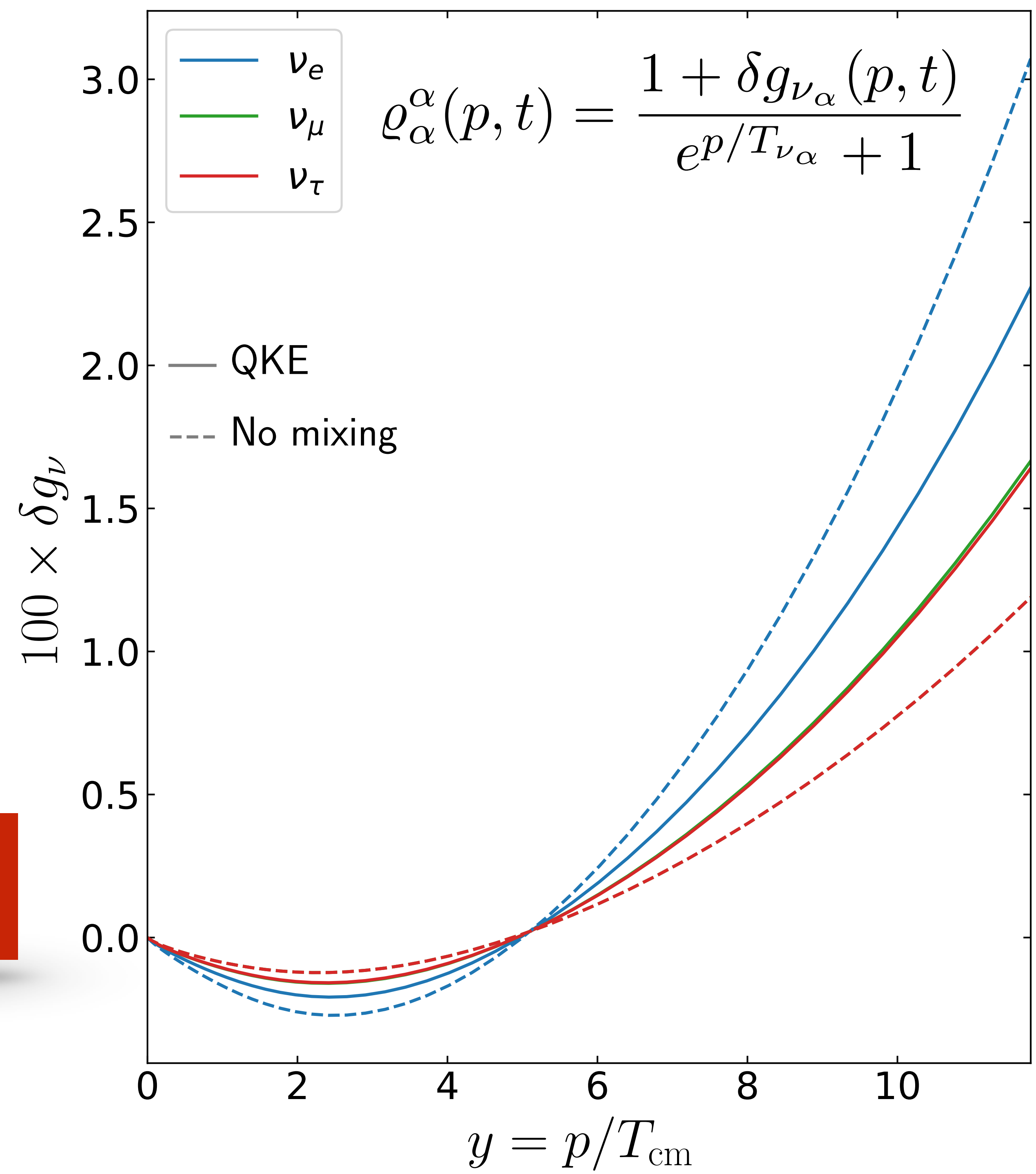


$N_{\rm eff} \simeq 3.0440$

Effective temperatures



$$\varrho_\alpha^\alpha(p,t) = \frac{1+\delta g_{\nu_\alpha}(p,t)}{e^{p/T_{\nu_\alpha}}+1}$$

Effective distortions

Decoupling with flavor oscillations - Comments

- Excellent accuracy of ATAO approximation ($<10^{-6}$).
- Slight increase of N_{eff} (3.0434 $\rightarrow$ 3.0440)

  flavor conversion of ν_e $\Longrightarrow$ more phase space for $e^{\pm}$ annihilations

- Small dependence with the mass hierarchy ($\Delta N_{\mathrm{eff}} \lesssim 10^{-5}$)
- Higher precision?
  - Full QED corrections
  - Inhomogeneous cosmology

$\Delta N_{\mathrm{eff}} \sim 10^{-5}$

Conclusion

Neutrino decoupling

- Neutrinos capture part of the entropy released by $e^{\pm}$ annihilations
- Increased effective temperatures + spectral distortions
- Exact or approximate treatment of neutrino mixing
- First calculation with the full collision term
- $N_{\mathrm{eff}} \simeq 3.044$

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Consequences on BBN, CMB…

[**JF**, C. Pitrou, *Phys. Rev. D* 101, 043524 (2020)]
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Variation of abundances	$\delta(\mathrm{D/H})\ (\%)$	$\delta([^3\mathrm{He}+\mathrm{T}]/\mathrm{H})\ (\%)$	$\delta([^7\mathrm{Li}+{}^7\mathrm{Be}]/\mathrm{H})\ (\%)$
Without mixing	0.368	0.120	-0.412
With mixing	0.387	0.126	-0.404

Quantum Kinetic Equations: collision term

$$i\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right]\varrho(p,t) = \left[U\frac{\mathbb{M}^2}{2p}U^\dagger, \varrho\right] - 2\sqrt{2}G_F p\left[\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2}, \varrho\right] + i\,\mathcal{C}[\varrho, \bar{\varrho}]$$

Vacuum Mean-field Collisions

$$\mathcal{C} = \mathcal{C}^{[\nu e^- \to \nu e^-]} + \mathcal{C}^{[\nu e^+ \to \nu e^+]} + \mathcal{C}^{[\nu\bar{\nu} \to e^- e^+]} + \mathcal{C}^{[\nu\nu]}$$

$$\begin{aligned}
\mathcal{C}^{[\nu e^- \to \nu e^-]} =& \frac{1}{2}\frac{2^5 G_F^2}{2E_1}\int \frac{\mathrm{d}^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{\mathrm{d}^3\vec{p}_4}{(2\pi)^3 2E_4} \\
&\times (2\pi)^4\delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\
&\times \Big[4(p_1\cdot p_2)(p_3\cdot p_4)F_{\rm sc}^{LL}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) \\
&\quad + 4(p_1\cdot p_4)(p_2\cdot p_3)F_{\rm sc}^{RR}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) \\
&\quad - 2(p_1\cdot p_3)m_e^2\left(F_{\rm sc}^{LR}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) + F_{\rm sc}^{RL}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)})\right)\Big]
\end{aligned}$$

Computationally expensive

Quantum Kinetic Equations: collision term

$$i\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right]\varrho(p,t) = \left[U\frac{\mathbb{M}^2}{2p}U^\dagger, \varrho\right] - 2\sqrt{2}G_F p\left[\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2}, \varrho\right] + i\,\mathcal{C}[\varrho, \bar{\varrho}]$$

Vacuum Mean-field Collisions

$$\mathcal{C} = \mathcal{C}^{[\nu e^- \to \nu e^-]} + \mathcal{C}^{[\nu e^+ \to \nu e^+]} + \mathcal{C}^{[\nu\bar{\nu} \to e^- e^+]} + \mathcal{C}^{[\nu\nu]}$$

$$\begin{aligned}\mathcal{C}^{[\nu e^- \to \nu e^-]} =& \frac{1}{2}\frac{2^5 G_F^2}{2E_1}\int \frac{\mathrm{d}^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{\mathrm{d}^3\vec{p}_4}{(2\pi)^3 2E_4} \\ &\times (2\pi)^4\delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ &\times \Big[4(p_1\cdot p_2)(p_3\cdot p_4)F_{\mathrm{sc}}^{LL}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) \\ &+ 4(p_1\cdot p_4)(p_2\cdot p_3)F_{\mathrm{sc}}^{RR}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) \\ &- 2(p_1\cdot p_3)m_e^2\left(F_{\mathrm{sc}}^{LR}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) + F_{\mathrm{sc}}^{RL}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)})\right)\Big]\end{aligned}$$

Computationally expensive

Statistical factor

$$F_{\mathrm{sc}}^{AB}(\nu^{(1)} + e^{(2)} \to \nu^{(3)} + e^{(4)}) = f_4(1-f_2)\left[G^A\varrho_3 G^B(1-\varrho_1)\right] - (1-f_4)f_2\left[G^A(1-\varrho_3)G^B\varrho_1\right] + \mathrm{h.c.}$$

"gain" "loss"

Quantum Kinetic Equations: collision term

$$i\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right]\varrho(p,t) = \left[U\frac{\mathbb{M}^2}{2p}U^\dagger, \varrho\right] - 2\sqrt{2}G_F p\left[\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2}, \varrho\right] + i\,\mathcal{C}[\varrho, \bar{\varrho}]$$

Vacuum — Mean-field — Collisions

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Computationally expensive

Statistical factor

Pauli-blocking

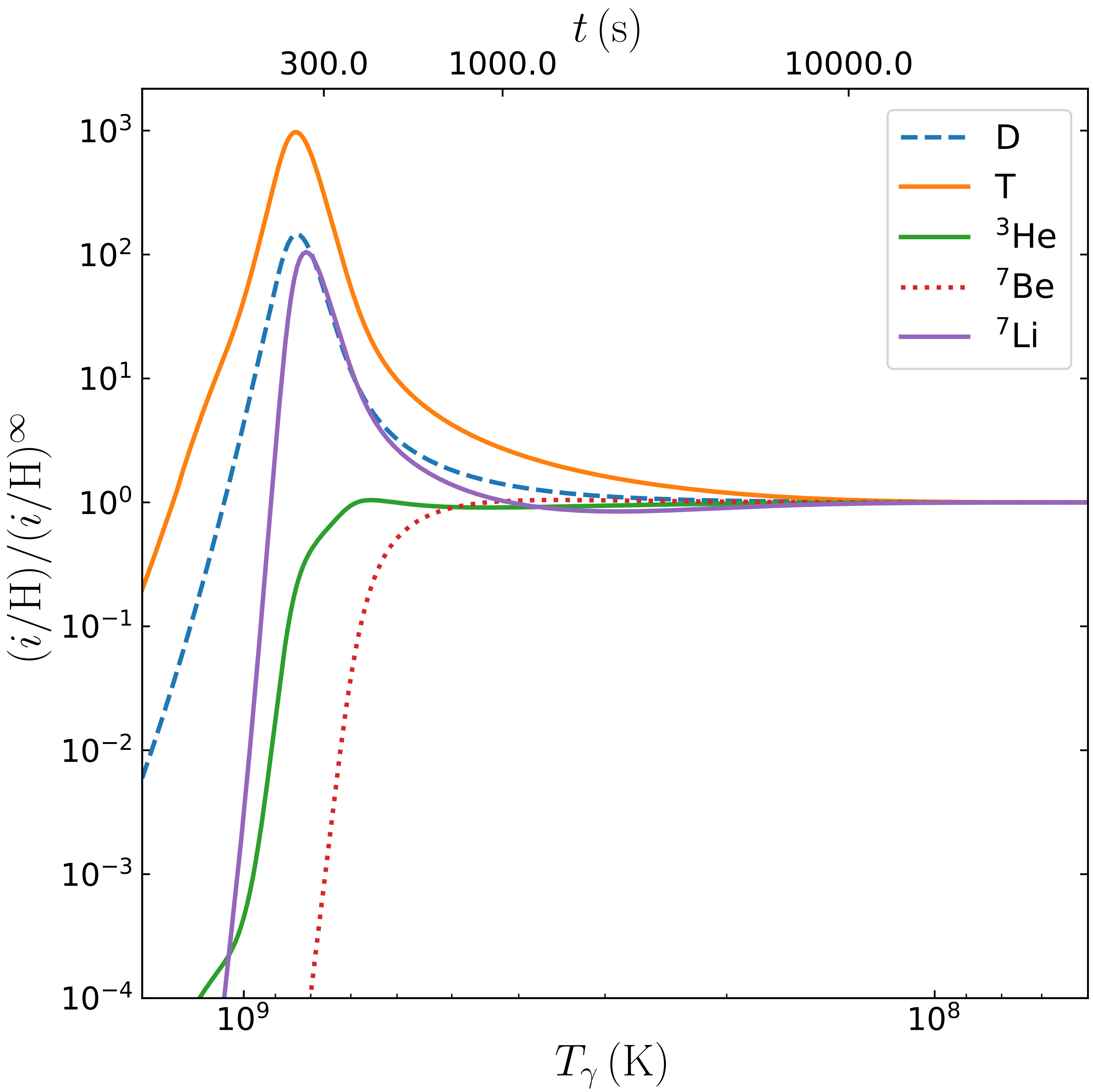
$$F_{\rm sc}^{AB}(\nu^{(1)}+e^{(2)}\to\nu^{(3)}+e^{(4)}) = f_4(1-f_2)\left[G^A\varrho_3 G^B(1-\varrho_1)\right] - (1-f_4)f_2\left[G^A(1-\varrho_3)G^B\varrho_1\right] + \text{h.c.}$$

"gain" — "loss"

Incomplete neutrino decoupling and BBN

$N_{\rm eff} > 3 \implies$ higher expansion rate

"Clock effect"



[**JF**, C. Pitrou, *Phys. Rev. D* 101, 043524 (2020)]
[**JF**, C. Pitrou, M.C. Volpe, 2008.01074]

Incomplete neutrino decoupling and BBN

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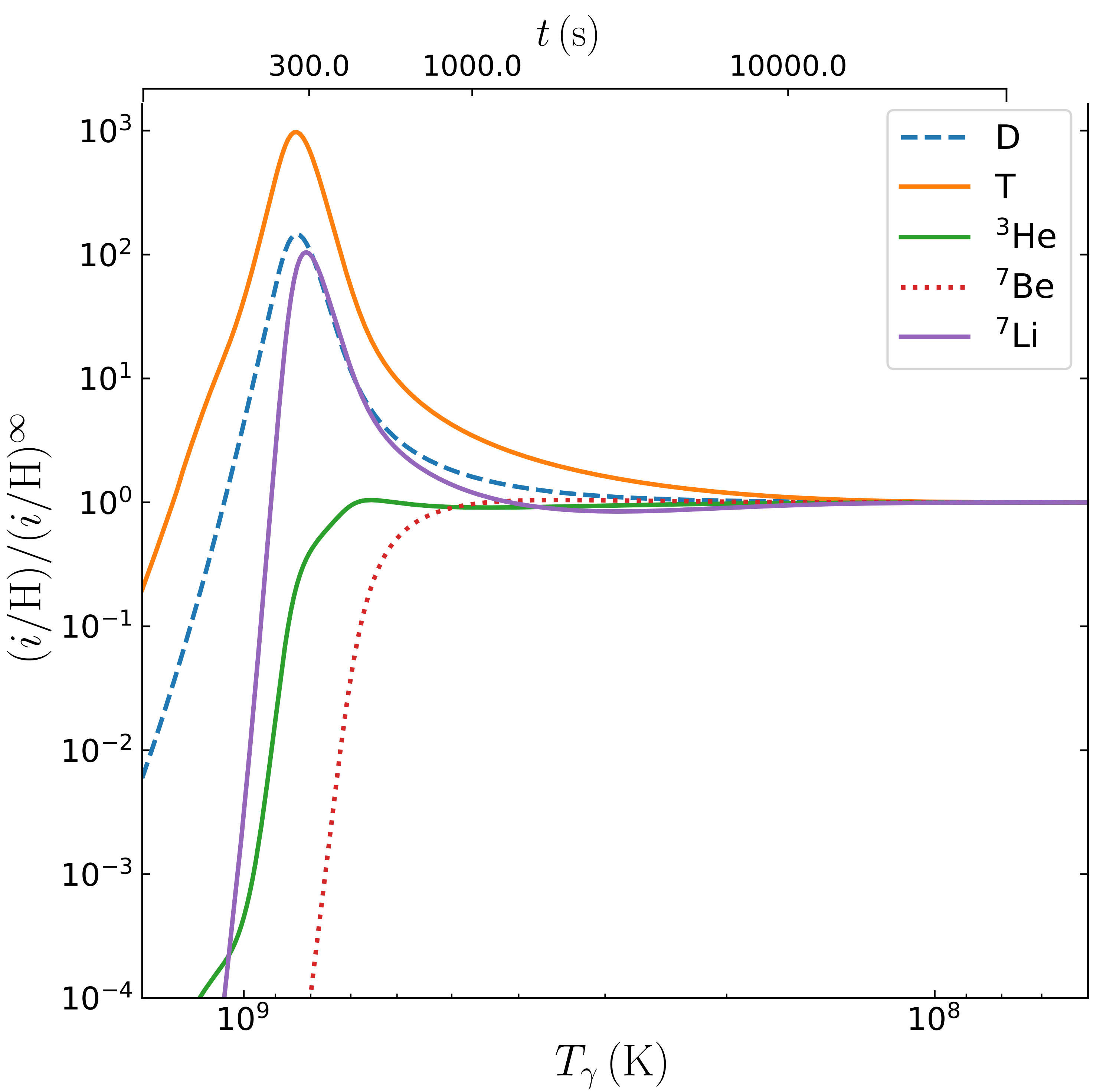

$\delta(\mathrm{D/H}) > 0$

$\delta(^3\mathrm{He/H}) \gtrsim 0$

$\delta(\mathrm{T/H}) > 0$

$\delta(^7\mathrm{Be/H}) < 0$

$\delta(^7\mathrm{Li/H}) > 0$

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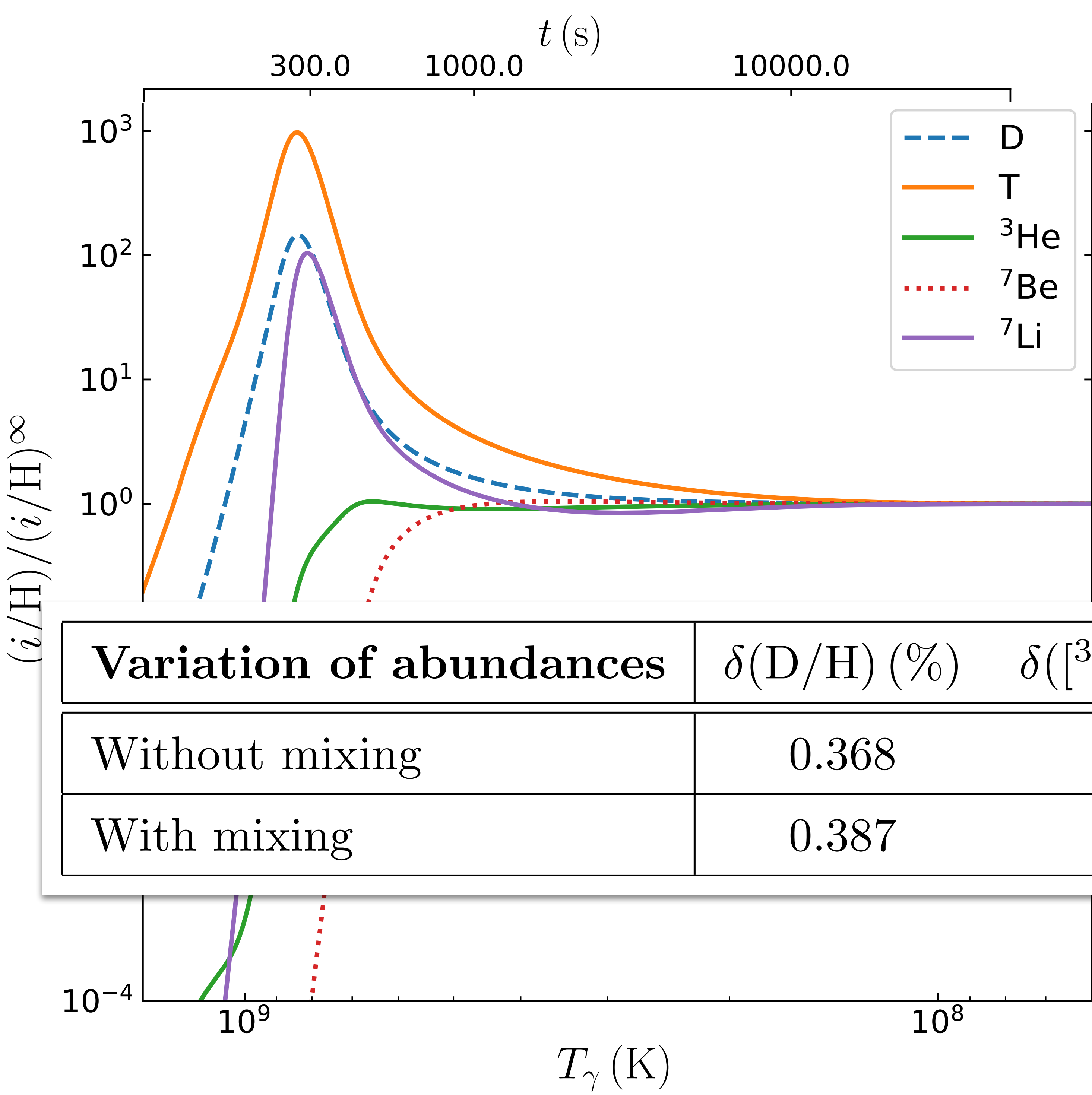

"Clock effect"

$\delta(\mathrm{D/H}) > 0$

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$\delta(^7\mathrm{Be/H}) < 0$

$\delta(^7\mathrm{Li/H}) > 0$

Variation of abundances	$\delta(\mathrm{D/H})\,(\%)$	$\delta([^3\mathrm{He}+\mathrm{T}]/\mathrm{H})\,(\%)$	$\delta([^7\mathrm{Li}+{}^7\mathrm{Be}]/\mathrm{H})\,(\%)$
Without mixing	0.368	0.120	−0.412
With mixing	0.387	0.126	−0.404

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Incomplete neutrino decoupling and BBN

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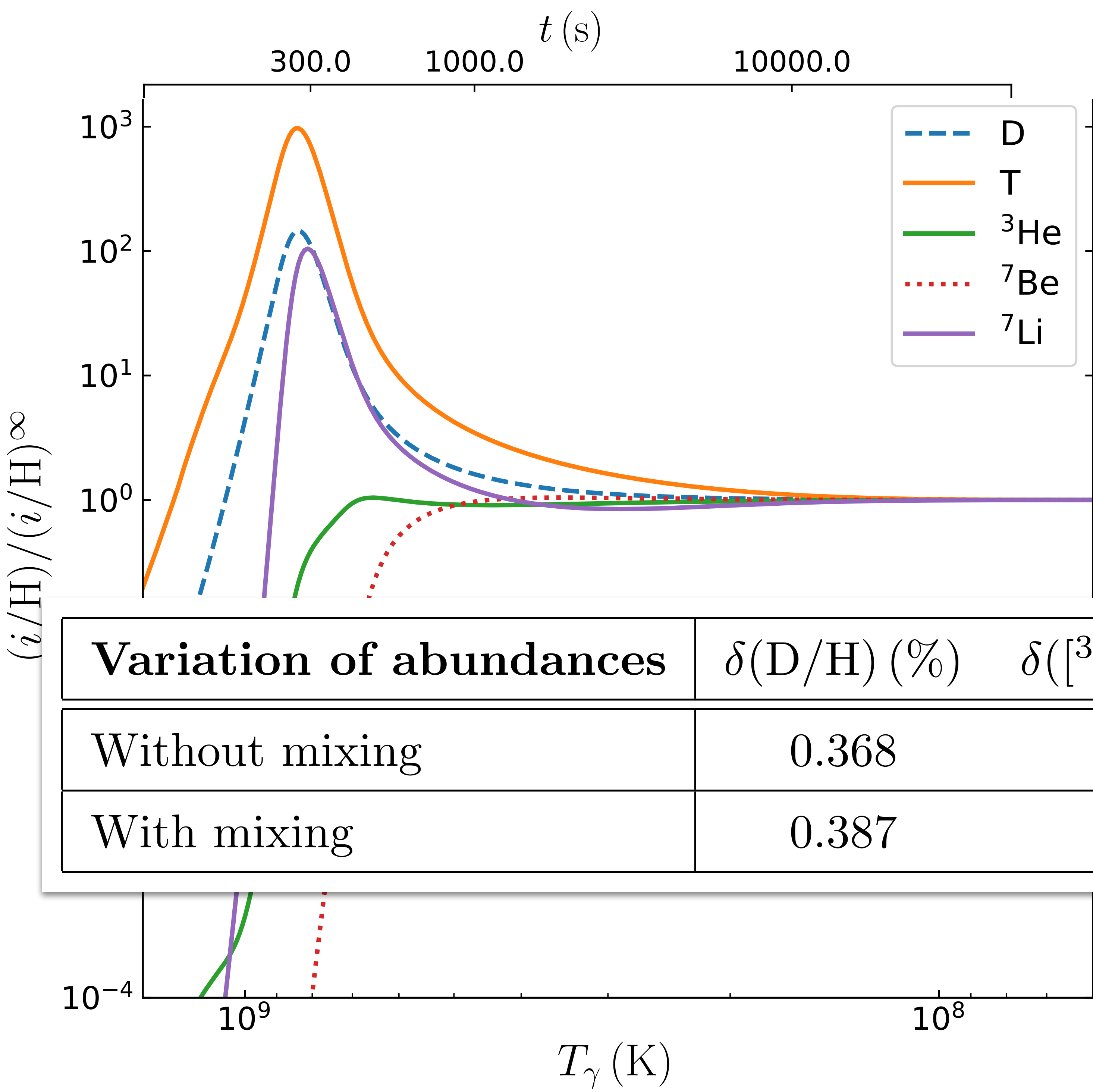

^{4}He depends mostly on the n/p ratio, controlled by ν_e

$\implies$ effect of $z_{\nu_e}, \delta g_{\nu_e}$ (+ clock effect)

Overall, relative variation $\sim 0.04\,\%$

Variation of abundances	δ(D/H) (%)	$\delta([^3\mathrm{He}+\mathrm{T}]/\mathrm{H})$ (%)	$\delta([^7\mathrm{Li}+{}^7\mathrm{Be}]/\mathrm{H})$ (%)
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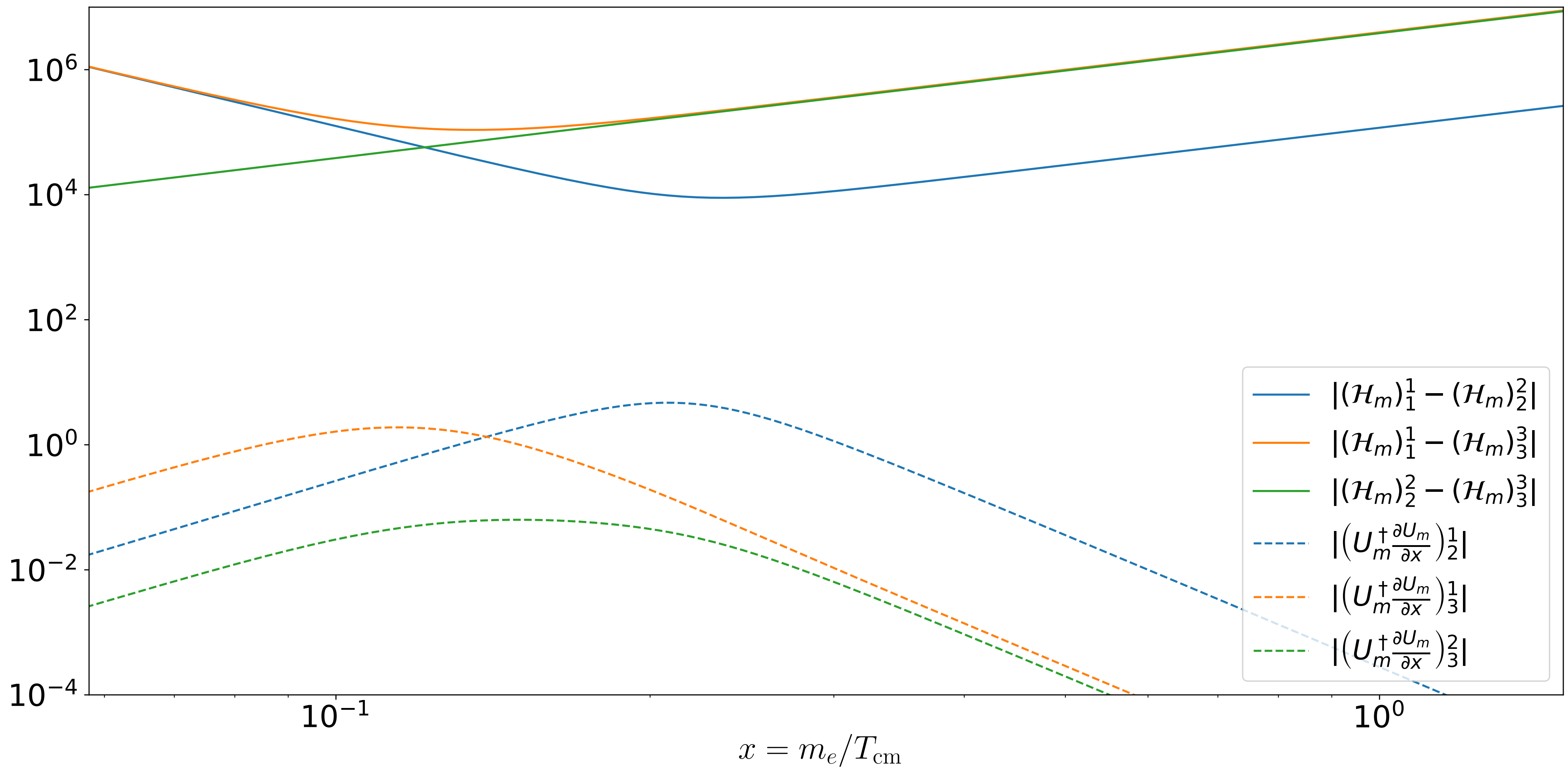
Checking the adiabatic approximation

$$\frac{\partial \varrho_m}{\partial x} = -i[\mathcal{H}_m, \varrho_m] - [U_m^\dagger \frac{\partial U_m}{\partial x}, \varrho_m] + \mathcal{K}_m$$

Effective oscillation frequencies



Checking that oscillations are averaged

$$\frac{\partial \varrho_m}{\partial x} = -i[\mathcal{H}_m, \varrho_m] + \mathcal{K}_m$$

Effective oscillation frequencies

Relative variation of collision term

Collision rate